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Page 775, line below Fig. 15, change "$(71.5)^2 =$ " to "$\frac{71.5^2}{8} = .$"

Page 782, line 32, change "$S_n + 1$" to "S_{n+1}."

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PROBLEMS AND CONTROL OF DECENTRALIZATION IN URBAN AREAS

A SYMPOSIUM

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AND DONALD M. BAKER.

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PROBLEMS OF UNITED STATES CITIES

By John M. Picton,[1] M. ASCE

Effective action is needed to help cities face the multitude of problems presented by decentralization. The spread of residential areas beyond city limits, whereby people seek to enjoy the benefits of the city without paying their just share of the taxes, poses problems for the future as well as for the present. A diagnosis of the present situation of United States cities and some explanation of what these problems are is given in this paper; the one that follows offers suggested approaches to the solution of these problems.

Much thoughtful attention has been given to the problems of cities during recent decades. Among other studies on this subject the good and bad characteristics of urban areas in the United States have been thoroughly explored in reports by the National Resources Committee, and more recently by the National Interregional Highway Committee.

Until recent years, however, most cities have had no adequate over-all plans for future development. Narrow streets built for horse and buggy use are unable to cope with current traffic demands, and traffic circulation systems are inadequate because of indirect, narrow routes. Mass transportation systems with outmoded, early day patterns and inadequate equipment did a remarkable job during World War II. However, because of the inconveniences of public transit a vicious cycle will cause an increase in the use of private vehicles, with more and more congestion of narrow streets having inadequate off-street parking facilities.

Little or no attention has been given to the appropriate development of waterfront areas. Industrial areas are clogged by the conflict of street traffic with railroad traffic, these areas often being choked by the rail facilities which originally led to their development. In addition, piecemeal development of acreage tracts without proper guidance or control has resulted in stereotyped gridiron patterns with dead-end streets, offset streets, and poorly laid out subdivisions. In many instances these layouts have ignored topography and have not resulted in appropriate home sites.

Uneconomic patterns of land use have resulted from indiscriminate mixtures of industrial, business, multifamily dwelling, and residential uses in older sections of cities. Newer sections are generally better developed but have too much area zoned for business use, too much vacant land, and too little recreational and other community facilities. Delightful environments are found in isolated sections of small neighborhoods, but these usually are too small and tend to be crisscrossed by major traffic arteries. There has been a decline of the central business districts; more and more shoppers prefer to trade in the outlying business areas. As a result the stability of the downtown areas has been undermined.

Many cities during the decade from 1930 to 1940 suffered losses in population, whereas a few showed population gains. Nevertheless, their metropolitan areas increased in size because of the "flight to the suburbs." The major part of residential building in the last few years has been beyond the central city limits, requiring the extension of utilities. By contrast, one third of the city areas are still vacant even though they are already provided with sewers, water, paved streets, and utilities. Curtailment of residential building activities during World War II resulted in a critical shortage of housing accommodations. Little relief is in sight until the middle 1950's at the 1946 and 1947 rate of activities.

---

[1] Chf. Planning Engr., City Plan Comm., Kansas City, Mo.

Over one third of the population is poorly housed in obsolete structures needing major repairs and lacking sanitary facilities. Additional blighted areas are rapidly being created in newer sections and beyond the city limits because of poor initial subdivision layouts, poor septic tank installations, inadequately sized lots, and similar reasons. The congestion and blight which these people sought to avoid in the cities is rapidly appearing in these outlying areas where they had hoped to have permanently more light, air, and open space.

Urban redevelopment bills have been passed in many of the states, and the means are now available for private capital to build redevelopment projects. Nevertheless, persons with deep-seated prejudices spend the major part of their time fighting public housing projects, which are badly needed to accompany private urban redevelopment. Land costs in blighted areas are excessive, requiring high rents or larger subsidies than are available under private urban redevelopment procedures. Only a few redevelopment projects have been started, except where the private developer proposed to house many times the present population and thus, by means of vertical expansion, to overcrowd the land hopelessly. Undoubtedly this will result in an extensive redevelopment program in the 1960's and 1970's to rebuild the slum clearance structures whose construction is being attempted in the 1940's and 1950's.

It is generally accepted that cities cannot continue without end to widen major streets to accommodate an ever-increasing flow of traffic. Express highways seem to be the answer to this pressing problem, but the funds available from state and federal governments are pitifully inadequate to finance the entire job and the cities appear unable to provide the necessary funds.

As more and more residents move out, increasing numbers and sizes of blighted areas are created, tax delinquency grows, and the cost of city services continues to increase rather than to decrease. The city tax base continues to shrink, and even where real estate bears less than 40% of the burden (as in Kansas City, Mo.) the additional funds collected are barely enough for the cities to provide minimum services.

Eleemosynary institutions are overcrowded and are seriously in need of new buildings and better facilities. Health and welfare programs need expansion and all the cities have a definite need to acquire and develop more playground space conveniently located for each neighborhood. Additional funds are required for community center buildings, and for the provision of better coordinated recreational programs to combat increasing crime, juvenile delinquency, and health problems.

Nevertheless, an ever-increasing percentage of people hope to reside outside the city while comprising its daytime population as workers in business and industrial districts. They wish to enjoy fire protection and other city services for which they do not want to pay and, of course, expect to use the parks, zoos, art galleries, and auditoriums with the same desire to escape the responsibility of paying. Annexation to the city of adjoining areas or an income tax to be paid the city by workers from these outlying areas is fought bitterly.

Schools face increasing difficulties in providing adequate services as the school plant rapidly becomes obsolescent. Many school buildings are outmoded, having been built before the turn of the century on inadequate sites. Population shifts indicate that others should be relocated. Present school locations along major traffic arteries in most instances increase the problems of providing good residential neighborhoods with the best possible living conditions for enjoyment and for creating good citizenship.

Studies of the National Resources Committee indicate the importance of cities in the national economy. Despite the difficulties previously mentioned, cities have many advantages which will continue to attract population. These comments are not to be construed to mean everything is bad in urban areas. Many delightful and charming residential areas are found in new and old sections of cities. Well-developed park and boulevard systems, museums, auditoriums, and cultural centers are available.

Cities are definitely an important part of the national pattern and must be preserved. However, a current diagnosis would indicate that they are suffering from decay, internal pains, poor circulation, congestion, stagnation, and hardening of the arteries, as well as from cancerous growths of slums, which should be cut out and replaced with new tissues lest they spread and absorb unaffected areas.

A more satisfactory urban life must be provided.

APPROACHES TO DECENTRALIZATION CONTROL

By Russell H. Riley[2]

The problem of controlling decentralization in cities is a most difficult one. Several steps now appear essential—others may evolve in the future. In general, the character of the problem is similar in all communities, but naturally it is more difficult and more serious in the larger cities. Consequently, the possible solutions are also similar, but the extent of the necessary improvements will vary in cities of different sizes.

Streets, Transit Facilities, and Parking

Since transportation is one of the basic causes of the present problem, it is one of the first fields in which improvements should be considered. Before the advent of the automobile cities were compact, business centers were stabilized, there was little vehicular congestion, and there were no serious depreciating influences on residential property. The automobile enabled the city to expand and precipitated many of the present problems. The automobile cannot be cast aside, however, for people are interested in rapid and convenient movement. Furthermore, the automobile is an essential part of modern economy. It is therefore essential to make such improvements that the motor car can take its proper place in the American community without destroying the desirable and economical form of urban growth. The improvement of streets and transit facilities must be coordinated. Each must perform maximum service in moving people about the city.

Streets.—The majority of existing streets were not planned for use by automobiles. Widenings, new connections, and extensions are essential, but whereas the widening of main thoroughfares will generally serve the traffic needs of the smaller cities (communities comprising not more than 100,000 persons), many widenings are needed in the larger cities and such widenings will not completely solve traffic problems in metropolitan areas. Experience has revealed that any widened street is soon used to capacity, and congestion still exists.

A new type of street—namely, the express highway—must be utilized in larger cities. It is not expected that enough of these highways can be improved to carry all the automobiles, as the cost would be prohibitive. However, a few should be developed to provide convenient, rapid, and especially safe access between the residential and the business and industrial districts. Only by making the business and industrial centers conveniently accessible to the citizens can their high values be stabilized. Considerable study must be given to the location of these expressways. One major terminus is the central business district; yet, if they are not to encourage further decentralization they should not be extended far beyond the thickly populated part of the urban area. Invariably, the greatest delays are encountered in traveling through the older parts of cities, and here the expressway can provide maximum service.

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[2] Partner, Harland Bartholomew and Associates, St. Louis, Mo.

Transit Facilities.—Use of transit facilities declined with the advent of the automobile and, in general, the facilities did not keep pace with other modern improvements. The emergency of World War II clearly revealed the important function that transit can and should still perform. It is obvious, however, that transit vehicles must be modernized and the service so improved that more and more persons will travel by this method. A recent study in a midwestern city (only slightly smaller than Kansas City, Mo.) revealed that 54% of the persons entering the business district during the average day used the transit facilities. The 146,900 that entered by automobile utilized 85,263. The 173,000 that entered via the transit system required only 5,129 vehicles. Whereas the congestion and parking problems could be completely solved by total use of transit vehicles, there is no hope for, and probably no desirability of, entirely replacing the automobile.

The improvement of the transit system will gradually effect a proper adjustment between the use of transit vehicles and automobiles so that a greater number of persons can conveniently enter the downtown business and industrial areas. This is a basic step in stabilizing these centers. The essential improvements will involve new and modern equipment, direct routing on wide streets, and, in the medium size and larger cities, the installation of express service. It is also imperative that transit service be confined to the normally developed urban areas and not serve the sparsely settled outlying sections. This not only benefits the operating company, but also is an important factor in checking decentralization. Express routes must be designed so that the transit facilities can make maximum use thereof, providing rapid movement through the congested part of the city. Normal transit operations can then be provided on surface streets in the outlying sections of the urban area. This is one of the most important means of expediting the movement of workers and purchasing power to the downtown areas.

Traffic Regulations.—Movement of both automobile and transit vehicles can be greatly expedited by modern traffic regulations. This is especially true of movements within the business district and in other congested parts of the city. The primary function of streets is to accommodate the movement rather than the storing of vehicles. Elimination of parking, especially during rush hours, establishment of one-way streets, and modernization of traffic signals can accomplish outstanding results in moving more vehicles and in moving them rapidly. Experiences in certain cities, such as Philadelphia, Pa., reveal the potential benefits, although it is frequently difficult to secure the support of businessmen and city officials in making such changes. The staggering of working hours is another important way of reducing congestion during peak periods.

Improvements must also be made in the loading and unloading of commodities in the central areas. A single truck can practically destroy one traffic lane for an entire block. Alleys and off-street loading zones are essential in providing a satisfactory solution. In recognition of these facts certain communities have adopted regulations that will gradually improve this condition.

Parking.—To bring more automobiles to the business center will not solve the problem: There must be provisions to accommodate them. Only a small proportion can be parked on the streets, and off-street parking facilities must be provided. As yet there is no uniform agreement as to the location or type of such facilities, or as to who should be financially responsible therefor. From the standpoint of solving the major problem, it is immaterial whether the parking facilities are lots or garages, publicly or privately owned, or where they are located—as long as they are within reasonable distance and can be conveniently used by the traveling public.

Consideration is now being given in several large cities to acquiring parking lots some distance from the shopping center and providing shuttle bus service to the downtown area. This may be a sound solution in larger communities, but it would not be satisfactory in the smaller areas. Each city must study and develop the type of parking that seems best suited for its particular requirements. The important factor is that adequate off-street parking facilities be provided.

ZONING AND REHABILITATION

Another important reason for decentralization is the intermingling of property uses and the undesirable living conditions in the older sections of cities. Not only was zoning begun too late to prevent this condition in larger cities, but also, all too frequently, the present zoning regulations permit further depreciating influences in these older areas. The majority of zoning ordinances are not in scale with current needs. There is far too much commercial and industrial zoning in sections that are and should continue to be, primarily residential.

Zoning can be one of the most important factors in checking decentralization, but it must be a positive rather than a negative force. It must completly protect the good and fair residential districts so that they will not be injured by inappropriate uses, and it must also protect the older blighted districts that can only be logically used for residential purposes. It must be a positive guide for the future organization of the city and must give assurance that the older areas can be rehabilitated or redeveloped for housing purposes without recurrence of the same damaging influences that caused their depreciation.

Rehabilitation or redevelopment is a comparatively new field in cities of the United States. There are few actual examples indicating how blighted areas can be cleared and rebuilt for residential purposes. However, these few examples reveal great potentialities, and it is encouraging to note that many states have recently enacted laws facilitating redevelopment, especially by private capital. Comprehensive studies are also being made for redevelopment projects in many cities.

It is unthinkable that a country as young and as powerful as the United States would continue to stand the economic waste and the undesirable living conditions that are found in extensive parts of the larger cities. Both private and public capital must adopt widespread programs for the gradual rebuilding of these blighted areas in a manner that will prevent a recurrence. This is a large and difficult task, yet the few examples indicate that it is not impossible.

It is sound business to remove an uneconomical section of the urban area and to replace it with an attractive, livable, and revenue-producing development. The rebuilding of blighted areas is a fundamental step in checking decentralization.

Other Physical Improvements

Schools, parks, sewers, public buildings, and other public improvements are seriously needed in practically all communities. Cities grew so rapidly that these facilities seldom kept pace with the growth. Furthermore, new standards are frequently being evolved so that extensive improvements are needed to meet modern requirements. These physical facilities are not mere luxuries—they are minimum essentials. Although frequently not found in the newly developing outlying sections, the population soon presents a strong demand for them. Much can be done to stabilize residential growth if adequate and modern systems of these improvements are provided in the existing city. There should also be a definite policy that no additional facilities will be developed until such time as the area served by the existing facilities has been completely absorbed by urban growth. The location and character of improving essential public facilities can thus be very important factors in checking outlying growth, and can also enable the city to develop a long-range public works plan and to budget its expenditures accordingly. The cost of providing such facilities is admittedly large, yet it is small in comparison with the cost of decentralization or with the cost of continuously extending improvements over a never-ending area.

Guidance of New Growth

Irrespective of the effectiveness of the methods previously outlined, there will always be some outlying growth and development—it is the inherent tendency of American people to move to new locations. While an excessive amount of decentralization must and can be controlled, it is equally important that such outlying growth as does occur conform to modern desirable standards. It is particularly important that the new growth be compact rather than widely scattered. It is the scattered development in which the cost of improvements and services is abnormally high that cannot be economically justified. On the other hand, the development of complete, compact communities containing a population that can be economically provided with public improvements and services does not present serious problems of decentralization. Such communities even have an important advantage in that they can be surrounded by rural development, which should be an important means of protecting living conditions and property values.

County zoning regulations and good subdivision control are the most effective methods of bringing about a compact and economical pattern of outlying growth. Subdivision regulations that require the installation of all necessary improvements before lots are sold can be particularly effective in checking unwarranted outlying growth and in securing a desirable standard of improvements. Likewise, county zoning regulations, based on careful studies of the population that should eventually be located in the outlying areas, and which provide for the compact grouping of this growth, are valuable aids. Under such zoning the remainder of the unincorporated area can be restricted to rural uses and large lots, which are its most logical use.

Distribution of Cost and Taxes

A reason frequently advanced for persons leaving the older city for the outlying areas is to escape the higher taxes. This is understandable. It is seldom realized over a long period, however, for eventually the cost of providing improvements and services in the outlying areas to the standards to which the people were accustomed frequently involves taxes higher than those paid in the large city. It is also fundamentally inequitable for persons living beyond the corporate limits of the cities to utilize and benefit from the facilities of the city without contributing to their improvement and maintenance. Such facilities include major streets, parks, libraries, and other similar conveniences. There appears to be logical justification for a metropolitan form of government comprising all the urban area, and which would tax this area for such improvements as streets, libraries, parks and other facilities that are used by the entire population. This would not only insure the center city of receiving some assistance for the improvements used by all persons, but it would probably result in a better standard of such facilities in the outlying areas. At the same time, it would permit the continuance of smaller local governments, which would provide improvements and services used primarily by the citizens in the particular community.

Public Understanding

In the final analysis it is public understanding that will, to a large extent, determine whether decentralization will continue as at present or whether it will be effectively controlled. The people must know how they are affected by decentralization, they must understand the cost of the blighted areas—the cost that will result in installing a satisfactory standard of improvements and services in outlying areas—and the hazards and inconveniences that will result from inadequate improvements. They must realize that no matter where they live, they have a definite personal interest in the entire metropolitan area and that each one will be affected by what happens in any part of this large area. There must be widespread dissemination of data and facts regarding the problem, the needs, and the possible solutions.

Of particular importance is the fact that there must be aggressive leadership. Only by widespread understanding on the part of the citizens and by aggressive leadership can the essential results be obtained. Admittedly, the problem is extremely difficult and serious, yet the potential benefits are so great that every likely solution must be examined and every possible effort must be exerted.

DISCUSSION

William T. Hooper,[3] Assoc. M. ASCE.—The authors of these two papers take the same view that many city governments in the United States are taking—that cities as we know them are indispensable and provide facilities and advantages that can be obtained nowhere else. Is it not possible that the remedy for the ills to which most cities are heirs is to perform a dissection and allow the individual parts to function as parts of the whole? Instead of trying to correct entirely the mistakes of the founding fathers, it should be possible to guide new growth with intelligent foresight so that new errors are at least separated from the old. The cost will be less in the long run.

Energy, time, and money are being devoted to attempts to prevent decentralization of American cities in order to stop their decay. The programs of rejuvenation consist primarily of expenditures of astronomical sums of money for "super highways" leading from the suburban residential areas into the hearts of the business districts. These sunbursts of expensive pavement are expected automatically to clean up slums, solve traffic problems, and provide for the building of great metropolises.

It seems apparent that the problem of rehabilitation is being attacked from the wrong angle because of the prejudices and shortsightedness of the interests fostering the work. The primary consideration is the desire to protect the huge real estate investments in the business sections of the cities at the expense of the entire taxpaying public.

It can be seen from a study of the growth trends of the average American city that the need for ever-increasing concentrations of population is past. These concentrations are dangerous from the standpoint of defense in war, as well as being inefficient for peacetime operation. The location of businesses in close proximity to antiquated public transportation facilities—freight and passenger, cross-country and local—is unnecessary because of rapid developments in both communication and private transportation.

American cities began as convenient trading and transportation centers on a small scale. The businessman or tradesman lived either in the same building in which he worked, or within walking distance of it. As business increased and individual enterprises became more dependent on others, a centralized commercial area began to form and residences were moved to the outskirts of the district. As public transportation improved, it was possible for the more affluent members of society to move some distance out of the district.

In order to serve the domestic needs of those who were not within walking distance of the larger stores, small shopping centers were formed, which in time were incorporated as municipalities as they attracted more dwellers. Lower salaried workers moved into the vacated dwellings owned by the suburban residents, until gradually urban residential areas were forced farther afield by the expanding businesses. Eventually, the property formerly used for living quarters became more valuable for commercial use and was rented to these enterprises.

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With the advent of the multistoried buildings, business ceased growing horizontally and moved upward and inward toward the center again, thus vacating buildings which, in turn, were left to decay by absentee owners. Industries began to grow in the vicinity of the commercial and transportation concentrations, and housing for the swarms of low-paid workers was provided by the old, dilapidated buildings vacated now by both the higher-paid tradesmen and the businesses that had followed them. The aging and the spreading of the blight have continued until in some cases few desirable residential areas remain between these blighted areas and the city limits as defined by the suburbs.

Residents of the small communities have refused to be annexed by the larger cities and thereby be forced to pay for the conditions existing therein. Consequently, the cities have lost the higher real estate taxes on the decent residential property and, in order to obtain revenue, have increased taxes on commercial and industrial real estate. The owners of the business property now feel that their activities must be increased in order to meet their higher costs of operation.

While these adjustments were being made, public transportation was built and expanded in much the same manner as the real estate interests had developed their sphere of influence. No coherent plans were made for future development and, above all, no funds were provided for maintenance and improvement. The result is that public transportation has deteriorated and has become so inadequate that increasing numbers of suburbanites are relying upon private transportation, which in turn has made such rapid development that it is changing entirely the aspect of our modern life. The congestion resulting from the many private vehicles driven to and from work each day has caused traffic problems and created serious hazards which must be eliminated.

Unfortunately, the first and most spectacular solution to these problems to present itself is the express highway, pouring thousands of automobiles into an already overcrowded area each morning and spilling them out each evening. The hope is that business people and shoppers will be enticed into the city for their work and recreation, thus maintaining the business interests established there. Appropriations are being made by which state governments are subsidizing the construction of these arterial highway systems at the expense of citizens who have a steadily decreasing interest in, and need for, the facilities provided by the city. More money will have to be spent to provide parking space on land whose price has been enhanced by the preceding outlay for highways.

It seems inevitable that we must accept the decentralization movement and improve upon its results to spread the burden of taxation and the furnishing of governmental services more equitably. Instead of inhibiting growth by attempting to limit zones in which commerce and industry may grow, it would be more reasonable to permit economic expansion into less densely populated and more conveniently serviced semirural areas.

Existing business could be serviced by the renovation and reconstruction of the substandard housing facilities that in many cases are within walking distance of the central commercial and manufacturing districts. Money spent for express highways could better be spent for cheaper crosstown and interurban access highways planned to facilitate intercourse between small business and residential centers being established outside the urban development.

Donald M. Baker,[4] M. ASCE.—The United States, since its founding in 1787, steadily has shown a continued trend toward greater urbanization (see Table 1).

By 1920 slightly more than one half of the population of the United States lived in cities, one quarter of the people centering in cities with a population of more than 100,000. Although the increase in proportion of urban population between 1930 and 1940 was negligible—from 56.2% to 56.5%—and undoubtedly was due to the "back to the farm" movement during the depression of that period, the 1950 census should show a continuation of the major trend as a result of activity in war industry in urban areas.

TABLE 1.—Trend of Urbanization of Population in the United States

Census year	Percentage of Population		Cities with Populations in Excess of 100,000	
	Urban	Rural	No.	Percentage of national population
1790.......	5.1	94.9	....	....
1860.......	19.8	80.2	....	....
1890.......	35.1	64.9	28	15.4
1920.......	51.2	48.8	68	26.0
1940.......	56.5	43.5	92	29.8

These data do not present a complete picture either as to increasing urbanization or as to decentralization of urban areas. In 1930, 44.7% of the national population lived in metropolitan districts each of which had a population within its central city or cities and satellite area in excess of 100,000; in 1940 this proportion had increased to 44.8%. Between 1920 and 1930 the population of these districts increased 28.3%, as against a national rate of increase of 16.1%. Population in the central cities of these districts increased 22.3%, whereas that in satellite areas increased 44.0%. Between 1930 and 1940, the increase in national population was 7.3%; in these same metropolitan districts, 7.7%; in the central cities of these districts, 4.7%; and in their satellite areas, 14.4%.

The continued trend has been for people to move from rural to urban areas, but for the greatest population increase to occur in peripheral sections of these areas. Whether the added population located in the peripheral areas because space was available there, or whether the older residents of these areas moved outward, creating a vacuum in the central area which attracted the new population, is primarily a problem for sociologists. Prior to about 1910 much of the increase in urban living undoubtedly came from foreign immigration, but since that date a large proportion of it has come from rural areas.

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[4] Ruscardon Engrs., Los Angeles, Calif.

Mechanization of farming operations has no doubt been an important factor, and increase in industrialization in urban areas has provided a means of livelihood for new inhabitants.

Three distinct stages have occurred in the development of the American city, as a result of increasing population and geographical size. These have, to a large extent, been governed by the then existing modes of transportation. During the first part of the nineteenth century, urban populations were small and were generally grouped around a central district occupied by offices, stores, and shops. Such industry as existed was usually located close to the central district. The developed area was small in extent and population densities were low because daily travel distances were not great. Travel was by private carriage, on horseback, or on foot—at speeds ranging from 3 miles per hr to 6 miles per hr. Public transportation was not required. When it did become necessary, it was provided by horse-drawn omnibuses, which had travel speeds of from 4 miles per hr to 6 miles per hr. These travel speeds limited residential development to a distance of about five miles from the central district.

In the 1870's horse-drawn cars superseded the omnibus, with slightly greater speeds of travel, and allowed settlements to spread out somewhat. Settled areas still remained compact with only slightly greater population densities. People in the higher income brackets moved outward to peripheral areas, and those in the lower brackets moved into their vacated residences.

The second stage commenced around 1890. Cable cars superseded horsecars, and these were soon superseded by electric streetcars having travel speeds of from two to three times those of the outmoded vehicles. Use of electric cars allowed the population to spread out more, but densities increased. Extensions of streetcar lines into undeveloped areas usually were made only after the areas served were fully developed. This had a tendency to increase population densities and to cause development to remain in a compact form. Daily movements of people were still along radial lines, between homes and the central or industrial district. In the larger cities, because travel between homes and stores in the central district required too much time, stores supplying necessity shoppers (food, drugs, hardware, and some service trades) sprang up along streetcar lines, especially near transfer points.

About this time, commuter trains commenced to make their appearance on steam railroad lines, and satellite residential communities sprang up, separated from the central city by vacant or farm land. These were inhabitated largely by people in the upper income brackets, who could afford the cost of daily transportation, and those serving these people. Retail stores to serve the suburbanites developed along these lines, but most of the trade and business was still carried on in the central districts. During the decade prior to World War I, electric interurban lines were constructed in a number of communities, and New York (N. Y.), because of its peculiar size and shape and its surrounding water area, developed off-surface rapid transit.

Following World War I, construction and operating costs on rail transit lines increased, while the traveling public still insisted on a prewar fare. As a result, operation of transit facilities became less and less profitable, and in many cases definitely unprofitable. Equipment and service deteriorated and new capital for extensions and improvements was difficult or impossible to secure. Extensions and improvements, therefore, were not made.

Prior to 1920 communities had grown in a more or less hit-or-miss fashion. Such through streets as existed were laid out along roads that connected the central district and the outlying suburban or farming centers, or had followed property boundaries. Sometimes these were made wider than interior streets, sometimes not. Streetcars and horse-drawn carriages did not cause traffic congestion, even on narrow streets, and street jogs caused no inconvenience in travel in the existing slow speed of movement.

Following 1920, however, the private automobile came into common use. The national rate of motor vehicle registration increased from 35 in 1,000 (1916) to 87 in 1,000 (1920), and to 215 in 1,000 (1930). People found that they could live half a mile from a streetcar line if they owned and used an automobile, and still get around. Urban population commenced to decentralize at a rapid rate, and developed urban areas commenced to assume a far less compact shape.

At this time also city planning began to be recognized. Planning commissions were established in many communities. Land was subdivided in peripheral areas in a more intelligent manner. Attempts were made to provide arterial highways of proper width and in suitable locations in newly developed areas. Street jogs were eliminated, and street widths were given more careful attention. Population densities were lowered, as a result of zoning in newer areas. These zoning ordinances controlled land use, and segregated various types to more suitable locations. In the studies made to establish these ordinances, basic data relative to land use were obtained, as was also knowledge regarding city patterns and structure. As a result, the newer sections of urban areas developed after 1920 did not encounter all the problems found in the older sections.

Even by 1920, however, there had been little decentralization of central districts. Stores serving luxury and selective shoppers were still located there, but during the 1920–1930 decade the increase in motor vehicle ownership gave rise to problems of traffic congestion, particularly on approach streets to central districts. Heavy motor vehicle traffic slowed down the speed of streetcars, and more and more riders began to use their own automobiles. The habit of riding on transit facilities dropped, net revenues fell, and a vicious spiral commenced.

Extensive programs of street openings and street widenings were carried out in the 1920–1930 decade, but as fast as new arterial highways were constructed more automobiles were attracted to them; little or no relief from traffic congestion resulted. By 1930 most stores serving necessity shoppers had moved to outlying areas, although those serving luxury and selective shoppers still remained at locations in the central district.

Rail transit companies entered the 1930 depression with low riding habits, low net income, and low morale. Such extensions of lines as were made were bus lines, which did not require heavy investment in rails, rights of way, or power equipment, and which had far greater flexibility in routing. During the 1930 decade the American Transit Association, through its committees, developed the design of what was known as the President's Conference Car, a streamlined electric transit vehicle, having far easier and more comfortable riding qualities, being lighter in weight, and having greater acceleration—thus allowing an increase in scheduled speeds. Its use on a few lines spurred patronage somewhat, but riding habit and net income showed little or no over-all improvement. Motor vehicle registration increased to 250 per 1,000 population in 1940, and use of motor vehicles as a mode of mass transportation increased at a much greater rate.

Between 1930 and 1940 the rate of population increase in urban communities dropped to about that of the national increase. Depressed business conditions discouraged the construction of buildings in central districts and the expansion of retail trade to outlying areas. Such public improvements as were carried on were primarily to provide employment; those which sought to alleviate traffic congestion and improve circulation in urban areas were relatively few in number. Much constructive work was done, however, in the collection of basic data pertaining to traffic, circulation, and land use, and in the preparation of plans to remedy the problems thereof.

World War II more or less "froze" conditions to the status of the 1930–1940 decade, but it created a prosperous period for public carriers, which lasted throughout the war and for a year or two following its termination.

The elimination of gasoline rationing and the increase in motor vehicle production quickly caused traffic congestion of a most serious character to occur, and also caused the parking problem in central districts to become extremely acute. New housing, located for the most part in peripheral areas where land had been developed in a more intelligent manner, served to decentralize population further. Because of prosperous business conditions many retail stores in central districts gave serious thought to the problem and actually moved to outlying sections or opened large branches there, with large parking areas for their customers.

Parking in central districts has become an even more serious problem than traffic congestion on streets approaching these districts. Many cities now require that all new buildings constructed in their limits provide automobile parking space. Los Angeles (Calif.) has a provision in its zoning ordinance which requires that parking space for one automobile be furnished for every 1,000 sq ft of gross floor area in a new building, such space being located at the building site or within 1,500 ft of it. This means that about 200 sq ft of parking space must be provided for every 1,000 sq ft of gross floor area. Such a provision greatly adds to construction cost in central districts. In the case of retail stores and many office buildings it may make new construction in central districts uneconomical, or at least retard it greatly, forcing new development to the outlying sections where ground-level parking on cheaper land can be had.

All the forces which tend to mold the pattern of development of urban areas would appear to encourage future decentralization of population and retail trade. This does not mean that central districts will eventually disappear, but that the future activities of these districts will be of a different character. Business operations, requiring people to work in close contact with large numbers of other people, and small, light industries, will ultimately predominate. Such districts will not grow at a rate comparable to that of the urban areas they serve; many people will still have to work there, and travel daily between the central district and their residences. It will also take considerable time for this transition in activities to occur. In the meantime, people will continue to move to peripheral areas—not to save taxes so much as to be able to live under less congested conditions—to own, keep, and use an automobile, or even two; to have a yard for their children and a garden for themselves; and to be able to enjoy the other amenities of life, which go with higher living standards and which cannot be achieved through living in apartments or tenements.

Expressways leading to central districts, and to other sections of urban areas, will be necessary. Whether transit companies will be able to operate profitably through increased fares, increased riding habit due to improvements in equipment, and faster schedules must remain to be seen. They will have an opportunity to do this in some localities where the scarcity and the high cost of parking may force more people to ride on transit lines.

Whatever decentralization of central districts and of population occurs in the future, most of the problems discussed in the Symposium will remain, and their satisfactory solution will involve large sums of money. The average American citizen, however, is like a man with poor teeth. He will readily admit that from the standpoint of economy and health it is advisable to "brush his teeth twice a day and see his dentist twice a year," yet he continues to stay away from his dentist until he has an excruciating toothache, and finally needs a complete set of dentures long before he should have them. Radio, undoubtedly, has done much to change the public's attitude toward dental care, but it will require much more than the studies and reports of social workers and city planners to convince the general public that the cost of playgrounds is more than repaid in better community health and in reduction in crime and juvenile delinquency, or that the cost of expressways is offset by reduction in automobile accidents, operating expenses, and travel time.

The person who invented the gasoline tax had the right idea; this tax is painless, it is collected in small installments, and it immediately produces tangible returns in miles of hard-surface highways. Its present rate of receipt, however, is inadequate to finance construction of the needed expressways in time for them to be of value to this generation. With present low interest rates, this tax, or part of it, say, that which is collected in urban areas, could be utilized to pay principal and interest on bonds that could be issued in amounts sufficient to pay for the construction of substantial mileages of expressways at an early date. This would do much to solve some of the most pressing problems of urban areas.

The greatest task now facing political scientists—including social workers, planners, and engineers—who are interested in the problems facing urban areas, is not that of investigating and stating the existing problems and suggesting solutions for them, as much as it is that of developing adequate and painless methods of financing these solutions, and of selling those who must pay for these solutions on the fact that money spent for these purposes is a sound investment.

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Paper No. 2361

ANALYSIS OF FRAMES WITH ELASTIC JOINTS

By Ralph W. Stewart,[1] M. ASCE

With Discussion by Messrs. L. E. Grinter, Arthur B. McGee, C. W. Jenkins, George W. Lamb, A. A. Eremin, J. Charles Rathbun, and Ralph W. Stewart.

Synopsis

A method is presented for analyzing structural frames with elastic joints. With this method the computations are generally such as to be classed as arithmetical rather than algebraic. No approximations other than those inherent to the common theory of flexure are involved. In the presentation of the method no algebraic equations are used either as a reference or as a means of performing computations.

Introduction

Full continuity is not accomplished in the usual welded or riveted joints of a steel frame. Nevertheless, their resistance to flexure is substantial and it is desirable to provide a method of computing their effect on the bending moments in a structure. Considerable pioneering work has been done[2] in this field. The application of end moment distribution and slope deflection to the analysis of structures with semi-rigid joints has resulted in computations which, as stated elsewhere,[2,3] are complex and tedious. The object of this paper is to simplify the analysis by the use of the traverse method, which is described in detail elsewhere.[4]

Analysis

In Fig. 1 is represented a cantilever beam whose connection to its support will deform and rotate when subjected to a moment. When loaded at the free end the elastic curve of the beam takes the position indicated by the dotted line. Deformation of the connection causes the joint to rotate, as represented

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Note.—Published in December, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[2] "Analysis of Building Frames with Semi-Rigid Connections," by Bruce Johnston and Edward H. Mount, *Transactions*, ASCE, Vol. 107, 1942, p. 993. (This paper contains an extensive bibliography of earlier publications in this field.)

[3] *Ibid.*, p. 1017.

[4] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *ibid.*, Vol. 104, 1939, p. 521.

by the angle θ. The angular change in the direction of the elastic curve due only to flexure of the beam is represented by Δ. The angle ϕ represents the over-all change in direction of the elastic curve due to both joint deformation and flexure along the beam. This interpretation of symbols for angles is used throughout the paper.

Considerable laboratory work has been done to determine for various joints how θ varies with changes in applied moment. A published stress-strain curve[5] indicates that the joint rotation does not accurately follow Hooke's law, but that the error introduced by assuming the joint to have a definite modulus of elasticity falls within a reasonable design tolerance. As a parallel situation it may be noted that in reinforced concrete design the constant, n, is used to express the ratio of the modulus of elasticity of steel to that of the concrete. This ratio is recognized to be only approximately accurate as is shown by the fact that it is usually taken as 8, 10, 12, or 15—any possible intermediate values between these figures being disregarded. It is also assumed that the modulus of elasticity for the concrete remains constant for different intensities of stress. This assumption is known to be inaccurate but the error can be, and is, tolerated.

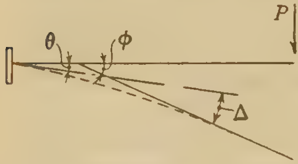

Fig. 1

In order to analyze frames with elastic joints a linear stress-strain relationship is assumed for the joints. To facilitate the work the rotation of a joint due to a moment is expressed as a percentage, or fraction, of the elastic curvature caused by the same moment in a connecting member. The interpretation of these assumptions with reference to Fig. 1 is that both θ and Δ vary directly as the bending moment in the beam and that θ will be expressed as a percentage of Δ. This is somewhat analogous to the assumption of a constant, E, for concrete and a constant ratio, n, between the deformations of concrete and steel used in the design of reinforced concrete.

The ratio between θ and Δ may not be the same for each joint in a frame with several joints. It will then be necessary for the designer to evaluate this ratio from joint to joint, basing his judgment on whatever test data are available.[2] The analytical procedure set forth herein is adaptable for use with a changing $\frac{\theta}{\Delta}$ - ratio from joint to joint.

Application of the elements of flexure (Fig. 1) to an actual problem is illustrated in Fig. 2, wherein the flexible joints are considered to be formed by riveting the ends of the members to gusset plates. The bases of the lower columns are considered fixed and the top of the upper column, hinged. Joints in this frame are assumed to be semi-rigid; that is, the angle of yield at each joint is assumed for this problem to be one half of the angular change in direction of the elastic curve of the member caused by the moment that produces the joint yield—$\theta = \frac{1}{2}\Delta$. Each member is considered as having a constant cross section, which will fix all Δ-angle points at the third points of the members.[4] Making $\theta = \frac{1}{2}\Delta$, in Fig. 1, and applying the law that the

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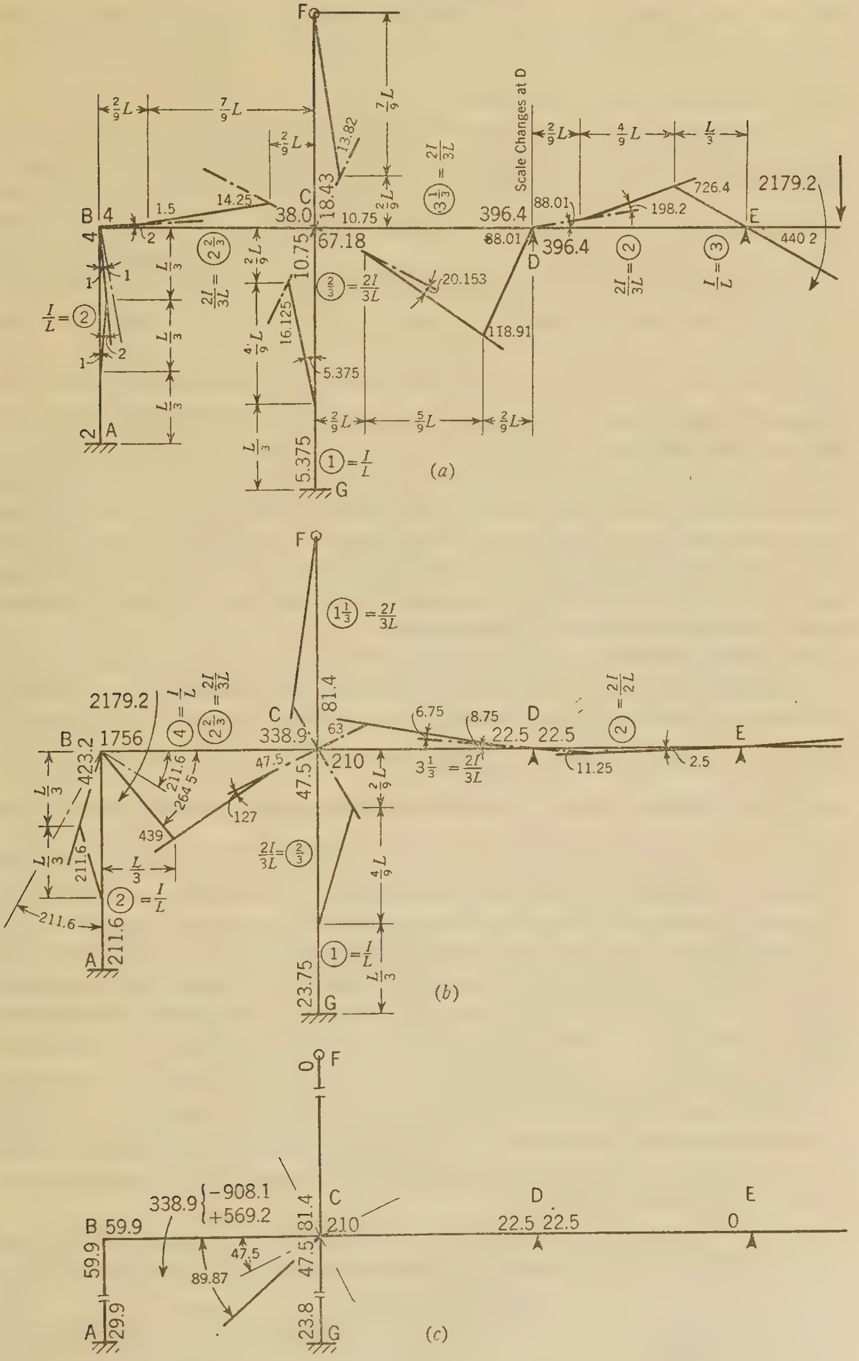
[5] "Analysis of Building Frames with Semi-Rigid Connections," by Bruce Johnston and Edward H. Mount, *Transactions*, ASCE, Vol. 107, 1942, p. 996.


F
$\frac{2}{9}L$
$\frac{7}{9}L$
$\frac{2}{9}L$
$\frac{7}{9}L$
13.82
$\frac{2I}{3L}$
Scale Changes at D
$\frac{2}{9}L$
$\frac{4}{9}L$
$\frac{L}{3}$
726.4
2179.2
B 4
1.5
14.25
C
38.0
18.43
10.75
$\frac{2}{9}L$
③⅓ =
396.4
88.01
198.2
E
440 2
4
2
67.18
88.01
396.4
D
②
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$\frac{2I}{3L}$ =
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1
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2
1
$\frac{I}{L}$ = ②
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2⅔
$\frac{2I}{3L}$ =
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10.75
16.125
(⅔) = $\frac{2I}{3L}$
20.153
118.91
5.375
$\frac{2}{9}L$
$\frac{5}{9}L$
$\frac{2}{9}L$
2 A
$\frac{L}{3}$
5.375
① = $\frac{I}{L}$
G
(a)
F
(1⅓) = $\frac{2I}{3L}$
2179.2
$\frac{I}{L}$ = ④
$\frac{2I}{3L}$ = (2⅔)
81.4
C
338.9
63
6.75
8.75
D
22.5 22.5
$\frac{2I}{2L}$ = ②
E
B
423.2
1756
211.6
264.5
47.5
210
47.5
11.25
2.5
3⅓ = $\frac{2I}{3L}$
$\frac{L}{3}$
$\frac{L}{3}$
127
439
$\frac{2}{9}L$
211.6
$\frac{L}{3}$
$\frac{2I}{3L}$ = (⅔)
$\frac{4}{9}L$
211.6
② = $\frac{I}{L}$
211.6
A
① = $\frac{I}{L}$
23.75
$\frac{L}{3}$
G
(b)
0 F
B 59.9
338.9
−908.1
+569.2
81.4
C
210
D
22.5 22.5
E
0
59.9
47.5
89.87
47.5
A 29.9
23.8
G
(c)

FIG. 2

lengths of the sides of a traverse triangle are proportional to the opposite angles, it is found that the ϕ-angle is located at a point $\frac{2}{9}L$ from the end of the beam. All ϕ-angles in Fig. 2 are so shown.

The over-all stiffness or "assembly stiffness" of a span having yielding joints is defined as the ratio of an end moment to the angle ϕ which results. The stiffness of a member without yielding joints is the ratio of an end moment to the change it produces in the direction of the elastic curve—that is, $\frac{M}{\Delta}$. In Fig. 2 the assembly stiffness of a span connected to a yielding joint will be two thirds of the member stiffness because $\phi = \frac{3}{2}\Delta$. The stiffness of a member of constant section has the value $\frac{2\,E\,I}{L}$, which will be recognized as the coefficient preceding the parenthesis in the usual slope deflection equation. It is different from the stiffness as defined for end-moment distribution.

To solve Fig. 2(*a*), begin at joint E and sketch the traverse. Dot-dash lines show the rotated position of gusset plates. The beam is taken as fully continuous over joint E so the angle of flexure at the left of joint E is a Δ-angle, located at $\frac{1}{3}L$ from joint E. For column AB draw all lines of the traverse showing both the rotation of the gusset plates and the angular change at point B in the elastic curve of the member.

At point B assume unit rotation for the top of the column. By the laws of a traverse[4] the upper and lower Δ-angles will then have values of 2 and 1, respectively. From the statement of the problem, the column connection yield under joint B will be one half of the upper Δ-angle, or 1. Using $\frac{I}{L}$ as the relative stiffness value the moments at the top and bottom of column AB will be the $\frac{I}{L}$-value of the column multiplied by the respective Δ-angles. These moments are written in Fig. 2 and the moments are balanced about joint B by writing the moment, 4, over the left end of beam BC.

Use of the ϕ-angles for the traverse to the right of point B calls for the use of the assembly stiffnesses, which are two thirds of the member stiffnesses. These are written in circles as shown in Fig. 2(*a*). Evaluating the traverse from joints B to E is an easy routine procedure utilizing the traverse properties that:

(1) An angle of flexure multiplied by the appurtenant stiffness equals the end moment which causes the flexure; and, conversely, the end moment in a member divided by the appurtenant stiffness is equal to the angle of flexure which is produced by a moment.

(2) The angles and the lengths of the lines between the angles must close geometrically. In other words, the traverse must pass through the joints or supports at the ends of the individual spans.

(3) In any traverse triangle the angles are proportional to the lengths of the opposite sides.

Applying these properties to traverse BC the steps are as follows:

The moment at B(= 4) is divided by the assembly stiffness $2\frac{2}{3}$ to obtain $\phi(= 1.5)$ at a distance $\frac{2}{9}L$ from B. This is recorded in Fig. 2(*a*).

Three procedures are available for obtaining the unknown joint rotation at joint C and the unknown, ϕ, at $\frac{2}{9}L$ from C. These are all similar to profile grade computations and are of such an elementary nature that they need no description. The method favored by the writer is to compute the unknown, ϕ, by $\left(2 \times 1 + \frac{7}{9} \times 1.5\right) \times \frac{9}{2} = 14.25$. This value of ϕ is entered in Fig. 2(*a*) and multiplied by the stiffness $(= 2\frac{2}{3})$ to obtain the moment (= 38) which is recorded over the right end of span BC. The numerical value of the joint rotation at C is $14.25 - (2 + 1.5) = 10.75$. With this joint rotation and the lengths of the traverse courses in members CF and CG known, the flexure angles in these members are determined by the third of the previously cited traverse properties. These angles are then multiplied by the appurtenant stiffnesses to obtain the moments, which are written at the ends of these members. Note that the lower flexure angle in member CG is a Δ-angle and not a ϕ-angle, and that the bottom moment in member GC is exactly half of the top moment. This verifies the carry-over factor used in the end moment distribution method. A further check is available by comparing the moments in column CG with those in column AB. The top joint rotations of these columns are in the ratio $\frac{10.75}{2}$. Stiffness ratios for the two columns are in the ratio $\frac{1}{2}$. Application of the two ratios to the moments in column AB gives: $\frac{10.75}{2} \times \frac{1}{2} \times M_{AB} = M_{CG}$ (check).

The three moments now written about joint C are all resisting moments activated by the moment at the left end of member CD. Addition of these resisting moments gives $M_{CD} = 67.18$. Also, note that the lines in the traverse show that M_{CD} is clockwise, whereas the other three moments about joint C are counterclockwise.

Traverse computations are carried from joint C to joint D by the same procedure used from joint B to joint C. The cantilever to the right of joint E is taken as fully continuous with member DE, therefore the angle which measures the moment at E is a Δ-angle. This angle is located at $\frac{1}{3}L$ from joint E and is multiplied by the $\frac{I}{L}$-value of the member to reduce it to the moment E. For member DE the traverse computations are $\frac{88.01 + \frac{7}{9} \times 198.18}{\frac{1}{3}} = 726.41 = \Delta_{DE}$, and $M_E = 726.41 \times 3 = 2{,}179.23$.

An interesting and useful property of this traverse is that all angle values are coefficients of the $\frac{L}{EI}$-value for column AB. This occurs because the true stiffness of AB is $\frac{2EI}{L}$ and using 2 as the stiffness simply eliminates $\frac{EI}{L}$ from the written values. Restoring this $\frac{EI}{L}$-value will enable the deflection of the cantilever end beyond joint E to be computed very quickly, as follows: The end slope at E is equal to $440.22 \times \frac{L}{EI}$ for member AB. The deflection at the end of the cantilever is the product of the end slope at E and the length of the cantilever, plus the ordinary cantilever deflection which would occur if there were no rotation over support E.

To the beginner Fig. 2(*a*) may resemble a maze of angles and numerals. It should be borne in mind, however, that there are no supplementary computation sheets to bother with as all computations are shown in Fig. 2(*a*).

To obtain the effect in member BC of a fixed-end moment at B, and also to obtain an automatic check on the computations heretofore made, the traverse shown in Fig. 2(*a*) is sketched. Evaluation of this traverse begins by assigning a value of $2\frac{1}{2}$ to the rotation at joint E. This value is selected because it is the amount by which end B of member BC rotates in Fig. 2(*a*). (It should be noted that the rotation of 2 at B (Fig. 2(*a*)) is the rotation of the gusset plate. To determine the rotation of the member, the angle of joint yield, which is one half of the Δ-angle or one third of the ϕ-angle in member BC, is added.) The computations for Fig. 2(*b*) follow the usual pattern until joint B is reached. At that point the complete detail of the flexure is drawn.

The Δ-angle which measures the moment at B in member BC is computed by the usual method. Also, the end rotation at B of member BC is computed by the usual angle summation from points C to B, and its value, 264.5, is recorded (Fig. 2(*b*)).

Multiplication of the Δ-angle value (439) by the $\frac{I}{L}$-ratio (4) gives the value of the resisting moment (1,756), which is shown in Fig. 2(*b*). To evaluate the remaining elements of the flexure about joint B, the upper Δ-angle in column AB is designated as the independent unknown quantity. By the properties of a traverse, the rotation of the top of the column at B will be $\frac{1}{2}\Delta$; by the terms of the problem, the joint yield at the top of the column will also be $\frac{1}{2}\Delta$. The gusset plate rotation at point B will therefore equal Δ. At the connection of beam BC to the gusset plate at B the joint yield will be half as much as the joint yield at the column connection because the beam is twice as stiff as the column and will have a Δ-angle and a θ-angle only half as large. Therefore, $\Delta + \frac{\Delta}{4} = 264.5$. Solving, the upper Δ-angle of column AB is 211.6 and the moment at the top of the column is 423.2 as recorded in Fig. 2(*b*).

Adding the two resisting moments about joint B, it is found that the applied external moment, or fixed-end moment due to loading, required to produce them is 2,179.2. Reference to Fig. 2(*a*) shows that the external moment is the same as the activating moment at joint E. It is known, therefore, by the Maxwell theorem of reciprocal deflections (angular) that all computations are correct and it is unnecessary to review the arithmetic. The closure about joint B can be further checked by noting that all column angles, the gusset plate rotation, and the moment for column AB are exactly proportional in Figs. 2(*a*) and 2(*b*).

After Figs. 2(*a*) and 2(*b*) have been completed the work of computing the effect of a moment at any intermediate joint is much reduced. To get the effect of a fixed-end moment applied at joint C, the moments in members CF, CG, CD, and DE, all taken from Fig. 2(*b*), are first copied on an outline of the frame (Fig. 2(*c*)). By the laws of the traverse, the gusset plate rotation of 47.5 at C in Figs. 2(*b*) and 2(*c*) will be caused by a rotation of 89.87 at end C of member BC. In Fig. 2(*a*) the gusset plate causes a rotation of 6 at end C of member BC. The moments in AB and BC (Fig. 2(*a*)) are multiplied by $\frac{89.87}{6}$ and the results recorded in Fig. 2(*c*). Next, the resisting moments about joint C in Fig. 2(*c*) are added and the resulting activating moment which produces them is taken as applied through member BC. The net moment in BC will be this activating moment less the resisting moment; that is, 908.08 − 569.18 = 338.9. These moments are recorded in Fig. 2(*c*). Signs, where shown, indicate whether the moment tends to produce clockwise (positive) or counterclockwise (negative) rotation about joint C.

To illustrate application of Fig. 2 in an actual problem, the moments in the frame are computed for a uniform load of 12 kips per ft on member BC, which is assumed to be 10 ft long. The fixed-end moments at B and C are each 100 kip-ft. Table 1(*a*) shows the computation of the moments throughout the frame for these fixed-end moments. Moments tending to produce counterclockwise rotation about the adjoining joint are shown as negative, the directions of the moments being obtained by inspection of Figs. 2(*b*) and 2(*c*). Table 1(*b*) shows the end moments for this frame if the joints were rigid and the same loading is applied.

TABLE 1.—MOMENTS, IN KIP-FEET, CAUSED BY UNIFORM 12 KIP PER FT LOAD ON MEMBER BC OF FRAME SHOWN IN FIG. 2

Factor	From Fig.	M_A	M_{BC}	M_{CB}	M_{CF}	M_{CG}	M_{CD}	M_{DE}	M_E	M_G
(*a*) SEMI-RIGID CONNECTIONS										
$\frac{100}{2{,}179.2}$	2(*b*)	−9.7	19.4	−15.6	3.7	2.2	9.6	−1.0	0	1.1
$\frac{100}{908.1}$	2(*c*)	−3.3	6.6	−37.3	9.0	5.2	23.1	−2.5	0	2.6
Total	....	−13.0	26.0	−52.9	12.7	7.4	32.7	−3.5	0	3.7
(*b*) RIGID CONNECTIONS (SHOWING FINAL MOMENTS ONLY)										
....	....	−21.1	42.2	−88.8	20.0	13.4	55.3	−10.4	0	6.7

SIDESWAY

Methods of including the effect of sidesway in the computation of bending moments in a rigid frame are presented in a number of textbooks. To correct the moments in Table 1(*a*) for the sidesway that would occur if supports D and E were on rollers involves only use of the type of traverse shown in Fig. 2 in the usual form of sidesway analysis procedure for rigid frames. Amplified slope deflection[6] can be readily applied to frames with elastic joints and is suitable for problems involving sidesway. The solution of such a problem by amplified slope deflection has been published.[7]

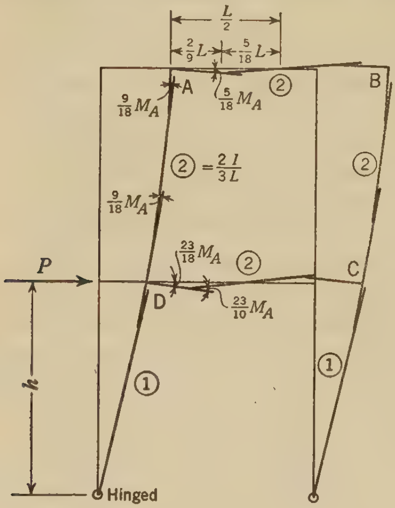

FIG. 3

The moments in a symmetrical two-story frame having elastic joints and subjected to a lateral force are easy to compute as evidenced by Fig. 3. In this figure the condition of joint yield is assumed to be the same as for Fig. 2, and the relative stiffness values shown have been corrected for joint yield by taking two thirds of the $\frac{I}{L}$-values of the members. Using the laws of a traverse, the values of all angles are written in terms of the moment at point A and, by balancing moments about D, M_A is found to equal $\frac{5}{56} P\,h$.

It will be of interest to users of the traverse method to note that the traverse lines in Fig. 2 have the same characteristics as the traverse of a frame having tapering members with rigid joints. In fact, it is theoretically possible to determine the design of a frame with rigid joints and tapering members which would have exactly the same traverse lines and same moments as those shown in Fig. 2(*a*). This is not to be interpreted as a general statement since, when the final closure of a traverse is at a joint having more than one elastic connection, the substitution of tapering members will prevent proper closure.

CONCLUSION

The properties of an elastic curve traverse facilitate computation of the bending moments in frames having semi-rigid connections. In computing deflections of such frames the advantages of the traverse become even more apparent than in computing moments. The underlying reason is that the traverse method utilizes all the angles that measure changes in direction of an elastic curve; for example, the flexure (Δ) angles, as well as the joint rotation (θ) angles.

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[6] "Amplified Slope Deflection," by Ralph W. Stewart, *Transactions*, ASCE, Vol. 110, 1945, p. 1401.

[7] "Analysis of Building Frames with Semi-Rigid Connections," by Bruce Johnston and Edward H. Mount, *ibid.*, Vol. 107, 1942, p. 1029.

DISCUSSION

L. E. Grinter,[8] M. ASCE.—The statement of the author (see "Introduction") that the "application of end moment distribution * * * to the analysis of structures with semi-rigid joints has resulted in computations which * * * are complex and tedious" must be considered in relation to the author's preferred procedure. There is no such thing as complexity or simplicity in structural analysis except as relative terms. A procedure[9] of alternately distributing moments and of balancing end angle changes, which requires no explanation for those who use these tools, is at least equally convenient as a method of solving the problems discussed in this paper. All ratios such as stiffness and the carry-over factors remain unchanged. The method has the added advantage that it may be used where the moment-rotation curve for the joint represents nonlinear elasticity. The distribution procedure again shows its natural advantage of keeping the designer closely in contact with the physical action of the structure as it deforms.

Arthur B. McGee.[10]—A practicable method for analyzing the effects of elastic, or yielding, joints has been presented by Mr. Stewart. However, the method of analysis as illustrated by the author in Fig. 2 is exceedingly "tricky" and subject to errors. Moreover, with transverse loads on all the columns and beams six additional traverse drawings would be needed to analyze the frame completely. For example, Fig. 2(*c*) is only one of four traverses required for joint C in order to distribute moments from all four directions.

Fortunately, a more general and easier procedure is available. A frame with rigid joints and with members having variable moments of inertia may be substituted for the original frame and solved by ordinary methods. The author mentions the possibility of such a frame. The actual shape of each member of such a frame is not important. The characteristics of the members can be determined easily and only one traverse is required for each joint; only three traverses, instead of eight, are required for the frame in Fig. 2 to handle any loading whatsoever. The author states that "* * * when the final closure of a traverse is at a joint having more than one elastic connection, the substitution of tapering members will prevent proper closure." The writer finds, however, that proper closure at such joints is accomplished by the method described hereinafter.

The procedure to be followed is to solve a substitute frame with members of variable I in the usual manner. The u-values and p-values (as defined by W. Ruppel,[11] M. ASCE) for a substitute member are

$$u = u_o \times \frac{\Delta_u}{\theta_u + \Delta_u} \quad \text{(1a)}$$

and

$$p = p_o \times \frac{\theta_u + \Delta_u}{\Delta_u} \quad \text{(1b)}$$

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[8] Research Prof. of Civ. Eng. and Mechanics, Illinois Inst. of Technology, Chicago, Ill.

[9] "Theory of Modern Steel Structures," by L. E. Grinter, Macmillan Co., 1937, Vol. 2, p. 262.

[10] Structural Engr., Bridge and Structural Design Div., City of Los Angeles, Los Angeles, Calif.

[11] *Transactions*, ASCE, Vol. 90, 1927, pp. 167–187.

Similarly,

$$v = v_o \times \frac{\Delta_v}{\theta_v + \Delta_v} \quad \text{(1c)}$$

and

$$q = q_o \times \frac{\theta_v + \Delta_v}{\Delta_v} \quad \text{(1d)}$$

in which the subscript *o* is used to designate the original member. Eqs. 1*a* and 1*c* are evident from Fig. 4. Eqs. 1*b* and 1*d* follow from

$$u\, p = u_o\, p_o = v_o\, q_o = v\, q \quad \text{(1e)}$$

Without knowing the shape of a member the fixed-end moments due to loads applied on the member cannot be found by ordinary methods. An end moment for a member with yielding joints is designated as a "yielding-end moment" (YEM) in this discussion. Yielding-end moments can be found readily and they are the same as the fixed-end moments for the substitute member. Several methods using the traverse are available for finding yielding-end moments when the fixed-end moments are known. End moment distribution may be used for this purpose also, as shown later. Following are general equations for finding yielding-end moments from fixed-end moments. The original member may be unsymmetrical and have different $\frac{\theta}{\Delta}$-ratios at the ends:

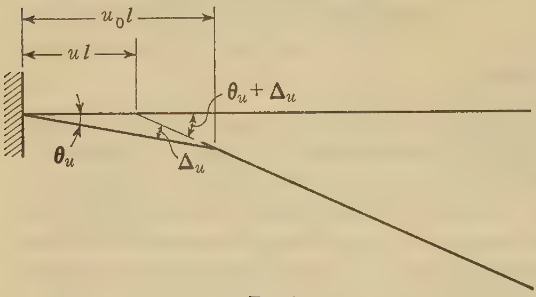


Fig. 4

$$\text{YEM}_{\text{AB}} = \frac{\text{FEM}_{\text{AB}}\,[n\,(1-u) + (1-u-v)] + \text{FEM}_{\text{BA}} \times u\,n}{m\,n + m\,(1-v) + n\,(1-u) + (1-u-v)} \quad \text{(2a)}$$

$$\text{YEM}_{\text{BA}} = \frac{\text{FEM}_{\text{BA}}\,[m\,(1-v) + (1-u-v)] + \text{FEM}_{\text{AB}} \times v\,m}{m\,n + m\,(1-v) + n\,(1-u) + (1-u-v)} \quad \text{(2b)}$$

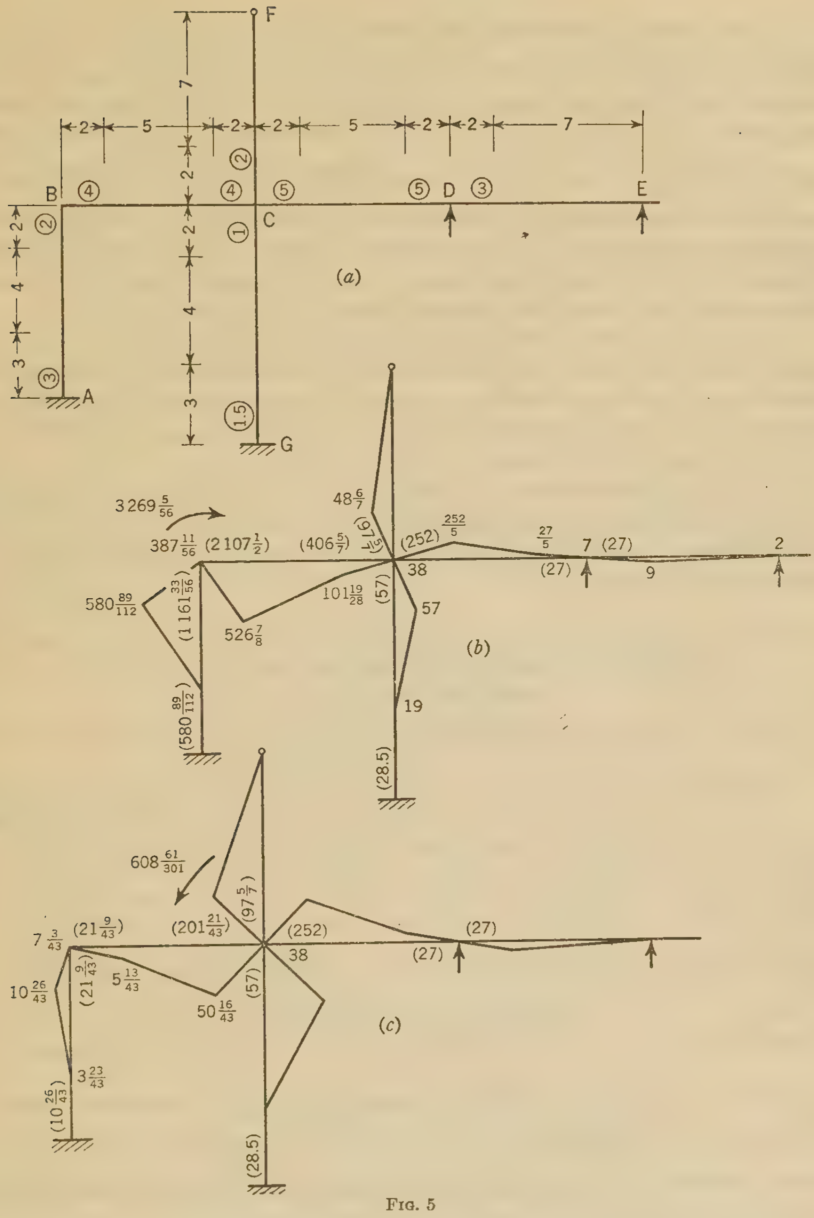
in which the subscript AB and the quantities u and m refer to the left end of the member, and subscript BA and the quantities v and n refer to the right end of the member:

$$m = \frac{\theta_u}{\Delta_u} \quad \text{(2c)}$$

and

$$n = \frac{\theta_v}{\Delta_v} \quad \text{(2d)}$$

Eqs. 2*a* and 2*b* are not as formidable as they appear at first glance.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①
C
2
2
4
4
3
3
③
A
(1.5)
G
(a)
3 269 5/56
48 6/7
252/5
27/5
7 (27)
2
387 11/56
(2 107 1/2)
(406 5/7)
(9 1/7)
(252)
38
(27)
9
580 89/112
(1 161 33/56)
101 19/28
(57)
57
526 7/8
(b)
19
(580 89/112)
(28.5)
608 61/301
(97 5/7)
7 3/43
(21 9/43)
(201 21/43)
(252)
(27)
38
(27)
10 26/43
(21 9/43)
5 13/43
50 16/43
(57)
(c)
3 23/43
(10 26/43)
(28.5)

Fig. 5

If a member is of uniform I, with identical fixed-end moments and with identical $\frac{\theta}{\Delta}$-ratios at the ends, then both yielding-end moments are the same and

$$\text{YEM} = \frac{\text{FEM}}{\frac{\theta}{\Delta} + 1} \qquad (2e)$$

In the example worked by Mr. Stewart $\frac{\theta}{\Delta} = \frac{1}{2}$. Therefore, by Eq. 2*e*, YEM = $\frac{2}{3}$ × FEM, and at ends where yielding occurs the *u*-values and *v*-values computed by Eqs. 1*a* and 1*c* equal $\frac{1}{3} \times \frac{2}{3} = 2/9$, and *p*-values and *q*-values by Eqs. 1*b* and 1*d* equal three halves of their former values. Traverse stiffnesses equal $\frac{E\,I}{p\,l}$ and $\frac{E\,I}{q\,l}$ at the two ends, respectively. Therefore, the stiffnesses of the substitute members will be two thirds of the stiffnesses in the original frame with the yielding joints, or, since stiffnesses need be proportional only, the stiffnesses at the yielding joints may be used unchanged—provided the two stiffnesses at the fixed column bases are increased by the factor 3/2.

Fig. 5(*a*) shows the substitute frame with stiffnesses encircled and *u*-values and *v*-values indicated. The traverses for externally applied moments at joints B and C are shown in Figs. 5(*b*) and 5(*c*), respectively. One additional traverse for a moment at joint D would permit the solution of any loading on any or all members of the frame.

The fixed-end moments for the uniform load of 12 kips per ft on member BC are 100 ft-kips, and the yielding-end moments, by Eq. 2*e*, are $\frac{2}{3}$ × 100 = 66$\frac{2}{3}$ ft-kips. Table 2 distributes the moments, giving totals that check the solution by Mr. Stewart.

TABLE 2.—Distribution of Moments, in Kip-Feet, Caused by Uniform Load of 12 Kip per Ft on Member BC of Substitute Frame Shown in Fig. 5

Description	End Moment at:									
	AB	BA	BC	CB	CF	CG	CD	DC	DE	GC
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Yielding-end moments.....			+66.67	−66.67						
$\frac{66.67}{3,269.09}$ × Fig. 5(*b*).......	−11.84	−23.69	−42.98	−8.29	1.99	1.16	5.14	0.55	−0.55	0.58
$\frac{66.67}{608.20}$ × Fig. 5(*c*)........	−1.16	−2.32	2.32	22.09	10.71	6.25	27.62	2.96	−2.96	3.12
Total...............	−13.00	−26.01	26.01	−52.87	12.70	7.41	32.76	3.51	−3.51	3.70

Of course, end moment distribution may also be used for solving the substitute frame. The carry-over factor is $\frac{u}{1 - u}$ or $\frac{v}{1 - v}$ and the end moment distribution stiffness is the basic stiffness, as used in the traverse, multiplied by $\frac{1 - u}{1 - u - v}$ or $\frac{1 - v}{1 - u - v}$, depending on the end of the member under consideration. Thus, carry-overs in both directions of members BC and CD equal 2/7, and column carry-overs to the fixed bases equal $\frac{1}{2}$. The stiffness at the top of column BA is $2 \times \frac{9 - 3}{9 - 2 - 3} = 3$; at the top of column CG, $1 \times \frac{9 - 3}{9 - 2 - 3}$

$= 1\frac{1}{2}$; at the bottom of column CF, $2 \times \frac{9-0}{9-2-0} = 2.571$; at the left end of beam DE, $3 \times \frac{9-0}{9-2-0} = 3.857$; at both ends of beam BC, $4 \times \frac{9-2}{9-2-2} = 5.6$; and at both ends of beam CD, $5 \times \frac{9-2}{9-2-2} = 7$. The distribution of the yielding-end moments by end moment distribution is carried out in convenient tabular form in Table 3, the results checking the other solutions.

TABLE 3.—DISTRIBUTION OF MOMENTS IN SUBSTITUTE FRAME IN FIG. 5 BY END MOMENT DISTRIBUTION

Description	Member							
	BA	BC	CB	CF	CG	CD	DC	DE
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Carry-over factor		↓ 2/7 ↓				↓ 2/7 ↓		
Relative stiffness	3	5.6	5.6	2.571	1.5	7	7	3.857
Distribution factor	0.349	0.651	0.336	0.154	0.090	0.420	0.645	0.355
Yielding-end moment		+66.67	−66.67					
	−23.27	−43.40	−12.40					
		+ 7.59	+26.58	+12.17	+7.12	+33.21	+9.50	
	− 2.65	− 4.94	− 1.41			− 1.75	−6.13	−3.37
		+ 0.30	+ 1.06	+ 0.49	+0.28	+ 1.33	+0.38	
	− 0.10	− 0.20	− 0.06			− 0.07	−0.25	−0.13
			+ 0.04	+ 0.02	+0.01	+ 0.05		
Total	−26.02	+26.02	−52.86	+12.68	+7.41	+32.77	+3.50	−3.50

$M_{AB} = \frac{1}{2} M_{BA} = -13.01$; $M_{GC} = \frac{1}{2} M_{CG} = +3.70$

Problems in partial fixation are in reality problems in elastic joints.

$$\frac{\theta_u}{\Delta_u} = \frac{(1-u-v)(1-r)}{r(1-v)} \quad \text{(3a)}$$

or

$$r = \frac{\Delta_u (1-u-v)}{\theta_u (1-v) + \Delta_u (1-u-v)} \quad \text{(3b)}$$

in which r is the fixation factor. Eq. 3*a* has been developed previously by the writer.[12] In the frame shown in Fig. 5, it was assumed that $\theta = \frac{\Delta}{2}$. Substituting this value of θ and inserting the values of u and v (namely, $\frac{1}{3}$) in Eq. 3*b* gives $r = 0.5$. Hence, the ends of member BC are 50% fixed. Placing the fixed-end moments of 100 at each end of the member and distributing by end moment distribution gives

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[12] *Transactions*, ASCE, Vol. 104, 1939, p. 549.

Carry-over = $\frac{1}{2}$

	B		C	
50%	50%	Distribution factor	50%	50%
	+100	FEM	−100	
−50	−50		+50	+50
	+25		−25	
−12.5	−12.5		+12.5	+12.5
	+6.25		−6.25	
....	....		....	....
....	....		....	....
....	....		....	....
....	....		....	....
$-66\frac{2}{3}$	$+66\frac{2}{3}$	YEM	$-66\frac{2}{3}$	$+66\frac{2}{3}$

This checks the yielding-end moments already found by Eq. 2*e*.

Fig. 6(*a*) shows a frame with one leg 25% fixed. From Mr. Ruppel's tables,[11] $u = 0.250$ and $p = 0.292$ at the lower end of the tapering leg; also, $v = 0.427$ and $q = 0.171$ at the upper end of the leg. This frame has been analyzed before by another method.[13]

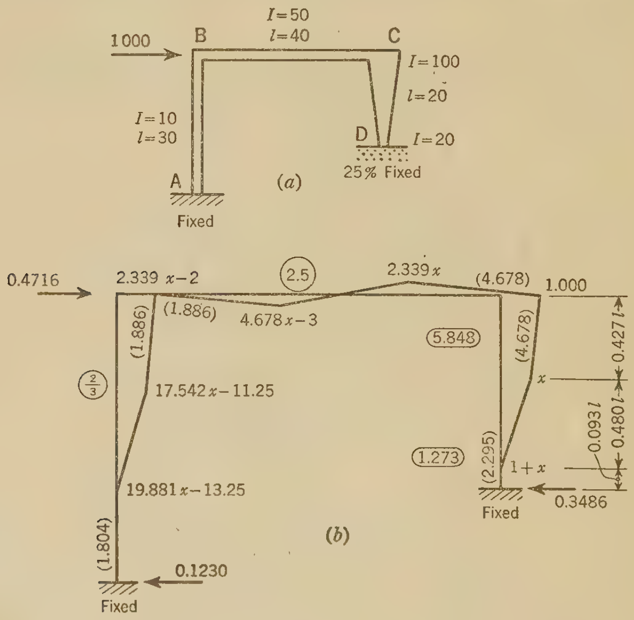
I=50
l=40
1000
B
C
I=100
l=20
I=10
l=30
D
I=20
25% Fixed
A
(a)
Fixed
0.4716
2.339 x−2
2.5
2.339x
(4.678)
1.000
(1.886)
(1.886)
4.678x−3
5.848
(4.678)
0.427
2/3
17.542x−11.25
x
0.0931
0.4801
1.273
(2.295)
1+x
19.881x−13.25
0.3486
Fixed
(1.804)
(b)
0.1230
Fixed

Fig. 6

By Eq. 3*a*, $\frac{\theta_u}{\Delta_u} = \frac{(1 - 0.250 - 0.427)\,(1 - 0.25)}{0.25\,(1 - 0.427)} = 1.691$ and $\frac{\theta_u + \Delta_u}{\Delta_u} = 2.691$. Therefore, by Eq. 1*a*, $u = 0.250/2.691 = 0.093$, and, by Eq. 1*b*,

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[13] *Transactions*, ASCE, Vol. 104, 1939, p. 550.

$p = 0.292 \times 2.691 = 0.786$ for the substitute leg, which will be fixed at the base. The relative stiffness at the bottom of the substitute leg DC is $\frac{\Delta}{M} = \frac{I}{p\,l} = \frac{20}{0.786 \times 20} = 1.273$, and at the top of the leg, $\frac{\Delta}{M} = \frac{20}{0.171 \times 20} = 5.848$.

In Fig. 6(*b*) the angles x and 1.000 are assumed, and the remaining angles are calculated by ordinary traverse procedure. Since the deflections in the legs are equal, the value of x must be 0.8026. A Δ-angle times the appurtenant stiffness gives the appurtenant end moment. The end moments for each member are shown in parentheses. The sum of the leg shears is the lateral force at the top of the frame. To find the effect of a lateral force of 1,000, multiply the moments shown by $\frac{1{,}000}{0.4716}$. This gives $M_A = 3{,}825$; $M_B = 3{,}999$; $M_C = 9{,}919$; and $M_D = 4{,}866$.

Frames with yielding joints can now be calculated in exactly the same manner as rigid joint frames with members of variable I.

C. W. JENKINS,[14] JUN. ASCE.—Use of the elastic curve traverse for the analysis of frames with elastic joints offers several profound advantages over methods previously presented. A study of the application of moment distribution,[15] slope deflection,[16] and the method of simultaneous balance of both moments and angles[17] will show each one to be more complex in the analysis of such frames for more than one condition of loading, or where the frame contains members having variable moments of inertia, or a combination of both. The method presented by Mr. Stewart is as basic and practical for these cases as it is for the analysis of a fully rigid frame. It offers the added advantages of an immediate self-check by Maxwell's law and a direct solution for the deflection of a frame.

A consideration of the elastic curve traverse will reveal its advantages. The traverse is a representation of the deflected frame, which expresses the joint rotation and the flexure angle due to moment at each end of a member.

The traverse is, therefore, a natural method of analysis of yielding joints. At the end of a member the joint yield is simply expressed as a percentage of the flexure angle caused by moment. It should be noted that expressing joint yield as a percentage of the flexure angle is a tool for analysis, and only that. For a given moment the flexure angle of the member depends on the elastic properties of the member, whereas the joint yield depends on the construction of the joint. The author's statements regarding the evaluation of the θ-ratio at each joint are of great importance.

As an illustration of the adaptability of this method to joint yield problems, the frame analyzed in the paper will be analyzed with another yielding joint added near the middle of a member. A semi-rigid splice has been inserted in column CG at a point 0.6 of the column height above G as shown in Fig. 7(*a*).

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[14] Structural Engr. Associate, Dept. of Bldg. and Safety, City of Los Angeles, Los Angeles, Calif.

[15] "Analysis of Building Frames with Semi-Rigid Connections," by Bruce Johnston and Edward H. Mount, *Transactions*, ASCE, Vol. 107, 1942, p. 997.

[16] *Ibid.*, p. 1001.

[17] "Elastic Properties of Riveted Connections," by J. Charles Rathbun, *ibid.*, Vol. 101, 1936, p. 561.

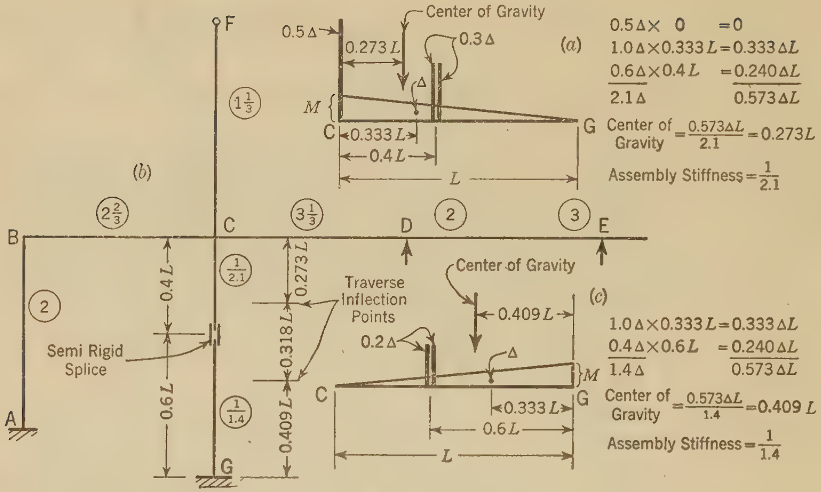
F
0.5Δ
Center of Gravity
0.3Δ
(a)
0.273L
Δ
M
C
0.333L
0.4L
L
G
0.5Δ× 0 =0
1.0Δ×0.333L=0.333ΔL
0.6Δ×0.4L =0.240ΔL
2.1Δ 0.573ΔL
Center of Gravity = 0.573ΔL / 2.1 = 0.273L
Assembly Stiffness = 1/2.1
(1⅓)
(b)
(2⅔)
(3⅓)
(2)
(3)
B
C
D
E
0.4L
0.273L
(1/2.1)
Traverse Inflection Points
Center of Gravity
(c)
0.409L
0.2Δ
Δ
(2)
Semi Rigid Splice
0.318L
M
1.0Δ×0.333L=0.333ΔL
0.4Δ×0.6L =0.240ΔL
1.4Δ 0.573ΔL
A
0.6L
(1/1.4)
0.409L
C
0.333L
G
0.6L
L
Center of Gravity = 0.573ΔL / 1.4 = 0.409 L
Assembly Stiffness = 1/1.4
G

FIG. 7

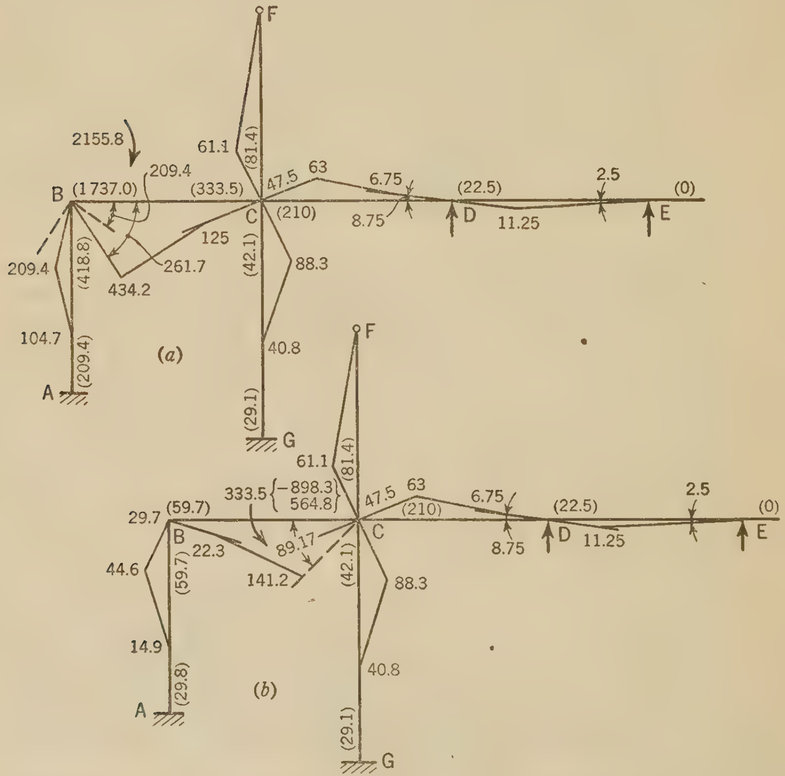
F
2155.8
61.1
(81.4)
209.4
B (1737.0)
(333.5)
47.5
63
6.75
(22.5)
2.5
(0)
C
(210)
8.75
D
11.25
E
125
209.4
(418.8)
261.7
434.2
(42.1)
88.3
104.7
(a)
(209.4)
40.8
A
(29.1)
G
F
61.1
(81.4)
333.5
−898.3
564.8
47.5
63
6.75
(22.5)
2.5
(0)
29.7
(59.7)
(210)
B
22.3
89.17
C
8.75
D
11.25
E
44.6
(59.7)
141.2
(42.1)
88.3
14.9
40.8
(29.8)
(b)
A
(29.1)
G

FIG. 8

The conjugate beam theorem has been found to be exceedingly useful in determining flexure factors. The conjugate beam load for member CG is shown in Fig. 7(*b*). Since the area under the $\frac{M}{E\,I}$-diagram is Δ, Δ is used as the load caused by flexure of the member. The load due to yield θ at joint C is $\frac{\Delta}{2}$. A semi-rigid splice would also have a yield of $\frac{\Delta}{2}$ on both ends of the splice plates if it were located at C. Inasmuch as a splice consists of two members

TABLE 4.—MOMENTS, IN KIP-FEET, CAUSED BY UNIFORM LOAD OF 12 KIP PER FT ON MEMBER BC (FIG. 7)

Factor	From	M_A	M_{BC}	M_{CB}	M_{CF}	M_{CG}	M_{CD}	M_{DE}	M_E	M_G
(*a*) SEMI-RIGID CONNECTIONS WITH SEMI-RIGID SPLICE										
$\frac{100}{2{,}155.8}$	Fig. 8(*a*)	−9.7	19.4	−15.5	3.8	1.9	9.7	−1.0	0	1.4
$\frac{100}{898.3}$	Fig. 8(*b*)	−3.3	6.6	−37.1	9.0	4.7	23.4	−2.5	0	2.2
Total		−13.0	26.0	−52.6	12.8	6.6	33.1	−3.5	0	4.6
(*b*) SEMI-RIGID CONNECTIONS (FINAL MOMENTS ONLY)										
Total	Table 1(*a*)	−13.0	26.0	−52.9	12.7	7.4	32.7	−3.5	0	3.7
(*c*) RIGID CONNECTIONS (FINAL MOMENTS ONLY)										
Total	Table 1(*b*)	−21.1	42.2	−88.8	20.0	13.4	55.3	−10.4	0	6.7

$$r_1 = \frac{10\,M_A\,h}{18}$$

$$r_2 = \frac{16\,M_A\,h}{18}$$

$$r_3 = \frac{11\,M_A\,h}{18}$$

$$r_4 = \frac{196\,M_A\,h}{45}$$

$$R = \frac{557\,M_A\,h}{90}$$

$$M_A = \frac{5\,P\,h}{56}$$

$$R = \frac{577}{90} \times \frac{5\,P\,h}{56} \times h \times \frac{3}{2} \times \frac{h}{2\,E\,I} = \frac{577\,P\,h^3}{1{,}344\,E\,I}$$

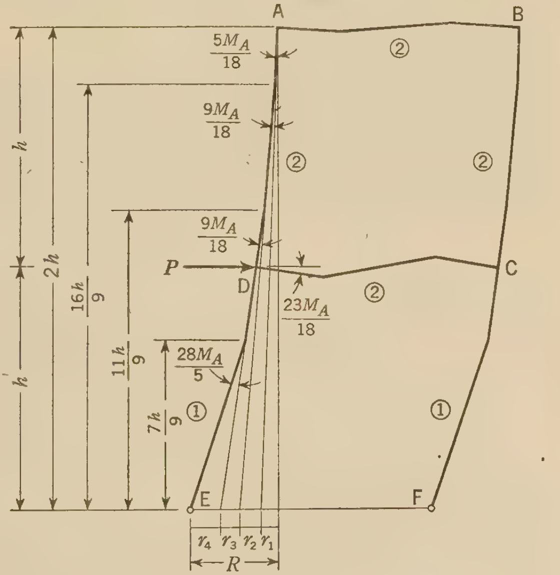

FIG. 9

connected by splice plates, it has a total yield of $2 \times \frac{\Delta}{2} = \Delta$. However, the splice transmits only 0.6 M where it is located on column CG. Thus, the splice yield is 0.6 Δ, the load on the conjugate beam caused by the splice yield.

The ratio of the assembly stiffness to the member stiffness is now equal to $\frac{\Delta}{\phi}$, ϕ being the total load on the conjugate beam. The location of the ϕ-angle (which is the center of gravity of the conjugate beam load) is also easily found from Fig. 7(*b*). The assembly stiffness and ϕ-angle location for column GC are determined in a similar manner from Fig. 7(*c*).

The frame has been analyzed in Figs. 8(*a*) and 8(*b*) by the procedure outlined by Mr. Stewart. The resulting moments are given in Table 4 for comparison with those of the original frame.

The flexure traverse affords a direct determination of deflections. As an illustration of this fact, the two-story frame with semi-rigid joints used as an example in the paper (Fig. 3) is analyzed for deflection in Fig. 9. It should be noted that the true assembly stiffness, $\frac{2}{3} \times \frac{2\,E\,I}{h}$, must be used instead of the relative assembly stiffness to determine the actual deflection, R. All computations are shown in Fig. 9.

Mr. Stewart's paper marks definite progress toward a practical method of designing steel or timber frames as they actually occur in practice with connections of partial rigidity.

George W. Lamb,[18] M. ASCE.—Use of the traverse method of analysis is well adapted to frames with elastic joints. However, there are several points in the paper that need clarification. Some of the reasons for certain parts of the analysis are not clear, due possibly to the fact that the writer is not as familiar with the procedure and the analysis as is the author.

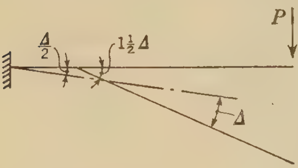

Fig. 10

Fig. 1 would be clearer if all the angles were expressed in terms of the $\triangle$-angle instead of using three symbols. For the conditions assumed in the paper (joint yield being equal to one half of the angular change in direction of the elastic curve of the member, caused by the moment that produces the joint yield), the angular displacements in Fig. 1 could be more advantageously shown as in Fig. 10

Some additional information would help in explaining Figs. 2(*a*) and 2(*b*). The total rotation at joint B is $2\frac{1}{2}$, as shown in Fig. 11, which shows all the rotations instead of groups of them. One reason for having the same rotation at joint E and joint B is to take advantage of Maxwell's theorem of reciprocal deflections (angular), and thus obtain a check on the final results. In addition, this is necessary, of course, to obtain final moments. Fig. 11 shows a unit rotation at the top of column AB. A moment of 4 is required to rotate the top of the column one unit. The stiffness used in Fig. 11 is the $\frac{I}{L}$-value and is not an assembly stiffness. This is done because the rotation angles are separated, not grouped into composite angles. Fig. 11 should be compared with Fig. 2(*a*).

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[18] Dist. Engr., Am. Inst. of Steel Constr., Inc., Topeka, Kans.

A comparison of the moments obtained in the structure having semi-rigid joints with the moments in the structure having rigid joints indicates that some economy could be obtained by considering semi-rigid joints in the design. This economy is obtained only if the joints are of such simplicity that little extra fabrication cost is involved.

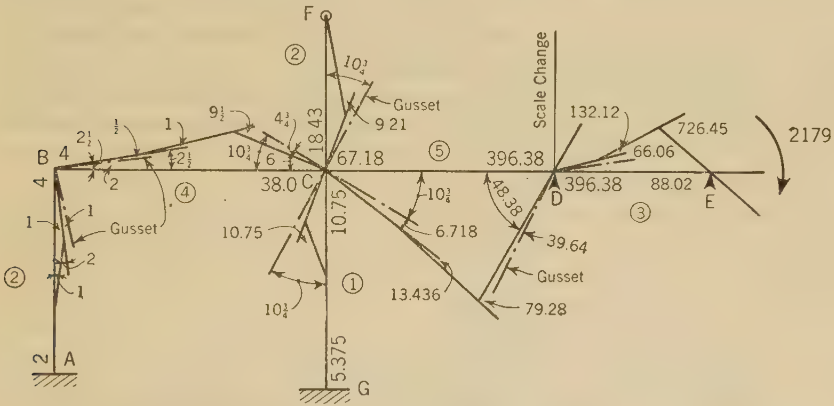
F
②
10¾
Gusset
9.21
Scale Change
132.12
726.45
2179
66.06
9½
4¾
18.43
67.18
⑤
396.38
1
½
2½
B
4
2½
10¾
6
38.0
C
396.38
D
88.02
E
4
2
④
10¾
48.38
③
1
1
Gusset
10.75
10.75
6.718
39.64
2
②
1
①
13.436
Gusset
10¾
79.28
2
A
5.375
G

FIG. 11

The author has selected for his analysis a type of joint in which the rotation occurring in the joint is a constant ratio to the total angular change in direction of the elastic curve of the member. A joint having these properties can be selected from a family of moment-rotation curves. Joints also could be selected

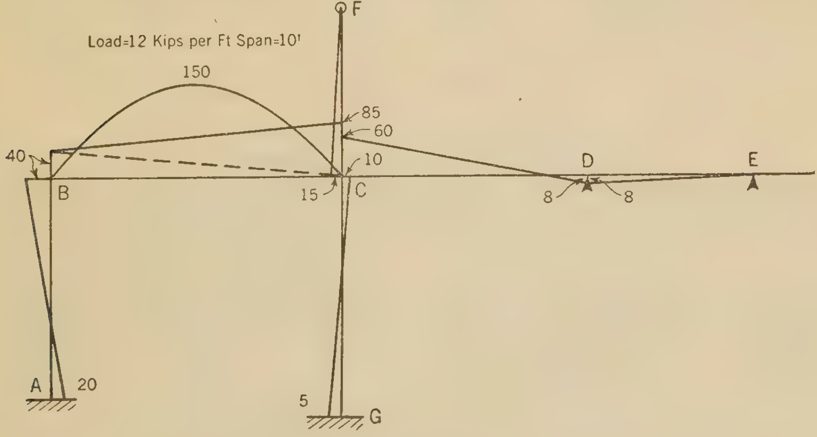
F
Load=12 Kips per Ft Span=10'
150
85
60
40
10
D
E
B
15
C
8
8
A
20
5
G

FIG. 12

from this same family of curves that would result in almost any joint yield for a given moment. With the properties of joints expressed as a family of moment-rotation curves from which many different joint yield (angles) can be obtained from one moment, a joint can be selected that would yield the proper amount to accommodate the moment and the slope of the joining members for many different ratio values.

If a joint works within the elastic limit the moments and rotations can be added for different loading conditions. If, however, the moment-rotation curve of the joint is a curved line, the maximum load conditions would be used.

When the ratio values between joint yield and curvature in the member can be selected by judgment from test data, the problem can be solved in the following manner. Using the frame given in the author's example (Fig. 2) and applying a load of 12 kips per ft on span BC, the moments in the ends of all members can be determined by inspection, and the slope between members determined by the traverse method. In this case the ratio between curvature and joint yield will not be constant. The moments selected are as shown in Fig. 12, and the slopes between members are obtained by the traverse method as shown in Fig. 13 and Table 5.

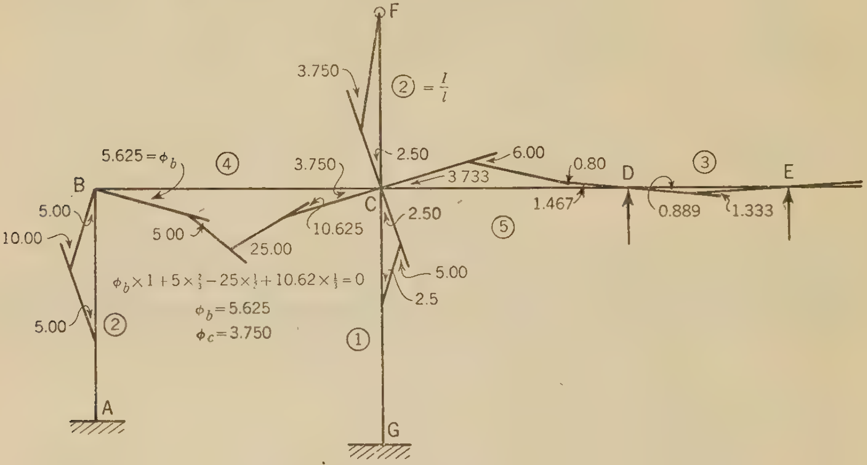
F
3.750
② = $\frac{I}{l}$
5.625 = ϕ_b
④
3.750
2.50
6.00
3 733
0.80
D
③
E
B
5.00
10.00
5 00
25.00
C
10.625
2.50
1.467
0.889
1.333
⑤
5.00
2.5
$\phi_b \times 1 + 5 \times \frac{2}{3} - 25 \times \frac{1}{2} + 10.62 \times \frac{1}{3} = 0$
$\phi_b = 5.625$
$\phi_c = 3.750$
5.00
②
①
A
G

FIG. 13

TABLE 5.—SUMMARY OF JOINTS

Members	Slope between members	Moment between members	COORDINATES ON MOMENT-ROTATION CURVE	
			M (kip-ft)	ϕ (radians)[a]
AB and BC.......	5.000 − 5.625 = 0.625	40.0	40.0	0.0002500
BC and CF.......	3.75 − 2.50 = 1.250	15.0	15.0	0.0005000
BC and CG.......	3.75 − 2.50 = 1.250	10.0	10.0	0.0005000
BC and CD.......	3.75 − 3.733 = 0.017	60.0	60.0	0.0000068
DC and DE.......	1.467 − 0.889 = 0.578	8.0	8.0	0.0002312
FC and CG......	2.50 − 2.50 = 0	0	0	0

[a] ϕ is the slope between members divided by 2,500.

From Table 5, joint B will have a connection delivering 40 kip-ft of moment with a joint yield of 0.00025 radian. Columns FC and CG have equal slopes at joint C and would be fully rigid, spliced columns, with member BC fastened to it by a connection that would yield 0.00050 radian under a moment of (10 + 15) or 25 kip-ft. In addition to the connection to column FC and CG member BC connects to member CD by a joint carrying 60 kip-ft and having a joint yield of 0.0000068 radian. The remaining joints are treated in the same manner. The joint yield is a property of the actual connection and its moment. When considering the construction of such a frame, the small rotations should be thought of as deformations caused by the additional stresses from the applied load.

The traverse method of analysis is relatively simple in application and is easily adapted to frames with any degree of flexibility at the joints. When the ratio of curvature of the member is fixed in relation to joint yield, its application is better visualized when all the rotations are shown, as in Fig. 11.

A. A. Eremin,[19] Assoc. M. ASCE.—By varying the distances between the deflection angles in continuous frames Mr. Stewart expresses the effect of elastic deformations at the joints. The method is simple and convenient for checking, by visual inspection. A further simplification of the method is possible by the graphical construction of moments. For example, let Fig. 14 represent span BCD of Fig. 2. Assume that each joint is rigid and that moments are obtained by "crosslines."[20] The elastic deformations at the joints were determined (see Fig. 14(*b*)) by considering that the distance from a support to its nearest point of inflection is one half the distance shown in Fig. 14(*a*). The moments corresponding to the elastic deformations at the joints were determined by interpolating the values shown in Fig. 14(*a*). When Fig. 14(*b*) is superposed on Fig. 14(*a*) the resulting values afford a convenient method for selecting a design moment visually.

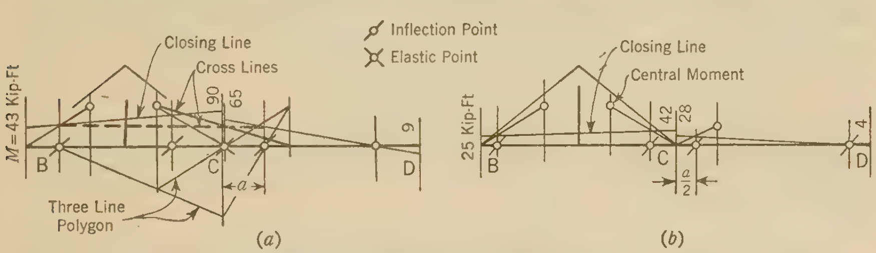

Fig. 14.—Distribution of Moments

By clarifying the effect of elastic deformations at the joints, Mr. Stewart has enlarged the range of application of rigid frames, and thereby has made an important contribution to the literature on the subject.

J. Charles Rathbun,[21] M. ASCE.—Once again Mr. Stewart has brought before the engineering profession the importance of the effect of the elasticity of the connections and, at the same time, has presented a practical and useful method of allowing for these properties. This has been done before by other methods but these do not have the advantage of giving the analyst the visual concept obtained by the author's diagrammatic method.

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[19] Associate Bridge Engr., Bridge Dept. of California State Highways, Sacramento, Calif.

[20] *Transactions*, ASCE, Vol. 111, 1946, p. 887.

[21] Prof., Civ. Eng., College of the City of New York, New York, N. Y.

In the routine analysis of frames it is quite customary to assume the joints as either hinged or rigid and to design them on the assumption that the stresses lie between these limits. The saving in material may not warrant a more careful analysis and in this case it would be academic. In a conference called in 1934 to consider the use of the elastic properties of the connections in analysis and design, the late Otis Hovey, M. ASCE, emphasized the necessity of reform and development of the theory. He prophesied that commercial design would adjust itself to the theory when perfected. The same situation prevailed a few decades before regarding statically indeterminate analysis. Mr. Stewart's paper is a step toward this goal.

That the proper design and the incorporation of the properties of a joint may result in a material saving can be shown by a simple problem of frequent occurrence: ABC is a simple two-span beam under a uniform load of w pounds per foot. The length of each span is l. If designed and built as two beams, the bending moment will be $+\frac{w\,l^2}{8}$ at the center of each span. If it is a continuous beam, the bending moment at point B, the central support, is $-\frac{w\,l^2}{8}$. An elastic connection can be placed at support B which will relieve the negative moment there and increase the positive moment until they are equal and of the value of $0.0859\,w\,l^2$—a material saving of 32%. In this particular problem the same effect may be obtained by lowering the support at point B an amount equal to $0.013\,\frac{w\,l^4}{E\,I}$ thus pre-stressing the beam.

Mr. Stewart's paper is of value in again calling attention to the traverse method and its adaptibility in incorporating the properties of connections in rigid-frame analysis.

RALPH W. STEWART,[22] M. ASCE.—The writer is pleased that the paper attracted the attention of exceedingly well-qualified analysts.

Professor Grinter stated that "complexity" is a relative term, and the writer agrees. The intent of the paper was to convey the idea that analyzing structures with yielding joints by the older methods is more complex than analyzing them by the traverse method and also to call attention to the automatic checking feature which Professor Grinter did not mention. Another important and exclusive asset of the traverse method is the direct computation of deflections from the geometry of the traverse as illustrated by Fig. 9 in Mr. Jenkins' discussion.

Professor Rathbun's paper,[23] published in 1935, was the first in known records on the analysis of frames with elastic joints. In the matter of keeping the designer closely in contact with the physical action of the structure as it deforms Professor Rathbun mentions that the other methods "do not have the advantage of giving the analyst the visual concept obtained by the author's diagrammatic method." Those who are interested should compare the last sentence of Professor Grinter's discussion with the first two paragraphs of Mr. Jenkins' discussion and also with Professor Rathbun's comment.

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[22] Los Angeles, Calif.

[23] "Elastic Properties of Riveted Connections," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 101, 1936, p. 524.

Mr. Lamb's Fig. 11 delineates each item in the deformation of a frame with yielding joints. The writer used this type of illustration in his first investigations of such frames but later combined the Δ-angles with the joint yield and showed ϕ-angles only except at the terminal points of a traverse. Mr. McGee's ingenious device of basing the computations on "yielding-end moments" instead of on fixed-end moments will permit consolidating angles at the terminal points also. This is a valuable variation of the method which, in some cases, will save a substantial amount of computing labor.

Mr. Lamb's Fig. 10 can be made general by using n to represent the ratio of the joint yield angle to the Δ-angle. The quantity $\Delta/2$ in Fig. 10 will then become $n\ \Delta$, and $1.5\ \Delta$ will become $(1 - n)\ \Delta$. Mr. Lamb's solution of a practical design problem indicates how the method can be used to advantage by designers of steel structures who have data on the properties of joints.

Mr. Eremin appends a supplementary graphical construction as an adjunct to the method. Various related types of graphical solutions[24] are used by engineers who learned them as part of their early training. It is questionable whether the use of these methods will grow.

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[24] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, ASCE, Vol. 90, 1927, p. 1.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2362

ELASTIC FOUNDATIONS ANALYZED BY THE METHOD OF REDUNDANT REACTIONS

By Zusse Levinton,[1] M. ASCE

With Discussion by Messrs. L. J. Mensch, E. P. Popov, Yi-Mai Yao, Jacob Feld, Victor R. Bergman, Joseph Gold, B. Levine, and Zusse Levinton.

Synopsis

A simplified method is presented for the general problem of a beam on an elastic foundation. The method consists in representing the pressure of the elastic foundation as a series of redundant reactions and in setting up a system of simultaneous equations in terms of the ordinates of the pressure diagram and the elastic constants of the beam and the foundation.

Applications of the method to various structures are presented, with particular reference to foundation slabs, pavements of highways and landing strips, piles and sheet piling, and floating dry docks and pontoon bridges. The equations are tabulated and their use is explained by numerical examples and by solutions of typical problems.

Introduction

The problem of a beam supported by a continuous elastic foundation received the early attention of investigators. In its simplest form, the problem is that of a beam of infinite length, supported by a homogeneous elastic body of infinite depth, or elastic "semispace," capable of developing tension as well as compression—that is, exerting either pull or pressure on the beam when the latter is loaded. For such a beam, loaded by any finite system of loads and moments, a comparatively easy solution by the theory of elasticity is possible if the following two assumptions are made:

(1) The foundation (or soil) is elastic, that is, its settlement at any point is proportional to the pressure at the point.

(2) The foundation modulus in tension is equal to that in compression. The modulus of the foundation is the elastic settlement resulting from unit pressure, or the elastic uplift that results from a unit tension.

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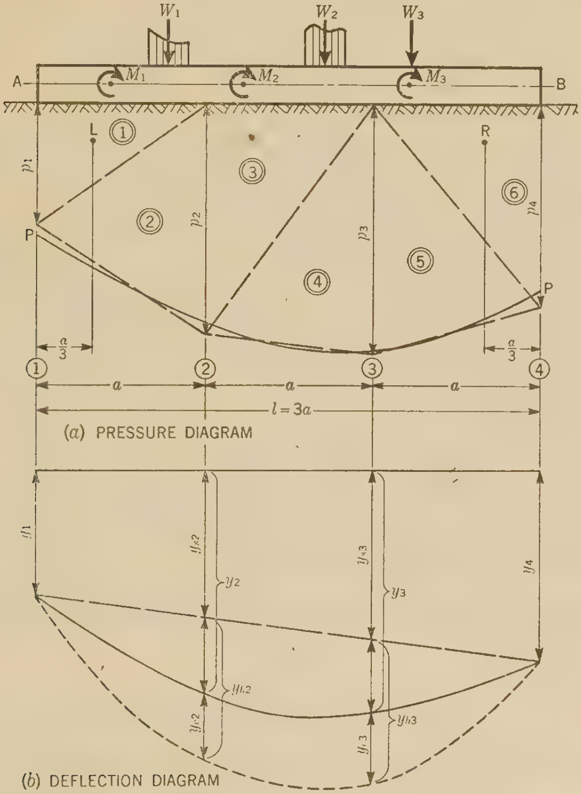
As an example of such a problem, the pressure distribution under continuous railway tracks subjected to concentrated wheel loads has been readily solved by the exact methods of analysis, using differential equations. However, when a beam of finite length is considered, the aforementioned methods result in such complex mathematical expressions and laborious procedures of solution as to make them inaccessible or impractical for many engineers and designers. In many cases of engineering design practice, a solution—at least an approximate one—is desirable for pressure distributions in a foundation of definite dimensions, which could be treated as a beam or one-way slab, supported on soil or on a group of closely spaced piles, acting as a continuous elastic foundation. Typical of this class of problems are bottoms of reservoirs, dry docks, locks, and pavement slabs of highways and landing strips. Another problem of similar nature is that of piles or sheet piling subjected to horizontal loads.



(a) PRESSURE DIAGRAM

(b) DEFLECTION DIAGRAM

Fig. 1

It is the writer's purpose to present a method of solution which he has used for a number of years and which is believed to be readily workable and accessible to the practical engineer. The method is essentially that of redundant reactions.

Approximate Solution

In Fig. 1, AB is a beam, or slab, of unit width and length l supported on a continuous elastic foundation. The symbols, W_1, W_2, W_3, . . ., W_n, are any concentrated or distributed loads and M_1, M_2, M_3, . . ., M_n, any moments acting on the beam. The unknown pressure distribution on the foundation is represented by a curve such as P–P, whose ordinates at any point are proportional to the pressure and also to the deflection (settlement or uplift) at the point.

If the span length l is divided into three equal panels, a, the entire pressure curve may be approximated by straight segments 1–2–3–4, with ordinates p_1 and p_4 at the ends and p_2 and p_3 at the third points and the area of the approximate diagram may be divided into the triangles numbered 1 to 6 in Fig. 1. The beam may be regarded as a simple beam, supported at the ends by unit areas with pressure intensities p_1 and p_4, which act as elastic yielding supports, and the loading of the beam will consist of the loads W and the moments M, together with the six triangular loads of the foundation pressure. The following convention of signs will be used: External loads W are positive when acting downward. Moments M are positive when acting clockwise, as shown. Foundation compression (acting upward on the beam) is positive. Downward settlements or deflections are positive.

Referring to the deflection diagram in Fig. 1(b), the total deflection at point 2 is $y_2 = k\,p_2$, in which k is the modulus of the foundation—that is, the settlement resulting from unit pressure (or uplift resulting from unit pull). This total deflection y_2 is made up of three parts, y_{s2}, y_{b2}, and y_{c2}.

1. Settlement y_{s2} represents the yielding of the end supports of the beam, as if the beam were rigid:

$$y_{s2} = \tfrac{1}{3}\,y_4 + \tfrac{2}{3}\,y_1 = \tfrac{1}{3}\,k\,p_4 + \tfrac{2}{3}\,k\,p_1 \quad\ldots\ldots\ldots\ldots\ldots(1)$$

2. Simple beam bending deflection, y_{b2} (Fig. 1(b)), results from loads W and moments M. This deflection may be obtained for any given loading by well-known formulas, by integration, or by the moment-area method.

3. The deflection y_{c2} represents the effect of the six triangular foundation pressure loads in Fig. 1(a). It can be shown (by the moment-area method) that the deflection at point 2 due to triangular load 1 is equal to

$$y_{1,2} = \frac{94\,a^4}{1{,}080\,E\,I}\,p_1 \quad\ldots\ldots\ldots\ldots\ldots(2a)$$

in which E is the modulus of elasticity; I is the moment of inertia of the beam; a is one third of the span, covered by the triangular load; and p_1 is the altitude of the triangular load. By the same method—the deflection at point 2 due to triangles 2, 3, 4, 5, and 6 can be shown, respectively, to be

$$y_{2,2} = \frac{176\ a^4}{1{,}080\ E\ I}\,p_2 \qquad \text{(2b)}$$

$$y_{3,2} = \frac{253\ a^4}{1{,}080\ E\ I}\,p_2 \qquad \text{(2c)}$$

$$y_{4,2} = \frac{242\ a^4}{1{,}080\ E\ I}\,p_3 \qquad \text{(2d)}$$

$$y_{5,2} = \frac{148\ a^4}{1{,}080\ E\ I}\,p_3 \qquad \text{(2e)}$$

and

$$y_{6,2} = \frac{77\ a^4}{1{,}080\ E\ I}\,p_4 \qquad \text{(2f)}$$

Adding Eqs. 2, the simple beam deflection at point 2 due to the entire foundation pressure is

$$y_{c2} = \frac{94\ a^4}{1{,}080\ E\ I}\,p_1 + \frac{429\ a^4}{1{,}080\ E\ I}\,p_2 + \frac{390\ a^4}{1{,}080\ E\ I}\,p_3 + \frac{77\ a^4}{1{,}080\ E\ I}\,p_4 \qquad (3)$$

With due regard to signs, the equation of deflections for point 2 can be written

$$y_2 = \tfrac{1}{3}\,k\,p_4 + \tfrac{2}{3}\,k\,p_1 + y_{b2} - \frac{94\ a^4}{1{,}080\ E\ I}\,p_1 - \frac{429\ a^4}{1{,}080\ E\ I}\,p_2 - \frac{390\ a^4}{1{,}080\ E\ I}\,p_3 - \frac{77\ a^4}{1{,}080\ E\ I}\,p_4 = k\,p_2 \qquad (4)$$

To simplify typography, let

$$N = \frac{1{,}080}{a^4}\,E\,I\,k \qquad (5)$$

$$(94 - \tfrac{2}{3}\,N)\,p_1 + (429 + N)\,p_2 + 390\,p_3 + (77 - \tfrac{1}{3}\,N)\,p_4 = \frac{1{,}080\ E\ I\ y_{b2}}{a^4} \qquad \text{(6a)}$$

Similarly, by equating deflections for point 3:

$$(77 - \tfrac{1}{3}\,N)\,p_1 + 390\,p_2 + (429 + N)\,p_3 + (94 - \tfrac{2}{3}\,N)\,p_4 = \frac{1{,}080\ E\ I\ y_{b3}}{a^4} \qquad \text{(6b)}$$

Since there are four unknowns, two more equations are necessary. These may be obtained from statics. Taking moments about point R in Fig. 1(*a*), which is the center of gravity of triangle 6:

$$7\,p_1 + 10\,p_2 + 4\,p_3 = -\frac{6\,M_R}{a^2} \qquad \text{(6c)}$$

and by taking moments about point L, the center of gravity of triangle 1:

$$4\,p_2 + 10\,p_3 + 7\,p_4 = \frac{6\,M_L}{a^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots(6d)$$

in which M_R and M_L are the moments of all external loads W about points R and L, respectively, together with the applied moments M.

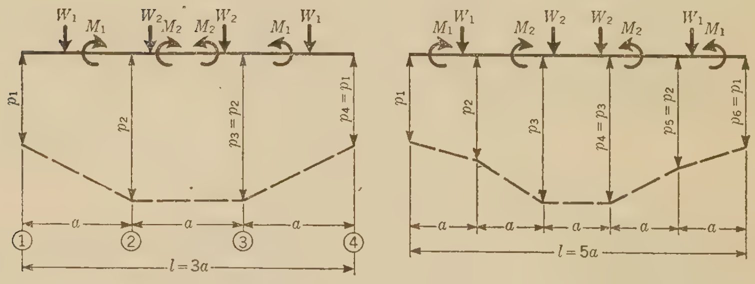
From these four simultaneous equations, the four unknown unit pressures may be obtained. The solution presupposes a knowledge of the foundation modulus. In many instances a fair idea of the foundation modulus may be obtained from soil tests or pile loading tests. In other cases, some critical or safe value for the modulus may be selected and the solution based on that value. Having once computed the constants y_{b2}, y_{b3}, M_R, and M_L, the coefficients may be easily adjusted for any value of N, as determined by the given or assumed value of k. The solution may be repeated for several values of k, thus permitting the study of the behavior of the structure on foundations with different compressibilities.

In the case of symmetrical loading, the solution is simplified. Since $y_{b2} = y_{b3}$ and $p_2 = p_3$, Eqs. 6 are reduced to the following two simultaneous equations:

$$(19 - N_s)\,p_1 + (91 + N_s)\,p_2 = \frac{120\,E\,I\,y_{b2}}{a^4} \quad\ldots\ldots\ldots\ldots(7a)$$

and

$$p_1 + 2\,p_2 = \frac{\Sigma W}{a} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7b)$$



(a) THREE PANELS (b) FIVE PANELS

FIG. 2.—SYMMETRICAL LOADING

in which ΣW is the summation of all external vertical loads W_1, W_2, . . ., W_n applied symmetrically to the beam, as shown in Fig. 2(a), and, to simplify typography,

$$N_s = \frac{120\,E\,I\,k}{a^4} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Example 1. Nonsymmetrical Loading; Three-Panel Solution.—The reinforced concrete slab shown in Fig. 3 is supported on soil of compressibility indicated by tests to be 0.01 ft for a load of 1 kip per sq ft ($k = 0.01$). The loads W_1 and W_2 and the moment M are per foot of width of slab; E (for concrete) = 3,000,000 lb per sq in. = 432,000 kips per sq ft; $I = \frac{b\,y^3}{12} = \frac{1 \times 4^3}{12} = 5.333$; and, by Eq. 5, $N = \frac{1{,}080 \times 432{,}000 \times 5.33 \times 0.01}{20^4} = 155.4$; Also, $M_L = (10 \times 8.33) + (15 \times 28.33) + 30 = 538.25$; and $M_R = -(15 \times 18.33) - (10 \times 38.33) + 30 = -628.28$. Simple beam deflection at point 2 due to

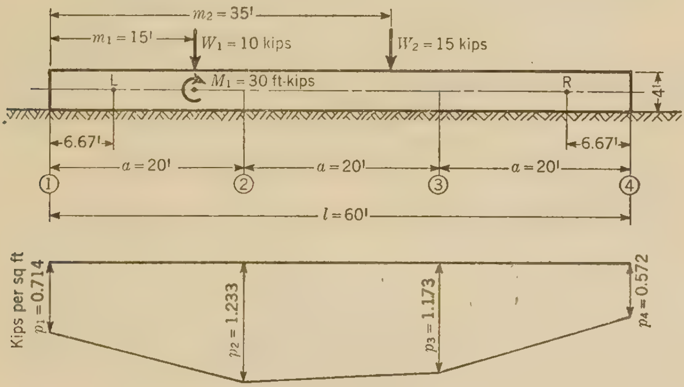


Fig. 3.—Nonsymmetrical Loading; Three-Panel Solution

a 10-kip load is given by formula (or by moment-area method): $E\,I\,y_{b2} = \frac{W_1}{9} \times (5\,a^2\,m_1 - m^3_1) = \frac{10}{9}(5 \times 20^2 \times 15 - 15^3) = 29{,}580$, due to a 15-kip load: $E\,I\,y_{b2} = \frac{W_2}{18}(19\,a^2\,m_2 - 9\,a\,m^2_2 + m^3_2 - 3\,a^3) = \frac{15}{18}(19 \times 20^2 \times 35 - 9 \times 20 \times 35^2 + 35^3 - 3 \times 20^3) = 53{,}650$, and, due to 30-ft-kip moment: $E\,I\,y_{b2} = \frac{M}{9}(5\,a^2 - 3\,m^2_1) = \frac{30}{9}(5 \times 20^2 - 3 \times 15^2) = 4{,}418$. The total of the three values of $E\,I\,y_{b2} = 29{,}580 + 53{,}650 + 4{,}418 = 87{,}648$.

Similarly, to evaluate total value of $E\,I\,y_{b3}$ for point 3 (Fig. 3):

$$\frac{W_1}{18}(8\,a^2\,m_1 - m^3_1) = \frac{10}{18}(8 \times 20^2 \times 15 - 15^3) = 24{,}794$$

$$\frac{W_2}{18}(8\,a^2\,m_2 - m^3_2) = \frac{15}{18}(8 \times 20^2 \times 35 - 35^3) = 57{,}609$$

$$\frac{M}{18}(8\,a^2 - 3\,m^2_1) = \frac{30}{18}(8 \times 20^2 - 3 \times 15^2) = 4{,}207$$

$$\text{Total value } E\,I\,y_{b3} = 86{,}610$$

From these two values of $E\,I\,y$: $\frac{1{,}080\,E\,I\,y_{b2}}{a^4} = \frac{1{,}080 \times 87{,}648}{20^4} = 591.6$;

and $\dfrac{1{,}080\,E\,I\,y_{b3}}{a^4} = \dfrac{1{,}080 \times 86{,}610}{20^4} = 585.0$. Substituting into Eqs. 6:

$$\left.\begin{aligned} -9.6\,p_1 + 584.4\,p_2 + 390\,p_3 + 25.2\,p_4 &= 591.6 \\ 25.2\,p_1 + 390\,p_2 + 584.4\,p_3 - 9.6\,p_4 &= 585.0 \\ 7\,p_1 + 10\,p_2 + 4\,p_3 &= 9.42 \\ 4\,p_2 + 10\,p_3 + 7\,p_4 &= 8.07 \end{aligned}\right\} \quad\text{(9)}$$

Solving the four equations: $p_1 = +0.1143$; $p_2 = +0.6325$; $p_3 = +0.5734$; and $p_4 = -0.0278$. Adding the weight of the reinforced concrete slab $4 \times 150 \dfrac{1}{1{,}000} = 0.600$ kips per sq ft, uniformly, the soil pressures, in kips per sq ft, are: $p_1 = 0.7140$; $p_2 = 1.233$; $p_3 = 1.173$; and $p_4 = 0.572$.

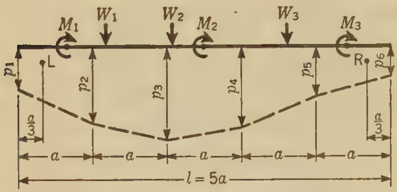


FIG. 4.—NONSYMMETRICAL LOADING

MORE ACCURATE SOLUTIONS

It is obvious that a closer approximation to the actual foundation pressure curve may be obtained by dividing the length of the beam (l) into a larger number of panels and by writing deflection equations for each of the panel points. Thus, by dividing span length l into five equal panels, as shown in Fig. 4, and by writing the deflection equation for point 2:

$$\left(116 - \frac{4}{5}N\right) p_1 + (605 + N)\,p_2 + 870\,p_3 + 780\,p_4 + 450\,p_5 + \left(79 - \tfrac{1}{5}N\right) p_6 = \frac{600\,E\,I\,y_{b2}}{a^4} \quad\text{(10)}$$

The six equations necessary for the solution of the six unknown pressure ordinates p_1, p_2, p_3, p_4, p_5, and p_6 are given in Table 1, for which a is one fifth of the span, and

TABLE 1.—SOLUTION OF FIVE-PANEL PROBLEM FOR NONSYMMETRICAL LOADING ($a = l/5$ AND VALUES OF N ARE DETERMINED BY EQ. 11)

Line No.	COEFFICIENTS						Absolute term
	p_1	p_2	p_3	p_4	p_5	p_6	
1	$116-4N/5$	$605+N$	870	780	450	$79-N/5$	$600\,E\,I\,y_{b2}/a^4$
2	$157-3N/5$	870	$1{,}385+N$	1,320	780	$138-2N/5$	$600\,E\,I\,y_{b3}/a^4$
3	$138-2N/5$	780	1,320	$1{,}385+N$	870	$157-3N/5$	$600\,E\,I\,y_{b4}/a^4$
4	$79-N/5$	450	780	870	$605+N$	$116-4N/5$	$600\,E\,I\,y_{b5}/a^4$
5	13	22	16	10	4	0	$-6\,M_R/a^2$
6	0	4	10	16	22	13	$6\,M_L/a^2$

$$N = \frac{600\,E\,I\,k}{a^4} \quad\text{(11)}$$

For the symmetrical loading shown in Fig. 2(b) these equations become simplified to those in Table 2.

Example 2. Symmetrical Loading: Five-Panel Solution.—Fig. 5 shows the same slab as shown in Fig. 3, loaded by symmetrical loads and moments. In a manner similar to Example 1: $N = \dfrac{120\,E\,I\,k}{a^4} = \dfrac{120 \times 432{,}000 \times 5.33 \times 0.01}{12^4} = 133.33$; and $\dfrac{\Sigma W}{a} = \dfrac{75 + 100 + 100 + 75}{12} = 29.17$. The total value of $E\,I\,y_{b2}$ is computed to be 669,920 and the total value of $E\,I\,y_{b3} = 1{,}073{,}733$. Therefore, $\dfrac{120\,E\,I\,y_{b2}}{a^4} = \dfrac{120 \times 669{,}920}{12^4} = 3{,}876.85$; and $\dfrac{120\,E\,I\,y_{b3}}{a^4} = \dfrac{120 \times 1{,}073{,}733}{12^4} = 6{,}213.73$. Substituting the foregoing values in the equations in Table 2:

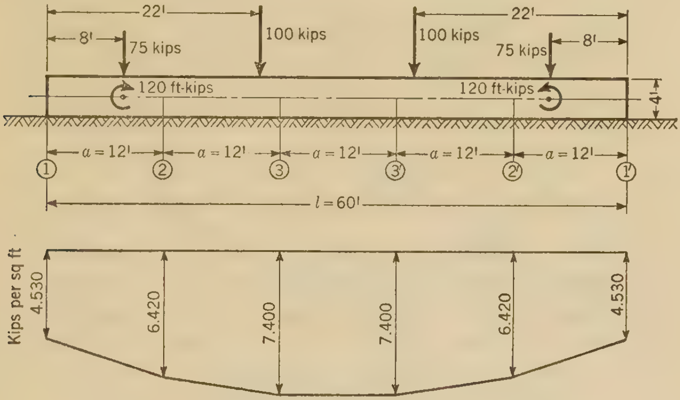


FIG. 5.—SYMMETRICAL LOADING; FIVE-PANEL SOLUTION

$$\left.\begin{array}{rl} -94.33\,p_1 + 344.33\,p_2 + 330\,p_3 &= 3{,}876.85 \\ -74.33\,p_1 + 330\,p_2 + 674.33\,p_3 &= 6{,}213.73 \\ p_1 + 2\,p_2 + 2\,p_3 &= 29.17 \end{array}\right\} \quad (12)$$

Solving Eq. 12: $p_1 = +\,3.930$; $p_2 = +\,5.820$; and $p_3 = +\,6.800$. Adding the uniform weight of slab, the pressures, in kips per sq ft, are: $p_1 = +\,4.530$; $p_2 = +\,6.420$; and $p_3 = +\,7.400$.

A still more accurate solution may be obtained by the use of Table 3 with N as determined by Eq. 8 and by Table 4, in which

$$N = \frac{1{,}200\,E\,I\,k}{a^4} \quad (13)$$

TABLE 2.—SOLUTION OF FIVE-PANEL PROBLEM FOR SYMMETRICAL LOADING ($a = l/5$ AND VALUES OF N ARE DETERMINED BY EQ. 8)

Line No.	Coefficients			Absolute term
	p_1	p_2	p_3	
1	$39 - N$	$211 + N$	330	$120\,E\,I\,y_{b2}/a^4$
2	$59 - N$	330	$541 + N$	$120\,E\,I\,y_{b3}/a^4$
3	1	2	2	$\Sigma W/a$

The length of the beam is divided into ten panels as shown in Fig. 6.

The three-panel solution is quite satisfactory for approximate and preliminary stress analysis, and the five-panel solution is sufficiently accurate for a great variety of elastic foundation problems such as may be encountered in practice. Since the physical properties of soil and pile foundations are never exactly determined, a precision greater than that given by the five-panel solution is seldom required. The accuracy of the approximate methods is illustrated in Fig. 7 in which three-panel and five-panel solutions are compared with the rigorous, or "exact" solution.

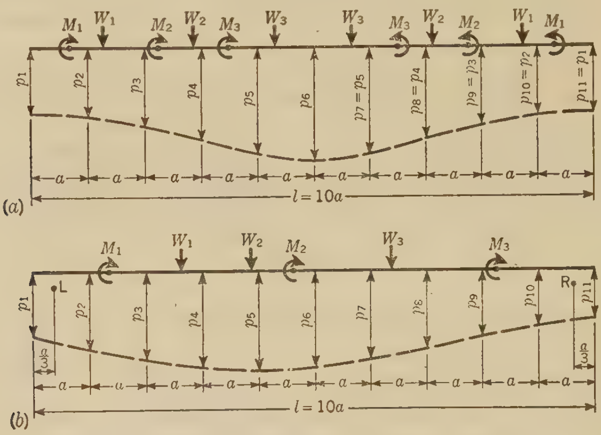

M_1 W_1 M_2 W_2 M_3 W_3 W_3 M_3 W_2 M_2 W_1 M_1
p_1 p_2 p_3 p_4 p_5 p_6 $p_7=p_5$ $p_8=p_4$ $p_9=p_3$ $p_{10}=p_2$ $p_{11}=p_1$
a a a a a a a a a a
$l=10a$
(a)
M_1 W_1 W_2 M_2 W_3 M_3
L R
p_1 p_2 p_3 p_4 p_5 p_6 p_7 p_8 p_9 p_{10} p_{11}
$\frac{a}{3}$ $\frac{a}{3}$
a a a a a a a a a a
$l=10a$
(b)

FIG. 6.—TEN-PANEL PROBLEM
(*a*) Nonsymmetrical Loading
(*b*) Symmetrical Loading

The assumption involved in this analysis is that the foundation is capable of exerting pull on the beam. Such is the case with friction piles with a value in tension nearly equal to that in compressive loading. If the beam rests on soil that does not develop tension; and, if the soil pressure diagram obtained by this analysis indicates part compression and part tension, the solution must be modified by applying to the beam a loading equal and opposite to the indicated soil tension. The effect of this loading will be to change the compressive reaction. By adding this effect the corrected reaction diagram may be obtained.

TABLE 3.—SOLUTION OF TEN-PANEL PROBLEM; SYMMETRICAL LOADING
($a = l/10$ AND VALUES OF N ARE DETERMINED BY EQ. 8)

Line No.	Coefficients						Absolute term
	p_1	p_2	p_3	p_4	p_5	p_6	
1	$89-N$	$511+N$	930	1,230	1,410	735	$120\,E\,I\,y_{b2}/a^4$
2	$159-N$	930	$1{,}741+N$	2,340	2,700	1,410	$120\,E\,I\,y_{b3}/a^4$
3	$209-N$	1,230	2,340	$3{,}211+N$	3,750	1,965	$120\,E\,I\,y_{b4}/a^4$
4	$239-N$	1,410	2,700	3,750	$4{,}441+N$	2,340	$120\,E\,I\,y_{b5}/a^4$
5	$249-N$	1,470	2,820	3,930	4,680	$2{,}476+N$	$120\,E\,I\,y_{b6}/a^4$
6	1	2	2	2	2	1	$\Sigma W/a$

It also must be borne in mind that the basic assumption, as in other elastic theory methods, is that the settlement at a point is proportional to the pressure at the point. Accordingly, under a uniformly distributed load the foundation pressure is assumed also to be uniform. This condition is not always realized in soil foundations, their behavior being determined by a number of factors of soil mechanics. The effective value of k near the center of the beam is often greater than at its ends. This is evidenced by concavity of foundation slabs under uniform loading.

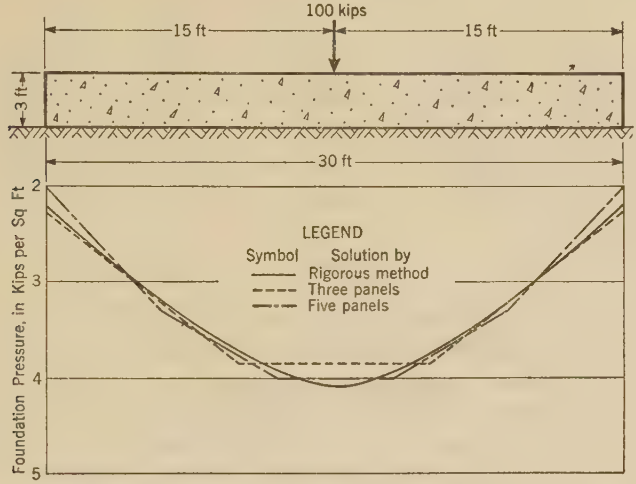
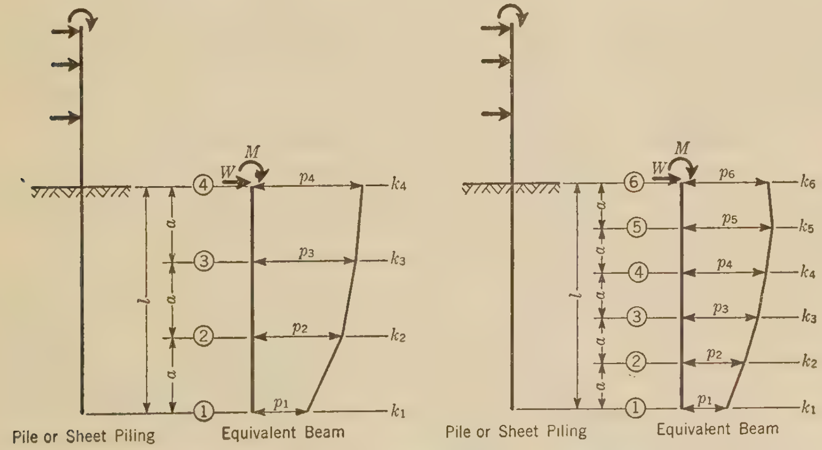


Fig. 7.—Comparison of Pressure Diagrams (Weight of Beam Not Included)


M
W
p_4
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Pile or Sheet Piling
Equivalent Beam
M
W
p_6
k_6
p_5
k_5
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Pile or Sheet Piling
Equivalent Beam

(a) THREE PANELS ($l = 3a$) (b) FIVE PANELS ($l = 5a$)

Fig. 8.—Vertical Piles or Sheet Piling Subjected to Horizontal Loading

Variations of k along the beam can be incorporated in the foregoing systems of equations in a manner similar to that discussed for vertical piles subjected to horizontal loading.

TABLE 4.—SOLUTION OF TEN-PANEL PROBLEM; NONSYMMETRICAL

Line No.	COEFFI					
	p_1	p_2	p_3	p_4	p_5	p_6
1	$561-9\,N/10$	$3{,}160+N$	5,520	6,930	7,500	7,350
2	$952-8\,N/10$	5,520	$10{,}090+N$	13,020	14,280	14,100
3	$1{,}183-7\,N/10$	6,930	13,020	$17{,}440+N$	19,620	19,650
4	$1{,}274-6\,N/10$	7,500	14,280	19,620	$22{,}810+N$	23,400
5	$1{,}245-N/2$	7,350	14,100	19,650	23,400	$24{,}760+N$
6	$1{,}116-4\,N/10$	6,600	12,720	17,880	21,600	23,400
7	$907-3\,N/10$	5,370	10,380	14,670	17,880	19,650
8	$638-2\,N/10$	3,780	7,320	10,380	12,720	14,100
9	$329-N/10$	1,950	3,780	5,370	6,600	7,350
10	14	26	23	20	17	14
11	0	2	5	8	11	14

PILES AND SHEET PILING

The method of redundant reactions can be applied to the case of a pile embedded in the soil and subjected to horizontal loading. The pile can be replaced by an equivalent vertical beam of length equal to the embedment and with shear and moment acting at the surface of the soil.

TABLE 5.—VERTICAL PILE OR VERTICAL SHEET PILING;
THREE-PANEL SOLUTION ($a = l/3$)

Line No.	COEFFICIENTS				Absolute term	N
	P_1	P_2	P_3	P_4		
1	$94-2\,N_1/3$	$429+N_2$	390	$77-N_4/3$	$-480\,M/a^2$	$N_4=1{,}080\,E\,I\,k_4/a^4$
2	$77-N_1/3$	390	$429+N_3$	$94-2\,N_4/3$	$-600\,M/a^2$	$N_3=1{,}080\,E\,I\,k_3/a^4$
3	7	10	4	0	$-6(M+W\,a/3)/a^2$	$N_2=1{,}080\,E\,I\,k_2/a^4$
4	0	4	10	7	$6(M+8\,W\,a/3)/a^2$	$N_1=1{,}080\,E\,I\,k_1/a^4$

TABLE 6.—VERTICAL PILE OR VERTICAL SHEET PILING;
FIVE-PANEL SOLUTION ($a = l/5$)

Line No.	COEFFICIENTS						Absolute term	N_x
	p_1	p_2	p_3	p_4	p_5	p_6		
1	$116-4\,N_1/5$	$605+N_2$	870	780	450	$79-N_6/5$	$-480\,M/a^2$	$600\,E\,I\,k_6/a^4$
2	$157-3\,N_1/5$	870	$1{,}385+N_3$	1,320	780	$138-2\,N_6/5$	$-840\,M/a^2$	$600\,E\,I\,k_5/a^4$
3	$138-2\,N_1/5$	780	1,320	$1{,}385+N_4$	870	$157-3\,N_6/5$	$-960\,M/a^2$	$600\,E\,I\,k_4/a^4$
4	$79-N_1/5$	450	780	870	$605+N_5$	$116-4\,N_6/5$	$-720\,M/a^2$	$600\,E\,I\,k_3/a^4$
5	13	22	16	10	4	0	$-6(M+W\,a/3)/a^2$	$600\,E\,I\,k_2/a^4$
6	0	4	10	16	22	13	$6(M+14\,W\,a/3)/a^2$	$600\,E\,I\,k_1/a^4$

Equations for three-panel and five-panel solutions (see Fig. 8) are given in Tables 5 and 6, respectively. The important consideration in this case is that k is taken to vary with the depth, and for each panel point there is its value of k and the corresponding value of N. In addition, the absolute terms of the equations, in this case, are expressed in terms of M and W, to represent the deflections $y_{b2}, y_{b3}, \ldots, y_{bn}$ at the panel points.

LOADING ($a = l/10$ AND VALUES OF N ARE DETERMINED BY EQ. 13)

CIENTS					Absolute term	Line No.
p_7	p_8	p_9	p_{10}	p_{11}		
6,600	5,370	3,780	1,950	$329 - N/10$	$1{,}200\ E\ I\ y_{b2}/a^4$	1
12,720	10,380	7,320	3,780	$638 - 2\ N/10$	$1{,}200\ E\ I\ y_{b3}/a^4$	2
17,880	14,670	10,380	5,370	$907 - 3\ N/10$	$1{,}200\ E\ I\ y_{b4}/a^4$	3
21,600	17,880	12,720	6,600	$1{,}116 - 4\ N/10$	$1{,}200\ E\ I\ y_{b5}/a^4$	4
23,400	19,650	14,100	7,350	$1{,}245 - N/2$	$1{,}200\ E\ I\ y_{b6}/a^4$	5
$22{,}810 + N$	19,620	14,280	7,500	$1{,}274 - 6\ N/10$	$1{,}200\ E\ I\ y_{b7}/a^4$	6
19,620	$17{,}440 + N$	13,020	6,930	$1{,}183 - 7\ N/10$	$1{,}200\ E\ I\ y_{b8}/a^4$	7
14,280	13,020	$10{,}090 + N$	5,520	$952 - 8\ N/10$	$1{,}200\ E\ I\ y_{b9}/a^4$	8
7,500	6,930	5,520	$3{,}160 + N$	$561 - 9\ N/10$	$1{,}200\ E\ I\ y_{b10}/a^4$	9
11	8	5	2	0	$-3\ M_R/a^2$	10
17	20	23	26	14	$3\ M_L/a^2$	11

For convention of signs, the right side of the pile is assumed to correspond with the bottom side of the horizontal beam discussed in preceding sections.

FLOATING STRUCTURES

In naval architecture, the longitudinal bending moments in ships and other vessels are generally computed from the buoyancy or water-pressure distribution for different conditions of trim of the vessel. The assumption generally made is that the trimmed ship remains straight and rigid. Variations in draft resulting from the ship's flexibility are disregarded.

In some structures, such as derrick barges, floating dry docks and certain types of pontoon bridges, the flexibility of the structure is such as to affect, greatly, the drafts and the buoyancy forces along the deflected structure. In such cases a more accurate analysis can be made by regarding the vessel (or unit width of vessel) as a beam floating on an elastic foundation, and the method of redundant reactions described herein can be used to advantage. It should be realized that water supporting the vessel behaves like an elastic foundation, since the increments of pressure are proportional to the settlements (that is, the increments of draft). For fresh water weighing 62.5 lb per cu ft, the modulus $k = 1/62.5 = 0.016$ cu ft per lb. For sea water $k = 1/64 = 0.0156$ cu ft per lb. The value of k, of course, is constant for all depths.

By taking a strip of unit width and dividing the length of the vessel into three or five equal panels, a solution can be obtained for deflections and bending moments in the vessel.

CONCLUSIONS

The difficulties generally met in problems involving elastic foundations are due to two main causes: (1) The uncertainty as to the physical properties of the foundation material, and (2) the complexity of the mathematical analysis. Although the former difficulty is overcome by the development of theory and the technique of testing the materials, the second difficulty can be overcome by a simplified method of analysis.

The advantages of the proposed method are amply demonstrated by the simple and straightforward solutions obtained in the illustrative examples.

Once computed and tabulated, the coefficients of the general equations given in this paper form a framework into which the data of various problems can be inserted. The remaining work is simply routine solution of simultaneous equations, with various degrees of accuracy, as the case may require.

ACKNOWLEDGMENTS

The derivation of the method and its formulation was made during the years from 1939 to 1940, while the writer was on the staff of Frederic R. Harris, M. ASCE, in New York, N. Y., and was engaged in the design of dry docks for the United States Navy. To the members of the design staff who gave the writer advice and encouragement, grateful acknowledgment is made. Valuable criticism and comment were given by Eugene E. Halmos, M. ASCE, and George T. Gilman, M. ASCE, formerly with the Dry Dock Engineers, in New York, N. Y. In the preparation of the final manuscript, useful suggestions were given by Robert W. Abbett, M. ASCE, of the Knappen Engineering Company, in New York.

DISCUSSION

L. J. Mensch,[2] M. ASCE.—Although this method of analyzing flexible supports is ingenious, it can be improved. For symmetrical loadings, as well as for many unsymmetrical loadings, the assumption of a parabolic line of deformation will simplify the analysis. This is demonstrated for the case in Fig. 7, as shown in Fig. 9. The following formulas apply:

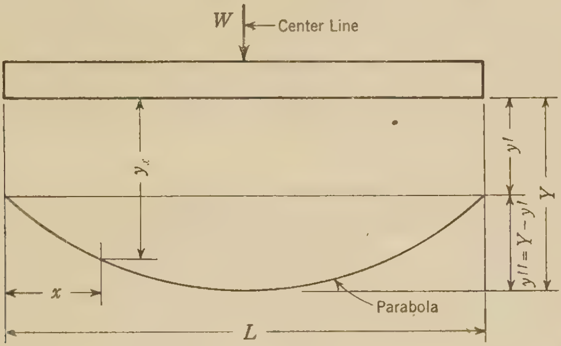

Fig. 9

$$y' = k\,p' \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

$$Y - y' = k\,p'' \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

$$Y = k\,p''' \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14c)$$

and

$$W = \frac{L\,y'}{k} + \frac{Y - y'}{k}\,\frac{2}{3}\,L \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

At any distance, x, from one end the ordinate is

$$y_x = \frac{4\,(Y - y')}{L^2}\,(L\,x - x^2) + y' \quad\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

It is not difficult to find the moment at such a point from the parabolic distribution and from the uniform load p', since

$$M_x = \frac{4\,(Y - y')}{k\,L^2}\left(\frac{L\,x^3}{6} - \frac{x^4}{12}\right) + \frac{y'\,x^2}{2\,k} \quad\ldots\ldots\ldots\ldots (17)$$

The deflection due to the parabolic distribution and the uniform load p' is

$$Y - y' = \int_0^{L/2} \frac{M_x \times dx}{E\,I} = \frac{(Y - y')\,L^4}{303\,E\,I\,k} + \frac{y'\,L^4}{128\,E\,I\,k} \quad\ldots\ldots (18)$$

For the values, $E = 432{,}000$ kips per sq ft, $I = \frac{27}{12}$, $k = 0.01$, and $L^4 = 810{,}000$,

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[2] Contractor. Evanston, Ill.

$\frac{L^4}{E I k} = \frac{1{,}000}{12}$, and Eq. 18 reduces to

$$\frac{12 (Y - y')}{1{,}000} = \frac{Y - y'}{303} + \frac{y'}{128} \qquad (19)$$

From Eqs. 15 and 19, it is easily found that $Y = 0.0404$ ft, $y' = 0.0213$ ft, $p''' = 4.04$ kips per sq ft, and $p' = 2.13$ kips per sq ft. In the writer's opinion the value of E should be taken as only 144,000 kips per sq ft when the initial modulus in cylinder tests is 3,000,000 lb per sq in., because of flow of the concrete due to long continued loading. In this case, Eqs. 15 and 19 give values of $p''' = 4.8$ kips per sq ft, and $p' = 0.395$ kips per sq ft. By setting $y' = 0$ in Eq. 18, the length L, at which the reactions at the ends become zero, can be found. For the values $E = 144{,}000$ and $k = 0.01$, L becomes 31.5 ft in a beam 3 ft deep.

E. P. Popov,[3] Assoc. M. ASCE.—An interesting subject—that of beams on elastic foundations—is presented in the paper by Mr. Levinton. Important applications of the theory to practical engineering projects are cited, and the point of view developed in the paper regarding the elastic foundation as equivalent to a series of redundant reactions is interesting. Likewise, the division of the total deflection of an elastic beam into three components, shown graphically in Fig. 1, is worthy of note. This interpretation clarifies the physical concepts involved in the problem, although the definition of y_{s2}, in Fig. 1, is a little ambiguous. The author's treatment of these physical concepts sheds some new light on the subject; but other phases presented in the paper seem open to criticism.

In the "Introduction," the author gives the impression that the beams analyzed were supported by a homogeneous elastic medium of infinite depth. Later, a remark is made to the effect that a solution for such beams of infinite length is available. Neither the author's solution nor the one usually available for an infinite beam is really that of a beam on an elastic continuum. A simplifying assumption, introduced in 1867 by E. Winkler,[4] is normally made—namely, that the reaction forces of the foundation are proportional to the deflection of the beam at that point. This type of an elastic foundation may be thought of as one composed of an infinite number of independent springs. Such independence of each "spring" capable of taking a force in either direction is definitely implied in assumptions (1) and (2) in the "Introduction." This cardinal assumption holds quite rigorously in a network of beams, pressure vessels, and shells; but, for soil foundations, it can only be regarded as a practical approximation.

The chief aim of the author is to make available a method of analysis for beams on elastic foundations to the "practical" engineer. The paper avoids references, but, it should be noted, nevertheless, that "theoretical" engineers have studied the problem exhaustively. In one such work by M. Hetenyi[5] ready "exact" formulas are available for a great many cases of simple loading on beams of finite length. For complicated loadings, solutions are possible by merely substituting tabular values into formulas and by superposition. Solutions for an infinite beam on an elastic foundation form only the fundamental cases. All the necessary higher functions, as well as sines and cosines of angles, are tabulated in Professor Hetenyi's work. This information, fully compiled and developed, is now more accessible than formerly.[6,7,8,9,10]

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[3] Asst. Prof. of Civ. Eng., Dept. of Eng., Univ. of California, Berkeley, Calif.

[4] "Die Lehre von der Elasticität und Festigkeit," by E. Winkler, Dominicus, Prag, 1867, p. 182.

[5] "Beams on Elastic Foundation," by M. Hetenyi, Univ. of Michigan Press, Ann Arbor, Mich., 1946

Professor Hetenyi presents a perfectly general method for the solution of almost any conceivable loading.[11] The general method is based on an exact solution for an infinite beam. Such a beam is conditioned by certain moments and shears applied on the infinite beam so as to create zero moment and zero shear at the points corresponding to the ends of the actual beam. In this manner a part of the infinite beam between the points of zero moment and shear corresponds to the actual beam. The writer has applied such computations to Example 1 of the paper to test the merits of Mr. Levinton's method.

In addition to the aforementioned procedures giving exact solutions, a third procedure is discussed by Professor Hetenyi based on the use of trigonometric series.[12] This method appears to be particularly adaptable to practical problems. Substituting values directly into simple formulas gives an equation for the deflection of the beam. As many terms of a series as desired may be used, but in most cases one term suffices. For example, for the beam shown in Fig. 5, the equation[13] for the elastic curve due to moments alone is

$$y = -\frac{M_o \frac{2\pi}{l} \cos\frac{\pi c}{l}}{\frac{\pi^4 E I}{2 l^3} + \frac{l k}{2} - \frac{4 l k}{\pi^2}} \left(\frac{2}{\pi} - \sin\frac{\pi x}{l}\right) \ldots\ldots\ldots\ldots (20)$$

In Example 2 of the paper $M_o = 120$ ft-kips; $c = 8$ ft; k is the reciprocal of Mr. Levinton's foundation constant; x is the distance to a particular point from the end of the beam in feet; and the deflection of the beam due to moments alone becomes

$$y = -0.01055 \left(\frac{2}{\pi} - \sin\frac{\pi x}{60}\right) \ldots\ldots\ldots\ldots (21)$$

Similar simple expressions are available to express the deflection caused by pairs of symmetrically placed concentrated loads. Adding—that is, superposing—three such equations (one for moments and two for the concentrated loads), the second example of the paper yields

$$p = k\,y + 0.60 = 4.56 + 2.95 \sin\frac{\pi x}{60} \ldots\ldots\ldots\ldots (22)$$

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[6] "Die Berechnung des Eisenbahnoberbaues," by H. Zimmermann, W. Ernst und Sohn, Berlin, 2d Ed., 1930.

[7] "Theorie des Trägers auf elastischer Unterlage und ihre Anwendung auf den Tiefbau," by K. Hayashi, Julius Springer, Berlin, 1921.

[8] "Analysis of Beams on Elastic Foundation," by A. A. Umansky, Central Research Inst. of Auto-Transportation, Leningrad, 1933.

[9] "Analysis of Beams on Elastic Foundations," by A. N. Krylov, Academy of Sciences of U. S. S. R., Leningrad, 2d Ed., 1931.

[10] "Strength of Materials," by S. Timoshenko, Pt. II, D. Van Nostrand Co., Inc., New York, N. Y. 2d Ed., Pt. II, 1941, p. 1.

[11] "Beams on Elastic Foundation," by M. Hetenyi, Univ. of Michigan Press, Ann Arbor, Mich., 1946 p. 47.

[12] *Ibid.*, p. 69.

[13] *Ibid.*, p. 72.

A more accurate solution of the same problem may be readily obtained by using two terms of a series, which then yields

$$p = k\,y + 0.60 = 4.53 + 2.99 \sin \frac{\pi x}{60} + 0.051 \sin \frac{\pi x}{20} \ldots\ldots (23)$$

To obtain this more accurate solution two sets of two simultaneous equations had to be solved. A comparison of values of p (in kips per square feet) computed by Eqs. 22 and 23 is as follows:

x	Eq. 22	Eq. 23
0	4.56	4.53
12	6.30	6.34
24	7.37	7.35

Thus, it must be concluded that even a one-term solution gives results sufficiently accurate for practical purposes.

In examining the method presented by Mr. Levinton and in comparing it with the foregoing procedures, the series solution appears to be superior in the opinion of the writer. No matter how complicated the given loading may be, nor how many points are selected where soil pressure is desired, the series solution gives the answer easily. All necessary calculations can be performed quickly with the aid of a slide rule. If the series solution is used, one would never be obliged to solve as many as eleven simultaneous equations, as required in Table 4; the solution of that many simultaneous equations is complex no matter how simply they may be set up.

This discussion is not to be construed as a severe criticism of Mr. Levinton's paper; it is commendable that such an important subject, well presented, can be offered for the comments of the profession. Whether one uses Mr. Levinton's method or the series solution will remain a matter of personal preference in many cases. However, it is hoped that some idea of the scope of available literature has been helpful in giving the novice, and others interested, some direction for additional study in this interesting subject.

Yi-Mai Yao,[14] Esq.—The fundamental differential equation for studying the deflection curve of a beam of infinite length embedded in a material capable of exerting equal tensive and compressive foundation modulus is

$$E\,I \frac{d^4y}{dx^4} = p = -\frac{y}{k} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

or

$$\frac{d^4y}{dx^4} = -(4\,\beta)^4\,y \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which p is the intensity of reaction, k is the foundation modulus, and the constant

$$\beta = \sqrt[4]{\frac{1}{4\,k\,E\,I}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

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In general, a discussion of the bending of a beam of finite length falls naturally into three groups:[15] (1) Short beams, $\beta L < 0.60$; (2) long beams, $\beta L > 5.00$; and (3) beams of medium length, $0.60 < \beta L < 5.00$.

In discussing beams of the first group (for example, the case of a 100-kip load at the middle, as shown in Fig. 7, assuming $L = 6.95$ ft and $\beta L = 0.50$) by the rigorous method developed by M. Hetényi[16]—in which the bending of a beam of finite length on an elastic foundation can be investigated by using the equations for an infinitely long beam, together with the method of superposition—the pressure intensities at the middle and at the end are 14.41 and 14.39, respectively. Bending can be entirely neglected and the beam considered infinitely rigid with very good accuracy; hence, $p = W/L = 14.40$.

In the case of the second group, it can be assumed in investigating one end of the beam that the other end is infinitely distant. For example (Fig. 7), assuming $L = 83.4$ ft and $\beta L = 6.00$ (by the rigorous method) the pressure intensities at the middle and at the end equal 3.66 and -0.755, respectively. Treating the beam to be of infinite length, the pressure intensities at the middle and at the end equal 3.60 and -0.152, respectively.

A beam of the third group has the characteristic that a force acting on one end of the beam produces a considerable effect at the other end. Thus, such a beam must be treated as one of finite length. This is the problem discussed in the paper.

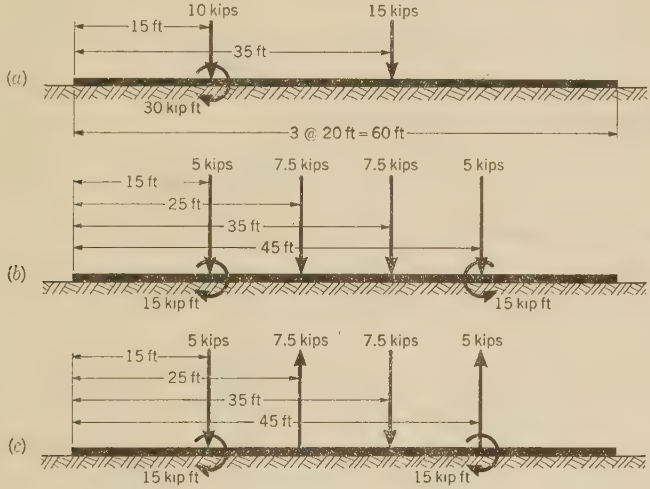

FIG. 10

Because of the uncertainty of the physical properties of the foundation materials, either the rigorous solution or the approximation solution is merely "mathematical play." Major advantages of the author's proposed method are the avoidance of complex mathematics and the ease with which it can be learned. The minor advantage is that it can solve the following problems:

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[15] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 1946, Pt. II, p. 20.

[16] "Final Report of the Second Congress of the International Association of Bridge and Structural Engineering," Berlin, 1938.

1. Bending of a beam of finite length on an elastic foundation with variable foundation modulus;
2. Bending of a beam of finite length on an elastic foundation with variable $E\,I$-values; and
3. Bending of a beam of finite length on an elastic foundation with zero tensive foundation modulus.

The disadvantage of the proposed method is the tedious work of solving the simultaneous equations, especially for nonsymmetrical loading. Having determined the symmetrical and the antisymmetrical loading, the solution for any kind of loading can be obtained readily by using the principle of superposition. Hence, routine solution of the simultaneous equations is much simplified. For instance, in Example 1, the nonsymmetrical case (Fig. 10(*a*)) is solved by superposing the solutions of the symmetrical and the antisymmetrical cases shown in Figs. 10(*b*) and (10(*c*).

For the symmetrical case, the three-panel solution (Eqs. 7*a*, 7*b*, and 8) is used. In the antisymmetrical case, since $p_1 = -\,p_4$, $p_2 = -\,p_3$, and $y_{b2} = -\,y_{b3}$, Eqs. 6 reduce to

$$(17 - N)\,p_1 + (39 + N)\,p_2 = \frac{1{,}080\,E\,I\,y_{b2}}{a^4} \ldots\ldots\ldots\ldots (27a)$$

and

$$7\,p_1 + 6\,p_2 = -\,\frac{6\,M_R}{a^2} \ldots\ldots\ldots\ldots\ldots\ldots (27b)$$

In Fig. 10(*b*), by Eq. 8, $N_s = \frac{120 \times 432{,}000 \times 5.33 \times 0.01}{20^4} = 17.28$; $E\,I\,y_{b2} = 14{,}790 + 26{,}825 + 2{,}218 + 12{,}397 + 28{,}804 + 2{,}104 = 87{,}138$; and $\Sigma W = 25$. Substituting these values in Eq. 7,

$$\left.\begin{aligned} 1.72\,p'_1 + 108.28\,p'_2 &= 65.4 \\ p'_1 + 2\,p'_2 &= 1.25 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

from which $p'_1 = +\,0.042$ and $p'_2 = +\,0.604$.

TABLE 7.—PRESSURE INTENSITIES (THREE-PANEL SOLUTION) BY SUPERPOSITION

Loading	PRESSURE INTENSITIES			
	p_1	p_2	p_3	p_4
Dead	+0.600	+0.600	+0.600	+0.600
Symmetrical	+0.042	+0.604	+0.604	+0.042
Antisymmetrical	+0.077	+0.023	−0.023	−0.077
Total	+0.719	+1.227	+1.181	+0.565

In Fig. 10(*c*), $N = 155.4$; $E\,I\,y_{b2} = 14{,}790 + 26{,}825 + 2{,}218 - 12{,}397 - 28{,}804 - 2{,}104 = 528$; and $M_R = +15 + 15 + (7.5 \times 10^{-5} \times 30) = -\,45$. Substituting these values in Eqs. 27,

$$\left.\begin{aligned} -\,34.8\,p''_1 + 194.4\,p''_2 &= 0.366 \\ 7\,p''_1 + 6\,p''_2 &= 0.675 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

from which $p''_1 = +\,0.077$ and $p''_2 = +\,0.023$.

The total pressure intensities p are obtained by simply adding p', p'', and the pressure intensities due to the weight of the reinforced concrete slab, as shown in Table 7. In conclusion, this approximate method is very workable and accessible to the practical engineer. In the ordinary condition, the three-panel solution is simple and gives good accuracy.

Jacob Feld,[17] M. ASCE.—The results obtained from a mathematical analysis are no more accurate than the assumptions used as the basis of the formulas. The author clearly outlines his assumptions. The reader must not expect the result to agree with the actual stresses within and below a foundation unless all the assumed conditions exist in the problem being investigated. For the rigid structures floating in water or in soils of liquid consistency, the method is consistent with the physical conditions.

To extend the method to the problems of soil bearing foundations, piles, and sheet piles is not to be recommended, because of the inconsistency of assumptions and of the physical action of soils. Proportionality between settlement and pressure reaction may exist in nonplastic soils within the lower range of values, but in such soils no tension resistance can be expected. On the other hand, where some tension from adhesion or tackiness is possible, the proportionality of settlement and pressure does not exist.

In the massive foundation slabs used in the author's examples, from 3 ft to 5 ft thick, the K-values of the soil will not be much less near the ends than near the center of the beam. Actually, the edge shear resistance in some soils may result in a greater value near the edge than in the interior areas of a footing.

The results shown in Fig. 8 for piles and sheet piling are contrary to the requirements of statics. The reaction at the bottom of an embedded pile must be in the same direction as the resultant lateral load to satisfy the necessary equality of lateral forces. The assumptions of reactions, as directly proportional to the magnitude of the soil displacement, give the impossible result of a large reaction at the surface of the soil. The more reasonable soil reaction distribution, almost always accepted, is zero at the surface. This example illustrates the necessary precaution against the use of the author's method in actual soil problems.

Victor R. Bergman,[18] Assoc. M. ASCE.—An approximate treatment of a problem which has long attracted the more mathematically-minded engineer is presented in this paper, the noteworthy feature of which is that the author has striven, with considerable success, to reduce the mathematical refinement of analysis to a degree consistent with the effect of errors inevitably introduced by the simplifying assumptions. The latter, as stated by Mr. Levinton, are as follows:

> "(1) The foundation (or soil) is elastic, that is, its settlement at any point is proportional to the pressure at the point.
>
> "(2) The foundation modulus in tension is equal to that in compression. The modulus of the foundation is the elastic settlement resulting from unit pressure, or the elastic uplift that results from a unit tension."

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[17] Cons. Engr., New York, N. Y.

[18] Vice-Pres., Godwin Constr. Co., New York, N. Y.

There is ample evidence in the paper to show that the author comprehends the implications of the assumptions as he has employed them. However, it seems to the writer that the phrasing of the first assumption (and to some extent, the second) is erroneous and misleading in that it implies that the settlement at any point will be proportional to the pressure at the point if the foundation (or soil) is elastic.

The two assumptions might better read simply:

(*a*) The settlement of the foundation (or soil) at any point is proportional to the pressure at the point.

(*b*) The foundation modulus in tension is equal to that in compression. The assumed constant ratio of pressure to settlement is called the foundation modulus—

$$k = \frac{p}{y} \qquad (30)$$

As a matter of fact, even perfect elasticity of the soil would not, in general, produce proportionality between pressure and settlement. The following comments may serve to clarify this matter.

It may be instructive to start with an engineering problem in which the assumption of a constant value of k is clearly justified. Fig. 11 depicts the two rails of a single-track railway resting on a steel floor system whose principal supporting elements are the transverse beams. These transverse beams are uniform in size, length, and spacing, and are supported at their ends by main (longitudinal) girders of such large relative rigidity that the deflections of the girders may be ignored with negligible error in analyzing the distribution of the track load among the transverse beams.

Here, then, is an instance in which the behavior of the foundation (the transverse beams) beneath a continuous footing (the rails) is such that the foundation reaction at any point (at any beam) is directly proportional to the deflection y at that point—that is,

$$p = k\,y \qquad (31)$$

Furthermore, with the rails clamped to the beams, "negative pressures" are possible, with foundation reactions still directly proportional to deflections. It is understood, of course, that the steel stresses remain comfortably below the yield point and, hence, below the proportional and elastic limits.

To obtain such a perfect example of strict proportionality between foundation reactions and deflections, it was necessary to borrow from the bridge engineering field. It is difficult to conceive of a similarly perfect example in foundation engineering design. In fact, regarding this very matter, K. Terzaghi,[19] Hon. M. ASCE, states that

> " * * * in order to establish agreement between assumption and reality, it it would be necessary to replace the soil support of the footing by a bed of equally spaced and equally compressible springs each one of which is independent of the others."

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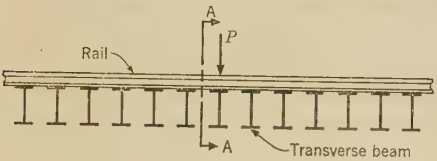
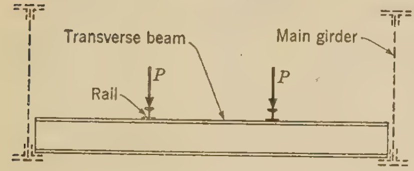
[19] "Theoretical Soil Mechanics," by K. Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 345.

Fig. 12 shows such an imaginary footing subjected to a uniform loading which, of course, produces a uniform settlement y, whether the footing is rigid or flexible.

It should be observed, in both Figs. 11 and 12, that the "foundations" are composed of discrete elements quite separate from, and independent of, each other, with no shearing interaction between elements. This state of zero shearing interaction does not occur in either an actual or an ideal soil subjected even to a uniform loading, if the latter is of finite extent. It does, however, exist in a liquid at rest, since shearing stresses are nonexistent in a motionless fluid. The pressure acting at any point on a body floating in a liquid is, of course, directly proportional to the depth of the point below the free surface of the liquid—that is, to the "settlement" of the point considered. Thus, if footings were supported, not on soil, but by flotation in a very dense liquid, there would be strict compliance with Mr. Levinton's assumption of proportionality between pressure and settlement. An interesting exposition[20] of this idea has been presented elsewhere by Stanley U. Benscoter, Assoc. M. ASCE.



(a) Longitudinal Section



(b) Section A-A

Fig. 11

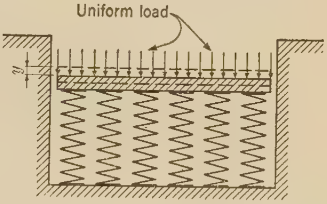


Fig. 12

Consider next (Fig. 13), a perfectly flexible, continuous footing of finite length, supported on a perfectly elastic soil and subjected to a uniform loading of w pounds per square foot. As a necessary concomitant of the complete flexibility of the footing, the contact pressure between footing and soil will also be uniform and equal to w pounds per square foot. However, as the load spreads downward and outward through the soil, the vertical soil pressure along a horizontal line lying parallel to, but at some distance below, the longitudinal axis of the footing, will no longer be uniform, but will instead have the distribution shown in Fig. 13(c). It is evident that a "soil column" under the footing at, for instance, point A, will be subjected to a smaller average vertical stress than one under the center B, and will therefore undergo a smaller compression. The final position and shape (Fig. 13(b)) of the completely flexible footing will be determined by the unequal compressions (settlements) of the various "soil columns" beneath the footing.

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[20] "Floating Block Theory in Structural Analysis," by Stanley U. Benscoter, *Journal*, A.C.I., January, 1946 ,p. 205.

Here, a uniform loading is accompanied by nonuniform deflections, and it is clear that no direct proportionality holds between footing contact pressures and corresponding settlements. Furthermore, this disproportionality would hold true even if actual foundation material (soil as found in nature) were perfectly homogeneous and elastically isotropic.

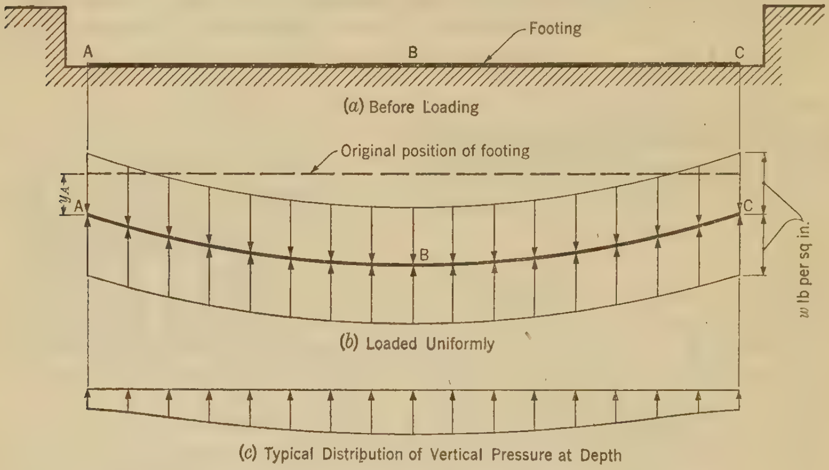
Footing
A
B
C
(a) Before Loading
Original position of footing
y_A
A
C
B
w lb per sq in.
(b) Loaded Uniformly
(c) Typical Distribution of Vertical Pressure at Depth

Fig. 13.—Perfectly Flexible Strip Footing, Uniformly Loaded

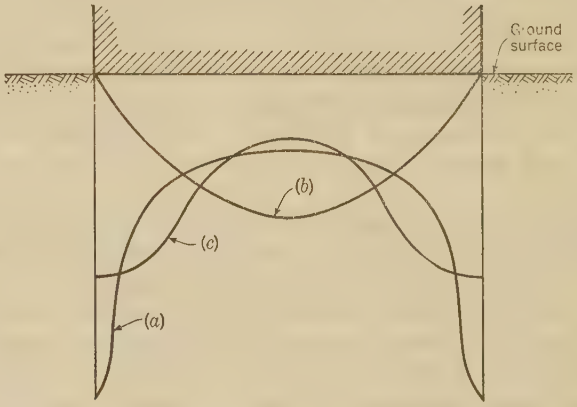
Ground surface
(b)
(c)
(a)

Fig. 14.—Distribution of Contact Pressures on Smooth Bottom of a Rigid Footing, for Three Different Types of Soil

Another limiting condition worthy of consideration is illustrated in Fig. 14, which depicts an extremely rigid footing (such as a bridge pier), resting on compressible soil and subjected to a centric load. Depending on the type of soil, cohesive or cohesionless, and on the depth of footing bottom below the adjacent ground surface, the base pressures will exhibit variations of the general character indicated. In Fig. 14, curve (*a*) is for a real, elastic soil ($\phi = 0$); curve (*b*), for a cohesionless soil ($c = 0$); and curve (*c*), for a soil intermediate between curves (*a*) and (*b*) ($\phi > 0$ and $c > 0$). (The footing is shown resting on the surface of the foundation merely in order to show the pressure variation in accentuated manner. Good practice, it hardly need be stated, dictates sinking the pier into the ground, under which condition the variations in contact pressure would be reduced to a considerable extent.) There may be some disagreement over the precise form of pressure variation, but there is little doubt that some such variations do occur. In contrast to the preceding example, which showed how uniform base pressures are accompanied by nonuniform settlements, here is an instance of uniform settlement (enforced by the rigidity of the footing), coupled with variable base pressure.

The foregoing discussion has been presented to show that the assumption of an elastic foundation does not lead to proportionality of base pressures to settlements. The examples presented show, furthermore, that in general the concept of such proportionality is theoretically unjustifiable. Nevertheless, such an assumption does provide a practical basis for mathematical treatment and consequently is frequently made. The errors thus introduced are minor enough to be acceptable, at least in some lines of investigation. S. Timoshenko states:[21]

> "* * * the simple assumption that the continuous reaction of the foundation is proportional to the deflection is a satisfactory approximation in many practical cases. For instance, in the case of railway tracks, the solution obtained is in good agreement with actual measurements."

Some writers on soil mechanics look with disfavor on the use of so artificial and so theoretically irrational a concept as the "coefficient of subgrade reaction." Nevertheless, what is the harassed structural engineer to do when confronted with a problem of the type treated by the author? He cannot "draw up a chair on a cloud" and discourse with Plato on ideal solutions. He must instead be pragmatist enough to accept and utilize the admittedly imperfect concept—judging its value by the practical consequences.

Professor Terzaghi and R. B. Peck,[22] Assoc. M. ASCE, state that:

> "* * * even a rough evaluation of the distribution of the real contact pressure is very cumbersome. Yet without some knowledge of the contact pressure footings or mats cannot be designed. Therefore, it is customary and necessary to estimate the contact pressure on the basis of simplifying assumptions and to compensate for the error due to these assumptions by an adequate factor of safety."

Mr. Levinton's method, although based on an assumption shown hereinbefore to be only a rough approximation to actual conditions, does provide a useful, and therefore welcome, contribution to footing analysis. For a difficult problem, he has provided a workable, practical method of analysis which takes into account the relative rigidities of footing and foundation bed. Results obtained by following the method should approximate reality more closely than do results from the two commonly adopted lines of attack—one of which treats the footing as merely an inverted, continuous, floor beam, whereas the other adopts the fictions that the footing is entirely rigid and that it produces a trapezoidal distribution of soil pressure. Of these two common methods, the first ignores the possible existence of important distributive bending moments, whereas the second may lead to calculated moments of excessive amount, resulting from failure to take into consideration the relief provided by footing flexibility.

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[21] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., Pt. II, 1941, p. 401.

[22] "Soil Mechanics in Engineering Practice," by K. Terzaghi and R. B. Peck, John Wiley & Sons, Inc., New York, N. Y., 1948, p. 214.

Joseph Gold,[23] Jun. ASCE.—The value of this paper lies in its introduction of a much needed approximate analysis for the design of elastic foundations. Unfortunately, the author's introduction of unnecessarily complicated relative deflections may be a deterrent to the wide application of his method. Solely on the basis of Mr. Levinton's paper, the design engineer is unable to decide whether the usual design procedure is adequate or whether a more accurate analysis of the type suggested is warranted. The author also fails to indicate the accuracy obtained in using a specific panel subdivision and neglects to define, adequately, the conditions under which a three-panel, five-panel, or ten-panel solution is sufficient.

Equations similar to those obtained by the author can be derived in a more direct and elementary manner by the method of finite differences. Another advantage of finite differences lies in the existence of various procedures to increase the accuracy of the solution without substantially increasing the calculations required.

Finite differences transform the solution of a differential equation into the solution of a finite number of simultaneous linear algebraic equations. This transformation is accomplished by expressing, approximately, derivatives of all orders of a given function in terms of function values at a discrete number of points. Thus, the first derivative or slope of y at the points x_i and x_{i+1} of Fig. 15 can be approximated by

$$\left.\frac{\Delta y}{\Delta x}\right|_{x_i} = \frac{y_i - y_{i-1}}{\Delta x};\ \left.\frac{\Delta y}{\Delta x}\right|_{x_{1+i}} = \frac{y_{i+1} - y_i}{\Delta x} \qquad \text{(32a)}$$

The second derivative or curvature of y at x_i is the rate of change of the first derivative and hence is approximated by

$$\left.\frac{\Delta^2 y}{\Delta x^2}\right|_{x_i} = \frac{\left.\frac{\Delta y}{\Delta x}\right|_{i+1} - \left.\frac{\Delta y}{\Delta x}\right|_{i}}{\Delta x} = \frac{y_{i+1} - 2\,y_i + y_{i-1}}{\Delta x^2} \qquad \text{(32b)}$$

Eq. 32b is the "first-order" central difference operator corresponding to the differential operator $\left.\frac{d^2 y}{dx^2}\right|_{x_i}$. The differential equation governing beam deflections is

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[23] Asst. Chf. Structural Engr., Tumarkin & Emerman, Cons. Engrs., New York, N. Y.

$$M = E\,I\,\frac{d^2y}{dx^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

in which the moments that produce tension in the upper fibers of a horizontal beam are considered positive, and the deflections downward are positive. The basic assumption of elastic foundation analysis is manifested by the equation:

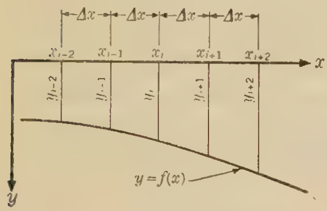

FIG. 15.—DEFLECTION DIAGRAM

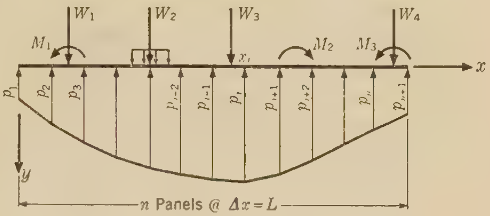

FIG. 16.—*n* PANEL SOLUTION

$$y = k\,p\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34)$$

in which foundation pressures are positive when they are compressions. Values of k, the modulus of the foundation,[24] have been tabulated by the Portland Cement Association, for various soils. For a beam on an elastic foundation Eq. 33 becomes

$$M = k\,E\,I\,\frac{d^2p}{dx^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(35)$$

If the beam (Fig. 16) is subdivided into n equal segments (or panels) of length Δx, and if $\frac{d^2p}{dx^2}$ is replaced by its equivalent finite difference operator

$$\left.\frac{\Delta^2 p}{\Delta x^2}\right|_{x_i} = \frac{p_{i+1} - 2\,p_i + p_{i-1}}{\Delta x^2}\ldots\ldots\ldots\ldots\ldots\ldots(36)$$

then, at the $n - 1$ intermediate points of subdivision, Eq. 35 yields

$$M_{xi} = \frac{k\,E\,I}{\Delta x^2}\,(p_{i+1} - 2\,p_i + p_{i-1})\ldots\ldots\ldots\ldots\ldots(37)$$

for which condition $i = 2, 3, \cdots, n$ and M_{xi} is the moment at the point x_i of the external loads and of the resulting unknown pressures p. It is ascertained that there are $n + 1$ unknown pressures to be determined when the beam is divided into n panels. The remaining two simultaneous equations required for the determination of the pressures p_i are the equations of equilibrium of vertical forces and of moments. Thus, $n + 1$ simultaneous equations for the $n + 1$ unknown pressures can be easily written.

Applying finite differences to the solution of Example 2 (see Figs. 5 and 17), the numerical value of $\frac{k\,E\,I}{\Delta x^2}$ required is $\frac{0.01 \times 432{,}000 \times 5.33}{12^2} = 160$. Symmetry of the loading leaves only three unknown pressures p_1, p_2, and p_3.

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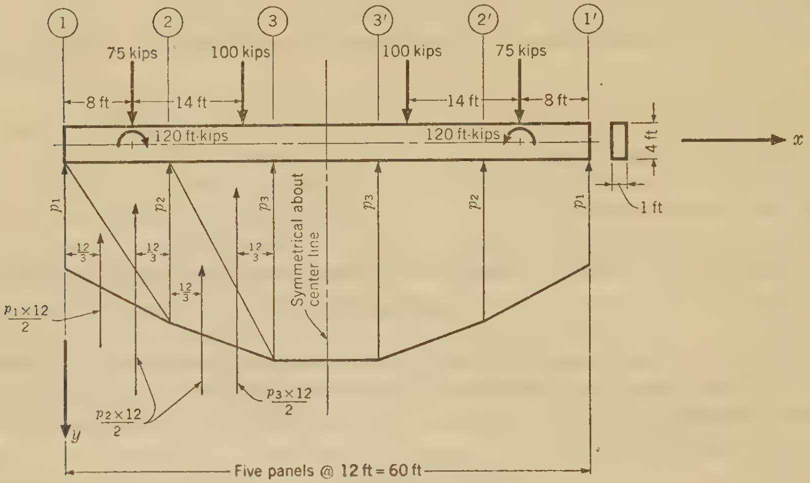
[24] "Concrete Pavement Design," Portland Cement Assn., Chicago, Ill., 1946.

The three equations required are obtained by using Eq. 37 at points 2 and 3, and by setting the sum of the vertical forces equal to zero. Thus, at point 2,

$$\left.\begin{aligned} &M_2 = 75 \times 4 - 120 - p_1 \times \frac{12}{2} \times \frac{2}{3} \times 12 - p_2 \times \frac{12}{2} \times \frac{12}{3} \\ \text{and} \qquad &M_2 = 160\,(p_3 - 2\,p_2 + p_1) \end{aligned}\right\} \ldots (38a)$$



1
2
3
3′
2′
1′
75 kips
100 kips
100 kips
75 kips
8 ft
14 ft
14 ft
8 ft
120 ft-kips
120 ft-kips
x
4 ft
1 ft
p1
p2
p3
p3
p2
p1
12/3
12/3
12/3
12/3
Symmetrical about center line
p1×12/2
p2×12/2
p3×12/2
y
Five panels @ 12 ft = 60 ft

Fig. 17.—Symmetrical Loading, Five-Panel Solution

at point 3,

$$\left.\begin{aligned} M_3 = {} & 75 \times 16 + 100 \times 2 - 120 - p_3 \times \frac{12}{2} \times \frac{12}{3} - p_2 \times \frac{12}{2} \times \frac{2}{3} \times 12 \\ & - p_2 \times \frac{12}{2}\left(12 + \frac{12}{3}\right) - p_1 \times \frac{12}{2}\left(12 + \frac{2}{3}\,12\right) \\ \text{and} \qquad M_3 = {} & 160\,(p_3 - 2\,p_3 + p_2) = 160\,(p_2 - p_3) \end{aligned}\right\} \ldots (38b)$$

and, by equilibrium of vertical forces,

$$75 + 100 + 100 + 75 = 2\left(\frac{p_1 + p_2}{2} \times 12 + \frac{p_2 + p_3}{2} \times 12 + p_3 \times 6\right) . (38c)$$

After simplification, this system becomes

$$\left.\begin{aligned} 208\,p_1 - 296\,p_2 + 160\,p_3 &= 180 \\ 120\,p_1 + 304\,p_2 - 136\,p_3 &= 1{,}280 \\ 12\,p_1 + 24\,p_2 + 24\,p_3 &= 350 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (39)$$

and its roots—$p_1 = 3.807$, $p_2 = 5.791$, and $p_3 = 6.890$—agree fairly closely with the author's values—$p_1 = 3.930$, $p_2 = 5.820$, and $p_3 = 6.800$—all in kips per sq ft. The positive values of p_1, p_2, and p_3 indicate that the pressures are all compressions, and that the deflections of the beam are all downward.

Knowing the pressures, the moments at points 2 and 3, $M_2 = -142$ ft-kips and $M_3 = -176$ ft-kips, are obtained by substitution in Eqs. 38*a* and 38*b*.

The author states in his conclusions, "The remaining work is simply routine solution of simultaneous equations, with various degrees of accuracy, as the case may require." In order to establish the factors determining the accuracy of the solution of a specific problem, a few pertinent remarks are in order. The conventional method for the design of concrete foundations on soil or piles is based on the premise that these foundations are infinitely rigid and do not settle unequally under load. In many cases this conventional analysis yields results that are within the accuracy of the determination of the physical properties and loadings of the structure, and a more refined analysis is not justified. A dimensionless parameter, λ, governing problems of this type is defined by the equation:[25,26]

$$\lambda = \sqrt[4]{\frac{1}{4\,k\,E\,I}}\,L \qquad (40)$$

in which L denotes the length of the beam. The value $\lambda = 0$ corresponds to the conventional assumptions of infinite rigidity. Large values of λ are possible as a result of flexible beams or stiff foundation supports (small k).

To estimate the accuracy of the finite difference method, the two-panel, three-panel, four-panel, and five-panel solutions are compared with the theoretical solution[25] for the beam with a concentrated load at its midpoint. The conclusions reached are also valid for the author's method of redundant reactions in the light of the close agreement between the two methods in the solution of Example 2 and other problems not included in the discussion. This discussion is limited to values of λ between 0 and 6, since a great variety of concrete foundations falls within this range. To maintain the desired dimensionless characteristic, ratios of the maximum pressure (p_{cl}) and maximum moment (M_{cl}) to the conventional values obtained with $I = \infty$, are plotted in Fig. 18. Referring to the ordinate captions of Fig. 18, the conventional values $(M_c)_{cl}$ and $(p_c)_{cl}$ (that is, the maximum center-line values) are, respectively, $\frac{W}{L}$ and $\frac{W\,L}{8}$. Examination of these curves clearly shows that for small values of λ the two-panel solution ($n = 2$) yields results that are close to the theoretical values. For larger values of λ, a greater number of panels is required to approximate the deflected shape and yield reasonably accurate results. As was to be expected, in general, the larger the number of panels, the greater will be the accuracy. As a result of the experience gained in this simple problem, the recommendations contained in Table 8 are made for the usual design problems.

A more refined derivation[27] of the various finite difference operators shows that in Eq. 36, only the first term of an infinite series for the finite difference operator corresponding to $\frac{d^2p}{dx^2}$ appears. Using two terms of this infinite series

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[25] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 1944 Pt. II, pp. 1–24.

[26] "A Symmetrically Loaded Base Slab on an Elastic Foundation," by S. U. Benscoter, *Transactions*, ASCE, Vol. 109, 1944, pp. 763–776.

[27] "Central Difference Formulae," by W. F. Sheppard, *Proceedings*, London Mathematical Soc., Vol. 31, 1899, p. 460.

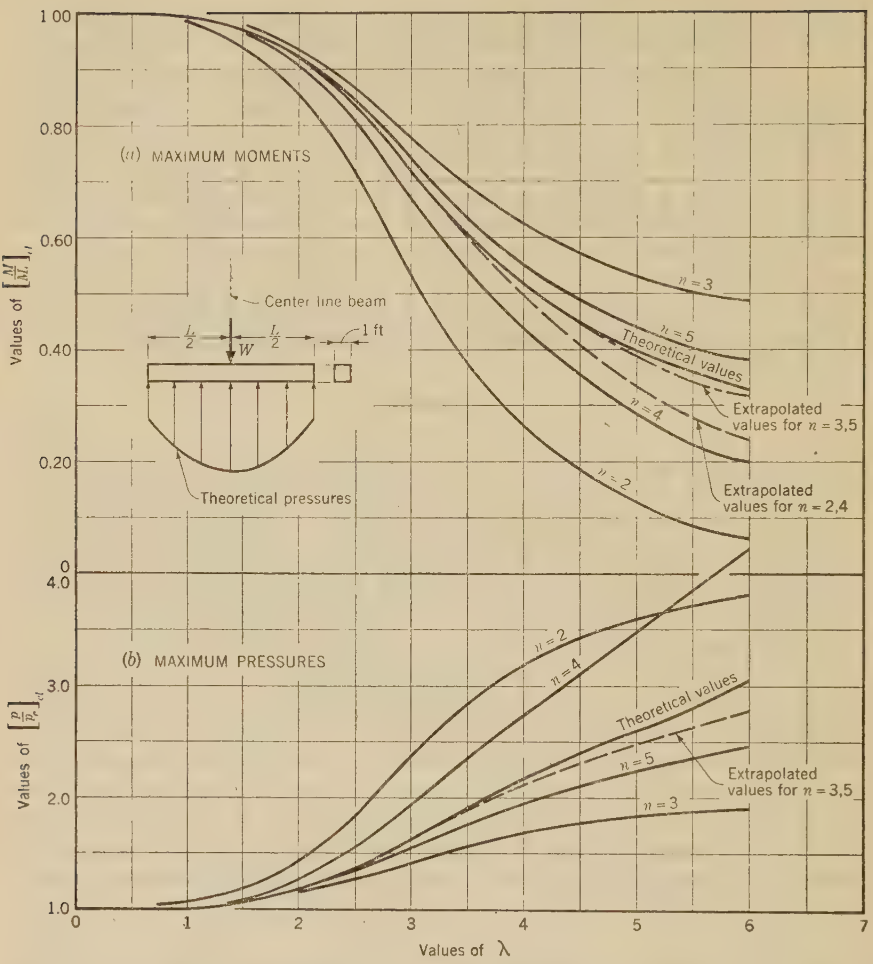

FIG. 18.—COMPARISON OF MAXIMUM PRESSURES AND MOMENTS

TABLE 8.—RECOMMENDED NUMBER OF PANELS FOR AN AVERAGE ACCURACY OF ± 10% IN THE DETERMINATIONS OF PRESSURES, SHEARS, AND MOMENTS

Recommended:	RANGE OF λ-VALUES					
	0 to 1	1 to 1.5	1.5 to 2	2 to 3	3 to 4.5	4.5 to 5
Number of panels	Conventional[a]	2	3	4	5	6

[a] Conventional method is sufficient.

results in a more accurate expression for $\frac{d^2p}{dx^2}$ depending on five values of p_i in comparison to the three values appearing in Eq. 36. Using this "second-order" central difference operator, Eq. 37 becomes

$$M_{x_i} = \frac{k\,E\,I}{\Delta x^2}\left(-\frac{1}{12}\,p_{i+2} + \frac{4}{3}\,p_{i+1} - \frac{5}{2}\,p_i + \frac{4}{3}\,p_{i-1} - \frac{1}{12}\,p_{i-2}\right)\ldots(41)$$

for which condition $i = 2, 3, \cdots, n$. Applying the second-order operators to Example 2 leaves Eq. 38*c*, representing vertical force equilibrium and the first part of Eqs. 38*a* and 38*b*, representing the moments of the external loads and unknown pressures p, unchanged. The only changes resulting are at point 2:

$$M_2 = 160\left(-\frac{1}{12}\,p_3 + \frac{4}{3}\,p_3 - \frac{5}{2}\,p_2 + \frac{4}{3}\,p_1\right)\ldots\ldots\ldots(42a)$$

and, at point 3:

$$M_3 = 160\left(-\frac{1}{12}\,p_2 + \frac{4}{3}\,p_3 - \frac{5}{2}\,p_3 + \frac{4}{3}\,p_2 - \frac{1}{12}\,p_1\right)\ldots\ldots(42b)$$

Simplification yields the system:

$$\left.\begin{array}{rcr} 261.33\,p_1 - 376\,p_2 + 200\,p_3 & = & 180 \\ 106.67\,p_1 + 344\,p_2 - 162.67\,p_3 & = & 1{,}280 \\ 12\,p_1 + 24\,p_2 + 24\,p_3 & = & 350 \end{array}\right\}\ldots\ldots\ldots\ldots(43)$$

and its roots in kips per square feet—$p_1 = 3.774$, $p_2 = 5.808$, and $p_3 = 6.888$ (which are within 1% of the previously computed values)—are more accurate. It is interesting to note that the value of λ for this beam is $\sqrt[4]{\frac{1}{4 \times 0.01 \times 432{,}000 \times 5.33}} \times 60 = 3.44$, and that the five-panel solution which was used is the one recommended in Table 8. As a result of the increased accuracy inherent in the use of second-order operators, the number of panels required to attain a certain accuracy is smaller than that necessitated by the use of first-order operators. L. Collatz[28] and M. G. Salvadori,[29] Assoc. M. ASCE, have shown the advantages of using the more accurate difference operators in the solutions of various linear boundary value problems.

The error introduced by using the various ordered central difference operators in place of the differential operators has been evaluated[30,31] and an extrapolation procedure has been established and used successfully elsewhere.[29,30,31,32] Generally, at least three approximations are calculated (for example, $n = 2, 3, 4$), and the extrapolation formulas developed are applied to those approximations forming a monotonic sequence. The dotted curves shown in Fig. 18 represent the extrapolated values for the odd and even approximations. The great increase in accuracy inherent in the extrapolation procedure, as compared with that suggested by Mr. Levinton, is obvious upon examination of these curves.

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[28] "Zusammendfassender Bericht über die genäherte Berechnung from Eigenverte," by L. Collatz, *Zeitschrift*, Angewandte Mathematik und Mechanik, Vol. 19, 1939, pp. 224–249 and pp. 297–318.

[29] "The Numerical Evaluation of Buckling Loads by Finite Differences," by M. G. Salvadori, paper presented at the Annual Meeting of the ASCE, New York, N. Y., January, 1948.

[30] "The Approximate Arithmetical Solution by Finite Differences of Physical Problems Involving Differential Equations," by L. F. Richardson, *Philosophical Transactions*, Royal Soc. of London, Vol. 210, 1911, pp. 307–357.

[31] "On the Evaluation of Characteristic Values by Finite Differences and Richardson's Extrapolations," by M. G. Salvadori, paper to be presented at the 7th International Cong. of Applied Mechanics, London, England, September, 1948.

[32] "Buckling Loads for Built-In Rectangular Plates by Finite Differences," by J. Gold, thesis presented to Columbia University at New York, N. Y., in June, 1947, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

B. LEVINE,[33] Assoc. M. ASCE.—In applying the author's method to a number of actual design problems, the writer has observed its simplicity and time saving features as compared with exact methods of analysis using differential equations. The following data will enable a designer to establish, quickly, the conditions under which it is advantageous to use the author's method.

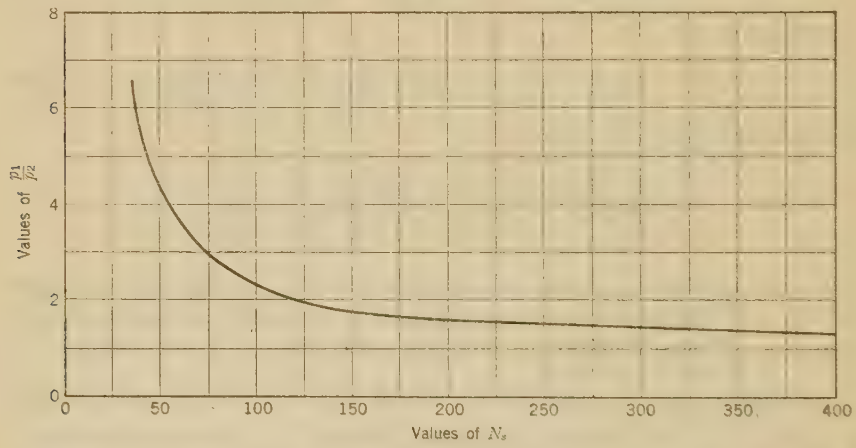

FIG. 19.—RELATION BETWEEN N_s AND $\frac{p_1}{p_2}$ FOR SYMMETRICAL LOADING; THREE-PANEL SOLUTION

Table 9 indicates a foundation slab loaded symmetrically and gives the author's solution for five different types of soils. The soils vary from a gravel-sand mixture having a modulus of subgrade reaction,[34] K, of 500 lb per sq in. per in. to a silt having a subgrade reaction modulus of 100 lb per sq in. per in. The corresponding values of k, defined by the author as "modulus of foundation material" are obtained by dividing the number $0.579 = \frac{1,000}{1,728}$ by the subgrade reaction modulus (introduced by H. M. Westergaard,[34] M. ASCE).

It is interesting to note the relation between the value of N_s and the ratio p_1/p_2. For case 1 (gravel-sand mixtures) the value of $N_s = 39.5$ and $p_1/p_2 = 6.35$. For case 5 ("silt") the value of $N_s = 196$ and $p_1/p_2 = 1.63$. In general, the larger the number N_s, the nearer does the pressure diagram approach the condition of uniform soil pressure under the entire length of the foundation beam.

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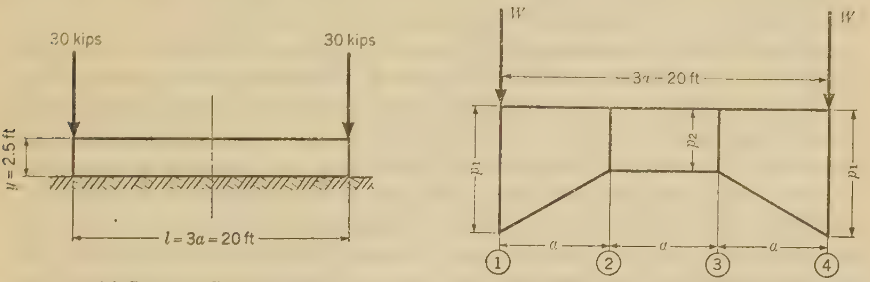
[33] Designing Engr., Knappen Tippetts Abbett Eng. Co., New York, N. Y.

A second item of interest to the designer is the value of p_1 as compared with the conventional soil pressure p. For case 1, $p_1 = 6.84$ kips per sq ft as compared with $p = 3$ kips per sq ft. In general, the smaller the number N_s, the greater will be the soil pressure under the ends of the foundation beam as compared with the soil pressure under the middle.

TABLE 9.—THREE-PANEL SOLUTION FOR FIVE TYPES OF SOIL; SYMMETRICAL LOADING



(a) LOADING DIAGRAM

(b) TYPICAL SOIL PRESSURE DIAGRAM

Symbol	Definition	Quantity
W	Applied loads, in kips	60
a	Segment of span length, in feet	6.67
E	Modulus of Elasticity—	
	Pounds per square inch	3,000,000
	Kips per square foot	432,000
I	Moment of inertia, $\frac{1 \times 2.5^3}{12}$ ft^4	1.30
b	Breadth of slab (unity)	1
p	Conventional soil pressure, W/l, in kips per square foot	3
l	Span length, 3 a, in feet	20
M_A	Conventional moment at center line of beam (point A) in foot-kips	150
M'_A	Moment at center line of beam (point A) as computed by three-panel solution, in foot-kips	...
K	Modulus of subgrade reaction	...
k	Modulus of foundation material	...
N_s	Substitution factor defined by Eq. 8	...
Y	Depth of foundation beam, in feet	2.5

Case No.	Type of soil	K	k	N_s	p_1	p_2	p_1/p_2	M'_A	M'_A/M_A
1	Gravel-sand mixtures	500	0.00116	39.5	6.84	1.08	6.35	97	0.64
2	Gravel-sand-clay mixtures	400	0.00145	49.4	6.28	1.36	4.6	104	0.69
3	Sand-clay mixtures	300	0.00193	65.6	5.64	1.68	3.35	112	0.75
4	Fine sand	200	0.00290	98.6	4.9	2.05	2.39	124	0.83
5	Silt	100	0.00579	196.0	4.04	2.48	1.63	136	0.91

A third item of interest is the reduction in maximum moment at the center of the foundation beam obtained by using the author's solution as compared with the moment obtained by using the conventional method. For case 1 the value of M_A is 64% of the value of M. For cases 1 and 2, this reduction in moment will justify using a foundation beam 2 ft deep, instead of 2.5 ft deep as obtained by conventional analysis.

The curve in Fig. 19 indicates the relation between the number N_s, by Eq. 8, and the ratio p_1/p_2. This curve is obtained as follows: For symmetrical loading the author gives Eq. 7*a*. For the loading shown in the diagrams of Table 9, the term y_{b2}, Eq. 7*a*, becomes zero and that equation may be written:

$$(19 - N_s)\, p_1 + (91 + N_s)\, p_2 = 0 \qquad (44a)$$

or

$$\frac{p_1}{p_2} = \frac{N_s + 91}{N_s - 19} \qquad (44b)$$

The value of N_s may be computed for any particular design problem by substituting the proper values in Eq. 8. Then, by using Fig. 19, the corresponding value of p_1/p_2 may be obtained from the curve.

A study of Fig. 19 indicates that in general a value of N_s of 200 or more corresponds to a foundation condition in which the beam is relatively stiff as compared to the soil. For this condition the conventional design method may be used, since the soil pressure diagram and the value of maximum moment are practically the same as those obtained by the author's solutions. However, for values of N_s less than 200, the three-panel solution enables the designer to visualize clearly (*a*) the distribution and intensities of soil pressures under the foundation beam, and (*b*) the maximum value of moment at the center of the foundation beam and the possibility of reducing the depth of the beam as compared with the depth required by conventional analysis.

The numerical work involved in obtaining the simple beam deflections resulting from loads and moments is quite appreciable; and this work could be reduced considerably by the use of influence lines.

Zusse Levinton,[34] M. ASCE.—The theoretical basis of the problem of a beam on elastic foundation has been fully developed by the proponents of the various rigorous methods of the theory of elasticity, and the statement of theoretical principles in this paper was necessarily brief. Unfortunately, this brevity has led to some misunderstandings; but, on the other hand, it served to stimulate discussion and round out the subject for the benefit of the interested reader.

Because the writer has introduced a novel method of solution, this part of the paper was treated more fully and has attracted comments on improvements and variations of the method.

Mr. Mensch proposes to improve the method by assuming a parabolic line of deformation. In the case of symmetrical loading with a simple convex or concave "line of deformation," the results will be close to those obtained by the writer's method, as illustrated in Mr. Mensch's example. However, in cases of more complicated deformations with reversed curves or sinuous curves (such as in a "ribbon foundation" with a row of columns), the parabolic assumption would be far from true. The writer's five-panel solution would more closely approximate the conditions.

Mr. Mensch also proposes that a reduction in E be made to account for the flow of concrete in long continued loading. This is justified, provided similar consideration is given to the foundation modulus k. The writer believes that an increase in the effective value of k will compensate to some extent for the decrease in E.

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[34] Chf. Structural Engr., Knappen Tippitts Abbett Eng. Co., New York, N. Y.

Professor Popov points out that the usual theoretical treatments of the problem are based on the Winkler hypothesis, and gives references to the literature on this subject. In 1939, M. I. Gorbunov-Posadov developed the subject along the lines of an elastic semiplane continuum in which the effect of a foundation beyond the ends of the beam is taken into consideration. He confined the analysis to two dimensions,[35] and also extended it to three dimensions, to account for the effect of the foundation along the sides of the beam.[36]

The writer's method is based on the Winkler hypothesis, as long as k is considered to be constant along the beam. However, when variation of k along the beam is considered, the method in effect departs from the Winkler method and is an approximation to the case of a continuum. The writer believes that a judicious assumption of a variable k in his method, in effect, accomplishes the same result as do the complicated theories of semiplane and semi-space continuums.

By using the term "practical engineer" the writer does not wish to imply that there are two kinds of engineers, "practical" and "theoretical." An engineer becomes "practical" when under pressure of work he is compelled to seek quicker and simpler solutions to his problem; and he uses them, with all due respect for other established methods and theories. The trigonometric series method[12] is one of such practical methods. The writer believes, however, that his method is simpler. Furthermore, the trigonometric series method does not seem to be applicable for the case of variable k.

Mr. Yao defines the limits of applicability of the writer's method and correctly states its advantages. The writer agrees with Mr. Yao that dividing the nonsymmetrical loading into a symmetrical loading and an antisymmetrical loading simplifies the computation. This simplification can be used except in the case of nonsymmetrical variation of k, such as in the vertical pile problem.

The writer agrees with Mr. Feld's statement that "The results obtained from a mathematical analysis are no more accurate than the assumptions used as the basis of the formulas." This principle should be extended to the conventional methods of analysis, as well as to new ones. Of the conventional methods, one is based on the fiction of perfect rigidity of the beam and the other regards the structure as an inverted frame, and the physical properties of the soil are completely disregarded.

The writer's brevity regarding "tension" in the foundation and its "modulus in tension" is probably responsible for Mr. Feld's erroneous interpretation of tension as implying adhesion or "tackiness." In the writer's method, as well as in the "rigorous" methods, based on the theory of elasticity, "tension" of the foundation is regarded differently. It is implied that upward deflections have the same modulus as downward deflections. In properly designed flexible footings, the soil settlement that results from direct compression is greater than the relieving effect of bending, and the ends of the beam do not tend to rise and "pull" on the soil. It is not important, then, whether the soil has, or has not, adhesion or "tackiness" since this property is not mobilized. The condition is similar to the case of loading a concrete block with an eccentric load. If the stress due to direct compression is greater than the extreme fiber stress due to bending, the tensile strength of the concrete is not mobilized and it is immaterial whether the concrete has it at all. In computing the extreme fiber stress the modulus in tension is assumed to be equal to the modulus in compression. A case in which "tension" is expressed as resistance to upward pull on friction piles is mentioned in the paper.

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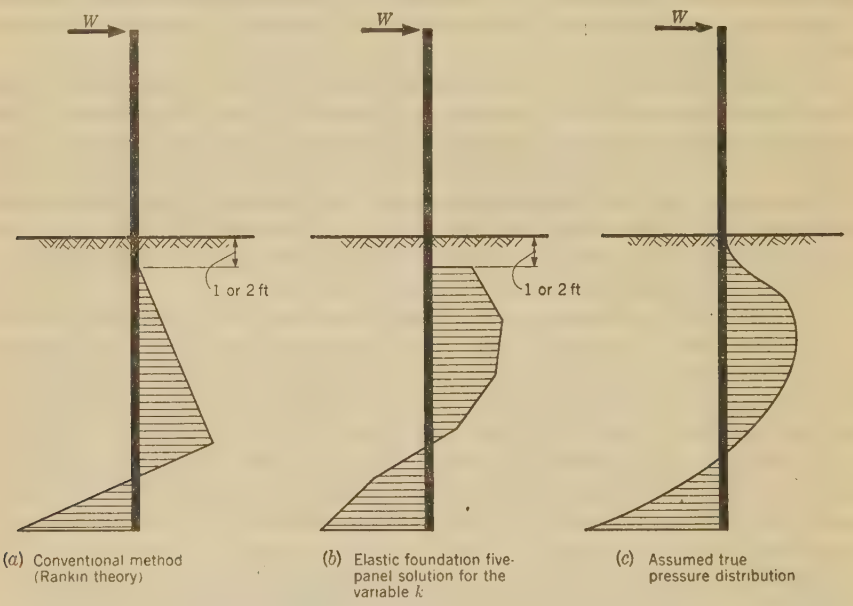
[35] "Tables for Beams on Elastic Foundation," by M. I. Gorbunov-Posadov, Gosstroisdat, Moscow, 1939.

[36] "Calculation of an Infinite Beam Resting on Elastic Semi-Space," by M. I. Gorbunov-Posadov, *Engineering Review*, U.S.S.R. Academy of Science, Vol. III, No. 2, 1947.

In the case of vertical sheet piling, the assumption of "tension" simply means that resistance to soil compression to the left of the piling (for negative deflections) is equal to that to the right of the piling, with the additional assumption that the modulus on either side varies with the depth.



(a) Conventional method (Rankin theory)

(b) Elastic foundation five-panel solution for the variable k

(c) Assumed true pressure distribution

FIG. 20.—COMPARISON OF PRESSURE DIAGRAMS, VERTICAL PILE WITH LATERAL LOADING

Mr. Feld interprets Fig. 8 as showing results contrary to statics. The writer did not intend Fig. 8 to show any results, it merely indicates in which direction the values of p are to be considered as positive in setting up the equations. When the results of an actual problem are discovered, the reactions at the bottom of the pile prove to be negative—that is, in the same direction as the resultant lateral load, as required by statics. In case of excessive depth of embedment, the pressure diagram crosses the vertical line twice and the reaction at the bottom becomes positive—results which are similar to those obtained by conventional methods of analysis.

The writer agrees with Mr. Feld that the soil reaction at the surface of embedded piling should be zero. To accomplish this, the virtual surface of the "equivalent beam" in Fig. 8 is to be taken 1 ft or 2 ft below the surface. Fig. 20 shows a comparison of pressure diagrams obtained by the conventional method

(Fig. 20(*a*)) and by the writer's method (Fig. 20(*b*)), as well as what is likely to be the true picture of soil reactions (Fig. 20(*c*)).

Mr. Bergman makes a restatement of the basic assumptions of the method, which avoids confusion and misunderstanding of the term "elastic." This term originally denoted both the ability to recover shape on removal of the load and the ability to deform in proportion to the applied load. When there is danger of confusing the two properties, a distinction in terminology must be made. Thus, a distinction is made between "elastic limit" and "proportional limit" in testing materials. The term "elastic" lends itself as an adjective to any noun, such as "elastic foundation," "elastic medium," etc. On the other hand, the semantics of the English language does not permit one to say "proportional foundation" or "proportional medium." Mr. Bergman avoids adjectives using nouns "elasticity" and "proportionality" instead, and it is to be noted that the term "elasticity" can stand alone, whereas the other requires an entire phrase: "Proportionality between stress and deformation."

In this paper, "elastic foundation" refers to a foundation that has a settlement at any point proportional to the pressure at the point; and it is not implied that the soil must also have the property of complete recovery on removal of the load. The constant of proportionality k may be assumed to be the same for every point, or it may be assumed to vary along the beam.

Mr. Bergman's references to authoritative statements from the literature on elastic foundations and soil mechanics[21,22] seem to confirm the writer's assertion that an approximate method based on simplified assumptions is desirable and useful.

Mr. Gold considers that the use of the simple beam deflections (y_b) in the absolute terms of the equations is unnecessarily complicated and may be a deterrent to the wide application of the method. He proceeds to develop similar equations by the calculus of finite differences and introduces the beam deflections in another form (Eqs. 33 to 37). In the final analysis, Mr. Gold's method is another variation of the same method of redundants, with equations for each redundant, together with the equations of statics. The difference is in form only. The writer leaves the beam deflection y_b to the individual computer, to be determined by the method most familiar to him (integration, moment area, or a ready formula from a handbook). Mr. Gold expresses the beam deflections in terms of moments taken at every panel point.

It is difficult to agree with Mr. Gold that leaving the choice to the individual would be a deterrent in the wide application of the method. The writer does not agree with Mr. Gold that the calculus of finite differences is "more direct and elementary" compared with the simple algebraic addition of deflections shown in Fig. 1.

With regard to accuracy of the method, Fig. 7 is intended to illustrate the close agreement with the rigorous method of analysis. By scaling the ordinates in Fig. 7, the percentage differences can be established, if desired. Mr. Gold develops the subject further. The curves in Fig. 18 apply to the case of a single central load on the beam, and Table 8 could be used for this case only. However, in the writer's opinion, the general recommendation for an accuracy of

±10% may be misleading. The assumed "accuracy" would apply with respect to "theoretical values," presumably derived by the "rigorous method" of the theory of elasticity, based on the Winkler hypothesis. Other, equally rigorous, methods based on semiplane and semispace continuums give values diverging more than 10% among themselves and from the Winkler hypothesis. Under conditions in which the theoretical yardstick itself is uncertain, it is futile to think about 10% "accuracy."

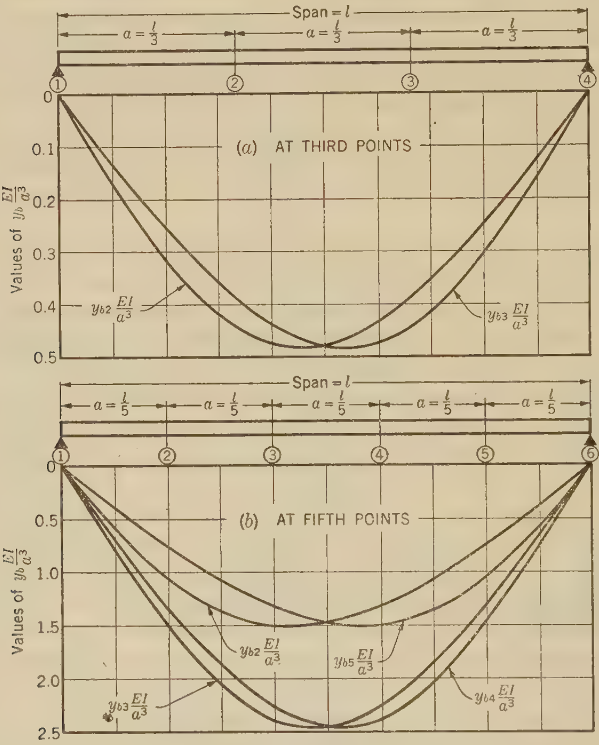


FIG. 21.—INFLUENCE LINES FOR SIMPLE BEAM DEFLECTIONS UNDER A UNIT APPLIED LOAD

The writer recommends that the approximate method of redundants be considered as "accurate" as the "rigorous" methods. When the nature of the loading indicates that the deformed beam has a simple convex or concave curvature, the three-panel solution is sufficient. When a reversal of a curve or a sinuous curve is to be expected, the five-panel solution will give a more desirable approximation. In case of doubt, the five-panel solution should be used; the labor involved is not great in either case.

The ten-panel solution will scarcely find any application in ordinary foundation problems. In the writer's experience, the ten-panel solution was used for long flexible floating dry docks. Under certain conditions of loading the deflection curve is sinuous, and with the ten panels it was possible to detect all positive and negative moments along the structure. With water as the foundation material, the theoretical assumptions are fully realized, and the precision of the solution is not illusory.

Mr. Levine discusses the application of the method to a problem with two symmetrical concentrated loads at the ends of the beam. (A number of structures may fall into this group, such as subway inverts, dry docks, locks, reservoirs, etc.) For this loading condition $y_b = 0$. Mr. Levine suggests that for other cases, where y_b must be evaluated, the work could be reduced by the use of influence lines.

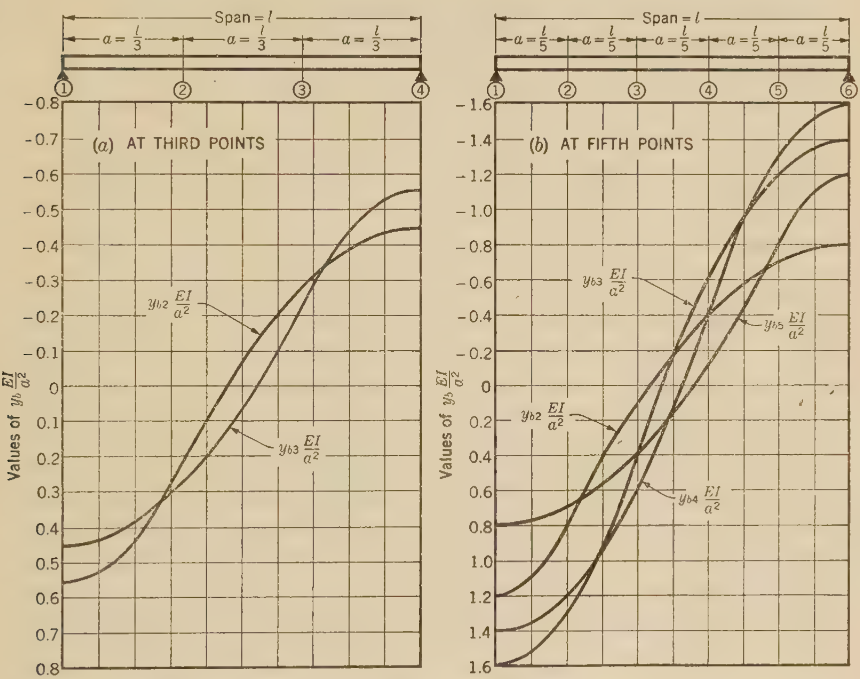


FIG. 22.—INFLUENCE LINES FOR SIMPLE BEAM DEFLECTIONS UNDER A UNIT APPLIED COUPLE

The writer has used influence lines for deflections of simple beams and, in Figs. 21 and 22, has constructed special influence lines that can be used with the three-panel and five-panel solutions. To illustrate their use: In Example 1 (Fig. 3), $m_1 = 15\text{ ft} = 0.25\,l$. For this position, Fig. 21(a) gives $y_{b2}\frac{E\,I}{a^3} = 0.37$ for a unit load, and for the position of the second load ($m_2 = 35\text{ ft} = 0.583\,l$) $y_{b2}\frac{E\,I}{a^3} = 0.45$. These values must be multiplied each by the load and by $\frac{1{,}080}{a}$; thus: $[(10^k \times 0.370) + (15^k \times 0.45)]\frac{1{,}080}{20} = 564$. For the 30-ft-kip moment ($m_1 = 0.25\,l$), Fig. 22(a) gives $y_{b2}\frac{E\,I}{a^2} = 0.36$. This value is multiplied by M and by $\frac{1,080}{a^2}$; thus: $(30 \times 0.36)\ \frac{1,080}{20^2} = 29$. Adding the two, $564 + 29 = 593$, which checks with 591.6 obtained by using standard formulas for beam deflections. By a similar procedure the value of $1,080\ y_{b3} \frac{E\,I}{a^4}$ can be determined by using the curve of $y_{b3} \frac{E\,I}{a^3}$ for unit loads, and the curve of $y_{b3} \frac{E\,I}{a^2}$ for unit moments.

In conclusion, the writer wishes to thank all participants for their interesting and valuable contributions to the discussion.

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TRANSACTIONS

Paper No. 2363

OPERATION OF THE CONOWINGO HYDROELECTRIC PLANT

BY ROBERT E. TURNER,[1] M. ASCE

WITH DISCUSSION BY MESSRS. JOEL D. JUSTIN, E. T. SCHULEEN, GORDON R. WILLIAMS, JOHN S. COTTON, AND ROBERT E. TURNER.

SYNOPSIS

Large run-of-river hydroelectric (or "hydro") developments on rivers with extremely wide variations in flow can only be justified on a sound economic basis provided such plants can be operated in conjunction with a large steam generating system so as to attain the greatest possible economic use of the hydroelectric power. By reducing the steam generating costs to a minimum, over-all economies can be effected to maximum advantage for the entire system. Allocation of hydroelectric energy to the best advantage requires determinative river flow forecasting, proper use of pondage, maintenance of head, and station efficiency so that maximum capacity and maximum energy can be obtained from the available water to replace the most expensive steam generation. This paper presents in detail the methods by which the various factors are analyzed to determine the most effective operation sequences for a large hydroelectric installation.

Full coordination of hydro and steam capacity in electric power systems has been expanded through interconnected lines on a regional basis to assure fullest utilization of available generating capacity for carrying increased loads, in scheduling maintenance, in meeting emergency outages of equipment, as well as for obtaining economies of generation. Hydro and steam power, rather than being competitive, are actually complementary and many advantages accrue when their use is coordinated in electric power systems.

CONOWINGO HYDROELECTRIC DEVELOPMENT

The Conowingo hydroelectric plant of the Philadelphia (Pa.) Electric System was placed in operation on March 1, 1928. It is located in Maryland on the Susquehanna River about 4 miles above tidewater and approximately

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NOTE.—Published in November, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Hydrographer, The Susquehanna Elec. Co., Conowingo, Md.

60 miles from Philadelphia. Each of the seven main generating units installed in the plant consists of a vertical shaft, Francis-type turbine of 54,000-hp capacity, directly connected to a 36,000-kw generator. Energy from the generators is fed to four 80,000-kva, 13.8/220-kv transformer banks, which supply the 220-kv switching station on the roof of the power plant, in turn, supplying two 220-kv circuits.

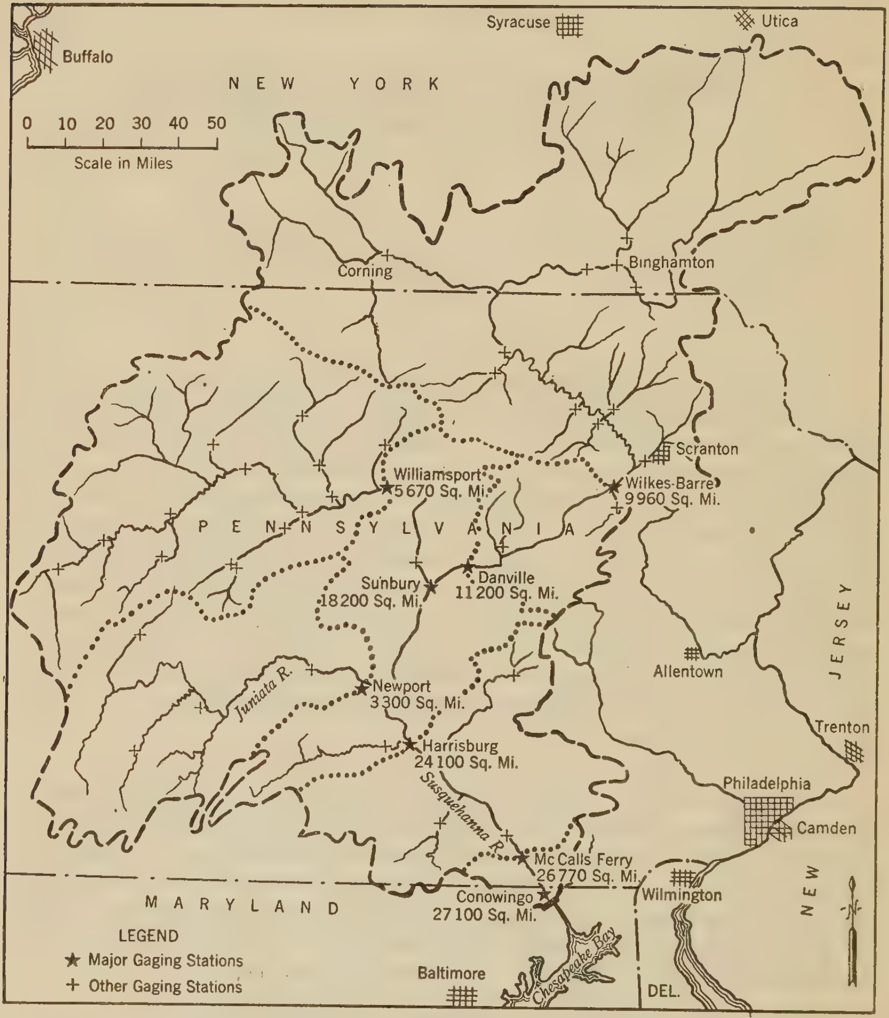
Syracuse
Utica
Buffalo
NEW YORK
0 10 20 30 40 50
Scale in Miles
Corning
Binghamton
Scranton
Williamsport
5670 Sq. Mi.
Wilkes-Barre
9960 Sq. Mi.
PENNSYLVANIA
Sunbury
18200 Sq. Mi.
Danville
11200 Sq. Mi.
Allentown
NEW JERSEY
Juniata R.
Newport
3300 Sq. Mi.
Harrisburg
24100 Sq. Mi.
Trenton
Susquehanna R.
Philadelphia
Camden
McCalls Ferry
26770 Sq. Mi.
Wilmington
Conowingo
27100 Sq. Mi.
MARYLAND
LEGEND
★ Major Gaging Stations
+ Other Gaging Stations
Baltimore
Chesapeake Bay
DEL.
N

FIG. 1.—GENERAL MAP OF SUSQUEHANNA RIVER BASIN

SUSQUEHANNA RIVER DRAINAGE AREA

The Susquehanna River has a total drainage area of more than 27,000 sq miles, which includes 47% of the total area of the State of Pennsylvania and 13% of the total area of the State of New York, besides a small area in the

State of Maryland (Fig. 1). With the exception of the Saint Lawrence River, in Canada, it is the largest and most important river on the Atlantic coast. The river has rather steep slopes and has very little storage, either natural or artificial.

In its lower reaches, where the river has cut its way through a range of tableland, the river channel has an average slope of almost 6 ft per mile and is walled on both sides by steep, rocky bluffs affording excellent foundation for water-power developments. Four run-of-river hydroelectric plants have been constructed in this stretch. Data for these plants are given in Table 1.

TABLE 1.—HYDROELECTRIC PLANTS ON THE LOWER SUSQUEHANNA RIVER

Plant	Distance from tidewater (miles)	Year of initial operation	No. of units	Head (ft)	Maximum discharge (cu ft per sec)	Effective capacity (kw)	Pondage[a]
Conowingo, Md.	4	1928	7	89	45,000	252,000	4,200
Holtwood, Pa.	19	1910	10	51	32,000	104,000	1,100
Safe Harbor, Pa.	27	1931	7	55	65,000	230,000	3,300
York Haven, Pa.	50	1904	20	20	18,000	20,000	....

[a] Approximate pondage in (cu ft per sec)—days per ft for normal regulation.

RIVER FLOW CHARACTERISTICS

The mean annual precipitation over the drainage area is almost 40 in. and the mean annual runoff averages about 47% of the precipitation. The runoff varies widely from day to day, week to week, and season to season; the recorded minimim, about 2,000 cu ft per sec average for a week, occurred during late summer of 1930, and the maximum recorded flood runoff, about 875,000 cu ft per sec, in March, 1936. The wide variation in runoff throughout the year can be illustrated best by reference to the monthly flow duration curves shown in Fig. 2. It is considered that the Conowingo plant can utilize all flows below 45,000 cu ft per sec. Runoff is at a minimum during August, September, and October, and is at its maximum during March, April, and May.

INTERCONNECTED LOAD

Practically all the output of the Conowingo plant is transmitted over two 220-kv circuits, about 58 miles in length, to the Plymouth Meeting (Pa.) substation located approximately 15 miles northwest of Philadelphia. From there the energy is transmitted into Philadelphia via 66-kv transmission lines and to suburban areas at both 66 kv and 33 kv. Plymouth Meeting substation also serves as the point of tie-in with the Pennsylvania-New Jersey Interconnection at 220 kv. The Pennsylvania-New Jersey Interconnection is made up of the Pennsylvania Power and Light Company, the Public Service Electric and Gas Company of New Jersey, and the Philadelphia Electric Company. Additional interconnection facilities with other companies at different points on these systems makes it possible to supply the large urban and industrial area reaching from New York, N. Y., to the District of Columbia.

The entire output from the Conowingo hydroelectric plant can be used advantageously on the Philadelphia Electric System alone, but actually it is used in combination with the entire interconnected system to reduce steam generating costs to a minimum. The Pennsylvania-New Jersey Interconnection has approximately a 3,000,000-kw peak, of which about 300,000 kw can be carried on hydro power. The total area load is about 4,000,000 kw with approximately 650,000 kw hydro power available. The load is served predominantly by steam plants, but efficient design of plants and transmission system, together with close cooperation and no elaborate inflexible contracts, allow the fullest coordination between hydro and steam generating facilities to carry the load at minimum cost.

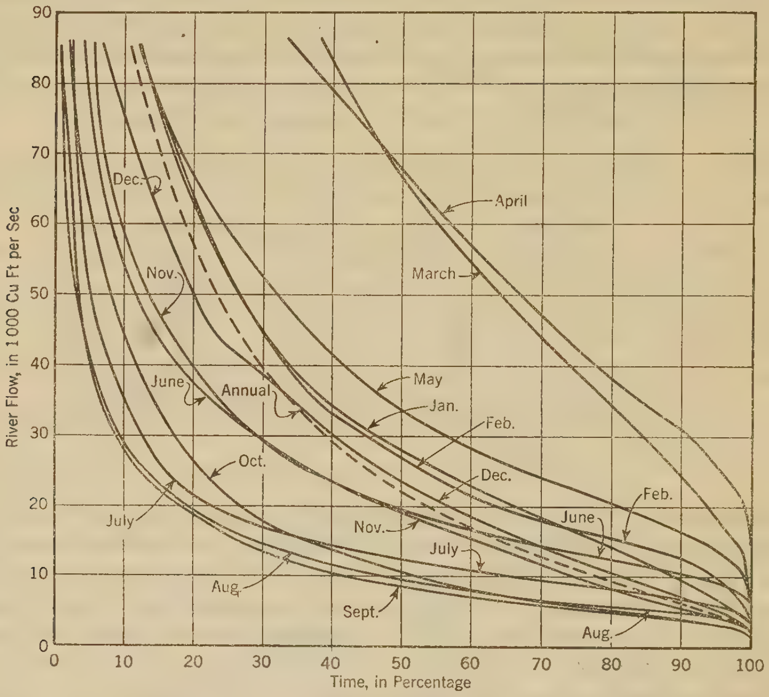

Fig. 2.—Flow Duration Curves of Susquehanna River at Conowingo, Md.

Allocation of Hydroelectric Energy

The operation of run-of-river hydro plants in a hydro-steam system has been frequently explained. It is sufficient to say that the available hydro capacity should be utilized on the system load so that the available hydro energy is used most effectively, drawing from pondage during the hours and days of heavy demand and refilling during periods of light load. The hydro plant is prepared to deliver up to its maximum capacity and the use of pondage is normally so scheduled as to have the level at top elevation on Monday mornings to meet the load demands of another week and, for all but the lower flows, top elevation is scheduled each weekday morning, particularly in the early part of the week.

In general, steam generating costs for the system vary directly with the load. Since the hydro production costs are almost constant for all loads, the value of the energy delivered is that of the marginal steam generation it replaces on the interconnected system. The load diagram shown in Fig. 3 illustrates three typical basic conditions for coordination of hydro with steam generation. It can be seen that the operation of equipment in the steam stations is governed, to an appreciable degree, by the estimated output of the hydro station.

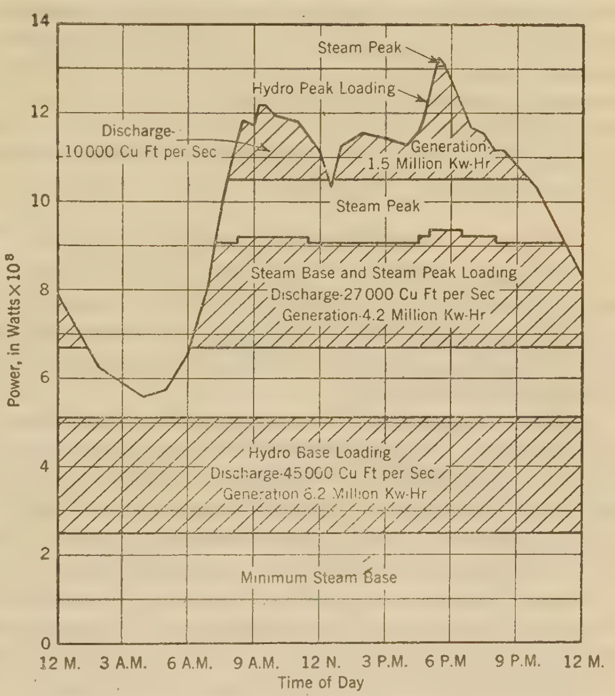

Fig. 3.—Coordinated Use of Hydro Power with Steam Generated Power

With low river flow, the steam base load is carried at an amount where the generation of hydro power over peaks (up to the hydro capacity) will correspond to the available energy from the river flow on a weekly basis—greater on heavy load days, less on light load days, with recuperation of storage over the week end. The available pondage at Conowingo is ample for weekly operation—5,000,000 kw-hr in the top 8 ft, or about double the available weekly energy for the minimum flows of record. Application of the energy to Pennsylvania-New Jersey Interconnection load curves allows fuller use of the available capacity with fewer hours of hydro operation than are indicated in Fig. 3.

When the river flow is greater, the steam base load is decreased so that more hours of hydro operation at greater loads are called for, using the available energy on a daily bais, or for longer periods over week ends. Hydro loads in excess of maximum efficiency on the number of units available may be justified for some periods by the difference in steam generating costs. With high river flows, Conowingo will generate to full capacity on base load 24 hours a day and the steam generating system assumes the burden of providing reserve capacity, of absorbing the variations in load, and of regulating the frequency. Superimposed on the base minimum steam load, and the hydro base load, the balance of steam generation will vary with the load. With these conditions, the maximum hydro capacity is most important, particularly over the system peak, since higher flows mean higher tailwater elevation and lower plant head, with consequent adverse effect on plant capacity.

The allocation of hydroelectric energy to best advantage requires determinative river flow forecasting, proper use of pondage, maintenance of head, and maintenance of station efficiency so that maximum capacity and the maximum amount of energy can be obtained from the available water to replace the most expensive steam generation. Head and station efficiency are basically designed qualities and can be controlled, however no control can be exercised over river flow, which varies greatly. Knowledge of river flow prospects is the most important consideration in the operation of a hydro plant.

Determinative River Flow Forecasting

It is not difficult to forecast probable river flows when flow is receding, as the recession rate is generally quite definite and, of course, the river flow actually is receding most of the time. To determine the probable increase in river flow, the sequence of meteorological and hydrologic factors must be anticipated progressively.

Active cooperation is carried on with governmental agencies interested in various phases of these problems, particularly with the United States Weather Bureau, which maintains the extensive facilities necessary for recording and gathering information on the air mass and water cycle, and with the United States Geological Survey, which maintains the stream gaging facilities. Some of the same river and rainfall observers are used in the Philadelphia Electric Company's skeleton network for obtaining necessary information. This work is unified through the Federal-State Flood Forecasting Service, a cooperative organization with the Water and Power Resources Board of Pennsylvania, which is unique in the United States and the result, in part, of recommendations made by the writer after the 1936 flood.

Accurate weather forecasts are always valuable for making anticipatory estimates of river flow and for many other uses in the operations of the system. Additionally, the peculiar uses made of specialized river flow forecasts and the particular economic value of such forecast dictate the need for independent river flow forecasting. The general public interest in river flows seems restricted to flood flows and governmental agencies naturally concentrate on these requirements. The interest of the electric system is mainly in the small flows increases and the extremely high floods.

Hydrographs of River Flow.—The method of plotting hydrographs from eleven river observation points on the Susquehanna River and its tributaries above Conowingo is shown in Fig. 4. These hydrographs are not plotted for the actual time of observation; they are transposed by what seems to be the normal time of travel of the crest from these points to Conowingo for the range of flows most valuable (from an economic standpoint) to forecast accurately. In general, the water flows at a rate slightly in excess of 4 miles per hr.

This method of plotting is very useful for forecasting flows when river stages are at crest or approaching their crest, and therefore stabilized, but it is of little assistance before and during a rain or in the first stages of runoff—an important deficiency in forecasting at a critical time.

Forecasts of Flow Increases.—To meet the deficiency already mentioned, the writer has developed a method for making an initial estimate of flows without stage verification. It is an empirical rather than a theoretical method for the particular conditions most useful for economic operation of the Conowingo hydro plant. While subscribing to the theoretical methods, the writer finds their application best when based on hindsight for actual conditions that have occurred, because in all practical applications insufficient information and lack of time make theoretical methods impracticable. More approximate but quicker practical methods are more useful and valuable for obtaining certain system economies of considerable magnitude.

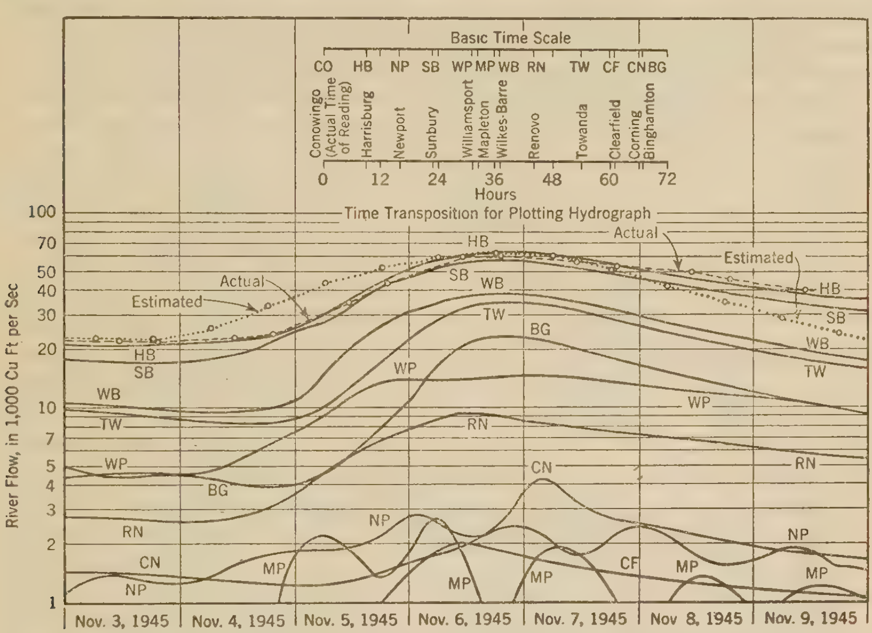

FIG. 4.—HYDROGRAPH OF A FLOW INCREASE ON SUSQUEHANNA RIVER TRIBUTARIES

Variables to Consider in River Flow Forecasts.—Determination of the probable river flows for predicted precipitation or from rain that has just fallen is the most complex, yet the most valuable, forecast of river flow that can be made. The sequence of meteorological and hydrologic factors must be anticipated, progressively, from the development and movement of storms, through rainfall and runoff, to river flow. Some idea of the complexity of such forecasting may be obtained from consideration of the variables involved.

1. Forecasts of Rainfall.—Determinative rainfall forecasting has been improved immensely, but there is still room for further improvement. Full reliance on forecasts of quantity and distribution over the Susquehanna River watershed is not yet justified but such forecasts are of prime importance,

and anticipatory estimates are made for scheduling loads. Close cooperation is maintained with the U. S. Weather Bureau for mutual understanding of the problems.

2. Areal Distribution of Rainfall.—Areal distribution of rainfall cannot be determined accurately, particularly in sections where the terrain is rugged and for summer thunderstorms, unless a large number of reporting stations are used. There is a practical limit to the number of stations used because the task of computation becomes laborious and time-consuming and the additional expense involved is not justified. Most of the reporting stations used are located along the rivers in the valleys. The U. S. Weather Bureau assists in the determination of the average rainfall distribution over the areas.

3. Rainfall-Runoff Ratio.—Because of the many subtractions and delays in transit it is difficult to forecast the amount of surface runoff resulting from a rain. The ratio varies during seasons of the year due to transpiration and evaporation losses, and the absorption and infiltration qualities of the ground. Also important are the effect of antecedent rainfall, the intensity of precipitation, and its duration.

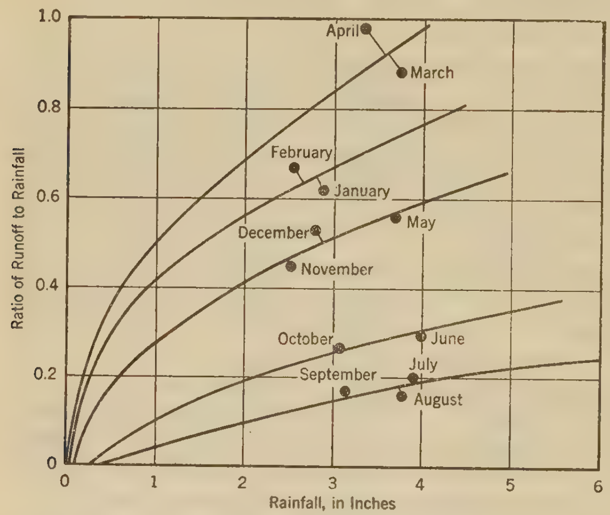

Fig. 5.—Rainfall-Runoff Ratio on Susquehanna Watershed

A guide for determining the amount of runoff to be expected from a rain during different seasons of the year is shown in Fig. 5. The cumulative rainfall for the storm should be used with these curves to determine the ratio of rainfall which may appear as runoff. No runoff is expected for light rains of different amounts, varying with the season. During wet periods or dry periods, it is necessary to use a higher or lower curve than that shown in Fig. 5 for the month in which the rain occurs, because these ratio curves are based on average conditions. Some judgment must be exercised in choosing the applicable curve. During winter months the runoff can be quite different from that indicated, due to temperature and type of precipitation.

4. Runoff Distribution.—The manner in which the surface runoff flows into the main stream channel depends on the physical characteristics of the drainage area, especially on the size, slopes, and locations of its tributaries. Use has been made of the unit-hydrograph principle that, for the same drainage area, surface runoff from rainfalls occurring within the same time interval, such as a part of a day, will generally produce hydrographs whose ordinates will vary

directly with the total amount of the surface runoff. It is rare, however, that isolated storms occur which produce consistent hydrographs from which an average distribution graph can be determined—the larger the drainage area, the less frequently will they occur. Likewise, it is just as infrequent in actual practice for the storm for which the runoff is forecast to meet the conditions from which the unit hydrograph was derived. Actually, the writer has never seen duplicate conditions result in identical hydrographs from the Susquehanna River basin. Each case must be considered on its individual characteristics.

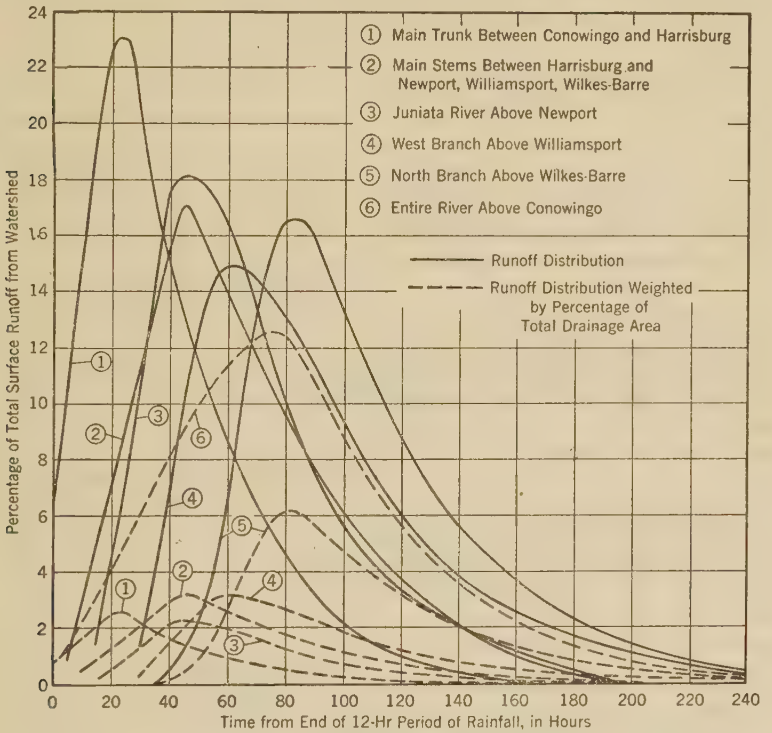

Fig. 6.—Runoff Distribution Curves for Subareas of Susquehanna Watershed

The solid curves in Fig. 6 show the normal distribution of runoff, expressed in percentage of total runoff, that may be expected from a 12-hr period of rainfall over the various sections of the watershed as indicated. In plotting they have been transposed in accordance with the basic time scale (Fig. 4) for the water travel time from the various gaging stations to Conowingo. The dashed curves in Fig. 6 show the runoff distribution graphs weighted by the percentage of total drainage area each represents. The sum of the ordinates of these five subarea curves equals the ordinate of curve 6 for the entire watershed above Conowingo. Curve 6, the over-all distribution graph of runoff, is what may be expected when a rainfall of uniform amount occurs over the entire watershed above Conowingo. The average percentages of total expected runoff contributed by each subarea to the river flow at Conowingo were obtained from the distribution curves (curves 1 to 6) for each 12-hr period after the rainfall by taking the midpoint value for each period. These values are listed in Table 2(*a*) for each reference.

TABLE 2.—DISTRIBUTION OF RUNOFF AND

Sub-area[a]	Total rain-fall (in.)	Runoff-rain-fall ratio	Rainfall in period (in.)	Runoff for period (in.)	Drain-age area ×$K^{(b)}$	Runoff volume for period[c]	NUMBER OF EACH					
							−1[d]	0	1	2	3	4
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
										(*a*) PERCENTAGE OF RUNOFF		
1	....	....	....	....	80,400	....	....	2.0	12.0	21.5	20.2	14.0
2	....	....	....	....	137,200	....	....	0	1.4	6.3	11.3	16.2
3	....	....	....	....	90,200	....	....	0	0	3.3	11.0	17.9
4	....	....	....	....	152,800	....	....	0	0	0	1.3	8.1
5	....	....	....	....	267,800	....	....	0	0	0	0	0.7
6	....	....	....	....	728,400	....	....	0	1.5	3.8	6.1	8.6
							(*b*) EXPECTED FLOW INCREASE FROM FIRST 12-HR					
1	0.06	0	0.06	0	80,400	0	....	....	....	....	....	....
2	0.34	0.13[e]	0.34	0.044	137,200	6,040	....	0	0.2	0.8	1.4	2.0
3	0.36	0.14[e]	0.36	0.050	90,200	4,520	....	0	0	0.2	1.0	1.6
4	0.59	0.20[e]	0.59	0.118	152,800	18,030	....	0	0	0	0.4	3.0
5	0.53	0.19[e]	0.53	0.101	267,800	27,020	....	0	0	0	0	0.4
Total	....	....	....	....	....	....	....	0	0.2	1.0	2.8	7.0
							(*c*) EXPECTED FLOW INCREASE FROM SECOND 12-HR					
1	0.21	0.08[f]	0.15	0.012	80,400	1,000	....	0	0.2	0.4	0.4	0.3
2	0.86	0.25[f]	0.52	0.130	137,200	17,820	....	0	0.4	2.2	4.0	5.8
3	0.76	0.24[f]	0.40	0.096	90,200	8,670	....	0	0	0.6	2.0	3.2
4	1.05	0.34[f]	0.46	0.156	152,800	23,830	....	0	0	0	0.4	3.8
5	0.86	0.31[f]	0.33	0.102	267,800	27,310	....	0	0	0	0	0.4
Total	....	....	....	....	....	....	....	0	0.6	3.2	6.8	13.5
							(*d*) EXPECTED FLOW INCREASE FROM THIRD 12-HR					
1	0.24	0.10[g]	0.03	0.003	80,400	240	....	0	0	0.2	0.2	0.1
2	1.05	0.36[g]	0.19	0.068	137,200	9,330	....	0	0.2	1.2	2.2	3.0
3	0.83	0.33[g]	0.07	0.023	90,200	2,080	....	0	0	0.2	0.4	0.8
4	1.14	0.39[g]	0.09	0.035	152,800	5,350	....	0	0	0	0.2	0.8
5	1.24	0.41[g]	0.38	0.156	267,800	41,760	....	0	0	0	0	0.6
Total	....	....	....	....	....	....	....	0	0.2	1.6	3.0	5.3
							(*e*) EXPECTED FLOW INCREASE FROM THE THREE 12-HR					
From third period							....	0	0.2	1.6	3.0	5.3
From second period							....	0.6	3.2	6.8	13.5	17.4
From first period							....	1.0	2.8	7.0	10.4	14.2
Total increase in surface runoff							....	1.6	6.2	15.4	26.9	36.9
Expected recession of present flow							23.0	21.1	19.5	18.2	17.0	16.0
Expected future river flow							23.0	22.7	25.7	33.6	43.9	52.9

[a] As described in Fig. 6. [b] $K = 26.89$. [c] In (cu ft per sec)-days. [d] 12-hr period preceding 12-hr rainfall December and January curve (Fig. 5).

It should not be expected that these distribution curves are exact or that actual conditions will be similar to those on which they are based, but it is expected that the accuracy is sufficient for the normal diversity of conditions in the several subareas to give, in the main, a reasonable expectation of the actual flow.

CALCULATION OF EXPECTED FLOW INCREASE AT CONOWINGO

12-HR PERIOD FOLLOWING 12-HR RAINFALL PERIOD												
5	6	7	8	9	10	11	12	13	14	15	16	17
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
CONTRIBUTED TO TOTAL FLOW												
10.2	7.2	4.9	3.2	2.0	1.2	0.8	0.5	0.2	0.1	0	....	....
15.2	12.5	10.0	7.7	5.8	4.3	3.2	2.4	1.7	1.1	0.6	0.3	0
17.4	14.6	10.6	7.6	5.4	3.7	2.9	2.2	1.5	0.9	0.6	0.3	0.3
14.2	14.7	13.3	11.3	9.0	6.9	5.3	4.0	3.1	2.4	1.9	1.4	1.4
3.5	10.7	16.2	15.9	12.7	10.0	7.7	5.9	4.5	3.5	2.7	2.0	2.0
10.5	12.0	12.6	11.0	8.5	6.6	5.0	3.8	2.9	2.1	1.6	1.1	1.1
RAINFALL PERIOD (7 A.M. to 7 P.M. 11/2/45)												
....	....	....	....	....	....	....	....	....	....	....	....	....
1.8	1.6	1.2	1.0	0.8	0.6	0.4	0.2	0.2	0.1	0	....	....
1.6	1.4	1.0	0.6	0.4	0.3	0.2	0.2	0.1	0	....	....	....
5.2	5.4	4.8	4.0	3.2	2.4	2.0	1.4	1.2	0.8	0.6	0.6	0.4
1.8	5.8	8.8	8.6	7.4	5.4	4.2	3.2	2.4	1.8	1.4	1.1	0.9
10.4	14.2	15.8	14.2	11.8	8.7	6.8	5.0	3.9	2.7/	2.0	1.7	1.3
RAINFALL PERIOD (7 P.M. 11/2/45 to 7 A.M. 11/3/45)												
0.2	0.2	0.1	0.1	0	....	....	....	....	....	....	....	....
5.4	4.4	3.6	2.8	2.0	1.6	1.2	0.8	0.6	0.4	0.3	0.2	0.1
3.0	2.6	1.8	1.4	1.0	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0
6.8	7.0	6.4	5.4	4.2	3.2	2.6	2.0	1.4	1.2	1.0	0.6	0.5
2.0	5.8	8.8	8.6	7.0	5.4	4.2	3.2	2.4	1.8	1.4	1.1	0.9
17.4	20.0	20.7	18.3	14.2	10.9	8.5	6.4	4.7	3.6	2.9	2.0	1.5
RAINFALL PERIOD (7 A.M. to 7 P.M. 11/3/45)												
0	....	....	....	....	....	....	....	....	....	....	....	....
2.8	2.4	1.8	1.4	1.0	0.8	0.6	0.4	0.3	0.2	0.2	0	....
0.8	0.6	0.5	0.4	0.3	0.2	0.2	0.1	0	....	....	....	....
1.6	1.7	1.4	1.2	1.0	0.8	0.6	0.4	0.4	0.3	0.2	0.2	0.2
3.0	9.0	13.6	13.2	10.6	8.4	6.4	5.0	3.8	3.0	2.2	1.7	1.5
8.2	13.7	17.3	16.2	12.9	10.2	7.8	5.9	4.5	3.5	2.6	1.9	1.7
RAINFALL PERIODS (7 A.M. 11/2/45 to 7 P.M. 11/3/45)												
8.2	13.7	17.3	16.2	12.9	10.2	7.8	5.9	4.5	3.5	2.6	1.9	1.7
20.0	20.7	18.3	14.2	10.9	8.5	6.4	4.7	3.6	2.9	2.0	1.5	1.0
15.8	• 14.2	11.8	8.7	6.8	5.0	3.9	2.7	2.0	1.7	1.3	0.8	0.6
44.0	48.6	47.4	39.1	30.6	23.7	18.1	13.3	10.1	....	....	....	....
15.0	14.2	13.5	12.8	12.2	11.7	11.3	10.9	10.5	....	....	....	....
59.0	62.8	60.9	51.9	42.8	35.4	29.4	24.2	20.6	....	....	....	....

period. • From November curve (Fig. 5). / From December curve (Fig. 5). ø From average between

5. Rate of Flow in Streams.—The rate of travel from various points on the river, as indicated by the basic time scale shown in Fig. 4, holds quite well for the flow values that are most critical from the standpoint of economy of use at Conowingo. This time scale must be tempered by judgment when applied to very small or very large increases in flow and the effect of channel storage and intermediate drainage area must be fully considered.

6. Ground-Water Flow.—The foregoing discussion has been concerned only with surface runoff. There is undoubtedly a hydrograph of ground-water flow, but to isolate it and to determine its value has been neither possible nor sufficiently valuable to warrant the effort. Ground-water flow has been likened to a large reservoir discharging through a V-notch weir. So far it has not been possible to determine accurate discharge coefficients or an accurate head gage for the simulated weir. Moreover, it is not of major importance in this problem except as to its effect on estimating minimum flows.

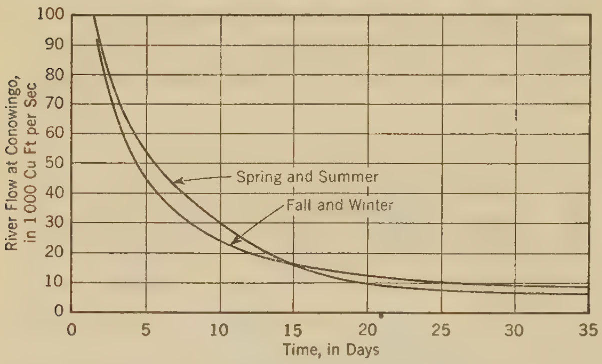

Fig. 7.—Normal Flow Recession Curves at Conowingo

7. Flow Recession.—It is necessary to know the river flow that would have occurred without the new runoff, that is, the base flow upon which the surface runoff is superimposed. Fig. 7 shows the normal flow recession curves for the Susquehanna River when no additional precipitation occurs. These were determined from actual past experience at Conowingo. The values used are read differentially from the curve.

8. Rainfall Along Main Stem of River.—Rain falling directly on the storage ponds and river channels, since its runoff is fast and almost 100%, can have a noticeable effect at low flows. This is particularly true on the main stem of the Susquehanna, when heavy summer thunderstorms travel down the river valley. Occurrence of such rains over week ends can affect operations to a considerable extent.

9. Effect of Temperature.—Temperature can have a large effect on river flows, particularly during the winter months. Precipitation may fall as rain, freezing rain, sleet, and snow. It may be a combination of these or it may be different in various areas of the watershed so that runoff values are indeterminative. Rain may fall on snow and be absorbed, so that runoff may be delayed or even arrested. On the other hand, rain or even warm temperatures alone may bring additional runoff from snow or ice already on the ground. River stages alone must be relied on to verify such indications.

Then, even when the water gets into the river channels, flow can be seriously impeded by severe subfreezing temperatures forming ice, which packs and jams, thus providing an astonishing amount of channel storage in a comparatively short time. These ice conditions affect the river gages, sometimes giving erroneous river flow readings for a protracted period. Some gages are affected more than others and the result is that true gaging is not possible in some cases—in fact, the only true measure is through the powerhouse units.

Calculation of Expected Flow Increases.—The method of calculating the expected increase in flow, either from forecast rainfall or from rain that has actually fallen, or both, is shown on the work sheet reproduced in Tables 2(*b*), 2(*c*), 2(*d*), and 2(*e*). A freshet occurring from November 3 to November 10, 1945, is used because it represents a separate clear-cut computation. The procedure is

1. Obtain the average rainfall for each subarea of the watershed for each 12-hr period, usually ending at 7 a.m. and at 7 p.m.
2. Obtain the ratio of runoff to rainfall from the curves shown in Fig. 5, considering the total rainfall for the periods immediately preceding, as well as the current period.
3. Multiply this ratio by the rainfall for the present period to get the expected runoff in inches. Multiplying this value by the drainage area in square miles and a constant of 26.89 will give the total runoff volume in (cubic feet per second)-days from this period rainfall.
4. Multiplication of this total runoff volume, for the area concerned, by the respective percentages from Table 2 gives the expected increase in total surface runoff contributed by this area for each subsequent 12-hr period. Multiplication of this runoff value by 2 determines the rate in cubic feet per second.
5. Addition of the runoffs thus obtained for each of the five drainage subareas gives the total additional surface runoff rate in cubic feet per second for each subsequent 12-hr period because of the rain that has fallen in the present period.
6. Addition, in proper succession, of these separate 12-hr period runoff rates gives the total increase in surface runoff in average (cubic feet per second)-days resulting from the storm.
7. Addition to this increase in surface runoff of the base recession flows corresponding to the flow occurring at the time of the precipitation (in some cases the increase in surface runoff only serves to counteract the natural recession if no rain has occurred) will result in the expected average rate of flow, for this particular freshet, of each subsequent 12-hr period after the rainfall measurement.

An actual example, plotted as short dashed lines on the hydrograph (Fig. 4), is the estimate made on Saturday evening, November 3, 1945, immediately after all rainfall had occurred. The probable effect of channel storage will be noted as it is more marked than usual in this case.

Appraisal of Method.—During 1945, which was a record output year for Conowingo, there were a number of occurrences when probable flow increases necessitated just such calculation as given by this approximate method. It cannot be expected that such an approximate method of calculation will always result in accurate flow estimates—first, the method was developed only for the range of river flows most important, from the system economy standpoint, to forecast quickly without stage verification; and, second, no method of calculation can be any better than the ability to determine the variables previously discussed.

Determinative forecasts of river flows is a complex problem that defies definite computation within practical limits of time and expense. Nevertheless, some basis for computation is most valuable, and good judgment obtained from experience, exercised and tempered with proper consideration of the probable effect on system economy, is an important ingredient.

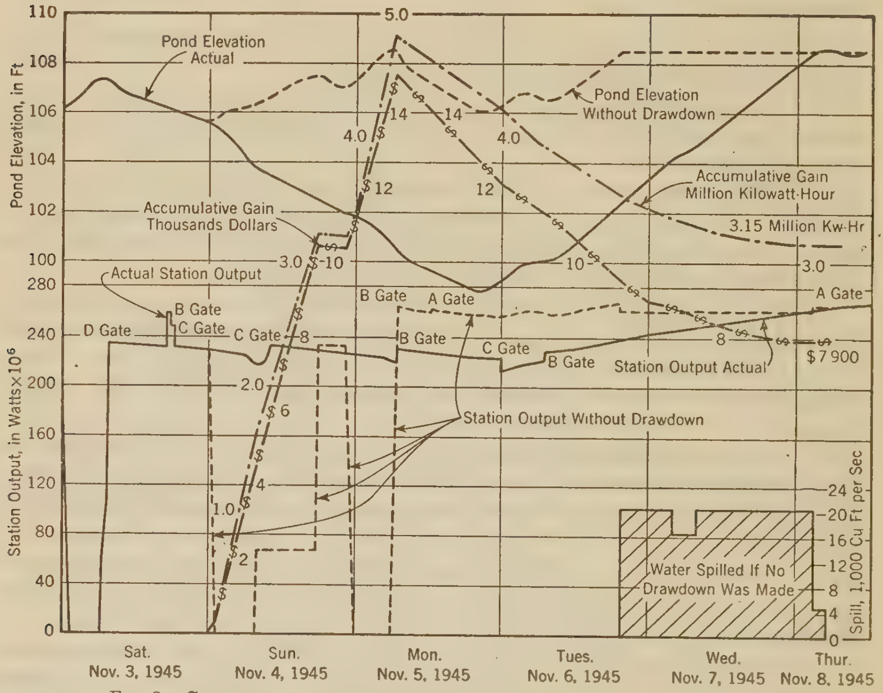

FIG. 8.—COMPARISON OF ACTUAL AND ALTERNATIVE METHODS OF OPERATION

Effects of System Economics.—That system economics play a large part in determining what course of action to take can be best illustrated by an actual case of operation. The basic operating information for the freshet occurring from November 3 to November 10, 1945, is shown in Fig. 8. Fundamentally, two courses of action were open on Saturday, November 3, 1945: Either to draw down the Conowingo pond, using the water therein to generate hydroelectric energy to displace low cost steam energy over the week end and sacrificing head and therefore energy and capacity on Monday and Tuesday, when steam replacement costs were higher, or, to carry the hydroelectric energy available from the river flow with recovery of storage by Monday morning, then operating at full output thereafter. After weighing the probable disadvantages of reduced capacity on Monday and Tuesday, the first alternative was chosen. This was the proper choice, and resulted in a net saving of approximately $7,900.

As a case in point, this precipitation could have occurred two days earlier in the week. The value of the drawdown would then have been greater on Thursday and Friday and the penalties from low head would have been less on Saturday and Sunday, resulting in a greater economic gain. Conversely, heavier loads on Monday and Tuesday, or forced outages of steam equipment, might have raised steam replacement costs to a point where such a drawdown would show little if any gain; or, more important, if the flow increase had not been great enough to fill the storage, a net loss, rather than a gain, would have resulted from the low head.

In any event, good estimates of river flow at the earliest possible moment, even based on predicted precipitation, may be vital if certain system economies of considerable magnitude are to be obtained or if uneconomical operation is to be avoided.

MAINTENANCE OF HEAD

After the determination of available river flow, the next most important considerations are the maintenance of maximum head and the maintenance of maximum station efficiency, so that the greatest possible capacity and energy may be available for replacement of the highest cost steam generation. These two considerations are interrelated in many ways.

Seiches in the Conowingo Pond.—By agreement with the owners of the Holtwood (Pa.) plant prior to the construction of the Conowingo development, a maximum elevation of 108.5 ft above mean sea level was adopted for the Conowingo Dam pond with the anticipation that it would be the normal elevation. However, from the standpoint of system economy, it is not desirable and, from a practical standpoint, it is not possible to maintain a fixed

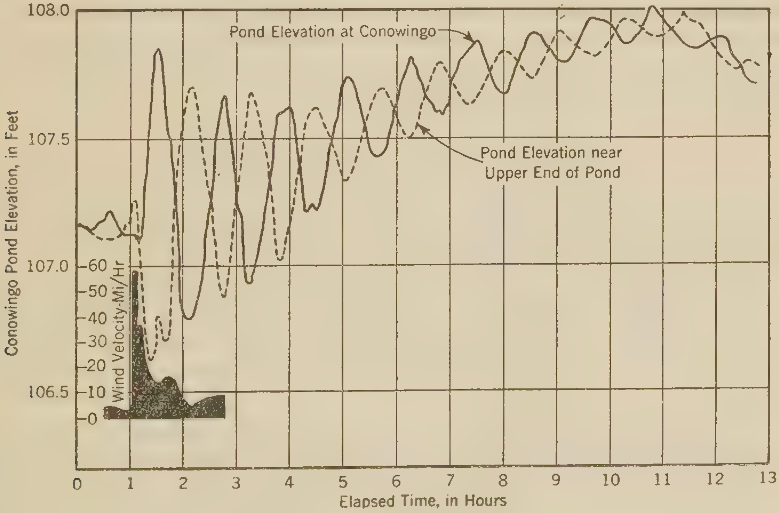

FIG. 9.—EXAMPLE OF SEICHE ON CONOWINGO POND CAUSED BY HIGH WIND

elevation. If 108.5 ft is the maximum elevation, then the average must be less, even when operating on base load and spilling water.

The Conowingo pond level has a regular oscillation whenever the equilibrium is upset, such as sudden increase or decrease of load on Conowingo, sudden change in discharge from the Holtwood plant, failure of the Holtwood flashboards under high flow, raising or lowering of spillway gates at Conowingo, or wind. A clear-cut occurrence caused by a sudden squall shortly after 8 p.m. on a summer day is illustrated in Fig. 9. A wind approaching 60 miles per hr blew almost directly downriver along the longitudinal axis of the pond, pushing ahead of it a wave of increasing size, which was followed by a trough of similar magnitude. The wave registered first on a gage near the upper end of the pond, before progressing downstream to the dam, from which it was reflected. The conditions presented an opportunity to determine the natural oscillation period of the Conowingo pond, which proved to be about 1 hour and ten min. In this instance the strong wind gust lasted about one quarter of the oscillatory period, which undoubtedly assisted in setting up a uniform oscillation of considerable amplitude. Knowledge of the natural period of the pond has been helpful to the operators in determining the probable effect of spillway gate operations so as to maintain high average elevation under the limiting maximum elevation.

Effect of Spill Water on Head.—During spill periods, station capacity is curtailed because of increased tailwater elevation. It has been valuable to know where along the dam this excess water may be spilled to cause the least increase in tailwater elevation at the plant for all surplus flows. Also, to avoid opening additional spillway gates at the time of the system peak or even to be able to close some at that time may be very desirable if the "peak prepared for" steam costs are high. For this purpose, some drawdown of the pond just prior to the time of the system peak may be justified.

Maintenance of Station Efficiency

Determination of the available river flow and maintenance of head are joined by the third important consideration, which is the efficiency of the station in converting the water power into electrical energy.

Unit Loading.—In maintaining maximum station efficiency, the unit loading is important. Operating characteristics of the various units have been established on the basis of tests of the individual machine. Periodically, check tests are made to detect any variation in performance from established standards.

If the economy of the entire system, steam and hydro, is not otherwise improved, the loading of Conowingo is such as to secure the highest hydro equivalent (kilowatt/cubic feet per second) consistent with the use of all the available water. To accomplish this, the minimum number of units are operated to meet the load demand and these units are normally not loaded in excess of maximum efficiency, the balance of their capacity being available for reserve. For loading in excess of maximum efficiency, whether to take care of higher river flows or to meet greater load demands, four different turbine gate settings have been established on an incremental basis to cover practically all conditions of load on seven units. The details of these settings are given in Table 3.

TABLE 3.—LOADING CHARACTERISTICS ON CONOWINGO UNITS

Turbine gate setting (1)	Station output (kw) (2)	Plant discharge[a] (cu ft per sec) (3)	Hydro equivalent[a] (kw per cu ft per sec) (4)	Incremental hydro equivalent from next lower gate setting (5)
A.......	267,000	45,400	5.88	1.13
B.......	264,500	43,000	6.15	3.30
C.......	250,000	38,600	6.47	4.81
D.......	236,500	35,800	6.60	....

[a] For 88-ft plant head.

A (maximum output) gate, giving the maximum possible output (Conowingo turbines are all over-gated), is used only when spilling water or when such possibility is certain to occur in 24 hours, and to meet unforeseen load demands.

B (maximum drawdown or peak output) gate is the normal limit when there is no imminent wastage of water or unforseen load demands.

C (economical limit) gate is considered the economical limit for purposes of obtaining energy other than for load peaks or for maximum drawdown in advance of a freshet. Unit efficiency drops rapidly above this point.

D (maximum efficiency) gate is the most economical point on all units.

The higher, more inefficient gate openings are used for the periods during the day when, by sacrificing Conowingo economy, greater monetary gains can be made on the system, or to take care of river flows greater than can be utilized by 24-hr operation at maximum efficiency. It can be seen from Table 3 that if the steam peak costs exceed the steam base costs by about 40% there is economic justification for operating at C gate instead of at D gate. If the steam peak costs are more than twice the steam base costs, B-gate operation is justified. The small amount of additional generation between B gate and A gate at a cost ratio of 6 to 1 limits its use to spill periods when the water may otherwise be wasted.

Use of Pondage.—Care must be exercised in the use of pondage by drawdown, as it is possible to cause head losses of considerable magnitude, reducing not only the available capacity (approximately 4,000 kw per ft), but also the energy generated from the water. This reduction in energy amounts to about $1\frac{3}{4}$ kw-hr per (cu ft per sec)-day per ft. In effecting economies on the connected steam system, Conowingo may be so loaded as to reduce the capacity requirements of the connected steam system or to absorb the daily system load variations, which may be the result of differences in the normal demand from day to day, either unforeseen or forecasted. If there is an overdraft on the Conowingo pond for any cause, replacement of the storage is effected as soon as possible by reducing the demand on Conowingo. This reimbursement is made, if possible, when the marginal cost of the connected steam system is less than it was when the drawdown was made.

1. Scheduled Drawdowns.—In general, scheduled drawdowns for the purpose of system economy are made late in the week to minimize head losses and capacity reductions when full capacity may be needed; the pond then is refilled over the week end when increment steam costs are lower. The available hydro generation is estimated for the 3 or 4 days remaining in the week and the daily output is then prorated to obtain such savings in the operation of the steam stations as difference in incremental steam generating costs, and reduction in boiler and machine hours. Generally, the greatest saving by drawdown is obtained when the steam base can be held the same for the days for which the energy estimates are made. The coördinated use of hydro storage allows maintenance outages of steam equipment to be scheduled to best advantage.

2. Drawdowns in Anticipation of Increased Flow.—When higher flows are expected, drawdowns may replace steam generation, sacrificing head at Conowingo to obtain the greatest monetary gains on the system. With well-defined possibilities of river flows in excess of the usable flow at Conowingo, full drawdown of the storage is begun and energy is generated by water that might otherwise be wasted over the spillway. It might be well to point out in this connection that the Susquehanna watershed is peculiarly situated, in that the normal paths of storms causing heavy precipitations are such as to result in heavy rains on the lower part of the watershed before the upper sections are affected. It is, therefore necessary to start a complete drawdown, if such is desired, even in advance of the rainfall, otherwise local inflow might make drawdown impossible. Then, if the expected flow increases do not materialize, due to meteorological changes, a quick correction is necessary to minimize head losses. A good gamble, with odds, is worth taking.

It is sometimes difficult to visualize the fact that drawdowns made in advance of excess water are not always profitable, because it appears that energy obtained from use of pondage that is later refilled by otherwise wasted water is an obvious gain. This, however, is not always true. Drawdowns are necessarily made by replacing "off peak" (and hence lower cost) steam generation, yet the head losses are even more effective on high cost than on low cost steam generation. Even without this consideration of difference in increment costs, the actual gain in kilowatt-hours alone diminishes quite rapidly for drawdowns when river flows are moderate or greater. Also, the higher the inflow, the less the drawdowns can be without taking undue risks.

An attempt to illustrate the interrelation of drawdown and possible gain at various flows is shown in Fig. 10. The four top curves show the energy available in the pondage when used through the Conowingo units without considering the losses from any other inflow which would be used at the lowered head. The three bottom curves show the losses in energy obtained from three different values of inflow, because of the loss of head, as drawdown of storage is made. These inflows are expressed in (cubic feet per second)-days to take into account the time element involved. The dashed curves show the net gains possible from drawdown, using most efficient turbine gate openings and with inflows of three different values. If the rate of inflow was 24,000 cu ft per sec, then all drawdown would be made off peak and operation at C gate would be justified most of the day. A curve of net energy at C gate is, therefore, also shown at this flow. Likewise, if the rate of inflow was 36,000 cu ft per sec, drawdown would have to be made at a higher gate so a curve of net energy at B gate is also shown for this flow.

These curves show that possible energy gains are greatest at the low flows and so attempts are made to reach the drawdown limit unless capacity is too valuable to risk. For higher inflows, drawdowns below El. 100 are rarely justified and in some cases monetary losses may result due to difference in incremental steam generating costs and capacity value. Of course, inability to forecast increases in flow far enough in advance to make possible these large drafts on pondage, particularly with the higher flows, limits the chances of making unprofitable drawdowns if care is taken to consider the replacement costs on steam.

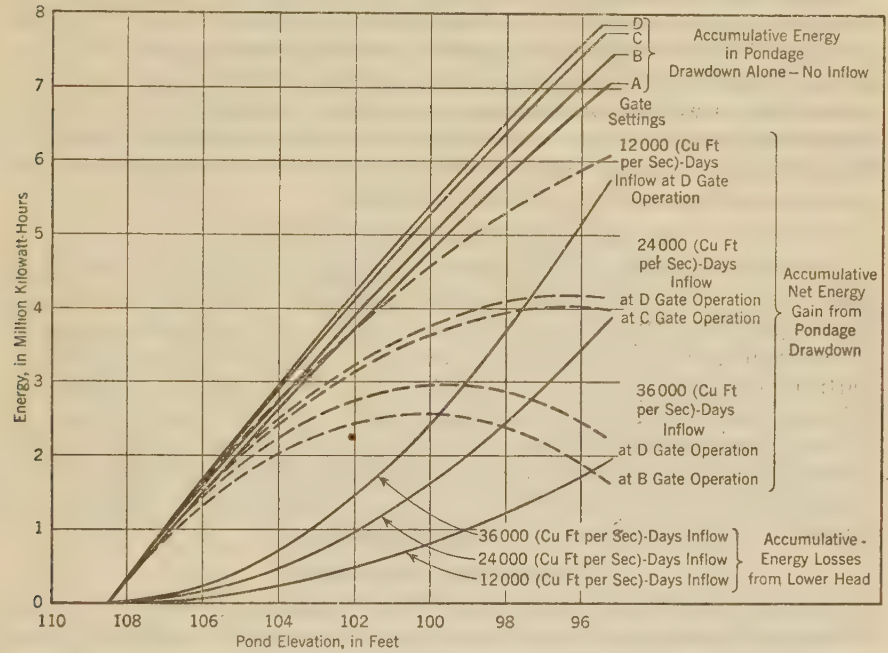

FIG. 10.—INTERRELATION OF DRAWDOWN AND ENERGY GAIN FROM CONOWINGO

System Reserve Capacity Requirements.—Hydroelectric generating plants have a peculiar value for system reserve. Energy is always available in pondage for immediate use and at times system requirements may dictate having more units running, for reserve purposes, than the load demands. These extra machine hours are kept at a minimum because the no-load water use is appreciable during low flows. Further economy can be effected by placing these generators in condenser operation by closing the turbine gates and, if tailwater level approaches the turbine level, by depressing the water level by the use of compressed air injected into the turbine head cover. Naturally, all unit capacity in excess of the gate opening at which each is operating is available immediately for system reserve and if additional units are required and available they can be synchronized very quickly.

UPRIVER HYDRO PLANT OPERATION

Pondage at the upriver hydroelectric plants (see Table 1) is an important factor and the interrelation of operation with that of the upriver plants is given careful consideration in order to eliminate excess pond elevations or head losses. If conditions are critical, river flow estimates are compared. Close cooperation and an understanding of all problems with which there is mutual concern have resulted in the most efficient utilization of river flow.

Coordinated operation of hydro and steam capacity in electric power systems on a regional basis, through interconnected tie lines, has been expanded to assure fullest utilization of available generating capacity in carrying increased loads, in scheduling maintenance, and in meeting emergency outages of equipment, as well as for obtaining economies of generation. An excellent presentation of the results of cooperative studies for coordinated operation of the hydro plants on the Susquehanna River with the large regional steam generating system has been published elsewhere.[2]

ESTIMATES OF CONOWINGO GENERATION

Estimates of energy available from Conowingo are needed for more than just a day in advance to obtain maximum economies in the system. Whereas day to day estimates are most important, weekly estimates are necessary to take advantage of incremental costs in scheduling less inefficient steam stations, particularly during low flow. Biweekly estimates are helpful in order that shipments of coal may keep pace with steam generating station demands. Monthly and even yearly estimates are helpful for budget purposes, as large deviations from the average Conowingo outputs affect the cash position of the company, as well as economies in scheduling maintenance. Naturally, the longer the period the less reliable these estimates are.

COOPERATIVE SUPERVISION

Control of the supply of power to meet the demand on all sections of the system is delegated to the load dispatcher and, therefore, allocation of load to attain the greatest economy must be under the same centralized control because the system load and the available sources of energy are subject to continual change. Because the Conowingo output affects system economy so definitely, it is as important sometimes to indicate possibilities as it is to actually forecast their occurrence. For this reason a quick method of determining river flow, as explained herein, is needed. Exchange of information regarding possible changes of system load and equipment outage is helpful to the forecaster as indicating what probabilities of economy may exist. It is imperative to maintain close cooperation and mutual understanding between the responsible parties so that, by constant vigilance, over-all economies can be effected to maximum advantage and uneconomical operation avoided. Hydro and steam power are not rivals, as is popularly believed, but are actually complementary and can be advantageously used together in a coordinated system for over-all economy.

ACKNOWLEDGMENTS

The author desires to express his thanks to the executives of the Philadelphia Electric Company for their courtesy in permitting use of the material contained herein, and to his associates in the company for their constructive criticism and assistance in assembling the data.

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[2] "The Co-Ordinated Operation of Hydro and Steam Capacity in Electric Power Systems," by G. W. Spaulding, *Transactions*, A.S.M.E., Vol. 66, 1944, p. 545.

DISCUSSION

JOEL D. JUSTIN,[3] M. ASCE.—The proper functioning of a run-of-river hydroelectric plant with ample pondage has been adequately covered by the author. In such a system as Mr. Turner describes, served largely by steam plants, a hydro plant like Conowingo in Maryland may be used to reduce the over-all cost of the steam generated energy. To a considerable extent the possession of a plant like Conowingo permits one to place base load on the steam plants and peak load on the hydro plant during the season of low stream flow. During the time of surplus stream flow the allocation of loads to the plants is changed and by preference the hydro plant takes the base load and the steam plants take the peak load.

The great aid which such a hydro plant may be to a system served largely by steam plants is considerable in reducing the over-all cost of steam generated energy. The steam plants are thus enabled to operate on a higher capacity factor. It is evident that with a plant such as Conowingo the pond should be drawn down only a small percentage of the head; otherwise, the decreased head may seriously affect the capacity available at time of peak load and may also decrease the energy generated from the available water supply.

For taking full advantage of rainfall on the watershed (27,000 sq miles), a rather complete warning system is essential—such as has been used at Conowingo, Holtwood, and Safe Harbor, which are companion plants located on the lower Susquehanna River. A thorough system of stream and rain gages is maintained. All gage keepers have telephones and are required to call with regard to any sudden changes. In turn, the gage keepers may be called at any time for special advice as to hydrological conditions. Telephoned records and hydrological advice are quickly tabulated and converted into probable pond inflow.

Knowing that sudden accretion of runoff is likely, the pond may be drawn down to store the freshet waters and avoid wasting kilowatt-hours of energy in flow over the dam. That the engineers at Conowingo have been successful in their predictions is indicated by the fact that the utilization factor at Conowingo (the ratio of the kilowatt-hours actually generated to the kilowatt-hours which theoretically might have been generated) has for many years exceeded 99.5%.

E. T. SCHULEEN,[4] M. ASCE.—The many factors relating to the operation of hydro plants such as those on the lower Susquehanna River which function as part of large steam generating systems have been very ably analyzed by the author. Table 1 calls attention to the large block of effective hydro capacity of 606,000 kw—in these four plants, which capacity, however, is only about 11% of the total capacity in the combined Pennsylvania-New Jersey and Baltimore (Md.)-Washington (D. C.) regional systems. Although use of these plants for peaking capacity and as reserve has been an important factor in their justification, sharply rising fuel costs are placing new emphasis on their energy value.

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[4] Hydrographic Engr., Pennsylvania Water & Power Co., Conestoga, Pa.

There can be no dispute with the author's statement (under the heading, "Allocation of Hydroelectric Energy") that: "Knowledge of river flow prospects is the most important consideration in the operation of a hydro plant." Decisions on practically all operating problems hinge on the estimated river flows. The Conowingo flow forecasting methods were developed primarily for a relatively narrow flow range. With this qualification, the use of constant time of peak travel, simple rainfall-runoff ratios, and combined ground and surface runoff recession curves is perhaps justified. When forecasts over a wider range of flows are required, "time of peak travel" curves which vary with stage should be used. Hydrograph plotting to a semilogarithmic scale (as in Fig. 4) possesses the advantage of holding the plot to a convenient size with all probable flows, but it does not permit a ready visual check of the variation in inflow between stations, nor does it allow hydrographs above a confluence to be combined graphically to aid in projecting ahead the hydrograph at a station below the confluence.

Forecasting flows from rainfall is a hazardous procedure, at best, but there is no alternative in obtaining an early forecast when the rain has just stopped or is still falling. After the headwater stations begin to peak, forecasting becomes progressively simpler and more reliable. The most important single factor affecting the accuracy of a flow forecast from rainfall is the determination of the depth of runoff per time interval of rainfall. The author's use of the simple ratios in Fig. 5 may appear to some engineers as a step backward in the science of hydrology. In a theoretical sense this is true, but the practical forecaster, at the time of making his forecast, very seldom has at hand all the data that ultimately become available. Neither does he have time for laborious computations. Hence, a short cut solution such as Mr. Turner has suggested, very well suits the conditions outlined in the paper. However, the author will probably agree that such a solution is not the best obtainable and that the search for better methods should continue.

The author may have minimized the importance of determining the ground-water flow because, under the conditions at Conowingo, ground water does not affect the peak capacity to the extent that it does at Holtwood and Safe Harbor (in Pennsylvania). With greater incentive in this direction, serious study has been given to the determination of the ground-water component of the total flow, which apparently behaves as though released from a reservoir about 300 sq miles in area, with a season variation in height of between 4 ft and 5 ft, and with a notched weir which controls the outflow entirely as a function of the reservoir height. Consequently, if the elevation of the water surface in this imaginary reservoir (sometimes called the ground-water index[5,6]) is known, and if it is assumed that under the worst possible conditions there would be no more inflow, then the assured outflow hydrograph for a number of weeks to come can be definitely determined.

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[5] "Analysis of Natural Fluctuations in Ground-Water Elevation," by Carroll F. Merriam, *Transactions*, Am. Geophysical Union, Vol. 23, 1942, pp. 598–603.

[6] "Interpretation of Natural Fluctuations of Ground Water in Terms of River Flow," by Carroll F. Merriam, *Journal*, A.W.W.A., July, 1945, p. 632.

Referring to Fig. 4 it appears that at the time the rainfall forecast was made on the evening of November 3 sufficient stage data on the upriver hydrographs were available to indicate that the estimates for November 4 and November 5 were too high. The rainfall between Conowingo and Harrisburg (Pa.), subarea No. 1 in Table 2, was relatively low, which probably accounts for no rise in actual flow until the evening of November 4.

The author's example covers a freshet with drawdown possibilities, where accurate forecast of flow makes possible a considerable additional generation. Reliable forecasts are also important for these Susquehanna River plants when the river is relatively stable—particularly if operating schedules indicate the desirability of certain forebay elevations at specified times. Referring to Fig. 3, assume an actual flow of 10,000 cu ft per sec. If the estimated rate of inflow for the period of hydro shutdown, 9:30 p.m. to 8:00 a.m., were 11,000 cu ft per sec, or 1,000 cu ft per sec higher than the actual, the forebay level at Conowingo at 8:00 a.m. would have been only 0.1 ft lower than scheduled, which would not have been serious. At Holtwood, however, where the storage per foot is considerably less than at Conowingo, a similar forecasting error would have made a difference of 0.4 ft in the forebay level.

One of the puzzles presented to a forecaster on the Susquehanna River, as on many rivers in the northern United States, is the effect of ice on river stages. Under ice conditions a rising stage at a particular station may be a legitimate indication of an impending increase in flow downstream. It may also be the the result of backwater behind an ice jam or of ice on the control, in which case the downstream flow would be temporarily decreased. This situation requires constant vigilance and the exercise of good judgment in studying the sequence of gage reports at the affected station and at other stations on the same stream.

Although the author has shown that system economy is affected by the reliability of the river flow estimates, it should be remembered that economy may also be affected by errors in the load forecasts. The errors in the flow and load estimates can be compensating, but they also can be cumulative. Should the load be underestimated and the flow overestimated, or the reverse, considerable readjustment in steam generation schedules might be required.

The author's observation regarding the effect of location of spill along the dam on plant tailwater is pertinent. At Safe Harbor, spill near the powerhouse raises the tailwater twice as much as the equivalent spill 2,000 ft farther away, which flows for some distance through channels separating it from the tailwater.

Small variations in inflow above power plant capacity are not so troublesome at Safe Harbor as they are at Conowingo because the use of remote controlled regulating gates, instead of crane operated gates, permits closer maintenance of maximum forebay level.

The operators of hydro plants in large steam systems may not always have control over the size of the total load assigned to the station, but they usually do have full responsibility as to the distribution of this load between available units. The author has very clearly explained how the best unit loadings on the Conowingo units are determined. Actual metering of the turbine discharge provides additional opportunities for improving station economy by detecting irregulari-

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ties in machine performance. Where Kaplan turbines are used, continuous water measurements are helpful in the detection of losses resulting from operation with incorrect relations between gate opening and blade pitch. At Safe Harbor, flow meters connected to scroll case piezometers not only provide a large part of the record of river flow, but also aid in checking unit performance and are the basis of a scheme whereby the operator receives a light signal when the incremental equivalent justifies a change in the number of units operating.[7]

Working on the same river with Mr. Turner, the writer is impressed with the importance of cooperation. By proper coordination considerable mutual assistance can be rendered from one company to another. When plants are closely located, as in this case, so that the time of wave travel from one to another is very short, coincidence in hours of daily water use permits much smaller fluctuations in forebay level in the lower plants than would be possible if each plant were the only plant on the river.

Gordon R. Williams,[8] Assoc. M. ASCE.—Hydrologic engineering is not only of vital importance in the original design of hydroelectric projects, but it can play an important part in the profitable operation of such projects. The author has used modern hydrologic techniques effectively to develop forecasts of mean 12-hr flows for 10 days or more following a storm. The problem differs from ordinary flood forecasting in that the entire runoff hydrograph must be estimated rather than merely an instantaneous peak stage and discharge.

The statement is made that the methods adopted are empirical rather than theoretical. It is safe to state that all hydrologic relationships are empirical in that no exact mathematical law can be devised to express such relationships. The degree of refinement used in developing and applying the relationships depends on the accuracy of the basic data, the time and funds available for computing the results, and the benefits to be obtained from a greater degree of refinement. The test of any empirical procedure, hydrologic or otherwise, is whether or not it gives the desired results. Apparently the river flow-forecasting method as developed and as modified by judgment and experience has proved satisfactory. The writer offers the following suggestions with the hope that they may be helpful if later refinements are necessary.

Runoff is considered a residual of rainfall after losses have been satisfied. Losses for various seasons are relatively constant; and, therefore, if runoff is expressed as a ratio to rainfall as shown in Fig. 5, there is a wide variation in the ratio with the depth of rainfall. Fig. 5 is based on a study of the relation between rainfall and runoff for the full duration of the storm and makes no allowance for the fact that percentage of runoff may also vary inversely with the duration of the storms. More consistent results could be obtained by computing losses as definite quantities and relating them to the duration of the storm. For example, if the basin rainfall were 3 in. in 24 hours and the total surface runoff were 1 in., the loss rate for that particular storm and season would have been 1 in. in 12 hours. The loss rate might also be correlated with the total antecedent rainfall for periods up to 10 days or more prior to the storm. If the foregoing procedure were substituted for that used in deriving Fig. 5, it is possible that a more consistent correlation would be obtained and that there would be less need for depending on judgment in applying the relationships to future storms.

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[7] "More Power Through High Operating Efficiencies of Hydro Electric Plants," by W. B. Hess and J. M. Mousson, *Bulletin*, Edison Electric Inst., July, 1941.

[8] Associate, Knappen Eng. Co., New York, N. Y.

In determining river discharge from rainfall excess, distribution graphs with runoff expressed as a percentage of the total runoff are illustrated in Fig. 6. There does not seem to be any advantage in the use of the distribution graph rather than the normal unit hydrograph from which the discharge for any storm is obtained directly by multiplying the ordinates by the inches of runoff. Since the desired results should be in average flows for 12-hr periods, typical unit hydrographs could be set up as "hystograms," or bar graphs, in which each bar represents the average discharge for a 12-hr period and the total of the bars is equal to 1 in. of runoff from a 12-hr period of rainfall excess.

The author mentions the difficulty of obtaining an average distribution graph which is representative of various storm distributions over the subareas. This difficulty is understandable when it is realized that subareas as large as one of 9,960 sq miles (that of the North Branch above Wilkes-Barre, Pa.) have been considered. Two courses of action are suggested—one is to increase the number of subareas and the other is to derive several distribution graphs for each subarea. The purpose of having several distribution graphs (or unit hydrographs) would be to have one applicable to the case where the rainfall was concentrated near the outlet, another for the case where the rainfall was concentrated near the headwaters, and possibly a third to take care of the case where the rainfall was concentrated in the center of the subarea.

The recession curves shown in Fig. 7 have been separated on a seasonal basis; and, without access to the basic data, it is not clear why there should be this difference. The recession for the first 10 days to 15 days represents depletion of channel storage and except for the volume of storage taken up by ice the amount of storage available in the winter should be essentially the same as that in the open-water months. Periods of heavy runoff in the winter are often followed by cold waves that cause runoff to be temporarily ponded in numerous small ice jams. This runoff is later released and may account for the fact that the curve for fall and winter is higher than the curve for spring and fall after an interval of 15 days. For periods of from 15 days to 35 days, the curves would normally represent ground-water flow rather than surface runoff coming out of channel storage.

JOHN S. COTTON,[9] M. ASCE.—The highly successful operation of the Conowingo plant in Maryland has been well described by Mr. Turner. Examination of the performance indicates that almost all available water and head are utilized effectively. The secret of this record is sufficient data on rainfall and runoff, and cooperation by all concerned. Because of rising fuel costs, such run-of-river plants are becoming more attractive.

Such a run-of-river hydro project, as exemplified by Conowingo, is highly economical and desirable. This type of project usually operates in conjunction with a large steam generating system, has considerable pondage, and usually has short transmission lines. If fuel is relatively costly, the installed capacity of hydro power is based primarily upon supplying large amounts of energy to the load curve. Such plants have moderate annual capacity factors and operate continuously to utilize the available flows as far as it is economically feasible. If fuel is relatively low priced, the installed capacity of hydro power is based primarily upon the relative capacity and energy demands of the load curve peak. Such plants have low fixed charges and annual capacity factors, and are used to carry system peaks. Usually, a compromise is made in order to obtain the greatest return, and the plant is operated at various positions in the load curve in order to utilize the seasonal water flow efficiently.

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[9] Cons. Engr., San Anselmo, Calif.

This utilization for Conowingo is demonstrated by the integrated load curve (Fig. 11). The integrated load curve shows clearly the relation between kilowatts and kilowatt-hours for each position of the hydro plant on the load curve. Table 4 shows daily capacity factors (line 2) and daily system load factors (line 4) derived from the integrated load curve. The hydro plant also furnished the necessary kilowatt-hours to the load for cases I, II, and III, as shown by lines 5 and 6, Table 4.

It will be noted that, for cases I, II, and III, the hydro at all times furnished its full kilowatt rating to the system load thus obviating the necessity of constructing steam plant capacity amounting to 260,000 kw. The annual value of capacity in the form of a steam generating plant, "hot and spinning," is at least from $18 to $20 per kw.

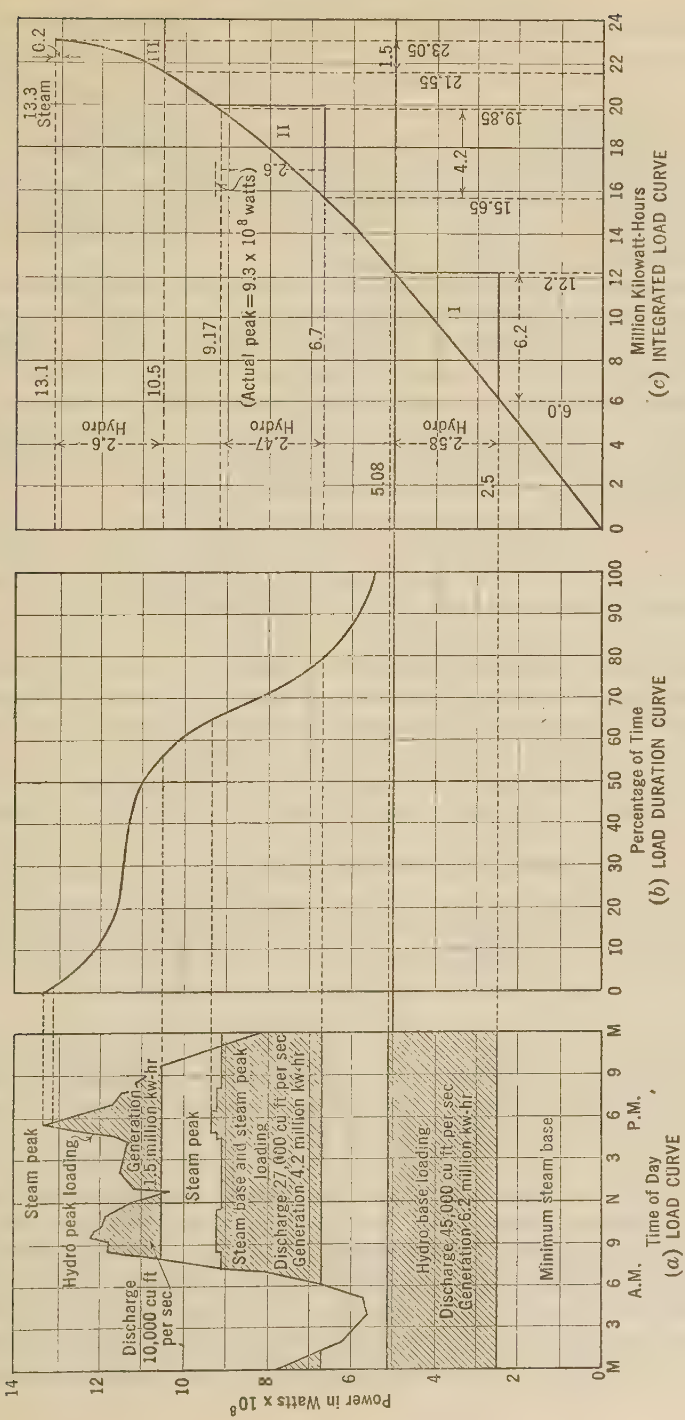
TABLE 4.—FACTORS DERIVED FROM THE INTEGRATED LOAD CURVES, FIG. 11(c)

Line	Factors (daily)	Case I	Case II	Case III	Case IV	Case V
1	Flow, in cubic feet per second.......	High	Medium	Low	4,000[a]	2,000[b]
2	Hydro capacity— 260,000 kw	100%	67.2%	24.0%	9.8%	5.2%
3	Steam capacity— 1,070,000 kw	65.3%	72.2%	83.7%	87.0%	89.0%
4	Load factor (system)— 1,330,000 kw	72.0%	72.0%	72.0%	72.0%	72.0%
5	Daily hydro energy output, in thousand kilowatt-hours...............	6,200	4,200	1,500	640	320
6	Dependable hydro capacity, in thousand kilowatts....................	260	260	260	....	...

[a] 90% of the time. [b] 100% of the time.

The hydro energy supplied would cost at least from 3 mills to 4 mills per kw-hr to produce in a steam generating plant. At times of low or moderate energy output from the hydro plant, the steam generating plant must generate more kilowatt-hours, the steam capacity remaining the same. This is indicated by the daily load factors for hydro and steam in lines 2 to 4, Table 4.

Since hydro could not fully supply the energy demands in cases IV and V, the full capacity of the plant could not be utilized and the hydro capacity factors shown for cases IV and V (lines 5 and 6, Table 4) could not be realized.



(a) LOAD CURVE

(b) LOAD DURATION CURVE

(c) INTEGRATED LOAD CURVE

FIG. 11

Working back in Fig. 11 from the available hydro energy, the dependable hydro capacities for cases IV and V would be:

Basis	Approximate dependable hydro capacity
90% time flow	215,000 kw
100% time flow	155,000 kw

It is obvious that, if hydro with pondage is given the very favorable position at the top of the load curve peak, a large dependable hydro capacity can be made available with very little hydro energy.

Except in very small isolated plants the basis of 100% time flow is not a reasonable criterion for dependable hydro capacity. For larger hydro plants with simple interconnections 90% time flows are commonly used. In systems as large as that in which the Conowingo plant operates, bases, of either 100% time flow or 90% time flow, are rather severe because: (*a*) Power systems are rarely loaded to maximum capacity; (*b*) spare steam equipment is available for use in infrequent emergencies; (*c*) the power construction program is ordinarily ahead of actual need; and (*d*) the systems are interconnected.

A run-of-river hydro plant in a small system usually is capable of generating energy in excess of load requirements. In large systems however, there is a greater amount of energy in the top section of the load curve per kilowatt of peak, and usually all the hydro energy can be utilized. Hydro utilization under the load curve increases roughly as the square root of the system load.

In a large integrated power system, most of the run-of-river hydro capacity, as well as the hydro energy, can be utilized under the load curve. Therefore, hydro capacity and energy should have the same value as steam capacity and energy, and not merely fuel saving (kilowatt-hour) value—provided that, when the hydro capacity is assigned to the peak of the load curve during low flows, hydro can (except on very rare occasions) furnish the corresponding energy called for in this part of the peak. If hydro, as the result of ordinary low flows and insufficient pondage, cannot supply enough energy to satisfy its full capacity at the peak loading, then only that part of the hydro capacity which has sufficient energy for that area of the peak can be regarded as firm. If greater flows exist, then the hydro capacity can be assigned a lower position on the load curve depending upon the available hydro energy.

Hence, it is primarily a matter of system experience in arriving at the practical dependable capacity of a run-of-river hydro project in a large integrated system. It appears then that the dependable capacity of Conowingo is equal to about 215,000 kw provided that hydro always has the most favorable positions in the load curve. This corresponds to about the 94% time flow on an annual basis.

The writer is of the opinion that certain standard power definitions are not applicable for large systems. Firm power is defined as "the power intended to be always available even under emergency conditions." A steam generating plant meets this condition, but a run-of-river hydro project in an integrated power system does not, although it furnishes considerable dependable kilowatt capacity and variable energy to the system load, in an acceptable manner. This definition is satisfactory if it is interpreted that furnishing power at variable capacity factors is allowable. In an isolated project the hydro capacity factor would be fixed by the total load whereas in an integrated system it need not be. A more logical definition in this case is, as follows:

> "Firm hydro power under system operation is that portion of the total installed capacity of the hydro plant which under existing conditions of load, stream flow, and pondage, is capable of performing the same function in serving the part of the load allocated to it that an alternative steam plant could perform."

On this basis, the firm power of Conowingo would be about 155,000 kw, or more

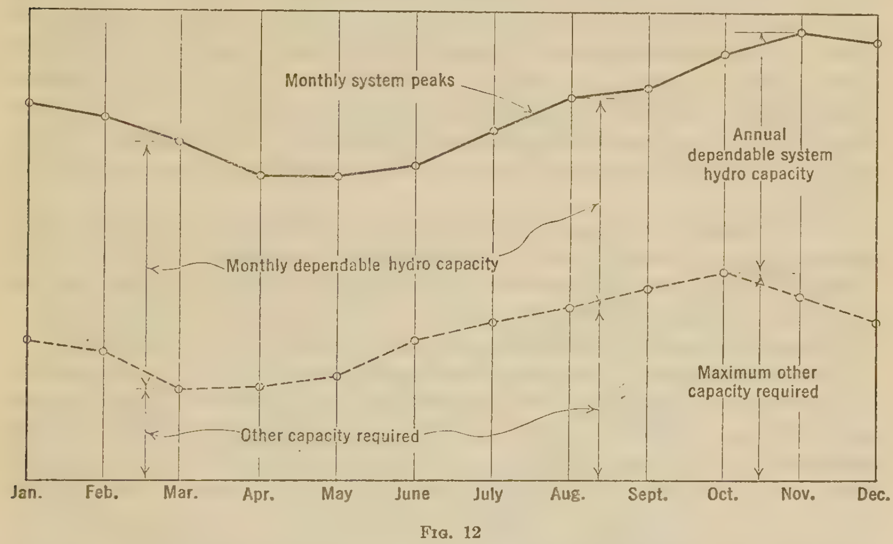


FIG. 12

Probably the fairest criterion in the case of hydro in an integrated system is "dependable hydroelectric capacity," as defined by the Federal Power Commission:

> "Dependable hydroelectric capacity is intended to be the capacity value of the system hydroelectric plants in serving, together with the other available system capacity, the maximum annual system peak load. For any specified period it represents, on the basis of complete utilization of available storage energy over the critical flow periods, the difference between the peak load for that period and the maximum other capacity required. Where a portion of storage energy is scheduled to be held as a reserve for emergency use only, the dependable capacity should also include the reserve capacity value of such energy reserve. The maximum monthly dependable capacity is not necessarily the same as the annual dependable hydroelectric capacity as the annual peak demand may not occur in the same month, requiring the maximum capacity from other than system hydroelectric plants."

This is illustrated in Fig. 12. The writer does not have sufficient data to determine the dependable capacity of Conowingo.

Robert E. Turner,[10] M. ASCE.—Mr. Justin has emphasized the point, concurred in by Messrs. Schuleen and Cotton, that the operation of electric generating systems supplied largely by steam plants can be aided greatly by a hydro plant of proper size and character not only because the hydro plant reduces the over-all cost of power generation but also because it supplies additional power capacity on extremely short notice to meet unforeseen load demands, load variations, and forced outages.

In striving for over-all system economy, the sacrifice of most efficient operation of the hydroelectric plant can result in relatively large steam electric operating savings. Thus, the energy generation of the hydro plant is reduced, but the value of the energy generated is increased proportionately. With such operation, the utilization factor defined by Mr. Justin (as the ratio of kilowatt-hours actually generated to the kilowatt-hours which theoretically might have been generated) is not a fair measure of performance. Even if the utilization factor is defined as the ratio of the river flow advantageously used to the river flow available for use, the evaluation of the term, "adventageously used," makes it difficult to compare plants on different systems on the basis of this factor.

Mr. Schuleen's statement that the sharply rising fuel costs place new emphasis on the energy value of hydroelectric generation is indisputable. Certainly during war and postwar periods when all available steam electric generating capacity is pressed into service, the best use of the available hydroelectric generation on an over-all basis is even more important and valuable.

Further extension of the river flow forecasting techniques and refinements in the methods as outlined by Mr. Schuleen and Mr. Williams are more important when it is necessary to set up definite flow estimates for specific periods, particularly when these estimates are used by others to obtain operating economies. However, when the potentialities of all the phases of the situation can be mutually recognized, discussed, and understood, it is more valuable to have a quick reliable estimate of river flows to obtain the most probable savings available than to take considerably more time to calculate rainfall losses, and to route the probable flows from a greater number of drainage areas—especially if complete information of precipitation is not immediately available.

Mr. Schuleen mentions the comparatively narrow flow range which is most important to forecast accurately for the purpose of obtaining system operating economies from the Conowingo plant in Maryland. The writer believes there is a similar narrow range of river flows which effect the greatest economy possibilities for every hydro plant on its interconnected system. On a potentially rising river it is usually sufficient to make a preliminary forecast as to whether flows will exceed certain values and then to follow up with more accurate estimates after rainfall ceases, headwaters begin to peak, and forecasting becomes progressively simpler and more reliable. When rainfall is predicted, it is well to analyze the possibilities and to make preliminary estimates of how much flow increase may be expected from several assumed values of rainfall.

It is granted that there would be some advantages in making the more detailed calculations mentioned; but more often, with the many indeterminate factors involved, it is as well to estimate the final answer by a short cut method as it is to calculate all the different individual approximations for which, many times, the complete data are not available when needed. Speed is important and is the major advantage of the short cut method.

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[10] Hydrographer, The Susquehanna Elec. Co., Conowingo, Md.

Mr. Schuleen's comment that the estimated flow plotted in Fig. 4 could have been revised in light of stage data and rainfall distribution is true, and in actual practice this revision was made, but the fact remains that the basic reason for the difference between the actual and the estimated flows was the effect of channel storage. The flow increase came more from the upper parts of the watershed, thereby bringing into effect more channel storage as the crest moved down to the mouth. The reverse effect of depletion of channel storage on the decreasing river flow is apparent, giving a difference in the opposite direction between estimated and actual flows. The actual volume of water checked quite closely with the estimated volume.

Mr. Schuleen's observation that an error in flow estimates results in greater change of pond level at Holtwood (in Pennsylvania) than at Conowingo is true but in actual practice, since the Safe Harbor plant (also in Pennsylvania) operates on the same system, the Safe Harbor pond can be used to minimize this wider variation; so the effect is about equal. The sum of the pondage per foot at both plants is about equal to that at Conowingo (see Table 1).

Part of the criticism Mr. Williams expresses in regard to the simple rainfall-runoff relations used in Fig. 5 is answered by the fact that the data for each 12-hr period are computed separately before being added to the previous period runoff. Antecedent conditions and character of rainfall can be considered.

Distribution graphs were included to illustrate the method but computation sheets obviate the use of the actual graphs. The suggestion of Mr. Williams that three distribution graphs be supplied for the larger areas to fit different types of rainfall concentrations has the major disadvantage that, unless the observation and communication facilities are greatly expanded and improved, too often information would be lacking to make the proper choice without delay and an improper choice may be worse than the use of average conditions.

The writer believes the difference in the two recession curves (Fig. 7) is due to greater evaporation losses during the summer than during the winter in the main and the tributary channels of the river. Actually during low flows, the lower part of the watershed seems to deliver more than its normally expected share during the greater evaporation season. However, the writer does agree with Mr. Williams that because data, unaffected by interim precipitation, were meager it is best to use only one recession curve since the inherent inaccuracies of the data chosen may account for much of the difference shown and, finally, the actual use of the curves does not justify the refinement.

Mr. Cotton stresses the importance of the "dependable" capacity of the Conowingo hydroelectric plant and states that insufficient data were given in the paper for the determination of this capacity. For purposes of presentation, the writer limited his paper to a discussion of how the most effective operation of the Conowingo plant on the load of the Philadelphia Electric System at the time was obtained. Actually the plant was, and is, operated on a much larger system, including Federal Power Commission areas 5 and 6, in conjunction with the hydroelectric plants on the lower Susquehanna River listed in Table 1, as well as a 40,000-kw seasonal storage hydroelectric plant and smaller nominal run-of-river hydro plants in interconnected areas.

The writer agrees with Mr. Cotton that firm hydro capacity should be defined as related to alternative steam capacity on the system and not as related only to low river flows. Much confusion is introduced in discussions relating to this matter by indiscriminate use of the terms "firm power" and "firm capacity" as synonymous. Actually "power" and "capacity" do not have the same meaning—power is the rate at which energy is supplied and capacity is the maximum ability to supply power.

In addition, the use of the term "dependable" requires definition or qualification. The determination of "dependable" hydro capacity involves many factors; some are limited to individual systems and some are controversial and not considered in the paper. The writer would like to refer those interested in these matters to a paper and its subsequent discussion published elsewhere.[11]

The writer wishes to express his thanks to Messrs. Justin, Schuleen, Williams, and Cotton for bringing out corollary ideas and for assisting in the clarification of parts of the paper.

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[11] "Probability Methods Applied to Generating Capacity Problems of a Combined Hydro and Steam System," by E. S. Loane and C. W. Watchorn, *Transactions*, A.I.E.E., Vol. 66, 1947, pp. 1645–1657.

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EXPERIMENTAL DETERMINATION OF VIBRATION CHARACTERISTICS OF STRUCTURES

By Samuel J. Loring,[1] Assoc. M. ASCE

With Discussion by Messrs. Alexander Klemin, R. K. Bernhard, Carl A. Ludeke, M. J. Turner, F. B. Farquharson, J. J. Slade, Jr., and Samuel J. Loring.

Synopsis

A method is described in this paper for experimentally determining vibration characteristics by observing the characteristic motion of a small test mass attached to the structure by a linear spring. The specific motions that are observed are those following the release of the test mass from rest in a displaced position and for a series of closely spaced frequencies of the test mass on its spring. A possible arrangement of the test mass is shown in Fig. 1. The outer frame to which the springs are attached is itself attached to the structure by the fitting provided at the top. Small adjustments in frequency are made by removing various parts of the segmented mass; larger variations can be attained by changing the springs. Since observations are restricted to vibration amplitudes they can be made visually on the wedge-shaped pointer attached to the mass as shown in Fig. 1.

The analysis presented shows that the ratio of the test mass weight to the structural weight may range from about 10^{-2} to 10^{-6}. Consequently, the test mass will be relatively small. For this reason and also because of the extreme simplicity of the device, the method introduced should be particularly adaptable to the testing of large structures.

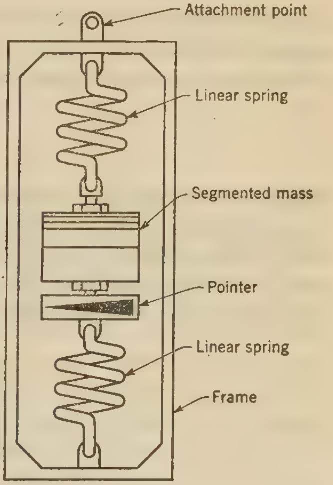

Fig. 1.—Schematic Arrangement of Test Mass

Note.—Published in December, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Consultant on Technical Problems, Stratford, Conn.

In Section 2 a preliminary review of the general vibration characteristics of structures is presented in order to place a precise numerical meaning on "vibration characteristics of structures." This is essential, of course, if precise measurements are to be made. The review also forms a basis for the analysis of the observations on the test mass described in Sections 3 and 4.

The theory has been verified by tests with an instrument designed to exploit this method of vibration testing. The instrument and the results obtained with it are described in Section 5. Several uses of this method of testing for the determination of quantities other than vibration characteristics, such as loads in columns or ties, weights, and effective stiffnesses of structures, are also suggested in Section 5.

NOTATION

The letter symbols in this paper are defined in the text where they first appear and are assembled for convenience of reference in the Appendix.

2. SPECIFICATION OF VIBRATION CHARACTERISTICS OF STRUCTURES

The method of specifying vibration characteristics of structures, described herein, is based on results of the general theory of elastic vibrations of systems with small damping. An excellent account of this general theory has been given by J. W. Strutt, Baron Rayleigh.[2]

The state of distortion of any system from a position of equilibrium is specified by the field of the displacement vectors δ of all its points. The general displacement field δ can be expressed as a linear combination of a set of fixed and completely specified displacement fields ϕ_i; thus:

$$\delta = \sum_i (a_i \phi_i) \qquad \text{(1)}$$

Since the fields ϕ_i have the dimension length of a displacement, the amplitudes a_i are dimensionless scalar quantities; and they will generally be functions of time. According to the general theory of small vibrations there is a unique set of displacement fields ϕ_i for each structure for which the expressions for the kinetic energy T and potential energy V take the forms:

$$T = \tfrac{1}{2} I_r \sum_i \left(\frac{da_i}{dt}\right)^2 \qquad \text{(2a)}$$

and

$$V = \tfrac{1}{2} K_r \sum_i (f_i a^2{}_i) \qquad \text{(2b)}$$

The quantities I_r and K_r are dimensional constants the values of which should be chosen to reduce the other quantities to convenient magnitudes; the dimensions of these quantities are, respectively, force × length × time × time (FLT^2) and force × length (FL). The coefficients f_i are dimensionless quantities.

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[2] "The Theory of Sound," by J. W. Strutt, Baron Rayleigh, Macmillan & Co., London, 1937.

The particular set of displacement fields ϕ_i which result in Eqs. 2 are the normal coordinates of the structure, which are fundamental characteristics of vibratory systems. In the sequel the normal coordinates will be chosen as the ϕ_i-values of Eq. 1. With this choice the La Grangian equations of motion of any system reduce to

$$I_r \frac{d^2 a_i}{dt^2} + K_r f_i a_i = Q_i; i = 1, 2, \cdots \ldots\ldots\ldots\ldots (3)$$

The quantities Q_i in these equations are the generalized external forces on the system corresponding to the respective normal coordinates. The approximate effect of small damping forces in a structure can be represented by introducing into the equations of motion (Eq. 3) a set of angles g_i ($i = 1, 2, \cdots$) by which the restoring force leads the displacement in the ith normal coordinate. This is accomplished by the use of the complex notation and by replacing the quantities f_i in Eq. 3 by $f_i e^{jg_i} \approx f_i (1 + j g_i); j = \sqrt{-1}$. The approximation used is justified since only small damping is considered, for which g_i is less than about 0.10. The equations of motion including the effect of small damping are, therefore,

$$I_r \frac{d^2 a_i}{dt^2} + K_r f_i (1 + j g_i) a_i = Q_i; i = 1, 2, \cdots \ldots\ldots\ldots\ldots (4)$$

Eq. 4 shows that the motion in each normal coordinate of a structure behaves just as the motion in a single degree of freedom system; there is, therefore, a natural frequency ω_i and a damping factor g_i associated with each normal coordinate ϕ_i. The expression for the frequency ω_i in terms of the quantities appearing in Eq. 4 is

$$\omega_i = \sqrt{\frac{K_r f_i}{I_r}} = \omega_r \sqrt{f_i} \ldots\ldots\ldots\ldots (5)$$

in which ω_r, the reference frequency, is equal to $\sqrt{\frac{K_r}{I_r}}$. Although the dimensional constants I_r and K_r may be chosen arbitrarily, their actual values have no intrinsic effect upon these results and their introduction is merely a convenience. Actually the choice of the value of I_r has only the effect of setting the scale of the normal coordinates ϕ_i, whereas the value of K_r sets the value of the reference frequency ω_r. When the values of I_r and K_r have been chosen, the displacement fields ϕ_i, the frequencies ω_i, and the damping factors g_i associated with each of the normal coordinates completely describe the vibration characteristics of a structure. For all real structures, of course, there are an infinite number of normal coordinates which are ordered in sequence by their associated frequencies. In practice, however, knowledge of a relatively few normal coordinates whose frequencies lie below some specified limit is sufficient. It is also sufficient to describe the continuous displacement fields ϕ_i by the values of relatively few components $(\phi_i)_k$ at a few key points on the structure.

It will be of interest to investigate further the property of the normal coordinates acting independently as single degrees of freedom, particularly with reference to displacements and applied forces at a specific point of a structure and in a specific direction. Let the component of the normal coordinate ϕ_i for a particular point and in a particular direction be denoted by $(\phi_i)_k$. In this notation the subscript i identifies the normal coordinate; and k, the point and direction in question. Following the representation of Eq. 1, the total displacement component δ_k at this point and in this direction has the value:

$$\delta_k = \sum_i a_i\,(\phi_i)_k = \sum_i (\delta_i)_k \qquad (6a)$$

and is evidently the algebraic sum of the components—

$$(\delta_i)_k = a_i\,(\phi_i)_k \qquad (6b)$$

—attributable to motion in each of the normal coordinates. The values of the generalized forces Q_i corresponding to a force component P_k applied to the structure along the line of $(\phi_i)_k$ and $(\delta_i)_k$ are given by

$$Q_i = (\phi_i)_k\,P_k \qquad (7)$$

Multiplication of each of Eqs. 4 by the corresponding value of $(\phi_i)_k$ and use of Eqs. 5, 6*b*, and 7 leads to the following equations of motion:

$$(m_i)_k\,\frac{d^2(\delta_i)_k}{dt^2} + (k_i)_k\,(1 + j\,g_i)\,(\delta_i)_k = P_k;\, i = 1, 2, \cdots \qquad (8a)$$

in which

$$(m_i)_k = \frac{I_r}{(\phi_i)^2_k}\,; \quad (k_i)_k = (m_i)_k\,\omega^2_i \qquad (8b)$$

The new formula (Eq. 8*a*) demonstrates the fact that, as far as forces P_k are concerned, the motion in each component $(\delta_i)_k$ of the total deflection δ_k behaves exactly like motion in a single degree of freedom system with mass $(m_i)_k$, damping, g_i, and natural frequency ω_i. The relationship between the "effective" mass $(m_i)_k$ and the normal coordinate component $(\phi_i)_k$ is shown in Eq. 8*b*.

Eq. 8*a* represents a type of equation of motion that describes completely the manner in which the response characteristics of a structure along a particular line of action and at a single point are related to the vibration characteristics of the entire structure. This relation will form the basis of the analysis presented in Section 3.

3. GENERAL BEHAVIOR OF TEST MASS MOTIONS

The method proposed for measuring the vibration characteristics of a structure consists in observing the character of motion of a small test mass which is attached to a point on the structure by a linear spring. Specifically, the observations are to be made on the motions of this test mass following its release from a displaced position, for a series of closely spaced natural frequencies of the mass on its spring. The analysis presented herein is directed toward finding the relationship between these observed motions and the vibration characteristics of the structure to which the test mass is attached. The motions to be observed are determined analytically in terms of the vibration characteristics of the structure itself and of the test mass spring system. General expressions are obtained for the free vibration modes of the vibratory system formed by the combination of the structure and the test mass; and these general expressions are specialized for the appropriate initial conditions to represent the motions to be observed. The required relationship between the observed motions and the vibration characteristics of the structure itself are then deduced from these specialized expressions.

The characteristics of the test mass and spring are the mass itself, m_o; the spring stiffness, k_o; and the effective spring damping factor g_o. The "frequency" ω_o of the test mass is defined as

$$\omega_o = \sqrt{\frac{k_o}{m_o}} \qquad (9)$$

and is equal to its natural frequency when the end of the linear spring is rigidly fixed. Since all interaction between the test mass and the structure takes place along a single line of action and at a single point, only the response characteristics at this point and in this direction are effective in shaping the character of motion of the mass. Equations of motion of the type of Eq. 8*a* are convenient for appraising the required characteristics of the structure. By use of these equations with—

$$P_k = -m_o \frac{d^2x}{dt^2} \qquad (10)$$

—the equations of motion of the combined structure and test mass may be written

$$m_o \frac{d^2x}{dt^2} + k_o (1 + j\, g_o)[x - \sum_i (\delta_i)_k] = 0 \qquad (11a)$$

and

$$(m_i)_k \frac{d^2(\delta_i)_k}{dt^2} + (k_i)_k (1 + j\, g_i)(\delta_i)_k + m_o \frac{d^2x}{dt^2} = 0;\ i = 1, 2, \cdots \qquad (11b)$$

The quantity x in Eqs. 11 represents the displacement of the test mass. The point and direction to which the subscript k refers are, of course, the point of attachment of the test mass spring and the direction of motion of the test mass.

Following the usual method of treating equations in the form of Eqs. 11, expressions of the known form of the solution—

$$(\delta_i)_k = (X_i)_k\, e^{j\lambda t};\quad x = X_o\, e^{j\lambda t} \qquad (12)$$

—are substituted into the equations. The exponential factor λ and the amplitudes $(X_i)_k$ and X_o are determined from the resulting algebraic equations:

$$\left[1 + j\, g_o - \left(\frac{\lambda}{\omega_o}\right)^2\right] X_o - \sum_i (X_i)_k = 0 \qquad (13a)$$

and

$$-\frac{m_o}{(m_i)_k}\left(\frac{\lambda}{\omega_i}\right)^2 X_o + \left[1 + j\, g_i - \left(\frac{\lambda}{\omega_i}\right)^2\right](X_i)_k = 0;\ i = 1, 2, \cdots \qquad (13b)$$

The corresponding determinantal equation for λ has the form:

$$\begin{vmatrix} 1+jg_o-\left(\frac{\lambda}{\omega_o}\right)^2 & -1 & -1 & -1 & \cdots \\ -\frac{m_o}{(m_1)_k}\left(\frac{\lambda}{\omega_1}\right)^2 & 1+jg_1-\left(\frac{\lambda}{\omega_1}\right)^2 & 0 & 0 & \cdots \\ -\frac{m_o}{(m_2)_k}\left(\frac{\lambda}{\omega_2}\right)^2 & 0 & 1+jg_2-\left(\frac{\lambda}{\omega_2}\right)^2 & 0 & \cdots \\ -\frac{m_o}{(m_3)_k}\left(\frac{\lambda}{\omega_3}\right)^2 & 0 & 0 & 1+jg_3-\left(\frac{\lambda}{\omega_3}\right)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0 \tag{14}$$

The expanded form of Eq. 14 can be written

$$1 + j g_o - \left(\frac{\lambda}{\omega_o}\right)^2 - \sum_i \frac{m_o}{(m_i)_k} \frac{\left(\frac{\lambda}{\omega_i}\right)^2}{1 + j g_i - \left(\frac{\lambda}{\omega_i}\right)^2} = 0 \tag{15}$$

Finally, the expressions for the amplitude ratios are obtained directly from Eqs. 13:

$$\frac{(X_i)_k}{X_o} = \frac{m_o}{(m_i)_k} \times \frac{\left(\frac{\lambda}{\omega_i}\right)^2}{1 + j g_i - \left(\frac{\lambda}{\omega_i}\right)^2} ; i = 1, 2, \cdots \tag{16}$$

The solutions of Eq. 15 for λ are substituted into Eq. 16 to obtain numerical values for the amplitude ratios.

Each term in the summation of Eq. 15 contains a factor $m_o/(m_i)_k$ which is very small because the test mass m_o is very small compared to each of the effective masses $(m_i)_k$. Consequently, for a given value of λ, all the terms of the summation are very small with the possible exception of a single term:

$$\frac{m_o}{(m_h)_k} \frac{\left(\frac{\lambda}{\omega_h}\right)^2}{1 + j g_h - \left(\frac{\lambda}{\omega_h}\right)^2} \tag{17}$$

—if the value of λ is such the $1 + j g_h - \left(\frac{\lambda}{\omega_h}\right)^2 \approx 0$. Therefore, the roots of Eq. 15 may be determined by neglecting all but one term in the summation and solving the resulting quadratic equation:

$$1 + j g_o - \left(\frac{\lambda}{\omega_o}\right)^2 - \frac{1}{r} \frac{\left(\frac{\lambda}{\omega_h}\right)^2}{1 + j g_h - \left(\frac{\lambda}{\omega_h}\right)^2} = 0 \tag{18}$$

in which the quantity r equal to $(m_h)_k/m_o$ will be called the "mass ratio," and is equal to the ratio of the effective mass of the structure in the mode under consideration to the test mass. This ratio is assumed to be a large quantity; and its reciprocal, a correspondingly small quantity.

There is an equation of the form of Eq. 18, of course, representing the interaction of the test mass with each structural mode; but it will be noted that the treatment of the one equation in its general form will reveal the complete nature of the test mass responses.

In the solution to the quadratic Eq. 18 only the lowest power in the small quantities g_o, g_h, and $1/\sqrt{r}$ is retained. For the expression of this solution it is convenient to introduce two additional quantities which express the relation of the test mass vibration characteristics to those of the structural normal mode under consideration. These quantities and a suitable terminology are as follows: The "tuning index" of test mass natural frequency relative to the structural natural frequency ω_h is equal to

$$\Delta = \left(\frac{\omega_o}{\omega_h} - 1 \right) \sqrt{r} \quad \text{(19}a\text{)}$$

and the "damping index" equals

$$G = \tfrac{1}{2} \sqrt{r}\, (g_h - g_o) \quad \text{(19}b\text{)}$$

In terms of the tuning and damping indexes the solution of the quadratic Eq. 18 may be written

$$\left(\frac{\lambda}{\omega_o} \right)^2 = 1 + j\, g_o + \frac{-\Delta + j\, G \pm \sqrt{1 + (\Delta - j\, G)^2}}{\sqrt{r}} \quad \text{(20)}$$

It is convenient to write Eq. 20 in a form which separates the real and imaginary parts explicitly. This may be accomplished by introducing quantities α and β given by:

$$\alpha = \sqrt{+ \left(\frac{1 + \Delta^2 - G^2}{2} \right) + \sqrt{\left(\frac{1 + \Delta^2 - G^2}{2} \right)^2 + \Delta^2 G^2}} \quad \text{(21}a\text{)}$$

and

$$\beta = \sqrt{- \left(\frac{1 + \Delta^2 - G^2}{2} \right) + \sqrt{\left(\frac{1 + \Delta^2 - G^2}{2} \right)^2 + \Delta^2 G^2}} \quad \text{(21}b\text{)}$$

The quantities α and β are determined so that

$$\alpha - j\, \beta = \sqrt{1 + (\Delta - j\, G)^2} \quad \text{(22)}$$

Accordingly, Eq. 20 may finally be written

$$\left(\frac{\lambda}{\omega_o} \right)^2 = \left(1 - \frac{\Delta \mp \alpha}{\sqrt{r}} \right) + j \left(g_o + \frac{G \mp \beta}{\sqrt{r}} \right) \quad \text{(23}a\text{)}$$

and

$$\frac{\lambda}{\omega_o} = \left(1 - \frac{\Delta \mp \alpha}{2\sqrt{r}} \right) + j \left(\frac{g_o}{2} + \frac{G \mp \beta}{2\sqrt{r}} \right) \quad \text{(23}b\text{)}$$

The amplitude ratios of Eq. 16 are equal to the terms of the summation of Eq. 15, and are all very small, with the possible exception of the one corresponding to the term retained in Eq. 18. By use of Eq. 18 and Eqs. 23, this one remaining amplitude ratio $(X_h)_k/X_o$ is found to be

$$\frac{(X_h)_k}{X_o} = \frac{(\Delta \mp \alpha) - j\,(G \mp \beta)}{\sqrt{r}} \qquad (24)$$

The general expressions for the motions of the test mass and structure can be written by the use of Eqs. 23 and 24. Only two degrees of freedom are significant; accordingly, the general expressions for these motions will contain four arbitrary constants. The constants are adjusted to meet the initial conditions of release of the test mass from rest at an initial displacement A_o from equilibrium, which are

$$t = 0;\ (\delta_h)_k = \frac{d(\delta_h)_k}{dt} = \frac{dx}{dt} = 0;\ x = A_o \qquad (25)$$

When conditions defined by Eqs. 25 are applied, with g_o and $1/\sqrt{r}$ treated as small quantities, the resulting expression for the test mass motion is

$$x = A_o\, e^{-og_o} \Bigg(e^{-ou} \Bigg\{ \Bigg[\tfrac{1}{2} + \frac{\alpha\Delta + G\beta}{2\,(\alpha^2+\beta^2)} \Bigg] \cos\left(1 - \frac{\Delta - \alpha}{2\sqrt{r}}\right) \omega_o t$$

$$+ \Bigg[\frac{G\alpha - \beta\Delta}{2\,(\alpha^2+\beta^2)} \Bigg] \sin\left(1 - \frac{\Delta - \alpha}{2\sqrt{r}}\right) \omega_o t \Bigg\} + e^{-ov} \Bigg\{ \Bigg[\tfrac{1}{2} - \frac{\alpha\Delta + G\beta}{2\,(\alpha^2+\beta^2)} \Bigg]$$

$$\times \cos\left(1 - \frac{\Delta + \alpha}{2\sqrt{r}}\right) \omega_o t - \Bigg[\frac{G\alpha - \beta\Delta}{2\,(\alpha^2+\beta^2)} \Bigg] \sin\left(1 - \frac{\Delta + \alpha}{2\sqrt{r}}\right) \omega_o t \Bigg\} \Bigg) \qquad (26)$$

In Eq. 26, and in the equations that follow, the symbols o, u, and v have been introduced as substitution factors to simplify typography, as follows:

$$o = \tfrac{1}{2}\,\omega_o t \qquad (27a)$$

$$u = \frac{G - \beta}{\sqrt{r}} \qquad (27b)$$

and

$$v = \frac{G + \beta}{\sqrt{r}} \qquad (27c)$$

With some rearrangement and by use of trigonometrical identities, Eq. 26 for the test mass motions can be rewritten in a more illuminating form. The assumption that $1/\sqrt{r}$ is a small quantity is retained:

$$x = A_o\, e^{-og_o} \Bigg[\Bigg(\Bigg(\Bigg\{ \Bigg[\tfrac{1}{2} + \frac{\alpha\Delta + G\beta}{2\,(\alpha^2+\beta^2)} \Bigg] e^{-ou} + \Bigg[\tfrac{1}{2} - \frac{\alpha\Delta + G\beta}{2\,(\alpha^2+\beta^2)} \Bigg] e^{-ov} \Bigg\}$$

$$\times \cos \frac{\alpha}{2\sqrt{r}}\,\omega_o t + \frac{G\alpha - \beta\Delta}{2\,(\alpha^2+\beta^2)}\,(e^{-ou} + e^{-ov}) \sin \frac{\alpha}{2\sqrt{r}}\,\omega_o t \Bigg) \cos \omega_o t$$

$$+ \Bigg(\Bigg\{ - \Bigg[\tfrac{1}{2} + \frac{\alpha\Delta + G\beta}{2\,(\alpha^2+\beta^2)} \Bigg] e^{-ou} + \Bigg[\tfrac{1}{2} - \frac{\alpha\Delta + G\beta}{2\,(\alpha^2+\beta^2)} \Bigg] e^{-ov} \Bigg\} \sin \frac{\alpha}{2\sqrt{r}}\,\omega_o t$$

$$+ \frac{G\alpha - \beta\Delta}{2\,(\alpha^2+\beta^2)}\,(e^{-ou} - e^{-ov}) \cos \frac{\alpha}{2\sqrt{r}}\,\omega_o t \Bigg) \sin \omega_o t \Bigg] \qquad (28)$$

It is evident from Eq. 28 that the test mass motion is a vibration of frequency ω_o which has a slowly varying amplitude A given by

$$A = A_o e^{-\sigma g_o} \psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

and the factor ψ is defined as

$$\psi^2 = \frac{(\Delta + \alpha)^2 + (G + \beta)^2}{4 (\alpha^2 + \beta^2)} e^{-2\sigma u} + \frac{(\Delta - \alpha)^2 + (G - \beta)^2}{4 (\alpha^2 + \beta^2)} e^{-2\sigma v}$$
$$+ e^{-2\sigma z} \left\{ \left[\tfrac{1}{2} - \frac{\Delta^2 + G^2}{2 (\alpha^2 + \beta^2)} \right] \cos \frac{\alpha}{\sqrt{r}} \omega_o t + \left(\frac{G \alpha - \beta \Delta}{\alpha^2 + \beta^2} \right) \sin \frac{\alpha}{\sqrt{r}} \omega_o t \right\} \ldots (30)$$

in which, to simplify typography,

$$z = \frac{G}{\sqrt{r}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

The amplitude of the test mass vibration subsequent to its release is equal to the amplitude:

$$A_o e^{-\sigma g_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

which it would have if mounted on a fixed base, multiplied by the factor ψ. The factor ψ is a function of the indexes G, Δ, and r which relate the test mass vibration characteristics to those of the structure associated with the natural frequency ω_h.

It may be observed by inspection of Eqs. 21 that, when the tuning index Δ assumes large values, the quantity α approaches the value Δ, whereas the quantity β approaches the value G. Upon substitution of these limiting values into Eq. 30 it becomes evident that the corresponding value of ψ approaches unity; that is,

$$\Delta \to \infty ; \alpha \to \Delta, \beta \to G, \psi \to 1.0 \ldots\ldots\ldots\ldots (33)$$

This fact leads to the conclusion that the test mass motions are unaffected by the structure except when the tuning index is small—that is, except when the test mass frequency is tuned close to a natural frequency of the structure.

Subsequent analysis shows that the test mass frequency must lie within about 5% of a structural natural frequency before the effect of the factor ψ becomes observable. This fact justifies the use of the approximation, Eq. 18, separately for each structural mode.

4. INTERPRETATION OF TEST MASS MOTIONS

It has been shown that, except when the test mass is tuned very close to a natural frequency of the structure on which it is mounted, the test mass behaves substantially as if it were mounted on a rigid support. The procedure for interpreting the indications of the test mass, therefore, is to observe its characteristic motions for a set of closely spaced frequencies until a deviation from its rigid support motions is noted. Such a deviation is evidence that the test mass is tuned close to a natural frequency of the structure.

The next step is the recognition of the condition of exact tuning for which the structural frequency is just equal to the test mass frequency. This condition of exact tuning corresponds to $\Delta = 0$. In evaluating the parameters

α and β by Eqs. 21 with $\Delta = 0$, it becomes evident that two cases must be distinguished, respectively, for $G < 1.0$ and for $G > 1.0$. These two cases will be discussed separately.

Case I: $G < 1.0$.—In this case the limiting values for α and β, when $\Delta = 0$, are: $\alpha = \sqrt{1 - G^2}$ and $\beta = 0$. With these values Eq. 30 for ψ reduces to

$$\psi = e^{-oz} \left(\cos \frac{\sqrt{1 - G^2}}{2\sqrt{r}} \omega_o t + \frac{G}{\sqrt{1 - G^2}} \sin \frac{\sqrt{1 - G^2}}{2\sqrt{r}} \omega_o t \right) \quad \ldots .(34)$$

A test mass motion corresponding to Eq. 34 is readily recognizable since the amplitude is oscillatory and periodically becomes zero. Alternating with the zeros of the test mass amplitudes are peaks, corresponding to the peaks of Eq. 34. Further observations of a test mass response of this type readily lead to a determination of the parameters G and r. The ratio of two successive peak values of the factor ψ is

$$e^{-\pi G/\sqrt{1-G^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35a)$$

and the time interval between successive zeros of ψ is

$$\frac{2\pi\sqrt{r}}{\omega_o\sqrt{1 - G^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35b)$$

By use of these relations the parameters r and G can be determined from observations on the test mass amplitude when tuned to a structural natural frequency. The effective mass and damping of the structure can then be determined from the relations defining r and G.

Case II: $G > 1.0$.—The treatment of this case will include the limiting case $G = 1.0$. Setting $\Delta = 0$ in Eqs. 21: $\alpha = 0$ and $\beta = \sqrt{G^2 - 1}$. With these values Eq. 30 for ψ reduces to

$$\psi = \tfrac{1}{2}\left(1 + \frac{G}{\sqrt{G^2 - 1}}\right) e^{-(G-\sqrt{G^2-1})o/\sqrt{r}} + \tfrac{1}{2}\left(1 - \frac{G}{\sqrt{G^2 - 1}}\right) e^{-(G+\sqrt{G^2-1})o/\sqrt{r}} \quad \ldots\ldots\ldots (36)$$

In this case the amplitude variation is not oscillatory, but continuously decreasing. On this account it has been found impracticable to make observations on the shape of the amplitude variation at exact tuning to determine the two parameters r and G. The form of Eq. 36 does show, however, that the amplitude of the test mass decreases more rapidly at exact tuning in this case than on a rigid mount. The rate of decay of the amplitude of the test mass can be readily measured, of course, but to obtain a second observation with which to determine both parameters r and G the approach to exact tuning will be considered.

For $G \gtreqless 1.0$ and $\Delta^2 + G^2 > 1.0$, Eqs. 21 for α and β are very closely approximated by the simplified formulas:

$$\beta \approx G\left[1 - \frac{1}{2(\Delta^2 + G^2)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (37a)$$

and

$$\alpha \approx \Delta \left(\frac{\Delta^2 + G^2}{\Delta^2 + G^2 - \frac{1}{2}} \right) \qquad (37b)$$

Inspection of Eq. 30 in the light of these approximations shows that the first exponential term diminishes very much more slowly than the others, and also that its coefficient is very nearly equal to unity. Accordingly, for sufficiently large values of t the expression for ψ reduces approximately to

$$\psi \approx e^{-o\eta} \qquad (38)$$

and the test mass amplitude A from Eq. 29 is given by

$$A \approx A_o\, e^{-o(g_o+\eta)} \qquad (39)$$

in which, to simplify typography,

$$\eta = \frac{G}{2\sqrt{r}\,(\Delta^2 + G^2)} \qquad (40)$$

It is easy to measure the time interval t_θ required for the test mass amplitude to decrease from an initial value of A_o to a value $\theta\, A_o$ ($\theta < 1.0$). According to Eq. 39, this time interval is given by

$$t_\theta = \frac{1}{\omega_o} \; \frac{-2 \log_e \theta}{g_o + \dfrac{G}{2\sqrt{r}\,(\Delta^2 + G^2)}} \qquad (41)$$

The value of the time interval $(t_\theta)_o$ for the same amplitude range but with the test mass on a fixed base is

$$(t_\theta)_o = \frac{1}{\omega_o} \times \frac{-2 \log_e \theta}{g_o} \qquad (42)$$

This latter time interval is characteristic of the test mass and can be considered as known. Therefore, the ratio of t_θ to $(t_\theta)_o$ (which will be denoted by χ) can readily be measured for any given value of θ for a series of test mass frequencies ω_o. Dividing Eq. 41 by Eq. 42 the value of χ is given by

$$\chi = \frac{1}{1 + \dfrac{G}{2\, g_o\, (\Delta^2 + G^2)\sqrt{r}}} \qquad (43a)$$

Eq. 43*a* may be rearranged in the following form:

$$\frac{2\, g_o\, (\Delta^2 + G^2)\sqrt{r}}{G} = \frac{\chi}{1 - \chi} \qquad (43b)$$

It is evident from the form of Eq. 43*a* that the minimum value, χ_m, occurs at $\Delta = 0$—that is, χ_m occurs at exact tuning. Since the minimum of χ is readily observable, the condition of exact tuning is easily recognized. Eq. 43*b* for the condition of exact tuning reduces to

$$2\, g_o\, G \sqrt{r} = \frac{\chi_m}{1 - \chi_m} \qquad (44a)$$

Using Eq. 44*a* to eliminate the quantity G in Eq. 43*b*,

$$(2\,g_o\,\Delta)^2\,r = \frac{\chi_m}{1-\chi_m}\left(\frac{\chi}{1-\chi} - \frac{\chi_m}{1-\chi_m}\right) \tag{44b}$$

Eliminate g_o from Eq. 44*b* by Eq. 42 and Δ by use of the defining relation (see Eq. 19*a*); thus:

$$\Delta = \frac{\omega_o - \omega_h}{\omega_h}\sqrt{r} \tag{45}$$

It is convenient to introduce the number n_m given by

$$n_m = \frac{\chi_m\,(t_\theta)_o\,\omega_o}{2\,\pi} \tag{46}$$

which represents the number of cycles of the test mass motion in the time interval t_θ at which the minimum χ_m is observed. This number is readily determined from observations at exact tuning. Eq. 44*b* may now be written

$$\left[\frac{-\,2\,\chi_m\log_e\theta}{\pi\,n^2_m}\left(\frac{\omega_o-\omega_h}{\omega_h}\right)n_m\,r\right]^2 = \frac{\chi_m}{1-\chi_m}\left(\frac{\chi}{1-\chi} - \frac{\chi_m}{1-\chi_m}\right) \tag{47}$$

The value of the structural natural frequency ω_h has been determined as that value of the test mass frequency at which χ_m is observed. The value ω'_o of the test mass natural frequency at which $\chi = (1 + \chi_m)/2$ can also be determined readily by observation, and the value of the quantity—

$$Y = n_m\sqrt{\left(\frac{\omega'_o - \omega_h}{\omega_h}\right)^2} \tag{48}$$

—can be computed. Eq. 47 can now be solved for the mass ratio r in terms of known quantities:

$$r = \frac{n^2_m}{\kappa^2\,Y} \tag{49}$$

with

$$\kappa^2 = \frac{-\,2\log_e\theta}{\pi}\,(1-\chi_m)\sqrt{\chi_m} \tag{50}$$

and, by eliminating g_o and $\sqrt{r}$ from Eq. 44*a*,

$$G = \frac{\sqrt{\chi_m\,Y}}{\kappa} \tag{51}$$

Eqs. 49 and 51 express the parameters r and G in terms of quantities which are easily determined in case II.

Although there is no definite restriction on the value of θ to be used in making these observations, its value must not be too large because Eq. 39 is only valid for sufficiently large values of t. Errors have resulted from the use of $\theta = \frac{1}{2}$, but the value $\theta = \frac{1}{10}$ has been found to give good results. The latter value is convenient and is recommended.

In general, the responses of the type of case I are observed with the smaller values of mass ratio r. The lower limit of the mass ratio is limited only by the assumption that $1/\sqrt{r}$ shall be a small quantity. Effectively, this lower limit is probably $r = 100$, more or less.

Responses of the type of case II are observed with the larger values of mass ratio. The upper limit may be estimated from Eq. 44a by making the substitution:

$$G = \tfrac{1}{2}\,(g_h - g_o)\,\sqrt{r} \qquad (52)$$

Accordingly,

$$r = \frac{\chi_m}{g_o\,(g_h - g_o)(1 - \chi_m)} \qquad (53a)$$

Eq. 53a shows that the mass ratio range is inversely proportional to the test mass damping, which thus should be kept as low as possible. If g_o is very small, Eq. 53a may be replaced by

$$r = \frac{\chi_m}{g_o\,g_h\,(1 - \chi_m)} \qquad (53b)$$

A value of g_o as low as 10^{-3} can easily be attained in an actual instrument. If it is assumed that values of χ_m as great as 0.80 can be readily observed and recognized, the upper limit of the mass ratio according to Eq. 53b becomes

$$r = \frac{4{,}000}{g_h} \qquad (53c)$$

Accordingly, for a structure with a damping factor $g_h = 0.01$, mass ratios to 400,000 can be measured as has been verified by experiment.

It may now be observed that, although the development of the theory of this method of vibration testing may be considered somewhat involved, actual use of the method is extremely simple. It is only necessary to make simple observations on the test mass amplitude and to apply the simple formulas given for the interpretation of the response (Eq. 34) for case I or Eqs. 49 and 51 for case II.

5. Verification of the Theory

An instrument has been constructed to check and to make use of the foregoing theory. This instrument, which the writer calls the "vibration mode analyzer" is shown in Fig. 2. General specifications of the instrument are:

Description	Value
Frequency range, in cycles per minute	150 to 3,600
Test mass range, in pounds	0.050 to 0.750
Height of frame, in inches	18
Approximate weight of instrument, in pounds	1.5

This instrument is capable of measuring effective masses from about 5 lb to well in excess of 100,000 lb. Therefore, it would be usable for measuring vibration characteristics of aircraft, heavy machinery, or parts of a building structure.

Numerous tests have been conducted with this instrument—on simple structures with known vibration characteristics and on typical complex structures—both for the purpose of checking the theory and for determining its general usefulness. The interpretation formulas and mass ratio range formula of Section 4 have been verified by these tests.

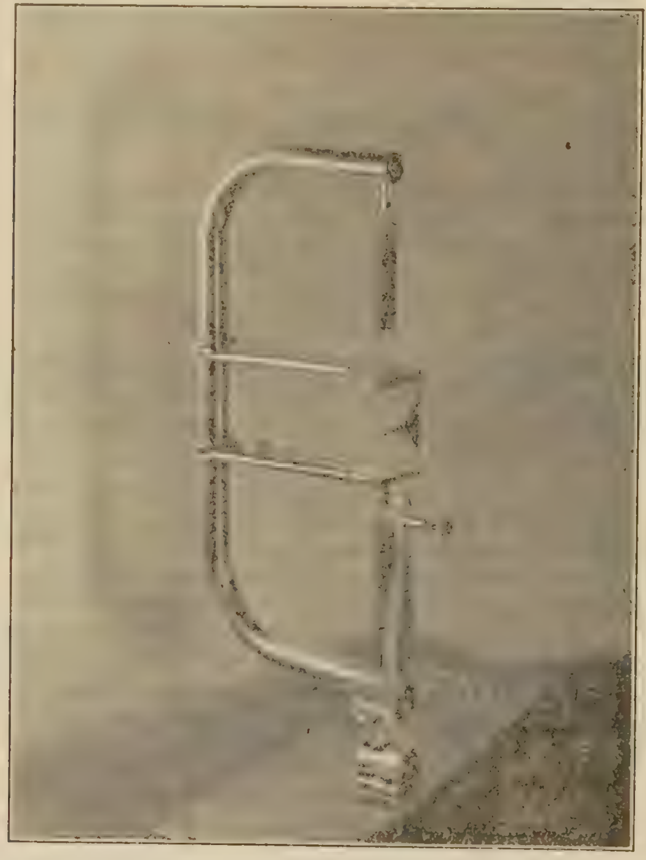

Fig. 2.—Vibration Mode Analyzer

It has been concluded from these tests that the instrument behaves in accordance with the foregoing theory; and the applicability of (Eqs. 34, 49, 51, and 53*c* has been specifically checked. The tests have also demonstrated the relative ease with which vibration tests may be made by this method.

The readings and their interpretation are presented herein for two examples selected from tests run with the "analyzer." These two tests were run on a test beam set up especially for the tests; it was possible to determine approximate vibration characteristics of this beam by direct observations and computation. The precision of the determinations by direct observation and calculations is believed to be less than that obtained from the instrument readings;

accordingly, comparison of the test results with these determinations is not believed to indicate the precision of the instrument. The first example illustrates the oscillatory response of the test mass treated as case I of Section 4; the second example illustrates the nonoscillatory resonant response of case II.

Example I.—An oscillatory amplitude variation of the test mass was observed with the following data for the test mass:

Natural frequency, ω_o, in radians per second........	55.2
Test mass weight, m_o, in pounds..................	0.830
Test mass damping, g_o..........................	0.63×10^{-3}

The measured data for the oscillatory variation of the test mass amplitude are presented in Table 1. The entries in Cols. 1, 2, and 3, Table 1, are self-explanatory. Entries in Col. 4 are the amplitudes which the test mass would have on a fixed mount; the observed amplitudes divided by these values are entered in Col. 5 and represent the function ψ (compare Eq. 29).

TABLE 1.—OSCILLATORY VARIATION OF THE TEST MASS AMPLITUDE

Line	Event (1)	Time (sec) (2)	Amplitude (in.) (3)	$A_o e^{\omega g_o}$ (4)	ψ (5)
1	Release of test mass...........	0.0	0.50	0.500	1.00
2	First zero amplitude...........	8.7	0.00	0.430	0.00
3	First peak amplitude..........	13.6	0.085	0.394	0.214
4	Second zero amplitude.........	21.6	0.00	0.344	0.00

Comparison of lines 1 and 3, Col. 5, Table 1, indicate a value of 0.214 for the ratio of successive peak values of the function ψ. According to the theory for case I this ratio is identified with the expression of Eq. 35*a*, thus: $\frac{G}{\sqrt{1 - G^2}} = \frac{1}{\pi} \log_e \left(\frac{1}{0.214} \right) = \frac{1.54}{\pi} = 0.49$; and $G = 0.44$. The time interval between successive zeros of the function ψ is the difference between lines 2 and 4, Col. 2, Table 1, and is 12.9 sec. According to the theory, this time interval is identified with the quantity $\frac{2\,\pi\,\sqrt{r}}{\omega_o \sqrt{1 - G^2}} = 12.9$. Consequently, with $G = 0.44$ and $\omega_o = 55.2$, the value of $\sqrt{r}$ is 102 and $r = 10{,}400$. The damping factor of the structure is given by $g_h = \frac{2\,G}{\sqrt{r}} + g_o = 0.0092$, and the effective mass of the structure at the point at which the test mass was mounted, in weight units, is $r \times (m_o) = 10{,}400 \times 0.830 = 8{,}640$ lb. The corresponding values determined by direct observation and calculation were

Natural frequency, in radians per second................	55
Damping factor, g_h..................................	0.01
Effective mass, m, in pounds..........................	9,000

Example 2.—With the test mass located very close to a node on the test beam a nonoscillatory resonant response was observed. For a set of closely spaced frequencies, it was required to find the time intervals during which the test mass amplitude decreased from an initial value of 0.50 in. to 0.05 in. ($\theta = \frac{1}{10}$). These data are presented in Cols. 1 and 2, Table 2. In Col. 3 are entered the time intervals for a similar decay of amplitude with the test mass mounted on a rigid support; the ratio χ appearing in the theory for case II (which is the ratio of these two time intervals) is entered in Col. 4, Table 2.

TABLE 2.—TIME INTERVAL, IN SECONDS, REQUIRED FOR THE TEST MASS AMPLITUDE TO DECREASE FROM 0.5 IN. TO 0.05 IN.

ω_o (1)	Test beam (2)	Fixed support (3)	χ (4)
54.75	148.0	163.9	0.90
54.86	142.8	163.3	0.87
54.97	132.7	162.8	0.82
55.08	117.5	162.2	0.72
55.19	106.9	161.7	0.66
55.30	103.8	161.1	0.64
55.41	112.4	160.6	0.70
55.52	118.3	160.0	0.74
55.63	135.0	159.5	0.85
55.74	142.8	158.9	0.90
55.85	143.3	158.4	0.91
55.96	145.9	157.9	0.92
56.07	147.4	157.3	0.94

The following values are determined by the data in Table 2: Natural frequency $\omega_h = 55.28$ radians per sec; minimum value of $\chi = \chi_m = 0.63$; and $n_m = \dfrac{0.63 \times 161.4 \times 55.28}{2\pi} = 895$. Also, from Table 2, it can be noted that the frequencies ω'_o at which $\chi = (1 + \chi_m)/2 = 0.815$ are 54.98 and 55.60 radians per sec. Accordingly, the value of the parameter Y is $\dfrac{0.31 \times 895}{55.28} = 5.0$.

The value of the constant κ^2 in Eq. 50 is $\dfrac{2 \log_e 10}{\pi} \times 0.37 \times \sqrt{0.63} = 0.430$. Thus, from Eq. 49, $r = \dfrac{(895)^2}{0.430 \times 5.0} = 370{,}000$; and Eq. 51 gives $G = \dfrac{\sqrt{0.63 \times 5.0}}{0.655} = 2.7$. The test mass weight was 0.830 lb and the indicated effective mass of the beam is 370,000 × 0.830 = 310,000 lb. The indicated damping factor is $g_h = \dfrac{2G}{\sqrt{r}} + g_o = 0.0095$. The values of natural frequency and damping factor indicated by direct observation and calculation are, as before, 55 radians per sec and 0.01; the effective mass for this point was calculated as about 300,000 lb.

Because of the ease of testing, this method may be applicable in making measurements other than those of vibration characteristics. For example, the natural frequency of a column is dependent on the column load, and, accordingly, column loads might be measured quite easily by precise measurements of natural frequency; in the same manner the tension in ties, rods, and cables could be determined by frequency measurements. Since there is a relation between the mass of a structure and its effective mass in a vibration, it would be possible to determine the weights of structures by measurement of their effective masses. Also the over-all effective stiffness of composite members in place could be deduced from frequency and effective mass measurements.

ACKNOWLEDGMENTS

Patents are pending on certain instruments based on the ideas advanced in this paper. Nevertheless, the writer offers the results of his experience, together with patent rights, for the free use of the profession. It is expected merely that suitable acknowledgment or recognition will be made by those who use the patentable principles involved.

APPENDIX. NOTATION

It has been found necessary to introduce a nomenclature, in this paper, that includes a large number of letter symbols. The complete definition of each quantity appears in the text where the symbol is introduced.

For additional reference the physical dimensions of each quantity represented by the sumbols are given. The fundamental dimensions, force (F), length (L), and time (T), appear in parentheses. Mass, which is usually considered a fundamental dimension, is replaced by force since the former seldom enters directly into engineering calculations, whereas the latter does very often.

A = amplitude or displacement (L) of the test mass; A_o = initial amplitude or displacement;

a_i = amplitude of displacement field ϕ_i;

e = base of natural or Naperian logarithms;

f = a dimensionless coefficient in Eq. 2*b*; a subscript i denotes the mode;

G = a dimensionless damping index (see Eq. 19*b*);

g = a dimensionless damping factor in a given structure:

  g_h = damping factor in mode h where attention is fixed;

  g_i = damping factor in unspecified mode i;

  g_o = damping factor of test mass;

  g_1, g_2, g_3 = sequence of damping factors g_i used in a determinant expression (see Eq. 14);

h = dimensionless mode number where attention is fixed;

I = inertia moment (FLT^2), a dimensional constant; I_r = a reference inertia moment (see Eq. 8*b*);

i = dimensionless unspecified mode number;

j = the imaginary radical $\sqrt{-1}$;

K = a moment stiffness (FL); K_r = reference stiffness;

k = a linear stiffness (FL^{-1}); also subscript denoting point and direction on structure:

  $(k_i)_k$ = equivalent spring stiffness in mode i at a point and in a direction denoted by k;

  k_o = spring stiffness of the test mass spring;

m = mass ($FL^{-1}T^2$):

$(m_h)_k$ = effective mass of a structure in mode h, and at a point and in a direction denoted by k;

$(m_i)_k$ = effective mass of a structure in mode i, and at a point and in a direction denoted by k;

m_o = test mass;

$(m_1)_k$, $(m_2)_k$, $(m_3)_k$ = sequence of mass designations used in a determinant expression (see Eq. 14);

n = number of cycles; n_m = the number of cycles of test mass vibration in the time interval t_θ corresponding to an observation of χ_m;

o = a dimensionless substitution factor defined by Eq. 27*a* and introduced to simplify the typography (see also u, v, η, and z);

P = point force; P_k = force component; at a point and in a direction denoted by k;

Q = a generalized external force (FL); in Eq. 3; Q_i = the value of Q applied to a structure in mode i, corresponding to the respective normal coordinates;

r = a dimensionless mass ratio;

T = kinetic energy (see also V);

t = time (T):

t_θ = time interval required for a test mass amplitude to decrease from its initial value A_o to θA_o;

$(t_\theta)_o$ = value of t_θ for a test mass on a rigid support;

u = a dimensionless substitution factor defined by Eq. 27*b* and introduced to simplify the typography (see also o, v, η, and z);

V = potential energy (see also T);

v = a dimensionless substitution factor defined by Eq. 27*c* and introduced to simplify the typography (see also o, u, η, and z);

X = amplitude (complex) of vibration:

X_h = amplitude in mode h;

X_{i_k} = amplitude at the point and in the direction indicated by k;

X_o = amplitude (complex) of vibration of the test mass;

x = displacement of the test mass from its equilibrium position;

Y = a dimensionless quantity defined by Eq. 48 and derived from observations of test mass behavior;

z = a dimensionless substitution factor defined by Eq. 31 and introduced to simplify typography (see also o, u, η, and v);

α = a dimensionless parameter dependent on Δ and G (see Eq. 21*a*);

β = a dimensionless parameter dependent on Δ and G (see Eq. 21*b*);

Δ = tuning index of a test mass (see Eq. 19*a*):

δ = displacement vector of a point on a structure:

δ_h = displacement vector in mode h;

δ_i = displacement vector in mode i;

δ_k = displacement vector component at a place and in a direction denoted by k (subscripts h and i used only in connection with k, thus $(\delta_i)_k$);

η = a dimensionless substitution factor defined by Eq. 40 and introduced to simplify typography (see also o, u, v, and z);

θ = ratio of final to initial test mass amplitude for a measured time interval t_θ;
κ = dimensionless factor defined by Eq. 50;
λ = a complex frequency of natural vibrations in a test mass structure combination;
ϕ = normal mode vector displacement field (L):
  ϕ_i = value of ϕ for all points of a structure, for the normal mode i;
  ϕ_k = value of ϕ at a point and in a direction indicated by k;
χ = a time ratio defined by Eq. 43a: χ_m = a minimum value of χ;
ψ = a dimensionless factor containing the effect of the structure on test mass vibration amplitude, defined by Eq. 30; and
ω = natural circular frequency (T^{-1}):
  ω_h = frequency of a structure in mode h;
  ω_i = frequency of a structure in mode i;
  ω_o = frequency of a test mass;
  ω_r = reference frequency.

DISCUSSION

ALEXANDER KLEMIN,[3] ESQ.—This paper is in the best traditions of scientific instrumentation. Although the theory is soundly conceived, it appears complex at first. However, it no longer appears complex if one checks back against the Rayleigh theory, and the analyzer itself is simple (though ingenious) in construction and operation.

The investigation of flutter and vibration is a vital and difficult element of airplane design. Before any calculations can be made, a number of tests are necessary—tests concerned with the natural frequencies of different parts of the airplane, the location of nodes and axes, and many other pertinent data. Frequencies are formed by setting up vibrations, most often by applying a periodic force and finding a resonant condition. Electro-mechanic oscillators of some complexity are used. The instrumentation is not too portable, and there is some hazard in setting up resonance. Mr. Loring's instrument appears superior in portability, in simplicity of operation, and in absence of any hazard. It is an added advantage that the instrument is so light that it should be possible to produce flexural vibration without torsional vibration. The writer can vouch for its great utility in the aircraft field.

R. K. BERNHARD,[4] M. ASCE.—The method described by the writer presents an ingenious procedure for determining certain vibration characteristics of actual structures. The outstanding simplicity of application is one of its most important advantages.

The question the writer would like to raise is: Which of the many significant and complex dynamic phenomena, occurring in numerous practical cases, can be determined by the new method? Any convenient nondestructive procedure which permits the determination of static and dynamic characteristics by observations on the structure itself is preferable, in many cases, to methods involving either undersized or oversized models. The change in size from the actual structure to the model very often includes, besides other disadvantages, an undesirable compromise with respect to the various similarity scales.[5]

However, any method involving the use of the prototype structure must simulate actual service conditions as closely as possible. This stipulation can be explained by a few practical examples referring to the magnitude of excited force amplitudes, excited displacement amplitudes, and the required energy input to produce these amplitudes.

Nonharmonic Vibrations.—If the writer understands this paper correctly, the method induces extremely small sinusoidal vibrations in the structure under investigation. Exciter mass and excited mass are coupled loosely by the elasticity of the pendulum springs.

Numerous investigations by means of induced sinusoidal vibrations,[6] however, with a rigid coupling between exciter and excited mass, have shown a logarithmic damping decrement (θ) in bridges, for example, of the order: $\theta = 0.1$. For an exciter mass (pendulum) of, say ,1 lb, the amplification factor at resonance ($A = \frac{\pi}{\theta} = 30$) would yield an effective dynamic force of 30 lb. For simplicity, consider the exciter and excited masses as a two-mass system without damping, and assume a medium size structure weighing 200 tons. In the case of a loosely coupled two-mass system a deflection amplitude of $\frac{1}{2}$ in. of the exciter mass (pendulum) may build up maximum amplitudes in the excited mass (structure) of $\frac{1}{2} \times \frac{30}{400{,}000}$, which is approximately $\frac{1}{27{,}000}$ in. To the writer, this value appears smaller than the deflection amplitudes to be expected in many cases under field service conditions.

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[5] "Introduction to Engineering Analysis," by J. J. Slade, Edward Brothers, Inc., Ann Arbor, Mich., 1947.

[6] "Determination of the Static and Dynamic Constants by Means of Response Curves," by R. K. Bernhard, *Journal of Applied Physics*, December, 1941, pp. 866–874.

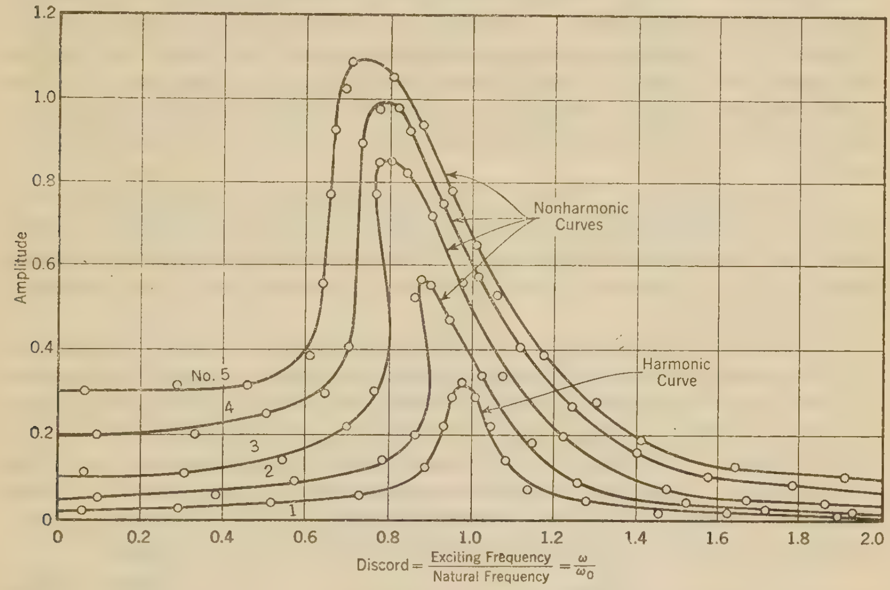

Fig. 3.—Resonance Curves for Nonharmonic Vibrations Excited by Induced Vibrations (Amplitudes Versus Discord for Varying Magnitudes of the Exciting Forces)

Larger deflection amplitudes, however, may produce nonharmonic vibrations, which do not represent exceptional cases, as is often assumed. The five resonance curves, shown in Fig. 3, are taken from experiments on actual structures, excited by means of induced sinusoidal vibrations with increasing displacement amplitudes, which seem to prove this statement.[7] For a simple girder a series of resonance curves has been determined, showing the relation between the discord (that is, the ratio of exciter frequency to natural frequency of the girder) and the corresponding excited deflection amplitude of the girder. The magnitude of the exciter forces has been varied.

Curve No. 1 is plotted for a small exciter force in order to produce a maximum deflection amplitude within the linear range—that is, for constant spring elasticity and constant damping. Hence, this particular curve still has a normal form. Only with larger deflection amplitudes will nonlinear elasticity and nonlinear damping appear, mainly because of minute slipping action in the connections or the supports or both. Thus, the next curve (No. 2) shows a slight tilting effect. This effect is increased in curve No. 3, which is produced by a substantially larger exciter force. A wider gap at the top of curve No. 3 (still more pronounced in curve No. 4) indicates a considerable increase in damping. The tilting of the three curves (Nos. 2, 3, and 4) toward lower frequencies is characteristic of nonharmonic vibrations.

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[7] *Journal of Applied Mechanics*, A.S.M.E., June, 1939, p. 91, Fig. 1

Finally, in curve No. 5, this tilting effect gradually disappears because of larger damping. However, the broadening of this curve in the resonance range, combined with the steep incline in the rising branch, produces also a nonsymmetrical shape. With increasing frequencies, an abrupt step up from small to high amplitudes takes place. This step up indicates a state of unstable equilibrium. With decreasing frequencies, a similar abrupt step down from high to low amplitudes occurs, although it takes place at a point of larger forces as compared with the step up. Hence, for curves No. 2 and No. 3, at discords around 0.9 and 0.75, respectively, three different ranges of unstable dynamic equilibrium exist.

In this connection it might be mentioned that the Tacoma Bridge failure[8,9,10,11,12,13,14,15,16] was caused also by a state of unstable equilibrium, in that case, due to aerodynamic instability. These characteristic dynamic phenomena will not appear under small deflection amplitudes, producing curves of type No. 1 only.

Coupling and Beat Effects.—The active participation in the vibration of supports, adjacent structures, and surrounding soil may influence the dynamic characteristic substantially. Fig. 4 shows frequency versus energy-input curves for a railroad bridge, consisting of several spans of 230 ft each.[17] The curves on the right side of Fig. 4 represent resonance curves for one of the excited spans. These curves indicate a pronounced coupling effect (two-wave resonance curves) between the excited span and the adjacent span, loosely coupled by way of the intermediate pillar and foundation, as follows:

Description	Unloaded condition	60-ton locomotive
Natural frequency, in cycles per second	3.47	2.97
Logarithmic damping decrement	0.063	0.063
Energy input, in watts	460	160
Spring constant, in tons per inch	216	....
Reduced mass, in tons	179	....
Coupling coefficient between adjacent spans	0.015	....
Vibrating mass of foundation, plus adjacent soil, in tons	12,600	....

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[8] "Rigidity and Aerodynamic Stability of Suspension Bridges," by D. B. Steinman, *Transactions*, ASCE, Vol. 110, 1945, p. 439.

[9] *Ibid.*, p. 476.

[10] *Ibid.*, p. 485.

[11] *Ibid.*, p. 487.

[12] *Ibid.*, p. 490.

[13] *Ibid.*, p. 522.

[14] *Ibid.*, p. 532.

[15] "Failure of the Tacoma Narrows Bridge: Report of the Special Committee of the Board of Direction," *Proceedings*, ASCE, December, 1943, p. 1555.

[16] Discussion by Louis Balog of "Failure of the Tacoma Narrows Bridge: Report of the Special Committee of the Board of Direction," *ibid.*, November, 1944, p. 1475.

[17] "Purely Dynamic Methods for Determination of Stresses in Structures," by R. K. Bernhard and W. Späth, *Der Stahlbau*, March, 1929, pp. 61–68.

A simple calculation yields that the dynamically effective mass between the adjacent spans amounted to approximately 12,600 tons. This dynamically effective mass consisted of the mass of the pillar, the concrete foundation, and the soil surrounding this foundation, all three masses participating in the vibrations. The vibrating soil mass was approximately ten times larger than the enclosed concrete foundation.

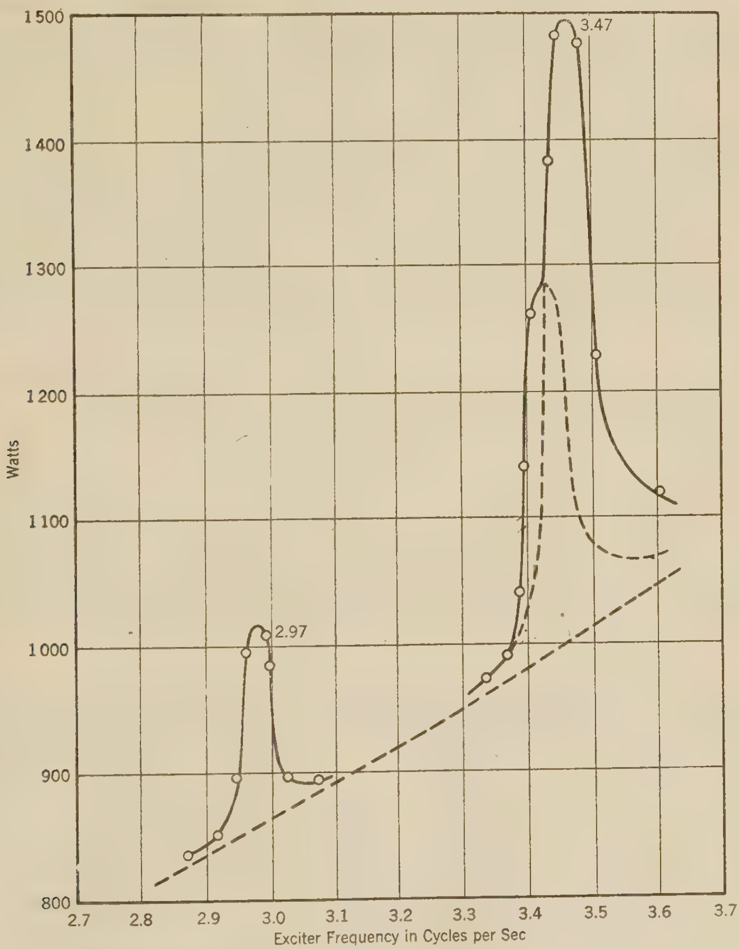

Fig. 4.—Frequency Power Input Curves of 230-Ft Railroad Bridge

Detuning the excited span by loading it with a light locomotive (60 tons) in the center yielded undistorted resonance curves of the excited span as shown on the left side of Fig. 4. The reduced natural frequency of the loaded span prevented effectively any observable coupling effects from the adjacent span. Stress-time diagrams of observations in the upper chords of highway trusses[18,19] during the passage of trucks have revealed similar coupling phenom-

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[18] "Dynamic Relations Between Moving Loads and Structures," by R. K. Bernhard, *Mechanical Engineering*, September, 1938, pp. 696–702.

[19] *Ibid.*, p. 701 Fig. 11.

ena. Pronounced "beat" effects in such diagrams indicate the characteristic transfer of energy from the exciter mass (vehicle) to the excited mass (truss), and, conversely, this holds true. In all cases, particularly where dynamically effective masses in the order of 10,000 tons and more are participating in the vibrations and must be excited, a substantial amount of energy is required to produce these phenomena.

In the writer's opinion these few examples demonstrate that, in many cases, the determination of the dynamic characteristics by tests on actual structures yields rather complex phenomena. These can be neither obtained nor interpreted correctly, unless actual service conditions are reproduced faithfully. This is true particularly as far as the magnitude of the excited deflection amplitudes or as far as the energy input, required to excite the desired vibrations, are concerned.

With respect to the application of the author's method, the writer would like to ask the following questions:

1. Keeping in mind the necessity of simulating actual service conditions, how heavy a structure can be investigated by increasing the size of the pendulum?
2. Can nonharmonic phenomena (which are often predominant in actual cases), including dynamically unstable conditions, be evaluated?
3. What methods have been used to record the decay curves of the vibrating pendulum? and
4. If an air gap of an electrostatic or an electromagnetic type of pickup unit forms the only coupling link between pickup and pendulum, can the restoring force of the recording mechanism be reduced sufficiently? In other words, does not the restoring force of the pickup unit distort the decay diagram (or "beat-diagram") of the pendulum? Photographing the pendulum indicator with a high speed motion picture camera might avoid these difficulties.

Carl A. Ludeke,[20] Esq.—By an interesting application of D. Bernoulli's principle of the superposition of vibrations Mr. Loring reduces the testing of the vibration characteristics of structures to a set of problems in two degrees of freedom. Each of these problems involves the manner in which the rectilinear motion of the damped vibrations of a small test mass is affected by the vibration characteristics of the structure in the direction of motion of the test mass and at the point of testing. A splendid application of the theory of elastic vibrations for systems with small damping shows that the motion of the test mass can be considered as a vibration of frequency ω_o, which has a varying amplitude A. It is obvious that A must account for the damping in the analyzer, the damping in the structure, and the "wandering" of energy between the test mass and the structure, which is to be expected in a system having two degrees of freedom. Results of the analysis show that A can be considered in the form,

$$A = A_o\, e^{-\frac{1}{2}\omega_o g_o t}\, \psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

If the structure is rigid or if the natural frequency of the test mass is more than slightly different from the natural frequency of the structure at the point of test and in the direction of the test mass motion, then Eq. 54 is of such a form that ψ must equal one. However, ψ has very interesting characteristics when the structure is not rigid and these two natural frequencies are "tuned"—that is, they are approximately equal. The function ψ is defined by Eq. 30, and, depending on the value of the damping index, G, it assumes three distinct forms when exact tuning is obtained. These forms are shown in Eqs. 34, 36, and 38. Since the characteristics of ψ determine the equations which are used when operating the analyzer, it seems of interest to point out that ψ itself can be considered as a free damped vibration; that is, it satisfies a differential equation of the form,

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[20] Associate Prof. of Mechanics, Univ. of Cincinnati, Cincinnati, Ohio.

$$a\psi'' + b\psi' + c\psi = 0 \quad\ldots\ldots(55)$$

The three forms of ψ correspond to the solutions of Eq. 55 for the well-known cases in which damping is less than, greater than, or equal to critical damping.

To show this, it should be noted that, when the damping is less than critical, the solution of Eq. 55 satisfying the initial conditions, when $t = 0$, $\psi = 1$, and $\psi' = 0$, can be written in the form,

$$\psi = e^{-\frac{1}{2}Kt}\left(\cos n't + \frac{K}{2n'}\sin n't\right) \quad\ldots\ldots(56)$$

in which

$$K = \frac{b}{a} \quad\ldots\ldots(57)$$

$$n' = \sqrt{n^2 - \tfrac{1}{4}K^2} \quad\ldots\ldots(58)$$

and

$$n^2 = \frac{c}{a} \quad\ldots\ldots(59)$$

Eq. 34 is in this form, with the values of a, b, and c such that Eq. 55 becomes

$$m_o\psi'' + \sqrt{\frac{m_o k_o}{r}}\,G\psi' + \frac{k_o}{4r}\psi = 0 \quad\ldots\ldots(60)$$

When the damping is greater than critical damping, Eq. 55 has a solution,

$$\psi = C_1 e^{s_1 t} + C_2 e^{s_2 t} \quad\ldots\ldots(61)$$

in which

$$s_{1,2} = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}} \quad\ldots\ldots(62)$$

If Eq. 55 is again taken in the form of Eq. 60, and if the same initial conditions are satisfied, then C_1, C_2, s_1, and s_2 take the forms required to make Eq. 61 the same as Eq. 36.

If the damping equals the critical damping, the solution of Eq. 55 assumes the form,

$$\psi = (C_1 + C_2 t)\,e^{-bt/2a} \quad\ldots\ldots(63)$$

If Eq. 55 is in the form of Eq. 60 with G equal to 1, and if the same initial conditions are satisfied, then Eq. 63 becomes

$$\psi = \left(1 + \frac{1}{2}\frac{\omega_o}{\sqrt{r}}\,t\right) e^{-\omega_o t/2\sqrt{r}} \quad\ldots\ldots(64)$$

which for large values of t assumes the form of Eq. 38.

Thus, the amplitude of motion of the test mass is given by Eq. 54, in which, at exact tuning, the function ψ must satisfy the conditions when $t = 0$, $\psi = 1$, and $\psi' = 0$, and must be a solution of the differential equation

$$m_o \psi'' + \left(\frac{\beta - \beta_o}{2}\right) \psi' + \frac{k_o}{4r} \psi = 0 \ldots\ldots\ldots\ldots\ldots (65)$$

in which β is the coefficient of viscous damping of the structure in the mode under observation, and β_o is the coefficient of viscous damping of the test mass.

M. J. Turner,[21] Esq.—During 1947 an instrument of the type described by the author has been used to determine vibration characteristics of the tail surfaces on a small airplane. Although experience is rather limited, the analyzer appears to have excellent potentialities for the accurate determination of natural frequencies and structural damping. The simplicity of the instrument and its relatively small size are particularly attractive when vibration surveys of limited scope have to be made outside the laboratory. For large-scale laboratory testing, in which vibration measurements at many points on the structure are required, the more elaborate electronic instrumentation in current use is definitely preferable. Even in this sort of work the analyzer is very useful for supplementary checking.

In the vibration testing of airplane structures, the frequent occurrence of several vibration modes with closely spaced natural frequencies is a source of great difficulty, and it is felt that the potentialities of the analyzer for resolving such difficulties should be systematically explored. Possibly the scheme presented by Mr. Loring for interpretation of test mass motions is adequate in almost all practical situations, but further investigation will be required to confirm that point. In view of the analytical complications which occur when only one mode of the structure is considered, it appears that a theoretical study of test mass motion under the simultaneous influence of two or more structural modes would be very difficult. However, it would be entirely feasible to determine experimentally the potentialities of the instrument for measurement of closely spaced natural frequencies. Probably it would be desirable to build a special structure for the purpose, with provision for adjustment of two of its natural frequencies to any desired degree of proximity. Then, by locating the analyzer at a nodal point of one of the interfering modes, it would be possible to determine the natural frequency of the other mode precisely. Further tests with the analyzer mounted at other points, where both the modes can be excited, would serve to determine the limits of its resolving power.

Finally, it should be noted that the test mass motion may be influenced by the excitation of vibration modes of the analyzer itself. The frequencies of these modes are indicated by very sharp dips in the plot of decay time, $(t_\theta)_o$, against frequency, obtained with the analyzer mounted on a rigid base. The existence of these extraneous modes has not proved to be a serious handicap in the use of the instrument.

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[21] Supervisor of Dynamic Analysis, Chance Vought Aircraft Div., Stratford, Conn.

F. B. FARQUHARSON,[22] M. ASCE.—The problem of the dynamic behavior of structures is one which should arouse more interest in the civil engineering profession than has hitherto been the case; and, by and large, it seems to be mainly those who have become associated with the aircraft industry who have shown the principal interest. The author is to be congratulated on the simplicity of his instrument and the ease with which it can be operated.

A troublesome matter involving the attention of a few engineers relates to the damping inherent in the modern suspension bridge. Laboratory study and mathematical analysis have both indicated that the oscillation of a suspension bridge under the action of wind may be greatly affected by the over-all damping (both structural and aerodynamic) existing on the structure.

The aerodynamic portion of the total damping can be obtained from model tests in a wind tunnel, but that portion arising from the interaction of the various parts of the structure and from the internal damping (solid friction) in the material of which the structure is built cannot be obtained from a model, nor can it be calculated. It appears that the solution for the latter part of this problem lies in the artificial vibration of the prototype itself in still air. This procedure is not new on bridges[23] but in the past it has been applied principally to railway structures of high stiffness and, consequently, of relatively high natural vibration frequencies. The instrument described by the author covers a frequency range from 150 cycles per min to 3,000 cycles per min, whereas it is estimated that a complete compilation of field data from existing suspension bridges could be confined within a range of from 5 cycles per min to 20 cycles per min.

Not the least attractive feature offered by the author's proposal is the fact that extreme excitation of the structure is not required. All the alternative methods discussed for use on the suspension bridge have involved significantly large amplitudes of oscillation. Furthermore, the devices under consideration involve serious interruption of the traffic which, together with a required amplitude of from 12 in. to 15 in. of vertical motion, might make it difficult to obtain permission for a clinical study on some of the most important bridges.

The application of the author's system to a suspension bridge involves complications, arising out of low frequency and great weight, which would require a radical departure from the simple design of the existing vibration mode analyzer. An example of the difficulty involved arose through correspondence with the author some months ago, when it was pointed out that a simple extension of the existing instrument would require the spring suspension of a mass capable of executing linear vertical vibrations at a natural frequency of 6 cycles per min. This condition implies a static deflection of the spring of 81 ft.

The special problem surrounding the suspension bridge involves the need of measuring both vertical and torsional vibration characteristics. In the course of the continued study of the application of the theory contained in the paper, it seems that it might be possible to provide an instrument not exceeding 2 tons in weight which might be mounted on a truck and used in the investigation of a series of representative structures.

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[22] Director, Eng. Experiment Station, and Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

[23] "Dynamic Tests by Means of Induced Vibrations," by Rudolf K. Bernhard, *Proceedings*, A.S.T.M., Vol. 37, 1937, Pt. II, p. 634.

J. J. SLADE, JR.,[24] ESQ.—This is an interesting method of detecting the normal modes of vibrations of full-scale structures that, because of the lightness of the equipment required, should prove useful in field tests. The interpretation of the results obtained by this method, however, raises a question which, in the writer's opinion, deserves investigation.

The classical theory of small oscillations, on which the author's theoretical work is based, assumes that coupling of the various members of a structure, to the motion of which the theory applies, may be considered perfect for sufficiently small displacements and velocities. If this assumption is made, then, if a point in a structure suffers a displacement x, the force $F(x)$ that opposes this displacement may be taken to be an analytic function of x near the zero value of the displacement and therefore may be written

$$F(x) = a_0 + a_1 x + a_2 x^2 + \cdots \qquad (66)$$

If x is measured from the unstrained position, $a_0 = 0$ since $F(0) = 0$, and if x is restricted to sufficiently small values, the powers higher than the first may be neglected. Under these conditions the expression for the restoring force becomes

$$F(x) = a_1 x \qquad (67)$$

in which a_1 is the generalized Hooke's constant. Thus, the essential nonlinearity of the force (Eq. 66) is eliminated.

Let the "upper nonlinearity" be the nonlinearity suppressed by the theory of small oscillations. The writer wishes to call attention to a "lower nonlinearity" that exists whenever coupling is imperfect and which may show its effects when the amplitude of the displacement is too small. When such a condition exists, Eqs. 66 and 67 are invalid for extremely small values of x, although they may still be used for ordinarily small values of the displacement when these are greater than those required to insure complete coupling. The smallness of a displacement for dynamical purposes must be estimated with respect to the intrinsic geometry of a structure, of course, and not with respect to the observational limit imposed by a measuring technique.

Consider, for example, a simple mass-spring system in which the spring coupling occurs gradually through small displacements about the unstrained position. A possible form of the restoring force in such a system is shown in Fig. 5. This type of restoring force may be represented by an expression such as

$$F(x) = k\,(\sqrt{\delta^2 + x^2} - \delta)\,\operatorname{sgn} x \qquad (68)$$

in which sgn denotes signum, so that sgn x is $+1$ when x is positive and -1 when x is negative; k is the spring constant after complete coupling has resulted; and δ is a linear quantity of the order of magnitude of the displacement required to insure substantial coupling. If m is the mass of the system, and

$$\lambda^2 = k/m \qquad (69)$$

the equation for the free oscillations is

$$\ddot{x} + \lambda^2\,(\sqrt{\delta^2 + x^2} - \delta)\,\operatorname{sgn} x = 0 \qquad (70a)$$

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[24] Director, Eng. Experiment Station, Rutgers Univ., New Brunswick, N. J.

When δ is negligibly small or, what amounts to the same thing, when x is not too small, Eq. 70a reduces to

$$\ddot{x} + \lambda^2 x = 0 \qquad (70b)$$

and λ is the natural frequency of the system. Eq. 70b is valid for $\alpha < x < \beta$, in which α is a sufficiently large multiple of δ and β is a displacement at which upper nonlinearity becomes evident.

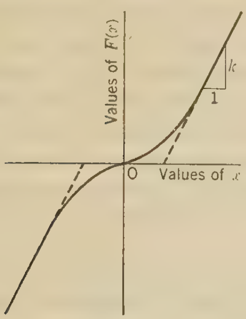

Fig. 5.—Restoring Force in Gradually Coupling Mass-Spring System

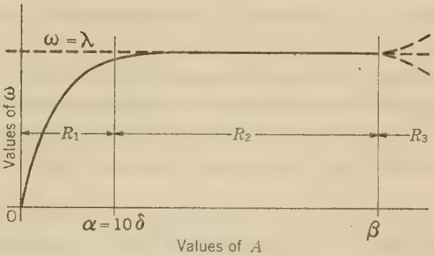

Fig. 6.—Dependence of Frequency on Amplitude

If $x < \alpha$, an approximate solution may be assumed of the form,

$$x = A \cos \omega t \qquad (71)$$

in which ω is the frequency to be determined as a function of the amplitude. The result of substituting this assumed value of x in the left-hand member of Eq. 70a is

$$-\omega^2 A \cos \omega t + \lambda^2 \left(\sqrt{\delta^2 + A^2 \cos^2 \omega t} - \delta\right) \operatorname{sgn} \cos \omega t = \psi(A, t) \qquad (72)$$

in which $\psi(A, t)$ is merely an abbreviation of the expression to the left. Eq. 70a is satisfied as nearly as possible by this assumed form of x when A is determined for a given ω so that ψ is as nearly zero as possible throughout a cycle. Therefore, let

$$I = \int_0^{2\pi/\omega} \psi^2 \, dt \qquad (73)$$

be a minimum, which requires that

$$\frac{dI}{dA} = 0 \qquad (74)$$

The resulting integrals are readily evaluated and a compact approximate expression for the frequency is found to be

$$\omega^2 = \frac{2}{\pi} \lambda^2 \tan^{-1} (A/\delta) \qquad (75)$$

This relation is illustrated in Fig. 6, in which are shown three ranges of the amplitude: R_1, in which A varies from 0 to about 10 δ, the range of lower nonlinearity; R_2, the linear range; and R_3, the range in which upper nonlinearity occurs.

As an illustration of the practical occurrence of conditions such as those discussed herein, consider a beam built into massive walls. If an imperfection in the connections allows the ends of the beam to rotate through a small angle before the built-in feature is established, the beam can oscillate with any frequency from that corresponding to pin connections to that corresponding to rigid connections, depending on the amplitude of the oscillations. The opposite condition would be encountered in a pin-supported beam, since friction could prevent the small rotations corresponding to very small amplitudes.

Inasmuch as the amplitudes of the oscillations excited by the author's method will generally be intrinsically small, it seems likely that inhomogeneities in the connections of the various parts of a structure will affect the observed results. The possibility of detecting modes of vibration by the use of a small test mass is not questioned; it is merely suggested that the use of micro-amplitudes to determine the dynamic characteristics of a structure introduces problems that transcend the theory of small oscillations.

Samuel J. Loring,[25] Assoc. M. ASCE.—The writer wishes to thank all the discussers, who have brought out a number of points of practical interest not treated in the paper.

Mr. Klemin has affirmed several of the advantages of the new method which the writer sought to gain by this development. His statement—"* * * the instrument is so light that it should be possible to produce flexural vibrations without torsional vibration"—is considered by the writer to be a specific, although graphic, statement of the fact that the method has considerable selectivity in its ability to isolate and respond to only a single normal mode at a time.

In connection with the testing of actual structures, Professor Bernhard brings out several important points, which certainly deserve attention. The paper was restricted to the use of the method to determine the vibration characteristics of structures according to the classical linear theory of small vibrations—that is, natural frequencies, damping factors, and normal mode shapes. Since these fundamental characteristics are, according to the classical linear theory, fixed structural properties independent of amplitude or specific excitation, the question of amplitudes and probable service conditions does not arise in the restricted treatment of the paper.

These linear characteristics, although admittedly approximations, are themselves of great value in predicting the dynamic behavior of airplane structures under service conditions and especially in investigating aerodynamic instability or "flutter." They might be expected to have the same value in some degree for other structures. The writer agrees, however, that actual structures may deviate somewhat from the linear theory and that it is desirable to give attention to the amplitudes produced by a measuring instrument.

It should be particularly stressed that the amplitudes induced in a structure by this method need not be "extremely small." In testing the 200-ton bridge mentioned by Professor Bernhard it would be wholly feasible to employ, for example, a 100-lb test mass and to permit 5-in. amplitudes of this test mass. This procedure would lead to bridge amplitudes at resonance of the order of

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[25] Consultant on Technical Problems, Stratford, Conn.

1/27 in. Considerably larger amplitudes could be produced by further enlargements in the size of the test mass or its movement. Nonlinearities of the types discussed by both Professor Bernhard and Mr. Slade could be thoroughly studied by varying the induced amplitudes in this manner.

The discussion of amplitudes induced in the structure really includes the question of energy input. It may be remarked, however, that the method is extremely economical of "wasted energy." With the small instrument described, adequate responses have been obtained in airplane structures of 100,000-lb gross weight. Testing similar structures by mechanically forced vibrations requires from 2 hp to 5 hp—although not all this power is realized as structural vibration.

Coupling and beat effects are characterized in the linear theory by two or more natural frequencies lying relatively close together. As noted in Section 3, natural frequencies lying 5% apart or less can readily be isolated and measured. The resulting beat phenomena could be predicted if the modes and the frequencies are known.

Observations on the experimental instrument described in the paper were made entirely by direct visual methods and timed with a stop watch. Although the writer has not as yet made experiments in the direction of recording movements of the test mass, it appears that, with the various readily available recording instruments, this problem should offer no real difficulty.

The observation of Professor Ludeke concerning the analogy between the response characteristic of the test mass at resonance and that of a simple single degree of freedom system with damping is very useful for visualizing the character of motions which must be observed in using the method.

Mr. Turner's remarks are of particular interest since he has had occasion to use the principles set forth in the paper in a practical application. However, his unqualified conclusion—"* * * the more elaborate electronic instrumentation in current use is definitely preferable"—does not seem to follow directly from his previous observations that:

> * * * the analyzer appears to have excellent potentialities for the accurate determination of natural frequencies and structural damping. The simplicity of the instrument and its relatively small size are particularly attractive * * *."

Closely spaced natural frequencies can present unusual difficulties. However, from the remarks in Section 3 and from Example 2, it is noted that resolution of natural frequencies as closely spaced as from 2% to 5% apart is readily accomplished. When natural frequencies are more closely spaced, they might be considered substantially identical. There is a straightforward and general method for detecting the presence of two or more identical natural frequencies at a resonance point by examining the observed effective masses at a number of points on the structure.

Consider, for example, the simple case of the in-plane vibrations of a uniform circular ring. It is known that the ring has two identical natural frequencies corresponding to the two modes shown in Fig. 7(*a*) and 7(*b*). The nodes of one of these modes are at the antinodes of the other. When tested with an instrument located at any point on the ring, these two modes will combine with proportions that result in an antinode at the instrument location, as shown in Fig. 7(*c*). In the radial direction the instrument would show the same effective mass, equal to one half of the mass of ring, at all points of the ring. This observed effective mass would be one half of the mass of ring because the average square of the displacement amplitudes in the sinusoidal mode shape is one half of the square of the peak value at the instrument.

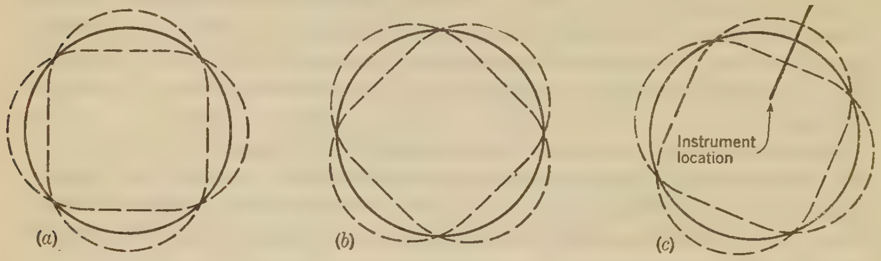

FIG. 7.—MODES OF IN-PLANE VIBRATION OF A CIRCULAR RING

However, if the ring had a single mode with the same effective mass at all points, that effective mass would have to be equal to the whole ring mass not to one half of the ring mass—because a uniform effective mass for a single mode implies a mode with uniform displacement amplitude and all elements of mass of the ring would act with unit effectiveness. It could be deduced from this apparent anomaly in the observed effective masses, therefore, that the ring must have two normal modes of equal frequency at its resonance. These two modes must combine in varying proportions, depending on the instrument location, to produce the observed effective masses.

Such apparent anomalies in observed effective masses will always show up if two or more modes with very closely spaced natural frequencies are being mistaken for a single mode in reading an instrument. If systematic use is made of this fact, no elaborate testing techniques for "resolving" closely spaced frequencies are necessary.

Resonance of the frame of an instrument (not the test mass) must, of course, be avoided. The writer believes, however, that what Mr. Turner has interpreted as the vibration modes of the analyzer itself on a "rigid base" are, in actuality, the response of the analyzer to natural frequencies of Mr. Turner's "rigid base."

In this connection, it is very difficult to find a structure to which the instrument will not respond at resonance. For purposes of calibration, however, a large structure not at resonance is wholly satisfactory.

Professor Farquharson interestingly suggests a specific problem in which the new method may be useful. The solution offered in the final paragraph of Professor Farquharson's discussion is believed by the writer to be a feasible one.

In connection with Mr. Slade's discussion the writer would like to reiterate two points already made: The amplitides excited by the new method need not be "intrinsically" small; and the "lower nonlinearity" mentioned by Mr. Slade could readily by detected or avoided by making tests with two or more induced amplitudes. In this connection, a variation in amplitude can be produced even by a single instrument by varying the amount of initial test mass movement.

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TRANSACTIONS

Paper No. 2365

DETERMINATION OF POSITION AND AZIMUTH BY SIMPLE AND ACCURATE METHODS

By T. F. Hickerson,[1] M. ASCE

With Discussion by Messrs. John C. Penn, Paul E. Wylie, P. V. H. Weems, and T. F. Hickerson.

Synopsis

Determination of latitude, longitude, and azimuth by stellar observations is outlined in this paper by simplified methods employing only the secant and cosecant functions. Also presented is a discussion of the effect of variations, or possible errors, in certain elements of the astronomical triangle, from which may be drawn certain conclusions as to the most favorable conditions for accuracy in the determination of latitude, longitude, and azimuth.

Introduction

The surveyor or engineer aims to establish the azimuth of a line, or to locate the true meridian, to the nearest minute or fraction thereof. The marine navigator, on the other hand, may plot his lines of position satisfactorily when the azimuths are accurate only to the nearest degree, and by the analytical method, as given herein, may determine his position (latitude and longitude) without knowing azimuth. This paper, therefore, is presented primarily for the consideration of the engineer and only incidentally for that of the navigator.

In all cases of precise azimuth determination the latitude must be known closely unless the observed body is on or near the 6-hr circle, as is shown hereafter. The longitude must also be known, unless the body is at or near elongation. Hence, to be sure of accurate azimuths it is desirable that the engineer be able to determine latitude and longitude independently and that he not be forced to rely on obtaining these data from maps—which may not be available.

True azimuths are frequently found from solar observations, but these readings are subject to errors due to two main causes. The first of these is that the sun may be in unfavorable positions; the second, that the average of

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Note.—Published in November, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

readings on the upper and lower limbs and on the right and left limbs may fail appreciably to give what is desired—namely, a representative horizontal and vertical circle reading at the known instant the line of sight is directed to the sun's center.

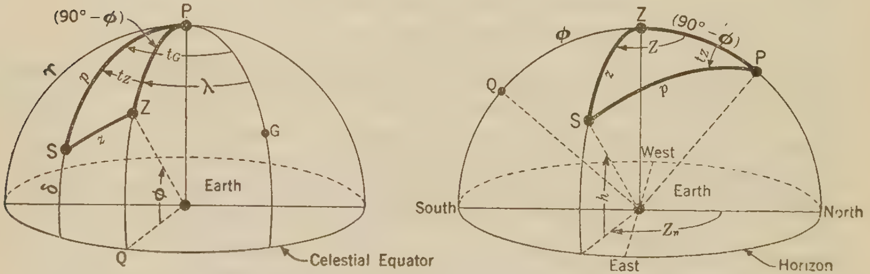
The number of so-called "navigational" stars that are also circumpolar at a particular latitude is quite limited. Among the bright and easily identified circumpolar stars always visible at many places in the northern hemisphere are Capella, Vega, Aldebaran, and Regulus. These stars are much brighter than Polaris, in fact they may be visible when Polaris is obscured by haze or by a cloud. For observation in the southern hemisphere three prominent circumpolar stars with large declinations are Acrux, Canopus, and Sirius, the last being the brightest star in the sky.

Too often engineers wait until the early hours of the morning to observe Polaris at elongation, whereas other stars might serve the same purpose at a convenient time.

NOTATION

The following letter symbols and definitions applying to the astronomical or celestial triangle (triangle PZS in Figs. 1, 2, and 3) are adopted for use in the paper and for the guidance of discussers.

A = a substitution factor = log cosecant × 10^5;
B = a substitution factor = log secant × 10^5;
F = point of intersection of observer's meridian with perpendicular from point S to the meridian;



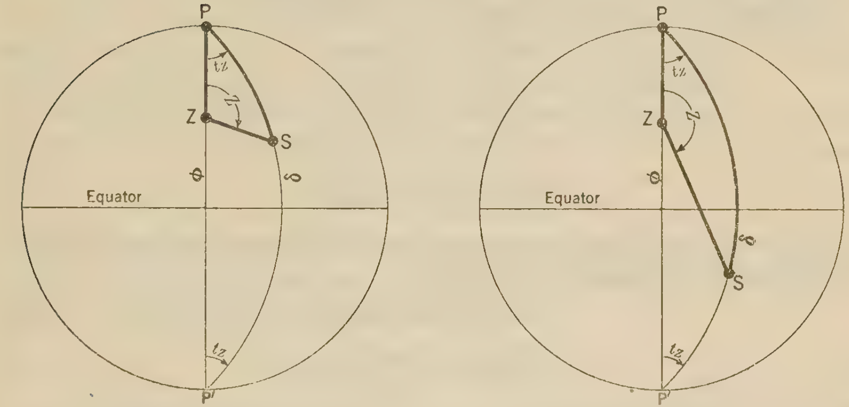
(a) EQUATOR SYSTEM (b) HORIZON SYSTEM

FIG. 1.—RELATION OF POINTS AND ANGLES IN CELESTIAL COORDINATE SYSTEMS

G = Greenwich (reference point);
h = altitude; angle of elevation to the sun or star:
  h_o = observed (corrected);
  h_s = as given by surveyor's transit or sextant (uncorrected);
P = projection of the north (or south) pole on the celestial sphere;
p = polar distance = $90° - \delta$;
Q = intersection of observer's meridian with celestial equator;
S = projection of the sun or star on the celestial sphere;
s = substitution factor = one half the sum of the sides of a triangle;

t = the hour angle:

t_G = at Greenwich, measured westward from the Greenwich meridian to the sun or star;
t_S = sidereal, measured westward from vernal equinox to sun or star;
t_Z = local, measured westward from the observer's meridian to the sun or star;



(a) NORTH DECLINATION (b) SOUTH DECLINATION

Fig. 2.—Relation of Points in PZS Triangle for Sun or Star in Eastern Sky

Z = the zenith at the observer's position;
Z = azimuth angle, reckoned from 0° to 180° east or west of the observer's meridian (sun or star in eastern or western sky); Z_n = azimuth, reckoned from 0° to 360° clockwise from north;
z = zenith distance = $90° - h$;
α = right ascension of sun or star = $360° - t_S$;
γ = arc SF (Fig. 3); perpendicular distance from S to the observer's meridian;
δ = declination; position of the sun or star north or south of the celestial equator;
λ = longitude; the observer's position east or west of the Greenwich meridian;
ϕ = latitude; the observer's position north or south of the equator; ϕ_F = latitude of point F (Fig. 3) north or south of the equator;
♈ = vernal equinox (reference point for stars).

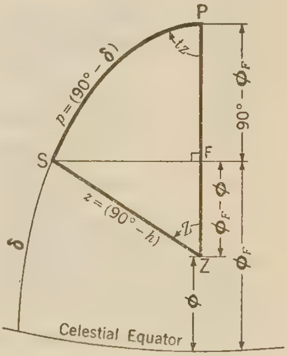


Fig. 3.

Determination of Latitude

Well-known methods of determining latitude are: (1) By meridian altitude of the sun (noon sight); (2) by altitude of Polaris; and (3) by meridian altitude of a star.

Another method applying more readily to either hemisphere is presented here. In Fig. 3 is shown the PZS triangle, in which arc SF ($= \gamma$) is drawn from S perpendicular to arc PZ at F, thus forming right triangles SFP and SFZ. Two cases arise, depending on whether the perpendicular SF falls inside or outside the triangle. If ϕ_F is the latitude of point F, it follows that $(\phi_F - \phi)$ is positive if the perpendicular is inside the triangle, and negative, if outside. Only the first of these two conditions is shown herewith.

Application of the Napier rules to Fig. 3 gives:

$$\sin\gamma = \sin t_Z \cos\delta \quad \text{(1)}$$

$$\sin\phi_F = \frac{\sin\delta}{\cos\gamma} \quad \text{(2)}$$

and

$$\cos(\phi_F - \phi) = \frac{\sin h}{\cos\gamma} \quad \text{(3)}$$

Since the cosecant and secant are reciprocals of the sine and cosine, respectively, these formulas may be rewritten:

$$\csc\gamma = \csc t_Z \sec\delta \quad \text{(4)}$$

$$\csc\phi_F = \frac{\csc\delta}{\sec\gamma} \quad \text{(5)}$$

and

$$\sec(\phi_F - \phi) = \frac{\csc h_o}{\sec\gamma} \quad \text{(6)}$$

It follows that ϕ can be computed if t_Z, δ, and h_o are known. The quantity $(\phi_F - \phi)$ may be positive or negative. It is positive if the body is in the northern sky and negative if the body is in the southern sky. The sign of ϕ_F is the same as that of δ.

Later it will be shown that favorable positions of the body for accurate latitude determination occur when it is on or near the observer's meridian. In the example which follows, the star Procyon is in the southern sky shortly after it has crossed the observer's meridian. (The time of meridian transit of all navigational stars for any date is found in the nautical almanac.)

$t_G = 49°26.5'$ $\quad A = \log \csc \times 10^5$

$\lambda = 48°00.0'$ $\quad B = \log \sec \times 10^5$

$t_Z = 1°26.5'$ (approximately)

		Add	Subtract	Subtract
$h_o =$	59°27.2′ →			A 6489
$t_Z =$	1°26.5′ →	A 159930		
$\delta =$	5°21.8′ →	B 191 →	A 102932	
$\gamma =$		A 160121 →	B 14 →	B 14
$\phi_F =$	5°21.9′		← A 102918	
	Subtract			
$(\phi_F - \phi) =$	− 30°31.0′[a]			← B 6475
$\phi =$	35°52.9′N = Latitude			

[a] $(\phi_F - \phi)$ is negative because star is in the southern sky.

FIG. 4.—WORK FORM FOR EXAMPLE 1

Example 1.—On April 10, 1946, at zone time 18 hr 40 min 12 sec, the corrected altitude to the star Procyon is 59°27.2′. The dead reckoning position of the observer is latitude 36°20′N, longitude 48°00′W. The Greenwich hour angle t_G of Procyon is 49°26.5′ and its declination δ is 5°21.8′. It is required to calculate the correct latitude by means of Eqs. 4, 5, and 6. The work form shown in Fig. 4 is believed to be self explanatory.

DETERMINATION OF LONGITUDE

Time, or longitude, may be determined when the declination, altitude, and latitude are known, the latitude having been found previously from pointings to bodies on or near the meridian. Also, if the observer is on a ship or plane, this latitude would be combined with a determination made by dead reckoning. It will be shown later that observations on bodies on or near the prime vertical—that is, east or west of the observer, lead to accurate values of the local hour angle t_Z or the longitude λ, even when the latitude is approximate.

The local hour angle t_Z from solar or star observations may be calculated accurately, and without rules as to quadrant, from the formula:

$$\sec^2 \tfrac{1}{2} t_Z = \frac{\sec s \sec (s - z)}{\sec \delta \sec \phi} \qquad (7)$$

in which $s = \frac{1}{2}(z + \delta + \phi)$. Eq. 7, which is unusual in that it requires the use of only one trigonometric function, the secant, whose logarithm is always positive and greater than unity, was derived from

$$\cos^2 \tfrac{1}{2} A = \frac{\sin s \sin (s - a)}{\sin b \sin c} \qquad (8)$$

—which applies to a spherical triangle whose angles are A, B, and C and whose opposite sides are a, b, and c. By letting $A = t_Z$, $a = 90° - h$, $b = 90° - \delta$, and $c = 90° - \phi$; and by substituting secants for reciprocals of the cosine, Eq. 8 is transformed into Eq. 7, which represents a solution of the astronomical triangle (Fig. 2).

Example 2.—At sea, on August 26, 1946, at 7 hr 30 min a.m., the corrected altitude to the sun's center is 24°12.0′. The dead reckoning position of the ship is latitude 28°34′N, longitude 44°20′W. The Greenwich hour angle of the sun is 337°01.5′ and its declination is 10°33.6′. It is required to calculate the longitude by use of Eq. 7. The work form shown in Fig. 5 is self explanatory except that the preliminary additions and subtractions can be checked before entering the table (a table of log secants), since $(s - z) + (s - \delta)$ always gives ϕ.

$B = \log \sec \times 10^5$

		B
$h_o =$	24°12.0′	
$z =$	65°48.0′	
$\delta =$	10°33.6′ →	742
$\phi =$	28°34.0′ →	5638
$2s =$	104°55.6′	6380 →
$s =$	52°27.8′ →	21519
$(s - z) =$	− 13°20.2′ →	1187
$(s - \delta) =$	41°54.2′	22706
$\phi =$	28°34.0′ (check)	6380 ←
		2)16326
$\frac{1}{2} t_Z =$	− 34°02.4′	← 8163
$t_Z =$	− 68°04.8′	
$t_Z =$	291°55.2′	
$t_G =$	337°01.5′	
$\lambda =$	45°06.3′ (= west longitude)	

FIG. 5.—WORK FORM FOR EXAMPLE 2

DETERMINATION OF AZIMUTH

Precise determination of the meridian or the true azimuth of a line is, or should be, an integral part of all important land survey and development operations. In making this determination three conditions are encountered. The first is that in which δ, ϕ, and h_o are given and it is required to find Z.

Replacing t_Z with Z in Eq. 7 and substituting the proper values from Fig. 1(*b*) results in—

$$\sec^2 \tfrac{1}{2} Z = \frac{\sec s \sec (s - p)}{\sec h_o \sec \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

—in which Z is the azimuth angle reckoned east or west of the observer's meridian depending on whether the body is in the eastern or western sky, and $s = \frac{1}{2}(p + h_o + \phi)$.

Like Eq. 7, Eq. 9 involves only one trigonometric function, the secant. Since Z is never near 0°, but always about 90°, $\frac{1}{2} Z$ will be approximately 45°. Hence, Eq. 9 is satisfactory for all cases that arise.

Example 3.—On July 31, 1946, at 10 hr 00 min p.m. Eastern Standard Time, the corrected altitude to the star Arcturus, in the western sky, is 34°35.0′. The observer's position is latitude 35°10′N, longitude 79°40′W. The star's declination is 19°27.8′N. The work form for the calculation of the azimuth by Eq. 9 is shown in Fig. 6. Here again the preliminary additions and subtractions can be checked before entering the table (a table of log secants). This is true because $(s - p) + (s - h_o) = \phi$.

B = log sec × 10^5

		B
$\delta =$	19°27.8′	
$p =$	70°32.2′	
$h_o =$	34°35.0′ →	8444
$\phi =$	35°10.0′ →	8752
$2s =$	140°17.2′	17196 →
$s =$	70°08.6′ →	46895
$(s - p) =$	− 0°23.6′ →	1
$(s - h_o) =$	35°33.6′	46896
$\phi =$	35°10.0′ (check)	17196 ←
		2)29700
$\frac{1}{2} Z =$	44°44.0′	← 14850
$Z =$	89°28.0′NW	
$Z_n =$	270°32.0′	

FIG. 6.—WORK FORM FOR EXAMPLE 3

The second condition is that in which δ and ϕ are given, and it is required to find Z when the body is at elongation. The most favorable position for accurate azimuth determination occurs when the star appears to be moving vertically for some time. In this position it is at east or west elongation, depending on whether the body is east or west of the observer's meridian. In these positions the PZS triangle becomes a right triangle in which the angle at S (at the star) equals 90°. Hence, at the instant of elongation,

$$\csc Z = \frac{\sec \delta}{\sec \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$\sec t_Z = \frac{\tan \delta}{\tan \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and

$$\csc h_o = \frac{\csc \phi}{\csc \delta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Because the cosecant is always greater than unity, Eq. 10 is impossible unless δ is numerically greater than ϕ; and since h_o is always positive, Eq. 12 is impossible unless ϕ and δ have the same sign. Hence, for any latitude, elongation does not occur except for these circumpolar stars whose declination is greater than the observer's latitude and of the same sign. Only δ and ϕ are necessary, hence Eq. 10 alone will give the azimuth when the time of elongation is known. Eq. 11 is useful in predicting the time at which elongation will occur on any given date. Eq. 12 is of no practical use, since azimuth in the region of elongation may remain almost constant for large variations in t_Z and h_o.

The field work for this problem is simple. The transit telescope (vernier reading 0°00′) is directed to the star a few minutes ahead of the predicted time of elongation. With the instrument set in this position, the vertical cross wire is kept centered on the star by the lower clamp tangent screw. When the horizontal motion ceases and the star moves vertically, the telescope is plunged and a point is marked 200 ft or more ahead. This marked line makes an angle Z (given by Eq. 10) with true north. If the star was at east elongation, true north is to the left; if at west elongation, true north is to the right.

TABLE 1.—CHANGES IN ALTITUDE AND AZIMUTH FOR SELECTED STARS AT VARIOUS LATITUDES AND LOCAL HOUR ANGLES (IN DEGREES OF ARC)

Latitude,[a] ϕ (1)	Local hour angle,[b] t_Z (2)	Altitude, h_o (3)	Azimuth angle, Z, from north (4)
(a) Capella, $\delta = +45°57'$[c]			
1	63–76	19.8–10.4	40.9–43.3
5	62–80	22.8–10.5	41.7–44.1
10	59–85	28.5–10.6	42.7–44.8
15	54–90	35.5–10.7	43.6–45.0
20	50–90	41.7–14.2	45.5–45.8
25	47–90	47.2–17.7	48.4–46.8
30	41–90	54.4–21.1	51.7–48.1
35	35–73	61.5–35.4	56.6–54.7
40	28–56	68.1–49.5	64.5–62.4
(b) Vega, $\delta = +38°44'$			
1	65–78	20.0–10.0	49.0–51.1
5	60–81	26.4–10.2	49.2–51.7
10	57–85	31.9–10.1	50.6–52.4
15	52–89	38.8–10.0	52.3–52.6
20	46–90	46.4–12.3	54.7–53.2
25	39–90	54.5–15.3	58.1–54.2
30	32–90	62.4–18.1	63.5–55.4
35	29–49	66.6–51.1	72.8–70.1
(c) Aldebaran, $\delta = +16°24'$			
1	50–79	38.4–10.8	69.6–73.4
5	42–81	47.4–10.0	71.0–74.1
10	32–82	58.2–10.4	74.8–74.9
15	25–83	65.9–10.7	83.0–75.6
(d) Regulus, $\delta = +12°14'$			
1	50–80	39.2–10.0	75.3–78.0
5	36–80	53.7–10.8	76.5–78.7
10	25–81	65.4–10.8	83.0–79.6
15	45–82	36.6–10.7	86.0–80.3

Latitude,[a] ϕ (1)	Local hour angle,[b] t_Z (2)	Altitude, h_o (3)	Azimuth angle, Z, from south (4)
(e) Acrux, $\delta = -62°48'$			
−1	60–70	14.0– 9.8	23.9–25.7
−5	60–77	17.7–10.3	24.4–26.7
−10	55–87	24.3–10.3	24.1–27.4
−15	55–90	28.8–13.5	25.1–27.8
−20	52–90	34.6–17.8	25.8–28.5
−25	53–90	38.6–22.1	27.7–29.3
−30	50–90	44.3–26.5	29.1–30.5
−35	50–90	38.6–30.7	31.7–31.9
−40	47–90	54.1–34.9	34.5–33.6
−45	44–90	59.4–39.3	38.3–35.8
−50	40–75	65.0–49.3	43.6–42.3
−55	55–62	61.6–55.5	51.3–50.0
(f) Canopus, $\delta = -52°40'$			
−1	65–74	15.7–10.5	35.0–36.5
−5	61–80	21.3–10.1	34.9–37.5
−10	60–86	26.0–10.3	35.9–38.1
−15	56–90	32.3–11.8	36.7–38.5
−20	54–90	37.4–15.8	38.3–39.2
−25	49–90	44.2–19.6	39.9–40.3
−30	47–90	49.1–23.4	43.0–41.6
−35	45–90	53.9–27.1	46.9–43.2
−40	38–74	61.3–39.7	51.4–49.5
(g) Sirius, $\delta = -16°38'$			
−1	50–80	38.4– 9.9	69.6–73.4
−5	44–81	45.4–10.0	71.5–74.1
−10	32–82	58.2–10.4	74.8–74.9
−15	30–83	61.1–10.8	82.9–75.6

[a] Positive latitudes are north; negative, south.
[b] East or west of observer's meridian.
[c] Positive declinations are north; negative, south.

The third condition occurs when δ, ϕ, and t_Z are given, and it is required to find Z when the body is near elongation. With the increasing facilities of radio, standard time or zone time can be known accurately at any date. Thus, with δ, ϕ, and the local longitude known, one may determine Z independently of h_o, and with greater accuracy, if the star is within the region of elongation.

Table 1 gives an approximate range of values of t_Z, h_o, and Z at various latitudes for a few circumpolar stars. Thus, to an observer at latitude 35°N, the value of Z to the star Capella varies from about 56.6° to 54.7°, as t_Z ranges from 35° to 73°, and h_o varies from 61.5° to 35.4°. Hence, within this region a small error in t_Z will cause no appreciable error in Z. It should be noted that the range of values of the local hour angle refers to the period either before or after the transit of the star across the observer's meridian. Hence t_Z is either east or west and the corresponding values of Z are northeast or northwest if the latitude is north, southeast or southwest if the latitude is south.

Application of the Napier rules to each of the right triangles in Fig. 3 gives, after reducing,

$$\tan \phi_F = \tan \delta \sec t_Z \quad\quad (13)$$

and

$$\cot Z = \frac{\cot t_Z \sec \phi_F}{\csc (\phi_F - \phi)} \quad\quad (14)$$

(*a*) DETERMINATION OF t_Z

Local Civil Time = 21 hr 02 min 20 sec 7/1/46
Time Zone = 6 hr
Greenwich Civil Time = 3 hr 02 min 20 sec 7/2/46
(From Nautical Almanac) $\begin{cases} 0°38.5' \\ 45°37.5' \\ 0°05.0' \end{cases}$
t_G = 46°21.0′
λ = 96°20.0′
t_Z = − 49°59.0′

(*b*) SOLUTION BY EQS. 13 AND 14

$\delta = 38°43.9' \rightarrow \log \tan = 9.90420$
$t_Z = 49°59.0' \rightarrow \log \sec = 0.19178 \rightarrow \log \cot = 9.92407$
$\phi_F = 51°16.8' \leftarrow \log \tan = 0.09598 \rightarrow \log \sec = 0.20376$
$\phi = 32°05.4'$ $\quad 0.12783$
$(\phi_F - \phi) = 19°11.4' \rightarrow \ldots\ldots \log \csc = 0.48320$
$Z = 66°11.6'\text{NE} \ldots\ldots \leftarrow \log \cot = 9.64463$
$Z_n = 66°11.6'$

(*c*) ALTERNATE SOLUTION BY EQS. 4, 5, 6, AND 15

$t_Z = 49°59.0' \rightarrow A\ 11585$
$\delta = 38°43.9' \rightarrow B\ 10786 \rightarrow A\ 20365.5$
$\gamma = \ldots\ldots A\ 22371 \rightarrow B\ \ 9587 \ \rightarrow B\ \ 9587 \rightarrow A\ 22371$
ϕ_F[a] $= 51°16.8' \ldots\ldots \leftarrow A\ 10778.5$
$\phi = 32°05.4'$
$(\phi_F - \phi) = 19°11.4' \rightarrow \ldots\ldots B\ \ 2483$
$h_o = \ldots\ldots A\ 12070 \rightarrow B\ 18508.5$
Z[b] $= 66°11.6'\text{NE} \ldots\ldots \leftarrow A\ \ 3862.5$
$Z_n = 66°11.6'$

$A = \log \csc \times 10^5$
$B = \log \sec \times 10^5$

[a] ϕ takes the same quadrant as t_Z, and the sign of δ. [b] Z is acute if $(\phi_F - \phi)$ is positive; obtuse if $(\phi_F - \phi)$ is negative.

FIG. 7.—WORK FORM FOR EXAMPLE 4

The angle Z may approach 90°, but never 0°, hence the cotangent was selected instead of the tangent in deriving Eq. 14. For convenience in applying Eqs. 13 and 14, the work form shown in Figs. 7(*a*) and 7(*b*) was devised.

As an alternate solution, Eqs. 4, 5, and 6, and

$$\csc Z = \frac{\csc \gamma}{\sec h_o} \quad\quad (15)$$

may be used. These equations are derived by applying the Napier rules to Fig. 3. By the aid of the work form shown in Fig. 7(*c*) this set of formulas is adapted to a table of log secants and cosecants. This alternate solution, although apparently longer, is believed to be much easier than that by Eqs. 13 and 14. Example 4 illustrates both methods.

Example 4.—During the evening of July 1, 1946, a pointing is made to the star, Vega, in the eastern sky. The horizontal motion of the transit is clamped at 21 hr 02 min 20 sec, at the instant the vertical cross hair is on Vega. The Greenwich hour angle of Vega is 46°21.0′, and its declination is 38°43.9′. It is required to calculate the azimuth for an observer at latitude 32°05.4′N, longitude 96°20.0′W by (*a*) Eqs. 13 and 14 and (*b*) Eqs. 4, 5, 6, and 15.

Inasmuch as altitude observations read by the vertical arc of the engineer's transit are scarcely accurate to the nearest minute, it is highly desirable to determine azimuths independently of the altitudes. The second and third of the foregoing cases satisfy this condition and may be classed as favorable methods for accurate azimuth determination.

MOST FAVORABLE CONDITIONS FOR ACCURACY

Certain fundamental relations applying to a spherical triangle are:

$$\sin h_o = \sin \delta \sin \phi + \cos \delta \cos \phi \cos t_Z \quad \ldots\ldots (16)$$

$$\cos h_o \cos Z = \sin \delta \cos \phi - \cos \delta \sin \phi \cos t_Z \quad \ldots\ldots (17)$$

$$\sin t_Z \cos \delta = \sin Z \cos h_o \quad \ldots\ldots (18)$$

and

$$\sin \mathrm{ZSP} \cos \delta = \sin Z \cos \phi \quad \ldots\ldots (19)$$

The law of cosines gives Eq. 16; Eqs. 18 and 19 are applications of the law of sines. Although Eq. 17 is unusual, it is derived elsewhere[2] and will not be derived here.

Latitude.—To evaluate the exact effect on ϕ of any error in t_Z, ϕ may be differentiated with respect to t_Z, assuming h_o and δ to be constant. Accordingly, from Eq. 16,

$$0 = \sin \delta \cos \phi \frac{d\phi}{dt_Z} + \cos \delta \left(- \sin \phi \cos t_Z \frac{d\phi}{dt_Z} - \cos \phi \sin t_Z\right) \quad \ldots\ldots (20)$$

and

$$\frac{d\phi}{dt_Z} (\sin \delta \cos \phi - \cos \delta \sin \phi \cos t_Z) = \cos \delta \cos \phi \sin t_Z \quad \ldots\ldots (21)$$

From Eqs. 17, 18, and 21,

$$\frac{d\phi}{dt_Z} = \tan Z \cos \phi \quad \ldots\ldots (22)$$

This shows that ϕ is entirely independent of t_Z when $Z = 0°$, or when the body is on the observer's meridian. Favorable positions occur when the body is near the meridian—and all the more so for large latitudes.

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[2] "Textbook on Practical Astronomy," by George L. Hosmer, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., p. 256.

Referring to Example 1, it follows from Eqs. 18 and 22 that an error of 10′ of angle in t_Z will cause an error in ϕ of only 0.3′ of arc. Differentiating ϕ with respect to h_o while treating t_Z and δ as constant gives, by Eqs. 16 and 17,

$$\frac{d\phi}{dh_o} = \sec Z \ldots\ldots\ldots\ldots\ldots (23)$$

which shows that the most favorable position is on or near the meridian, but that an error in h_o causes an equal error in ϕ.

Time or Longitude.—Inversion of Eq. 22 gives:

$$\frac{dt_Z}{d\phi} = \frac{1}{\tan Z \cos\phi} = \frac{\sec\phi}{\tan Z} \ldots\ldots\ldots\ldots\ldots (24)$$

which shows that at the instant when $Z = 90°$, $\frac{dt_Z}{d\phi} = 0$; hence, when the body is in that position, an error in ϕ has no effect on t_Z. In other words, the most favorable position of the body is on the prime vertical. It also shows that this statement is all the more true for observers nearer the equator. Thus, for $\phi = 30°$, suppose Z is within 2.5° of the prime vertical; that is, Z_n lies between 87.5° and 92.5°, or between 267.5° and 272.5°. Then $\frac{dt_Z}{d\phi}$ is less than 0.05′. Hence, if the dead reckoning latitude is in error by 20′ the calculated value of t_Z will not be more than 1′ of angle in error. Actually, the latitude should be known within narrow limits if it is determined from a previous or nearly simultaneous sight to a body on or near the meridian.

Longitudes may therefore be determined most exactly when the sun or star is on or near the prime vertical, even though the assumed latitude is somewhat in error.

Differentiating t_Z with respect to h_o while treating ϕ and δ as constant gives, by Eqs. 16 and 18,

$$\frac{dt_Z}{dh_o} = -\csc Z \sec\phi \ldots\ldots\ldots\ldots\ldots (25)$$

which indicates that when $Z = 90°$, $\csc Z$ is least and hence $\frac{dt_Z}{dh_o}$ is least for any ϕ, and all the more so when ϕ is small. The most favorable position of the body is therefore on the prime vertical. The negative sign indicates that t_Z decreases as h_o increases, but if Z is obtuse this sign is reversed.

Using the data in Example 2 and applying Eqs. 18 and 24, it follows that $\frac{dt_Z}{d\phi} = 0.02′$, hence an error in ϕ of 1° causes an error in t_Z of only 1′. However, Eq. 25 shows that $\frac{dt_Z}{dh_o} = 1.1$, hence an error in h_o causes about the same error in t_Z.

Digressing for a moment, it will be seen that by inversion of Eq. 25,

$$\frac{dh_o}{dt_Z} = -\frac{1}{\csc Z \sec\phi} \ldots\ldots\ldots\ldots\ldots (26)$$

Therefore, the computed value of h is most favorably determined, in so far as an error in t_Z is concerned, when the body is near the meridian, and even more so for the large latitudes. This is of particular interest in connection with the "line of position" method of determining latitude and longitude.

Azimuth.—From Eq. 24 and a cyclic change of letters,

$$\frac{dZ}{dh_o} = \frac{\sec h_o}{\tan \mathrm{ZSP}} \quad \text{(27)}$$

This shows that an error in h_o causes a negligible effect on Z when the ZSP angle becomes 90° (that is, when the body is at elongation), which cannot be true except when δ is greater than ϕ so that the observed star is circumpolar. It follows that when δ is less than ϕ, or is in the opposite hemisphere, the most favorable position of the body for azimuth determination will depend on both h_o and the ZSP angle. The relative error in Z is smallest when angle ZSP is large and h_o is small.

(a) Field Notes

Sun Observations—Chapel Hill, N. C.—2/2/46—Clear, cold—T. Rose—Transit No. 6317

LATITUDE = 35°54.7′N LONGITUDE = 79°03.1′W TIME ZONE = 5

Telescope	Object	Time	Vertical circle	Horizontal circle	Magnetic needle
Normal	Line	P.M.	....	0°00′	N20°00′W
		2 hr 06 min	32°47.0′	230°36.0′	
		2 hr 10 min	31°54.0′	232°11.0′	
		4 hr 16 min	64°41.0′	462°47.0′	(= Sum)
		2 hr 08 min	32°20.5′	231°23.5′	(= Average)
Reversed		2 hr 14 min	32°02.0′	52°38.0′	
		2 hr 18 min	31°03.0′	54°18.0′	
		4 hr 32 min	63°05.0′	106°56.0′	(= Sum)
		2 hr 16 min	31°32.5′	53°28.0′	
				(232°28.0′)	(= Average)
Average of normal and reversed readings		2 hr 12 min	31°56.5′	232°25.75′	

Eastern Standard Time = 14 hr 20 min
Greenwich Civil Time (GCT) = 19 hr 20 min
Declination for 0 hr GCT 2/2/46 = − 17°02.8′
Correction (19.2 × 0.72) = 13.8′
δ = − 16°49.0′

Observed altitude (uncorrected) = 31°56.5′
Parallax and refraction = − 1.5′
Observed altitude (corrected) = 31°55.0′

(b) Work Form

		B
δ =	− 16°49.0′	
p =	106°49.0′	
h_o =	31°55.0′→..........	7119
ϕ =	35°54.7′→..........	9156
$2s$ =	174°38.7′	16275→
s =	87°19.35′→.........	133055
$(s - p)$ =	− 19°29.65′→.........	2564
$(s - h_o)$ =	55°24.35′	135619
ϕ =	35°54.7′ (check)	16275←
		2)119344
$\frac{1}{2}Z$ =	75°20.4′...........	←59672
Z =	150°40.8′NW	
Z_n =	209°19.2′	

$$\sec^2 \tfrac{1}{2} Z = \frac{\sec s \sec (s - p)}{\sec h_o \sec \phi}$$

$$s = \tfrac{1}{2}(p + h_o + \phi)$$

$$B = \log \sec \times 10^5$$

FIG. 8.—EXAMPLE OF AZIMUTH DETERMINATION WITH SUN IN UNFAVORABLE POSITION

From Eq. 19 it is seen that the maximum value of angle ZSP occurs simultaneously with the maximum value of Z; that is, when the body is on or near the prime vertical. This position of the body may or may not be near the horizon; if it is, altitudes should not be less than about 15° because of the uncertainty of horizontal refraction. Using the data in Example 3 and applying Eqs. 19 and 27, it follows that $\frac{dZ}{dh_o} = 0.7'$, and hence an error in h_o of 1′ causes an error in Z of 0.7′.

The field notes and reduction of an actual case of well-executed azimuth determination from sun observations are given in Fig. 8. It will be seen, however, that at this latitude, time, and date, the sun is not in a favorable position for the most accurate results. For greater precision it would have been advisable to take check readings at night on Capella or Polaris.

With Z_n to the sun known, a freehand sketch shows that the true bearing of the line is 232°25.75′ − 209°19.2′ = N23°06.55′W, or its azimuth referred to true north is 336°53.45′. Solving Eq. 19 for angle ZSP,

$$\sin \text{ZSP} = \frac{\sin Z \cos \phi}{\cos \delta} \qquad (28a)$$

or

$$\csc \text{ZSP} = \frac{\csc Z \sec \phi}{\sec \delta} \qquad (28b)$$

Substitution of the foregoing values in Eq. 28*b* gives ZSP = 24°28.6′. Then, from Eq. 27, $\frac{dZ}{dh_o} = 2.6'$. Hence, an error in h_o of, say 1′, would cause an error in Z of 2.6′. Evidently this position of the sun is unfavorable for the most accurate azimuth determination. By interchanging the letters t_Z and Z in Eq. 24, however,

$$\frac{dZ}{d\phi} = \frac{\sec \phi}{\tan t_Z} \qquad (29)$$

which shows that the least error in Z due to an error in ϕ will occur when the body is on the 6-hr circle (t_Z = 90° or 270°).

Combining Eqs. 17 and 18, to eliminate $\cos h_o$, and dividing the resulting expression by $\cos \delta$, gives:

$$\sin t_Z \cos Z = \tan \delta \tan \phi - \sin \phi \cos t_Z \qquad (30)$$

Differentiating Z with respect to t_Z, with δ and ϕ held constant, gives:

$$\frac{dZ}{dt_Z} = \frac{\cot t_Z \cos Z - \sin \phi}{\csc^2 Z} \qquad (31)$$

The least error in Z due to an error in t_Z occurs when the numerator of Eq. 31 equals zero, or when

$$\sin \phi = \cot t_Z \cos Z \qquad (32)$$

Eq. 32 holds true only if angle ZSP equals 90°, or when the body is at elongation. Therefore, the most favorable position of the body for azimuth determination, in so far as a possible error in t_Z (and h_o) is concerned, occurs at or near elongation.

DISCUSSION

John C. Penn,[3] M. ASCE.—From a superficial examination, it may appear that Professor Hickerson has presented new and simplified methods of finding latitude, time, and azimuth.

Actually, the methods are not new; and they are not so simplified as those presented by A. A. Ageton in 1942.[4] Like the author, Lieutenant Ageton introduces the idea of multiplying the log secants and log cosecants by 10^5; and the use of the resulting substitution factors A and B by Professor Hickerson is also suggestive of the forms devised by Lieutenant Ageton.

The tables of A and B were designed to enable instructors to speed the inculcation of simple navigation. Use of the tables avoided entirely the conceptions of logarithms and spherical trigonometry, and reduced the entire subject matter to filling in forms, and to simple addition and subtraction. It is, of course, hardly to be expected that anyone would keep a table of values of A and B handy for a surveying problem that occurs only occasionally.

There are simpler methods of representing the zenith-pole-object triangle than those given in Figs. 1, 2, and 3. Few students or engineers have the imagination or the ability to see objects in space. It is most difficult for an observer to draw pictures of things infinitely large, and to place himself outside this infinitely large celestial sphere, when he is actually at the center of the sphere looking out. However, a student can readily visualize the spot on the earth pierced by a line from the sun, star, or planet to the center of the earth. This spot will have a latitude equal to the declination of the celestial body; its longitude will equal the Greenwich hour angle. The motion of this point on the earth can be studied in detail. Right ascension, so difficult to explain, can even be avoided, as it is in modern navigation.

As a revival of some tricks in trigonometry this paper may have value. More emphasis on Greenwich hour angle, local hour angle, and corrections for transit altitudes would indeed be appreciated by the engineer and surveyor. To appraise the full potential value of the ideas presented, however, the careful student will need references to all prior work supporting the broad scope of this subject.

Paul E. Wylie,[5] M. ASCE.—The section under the heading, "Determination of Latitude," in this interesting paper appears to be an adaptation of a method of solution of the PZS triangle, which is widely used in navigation, and which is published elsewhere as "HO 211."[4] Tables of A and B to half minutes also may be found in the same publication.

The navigator solves his triangle for the altitude which would exist at an assumed geographical position—that is, with knowledge of ϕ and t_Z. Professor Hickerson, on the other hand, knowing t_Z and h_o, solves the same triangle for ϕ, using the same methods.

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[3] Prof., Civ. Eng., Illinois Inst. of Technology, Chicago, Ill.

[4] "Dead Reckoning Altitude and Azimuth Table," by A. A. Ageton, *Publication No. 211*, Hydrographic Office, U. S. Navy Dept., Washington, D. C., 1942, p. 18.

[5] Lecturer, Dept. of Astronomy, Univ. of California, Los Angeles, Calif.

It will be interesting to discover circumstances in which this method of latitude determination is of practical utility. To use it, knowledge of the longitude is required; but the cases in which longitude is known and latitude is unknown are rare indeed. On the contrary, cases in which the latitude is known and the longitude is unknown are common enough, and in these cases the PZS triangle is solved by methods involving altitude instead of hour angle. In Example 1 the author has selected a small value of t_Z so as to minimize the error involved through uncertainty in the value of the longitude. This is quite legitimate, of course, but one wonders why the simpler method of meridian altitude was not employed if the observation must be made so near to meridian transit.

As the writer has stated elsewhere,[6] the latitude at upper or lower transit of any astronomical body may be found by

$$\phi = 90^\circ - H + D \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

in which $H = h_o$ when the body lies south of the zenith; $H = 180^\circ - h_o$ when the body lies north of the zenith; $D = \delta$ when the body lies in the upper branch of the meridian; and $D = 180^\circ - |\delta|$ when the body lies below the elevated pole, D being given the algebraic sign of the declination. Eq. 33 is, of course, a variant of the familiar equation,

$$\phi = z + \delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

redesigned to involve altitudes, with which the surveyor and the navigator are familiar, instead of zenith distances, which are unfamiliar, and also to include all cases in one expression, inasmuch as Eq. 34 is limited to upper branch observations.

Eqs. 7 and 9 are new, as far as the writer has observed. They extend the use of "HO 211"[4] to the determination of t_Z and Z, respectively. Presumably, Eq. 7 would be limited in use to stars of low declination. It is probable that Professor Hickerson does not approve the use of Eq. 7 when t_Z is near zero, or the use of Eq. 9 on close circumpolar stars. These are correct limitations, but the advantage of the exclusive use of secants is not clear.

The writer shares Professor Hickerson's preference for hour angle methods over altitude methods for determining azimuths of close circumpolar stars; and he believes that the common preference for altitude methods on these stars is a carry-over from the days when it was difficult to determine the civil time of observation accurately. Since the advent of frequent radio time signals, it is a simple matter, given a reliable watch that has been rated for a short period, to know the time at any moment to the nearest second. The precision of the time determination is thus about four times as great as the precision of the corresponding altitude determination, assuming the use of an engineer's transit in perfect adjustment. By making the assumption, for a reasonably short period, that the change in azimuth of a close circumpolar star is proportional to the time interval (in other words, that the star's path is a straight line), the benefits of repetition on a horizontal angle may be employed; the observations may be much speeded (since no altitudes are read); and a superior precision of azimuth determination may be attained. The error of the assumption involved is negligible for a short arc of a circumpolar star's diurnal circle; moreover, it is involved to a greater degree in the slower altitude method.

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[6] *Popular Astronomy*, March, 1945, p. 108.

Solution of the triangle for azimuth, using the hour angle method, is accomplished by employing

$$\tan Z = \frac{\sin t_Z}{\tan \delta \cos \phi - \sin \phi \cos t_Z} \quad\ldots\ldots\ldots\ldots\ldots (35)$$

It should be noted that, for five-place accuracy, when this equation is applied to Polaris, δ must be known to 0.1 sec of arc. Tabulations of the declination of Polaris at daily intervals to 0.01 sec of arc are available.[7,8] More condensed ephemerides of Polaris are not suitable.

In well-mapped country, position may be found with all necessary precision for circumpolar hour angle observations by consulting a good map, such as one of the quadrangle sheets of the United States Geological Survey, or a chart of the United States Coast and Geodetic Survey. In poorly mapped regions, a program of observations will be required to secure the data necessary for a precise determination of azimuth. Such a program might be as follows:

1. Determine latitude by maximum altitude of a star of low declination.
2. Determine longitude by altitude of a star on or near the prime vertical (solving for t_Z).
3. Determine latitude by Polaris, or another circumpolar body, at culmination.
4. Determine azimuth by Polaris at elongation. If no elongation occurs during the dark hours, use another circumpolar star at elongation, or substitute an observation on Polaris at any time, using the hour angle method.
5. Set up a true meridian on the ground; redetermine longitude by observations of times of meridian passage of a series of equatorial stars.
6. If necessary, recompute step 4, using the revised longitude. This is in the nature of a check, and, since a small error in the longitude will have little effect on the azimuth of a close circumpolar body, the change indicated will usually be negligible. Consequently, steps 5 and 6 may be omitted, except in precise work.

Many engineers have become familiar with modern methods of navigation through wartime training. These persons may substitute for the foregoing program, a determination of position by the line-of-position methods used at sea, using a number of well-spaced stars, measuring the altitude of each (preferably with a sextant and artificial horizon) and noting the time of each observation. If "HO 211" is used for the computation of these observations, t_Z, as has been shown,[9] should not be near 90° for any star used.

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[7] "American Ephemeris and Nautical Almanac," U. S. Govt. Printing Office, Washington, D. C.

[8] "The Ephemeris," Bureau of Land Management, U. S. Dept. of the Interior, Washington, D. C.

[9] "The Accuracy of Ageton's Method in Celestial Navigation," by Samuel Herrick, *Publications o the Astronomical Society of the Pacific*, August, 1944, pp. 149–155.

It may be shown[10] that

$$dZ = \tan Z \cot t_Z \, dt \ldots\ldots\ldots\ldots (36)$$

which shows that an error in the time will have minimum effect on the azimuth when t_Z is near 90°, and that the most precise value of the azimuth will be found when Z is near 0° or 180°. In other words, the most accurate determination of azimuth is found when a close circumpolar star (Polaris) is observed —particularly when the observation of such a star is made at or near elongation. Those surveyors who use astronomical methods infrequently will no doubt continue to rise at some dark, frigid hour to observe Polaris at elongation, rather than troubling to learn other methods which, if understood, might permit a more conveniently timed observation. It may be a satisfaction to them to be reassured that the simple method with which they are familiar remains the most easily computed, the most accurate, and thus, the best.

P. V. H. WEEMS,[11] ESQ.—Dealing in terms of the civil engineer, and presenting convenient formulas for working accurately latitude, longitude, and azimuth, this paper should prove a valuable aid to practical surveyors and engineers. As explained by Professor Hickerson, the methods given are more accurate than are required by the marine navigator and for that reason are longer than those ordinarily used at sea. The author failed to mention his book,[12] which in a hundred pages gives not only a description of the methods, but also the actual tables necessary for accomplishing almost any survey or navigation problem.

The methods and principles involved are obviously correct and the symbols used are especially selected for the civil engineer. It is unfortunate that mariners use different designations for different parts of a triangle. This, apparently, is an excellent field for a national standards committee.

Since Professor Hickerson's explanations for the various problems involved are clear, most of the writer's comments are restricted to the background of the secant tables published and explained by the author. As far as is known, the first condensed tables using log secants multiplied by 10^5 for simplification were devised by S. Ogura, a hydrographic engineer of the Japanese hydrographic department. These tables were a combination of tables of A and B, the table of A being for a quick solution of the right triangle and the table of B for the usual solution. The writer copied the Ogura tables (with permission) in 1927.[13] Subsequent adaptations of these tables include those of A. A. Ageton[14] (which rearranged the secant tables) and other similar tables.

During World War II, as Convoy Commodore, the writer noted that some marine navigators insisted on using the "time sight," whereby the latitude is assumed and the longitude is computed from the declination, the altitude, and the assumed latitude. The method was tested to see why some navigators insisted on its use even though it was longer than other available methods.

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[10] "Textbook of Practical Astronomy," by Jason J. Nassau, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 141.

[11] Capt., U. S. Navy (*Retired*), Annapolis, Md.

[12] "Latitude, Longitude, and Azimuth by the Sun or Stars," by T. F. Hickerson, Chapel Hill, N. C., 1947.

[13] "Line of Position Book," by P. V. H. Weems, U. S. Naval Inst., Annapolis, Md., 1927.

[14] "Dead Reckoning Altitude and Azimuth Table," by A. A. Ageton, *Publication No. 211*, Hydrographic Office, U. S. Navy Dept., Washington, D. C., 1942.

Part of the reason appeared to be psychological because for an assumed latitude a definite longitude was computed, so that a definite point could be put on the chart. The only trouble with this method is that another assumed latitude gives another longitude, and, if two points are computed, it will give the usual line of position.

While perusing the formula for the time sights, it was noted that the formula could be rearranged so that secants and cosecants could be used throughout. It was decided that a short table would be published permitting the longitude to be computed using only nine pages of secants and cosecants. This is exactly what Professor Hickerson did independently.

The secant tables were available in fairly convenient forms in several publications.[12,13,14] However, it was believed that an article stating that the time sight could be calculated by tables already included in other methods would not impress the marine navigator as much as would a separate book. For this reason the tables[13] published by Naval Institute at Annapolis, Md., were reduced to a table giving nine pages of secants and cosecants conveniently arranged with auxiliary tables.[15] By test, the method proved to be about 7% faster than the older time-sight method, in spite of the fact that the method required more steps.

Obviously the advantage of these methods resides largely in the simplification of the logarithm as published by Professor Hickerson, by the writer, and by others. Without question the young man starting in engineering could, with Professor Hickerson's tables and explanations, determine accurately the values of latitude, longitude, and azimuth, not to mention many other operations.

T. F. Hickerson,[16] M. ASCE.—As explained in the "Introduction," this paper deals primarily with the precise determination of azimuth and only incidentally with the determination of latitude and longitude. It might also be stated that in trying to simplify azimuth calculations the motive is to encourage wider use of true meridian reference lines on the part of engineers and surveyors who remember little or nothing about the manipulation of trigonometric and logarithmic functions.

In calculating latitude the discussers are correct in stating that an adaptation has been made of methods used by navigators, particularly those of A. A. Ageton.

Captain Weems, sponsor of the Weems System of Navigation, was requested to present a discussion in order to represent the navigators' point of view and thus to give credit where credit is due. Captain Weems makes a welcome contribution in presenting the background of the log secant and cosecant tables. These quantities are relatively simple in being always positive. They are beginning to appear in textbooks on trigonometry, designated by their real names, of course, and not by the letters A and B.

The author endorses Captain Weems' suggestion for uniform symbols, and expresses the hope that those used by navigators will be adopted—that is, L

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[15] "The Secant Time Sight," by P. V. H. Weems, Weems System of Navigation, Annapolis, Md., 1944.

[16] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

for latitude instead of ϕ (a well-known symbol for angle of friction), and d for declination instead of δ.

Professor Penn, on noting that the latitude determination is similar to the Ageton method, concludes that nothing in the paper is new or simplified. The writer disagrees emphatically with Professor Penn's statement that "There are simpler methods of representing the zenith-pole-object triangle than those given in Figs. 1, 2, and 3."

Mr. Wylie's program of observations for poorly mapped regions is a valuable contribution to the paper, and it includes, among other suggestions, a good resume of the ideas presented. Mr. Wylie recognizes that Eqs. 7 and 9 are new, but he raises the question as to the advantage of the exclusive use of secants. The answer is based on a desire to promote the greatest possible simplicity.

The following equation (a well-known formula) would give the same result:

$$\tan^2 \tfrac{1}{2} Z = \frac{\sin (s - \phi) \sin (s - h_o)}{\cos s \cos (s - p)} \qquad (37)$$

However, it involves three trigonometric functions with possible negative characteristics (in case logarithms are used), whereas Eq. 9 deals entirely with one function whose values are always positive numbers greater than unity. To the expert this difference is trivial but to those who are unfamiliar with trigonometry and logarithms it assumes importance. Of course, Eqs. 9 and 37 apply mainly to solar observations where the altitudes are needed.

Eqs. 13 and 14 might be added to the list of new formulas. In comparison with Eq. 35 they possess the advantage of being adapted to logarithmic computation. As pointed out, the combinations of Eqs. 4, 5, 6, and 15 are easier to solve provided the work form given in Fig. 7 is followed. The letter h is used in these formulas only as a medium. Actually the altitudes are unnecessary.

To conform to north as zero azimuth, Eq. 35 should be rewritten

$$\tan Z = \frac{\sin t_Z}{\sin \phi \cos t_Z - \cos \phi \tan \delta} \qquad (38)$$

Strict attention must be paid to the algebraic signs when using Eq. 38. If a table of natural trigonometric functions and a calculating machine are available, Eq. 38 is a very acceptable formula. As an illustration, Example 4 will be solved. Given $t_Z = -49° 59.0'$, $\delta = 38° 43.9'$, and $\phi = 32° 05.4'$. Then $\sin t_Z = -0.76586$; $\cos t_Z = 0.64301$; $\sin \phi = 0.53125$; $\cos \phi = 0.84722$; and $\tan \delta = 0.80206$. Applying these data to Eq. 38,

$$\tan Z = \frac{-0.76586}{0.34160 - 0.67952} = \frac{-0.76586}{-0.33792} = +2.26639$$

Hence $Z = 66° 11.5'$ NE, and $Z_N = 66° 11.5'$. The two calculated values of Z in Fig. 7 are 66° 11.6', which is satisfactory agreement.

The writer is glad to have unanimous approval of hour angle methods over altitude methods for determining azimuths of close circumpolar stars; and the remarks by Mr. Wylie as to the precision of time from radio signals reinforce the paper to a considerable extent.

No comment was made on Table 1. This information came by scanning the pages of *Publication No. 214* (a Public Works Administration project conducted by the Hydrographic Office of the United States Navy Department), that monumental work of nine volumes comprising the complete solution in tabular form of approximately 1,500,000 astronomical triangles. These extracts in Table 1 are very enlightening. Suppose, for example, the latitude is 20°N and the star is Capella; then, as the hour angle changes from 50° to 90°, and the corresponding altitudes change from 41.7° to 14.2°, the azimuth varies from 45.5° to 45.8°. In other words, this star appears almost stationary in direction during the interval of 3 hours 40 min. Attention is called to the fact that these hour angles apply to periods both before and after transit.

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Paper No. 2366

SHEARING STRESS DISTRIBUTION IN BOX GIRDERS WITH MULTIPLE WEBS

By A. S. Neiman,[1] Assoc. M. ASCE

With Discussion by Messrs. John E. Goldberg, J. G. Clark and W. E. Hanson, and A. S. Neiman.

Synopsis

A method for finding the distribution of shearing stress in box girders with multiple webs is presented in this paper. In effect, the structure is allowed to distort under an assumed set of shearing forces. A correction is then introduced to satisfy conditions of geometry known to exist. The particular example solved indicates that in some parts of the structure the shearing stresses are considerably greater than the average.

Introduction

In computing the shear stress in a box girder with multiple webs, the formula frequently used is

$$v = \frac{V\,Q}{I\,t} \qquad (1)$$

in which v is the unit shear stress at any point; V is the external shearing force; Q is the statical moment of the sectional area above the plane considered, about the neutral axis; I is the moment of inertia of the area of the cross section; and t is the combined thickness of members cut at the section. Eq. 1 gives the shear stress for a beam with single web of thickness t, or for a symmetrical box girder section with two webs of combined thickness t. For a girder with more than two web plates, such as is used frequently in heavy grillage foundations, or for a ship with longitudinal bulkheads, Eq. 1 is inadequate because it gives an average shear-stress value for all web members at the section. The actual shear stress may vary considerably from the average value. This is particularly true where web members are not of equal thickness, as is frequently the case for ships with longitudinal bulkheads.

The distribution of shear stress throughout a multiple-web beam, such as a ship with longitudinal bulkheads or a vessel with bulkheads that do not extend

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Note.—Published in February, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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the full height, can be obtained by the mechanics of "shear flow" and the application of the principles of continuity to contiguous cells. The term "shear flow" as used herein denotes shear force per unit of linear measurement. Therefore, the shear flow at a point is the shear stress multiplied by the thickness of the metal at the point. The concept of shear flow is used extensively in the design of airplane structures.[2] The cross section of a vessel under external shearing forces is subjected to shearing stresses which may be considered as "flowing" through the section. This shear flow must be in equilibrium with the external shear force at the section, or with a combination of the external shear and the shear resulting from twisting moments. Moreover, the shear distribution must be such that contiguous cells undergo the same angular distortion. Therefore, the solution of the problem of shear distribution in a complex structure such as a ship with three or more shear-carrying webs, or with multiple decks, requires the application of the principles of geometry and statics. Two cases of shear distribution will be considered.

NOTATION

The following letter symbols adopted for use in this paper, and in its discussions, conform essentially with American Standard Letter Symbols for Structural Analysis, prepared by a committee of American Standards Association with Society participation:

A = area of one or more cells in the cross section of a tanker; ΔA = an increment of area of a web, or member = $t\,\Delta L$;
F = "function of"; F(I) and F(II) are simplifying substitution quantities defined by the left-hand side of Eqs. 4*a* and 4*b*, respectively;
G = modulus of elasticity in shear;
I = moment of inertia of the area of the cross section;
L = length of a web, or member; ΔL = an increment of length;
M = bending moment; M_t = external twisting moment;
Q = statical moment of the sectional area above the plane considered, about the neutral axis;
q = shear flow, in pounds (or kips) per unit of length:
    q_a and q_b = shear flows introduced into cells I and II;
    q_{BC} = shear flow at point B in member BC;
    q_{GH} = shear flow at point G in member GH;
    q_0 = known shear at any point;
    q_1 = assumed shear flow;
    q'_1 = average assumed shear flow;
t = thickness of a web or combined thickness of the members cut at a section;
V = external shearing force;
v = unit shear stress at any point;
z = moment arm; distance from the neutral axis to the centroid of the increment of area (ΔA) bounded by the plane upon which the shear is known and the plane upon which the value of shear is desired; and
θ = angle of twist.

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[2] "Airplane Design," by K. D. Wood, Purdue Univ., Lafayette, Ind., 1941.

CASE I. NO TORSIONAL MOMENT AT SECTION

If no twisting moments exist at the section, the shearing stress must be the result of increments of longitudinal flexure only. The procedure for determining the distribution of shear stress for this case can be described in four steps:

1. Assume a distribution of shear flow q_1 consistent with the formulas:

$$q_1 = \frac{V}{I} \Sigma z \, \Delta A \ldots\ldots\ldots\ldots (2a)$$

or

$$q_1 = q_0 \pm \frac{V}{I} z \, \Delta A \ldots\ldots\ldots\ldots (2b)$$

in which V is the total shear at the section; I is the total moment of inertia about the neutral axis; q_0 is a known shear at any point; and $z \, \Delta A$ is the statical moment, about the neutral axis of the increment of area bounded by the plane upon which shear is known and the plane upon which the value of shear is desired.

2. Compute

$$2 \, A_1 G \, \theta = \Sigma q_1 \frac{\Delta L}{t} \ldots\ldots\ldots\ldots (3)$$

in which A_1 is the area of one cell; G is the modulus of elasticity in shear; and $\frac{\Delta L}{t}$ is the ratio of increment of length to corresponding thickness. Eq. 3 gives the angle of twist, θ, under the assumed shear flow, q_1, in each cell.

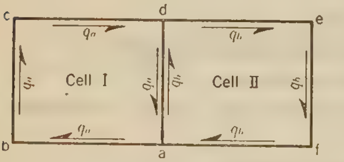


FIG. 1

3. Since in this example no torsion is acting on the structure, the angle of twist for each cell equals zero. Hence a shear flow acting around each cell can be introduced so that the resultant angle of twist of the cell is zero. This procedure can be followed for each cell by the use of equations similar to Eq. 3. For example, for a two cell structure (Fig. 1), the shear flows q_a and q_b are introduced into cells I and II, respectively, as corrections to the assumed shear flow q_1 so that the resultant angle of twist will be zero.

Hence the following formulas for the shear strain are obtained:

$$q_a \, \Sigma \frac{\Delta L}{t} \left[\begin{array}{c} \mathrm{a} \rightarrow \mathrm{b} \\ \mathrm{b} \rightarrow \mathrm{c} \\ \mathrm{c} \rightarrow \mathrm{d} \end{array} \right] + (q_a - q_b) \frac{\Delta L}{t} [\mathrm{d} \rightarrow \mathrm{a}] - \Sigma q_1 \frac{\Delta L}{t} \text{ (for cell I)} = 0 \,..\, (4a)$$

and

$$q_b \, \Sigma \frac{\Delta L}{t} \left[\begin{array}{c} \mathrm{d} \rightarrow \mathrm{e} \\ \mathrm{e} \rightarrow f \\ \mathrm{f} \rightarrow \mathrm{a} \end{array} \right] + (q_b - q_a) \frac{\Delta L}{t} [\mathrm{d} \rightarrow \mathrm{a}] - \Sigma q_1 \frac{\Delta L}{t} \text{ (for cell II)} = 0 \,..\, (4b)$$

in which the symbol [a → b], etc., denotes reference to the member extending from point a, Fig. 1, to point b.

If by symmetry, or by judicious choice of the assumed shear flow, q_a or q_b is equal to zero, Eqs. 4 would be reduced to a single equation:

$$q_a \text{ or } q_b = \frac{\Sigma q_1 \dfrac{\Delta L}{t}}{\Sigma \dfrac{\Delta L}{t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

4. From Eqs. 2 and 5, denoting q_a or q_b as q_2, the resultant shear flow at any point is found equal to

$$q_3 = q_1 + q_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

CASE II. WITH TORSION AT SECTION

The shear distribution throughout a beam section will be modified from that of case I if the beam is subjected to twisting moments in addition to flexural moments. A solution of this problem may be obtained by a procedure similar to that for case I except that steps 3 and 4 should be modified.

In a continuous structure the angles of twist of any two contiguous cells (see Fig. 1) are equal. Therefore, A_1 times the left side of Eq. 4*a*, step 3, must be equal to A_2 times the left side of Eq. 4*b*; thus:

$$A_1 F(\mathrm{I}) = F(\mathrm{II})\, A_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Another equation is obtained from statics by equating the moment of the internal shearing forces with the external twisting moment, M_t.

This procedure gives two equations from which the unknowns q_a and q_b can be determined. The final step giving the resultant shear flow at any point is obtained by combining q_a or q_b with q_1, as indicated in step 4, case I.

The signs of the various quantities $q \dfrac{\Delta L}{t}$ in Eqs. 4 to 7 may be kept consistent by assigning positive or negative signs to these quantities according to whether they cause clockwise or counterclockwise rotation in a particular cell.

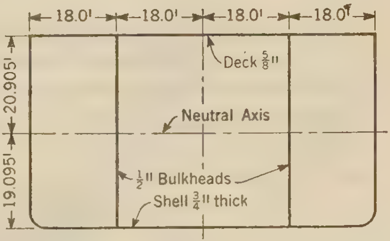


FIG. 2

EXAMPLE

Fig. 2 shows a cross section of a tanker subjected to a vertical shear of 12,000 kips. The moment of inertia about the neutral axis is 635,000 in².-ft². It will be assumed that the torque at the section equals zero.

Table 1 indicates, in an orderly manner, the arithmetical computations necessary to obtain the shear distribution for this example. Col. 5, Table 1, gives a set of assumed values of shear flow q_1 computed by Eq. 2*b*. These values of shear flow are plotted in Fig. 3. Because of symmetry the shear flow at points C and H equals zero; hence the shear flow (in pounds per linear inch) at point B in member BC is obtained directly from Eqs. 2:

$$q_{BC} = \frac{V}{I} z\,\Delta A = \frac{12{,}000{,}000}{635{,}000} 20.905 \times 18 \times \tfrac{5}{8} = -\,4{,}445$$

and, similarly,

$$q_{GH} = \frac{V}{I} z\,\Delta A = 18.9 \times 19.095 \times 18 \times \tfrac{3}{4} = -\,4{,}872$$

The sign of shear flow is negative since the rotation is clockwise.

TABLE 1.—SHEAR DISTRIBUTION, TYPICAL MIDSHIP SECTION OF A TANKER

Member[a] (Fig. 3)	ΔA (ft-in.)	z (ft)	$z\,\Delta A$ (ft²-in.)	q_1	q'_1	$\frac{\Delta L}{t}$	$q'_1 \frac{\Delta L}{t}$	q_3 shear flow[b]	Shear stress[c]
				(Lb per in.)					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
E.........	....	....	....	0	....	....	....	6,544	13,090
B(E)......	10.452	10.452	109.24	− 2,065	− 688	41.81	− 28,765	4,479	8,960
B(C)......	11.250	20.905	235.2	− 4,445	....	....	....	−4,445	7,110
B(A)......	....	....	....	− 6,509	− 8,732	28.8	−251,482	34	50
A(B)......	11.250	20.905	235.2	−10,954	....	....	....	−4,410	7,060
A(D)......	15.680	10.452	163.9	−10,954	−13,020	27.93	−363,649	−4,410	5,880
D.........	....	....	....	−14,052	....	....	....	−7,508	10,010
F(D)......	14.32	9.547	136.7	−11,469	−13,190	25.46	−335,817	−4,925	6,570
F(G)......	....	....	....	−11,469	....	24.0	....	−4,925	6,570
G(F)......	13.50	19.095	257.8	− 6,596	− 9,032	....	−216,768	− 52	70
G(H)......	13.50	19.095	257.8	− 4,872	....	....	....	−4,872	6,500
G(E)......	9.547	9.547	91.2	− 1,723	− 574	38.19	− 21,921	4,820	9,640
Σ.........						186.19	−1,218.402		

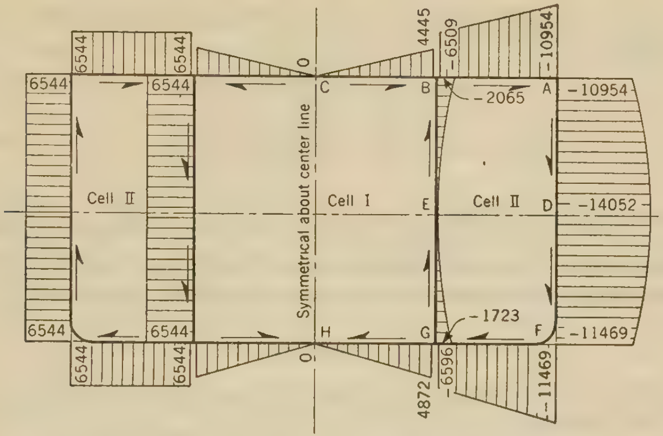
[a] Member or point; the designation B(E), for example, signifies the shear in member BE, Fig. 3, or the properties of member BE. [b] Shear flow, in pounds per inch. [c] Shear stress, in pounds per square inch.

By assuming a shear flow at point E in member BE the shear flow at point B in member BE may be obtained from Eq. 2*b*. Assuming the shear flow at point E equal to zero for convenience, the assumed shear flow is computed as in Cols. 3 and 4, Table 2. Similarly, by use of Eq. 2*b*, the assumed shear flow at each successive point may be computed. This shear flow is shown in Fig. 3.

TABLE 2.—COMPUTATION OF SHEAR FLOW AROUND CELL II, FIG. 3, IN POUNDS PER LINEAR INCH

Shear flow (1)	Member (2)	Computation (3)	Assumed flow (4)	Total flow (5)
q_E.............	....	q_e in Eq. 2*b*	0	6,544
q_B.............	BE	18.9 ×0.5 ×20.902 ×20.905 ×0.5	− 2,065	4,479
q_B.............	BA	−2,065 +(−4,445)	− 6,510	34
q_A.............	AB	−6,510 −(18.9 ×20.905 ×18 ×5/8)	−10,954	−4,410

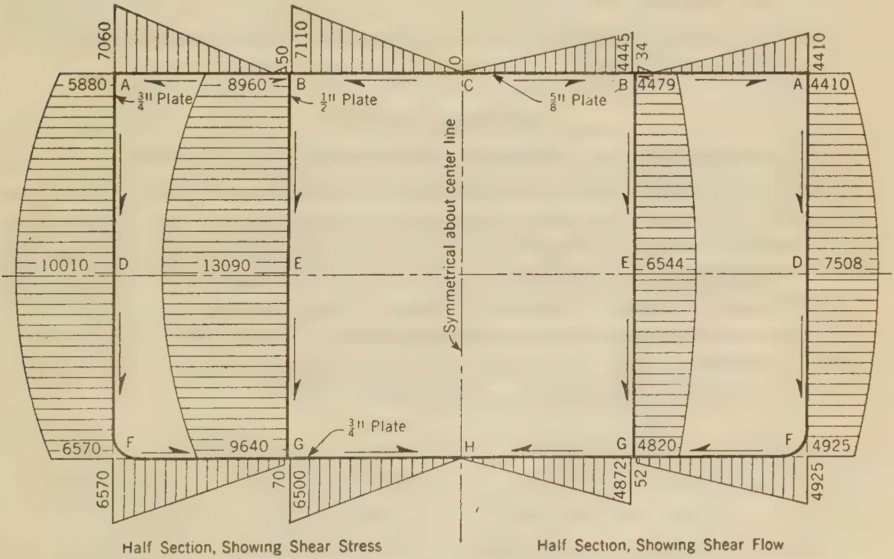
The assumed shear flow will result in a twist of cell II. Since the angle of twist θ must be zero, a shear flow, acting around the cell, will be introduced as a


6544
6544
6544
6544
0
4445
-6509
-10954
C
B
A
-2065
-10954
Symmetrical about center line
Cell II
Cell I
E
Cell II
D
-14052
6544
6544
-1723
H
G
F
-11469
6544
6544
0
4872
-6596
-11469

Half Section
Shear flow to correct for twist of Cell II

Half Section
Assumed shear flow

FIG. 3.—COMPUTED SHEARS IN VESSELS WITH LONGITUDINAL BULKHEADS, IN POUNDS PER LINEAR INCH (CLOCKWISE ROTATION IS NEGATIVE)


7060
50
7110
0
4445
34
4410
5880
A
$\frac{3}{4}$" Plate
8960
B
$\frac{1}{2}$" Plate
C
$\frac{5}{8}$" Plate
B
4479
A
4410
Symmetrical about center line
10010
D
13090
E
E
6544
D
7508
$\frac{3}{4}$" Plate
6570
F
9640
G
H
G
4820
F
4925
6570
70
6500
4872
52
4925

Half Section, Showing Shear Stress

Half Section, Showing Shear Flow

FIG. 4.—FINAL SHEAR STRESSES IN VESSELS WITH LONGITUDINAL BULKHEADS, IN POUNDS PER LINEAR INCH

correction to the assumed shear flow so that the resultant angle of twist will be zero. This can be done by Eq. 5, thus:

$$q_2 = \frac{1,218,402}{186.19} = 6,544 \text{ lb per in.}$$

Applying this constant correction to the assumed shear flow around cell II (Col. 4, Table 2) gives the final shear flow as shown in Col. 5, Table 2, and in Fig. 4. The unit shear stress at any point (in pounds per square inch) is obtained by dividing the shear flow at the point by the thickness of the plate at the point:

$$v_E = \frac{q_E}{t_E} = \frac{6{,}544}{0.5} = 13{,}090$$

$$v_D = \frac{q_D}{t_D} = \frac{7{,}508}{0.75} = 10{,}010$$

If computed by the ordinary formula, the shear stress, $v = \frac{V\,Q}{I\,t}$ and $v_E =$ 11,250 lb per sq in.—or an error in this case on the unsafe side of 16%. The distribution of the shearing stresses is shown in the right half section of Fig. 4.

SUMMARY

The distribution of the shearing stress in a multiple-web beam may be found by the following steps:

1. Assume a statically consistent set of shearing stresses;
2. Find the angular distortion that results from these forces;
3. Introduce a set of shearing forces of such magnitude that the resultant angle of twist will be equal to that known to exist in the structure; and
4. Superpose stresses found in step 3 on those of step 1 to give the desired stress distribution.

ACKNOWLEDGMENTS

The procedure for computing shear stress distribution and the particular application of the principles of mechanics to the solution of the problem as outlined in this paper were developed by the writer while engaged in the structural design of concrete barges for the United States Maritime Commission. The writer gratefully acknowledges the help given by members of the staff of the Research Section, Technical Division, of the commission, and particularly the encouragement given by John Vasta, assistant chief of research. Special mention is due C. C. Ragsdale of the Research Section for valuable assistance during the preparation of this paper.

DISCUSSION

JOHN E. GOLDBERG,[3] ASSOC. M. ASCE.—An interesting problem is presented in this paper. The author's statement that he has developed the principles of mechanics and the method which he has presented cannot be doubted. It is unfortunate, however, that the effort has been expended because not only current literature, but also an impressive list of textbooks[4,5,6,7,8,9,10] discuss and solve the problem in great detail.

The author presents, without derivation, only the simple case of a symmetrical two-cell box girder. This problem, as well as the much more general cases involving complete asymmetry, loads in any direction, the effects of taper, and greater numbers of cells, is practically routine in fields such as the aircraft industry. It has been handled so frequently during the past decade in this particular industry that many firms have developed standardized forms[7,9,10] to facilitate computations.

Certain limitations, not delineated by the author, should be mentioned. As presented in this paper the procedure is limited unnecessarily to symmetrical structures. The analysis as presented in the previously mentioned textbooks does not suffer from this restriction. Unless certain corrections are made, however, the general method is applicable under the following conditions only:

(*a*) When there are no restraints against warping at any cross section of box girder, or the method may be used only at sections remote from such restraints; and

(*b*) When the bulkheads, diaphragms, or ribs are assumed to be infinitely stiff in their own plane (but free to warp "out of plane"), or the method may be applied only at sections remote from the loads.

A word on the theoretical basis of the method of analysis may be in order, as it has not been included in the paper. The method is predicated upon the following:

(1) At every junction of two or more webs,

$$\Sigma q + \frac{V_x \, y \, \Delta a}{I_x} + \frac{V_y \times \Delta a}{I_y} = 0 \quad \text{.................} (8)$$

in which Σq is the algebraic sum of the shear flows in the webs approaching the junction; V_x and V_y are the applied shears perpendicular to the principal x-axis and y-axis, respectively; I_x and I_y are the moments of inertia of the bending material of the cross section about the x-axis and y-axis, respectively; and Δa is the elementary area of bending material located at the junction.

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[4] "Airplane Structures," by Alfred S. Niles and Joseph S. Newell, John Wiley & Sons, Inc., New York, N. Y., 1943, 3d Ed., Chapter XVI, p. 180.

[5] "Mechanics of Airplane Structures," by John E. Younger, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, pp. 188-194.

[6] "Airplane Structural Analysis and Design," by Ernest E. Sechler and Louis G. Dunn, John Wiley & Sons, Inc., New York, N. Y., 1942, pp. 345-352.

[7] "Analysis and Design of Airplane Structures," by Elmer F. Bruhn, John S. Swift & Co., Cincinnati, Ohio, 1943, pp. A15.8–A15.14 and pp. C9.16–C9.18.

[8] "Strength of Materials," by S. Timoshenko, Pt. II, D. Van Nostrand Co., New York, N. Y., 1941, 2d Ed., pp. 278–282.

[9] "Design of Wood Aircraft Structures," *Bulletin No. 18*, Army-Navy-Commerce Committee on Aircraft Requirements, Govt. Printing Office, Washington, D. C., 1942, Chapter 3, Section 3.135.

[10] "Unit Method of Beam Analysis," by F. P. Cozzone, *Lockheed Paper No. 51*, Lockheed Aircraft Corp., Burbank, Calif., 1941.

In the case where there is no bending element at the junction, Eq. 8 reduces to

$$\Sigma q = 0 \qquad (9)$$

The correctness of Eq. 9 is seen immediately (Fig. 5) if equilibrium in the z-direction is considered.

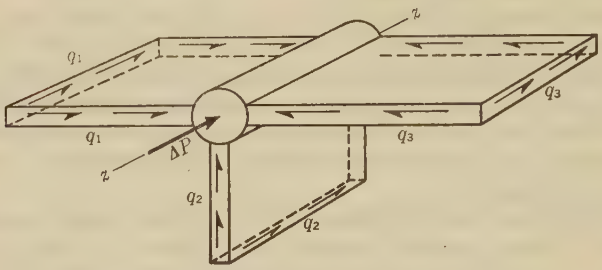


Fig. 5.—Forces at a Junction

(2) The twist of an individual cell—

$$\theta = \frac{1}{2\,A\,G} \oint q \frac{ds}{t} \qquad (10)$$

—is readily proved by strain-energy principles in the following manner:

At any point, the shearing strain is $\frac{q}{t\,G}$. Therefore, the total strain energy per unit length along the span is

$$U = \frac{1}{2\,G} \oint \frac{q^2\,ds}{t} \qquad (11)$$

in which the symbol $\oint$ signifies the line integral around the periphery of the cell. If

$$q = q_0 + \frac{T}{2\,A} \qquad (12)$$

in which $\frac{T}{2\,A}$ represents the shear flow due to a real or virtual torque of magnitude T, and q_0 represents all other shear flows, then

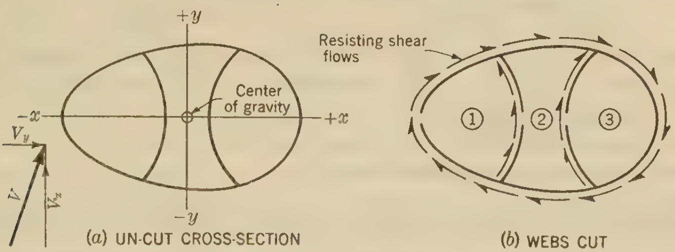
$$U = \frac{1}{2\,G} \oint \frac{\left(q_0 + \frac{T}{2\,A}\right)^2 ds}{t} \qquad (13)$$

and the twist per unit length along the span is

$$\theta = \frac{\partial U}{\partial T} = \frac{1}{2\,G} \oint \frac{2\left(q_0 + \frac{T}{2\,A}\right)\frac{1}{2\,A}\,ds}{t} = \frac{1}{2\,A\,G} \oint \frac{\left(q_0 + \frac{T}{2\,A}\right)}{t}\,ds \qquad (14)$$

If T is not real, but virtual, Eq. 14 reduces to Eq. 10.

(3) The twists of all cells are equal, but only in the special case where the transverse bulkheads are infinitely stiff in their own plane. In many actual box girders, this restrictive condition is practically realized and the equivalence of cell twists becomes a valid assumption. (The correctness of this point may also be proved by use of the principle of least work.)



(a) UN-CUT CROSS-SECTION (b) WEBS CUT

FIG. 6

With these points in mind, any cylindrical multiple-cell box girder may be analyzed. Taking, for example, a general asymmetrical girder, as shown in Fig. 6(*a*), the structure is cut back to a condition in which increments of shear due to bending elements may be evaluated (Fig. 6(*b*)) by cutting all but one web at convenient locations. A suggested procedure is as follows:

1. Locate the center of gravity of all bending elements, pass any two rectilinear axes through this point, and compute I_x, I_y, and I_{xy}, the moments of inertia about the x-axis, the y-axis, and the product of inertia about these axes, respectively. If the selected axes are the principal axes, then $I_{xy} = 0$.

2. Determine the distribution of static shear flows at every point in the cut-back structure which will be in equilibrium with the applied shear V, or its rectilinear components V_x and V_y. Note that this distribution, in conjunction with the external loads, will not, in general, satisfy the equilibrium condition $\Sigma T = 0$. Beginning with the shear flow equal to zero at each cut, the increments of shear flow due to bending elements may be obtained conveniently in the general case by the formula:

$$dq = \frac{V_y I_x - V_x I_{xy}}{I_x I_y - (I_{xy})^2} x \, da + \frac{V_x I_y - V_y I_{xy}}{I_x I_y - (I_{xy})^2} y \, da \ldots\ldots\ldots (15)$$

3. Set up the equilibrium equation for torsion, selecting any convenient reference point about which the torsions may be calculated,

$$\Sigma T = 0 = T_o + T_S + q_1 2 A_1 + q_2 2 A_2 + q_3 2 A_3 + \cdots + q_n 2 A_n \ldots (16)$$

in which T_o denotes the torsional moment of the external loads about the selected reference point in the plane; T_S represents the moment of the resisting static shear flows determined in step 2; and q_1, q_2, $\cdots$, q_n are the unknown closing shear flows that are to be added to the respective cells to satisfy the necessary conditions of continuity as well as equilibrium (Fig. 7).

It is pointed out, in connection with the equation of torsional equilibrium, that the moment of a shear flow of magnitude q lb per in., acting on a length of web or skin Δs inches in length, about any arbitrary reference point, O (Fig. 8), is

$$\Delta T = 2\, q\, \Delta A \qquad (17)$$

in which ΔA represents the area bounded by the element of skin and the two rays from point O to the two ends of the element.

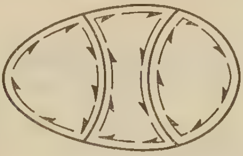


Fig. 7.—Closing Resisting Shear Flows

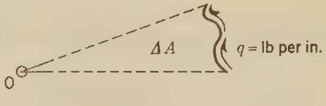


Fig. 8

4. Set up the continuity equations which, in the case of rigid bulkheads, are

$$\theta_1 = \theta_2 = \theta_3 = \theta_n \qquad (18)$$

using Eq. 10

5. Solve the equations of continuity simultaneously with the equilibrium equation to obtain the closing shear flows, $q_1, q_2, \cdots, q_n$.

6. Obtain the net shear flows, which at every point are the static shear flow plus the proper closing shear flows. Note that in webs common to two cells the two closing shear flows act in opposite direction if they are of the same sign.

J. G. Clark,[11] Assoc. M. ASCE, and W. E. Hanson,[12] Assoc. M. ASCE.—Designers of civil engineering structures can benefit greatly from knowledge of various methods of analysis developed primarily for use in airplane design. It is unfortunate that literature pertaining to civil engineering structures does not present more of these methods in detail; therefore, the author should be commended for presenting to civil engineers the concept of shear flow in multiple-cell structures with thin webs. There are many other references supplemental to the one given by the author, among which the work of Alfred S. Niles, Assoc. M. ASCE, and Joseph S. Newell,[13] and Elmer F. Bruhn[14] should be mentioned.

The author's omission of the development of Eq. 3 might imply that it is exact. It can be allowed only if the walls are thin so that the shear stress can be assumed uniformly distributed across the thickness. The equation also neglects the shear components normal to the center line of the walls, which are small in comparison with those tangent to the center line of the walls in any structure such as the example used by the author. One of the relationships used in the derivation of Eq. 3 is

$$q = \frac{T}{2\,A} \qquad (19)$$

which assumes that the member is free to warp (since there are no longitudinal stresses in the direction that tends to resist warping) and that the general shape of the section remains the same as the member warps. To make the common terminology of Eq. 19 agree with the notation of the paper, $T = M_t = 2\,A\,q$ as used subsequently in Eq. 21.

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[11] Associate Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

[12] Asst. Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

[13] "Airplane Structures," by Alfred S. Niles and Joseph S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1943.

[14] "Analysis and Design of Airplane Structures," by Elmer F. Bruhn, John S. Swift & Co., Cincinnati, Ohio, 1943.

If Eq. 3 is used to compute the angle of twist of a cell (or cells) when torsion exists at the section, it is important that θ be defined as the angle of twist per unit of longitudinal length. With the author's definition of θ, the left side of Eq. 3 should be divided by the longitudinal span of the structure.

Design loads for many structures may not pass through the shear center and the resulting torsion may be of considerable magnitude. It is clear from the symmetry about a vertical axis in Fig. 2 that the vertical shear must be applied along the vertical center line for zero torsion; but, if a transverse load is to be applied to the structure without torsion at the section, the line of action of the load is not immediately evident. An extension of the paper to include the calculations necessary to locate the vertical position of the shear center as part of an example of case II would add greatly to the value of the paper.

For any multicelled structure subjected to torsion in addition to flexure, such as shown in Fig. 2, the writers suggest the following procedure.

(*a*) Assume a distribution of shear flow, q_1, as outlined by the author in step 1.

(*b*) Correctional shear flows (constant within a given cell) must be applied to the assumed shear flows of step (*a*) in order to have the angle of twist equal for all cells. By the application of Eq. 3, write an equation similar to Eqs. 4*a* and 4*b* for each cell, keeping θ as an unknown along with the unknown correctional shear flows for each cell. Thus, the number of equations will equal the number of cells.

(*c*) One more equation is required since θ, the angle of twist per unit of span length, is also unknown in this case. This additional equation is obtained as the author states. The torque of the internal shearing forces (assumed and correctional) can be found from statics using any convenient moment center. Equating this internal torque to the torque of the external shear force about the same moment center will give the additional equation required.

(*d*) Solving for the correctional shear flows in the equations of steps (*b*) and (*c*) (preceding), and adding these corrections to the assumed shear flows of step (*a*), giving proper consideration to signs (direction of flow), the final shear flow, in each wall of all the cells, is obtained.

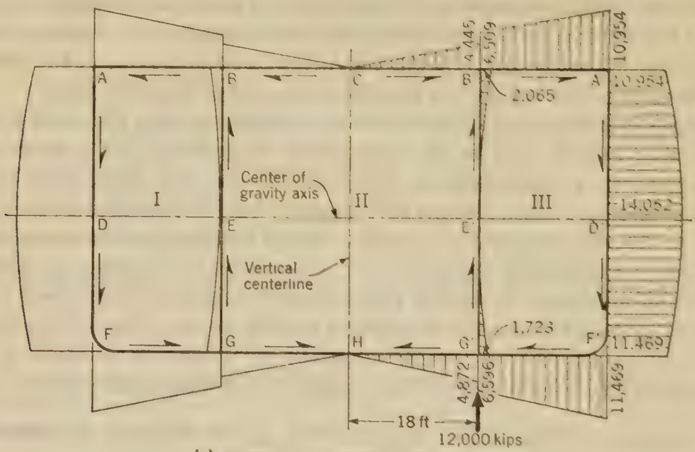
If the multicelled section is to be analyzed for various conditions of loading, the writers recommend the tabulation of the shear flows for the following:

(1) A convenient load (say, 1,000 kips) acting vertically through the shear center;

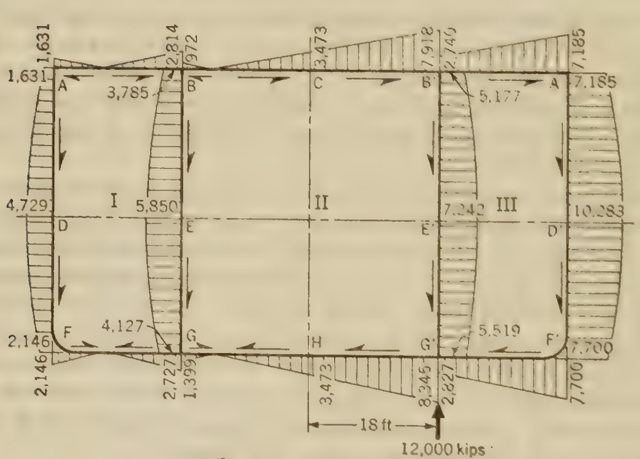
(2) A convenient load acting horizontally through the shear center; and

(3) A convenient torque at the section.

Then it is possible, by use of the proper ratios and by superposition, to obtain quickly the shear flows resulting from any torque producing load. If this method is used the location of the shear center must be known in order to determine the torque resulting from a load with both vertical and horizontal components. Consequently, the writers have shown the additional calculations necessary to locate the vertical position of the shear center in one of the following examples which considers a horizontal load on the structure.


4,445
6,509
10,954
A
B
C
B'
A'
2,065
10,954
Center of gravity axis
I
II
III
14,052
D
E
E'
D'
Vertical centerline
F
G
H
G'
F'
1,723
11,469
4,872
6,596
11,469
18 ft
12,000 kips

(*a*) AUTHOR'S ASSUMED SHEAR FLOWS


1,631
2,814
972
3,473
7,918
2,749
7,185
1,631
A
3,785
B
C
B'
5,177
A'
7,185
4,729
I
5,850
II
7,242
III
10,283
D
E
E'
D'
2,146
F
4,127
G
H
G'
5,519
F'
7,700
2,146
2,727
1,399
3,473
8,345
2,827
7,700
18 ft
12,000 kips

(*b*) FINAL SHEAR FLOWS

Fig. 9.—Diagrams of Shear Flows, in Pounds per Inch of Perimeter, for a 12,000,000-Lb Load Acting Vertically, 216 In. to the Right of the Center Line

The following examples consider, first, the author's vertical load of 12,000 kips applied along B'E'G', Fig. 9, and, secondly, a horizontal load of 1,000 kips applied along ABC, Fig. 10. Then, for both cases, it is evident by inspection that there is torque at the section.

In their work the writers have used definitions of terms that differ from the paper, as follows:

r = perpendicular distance, in inches from a wall to the center of moments;

q_1 = assumed shear flow at any point, in pounds per inch of perimeter, due to the applied load; and q_I = a constant correctional shear flow at any point, in pounds per inch of perimeter, in cell I, necessary for conditions of continuity and static equilibrium (when the q-values are clockwise, the sign is positive);

θ_I = angle of twist in cell I, in radians per inch of length along the span of the member; and

A_I = area of cell I, in square inches.

Similarly, q_{II}, θ_{II}, and A_{II} apply to cell II, and q_{III}, θ_{III}, and A_{III} apply to cell III. For continuity, $\theta_I = \theta_{II} = \theta_{III}$, in which

$$\theta_I = \frac{\sum q_1 \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{cell} \\ \text{I} \end{matrix} \right\} + q_I \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{cell} \\ \text{I} \end{matrix} \right\} + q_{II} \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{wall} \\ \text{BG} \end{matrix} \right\}}{2 A_I G} \quad \text{..(20a)}$$

$$\theta_{II} = \frac{\sum q_1 \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{cell} \\ \text{II} \end{matrix} \right\} + q_I \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{wall} \\ \text{BG} \end{matrix} \right\}}{2 A_{II} G}$$

$$+ \frac{q_{II} \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{cell} \\ \text{II} \end{matrix} \right\} + q_{III} \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{wall} \\ \text{B'G'} \end{matrix} \right\}}{2 A_{II} G} \quad \text{..(20b)}$$

and

$$\theta_{III} = \frac{\sum q_1 \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{cell} \\ \text{III} \end{matrix} \right\} + q_{II} \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{wall} \\ \text{B'G'} \end{matrix} \right\} + q_{III} \sum \frac{\Delta L}{t} \left\{ \begin{matrix} \text{for} \\ \text{cell} \\ \text{III} \end{matrix} \right\}}{2 A_{III} G} \quad \text{..(20c)}$$

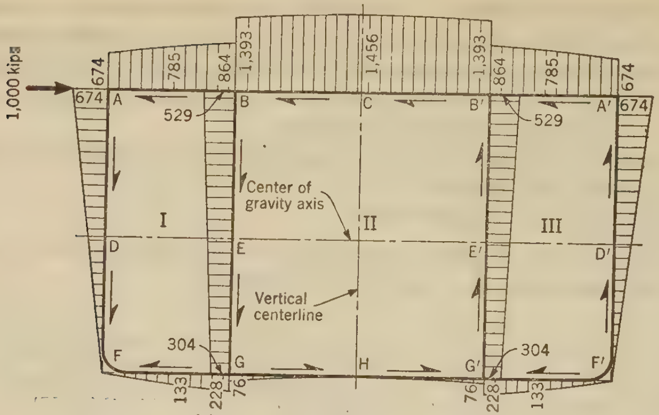
Computing moments about any convenient point, the fourth equation becomes

$$M_t = 2\, A_I\, q_I + 2\, A_{II}\, q_{II} + 2\, A_{III}\, q_{III} + \sum q_1\, r\, \Delta L \left\{ \begin{matrix} \text{for all walls} \\ \text{in cells} \\ \text{I, II, and III} \end{matrix} \right\} \quad \text{.(21)}$$

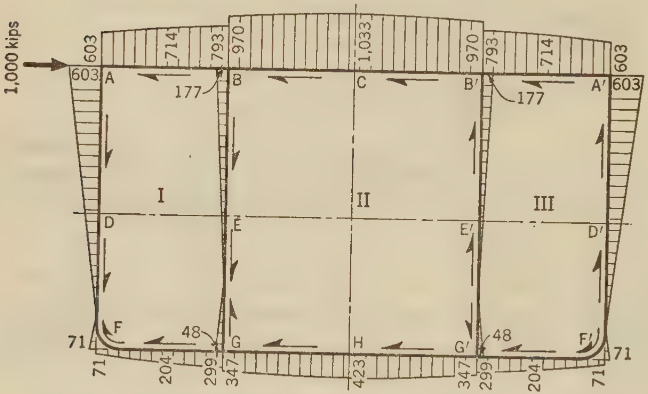
Other definitions not listed are the same as those given by the author, but it should be pointed out that the writers have used all dimensions, in inches, all areas, in square inches, and the moment of inertia, in inches to the fourth power. This was done to avoid confusing multiplication factors in the writing of Eqs. 20 and 21.

Except for special cases, it is necessary to include calculations for all three cells. Consequently, in the following examples, the structure used by the author is shown in Fig. 9(a), with the cells numbered I, II, and III, from left to right. Also, the letter symbols on the right half have a prime for the purpose of identification.

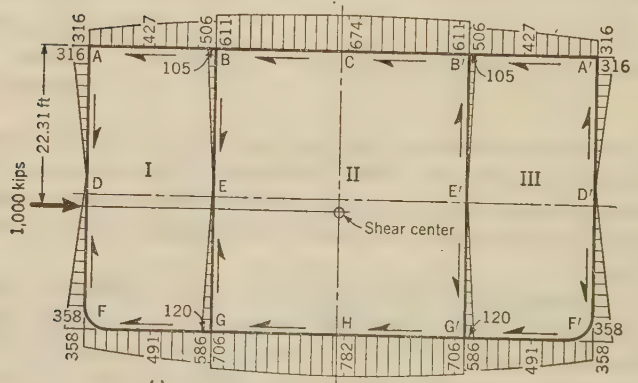
Vertical Load With Torque.—For Mr. Neiman's 12,000-kip load applied vertically, but at a distance of 18 ft to the right of the center line, the assumed


1,000 kips
674
785
864
1,393
1,456
1,393
864
785
674
674
A
529
B
C
B′
529
A′
674
Center of gravity axis
I
II
III
D
E
E′
D′
Vertical centerline
304
304
F
G
H
G′
F′
133
228
76
76
228
133

(*a*) ASSUMED SHEAR; LOAD AT THE TOP


1,000 kips
603
714
793
970
1,033
970
793
714
603
603
A
177
B
C
B′
177
A′
603
I
II
III
D
E
E′
D′
71
F
48
G
H
G′
48
F′
71
71
204
299
347
423
347
299
204
71

(*b*) FINAL SHEAR; LOAD AT THE TOP


316
427
506
611
674
611
506
427
316
316
A
105
B
C
B′
105
A′
316
22.31 ft
1,000 kips
I
II
III
D
E
E′
D′
Shear center
358
F
120
G
H
G′
120
F′
358
358
491
586
706
782
706
586
491
358

(*c*) FINAL SHEAR; LOAD AT THE SHEAR CENTER

FIG. 10.—DIAGRAMS OF SHEAR FLOWS, IN POUNDS PER INCH OF PERIMETER FOR A 1,000,000-LB LOAD ACTING HORIZONTALLY

shear flow is reproduced in Fig. 9(*a*). The assumed shear flow is in equilibrium with the external force both horizontally and vertically; however, the twisting moments are not in equilibrium. Also, the conditions of continuity are not satisfied—that is, the three cells do not have equal rotations. Therefore, it is necessary to compute correctional shear flows for each cell. Substituting in Eqs. 20 and 21, point H being the center of moments for M_t, the following four equations are obtained and can be solved simultaneously:

$$\theta_{\text{I}} = \frac{-14{,}620{,}824 + 2{,}233.6\, q_{\text{I}} - 960\, q_{\text{II}}}{207{,}360\, G} \qquad \text{(22a)}$$

$$\theta_{\text{II}} = \frac{0 - 960\, q_{\text{I}} + 3{,}187.20\, q_{\text{II}} - 960\, q_{\text{III}}}{414{,}720\, G} \qquad \text{(22b)}$$

$$\theta_{\text{III}} = \frac{+14{,}620{,}824 - 960\, q_{\text{II}} + 2{,}233.6\, q_{\text{III}}}{207{,}360\, G} \qquad \text{(22c)}$$

and

$$M_t = 12{,}000{,}000 \times 216 = 207{,}360\, q_{\text{I}} + 414{,}720\, q_{\text{II}} + 207{,}360\, q_{\text{III}} + 0 \qquad (23)$$

TABLE 3.—Assumed Shear Flows for 1,000,000 Lb Acting Horizontally

Member	ΔA (in.^{2})	z (in.)	$z\,\Delta A$ (in.^{3})	q_1(*a*) (Lb per In.)	q'_1 (Lb per In.)	$\frac{\Delta L}{t}$	$q'_1 \frac{\Delta L}{t}$ (cell I)	$q'_1 \frac{\Delta L}{t}$ (cell II)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
F	..	..	..	0	..	..	..	..
A(F)	360	432	155,520	− 674	− 337	640	−215,680	..
B(A)	135	324	43,740	− 864	− 779.7[b]	345.6	−269,460	..
G(F)	162	324	52,488	+ 228	+ 126.7[b]	288	+ 36,490	..
G(H)	162	108	17,496	− 76	− 25.3	288	..	− 7,290
B(G)	240	216	51,840	± 529	± 416.5	960	+399,840	−399,840
C(B)	135	108	14,580	−1,456	−1,435	345.6	..	−495,940
Σ							− 48,810	−903,070[c]

[a] q_1 is obtained by multiplying $\frac{V}{I}$ by $\Sigma z\, \Delta A$; the summation is from a point of assumed zero shear flow.

[b] To obtain this value it was necessary to compute the shear flow at midpoint of this wall (Fig. 10(*a*)).

[c] $-903{,}070 \times 2 = -1{,}806{,}140$.

In Eqs. 22*a* and 22*c* the value of 1,218,402 from Table 1 has been multiplied by 12 to give 14,620,824 since the writers wish to have ΔL in inches. From the solution of these simultaneous equations, the following correctional shear flows, in pounds per inch, are obtained: $q_{\text{I}} = +9{,}323$; $q_{\text{II}} = +3{,}473$; and $q_{\text{III}} = -3{,}769$. For a length of span of 100 ft, the value of $\theta_{\text{I}} = \theta_{\text{II}} = \theta_{\text{III}} = 0.0865°$. When correctional shear flows are added algebraically to the assumed shear flows the final shear flows are as shown in Fig. 9(*b*).

Horizontal Load With Torque.—For the second example using a horizontal load of 1,000 kips acting to the right along ABC, Fig. 10, the assumed shear flows are determined as based upon the assumption that the shear flows at F, F′, H are zero. Table 3 indicates the computations for the determination of the values of q_1, the assumed shear flows. The moment of inertia of the entire cross section about the vertical center line is 230,667,000 in.^{4} The values of q_1 are plotted in Fig. 10(*a*).

The values of the average assumed shear flows, q'_1, are given in Table 3, Col. 6. By multiplying these shear flows by their respective wall lengths the horizontal and vertical forces can be obtained. The sum of the vertical forces is equal to zero and the sum of the horizontal forces is equal to the applied load of 1,000,000 lb. However, the sum of the twisting moments of these vertical and horizontal forces is not equal to the twisting moment of the applied load about a center of moments. Also, the rotations of the three cells are not equal. By substituting in Eqs. 20 and 21, the following four equations involving the angle of twist and the three correctional shear flows, necessary to satisfy statics and continuity completely, can be obtained:

$$\theta_{I} = \frac{-48{,}810 + 2{,}233.6\, q_{I} - 960\, q_{II}}{207{,}360\, G} \qquad (24a)$$

$$\theta_{II} = \frac{-1{,}806{,}140 - 960\, q_{I} + 3{,}187.2\, q_{II} - 960\, q_{III}}{414{,}720\, G} \qquad (24b)$$

$$\theta_{III} = \frac{-48{,}810 - 960\, q_{II} + 2{,}233.6\, q_{III}}{207{,}360\, G} \qquad (24c)$$

and

$$M_t = M_B = 0 = -205{,}105{,}000 + 207{,}360\, q_{I} + 414{,}720\, q_{II} + 207{,}360\, q_{III} \qquad (25)$$

Point B is taken as the center of moments for M_t (Table 4). The moment of the applied load about point B is zero.

Solving Eqs. 24 and 25 simultaneously: $q_I = +71$; $q_{II} = +423$; and $q_{III} = +71$—all in pounds per inch.

These correctional shear flows, added algebraically to the assumed shear flows, give the final shear flows for the horizontal load of 1,000 kips applied along ABC (see Fig. 10(*b*)).

Horizontal Load Through Shear Center.—To locate the shear center of this cross section vertically, the assumed shear flows shown in Table 3, Col. 5, could be used for a horizontal load of 1,000 kips. Then, in order to have zero twist in all three cells, correctional shear flows must be added to these assumed shear flows. The correctional shear flows can be determined by substituting in Eqs. 20 and equating θ_I, θ_{II}, and θ_{III} to zero. These equations are the same as those for horizontal load with torque, except that the angle of twist is equal to zero. Solving these three simultaneous equations, the following correctional shear flows, in pounds per inch, are obtained: $q_I = +358$; $q_{II} = +782$; and $q_{III} = +358$. The final shear flows for 1,000 kips acting horizontally through the shear center are obtained by adding these correctional shear flows algebraically to the assumed shear flows (see Fig. 10(*c*)).

To determine the vertical location of the shear center, the moments about some point can be computed and the moment divided by the applied load will give the vertical distance from the center of moments to the shear center. In this case, point B has been taken as the center of moments because the moments about point B for the assumed shear flows were computed in Table 4. To these are added the moments about point B of the correctional shear flows listed in the preceding paragraph; thus: The vertical distance from point B to the shear center is

$$\frac{-205{,}105{,}000 + 207{,}360\ (358) + 414{,}720\ (782) + 207{,}360\ (358)}{1{,}000{,}000};$$

that is, 267.68 in. or 22.31 ft. As thus computed and shown in Fig. 10(*c*), the shear center lies on the vertical center line 22.31 ft below the top of the cross section.

TABLE 4.—MOMENTS ABOUT POINT B OF q'_1 FOR HORIZONTAL LOAD

Member (1)	q'_1 (lb per in.) (2)	ΔL (in.) (3)	r (in.) (4)	M_B (in.-lb) (5)
AB	779.7	216	0	0
BB′	1,435	432	0	0
B′A′	779.7	216	0	0
FA	337	480	216	− 34,940,000
GB	416.5	480	0	0
G′B′	416.5	480	432	− 86,365,000
F′A′	337	480	648	−104,820,000
FG	126.7	216	480	+ 13,133,000
GG′	25.3	432	480	− 5,246,000
G′F′	126.7	216	480	+ 13,133,000
Σ				−205,105,000

The shear flows (in pounds per inch) for a twisting moment of 1,000,000 lb times 267.68 in., or 267,680,000 in.-lb can be determined by taking the algebraic sums of the correctional shear flows for the horizontal load with torque and for the horizontal load through shear center as follows:

$$q_{\text{I}} = 71 - 358 = -287$$

$$q_{\text{II}} = 423 - 782 = -359$$

$$q_{\text{III}} = 71 - 358 = -287$$

This may be done because the assumed shear flows in both cases are equal. The shear flows for other twisting moments can be obtained by applying the proper ratio to these values. As part of the check on the solution of a vertical load of 12,000,000 lb applied 18 ft to the right of the center line, the foregoing shear flows should be multiplied by −12,000,000 × 216 = − 2,592,000,000 and divided by 267,680,000 which will give the following values (in pounds per inch):

$$q_{\text{I}} = -287\left(\frac{-2{,}592{,}000{,}000}{+267{,}680{,}000}\right) = +2{,}778$$

$$q_{\text{II}} = -359\left(\frac{-2{,}592{,}000{,}000}{+267{,}680{,}000}\right) = +3{,}475$$

$$q_{\text{III}} = -287\left(\frac{-2{,}592{,}000{,}000}{+267{,}680{,}000}\right) = +2{,}778$$

When these values are added algebraically to the author's final shear flows for a vertical load through the shear center, the results should be the same as shown in Fig. 9(*b*), or if these values are added to the author's correctional shear flows, the results should agree with the writers' correctional shear flows for vertical load with torque. This comparison is as follows:

$$\begin{aligned} q_{\text{I}} &= +\,6{,}544 + 2{,}778 = +\,9{,}322\ (+9{,}323) \\ q_{\text{II}} &= \phantom{+\,6{,}54}0 + 3{,}475 = +\,3{,}475\ (+3{,}473) \\ q_{\text{III}} &= -\,6{,}544 + 2{,}778 = -\,3{,}766\ (-3{,}769) \end{aligned}$$

A. S. NEIMAN,[15] Assoc. M. ASCE.—Discussers of the paper have contributed good derivations of formulas, have singled out certain limitations of the theory, and have expanded the solution to the more general cases involving torsion. Professors Clark and Hanson are especially to be commended for using arithmetic to show how the distribution of shear stress is obtained when torsion, in addition to bending, is to be considered.

Professor Goldberg's attention is drawn to the first sentence under the heading, "Acknowledgments," where the following is stated:

> "The procedure for computing shear stress distribution and the particular application of the principles of mechanics to the solution of the problem as outlined in this paper were developed by the writer while engaged in the structural design of concrete barges for the United States Maritime Commission."

No claim is made that principles of mechanics are developed. Professor Goldberg's statement that this problem is practically routine in the aircraft industry is undoubtedly true. Unfortunately, however, the large majority of structural engineers have not had experience in aircraft design. Moreover, as shown by Professors Clark and Hanson, the literature pertaining to civil engineering structures is somewhat scant in this regard. The problem was also complicated by the meager information available as to details for resisting shearing stresses at transverse frames, at transverse bulkheads near (and in the way of) large hatches, at corners, and at abrupt changes in thickness of plating. These subjects may furnish a field for further study and experimentation.

Full-scale hogging and sagging tests on concrete and steel ships were conducted by the United States Maritime Commission during 1943–1945. The ships were floated in still water and loaded by filling various holds with water ballast. Electrical strain gages in the form of rosettes were placed all around the girth of the vessel near points of maximum moment and shear. The vertical shearing stresses were computed from the strains. It will be of interest to engineers to know that the vertical shear computed to exist at the section from the diagram of loading checked within 10% of the total of the internal shearing forces as computed from the strains.

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[15] Chf. Structural Engr., Vacuum Concrete, Inc., Philadelphia, Pa.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2367

MATRIX ANALYSIS OF PIN-CONNECTED STRUCTURES

BY PEI-PING CHEN,[1] ESQ.

WITH DISCUSSION BY MESSRS. RUFUS OLDENBURGER, A. FLORIS, AND PEI-PING CHEN.

SYNOPSIS

Matrix methods of analyzing pin-connected structures are developed in this paper for single-joint frames, planar trusses, and space trusses. Numerical examples are given as an aid to visualizing application of the principles presented.

INTRODUCTION

A large proportion of the work in a structural analysis is involved in setting up and solving a set of simultaneous linear equations. It is desirable, therefore, to write these equations in a simpler but more systematic way. In general, a single equation in the matrix form may suffice for this purpose. In truss analysis, this matrix equation expresses a relation between the given force components and the unknown components of the joint displacements. To formulate this matrix equation is merely a straightforward process, no matter whether the truss is statically determinate or not.

A set of simultaneous linear, scalar equations, such as

$$\left.\begin{array}{l} p_1 = x_1\,a_{11} + x_2\,a_{21} + \cdots + x_n\,a_{n1} \\ p_2 = x_1\,a_{12} + x_2\,a_{22} + \cdots + x_n\,a_{n2} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ p_n = x_1\,a_{1n} + x_2\,a_{2n} + \cdots + x_n\,a_{nn} \end{array}\right\} \dots\dots\dots\dots\dots\dots (1)$$

may be written in matrix form for the purpose of simplicity. Thus,

$$[p_1, p_2, \cdots, p_n] = [x_1, x_2, \cdots, x_n] \begin{bmatrix} a_{11}\ a_{12} \cdots a_{1n} \\ a_{21}\ a_{22} \cdots a_{2n} \\ \dots\dots\dots\dots \\ a_{n1}\ a_{n2} \dots a_{nn} \end{bmatrix} \dots\dots\dots (2)$$

NOTE.—Published in December, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Brown Univ., Providence, R. I.; formerly, faculty member, National Central Univ., Nanking, China.

or $[p] = [x][a]$ in which $[p]$, $[x]$, and $[a]$ are the corresponding row matrices and square matrix in Eq. 2. Since $[a]$ is always nonsingular for a stable pin-connected truss, that is, the determinant formed by the coefficients, $[a]$, does not vanish, Eq. 2 may be solved either by expanding it into Eqs. 1 or by postmultiplying the reciprocal matrix, $[a]^{-1}$;[2] for example,

$$[x] = [p][a]^{-1} \quad \ldots\ldots\ldots (3)$$

The iteration method[3] is also applicable.

FUNDAMENTAL EQUATIONS FOR A BAR

If a prismatic bar is considered to be loaded as shown in Fig. 1(*a*), the following are easily obtained:

$$P_{Ai} = -P_{Bi} = \frac{E\,A}{L}(\Delta_{Ai} - \Delta_{Bi}) \quad \ldots\ldots\ldots (4a)$$

and

$$P_{Aj} = P_{Bj} = 0 \quad \ldots\ldots\ldots (4b)$$

in which P and Δ represent the forces and the displacements; A and B represent the ends of the bars; the subscripts i and j represent the axial and transverse components; and E, A, and L are the modulus of elasticity, the cross-sectional area, and the length of the bar, respectively. Referred to the standard x-axis and y-axis, as in Fig. 1(*b*), Eqs. 4*a* and 4*b* become:

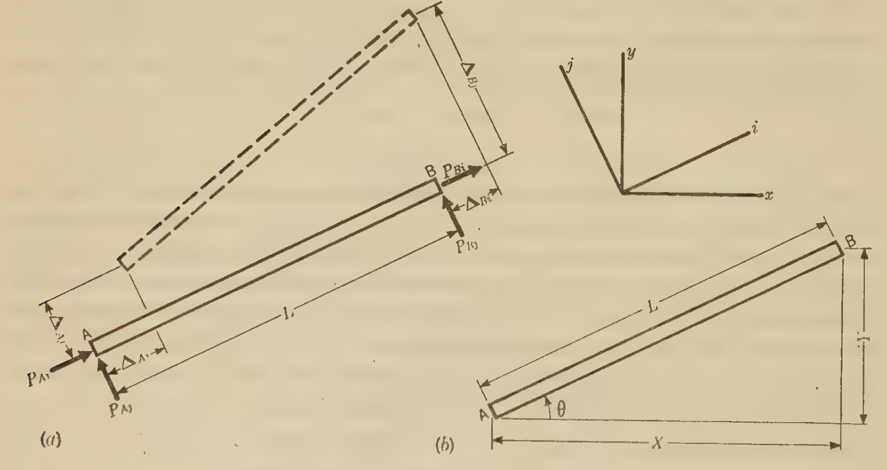

FIG. 1

$$P_{Ax} = \left[\frac{X^2}{L^2}(\Delta_{Ax} - \Delta_{Bx}) + \frac{X\,Y}{L^2}(\Delta_{Ay} - \Delta_{By})\right]\frac{E\,A}{L} \quad \ldots\ldots\ldots (5a)$$

and

$$P_{Ay} = \left[\frac{X\,Y}{L^2}(\Delta_{Ax} - \Delta_{Bx}) + \frac{Y^2}{L^2}(\Delta_{Ay} - \Delta_{By})\right]\frac{E\,A}{L} \quad \ldots\ldots\ldots (5b)$$

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[2] "Matrix Analysis of Continuous Beams," by Stanley U. Benscoter, *Transactions*, ASCE, Vol. 112, 1947, p. 1109.

[3] "Elementary Matrices and Some Applications to Dynamics and Differential Equations," by R. A. Frazer, W. J. Duncan, and A. R. Collar, Cambridge Univ. Press, Cambridge, England, 1946, p. 132.

By introducing

$$P_A = [P_{Ax}, P_{Ay}] \qquad (6a)$$

$$\delta_A = [\Delta_{Ax}, \Delta_{Ay}] \qquad (6b)$$

$$P_B = [P_{Bx}, P_{By}] \qquad (6c)$$

$$\delta_B = [\Delta_{Bx}, \Delta_{By}] \qquad (6d)$$

and the two-way matrices,

$$\gamma_{AB} = \gamma_{BA} = \begin{bmatrix} X^2 & X\,Y \\ X\,Y & Y^2 \end{bmatrix} \frac{E\,A}{L^3} \qquad (7)$$

the fundamental bar equation—

$$P_A = -\,P_B = (\delta_A - \delta_B)\gamma_{AB} \qquad (8)$$

—is obtained.

THE SINGLE-JOINT STRUCTURES

A planar, single-joint structure loaded by $P_A = [P_{Ax}, P_{Ay}]$ at joint A is shown in Fig. 2(*a*). Then, in Fig. 2(*b*), let P_{AN} or $[P_{AxN}, P_{AyN}]$ represent the forces acting at point A on the bar AN. Then, by Eq. 8,

$$\left.\begin{aligned} P_{AB} &= (\delta_A - \delta_B)\,\gamma_{AB} \\ P_{AC} &= (\delta_A - \delta_C)\,\gamma_{AC} \\ &\cdots\cdots\cdots\cdots\cdots \\ P_{AN} &= (\delta_A - \delta_N)\,\gamma_{AN} \end{aligned}\right\} \qquad (9)$$

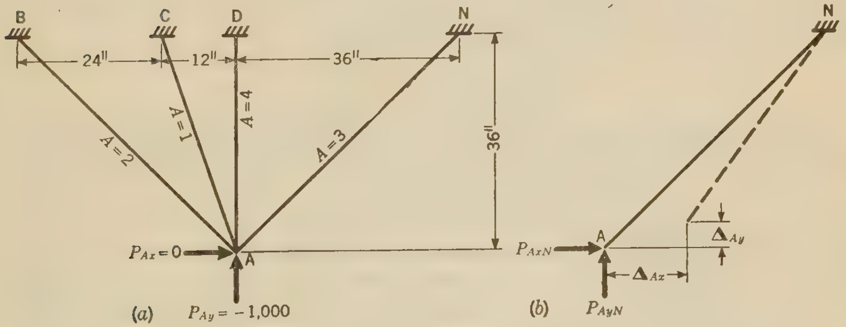

FIG. 2

The sum of Eqs. 9 is ΣP_{AN}, or P_A, which is the applied force matrix. If all the other ends (that is, B, C, $\cdots$, N) of these bars are hinged so that the displacements, δ_B, δ_C, $\cdots$, δ_N, are all zero, Eqs. 9 give:

$$P_A = \delta_A\,\Sigma\gamma \qquad (10a)$$

or

$$\delta_A = P_A\,(\Sigma\gamma)^{-1} \qquad (10b)$$

from which the joint displacement δ_A may be found. The load distributed to each bar may then be computed by means of Eqs. 9.

Example 1.—It is required to solve for the joint displacement of the truss shown in Fig. 2(*a*).[4] The quantity P_A is given as $[0, -1{,}000]$ and $E = 30 \times 10^6$ lb per sq in. Pound and inch units are used throughout this paper.

From Fig. 2 and Eq. 10*a*, the following values are found:

Bar	X	Y	L	γ
AB....	-36	36	$36\sqrt{2}$	$\begin{bmatrix} 0.01964 & -0.01964 \\ -0.01964 & 0.01964 \end{bmatrix} E$
AC....	-12	36	$12\sqrt{10}$	$\begin{bmatrix} 0.00264 & -0.00791 \\ -0.00791 & 0.02372 \end{bmatrix} E$
AD....	0	36	36	$\begin{bmatrix} 0 & 0 \\ 0 & 0.11111 \end{bmatrix} E$
AN....	36	36	$36\sqrt{2}$	$\begin{bmatrix} 0.02964 & 0.02964 \\ 0.02964 & 0.02964 \end{bmatrix} E$

from which $$(\Sigma\gamma)^{-1} = \begin{bmatrix} 0.05174 & 0.00191 \\ 0.00191 & 0.18392 \end{bmatrix}^{-1} \frac{1}{E} = \begin{bmatrix} 19.337 & -0.2016 \\ -0.2016 & 5.440 \end{bmatrix} \frac{1}{E}$$

By Eq. 10*b*, $\delta_A = [201.6, -5{,}440]\frac{1}{E}$; that is, $\Delta_{Ax} = 0.00000672$ in. to the right and $\Delta_{Ay} = -0.0001813$ in., the negative indicating downward displacement. Also, from Eq. 9, $P_{AN} = \delta_A\,\gamma_{AN} = [-154.3, -154.3]$; that is, 218.2-lb tension, for, as is seen in Fig. 2(*b*), the negatives mean that the force components should be reversed. Similarly, $P_{AB} = \delta_A\,\gamma_{AB} = [110.8, -110.8] = 156.7$-lb tension; $P_{AC} \doteq \delta_A\,\gamma_{AC} = [43.5, -130.6] = 137.6$-lb tension; and $P_{AD} = \delta_A\,\gamma_{AD} = [0, -604.3] = 604.3$-lb tension.

PIN-CONNECTED TRUSSES

In Fig. 3 is shown a stable plane truss (either statically determinate or not) consisting of n pinned joints. The forces acting on the individual bars are found by substitution in Eq. 8 to be:

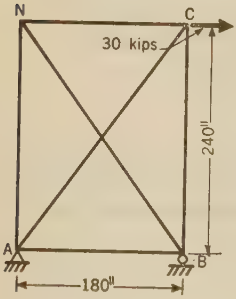

FIG. 3

$$\left.\begin{aligned} P_{AB} &= (\delta_A - \delta_B)\,\gamma_{AB} \\ P_{AC} &= (\delta_A - \delta_C)\,\gamma_{AC} \\ &\cdots\cdots\cdots\cdots \\ P_{NM} &= (\delta_N - \delta_M)\,\gamma_{NM} \end{aligned}\right\} \qquad (11)$$

If there is no bar connecting two joints, the corresponding values of γ and P vanish. By isolating joint A, it is easily seen that

$$P_A = P_{AB} + P_{AC} + \cdots + P_{AN} = \delta_A\,\Sigma_M\,\gamma_{AM} - \delta_B\,\gamma_{AB} - \delta_C\,\gamma_{AC} \cdots - \delta_N\,\gamma_{AN} \qquad (12)$$

and that similar relations hold for $P_B, P_C, \cdots, P_N$.

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[4] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1946, Vol. II, p. 13.

Introduction into Eq. 12 of the force matrix (including the applied forces and the reactions)—

$$P = [P_A, P_B, \cdots, P_N] \tag{13}$$

the displacement matrix—

$$\delta = [\delta_A, \delta_B, \cdots, \delta_N] \tag{14}$$

and the 2 n-way symmetrical matrix for the whole truss—

$$\alpha = \begin{bmatrix} \Sigma_M \gamma_{AM} & -\gamma_{AB} & \cdots & -\gamma_{AN} \\ -\gamma_{AB} & \Sigma_M \gamma_{BM} & \cdots & -\gamma_{BN} \\ \cdots & \cdots & \cdots & \cdots \\ -\gamma_{AN} & -\gamma_{BN} & \cdots & \Sigma_M \gamma_{NM} \end{bmatrix} \tag{15}$$

results in:

$$P = \delta\, \alpha \tag{16}$$

The matrices of Eqs. 14, 15, and 16 may then be expanded in terms of the scalar elements, namely:

$$P = [P_{Ax}, P_{Ay}, \cdots, P_{Nx}, P_{Ny}] \tag{17a}$$

$$\delta = [\Delta_{Ax}, \Delta_{Ay}, \cdots, \Delta_{Nx}, \Delta_{Ny}] \tag{17b}$$

and

$$\alpha = \begin{bmatrix} \alpha_{AAxx} & \alpha_{AAxy} & \cdots & \alpha_{ANxx} & \alpha_{ANxy} \\ \alpha_{AAyx} & \alpha_{AAyy} & \cdots & \alpha_{ANyx} & \alpha_{ANyy} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha_{NAxx} & \alpha_{NAxy} & \cdots & \alpha_{NNxx} & \alpha_{NNxy} \\ \alpha_{NAyx} & \alpha_{NAyy} & \cdots & \alpha_{NNyx} & \alpha_{NNyy} \end{bmatrix} \tag{17c}$$

Therefore, Eq. 16 may be expanded into 2 n scalar equations, such as:

$$P_{Ax} = \Delta_{Ax}\, \alpha_{AAxx} + \Delta_{Ay}\, \alpha_{AAxy} + \cdots + \Delta_{Nx}\, \alpha_{NAxx} + \Delta_{Ny}\, \alpha_{NAxy} \tag{18}$$

Usually three or more displacement components (for example, Δ_{Ax}, Δ_{Ay}, and Δ_{By} in Fig. 3) are zero at the hinged joint and the roller joint, and the corresponding force components are unknown reactions. All the elements in the corresponding rows and columns of the matrix, α, may be dropped out in order to modify the equations readily solvable for the unknown displacement components. The modified equation is of the form:

$$P' = \delta'\, \alpha' \tag{19}$$

in which α' is also a symmetrical matrix and must be nonsingular for a stable truss.

Example 2.—Compute the joint displacements and bar stresses due to the load, $P_{Cx} = 30{,}000$ lb, for all bars[5] in Fig. 3 if $\frac{E\,A}{L} = Q = 2.5 \times 10^6$. The following are obtained from Fig. 3 by inspection:

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[5] "Structural Theory," by Hale Sutherland and Harry Lake Bowman, John Wiley & Sons, Inc., New York, N. Y., 1942, 3d Ed., p. 255.

Bars	X	Y	L	γ
AB CN	180	0	180	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} Q$
AC	180	240	300	$\begin{bmatrix} 0.36 & 0.48 \\ 0.48 & 0.64 \end{bmatrix} Q$
AN BC	0	− 240	240	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} Q$
BN	180	− 240	300	$\begin{bmatrix} 0.36 & -0.48 \\ -0.48 & 0.64 \end{bmatrix} Q$

Hence,

$$\alpha = \begin{bmatrix} 1.36 & 0.48 & -1 & 0 & -0.36 & -0.48 & 0 & 0 \\ & 1.64 & 0 & 0 & -0.48 & -0.64 & 0 & -1 \\ & & 1.36 & -0.48 & 0 & 0 & -0.36 & 0.48 \\ & & & 1.64 & 0 & -1 & 0.48 & -0.64 \\ & & & & 1.36 & 0.48 & -1 & 0 \\ & & & & & 1.64 & 0 & 0 \\ & (s) & & & & & 1.36 & -0.48 \\ & & & & & & & 1.64 \end{bmatrix} Q$$

In the matrix expression, (s) indicates that the matrix is symmetrical. It is obvious that at the hinge and roller joints $\Delta_{Ax} = \Delta_{Ay} = \Delta_{By} = 0$.

Dropping out the corresponding elements gives $[0, 30{,}000, 0, 0, 0] = [\Delta_{Bx}, \Delta_{Cx}, \Delta_{Cy}, \Delta_{Dx}, \Delta_{Dy}]\,\alpha'$, in which

$$\alpha' = \begin{bmatrix} 1.36 & 0 & 0 & -0.36 & 0.48 \\ & 1.36 & 0.48 & -1 & 0 \\ (s) & & 1.64 & 0 & 0 \\ & & & 1.36 & -0.48 \\ & & & & 1.64 \end{bmatrix} Q$$

The roots of this expression are found to be: $\Delta_{Bx} = 12{,}300/Q = 0.00492$ in.; $\Delta_{Cx} = 80{,}630/Q = 0.03225$ in.; $\Delta_{Cy} = -23{,}600/Q = 0.00944$ in.; $\Delta_{Dx} = 68{,}330/Q = 0.02733$ in.; and $\Delta_{Dy} = 16{,}400/Q = 0.00656$ in. Likewise, from Eq. 8, $P_{BN} = (\delta_B - \delta_N)\,\gamma_{BN} = [\Delta_{Bx} - \Delta_{Nx}, \Delta_{By} - \Delta_{Ny}]\,\gamma_{BN} = [-56{,}100, -16{,}400]$ $\frac{1}{Q}\begin{bmatrix} 0.36 & -0.48 \\ -0.48 & 0.64 \end{bmatrix} Q = [-12{,}300, 16{,}400] = 20{,}500$-lb tension.

SPACE TRUSSES

By taking one more component into consideration, Eq. 16 is equally applicable to a space truss. Thus,

$$P_A = [P_{Ax}, P_{Ay}, P_{Az}] \quad \text{.....................} (20)$$

and

$$\gamma_{AB} = \gamma_{BA} = \begin{bmatrix} X^2 & X\,Y & X\,Z \\ X\,Y & Y^2 & Y\,Z \\ X\,Z & Y\,Z & Z^2 \end{bmatrix} \frac{E\,A}{L^3} \quad\ldots\ldots\ldots\ldots\ldots(21)$$

The remaining truss relations may be expanded in like manner.

Example. 3.—The given load at joint A (Fig. 4) is $P_A = [3{,}600,\ 2{,}400,\ 10{,}200]$ lb, and the value of $E\,A$ for every bar is 10^7 lb. It is required to find the joint displacements and the forces on all members.

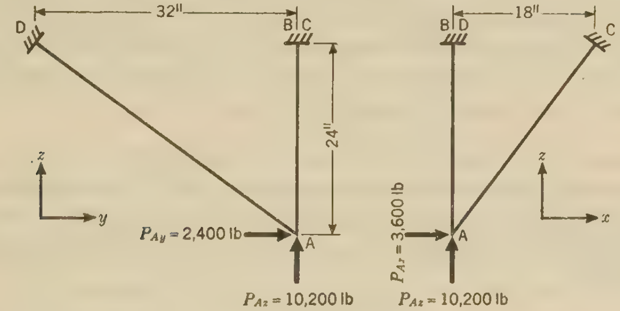

Fig. 4

In this example, A, B, and C are hinged; that is, $\delta_B = \delta_C = \delta_D = [0, 0, 0]$. Also, from Fig. 4,

Bars	X	Y	Z	L	γ
AB....	0	0	24	24	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.04167 \end{bmatrix} E\,A$
AC....	18	0	24	30	$\begin{bmatrix} 0.012 & 0 & 0.016 \\ 0 & 0 & 0 \\ 0.016 & 0 & 0.02133 \end{bmatrix} E\,A$
AD....	0	− 32	24	40	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.016 & -0.012 \\ 0 & -0.012 & 0.009 \end{bmatrix} E\,A$

from which

$$\Sigma\gamma = \begin{bmatrix} 0.012 & 0 & 0.016 \\ 0 & 0.016 & -0.012 \\ 0.016 & -0.012 & 0.072 \end{bmatrix} E\,A$$

and

$$(\Sigma\gamma)^{-1} = \begin{bmatrix} 126 & -24 & -32 \\ -24 & 76 & 18 \\ -32 & 18 & 24 \end{bmatrix} \frac{1}{E\,A}$$

By Eq. 10*b*, $\delta_A = P_A(\Sigma\gamma)^{-1} = [69{,}600,\ 279{,}600,\ 172{,}800]/(E\,A)$ in.; that is, $\Delta_{Ax} = 0.00696$ in., $\Delta_{Ay} = 0.02796$ in., and $\Delta_{AZ} = 0.01728$ in. Also, by Eqs. 9, $P_{AB} = [0, 0, 7{,}200] = 7{,}200$-lb compression; $P_{AC} = [3{,}600, 0, 4{,}800] = 6{,}000$-lb compression; and $P_{AD} = [0, 2{,}400, -1{,}800] = 3{,}000$-lb tension.

SUMMARY

The principal differences between the method herein suggested and existing methods are as follows:

(1) In the suggested method, the number of equations to be solved simultaneously is the same as the number of unknown displacement components, whereas in existing methods, the number of equations is equal to the degree of redundancy of the truss plus the necessary static equations.

(2) As the first stage in setting up the matrix equation, computations involve only the given dimensions of the members. The equation is then modified according to the nature of the supports. The given loads finally enter into the computations after the modified equation is obtained, and static equations are not needed. Thus, mental computation is largely reduced.

In conclusion, this method is theoretically applicable to all types of pin-connected trusses, either statically determinate or not. It is especially suitable to those trusses which are highly redundant but have only a few movable joints (see Example 1). Moreover, in dealing with various loading conditions for a given truss,

$$\delta' = P' (\alpha')^{-1} \qquad (22)$$

usually proves satisfactory.

DISCUSSION

RUFUS OLDENBURGER,[6] ESQ.—The use of matrices in the treatment of pin-connected structures clearly demonstrates the linear equation theory underlying the solution for unknown forces and displacements at the joints. If the number of unknowns is excessive, the problem of inverting matrices is a serious one and might handicap the analyst.

The matrix γ_{AB} of Eq. 21 is a practical example of the use of the direct product of vectors, in this case the direct product of a vector such as $(X\ Y\ Z)$ by itself; that is,

$$\gamma_{AB} = (X\ Y\ Z) \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad (23)$$

That α in Eq. 15 is a matrix with such direct products or sums of direct products as minors should simplify the treatment of α. The technique of setting up the matrix α for all components, and then dropping rows and columns corresponding to zero displacements is a convenient one.

The results of this paper are, of course, valid for first-order effects only, but this is generally sufficient. The assumptions underlying the paper are used in Eqs. 4 and 5. For Eqs. 4 Hooke's law is assumed; for Eqs. 5 it is supposed that the change in the angle θ in going from the unloaded to the loaded state of the bar is negligible. If x and y and u and v are the coordinates of the points A and B, respectively, so that the length of the bar is

$$L = \sqrt{(u-x)^2 + (v-y)^2} \qquad (24)$$

it follows that

$$dL = \frac{\partial L}{\partial u}\,du + \frac{\partial L}{\partial x}\,dx + \frac{\partial L}{\partial v}\,dv + \frac{\partial L}{\partial y}\,dy \qquad (25)$$

in which

$$\frac{\partial L}{\partial u} = -\frac{\partial L}{\partial x} = \frac{u-x}{L} \qquad (26a)$$

and

$$\frac{\partial L}{\partial v} = -\frac{\partial L}{\partial y} = \frac{v-y}{L} \qquad (26b)$$

Assuming that the increment ΔL is the same as dL for practical purposes, Eqs. 5 are obtained from Eqs. 4.

When any one of the foregoing assumptions is no longer valid, the matrix method of approach breaks down.

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[6] Chairman, Mathematics Dept., De Paul Univ., Chicago, Ill.

A. FLORIS.[7] ESQ.—In an abstract mathematical form the author develops an elegant method of analyzing stresses in framed structures. The concise presentation of the subject, coupled with the advanced technique used by the author, renders the understanding of the paper rather difficult. Nevertheless, the method is distinctly new and interesting, and as such should be appreciated.

The proposed theory follows paths opposite to those in common use. The true displacements of the bars are first determined, and the stresses in the bars are determined afterward. In the standard methods the opposite is true.

The problems with which the author illustrates his theory are solved much easier by the well-known methods. In Example 2, for instance, one equation is needed to determine the stress in the hyperstatic bar of the frame (Fig. 3) instead of the formidable matrix α (Eq. 17*c*). Using the standard methods, Example 1, is more tedious than Example 2, since it involves the solution of two simultaneous equations. If the author's method is applied, the solution of Example 2 is more complicated than the solution of Example 1. In hyperstatic structures of high degree of redundancy, the proposed method will show perhaps its superiority over the older methods. It is left to the discretion of the author to demonstrate this superiority in his closing discussion.

The elements of matrix for planar frames (Eq. 7) and those of matrix for space frames (Eq. 21) are symmetrically arranged about their main diagonal—thus showing a striking analogy to the elements of the determinants of the elastic equations obtained by other methods.

By removing the hyperstatic constraints or the bars of the structure and by applying the principle of virtual displacements or Castigliano's theorems, the elastic equations are derived. The coefficients of the hyperstatic quantities are usually of the form—

$$\begin{vmatrix} \alpha^2 & \alpha\,\beta & \alpha\,\gamma & \cdots \\ \alpha\,\beta & \beta^2 & \beta\,\gamma & \cdots \\ \alpha\,\gamma & \beta\,\gamma & \gamma^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

—which is the common denominator of the hyperstatic quantities.

The elements in the foregoing determinant represent virtual displacements, whereas the elements in Eqs. 7 and 21 are pure coordinants.

PEI-PING CHEN.[8]—Professor Oldenburger's comments concerning the restriction of the matrix approach to linear, first-order effects are well taken. Although it is not inconceivable that matrix technique could be adapted to the treatment of nonlinear problems, in the present application the matrix equations serve only as a convenient (and powerful) shorthand for sets of simultaneous linear equations. The statement that the matrix γ is a direct product of a vector by itself is not strictly correct. The matrix γ is actually proportional to a Gibbs product of a unit vector by itself; thus:

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[7] Los Angeles, Calif.

[8] Fellow in Applied Math., Brown Univ., Providence, R. I.

$$\gamma = \frac{EA}{L}\begin{bmatrix} X/L \\ Y/L \\ Z/L \end{bmatrix}\begin{bmatrix} \frac{X}{L}, & \frac{Y}{L}, & \frac{Z}{L} \end{bmatrix} \qquad (27)$$

Mr. Floris' observation that the method presented is "distinctly new" is greatly appreciated, since it may be true, as Mr. Floris suggests, that the basic approach may not have been understood by the reader who is not on intimate terms with matrices. It may be well to discuss briefly the underlying philosophy of the present approach, to compare it with standard methods, and to make evident the superiority of the present method for certain types of problems. In standard methods the problem of analyzing trusses is roughly as follows:

1. The hyperstatic, or redundant, bars are "removed" and replaced by unknown forces;
2. The forces in the remaining bars are computed by statics (method of joints, method of sections, etc.) in terms of the unknown forces and the applied loads;
3. Simultaneous equations are set up and solved for the unknown forces in the redundant bars; and
4. The forces in the remaining bars are then found from the results of steps 2 and 3.

The chief difficulty generally occurs in step 3. Thus a structure with six redundant bars requires the solution of six simultaneous equations. In addition, if displacements of the truss are desired, further computations are required using one or another of the standard methods of finding truss deflections.

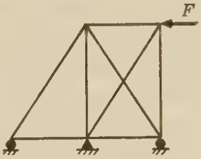

FIG. 5

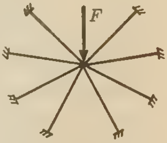

FIG. 6

In the present method the procedure is as follows:

a. The forces in the bars are computed in terms of the displacement components of each joint;

b. From equilibrium considerations, simultaneous equations are set up and solved for the displacement components; and

c. From the results of steps *a* and *b*, the forces in each bar are computed directly.

Again, the chief labor comes in solving the simultaneous equations of step *b*. However, in this case, both stresses and displacements will have been computed on completion of steps *a*, *b*, and *c*.

Although a truss with, say, two redundant members and six unknown joint displacement components (see Fig. 5) is best handled by standard methods, it is considerably more advantageous to use the writer's approach for analyzing a truss with six redundant members and only two unknown displacement components (see Fig. 6).

Furthermore, if the inverse matrix $(\alpha')^{-1}$ is computed at the outset, the truss may be solved for a large variety of loadings with little additional labor, whereas standard methods generally require the repetition of almost all computations for each set of loads.

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Paper No. 2368

THE SIGNIFICANCE OF PORE PRESSURE IN HYDRAULIC STRUCTURES

By L. F. Harza,[1] M. ASCE

With Discussion by Messrs. William P. Creager, J. S. Kendrick, Adolf A. Meyer, E. Montford Fucik, A. C. R. Albery, W. H. R. Nimmo, John S. Cotton, Serge Leliavsky Bey, A. H. Davison, R. E. Ballester, John S. McNown, V. T. Boughton, Ross M. Riegel, Douglas McHenry, Frederick L. Hotes, R. W. Carlson and Raymond E. Davis, W. G. Morrison, Raymond A. Hill, John V. Spielman, Paul E. Raes, and L. F. Harza.

Synopsis

By deductive reasoning it is shown that under and within any concrete structure founded below water level, or with water on one side (such as a gravity dam, even on rock foundation), hydrostatic uplift acts over the entire horizontal area, instead of over one half or two thirds of the area as commonly assumed. It is shown that it is the magnitude of the uplift force which is uncertain and which should be the subject of field observation and experimental investigation, not the area of application. It is also shown that the hydrostatic force against a dam is applied progressively along the route of seepage, instead of against the upstream face. Various implications of these fundamental principles are discussed.

Efforts to promulgate the doctrine of 100% effective uplift area under hydraulic structures, on the part of the few who have reached this conclusion by research or analysis, have never been convincing because of the indirect, and therefore not evident, character of any tests which could be made. In addition, the doctrine has always appeared to the reader as impossible because of two erroneous concepts, almost universally held: First, that 100% uplift area cannot be true because a large proportion of the base area must be in contact to support the load; and, second, that many or most existing dams would have failed if the 100% uplift area doctine were true. In fact, these views are almost universal and so strong that it has proved difficult to secure publica-

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Note.—Published in December, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication. The fundamental ideas contained in this paper were submitted by Mr. Harza in 1937 in a paper which was then declined. Changes and improvements have been made in the original paper, which was resubmitted and approved in 1947.

[1] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

tion space for any opposing viewpoint. This is not at all strange for, admittedly, the opposing principles seem axiomatic without resort to the demonstration which is hereinafter built up from elementary principles.

It is believed that the traditional controversy over the effective uplift area in dams is the result of a false conception of the internal characteristics of concrete. The writer has long maintained that uplift within or under a hydraulic structure can best be approached as a phase of the buoyancy phenomenon,[2] contending, on the basis of deductive reasoning, that pore pressure in submerged concrete (including dams) produces uplift equivalent to full buoyancy, which in turn is equivalent to uplift over 100% of horizontal area, and that pressure against a dam is not exerted largely against the upstream face, but progressively along lines of seepage and in the direction of seepage.

For presentation of the subject, it will first be necessary to visualize the porosity characteristics of concrete. It is customary to visualize concrete as a mass of loose and very porous sand and gravel made practically solid by filling the voids with nearly impervious, neat cement paste. Nevertheless, the usual commercial concrete contains voids of important proportion. These voids obviously do not occur in the aggregate particles, which are practically devoid of porosity. To some extent, in poor, lean concrete, they may result from incomplete filling of the aggregate voids with cement paste, thus leaving sand pockets and a honeycombed structure and also voids along the under surfaces of the larger aggregate particles. In good, well-proportioned and well-mixed concrete, however, they must necessarily occur chiefly in the cement paste.

A workable mix of concrete requires water far in excess of that required by the chemical action of setting, leaving this surplus water mixed throughout the cement paste, and the process of mixing inevitably entrains air bubbles. These two facts would seem to explain the porosity of the cement paste. Of late, air-entraining admixtures are becoming common for purposely introducing additional minute air voids. New problems or possible changes in treatment of the uplift problem, because of these voids, will be ignored as immaterial for the purpose of this paper. In any event, it has been found that these air voids will saturate with water after exhausting the air by a vacuum, indicating that they communicate, and it is possible or even probable that water will ultimately displace the air in these voids in concrete that is continuously subject to water pressure.

Normal commercial concrete has voids of the general magnitude of about 12% and the very best laboratory concrete has about 6%. Assuming these voids to be in the cement paste, and assuming six bags of cement per cubic yard of concrete, the voids in the cement paste would be 54% and 27%, respectively.

Porosity of cement paste has been studied[3] for the Bureau of Reclamation, United States Department of the Interior, in connection with studies of permeability of concrete. Table 1 shows the make-up of cement paste as determined by these studies, and compares it with the results obtained by F. R. McMillan,[4] M. ASCE, and T. Yoshioka, K. Kumagae, and H. Iijima.[5] These tests confirm the previous computations for concrete with voids of about 12%. Thus there will be assumed hereafter 50% voids in the cement paste, which in fact now becomes the porous element of the concrete, with voids of even greater proportion than those of the original aggregate although smaller in size, and yet known to yield to the passage of seepage water under pressure.

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[2] Discussion by L. F. Harza of "Stability of Straight Concrete Gravity Dams," by D. C. Henny, *Transactions*, ASCE, Vol. 99, 1934, p. 1085.

[3] "An Investigation of the Permeability of Mass Concrete with Particular Reference to Boulder Dam," by Arthur Ruettgers, E. N. Vidal, and S. P. Wing, *Proceedings*, Am. Concrete Inst., March–April, 1935, p. 382.

TABLE 1.—MAKE-UP AND POROSITY OF CEMENT PASTE

Item	PERCENTAGE OF TOTAL		
	At time of pour	After 48 days	McMillan and Yoshioka
(1)	(2)	(3)	(4)
Cement	35	46	48
Free water	59	42	38
Air	6	12	14
Total	100	100	100
Paste porosity	67	56	53

Assuming that the individual grains, minerals, and aggregate of concrete have a specific gravity, $G = 2.65$, and that the porosity of the concrete as a whole is 6% (equivalent to laboratory controlled manufacture), the saturated weight of concrete is $62.5 \times 2.65 \times 0.94 + 0.06 \times 62.5 = 159.44$ lb per cu ft, and the buoyant weight is $62.5 \times 1.65 \times 0.94 = 96.94$ lb per cu ft. If the porosity is 12% (more common in field concrete), then the saturated weight becomes 153.25 lb per cu ft and the buoyant weight 90.75 lb per cu ft. For 15% porosity, these weights become 150.16 lb per cu ft and 87.65 lb per cu ft, respectively.

Again, assume normal concrete with 12% voids and let y represent the average diameter of the pores. Then if all pores were concentrated in layers, equivalent to laminar voids of thickness y, occupying the entire cross section, the spacing, t, of such layers could be expressed by

$$\frac{y}{t} = \frac{12}{100} \quad \text{or} \quad t = \frac{100\,y}{12} \qquad (1)$$

Most of the pores in the cement paste will obviously be comparable in size with the cement grains. If pores were 0.01 in. in diameter, enough would thus occur in each 0.083 in. in height to form a full projected area in the concrete as a whole; if of 0.02 in. in average diameter, to form a full projected area in each 0.167 in.; and, even if the pores were 0.1 in. in diameter, to form a complete lamination across the section of the concrete every 0.85 in.

In the cement paste, considered by itself, the spacing of void laminations based on 50% voids would be only about $t = \frac{100\,y}{50}$ or, perhaps in the order of 0.02 in. for voids averaging 0.01 in. in diameter, or hardly more than a film of cement apart. As the aggregate particles themselves are relatively poreless such idealistic lamination of pores must divert around them.

Usual practice is to brush the foundation or construction joints of a structure with neat cement or mortar prior to a pour. Such practice obviously does not provide a perfect contact in the sense of one which will exclude water and be impervious, and the lowest pores, sufficient for a complete projected area of pores, would thus be separated from the bottom only by a mere film of cement paste, if at all.

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[4] "Basic Principles of Concrete Making," by F. R. McMillan, McGraw-Hill Book Co., Inc., New York N. Y., 1929.

[5] "Studies on Hydration of Cements," Pt. 7, "Hydro-Expansion of Cement Grains," by T. Yoshioka K. Kumagae, and H. Iijima, Japanese Soc. of Chem. Industry (Japan), October, 1931.

BUOYANCY

As the approach to the uplift problem is related to the principle of buoyancy, the ideas will be developed by a process of evolution from basic principles, at the risk of criticism for being elementary. This is excusable as necessary to clarify the subject. Consider an object, submerged in still water (Fig. 1). Assume the object divided into elementary vertical columns of cross section dA, of which ab is one. Then the downward hydrostatic pressure on the top of the element is $w\,h\,dA$ and the upward pressure on the bottom is $w\,H\,dA$, if w is the unit weight of the liquid. The uplift force by which the latter exceeds the former is $w\,(H - h)dA$, resulting in a reduction in submerged weight of this amount, known as buoyancy. The volume of the element is $(H - h)dA$, and the buoyancy is therefore w times the volume. The same holds true for all the elements combined; that is, w times the total volume, which is the weight of a volume of water equal to that of the object, or its displacement. This is true regardless of the depth of submergence.

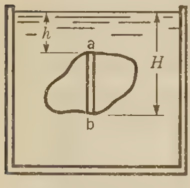
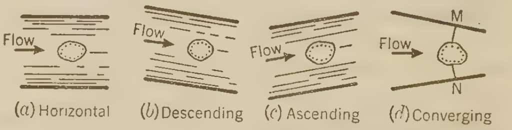


FIG. 1.—BASIC CONCEPT OF BUOYANCY OF OBJECT SUBMERGED IN STILL WATER

Again, if a body is submerged in water under the condition of uniform horizontal flow, buoyancy will be unchanged but there will also be a horizontal dynamic pressure on the approach side and reduced pressure on the receding side, as well as hydraulic friction acting on the body, as shown in Fig. 2(a) Under conditions of uniform flow in a direction with vertical component, as in Figs. 2(b) and 2(c), these forces cause an actual change in effective weight—an increase in Fig. 2(b) and a decrease in Fig. 2(c). Such dynamic force does not change the buoyant effect, but merely adds another force to be combined with the buoyancy to obtain the resultant effective force.



(a) Horizontal (b) Descending (c) Ascending (d) Converging

FIG. 2.—BODY SUBMERGED IN FLOWING WATER

If the flow is horizontal and converging (or diverging), however, as in Fig. 2(d), there will still be a differential between top and bottom pressure essentially equal to displacement, inasmuch as the static pressure along any line MN will increase as the depth, even though the static pressure is progressively reducing along the line of flow.

Flow in loose granular material, as shown in Fig. 3(a), is not different from the preceding illustrations, except that the whole waterway is filled with objects instead of one isolated object. There is a progressive loss of pressure through the mass; each void is under lower pressure than the void to the left, and each particle is therefore subjected to a greater pressure on the left or approach side than on the receding side. Therefore, these particles are subjected not only to buoyancy acting vertically, but also to a force tending to move them to the right.[6] If the pipe were to be turned to a vertical position (Fig. 3(*b*)) without flow, each particle would be buoyant. If any flow is directed upward through the mass, then the seepage forces (Fig. 3(*a*)) would be added to the buoyancy and both would tend to reduce the effective weight or to lift the particles.

This principle (that gravel, sand, and earth, submerged in still water, are fully buoyant—in other words, are reduced in weight to the full extent of the displacement of individual particles thereof) is quite generally accepted. The popular acceptance of this principle rests on the belief that the point to point contacts of each particle with the adjacent ones do not reduce the effective surface area of each particle subjected to hydrostatic pressure. In other words, each particle is fully surrounded by water like the submerged object in Fig. 1, regardless of these point to point contacts.

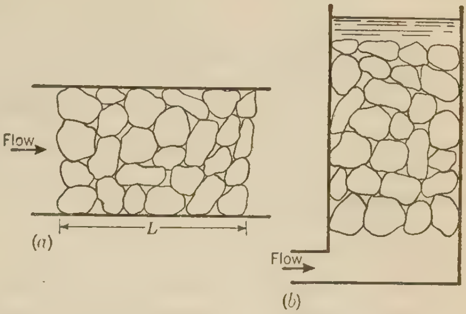


FIG. 3.—LIQUID FLOW THROUGH LOOSE GRANULAR MATERIAL

If it be assumed that the particles in Figs. 3(*a*) and 3(*b*) are all cemented together at their points of contact without increasing the areas of contact, then obviously no change in pressure will result and the solid body thus created would tend to move as a whole, not because of pressure against the upstream face, but because of progressively reducing internal pore pressure causing buoyant and seepage forces to act against each particle.

That all the principles previously enumerated would be just as true if the pores are filled with cement paste to form concrete is, however, not generally accepted by the profession and needs demonstration. Popular opinion, with few exceptions, regards as a fact the belief that the cementing together of particles to form concrete reduces the top and bottom areas of particles subjected to water pressure and thus excludes water pressure from so much of the surface area of each original particle as to preclude the action of buyoancy on the individual particles of the mass. It will be shown, on the contrary, that internal buoyancy continues to be fully as effective in concrete as in loose granular material.

In Fig. 4(*a*) it is first considered that there is no flow, so that only static pressure is effective. It has been proved herein that concrete is quite porous and it is well known that seepage under pressure passes through any concrete. It has likewise been proved experimentally[7] that a submerged specimen of concrete cannot be measurably compressed by subjecting the liquid in which it is submerged to a high pressure. This proves that all pores communicate and that the static pressure in Fig. 4(*a*) must exist in all pores. If the cross section of any one pore, such as a, is projected upward, it will eventually end in another pore, such as b, or in greater distance, as c to d; or a large pore, as at e, may require continuation until two or more pores, such as f and g, are encountered before the entire cross section of the cylinder from e is terminated in pores. In any event, the probability is so great, in any length of concrete greater than a fraction of an inch, as to become a certainty that each such cylinder will terminate in pores at both ends. Moreover, the entire volume of concrete will thus subdivide into such cylinders. Since the differences in hydrostatic head to which these pores will be subjected are equal to their differences in elevation, each column will be subjected to buoyancy just as is the elementary column in Fig. 1. Likewise, since each column is buoyant, and the whole object is composed of such columns, then the entire internal structure is buoyant, and the effect of cementation is nullified.

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[6] "Uplift and Seepage Under Dams on Sand," by L. F. Harza, *Transactions*, ASCE, Vol. 100, 1935, p. 1352.

[7] "Simple Tests Determine Hydrostatic Uplift," by Karl Terzaghi, *Engineering News-Record*, June 18, 1936, p. 872.

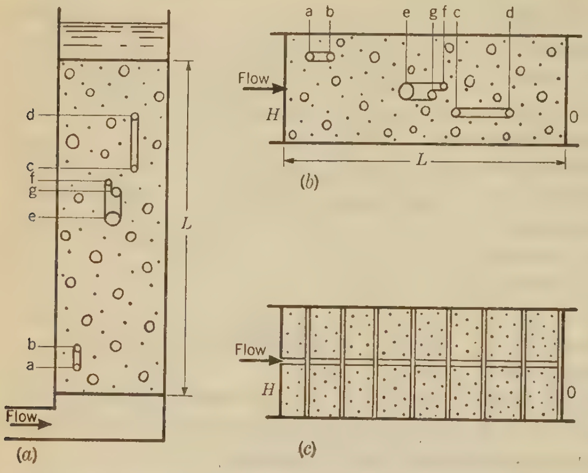


FIG. 4.—INTERRELATION OF PORES AND FLOW THROUGH CONCRETE

Even though the popular belief may be shared that internal buoyancy cannot exist in concrete because water pressure cannot be effective over the entire top and bottom areas of the original sand and gravel particles of aggregate because of the cement, yet it is only necessary thus to dismiss consideration of the original aggregate particles and visualize a new conception of a different elementary unit of material to obtain cylindrical particles that are exposed to pressure at top and bottom, and thus to restore the conception of full buoyancy, as in an uncemented mass. The entire volume of submerged concrete may be thought of, therefore, as built up of vertical cylinders averaging very much less than 1 in. in height, each terminating at a pore at top and bottom and subjected to buoyancy—the entire mass therefore weighing only from about 90 to 95 lb per cu ft.

Fig. 4(*b*) indicates the same pores as illustrated in Fig. 4(*a*). It has been previously shown that they are sufficient, even in the best of concrete, to form layers of pores filling the entire cross section at intervals of a fraction of an inch, as shown in Fig. 4(*c*). If water pressure is applied to the concrete filled pipe in Fig. 4(*b*), the pressure will reduce from H at entry to 0 at the point of escape. Intermediate pores will contain pressures reduced proportionally to distance, the pressure in these pores decreasing from left to right. Each solid cylinder between pores is therefore subject to a force equal to the differential pressure between the pores at the ends of such cylinders. Thus the differential pressure on cylinders ab and cd, if H is the total head lost in the distance L, will be, respectively, $\frac{\overline{ab}}{L} H$ and $\frac{\overline{cd}}{L} H$. The total volume under pressure is the sum of the elementary cylinder volumes; the total cross section is equal to the total cross section of the specimen; and the total summation of pressure is H. The total force therefore is $H A$, in which A is the cross section of the entire specimen. The volume can also be divided into vertical cylinders at right angles to the seepage forces, indicating internal buoyancy. If seepage forces are added to Fig. 4(*a*) by an applied water pressure, then they become additive to the buoyancy forces instead of normal, as in Fig. 4(*b*). Ideally the situation is the same as if the pores were to be concentrated into layers as in Fig. 4(*c*), and each layer of pores were connected by a fine seepage channel as shown, so that the water pressure would reduce in steps from H to 0 and be exerted over the entire area of each concrete disk in each layer of pores. Each disk would thus be subject to a thrust equal to the difference in pressure on its two faces.

The principle is identical with that of flow through cohesionless sand or gravel, and filling the voids with cement paste thus has no influence except to decrease the percolation factor—just as cementing the contact points had no effect on the loose granular material (Fig. 3(*b*)).

The only difference between buoyant and seepage forces is in the rate of dissipating pressure through the specimen. In buoyancy the rate of increase of pressure with increasing depth is fixed by the weight of water, whereas in Figs. 3(*a*) and 4(*b*) it is at the rate of $\frac{H}{L}$, whatever that may be. Also, the pressure is applied not at the entrance face, but along the route of seepage and in the direction of seepage.

It remains only to summarize the principles of static pressure and flow in concrete in comparison with these phenomena in loose granular material, based upon arguments herein and other known and recognized facts.

These principles are:

(*a*) Concrete is a porous material with all the pores communicating as in loose material.

(*b*) Hydrostatic pressure will develop in the pores of submerged concrete, equal in magnitude to the depth of a given pore below static water surface, except that a greater time element may elapse in reaching a static condition, analogous to the case of clay rather than to that of coarse loose material.

(*c*) Flow of water through concrete obeys Darcy's law as it does through loose material.

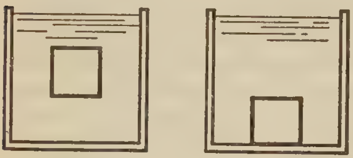
(*d*) Internal buoyancy exists in submerged concrete as truly as in loose material; to visualize this it is only necessary to conceive a different unit of material than the original aggregate pieces composing the concrete.

(*e*) There is no difference hydrostatically or hydraulically between the original loose aggregate and the finished concrete except for the reduction in the percentage and size of voids. These differences have no effect on the laws of flow, but merely on the permeability constants.

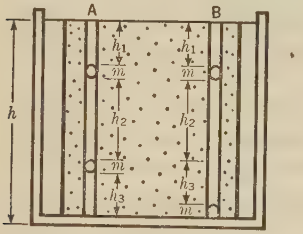
(*f*) The only difference in principles is that the particles adhere to each other in concrete and do not in the original aggregate, but the laws of hydraulics and hydrostatics do not concern themselves with this difference.

BOTTOM UPLIFT

Although it has been shown that submerged concrete is internally buoyant, the question will arise as to the buoyancy as a whole because of the belief that the weight of a concrete structure will exclude water pressure from the bottom on that part of the supporting surface with which it is in intimate contact. This traditional belief calls for critical analysis and will be considered in two phases: (1) When resting upon an impervious surface such as metal, rubber, or glass; and (2) when resting upon any natural mineral surface.



(*a*) Freely Submerged (*b*) Resting on Bottom



(*c*) Interrupting Pores

FIG. 5.—EFFECT OF INTERNAL VERSUS BOTTOM PORES ON BUOYANCY

Fig. 5(*a*) shows a cube suspended in water and therefore buoyant as a whole, as in Fig. 1, regardless of whether it is internally buoyant or not. In Fig. 5(*b*) this same cube is shown resting on the bottom. If the contact with the bottom is so perfect as to exclude water pressure over the whole or part of the contact area (which requires that the bottom be impervious), then prevailing opinion would not admit the cube to be buoyant over that much of the area. If the cube is porous, however, then it will be shown deductively that it will be buoyant, regardless of perfect contact with an impervious bottom, except perhaps below the lowest pores in the material.

For analysis of this principle reference is made to Fig. 5(*c*), which shows an enlarged view of the cube submerged in water and making contact with the bottom. Two elementary columns of the concrete cube, each of unit cross section, are shown: One, A, with two pores each of height, *m*, interrupting the continuity of the column, both above the base, these pores each communicating with the outside water through the general porosity of the specimen; the other, B, having two pores and differing from A only in that one of the pores is at the bottom and is assumed to prevent direct contact of this column with the supporting surface. For simplicity, the top of the specimen will be assumed to be at water level.

Each pore will be subjected to water pressure corresponding to its depth below the water surface and that part of the column above the upper pore will therefore be subject to an uplift pressure equal to h_1. The top of the column below this pore will be subjected to a downward pressure equal to $h_1 + m$. The water in each pore will exert a pressure upward on the top surface of the pore and downward on the bottom of the pore. Every part of an elementary column having a pore both at top and bottom will be buoyant, just as an element ab of the submerged object, in Fig. 1, is buoyant. Whereas the lowest section, h_3, of column A rests on the bottom and is assumed to be in water-exclusive contact with the impervious bottom, it will not be buoyant, but, on the contrary, its dry weight will be increased by the water pressure in the pore at its top.

In summarizing pressures and weights, expressed in units of depth of water, throughout the height of column, using G_c for the specific gravity of concrete, the total load placed on the foundation by column A is

$$W_A = G_c h_1 - h_1 + (h_1 + m) + G_c h_2 - (h_1 + m + h_2) + (h_1 + m + h_2 + m) + G_c h_3 \ldots\ldots\ldots\ldots (2a)$$

or

$$W_A = G_c (h_1 + h_2 + h_3) + 2m = G_c (h - P h) + P h = h[G_c (1 - P) + P] \ldots\ldots\ldots\ldots (2b)$$

in which W_A is thus found to be the saturated weight of the concrete, and P represents the aggregate height of pores in the column in proportion to the total height of the column.

The total load placed on the foundation by column B is

$$W_B = G_c h_1 - h_1 + (h_1 + m) + G_c (h_2 + h_3) - (h_1 + m + h_2 + h_3) \ldots (3a)$$

or

$$W_B = (G_c - 1)(h_1 + h_2 + h_3) = (G_c - 1)(h - P h) = h (G_c - 1)(1 - P) \ldots\ldots\ldots\ldots (3b)$$

in which W_B is thus found to be the buoyed weight of concrete.

If a is the horizontal cross-sectional area of the concrete block in Fig. 5(c) and r is the proportion of base area assumed to be in such intimate contact with the supporting surface as to prevent water pressure (water-exclusive contact), then the concrete load on the supporting surface, from Eqs. 2 and 3 will be

$$W = a h r [G_c (1 - P) + P] + a h [(1 - r)(G_c - 1)(1 - P)] \ldots (4)$$

Based on this analysis, the conclusion seems evident that the effective weight of concrete above those elementary columns, and hence those areas, in such intimate contact with an impervious foundation as to exclude water pressure, is the full dry weight of concrete plus pore water, or the saturated weight of concrete, whereas the effective weight above areas not in contact is only the buoyant weight of concrete. This tends to confirm commonly accepted opinion in this respect, on condition that the bottom contact on impervious foundation actually excludes water pressure.

Before accepting the foregoing principle, however, it should be subjected to further critical analysis of the internal buoyancy rather than of the body as a whole. Instead of analyzing the contact surface with the foundation, an analysis should also be made of a rough surface joining the lowest pore in each elementary column of the specimen at or near the base. If, for example, column A, Fig. 5(*c*), is analyzed only down to the lower pore, omitting h_3, Eq. 2*b* becomes

$$W_{A_p} = (G_c - 1)(h_1 + h_2) \qquad (5)$$

in which W_{A_p} is the buoyant weight of the superimposed concrete, as in column B, instead of the saturated weight obtained by including h_3. The weight of the specimen above a rough section joining the lowest pore in each elementary vertical column of concrete would thus be completely subjected to buoyancy and would have an effective weight of only from about 90 to 95 lb per cu ft.

Likewise, if the downward water pressure in the lowest pore of column B, acting directly on the foundation, is included in Eq. 3*b*, the result is Eq. 2*b*, as for column A, or the saturated instead of buoyed weight of concrete. Thus the total pressure (concrete plus hydrostatic) on the base is the same for both columns A and B. Column B, however, is not in contact with the base and its mechanical load of weight, carried to the base through the adjacent columns, is only the buoyant weight of the column. Thus, the downward water pressure on the base in the lower pore of column B is a load of no value to the stability of the structure.

The weight above the lowest full projected area of pores (using Eq. 3*b*) would be

$$w = a\,h\,(G_c - 1)(1 - P) \qquad (6)$$

Subtraction of Eq. 6 from Eq. 4 gives

$$W - w = a\,h\,r \qquad (7)$$

which is the summation of downward hydrostatic loads in the lowest pore of each elementary column in assumed water-exclusive contact with the base.

It has been shown that concrete contains enough pores in a small fraction of an inch to form a full projected area, and therefore that the rough surface through the lowest full projected area of pores is very close, in fact within a small fraction of an inch of the supporting surface, and that the volume above this surface is practically the same as the total volume. If neat cement paste is first applied to a foundation, then this surface will be within 0.02 in. or 0.03 in. of the foundation, and where there are bottom pores there will be no contact.

Thus a submerged block of concrete resting on an impervious bottom is apparently in a buoyant state except for the contact particles of a thin irregular layer at the bottom below the lowest full projected area of pores. Also, this thin layer, hardly more than a film of cement, suddenly adds to the effective weight the entire difference, indicated by Eq. 7, between the total effective weight and the buoyant weight. This great additional weight added by the bottom film of cement is the result of the unbalanced downward pressure in the lowest pores, which acts much as a vacuum cup by adding greatly to the effective adhesion or effective weight without having appreciable weight in itself. To function in this manner requires, as in the case of the vacuum cup, that the bottom be impervious to exclude pressure from underneath.

As discussed in the following paragraphs, it is claimed that neither concrete nor cement can make, with any natural mineral foundation, a contact capable of excluding water pressure, but if such contact is possible it would occur only on a plate of metal or rubber. The writer doubts that a water-exclusive contact of cement paste having voids of about 50% and a granular structure is possible even on rubber, metal, or glass. Common opinion assumes that it is, and on this assumption the difference between buoyant weight and actual foundation pressure (Eq. 7) is thus contributed only by this thin bottom layer.

The condition as to buoyancy of a submerged block of concrete resting on a natural rock foundation can be best visualized in Fig. 6, which illustrates a detached section of concrete near the base of a submerged structure, such as is shown in Fig. 5(*c*), with pores of irregular size, haphazardly placed, and resting upon a foundation of close-grained rock with smaller and fewer pores than are contained in the concrete. All natural rock foundations are porous, however, with the pores under hydrostatic pressure, and natural rock is always intersected by joint planes, bedding planes, and hair cracks, which make a foundation, in general, more pervious than concrete. If each rock pore below the base line in Fig. 6 is projected upward to form a column, it will soon terminate at a pore of the concrete and be subjected to buoyancy. Likewise, if the pores of the lowest layer of concrete are projected downward into the foundation, they will eventually terminate in sufficient rock pores to equal the projected sectional area of the column (or, more likely, will first intersect a hair crack, seam, fissure, joint plane, or bedding plane) also under a hydrostatic pressure corresponding to its elevation.

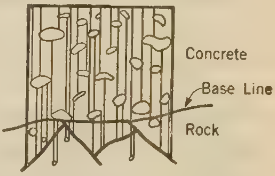


Fig. 6.—Relation of Pores in Concrete and Natural Rock Foundation

Thus, the base line of a submerged structure or contact line of concrete with rock is not significant of any change in principle of uplift or buoyancy which continues downward into the rock. The widely accepted principle, that uplift under a submerged structure depends on the area of the base in intimate contact with the rock, must therefore be abandoned entirely. The old theory would be valid, if at all, only on artificial foundation in a laboratory, such as a plate of steel or rubber, and this is very doubtful.

It is not denied that some strength exists along the rough surface of failure. It is conceivable and, in fact, a matter of observation, that a structure may fail along a surface partly through the concrete, partly through the rock, and elsewhere along the contact line. Conventional theory of design, against overturning of a dam for example, assumes no tensile strength or cohesion across the surface of failure. Strength does resist fracture along any rough surface containing sufficient pores to form a full projected area, thus subject to buoyancy, because it is obviously necessary to intersect solid material to obtain such a surface. The actual surface of fracture would be the weakest section, which would contain the minimum of solid material. Perhaps this surface would omit near-by pores in favor of solid fracture where the departure from a plane (necessary to include all pores in a full projected area) would require fracturing more area vertically to include pores relatively far away, than would need to be fractured to omit these pores and maintain a more nearly plane surface. This is the case when aggregate pieces are fractured. More likely, however, the fracture would localize in a horizontal pour plane, which also has cohesion and strength of less amount. The fact that the surface of fracture has strength does not disprove the thesis that the material above this surface is in a buoyant state.

Experimental Evidence

It next becomes necessary to reconcile this viewpoint with previous experiments, most of which were otherwise interpreted. It was found by Karl Terzaghi,[7] M. ASCE, however, that a submerged, bare specimen of concrete could not be measurably compressed by high pressure of the water in which it was submerged, although it was compressed when jacketed with metal foil to prevent water from entering the pores. This experiment, which arrived at 99% effective area, tends to indicate that the water pressure enters every pore, no matter how small, and practically surrounds each solid particle or poreless aggregation of material, as in the case of loose material, and would infer that there are no pores so small as to prevent the contained water from obeying the laws of static pressure. This would seem to offer complete and conclusive experimental proof of the viewpoint expressed herein, within experimental limits of observation. Unfortunately, however, more than ten years have elapsed since its publication and it has not been accepted by the profession in general, but only by a very few daring thinkers.

The latest reported research[8] indicates that the profession is still entirely in the dark as to the fundamental nature of the uplift phenomenon and is still trying to answer the question of what proportion of the base area of a dam is subject to uplift, although it was demonstrated by one experimenter[7] in 1936 to be nearly 100%, and by the writer in the same year to be always exactly 100%. The experiment by S. Leliavsky Bey, M. ASCE, shown in Fig. 7, demonstrates an ingenious approach to a supposed problem, which, however, is nonexistent.

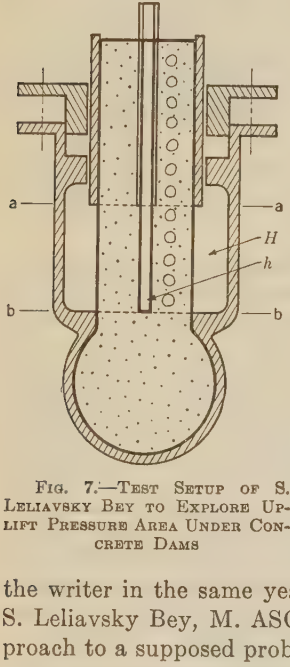


Fig. 7.—Test Setup of S. Leliavsky Bey to Explore Uplift Pressure Area Under Concrete Dams

In this experiment water pressure, *H*, is introduced in the outer annular space and escapes at the central drainage hole at controlled pressure, *h*, and at the top of the specimen with pressure zero. Mr. Leliavsky's basic assumption, as with previous writers and experimenters, was that water pressure in the pores between a and b serves as a wedge, acting only over part of the sectional area of the specimen to split the specimen at what may prove to be the weakest section. The assumed problem is to determine the proportional area over which the internal wedge pressure in the pores is effective, as analogous to the uplift area in dams.

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[8] "Experiments on Effective Uplift Area in Gravity Dams," by S. Leliavsky Bey, in "Uplift Pressure in and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 444.

By application of the reasoning expressed herein to Fig. 7, it can be shown that the pore pressure produces no wedge effect tending to split the specimen, and that the specimen is broken by plain tension, as in a mechanical testing machine, the results offering no possible significance toward the solution of the problem in hand. Thus, considering a line of pores in the concrete extending from section bb to the top of the specimen, pores from bb to aa will, ideally, all be under essentially the same pressure (except for minor differences as to elevation, nonuniformity of concrete, and pressure distribution, which can be ignored). The concrete cylinders joining adjacent pores will be subjected, therefore, to the same pressure at each end and will have no tendency to split apart or any tendency whatever to move as the result of water pressure in these pores. No wedging action or longitudinal stress can result from this pore pressure, as long as the pressure is the same at each end of any elementary cylinder. The pressure in a single pore might tend to split the specimen without considering the adjacent pores. However, the pressure in a single pore cannot exert any net splitting effect if the pressure in adjacent pores produces an equal opposing force at the other end of the elementary cylinder joining them.

From section aa to the top, the pressure in the pores will be gradually reducing to zero, as in Figs. 4(*a*), 4(*b*), and 4(*c*), and lifting effort will be applied gradually from section aa to the top because each elementary cylinder will have a higher pressure on its bottom than on its top and the total lift is the summation of such differentials. Any break in the specimen between sections aa and bb is therefore the result of tension applied above section aa, as though by a mechanical tension testing machine attached to the specimen above aa. The total amount of tension thus applied is the measure of the strength of the specimen when subjected to high hydrostatic pressure seeping radially toward the central drainage hole. It has no other significance and is not necessarily indicative of the strength of the concrete specimen in air. The only force of local origin tending to cause elongation of the specimen between sections aa and bb is that resulting from Poisson's ratio for the radially acting compressive force of the external water pressure on the cylinder.

Thus, Mr. Leliavsky's Eq. 2 should be written

$$p\,A = n\,z + F \qquad (8)$$

in which the unknown factor, n, is applied to the variable strength of the specimen, z, instead of to $p\,A$, the total tension applied to the specimen. Mr. Leliavsky's research, as well as that of all his predecessors (Rudeloff, Panzerbieter, Föppl, and others) sought to determine the percentage of area over which internal water pressure in concrete would act. The criticisms offered herein are applicable with variations to all these previous tests.

Those who have arrived by experiment at nearly 100% uplift area have hardly dared to believe their own experiments, and have sought explanation in complicated theories. They have not dared to go all the way to 100% uplift area because of failure to dissect and interpret correctly the porous nature of concrete. Thus, Professor Terzaghi found, in three series of experiments, values of 0.97, 0.97, and 0.998. According to his theory these large values are explained by the belief that—

> "* * * microstructure of concrete is an agglomeration of a large number of small blocks, with points of contact so minute as to belong to a still higher order of minuteness and that the surface of rupture is an irregular surface of least resistance which passes almost entirely through the micro voids."

Mr. Leliavsky quotes this explanation in defense of his average value of 91%, which he obviously has mental difficulty in justifying without the help of this theory.

It is seen by the previous analysis that this theory is unnecessary and that uplift over exactly 100% of the area, and no less, can be easily and clearly explained and requires no fantastic conception of the nature of concrete, but merely the fact that it is porous.

APPLICATION TO A CONCRETE DAM

It has been shown that concrete is internally buoyant as truly as loose granular material, and that it obeys all the hydraulic and hydrostatic laws of loose granular material. It is quite universally accepted that the material in an earth dam is all in a buoyant state below the line of pressure saturation, or free water surface, as in Fig. 8(*a*). Then, if the material in such a dam were cemented together into concrete, as in Fig. 8(*b*), it would remain internally buoyant as before, with the same line of saturation; in fact, it would remain buoyant with the same entire flow net. Also, although concrete dams are built with a different cross section than earth dams, the same principle applies, as will be demonstrated by another approach.

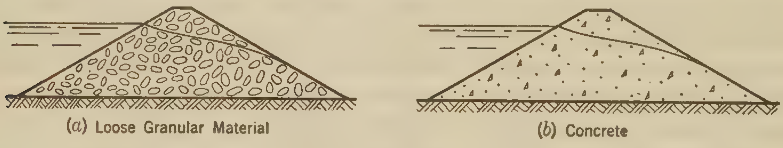
(*a*) Loose Granular Material (*b*) Concrete

FIG. 8.—SIMILARITY OF FLOW THROUGH EQUAL SECTIONS AND DIFFERENT MATERIALS

It is required to pass from the general principle of buoyancy of a completely submerged structure, or a structure of the section of an earth dam, to that of a concrete gravity dam submerged only on one side. Consider, initially, an ideal triangular gravity dam without artificial or natural internal drainage, seams, open cracks, pour planes, bedding planes, and monolith or other construction joints—in other words, an ideally perfect, homogeneous structure except for porosity.

Concerning these conditions, Professor Terzaghi states:[7]

> "If the dam consists of well-prepared concrete, the quantity of water which percolates through the body of the dam towards the downstream face is equal to the quantity of water which evaporates along the face. Therefore, the downstream face will appear to be dry. Nevertheless, there will be a continuous though imperceptible flow of seepage water from the reservoir toward the downstream face."

The late A. J. Wiley, M. ASCE, has stated:[9]

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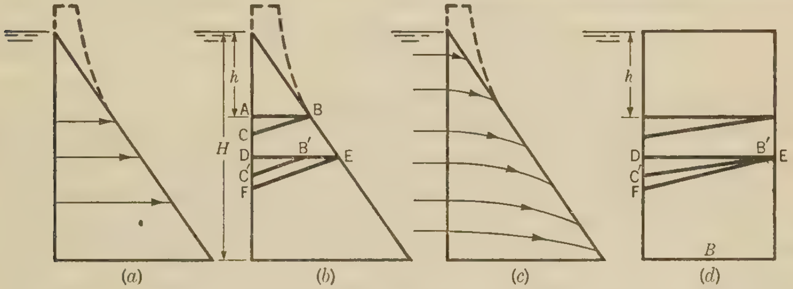
[9] "Past Experience with High Dams," by A. J. Wiley, in "High Dams: A Symposium," *Transactions*, ASCE, Vol. 95, 1931, p. 134.

> "It is the writer's observation that every void or seam in a masonry dam is filled with water under pressure. This was shown by the water which seeped out of the body of the St. Francis dam [California] for weeks after the failure. This seepage was apparently from the body of the concrete and was not confined to joints or seams."

It is scarcely to be questioned that any pores near the back, or water surface, of a dam will be under water pressure essentially corresponding to their depth below water surface. The simplest and most plausible assumption as to pore pressure farther removed from the reservoir would be that seepage through the dam follows along lines substantially as shown in Fig. 9(*a*), and that pressure reduces, from reservoir pressure at the back to zero at the face, at a uniform rate as shown by the triangular pressure diagram ABC (Fig. 9(*b*)); likewise, at a lower depth in the reservoir, as at D, the pressure triangle will be DEF. If the dam is considered triangular, then these two pressure diagrams will be geometrically similar, and if ABC is superimposed on DEF, as in triangle DB′C′, then B′C′ will be parallel to line EF. The differential of internal pressure as between sections AB and DE is thus constant (except from B′ to E) throughout the section, although the actual pressure reduces from maximum to zero. Since these sections would normally be separated only by pore spacings, the distance B′E becomes negligible. Since buoyancy depends only on maintaining a constant differential of pressure with increasing depth equivalent to that in open water, and not upon actual pressure, it may be concluded that buoyancy would be fully effective, in Fig. 9(*b*), from back to face of dam, since each elementary vertical cylinder between pores would be buoyant.



(*a*) (*b*) (*c*) (*d*)

FIG. 9.—PRESSURE RELATIONS IN CONCRETE DAM AND RECTANGULAR WALL (AS EXPLAINED IN TEXT)

This can also be demonstrated mathematically. At any depth, *h*, the pore pressure is represented by the triangle ABC (Fig. 9(*b*)). If *b* is the ratio of base width to height and *w* is the unit weight of water, then AB = *b h*, AC = *h*, and the differential uplift represented by the elementary strip between lines C′B′ and FE would be

$$\frac{dP}{dh} = w\,b\,h\,dh \qquad (9)$$

and the total pore pressure uplift would be

$$\int_0^H w\,b\,h\,dh = w\,\frac{b\,H^2}{2} \qquad (10)$$

—which is the weight of water equivalent to the volume of the dam, or its displacement, thus indicating full buoyancy.

Consideration of the rectangular dam or wall in Fig. 9(*d*), of width B and height H, shows that the differential increment of pore pressure at head, h, instead of a strip of uniform width will be a triangle C′EF (Fig. 9(*d*)), whose area is $\frac{dh}{2}$, or

$$\frac{dP}{dh} = w\,B\,\frac{dh}{2} \qquad (11)$$

The total uplift in the wall will be

$$P = \int_0^H w\,B\,\frac{dh}{2} = w\,\frac{BH}{2} \qquad (12)$$

—which is equal to one half of the displacement.

The same reasoning as applied to Figs. 9(*a*) and 9(*b*) would seem to apply if the pressure through the dam reduces along curved lines, as in Fig. 9(*c*), provided that the reduction of pressure at adjacent elevations follows a similar curve. Thus, if the usual attempt to obtain a relatively impervious back surface were to prove partially successful (by the use of richer concrete, copper water stops, cutoff walls, natural filter skin, grouting at heel, and even drainage wells at intervals), the result would probably be that pressure gradients CB and FE (Fig. 9(*b*)) would be curved, and somewhat convex upward near the back of the dam, instead of straight. Nevertheless, if all lines of pressure from back to face of dam are of nearly similar shape, the differential would still be approximately constant and the material therefore approximately buoyant. An exception should be made for the case of drainage wells near the heel at only one level, as at the foundation contact, causing upwardly convex pressure lines there and not at higher levels. This dam would not be homogeneous, however. Reverting to the analogy with an earth dam shows that in the latter all material below the line of saturation is considered to be in a buoyant state and yet the flow is converging toward the toe as in Fig. 2(*d*). No differences in hydraulic or hydrostatic principles exist.

Thus, from every point of view, one is forced to conclude that in an ideal homogeneous triangular dam, fully saturated, with water only on one side, where seepage can escape only from the downstream face, full buoyancy would theoretically exist as in the case of a completely submerged body of concrete. Full buoyancy is equivalent to a triangular uplift diagram with full water pressure at the back and zero at the face acting over any entire horizontal area.

It is possible that the natural and artificial expedients to be discussed later may modify the magnitude of the uplift forces, but no conditions whatever can modify the application of these forces to the entire gross area of the horizontal section of the dam, instead of to the one-half or the two-thirds proportion of the area usually assumed.

The conception of uplift, herein set forth, is thus based entirely on pore pressure and therefore, contrary also to prevailing opinion, is as applicable at any one horizontal section as another, instead of being confined to pour joints, base contact with rock, or other horizontal planes of greater permeability or weakness. The relative permeability at such joints could increase the volume of leakage, but not increase the total uplift pressure, which would already apply over 100% of the area because of pores alone, and would have a triangular distribution from full headwater at the back to zero at the face. A triangular dam would, moreover, be as weak at any elevation above the base as at the base, whereas efforts to reduce uplift are usually confined to the base. Failure would occur at a pour joint or base contact, not because of greater uplift, but because of greater weakness.

If the conception of a completely saturated and therefore buoyant dam be dismissed for the moment in favor of the old conception of water entering only along the pour joints and base contact, there is still complete uplift over 100% effective area if, instead of the actual pour joint, the rough surface through the nearest full projected area of pores is visualized. This surface would depart from the actual pour joint by an amount measured in terms of only a few hundredths of an inch at most, and therefore would be under the same pressure as the joint. Internal buoyancy is not, therefore, an essential basis of the conception of uplift over 100% area. The previously quoted observations of Mr. Wiley on the wrecked St. Francis Dam indicate that pore pressure is not confined to the joints.

If uplift of full reservoir pressure at the back reducing to zero at the face were to be effective over the entire horizontal section of a dam, it would imply a degree of safety against sliding and shearing less than that now believed to exist, and also that the resultant line of pressure is outside the middle third of the base on many existing dams. To keep the resultant within the middle third with fully buoyant concrete would require a base width of 0.831 H for 12% voids and 0.846 H for 15% voids. The question was raised by the late D. C. Henny,[10] M. ASCE, and others as to how to reconcile uplift in the range from 95% to 100% in view of the continued stability of many dams, some of whose base widths are even less than 0.7 H. The following discussion is offered in explanation.

Modern methods of construction of massive dams involve seasoning by sprinkling or flooding each lift continuously until the succeeding lift is placed, and by continuous wetting of the entire structure for a considerable period after construction. This should insure a high percentage of saturation of a dam when it goes into service. Despite these precautions there will be air voids, and the upstream side or back of the dam may have dried out to a depth of a few feet between completion of seasoning and submergence by the reservoir. Static conditions, which form the basis of the assumption herein, cannot be reached until enough time has elapsed to resaturate the back to the dried out depth, and until sufficient additional water has entered the voids of the structure to occupy the volume resulting from compression of the existing pore water, and solutions or compression of the contained air to the higher ultimate pressure.

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[10] "Stability of Straight Concrete Gravity Dams," by D. C. Henny, *Transactions*, ASCE, Vol. 99, 1934, p. 1041.

In storage dams subject to wide fluctuation of water surface, sufficient time may not elapse during a period of full reservoir to accumulate pore pressure throughout the section, as shown in Fig. 9(*b*). In fact, in very large dams it may require many years to achieve full saturation. In such cases, only the concrete below usual water level would seem to be fully buoyant, and that above usual water level, only partially so. However, it must be remembered that only one horizon of pores sufficient to form a full projected area need be under pressure, as in Fig. 9(*b*), to make the entire structure above that elevation buoyant. The pores in the base of the dam, always below water level and always saturated at nearly full pressure, may more rapidly assume full pressure as the reservoir fills. Especially may an imperfect horizontal pour joint more quickly develop full triangular pressure from back to face, thus causing nearly full buoyancy to result at each filling of the reservoir.

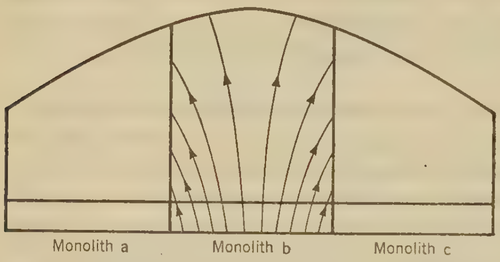

Fig. 10.—Part Plan of Gravity Dam to Show Reduction of Internal Pressure by Drain Action of Vertical Joints

Since the conception of uplift over 100% of horizontal area presupposes a rough surface comprising a complete horizontal projected area of pores; and, since these pores are staggered in elevation, solid material must be fractured between pores in the body of the concrete to encompass them. All horizontal pour joints also develop considerable cohesive bond strength and all contacts with rock foundation likewise develop considerable adhesion or bond. Both are ignored in conventional analysis, although they contribute substantially toward safety. The fact that any fracture surface has strength does not disprove the existence of buoyancy above that surface.

Vertical joints (such as shrinkage cracks, construction joints, and monolith joints) often, if not usually, serve as drains to reduce internal pressure as shown in Fig. 10, whereas full uplift requires that all seepage must reach the downstream face before relief of pressure. Therefore, theoretical pressure probably seldom exists, except near the upstream surface as measurements verify, and the theoretical is merely a mathematical limit of possibility. Such drainage reduces the magnitude of uplift pressures, but nothing can reduce the effective uplift area below 100% wherever pores are filled with water. Herein lies the fallacy of the usual assumption of one-half or two-thirds area for uplift, which it is the principal purpose of this paper to correct.

This correction does not necessarily imply that existing dams are weaker than intended. After all, uplift is the product of magnitude by area, and a reduction of either reduces the product in like proportion. Tests such as those at Hiwassee Dam (Tennessee Valley Authority) indicate that the magnitude of uplift, in this instance because of drainage, is even less than two thirds of the previously stated assumption at the level of measurement, so that the past error (made in assuming two-thirds area instead of full area) would seem to be compensated in this case by realizing only two thirds or less of the designed magnitude.

In practice it is believed that uplift is more effective at any internal horizontal section than at the foundation contact, if a good cutoff is provided by rock grouting at the heel, because natural rock (perhaps excluding shale), with its numerous jointing planes, usually furnishes better natural drainage and consequent release of internal pressure than does concrete. The usual expenditure of all anti-uplift efforts for the prevention of uplift only at the foundation contact is illogical. Curtain grouting of foundation to reduce seepage is valuable, but artificial drainage is usually not necessary because of naturally better drainage than in the dam above it. Little is gained by elaborate provisions toward making the foundation contact safer than the pour joints.

Hydrostatic pressure is conventionally assumed to be exerted against the upstream or back of a dam. The reasoning presented herein has proved that it is exerted progressively throughout the route of seepage. If the hydraulic gradient is uniform, as in Fig. 9(*b*), then the application of the thrust is uniformly distributed along the route of seepage from back to face. Actually, the seepage, viewed in three dimensions, will divert from a straight line in curves leading toward vertical shrinkage cracks, monolith joints, and any other drainage routes such as those shown in Fig. 10, in which the seepage through monolith b is shown diverting largely to the adjacent monolith joints instead of all escaping at the face. The thrust (and buoyancy as well) will be correspondingly complicated in detail of application, although the projection of the thrust forces into the plane of the dam section will total $w\frac{h^2}{2}$ as conventionally assumed.

It is remotely conceivable that horizontal layers of relatively very low permeability may exist within a dam and that a perched water table may result above such zones toward the face of the dam, with drained volume immediately below. The condition of internal buoyancy would thus be modified or destroyed near the face.

Furthermore, a dam will not necessarily fail because the computed line of pressure is outside the middle third and not, in fact, until it passes through the toe, or so close thereto, as to concentrate the load sufficiently at the toe to cause crushing. This ancient criterion is for the purpose of preventing tension at the heel, which in turn is directed toward the prevention of full uplift at the heel. The conception indicates, however, that full uplift exists at the heel in any event. Thus, for example, the worst situation which could be assumed is that the dam might be in the state of balancing on its toe, on the verge of tipping, with a completely open seam from back to face at pour joints or base, offering no resistance to entrance of water, and admitting full static pressure without loss of head through the entire joint or under the entire base. This is not a practical problem because of the extreme assumption, but considerable interest attaches to this limiting case. If the saturated concrete weight is 153 lb per cu ft, this condition would exist in a triangular dam with a base width of 0.726 H; if the saturated concrete weight is 150 lb per cu ft, the same condition would exist for a base width of 0.746 *H*. Very few dams have base widths much less than these figures.

Moreover, Ross M. Riegel, M. ASCE, determined by tests at Hiwassee Dam[11] that the load therein tended to concentrate largely at heel and toe instead of in the conventional triangular distribution, probably because of delayed cooling in the middle. This might indicate that little danger exists that compression would be relieved at the heel to admit water, even for a base width of 0.7 *H* or less.

The foregoing indicates that most existing dams would be safe against overturning under any reasonable assumption as to uplift or buoyancy even though the resultant might not be within the middle third, provided the unit load at the toe can be safely carried by the concrete and the foundation. The existence of dams with base width equal to 0.7 *H* would not seem to disprove the existence of full uplift area.

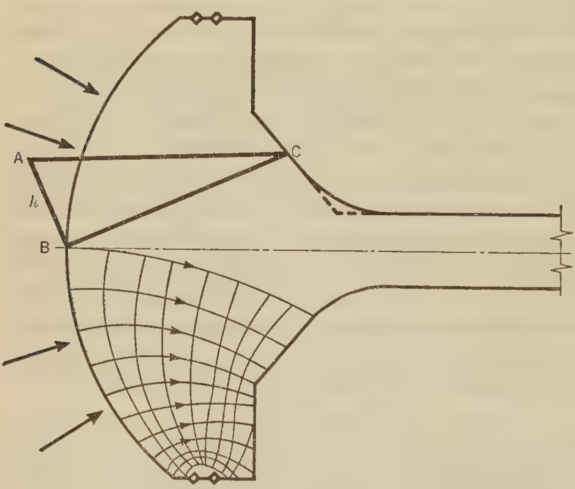

Fig. 11.—Flow Net in Round-Head Buttress Dam

The round-head buttress dam shown in Fig. 11 is a good example of a false principle of design caused by ignoring pore pressure. It is predicated on the theory that, because all lines of pressure are normal to the cylindrical surface and therefore converge at the buttress, they will produce no bending moment in the wings about the point of support at the buttress, and no tension can exist in the concrete and no steel is required. This theory is false; first, from the fact that pressure is not all applied at the surface, but progressively along the lines of seepage, and, secondly, because of the wedging action of pore pressure.

The lines of seepage and therefore the lines of pressure follow the shortest path, as illustrated by the flow net. None pass through the point of support and all produce a bending moment in the wings, thus creating tension in the curved face of the buttress head. The wings are, in fact, cantilever beams subjected to considerable bending moment from this cause.

Of much greater importance is the effect of pore pressure in causing a wedge effect analogous to that of uplift in a gravity dam. Thus, in Fig. 11, from previously discussed principles, the maximum wedge effect acting to produce a bending moment in the buttress wing would be full static pressure, *h*, at the curved face of the buttress head, reducing to zero along the line of seepage

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[11] "Structural Features of Hydraulic Structures," by Ross M. Riegel, in "Design Developments—Structures of the Tennessee Valley Authority: A Symposium," *Transactions*, ASCE, Vol. 111, 1946, p. 1169.

(triangle ABC), and acting over the full sectional area of the concrete. From this relation the bending moment about C along section BC for a unit height would be

$$M = w\frac{h\,(BC)}{2}\frac{2}{3}\,(BC) = w\frac{h}{3}\,(BC)^2 \ldots\ldots\ldots\ldots\ldots (13)$$

In addition to these forces, the weight of the wing would introduce some bending moment if the buttress head is sloped, as it usually is. Moreover, in case of shrinkage cracks in a general direction parallel to the axis of the cylindrical surface, the resisting strength of such a section as BC could be greatly reduced. There would be a great tendency to form such cracks, because of more rapid external cooling, unless cooling coils were provided in the interior of the buttress head; in fact, such cracks have been known to form.

The round-head buttress dam is thus predicated upon an entirely false conception that would only apply if the cylindrical surface were to be covered with an impervious metal surface. This type of dam can only be safe if the adjacent buttress heads make intimate contact, so that each one supports its neighbor against any spreading, which would need to occur if the bending moment in the wings were to cause local failure.

CONCLUSIONS

The following principles have been previously discussed or clearly inferred herein:

1. Uplift divides itself into two factors—magnitude, and area of application. A reduction in either factor reduces the product in like proportion. Present practice assumes full magnitude over reduced area whereas this should be reversed. The uplift area is always 100% and only the magnitude is subject to possible reduction.

2. Any submerged porous body, including concrete, is in a state of internal buoyancy equal to its volumetric displacement. This buoyancy results from pore pressure alone, regardless of the intimacy of the bottom contact.

3. A homogeneous concrete dam with water pressure only on one side, without cracks or joints, perfect except as to the necessarily porous nature of concrete, and in which all seepage must therefore reach the face to escape, is equally buoyant below the line of saturation.

4. Contact with the bottom, of such intimacy as to exclude uplift over all or part of the bottom area of a concrete structure, is impossible on any natural foundation such as earth, sand, or rock, all of which are porous or jointed. Such exclusion of uplift is doubtful on any foundation, but could only occur, if at all, on a nonporous foundation such as metal or rubber, and would not affect internal buoyancy, but merely would add an unbalanced downward pressure in the lowest pores.

5. The maximum magnitude of uplift in an ideal, homogeneous, and perfectly constructed triangular dam, from which seepage can escape only from the downstream face, is equal to full reservoir pressure at the back, or water side, reducing to zero (or tailwater) at the face exerted over 100% of horizontal area at any horizontal plane.

6. Mitigating circumstances that account for past satisfactory performance of existing dams are:

(*a*) The partial relief of magnitude (not area) of internal pressure by natural drainage through vertical shrinkage cracks, vertical construction joints, and monolith joints;

(*b*) The cohesive strength and necessity of fracturing solid material to cause failure, even at a pour joint or foundation contact;

(*c*) The time required to saturate a large dam, even with constantly full reservoir;

(*d*) The usual reservoir level fluctuation, which reduces the time when the reservoir is full; and

(*e*) The fact that location of the line of the resultant outside the middle third of the base is not indicative of failure, but only of a higher toe load.

7. Natural drainage through horizontal joints, such as imperfect contact at horizontal pour joints, does not mitigate uplift but conversely expedites the development of internal uplift in much shorter time than if dependent only on seepage through the pores in the body of the concrete.

8. The reasoning herein proves that the water pressure against a dam is not exerted solely against the back but progressively along the route of seepage. Each decrement of internal pore pressure represents hydrostatic pressure transferred by seepage forces into thrust against the internal concrete particles.

9. It is recommended that uplift be assumed to act always over 100% of the horizontal area, and of such magnitude or distribution as indicated by tests in similar existing dams.

10. Efforts to relieve uplift magnitude by drainage, if important at all, are as important at any elevation as at the foundation contact, in fact more so because of the usual joint planes and consequent better natural drainage of the foundation rock than of the concrete itself at higher elevation.

11. The value and success of drainage are questionable. Drainage shortens the path of, and thus encourages, seepage through the concrete and rock. Cement and even rock are altered and weakened by seepage, and consideration should be given to a stable section without drainage, thus maintaining as long a seepage path as possible.

DISCUSSION

William P. Creager,[12] M. ASCE.—A meritorious effort has been made by the author to induce the engineering profession to abandon a hidebound concept of uplift pressures, which, time and again, in current publication has been proved erroneous by deduction and conclusive tests. When a book[13] on this subject was published by the writer with others in 1945, it was hoped that its advocacy of 100% uplift area would be adopted. However, the writer has since been called upon repeatedly to review plans where less than 100% uplift area has been assumed.

The design of the Tennessee Valley Authority (TVA) dams embodied the assumption[14] that uplift pressures would be applied over only two thirds of the area of the base of the dam—despite the fact that, in some of its dams, the foundation consisted of calcareous rock with approximately horizontal bedding planes, considered favorable to uplift. In the writer's discussion of this paper,[15] he protested such an assumption to no avail, although the author of that paper confessed that it could not be defended experimentally and was adopted because it represented current practice.

Other engineers, including those of prominent government agencies, at present assume a full hydraulic gradient over one half or two thirds of the area of the base. When they are half convinced that this is erroneous, they explain that full hydrostatic pressure over half the area of the base is equivalent to half the full hydraulic gradient over 100% of the base area. This theory is not true at all, for the simple reason that, although the hydrostatic pressure at the upstream side can be reduced by grouting and drainage, the hydrostatic pressure at the downstream side cannot be reduced at all, but must remain equal to the tailwater pressure. Thus, if uplift is assumed to act over 100% of the area, the downstream toe is subjected to full uplift pressure of tailwater.

The paper has been confined to the subject of pore pressures. However, since pore pressures are induced by the hydrostatic pressures corresponding to the hydraulic gradient, the two are closely related and a short discussion of the latter feature would seem to be in order.

The writer's opinion[15] of the proper assumption for the hydraulic gradient is briefly as follows: For a dam without a cutoff, drainage, or upstream blanket, the hydraulic gradient varies from full headwater pressure at the upstream side to full tailwater pressure at the downstream side. The author has indicated reasons why thin dams have remained stable under such conditions and the writer will add his bit hereinafter. For a dam with a grouted cutoff supplemented by adjacent drainage, the hydraulic gradient should be considered to vary uniformly between: (a) Full headwater pressure at the upstream side; (b) tailwater pressure plus a fraction p of the differential head on the dam at the drains; and (c) full tailwater pressure at the downstream side.

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[12] Cons. Hydr. Engr., Buffalo, N. Y.

[13] "Engineering for Dams," Vol. II, by Julian Hinds, William P. Creager, and Joel D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1945.

[14] "Structural Features of Hydraulic Structures," by Ross M. Riegel, in "Design Developments—Structures of the Tennessee Valley Authority: A Symposium," *Transactions*, ASCE, Vol. 111, 1946, p. 1162.

[15] *Ibid.*, p. 1202.

The actual hydraulic gradient under ten dams with drains was published by Ivan Houk, M. ASCE, in 1932.[16] In all but one of the dams having grout curtains or drains the value of p ranged from about one sixth to two thirds. More recent tests on TVA dams showed the possibility of obtaining practically a zero value for p under best conditions. The actual value of p depends upon the nature of the foundation and the effectiveness of the grouting and drainage, which are not known beforehand. Conservatism is therefore necessary. The use of the values of p between one half and two thirds, depending upon conditions, should be adhered to.

The author has given several "mitigating circumstances" which account for the fact that thin dams, subjected to uplift, have not failed. The Barker Dam in Colorado is such an example. Before rehabilitation it had a base equal to about 70% of the height, at which there was no effective foundation treatment and in which there was pronounced leakage through the foundation and horizontal construction joints, all of which pointed to the probability of the existance of large uplift pressures, not considered in the original design.[17] In this dam, neglecting silt, ice, and earthquake pressures, any uplift in excess of about 30% would put the resultant outside the middle third.

The author's mitigating circumstances assisting stability are not credited assumptions in design. However, there is one item which is and must be tacitly assumed to exist. An orthodox design of a dam neglects consideration of possible tensile strength in either the dam or the foundation. For low dams on massive rock foundations grouting and draining the rock and draining the dam are not resorted to. In such a dam the hydraulic gradient may exhibit straight-line variation from headwater to tailwater, and full uplift over 100% of the area must be assumed to exist. However, such assumptions are never used under these circumstances.

In a well-built dam on massive rock foundations, with a continuously filled reservoir, the "cohesive" or tensile strength of the dam and the foundation is the only one of the author's mitigating circumstances which has any practical significance. It must be remembered that, if the resultant is outside the middle third, and tensile strength is not developed, full 100% uplift equivalent to headwater pressure will exist over the area in tension and failure will result by progression.

Actually, tension does exist in massive rock foundations and in the horizontal construction joints in the dam. Assuming this tension to have a value of 85 lb per sq in., it will resist 200 ft of hydrostatic uplift head. It would, therefore, seem that all dams more than 150 ft high should be provided with a grout curtain and complete drainage, even if erected on massive rock foundations.

J. S. KENDRICK,[18] ASSOC. M. ASCE.—That uplift acts on the entire submerged body of a hydraulic structure has been demonstrated in convincing fashion by the author. In so doing he has challenged, in the paragraph following the "Synopsis," two concepts which most authorities have treated as axiomatic:

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[16] "Uplift Pressures in Masonry Dams," by Ivan Houk, *Civil Engineering*, September, 1932, p. 578.

[17] "Grouted Gravel Fill and Precast Slabs Provide New Face for Barker Dam," by Raymond E. Davis, E. Clinton Jansen, and W. T. Neelands, *Civil Engineering*, February, 1948, p. 26.

[18] Dist. Engr., Water Rights Branch, Dept. of Lands, Victoria, B. C., Canada.

> "First, that 100% uplift area cannot be true because a large proportion of the base area must be in contact to support the load; and, second, that many or most existing dams would have failed if the 100% uplift area doctrine were true."

He had disposed of the first concept most effectively, but further examination of the second concept may be desirable. This examination discloses one or two modifying factors in the author's deductions, but it supports rather than detracts from his argument.

The concept that existing dams would have failed if the uplift area were 100% usually is based on the assumption that the foundation reaction is distributed in accordance with a linear pressure diagram. If the dam and foundation are considered as an elastic unit, this distribution cannot exist. However, to start with, even if it did exist, it does not follow that a dam might fail as a result. Uplift, or pore pressure, takes time to develop, and the stresses beyond the elastic limit may be taken care of by plastic flow, or by local adjustment of deformations in accordance with the elastic properties of the dam and foundation.

Quite apart from these adjustments, there is no reason to suppose that good concrete, or a well-made concrete-to-rock bond cannot resist some tension. A 200-ft-high dam with a base width 0.7 *H* and a full buoyancy would have a tension of only 50 lb per sq in. at the heel, with linear distribution of reaction and all water load applied at the upstream face.

However, Mr. Harza has shown that the water load is not all applied at the upstream face, although he has not followed his development to the conclusion indicated by his treatment of concrete as a porous medium, in which flow conforms with Darcy's law. Conditions of flow may be represented graphically by a flow net. Fig. 12 shows the flow net for a triangular section with a base width equal to 0.7 *H*. The base is assumed to be impervious. The flow line FF′ is intersected normally by equipotential lines similar to EE′. The element of water pressure dp is equal to $w\ h\ dh$ for a unit slice of the dam and is commonly assumed to have a moment about point A of $y_1\ dp$. Actually, the force dp is dissipated gradually over the flow path FF′, and the moment about point A is obviously less. This is the well-known but ill-understood "overturning moment."

Considering a vertical element EM of the dam itself with a cross-sectional area dA, the buoyant force on this element would be $w\ y_3\ dA$, if it were completely submerged. This force would be the equivalent of an "uplift" at point E produced by a triangular uplift diagram with full hydrostatic pressure at point B. However, line EE′ is an equipotential line, or a line of constant piezometric head in the flow net. It follows, therefore, that the actual buoyant force will be $w\ y_2\ dA$, or less than the full buoyant force by the ratio y_2/y_3. This corresponds to an uplift diagram of the shape ABU.

The author's reasoning, under the heading, "Application to a Concrete Dam," that leads him to the deduction of full buoyancy, depends on the assumption that "* * * the reduction of pressure at adjacent elevations follows a similar curve"—an assumption not supported by the flow net. The convex uplift diagram ABU exerts less moment about point A than does the author's triangular one, and helps to explain the satisfactory performance of existing dams where the base width is less than 0.7 H, thus lending additional support to Mr. Harza's deduction of 100% uplift area. If the foundation is assumed to be pervious, and the usual practice of curtain grouting at the heel is carried out, the freely draining base will cause all the flow lines of the flow net to assume steeper slopes, further reducing the "overturning moment," and even transmitting some of the horizontal water load directly to the foundation.

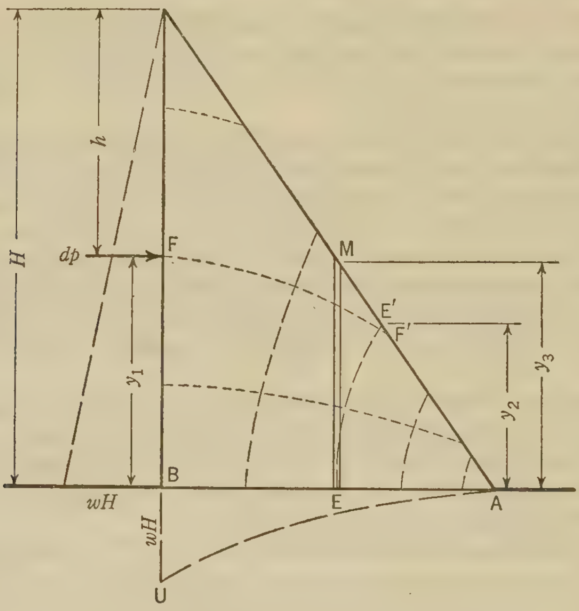

FIG. 12.—FLOW NET FOR TRIANGULAR DAM

The writer considers that further development of the analysis of gravity dams will depend on considerations of elastic characteristics of dam and foundation, and on applications of forces by the flow net principle. However, such considerations lie outside the proper limits for discussion of this important paper.

ADOLF A. MEYER,[19] M. ASCE.—Fourteen years ago Karl Terzaghi, Hon. M. ASCE, published his papers on the effective boundary porosity of concrete.[20,21] From a comparison of the compressive strength of concrete cylinders tested under confining pressure, with the confining pressure acting either on a jacket on the surface of the cylinders or in the pores of the cylinders, he concluded that the effective boundary porosity is near unity. He did not fail to point out the full implication of this conclusion on the concept of uplift in concrete dams, but a review of the textbooks published in recent years and of general design practice indicates that little credence was given to these findings. However, a certain uneasiness about the subject of uplift definitely exists in wide circles. For this reason the bold presentation by the author is to be welcomed very much. Its attention to the fundamental physical phenomena involved should help to bring forth clarifying discussion, with the result that more realistic assumptions will be introduced in the design of future dams.

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[19] Chf., Civ. Design Branch, TVA, Knoxville, Tenn.

[20] "Beanspruchung von Gewichtstaumauern durch das strömende Sickerwasser," by Karl Terzaghi, *Die Bautechnik*, July 6, 1934, p. 379.

[21] "Zur statischen Berechnung der Gewichtstaumauern," by Karl Terzaghi, *ibid.*, October 19, 1934, p. 589.

Much evidence is on hand to support the author's description of concrete as a porous and permeable material. The concept as such was introduced more than thirty years ago. Reliable permeability tests were made very much later, in particular those for the Boulder Dam (in Arizona-Nevada) investigation to which the author refers. Since air-entrained concrete has come into commercial use, much careful research has been conducted on the porosity of concrete. There should be no further difficulty in accepting mass concrete as a porous and permeable material, with the flow of water through a concrete structure following Darcy's law approximately. Saturation is a slow process. For instance, at the Fontana Dam of the Tennessee Valley Authority in North Carolina, a piezometer tip cell, placed only $3\frac{1}{2}$ ft from the upstream face, registered the first water pressure two months after the rising reservoir level had passed the level of the cell. By that time, the reservoir had risen almost 200 ft above the level of the cell. Another ten months were required to increase the pore pressure to the intensity that apparently corresponds to its stable value within the flow net, which is controlled by a drain curtain 30 ft from the water-exposed surface. The permeability coefficients approximated from these observations are $k = 0.75$ ft per yr for the first condition and $k = 3$ ft per yr for the second. The concrete at the upstream face contains 1.25 bbl of cement per cu yd, whereas the main body for the second condition contains 0.8 bbl per cu yd. These coefficients are somewhat higher than laboratory data tested on similar mixes.

It is easy to follow the author's reasoning on buoyancy of granular material and its application to almost all types of rock formations. Even when disregarding the boundary porosity of the rock itself, about which general statements should be avoided because of the radically different origin of rocks, there is no doubt that practically all rock structures are jointed and that the joints are pervious. However, when the author applies his reasoning to concrete, he introduces a concrete lacking in all tensile strength. This assumption certainly does not correspond to reality. The author actually refers in several instances to "cohesive strength" or to "some strength along the surface of failure," the existence of which he admits cannot be denied, but in his reasoning about pore pressures and buoyancy, all references to this quality of concrete are omitted. As Mr. Harza otherwise shows a keen interest in the basic physical characteristics of the materials involved, his departure from this course in this particular important instance is most unusual. Every concrete dam built by modern methods and with cement now commercially available depends for its action as a gravity cantilever on tensile strength. The writer knows of no large dam where the placing temperature plus the temperature rise from heat of hydration does not exceed the final stable temperature. The unavoidable cooling, whether natural of forced, brings about a decrease in volume which in the restraint zone of the dam where it cannot take its full course will be compensated for by internal tensile stresses. Therefore, in one important phase the tensile strength of concrete must be relied upon. How can it be considered nonexistent in another?

Professor Terzaghi, faced with a high effective boundary porosity, concluded that the real tensile strength of the effective area of cement paste must reach astounding values—in the order of from 10,000 lb per sq in. to 50,000 lb per sq in. He compares the action of the paste with threads of glass, which by experiment have shown tensile strengths of still much higher values. It is the writer's opinion that the promulgation of the use of an effective area for uplift much larger than has generally been accepted would best be served by carefully planned boundary porosity tests with confining pressures much higher than the tests available to Professor Terzaghi. According to Douglas McHenry,[22] M. ASCE, such tests are being carried out by the Bureau of Reclamation, and it will be of great interest to learn their results.

Although the author follows a line which differs very much from the experiences shown by triaxial load tests, his recommendation for design is the same as the conclusions derived from those tests. In its practical application, near unit boundary porosity requires 100% area application of uplift. It is true that the last few units of this percentage remain undefined. However, within the accuracy obtainable in the analysis of a dam, there is no practical gain in squabbling over the exact, specific value of the boundary porosity.

The author states that with this uplift assumption a base width of from 0.831 H to 0.846 H is required to keep the resultant within the middle third. After reading his explanation on flow through the dam, it is logical to ask to what type of flow net does this relation apply. It is obviously correct for the flow net shown in Fig. 9(*a*). It presupposes a dam without contraction joints, without galleries, without transverse cracks, and with a rock foundation at least as impermeable as the concrete above. Few dams built in the past approach such a condition. Since the permeability of even the best type of rock foundation is far greater than the concrete permeability, this difference will have a decisive influence on the flow net within the dam. Professor Terzaghi in 1934 showed by a few applications how the location of the resultant will be influenced by the foundation permeability. Much more direct and decisive will be the presence of drainage provisions within the body of the concrete itself. Many actual observations of pore pressures clearly show how readily the flow net corresponds to these opportunities for pressure relief. For future dams the designer can prepare for a flow net that will give him the maximum utilization of his materials.

In his final statement, the author shows himself to be in complete disagreement with this line of approach. "Consideration should be given to a stable section without drainage * * *," because "The value and success of drainage are questionable" and because seepage weakens concrete and rock. How the author can question the "value" of drainage, after explaining its action most effectively, is difficult to understand. As to the effectiveness, there are too many observations on successful installations to allow any doubt that effective drain curtains can be provided. A more realistic inquiry might ask: Will the effectiveness last for all times? If the drain curtain is not maintained, it might stop functioning properly, but there is no doubt that with proper care—and little care that is—it can always be kept functioning. Therefore, this periodic check and the possibility of small maintenance work at infrequent intervals cannot be used as binding arguments against the use of a drain curtain. Otherwise, it could just as well be concluded that it is unwise to build steel bridges in the future because they would fail some day if allowed to rust.

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[22] *Proceedings*, A.S.T.M., Vol. 45, 1945, p. 793.

That seepage weakens concrete and rock is, of course, a truism; but, like any statement of this type, its terms need qualifying definitions. What is harmful seepage? The author agrees that seepage occurs through a dam with a base width of 0.8 H. Many successful dams of the multiple arch and slab type have drainage passages with a length of from only 0.02 H to 0.03 H. Assuming the same concrete quality, their seepage volume must be from twenty-seven times, to forty times as much as the unit seepage in the nondrained gravity dam. If the experience record of the dams with high seepage rates is satisfactory, does it then show good judgment to add a safety factor of 27 or 40 on and above a satisfactory condition?

The writer does not advocate the use of drain curtains in gravity dams in a position which would anywhere reproduce the short drainage drops in slab dams. Reasons of temperature control usually make it undesirable to use a rich mix in gravity dams; the permeability coefficient will therefore be considerably higher than that for the rich mixes used in slab dams. However, based on the experiences gained from these slab dams, with conservative allowance for different permeability coefficients, location of a drain curtain can be selected which will not affect the permanence of the structure but will decisively improve its stability.

By his interesting presentation of concrete dams as porous and permeable bodies, the author is helping to extend a better understanding of the structural action of gravity dams. On studying his paper, the thought immediately follows how well the profession would be served if much of the pertinent data affecting this discussion could be made readily available—especially boundary porosity tests and actual measurements of pore pressures, uplift intensities, and permeability coefficients of mass concrete and of different rock foundations, both in natural condition and after grouting. Such data will help to clarify the zones where opinions differ. The writer believes, however, that there are enough factual data to adopt uplift assumptions which express unit boundary porosity. Furthermore, enough observations are available to justify the design of dams with planned drain curtains, extending into the foundation rock and up through the full height of the concrete body. With a flow net controlled in this fashion, durable structures with conservative safety characteristics can be built with a base width of from 0.7 H to 0.76 H, the range being controlled by unit weight and the location of drain curtain. This location in its turn must take into consideration the permeability coefficients of the selected concrete mix and of the treated and untreated foundation rock.

E. Montford Fucik,[23] Assoc. M. ASCE.—The imaginary conception of concrete as composed of vertical cylindrical elements with pores at top and bottom is apparently the basis for the theory and the conclusions expressed in the paper. If this conception is in error the conclusions must necessarily be so.

To date all recorded data have been concerned with the original sand, gravel, and cement particles. The cementing together of these particles obviously causes areas of cemented contact on top, bottom, and sides which can be assumed to exclude water pressure. Such contact areas when they occur on top and bottom would reduce the area of the original particles subject to uplift.

It would appear, however, that, once cementing has taken place, the original particles have lost their identity and are of no greater significance than any other subdivision of the material which can be logically imagined to exist. To argue otherwise would be to endow the original particles with the power of memory.

The author's imaginary subdivision of concrete into vertical cylinders with pores at top and bottom was based largely on the conception of pore space in the form of more or less globular volumes connected by microscopic channels of less importance. Apparently, relatively few (perhaps about 2% by volume) of more or less globular pores exist in concrete having about 12% of voids, of which the cement paste has about 50% voids; and most of the voids in the cement paste consist of microscopic channels leading between and around the cement grains.

Globular pores to the amount of 2% might be ample to support the author's reasoning, but this theory would seem to be improved by starting with contact areas of cement grains instead of globular pores. Suppose that such contact areas occur on the top or bottom projected area of the cement grains and that an imaginary vertical column incorporating each area of top or bottom contact is erected. Then such columns within a distance of a few cement grains will end in sufficient voids at top and bottom to terminate the column. Thus, the entire mass would be composed of vertical cylinders with top and bottom exposed to water pressure, as arrived at exactly by starting with the author's conception. Within a distance of a few cement grain diameters up or down, a surface of 100% projected uplift area would be formed. The imaginary particles making up the mass would now consist, just as before, of cylinders joined solidly to each other on their lateral faces instead of at top and bottom. The lateral contacts would not reduce the area of uplift below 100%, and only very small, almost microscopic, departure vertically would be needed to incorporate a 100% pore area.

Obviously a specimen would not break in this manner, but there need be no relation between surface of fracture and surface of effective uplift pressure. It is common knowledge that a fracture surface will pass through practically poreless aggregate pieces, although such a surface obviously would not be the surface of maximum porosity. This occurs when the aggregate pieces are weaker without pores than the cement paste is with its pores. In other words, a fracture picks the weakest section, whereas the uplift surface is determined by the maximum pore area, not necessarily related to the weakest section.

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[23] Harza Eng. Co., Chicago, Ill.

To arrive at 100% uplift area it is necessary to consider only joint seepage, rather than internal pore pressure in the mass of the concrete between horizontal joints. Conventional reasoning now assumes a pressure drop in horizontal joints reducing from reservoir pressure at the back to zero (or tailwater) at the face. The sole difference is that it is commonly assumed to be applied over, say, two thirds of the horizontal area. Joints are made with cement paste or mortar. To produce uplift over 100% of the area, it is only necessary for joint seepage pressure to spread through the cement or adjacent mortar above and below the joint for a sufficient distance (amounting to the diameter of a few cement grains) to incorporate pores of 100% horizontally projected area. A zone adjacent to the joint would doubtless be under pressure for sufficient thickness. Pressure need be built up in only one such joint in order to subject the entire volume above that joint to 100% uplift. This would be true if the pore pressure existed only in a single horizontal joint or in adjacent joints, because the elementary vertical cylinders of which the mass is assumed to be composed can extend from one horizontal joint to the next joint above or below it instead of from one pore to the next, without invalidating the reasoning.

Many modern, carefully built dams show efflorescence along horizontal joints, indicating that seepage often penetrates horizontal joints in a short time even if it may take many years to penetrate the body of the concrete. In fact, on uplift pressure tests made on several dams, usually at the horizontal joints, indicate that not many years are required to reach a state of equilibrium. The pressure line when determined is usually erratic and does not conform closely with the convenntional straight line from reservoir pressure at the back to zero at the face. The author, however, is concerned only with the area of uplift, not its magnitude.

The writer agrees with the author's conclusion that uplift must be considered as acting over 100% of the horizontal area. In view of the innumerable examples of successful concrete dams designed using only three fourths or two thirds of the area as affected by uplift, the conclusion can be drawn that the pressures assumed were higher than those that actually occurred. The net result is the same whether one uses two thirds of the area and 100% headwater pressure, or two-thirds headwater pressure and 100% of the area.

A. C. R. ALBERY,[24] ESQ.—While engaged on the design of a dam of the round-head buttress type, it was somewhat disconcerting to read the author's contention at the end of his paper that the round-head design was based on a fallacy. However, the writer has come to the conclusion that Mr. Harza's argument, although convincing for horizontal planes and for vertical planes at right angles to the lines of seepage flow, cannot be applied to vertical planes parallel to the lines of seepage flow.

The argument that uplift on any horizontal plane is really a buoyancy effect is a good one, and the writer agrees that the uplift is probably equivalent to the hydrostatic pressure acting on 100% area. The word "equivalent" is important. The uplift is not an actual hydrostatic pressure acting on 100% area, but is an aggregate buoyancy effect acting throughout the whole mass of concrete above the plane under consideration. A similar effect is produced by the steady drop in pressure along the lines of seepage flow. There is no such effect, however, on vertical planes parallel to the lines of seepage flow, and the author's argument cannot be used to prove it. On such planes the hydrostatic pressure will act only on the area of voids cut by any particular plane, which, as assumed in the paper, may be only about 12%.

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[24] London, England.

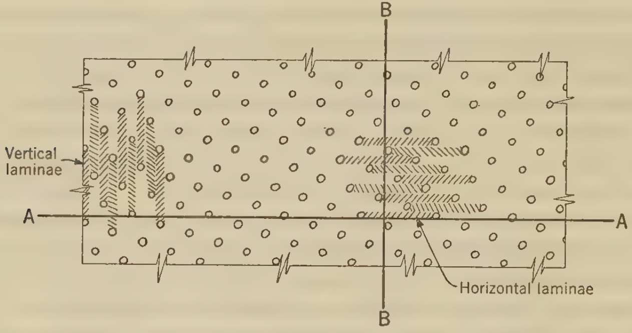

Fig. 13

The writer has found it easier to visualize the action of the uplift force by idealizing the conditions. Imagine a number of minute, straight, horizontal tubes passing through an otherwise impermeable dam in a symmetrical arrangement, as shown in Fig. 13. Water is forcing its way through so that the pressure drops from the full head at the upstream face to zero at the downstream face. Any horizontal plane such as A–A will only pass through a certain number of the tubes, and the hydrostatic pressure on that plane will only act on the area of the tubes cut by the plane. On the other hand, the dam may be divided into a number of vertical laminae, each having a tube above and a tube below. With water at the top and the bottom each lamina is, in effect, submerged and its effective weight will be its submerged weight. In the case of a triangular dam, the use of the submerged weight in the calculations gives the same result as using the actual weight and allowing for an uplift pressure varying from full head to zero and acting on 100% area. It is, nevertheless, not the same thing.

By assuming interflow between the tubes a similar argument can be used (see Fig. 4(*c*)) to show that there is a horizontal "buoyancy" effect on vertical planes perpendicular to the tubes because of the drop in pressure in the direction of flow.

Now consider any vertical plane parallel to the lines of flow, such as B–B. Once again the hydrostatic pressure will act on only the area of the tubes cut by the plane. However, if the dam is divided into a number of horizontal laminae, each having a tube at both sides, the water pressure on each side will be the same and there can be no resultant force due to buoyancy or differential pressures. The author himself put the argument very clearly when, referring to the experiments of S. Leliavsky Bey, M. ASCE, he stated (under the heading, "Experimental Evidence"):

> "The concrete cylinders joining adjacent pores will be subjected, therefore, to the same pressure at each end and will have no tendency to split apart or any tendency whatever to move as the result of water pressure in these pores. No wedging action or longitudinal stress can result from this pore pressure as long as the pressure is the same at each end of any elementary cylinder."

The writer believes that the probable effects of the pore pressure within concrete may be summarized as follows:

1. The water pressure within the pore spaces in concrete creates an aggregate buoyancy effect, which, in the case of a triangular dam, is equivalent to an uplift pressure acting on 100% area of horizontal planes.

2. Provided that a pressure gradient exists through the dam falling from full head to zero, there may be a similar effect on vertical planes perpendicular to the lines of flow, which amounts to hydrostatic pressure acting on 100% area. If there is no pressure gradient, there will be no such effect.

3. On vertical planes parallel to the lines of flow there is no differential pressure effect, and the hydrostatic pressure within the concrete acts on only the area of the pore spaces cut by any true plane, which with good concrete may be only about 12% of the total area.

The plane in Fig. 11 is almost parallel to the lines of flow, and Eq. 13 for the bending moment in the wing of a round-head buttress dam due to the so-called wedge effect should be more nearly

$$M = \frac{12}{100} w \frac{h\,(BC)}{2} \frac{2}{3} (BC) = \frac{w\,h}{25} (BC)^2 \ldots\ldots\ldots\ldots (14)$$

Such a force may reasonably be resisted by the inherent tensile strength of the concrete. Preferably, however, it should be provided for in the design of the head. Even if it is not agreed that the hydrostatic pressure acts over only a small percentage of the area, the total area affected by seepage flow can be greatly reduced by providing a series of small drains a few feet back from the upstream face and extending the entire height of the head.

The magnitude of the hydrostatic pressure may also be slightly reduced over the bulk of the head by employing a facing concrete that is rich compared with the remainder. The stresses in the head of a round-head buttress dam are usually very low, and the bulk of the head may be of a weak concrete. For the same workability the rich facing concrete can have a much lower water-cement ratio than the weak concrete. The total uncombined water per unit volume of concrete will also be less. Consequently, the percentage of voids will be less and the resistance to seepage flow greater. The pressure drop through the facing concrete will be relatively large, and the maximum intensity of pressure over the bulk of the head behind will be reduced.

In conclusion, the writer believes that this paper, far from condemning the round-head buttress dam, actually strengthens the argument for its adoption. If the author's theory of full buoyancy is accepted, the profiles of gravity dams may become more conservative, and any type of buttress dam will show a greater relative economy. Alternatively, the small total area over which uplift can act will give the buttress dam a greater relative factor of safety against sliding or overturning.

W. H. R. NIMMO,[25] M. ASCE.—Many engineers who have designed dams for a percentage of full theoretical uplift doubtless realize that the percentage applies to the intensity of pressure and not to the area on which it acts. Nevertheless, the author's demonstration of that fact is timely in the interest of clear thinking. The writer is entirely in agreement with the conclusions stated by the author, except the last one regarding the value of drains.

In combating a proposal to build a vertical wall supported by struts against the upstream face of a recently constructed gravity dam to prevent the possibility of leaching of the cement from the concrete, the writer investigated the percolation to be expected through the dam. The dam was built for the dual purpose of flood mitigation and water supply and the reservoir will usually be filled to 105 ft above the minimum level of tailwater, where the dam is 116.5 ft thick, with 12.5 ft of 6-bag concrete at the upstream face and with 4-bag concrete at the remainder of the section. There are 6-in.-diameter vertical drains, spaced 10 ft apart, 16 ft from the upstream face. The coefficients of permeability, for the 6-bag and 4-bag concrete, respectively, were found by experiment to be 20×10^{-12} cu ft per sec per sq ft and 800×10^{-12} cu ft per sec per sq ft for a head loss of 1 ft per ft.

Assuming that the concrete is homogeneous and that percolation is uniformly distributed, the time required to saturate the full thickness of the dam was found to be about 270 years. However, percolation along a horizontal construction joint might be much more rapid and lead to saturation of the dam at some levels in a comparatively short period. Computation indicated that the drains will intercept 96% of the water, thus reducing the saturation time (assuming percolation uniformly distributed) to about 5,000 years. In this case it seems that the drains may appreciably relieve uplift.

Denser concrete in the interior of a dam, or horizontal cracks extending inward from the upstream face, may entirely upset the usual assumption that uplift intensity varies linearly from headwater to tailwater. Thus, full reservoir head may act over an appreciable area at some levels. Under these circumstances, drains will be valuable both in reducing uplift pressure over the area downstream from them and in acting as indicators of any abnormal leakage, which may then be stopped by drilling and grouting.

JOHN S. COTTON,[26] M. ASCE.—This lucid demonstration of the mechanics and nature of pore pressure phenomena narrows down the uplift problem in dams to the determination of the magnitude and distribution of pore pressures for each specific condition of construction and loading. This is a great step forward in the very important and highly controversial uplift problem. It was not many years ago that some designers ignored uplift entirely.

Mr. Harza states (under the heading, "Application to a Concrete Dam"):

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[25] Chf. Engr., Stanley River Works Board, Brisbane, Queensland, Australia.

[26] Cons. Engr., San Anselmo, Calif.

> "Vertical joints (such as shrinkage cracks, construction joints, and monolith joints) often, if not usually, serve as drains to reduce internal pressure as shown in Fig. 10, whereas full uplift requires that all seepage must reach the downstream face before relief of pressure."

The writer noticed such action during the recent rehabilitation work on the huge Tsao-feng-men Dam on the Sungari River in Manchuria.

This concrete gravity dam, 3,400 ft long and 300 ft high, is in a hazardous condition for a number of reasons which will not be discussed here. The dam is not complete in that large craneway benches exist on most of the downstream face. The transverse contraction joints are ungrouted; most of the concrete is of poor quality, and there are many large rock pocket lenses and poor construction joints—hence leakage is general and severe. The main objection to the leakage in this case is the freezing of the leakage water in winter and the leaching of the cement. Some of the large concrete blocks were heaved in an upward, or downstream, direction by the freezing of the leakage.

TABLE 2.—SUMMARY OF FLOW NET STUDY OF UPLIFT PRESSURES FOR VARIOUS WATER SEAL ARRANGEMENTS
(59.2 Ft Between Transverse Joints)

				UPLIFT ON COMPLETE BLOCK		
Case (1)	Location water seal No. 2[a,b] (2)	Joints grouted[b] (3)	Drainage[b] (4)	Force, in kips (5)	Lever arm,[c] in feet (6)	Moment,[c] in foot-kips (7)
(a) HEAD IS 200 FT AND BASE LENGTH IS 166 FT						
1	D	C	D	61,400	27.7	1,700,000
2	N	N	T, D	14,200	55.5	790,000
(b) HEAD IS 100 FT AND BASE LENGTH IS 83 FT						
3	D	C	D	15,300	13.9	213,000
4	N	N	T, D	6,800	19.9	135,000
5	29.6D	N	S, D	10,000	20.6	206,000
6	29.6D	B	S, D	9,800	21.7	213,000
7	D	N	D[d]	13,400	8.4	113,000

[a] Water seal No. 1 is located on the upstream face in each case. [b] KEY: B = grouted between seals; C = grouted completely; D = downstream face; N = none; S = drainage of the joints below the second seals; and T = drainage of transverse joints. [c] About the neutral axis. [d] Drainage facilitated by ungrouted transverse joint between seals.

Careful examination of the leakage in the contraction joints, galleries, and shafts—as well as in the trenches, keys, and shafts excavated in the concrete for repair work—indicates that the leakage into the transverse contraction joints decreases rapidly toward the downstream face. Although it is likely that the upstream water seals contribute to the leakage, the writer is of the opinion that most of the leakage passes into the contraction joints through the concrete and the interconnected rock pocket lenses. The plan of repair is to grout the upstream face by vertical diamond drill holes spaced 4.5 ft center to center. It is estimated that at least 1 cu ft of grout per foot of hole will be required, the grouting pressure being 100 lb per sq in.

The writer favors open ungrouted transverse joints in gravity dams as a method of preventing accumulation of twist moments and longitudinal thermal forces. In addition, such open joints would help reduce uplift forces. By the use of flow nets, a study was made of the uplift pressures for various water seal arrangements. The flow nets were constructed by cut and try methods, as no electric analogy table was available. It was assumed that the concrete was uniform in porosity and that the pressure acted on 100% of the area; also, that pressure at the downstream face and in free-draining joints was 0. The results are shown in Table 2. The variation of pressure along the center lines of the blocks is shown in Fig. 14.

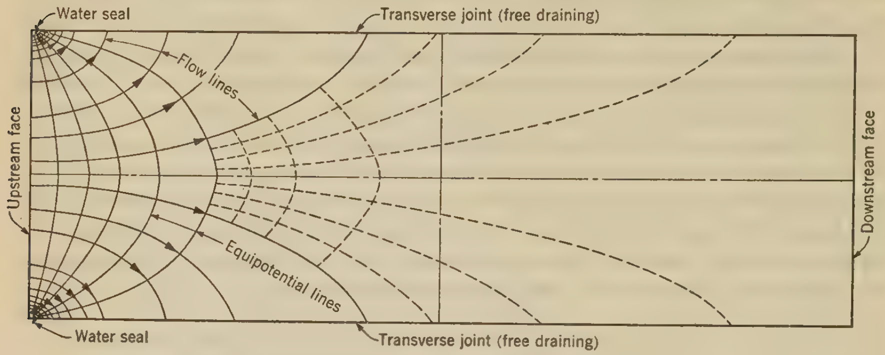
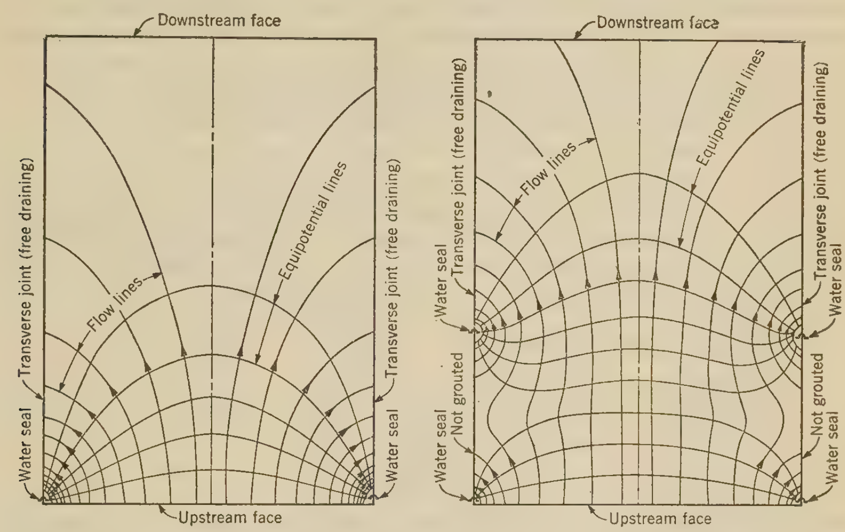


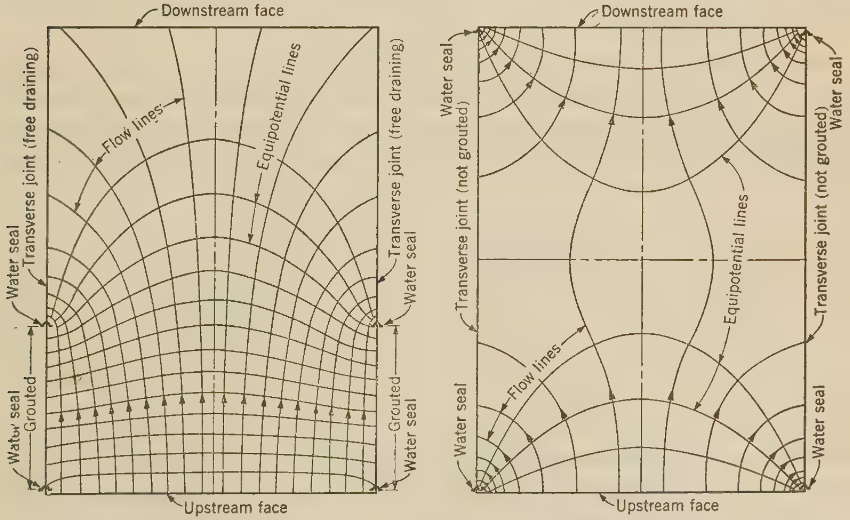
FIG. 14.—UPLIFT PRESSURES ON CENTER LINE OF BLOCKS

The uplift forces acting on 100% of the area of a completed horizontal section of dam subjected to a head of 200 ft of water are shown in Fig. 15. This is based on the installation of water seals at only the upstream face and the absence of grout in the joints. The uplift force and the moment are only 23.2% and 46.5%, respectively, of the force and moment of a similar section where the transverse joints are grouted. The possibility of leakage at the seals because of crowding of the flow and pressure lines is obvious.

Fig. 16 shows the uplift forces acting on 100% of the area of a completed horizontal section under 100 ft of head and various conditions of transverse joint sealing and grouting. In the case of upstream seals and open joints (Fig. 16(*a*)) the uplift force and the moment are only 44.3% and 63.5%, respectively, of the force and moment of a similar section where the transverse joints are grouted. As before, the likelihood of leakage at the seals is apparent. Figs. 16(*b*) and 16(*c*) show the effect of installing a second seal part way between the faces of the dam. In Fig. 16(*b*) the joints are ungrouted, and in Fig. 16(*c*) the joint is grouted between the seals. In these cases there is little reduction in uplift moment, but the uplift force is reduced to about 65%. An advantage of using two water seals in tandem in an ungrouted joint is that the twist in the block, caused by variable upstream water load, could be balanced by the unbalanced water pressure between the joints. Fig. 16(*d*) shows the effect of installing seals in the ungrouted joints at both faces of the dam, the uplift force and moment being 87% and 53%, respectively, of the uplift force and moment on a section having fully grouted joints. It will be noted that in this arrangement the variation in pressure is a reverse curve. Where there are leaky upstream seals and tight downstream seals the uplift pressure would be severe—that is, almost rectangular in shape. The flow nets for cases 1 and 3 are not shown because of their simplicity (straight lines).


Downstream face
Transverse joint (free draining)
Water seal
Flow lines
Equipotential lines
Transverse joint (free draining)
Water seal
Upstream face
Downstream face
Water seal
Transverse joint (free draining)
Not grouted
Water seal
Flow lines
Equipotential lines
Transverse joint (free draining)
Water seal
Not grouted
Water seal
Upstream face

(*a*) CASE 4 (*b*) CASE 5


Downstream face
Water seal
Transverse joint (free draining)
Flow lines
Equipotential lines
Water seal
Grouted
Transverse joint (free draining)
Water seal
Grouted
Water seal
Upstream face
Downstream face
Water seal
Transverse joint (not grouted)
Water seal
Flow lines
Equipotential lines
Water seal
Transverse joint (not grouted)
Water seal
Upstream face

(*c*) CASE 6 (*d*) CASE 7

FIG. 15.—UPLIFT FORCES UNDER 200-FT HEAD (CASE 2, TABLE 2)

The arrangements shown in Fig. 15 and Fig. 16(*a*) are the best, as far as reducing uplift and preventing the accumulation of twist moments and longitudinal thermal force, but they are conducive to leakage unless very high quality concrete is used in the vicinity of the seals. If the joints are to be grouted, it would be best to grout the transverse joints after the dam has aged and at a time when it is under appreciable water load.

The writer has long contemplated employing a continuous vertical zone of very porous concrete located behind a very dense and highly impervious concrete upstream dam face for eliminating uplift. This porous zone would extend into the foundation, and would be connected to galleries and tunnels to provide drainage. The principal objection appears to be the possible cracking of the impervious slab.

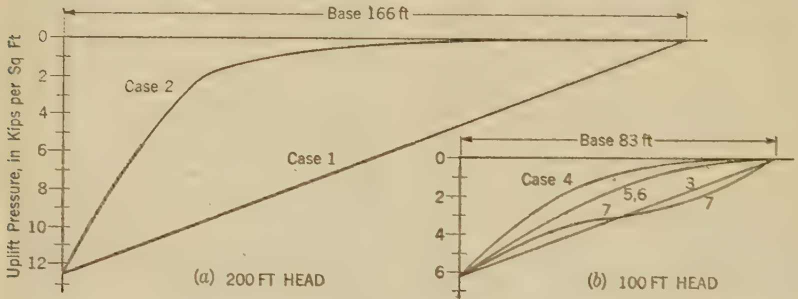
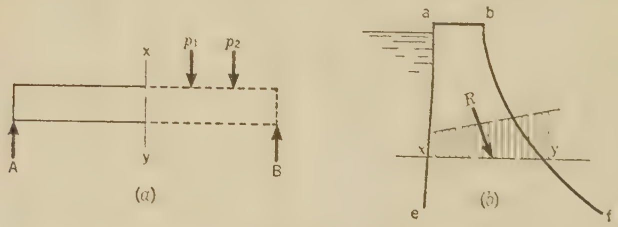


Fig. 16.—Uplift Forces Under 100-Ft Head for Various Conditions in Table 2
(a) Case 4
(b) Case 5
(c) Case 6
(d) Case 7

Serge Leliavsky Bey,[27] M. ASCE.—The postulate of the modern theory of resistance is that, in order to compute internal stresses, the designer must consider an element as entirely separated from the remainder of the structure by an imaginary plane, or potential surface of rupture, and he must determine the resultant of all the external forces (inclusive of such external forces as the weight and the interstitial uplift pressure) applied to this element. This resultant is then equivalent to the sum of the internal stresses acting on the assumed plane or surface of separation, from which the designer is able to determine these stresses quantitatively.



(a) (b)

Fig. 17

For instance, in order to compute the stresses in beam AB (Fig. 17(a)) the right side is separated from the left by the section xy, and then the inner stresses in this section are computed in such a way that their resultant and moment are equivalent to the geometrical sum of all the forces on the right side (inclusive of the "active" forces $p_1, p_2, \cdots, p_n$ and of the reaction at end B). The stresses in the dam ab (Fig. 17(*b*)) are determined in the same manner, by drawing a horizontal plane, xy, separating the part, abyx, from the remainder of the structure. The resultant, R, of all the forces above xy is computed, and the stresses follow. Neither the weight of the part exyf, nor any other force which that part resists below plane xy, can affect the stresses in this plane.

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[27] Director, Designing Service, Reservoirs and Nile Barrages Dept., Ministry of Public Works, Cairo, Egypt.

This is a well-known and well-established principle which governs the entire field of modern engineering design, with the possible exception of a few instances where pure elastic theory is applied in practice. In fact, in so far as the writer knows, the method was first used by Galileo as early as the seventeenth century,[28] and, in spite of the resistance afforded by the metaphysicists of the eighteenth century, has been accepted as a teaching principle in technical schools since Navier's time (the third decade of the nineteenth century).

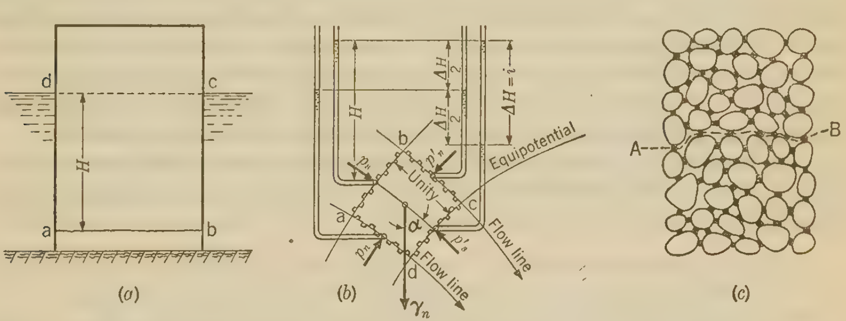

FIG. 18

Failure to use this principle leads the author to a basically erroneous conclusion regarding the mechanics of uplift action, particularly when he declares that the effective uplift area may be shown theoretically to be always 100%.

The literature on the uplift problem provides evidence that this is not the first unsuccessful attempt to obtain a solution from general considerations (deductive reasoning) and reveals, also, the various flaws in the conclusions thus derived. For instance, in 1934, Karl Terzaghi, Hon. M. ASCE, proved that, for the particular case of seepage in dams, the shape of the surface of separation to be adopted for the computation of stresses must depend on the microstructure of pores, and must coincide with a surface of rupture which is physically possible; in Fig. 18(*a*) Professor Terzaghi's idea[29] is applied to a pier, and in Fig. 18(*b*) the principle is applied to an elementary volume of porous structural material. In the first case the surface is represented by line ab and in the second, by line abcd.

The point is of considerable importance because it is thus proved that the "effective porosity," n_w, is greater than the volumetric porosity, n, and, therefore, n_w is an independent physical parameter, which cannot be arrived at by deductive reasoning. It must be found experimentally—"Der Wert n_w stellt eine unabhangige, empirsch zu bestimmende Material Konstante dar." An illustration of Professor Terzaghi's separation surface[30] is shown by section AB in Fig. 18(c).

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[28] "Discorsi e Dimonstrazioni Mathematiche," Leiden, 1638.

[29] "Die Wirksame Flächenporosität des Betons," by Karl Terzaghi, *Zeitschrift des Oesterreichischen Ingenieur-und-Architekten Vereins*, Vol. 86, 1934.

Whether the Terzaghi surface of separation, or any other type of surface of separation, is assumed, it must always intersect not only "microvoids" but "microsolids," as well. Therefore, part of the area of the surface of separation, or surface of rupture, will be inaccessible to the filtering water, for the simple reason that, in materials capable of resisting tension, solid contact between elementary particles is obvious. This is substantiated by experiments, which show that the percentage of the area which is thus inaccessible to uplift action, is a measurable, empirical quantity.

Since Mr. Harza does not use the clearly defined, standard rational method of stress analysis, based on the assumed surface of separation, which engineers employ in almost all cases during their daily design work, he is forced to resort, instead, to various comparisons and stratagems, which, by reason of their unorthodox nature and their somewhat imaginative character, cannot fail to create a certain amount of confusion, in regard to their scope, argument, and objective. In this discussion an attempt is made to systematize these various approaches, distinguishable in the paper, and to consider them seriatim.

For instance, in the text near Eq. 1 the pores are assumed flattened to form "laminations" with alternating layers of solid material and empty spaces. The object of this device is not obvious:

(*a*) The imaginary structure thus created does not resemble the true microstructure of porous solids (whatever the latter may be); and

(*b*) Such a stratified system could not resist tension at right angles to the strata, which is the main characteristic of porous solids as compared to granulated materials.

It follows then that this artificial assumption cannot serve as a starting point for deductive reasoning.

Another mental picture presented consists of projecting all the pores, contained in a volume of material, in one plane. That concept, of course, would yield an uplift on 100% of the area; but, if the stresses or pressures that occur in one plane can be projected at will into another plane, the process will never finish, and the stress intensities thus calculated will increase to infinity.

For instance, if the stresses in one section of a bridge were projected into and added to the stresses in another plane, then reprojected into a third section, and so on, the sum so obtained would be infinitely large without in any way representing the true state of stress in the material. In order to obtain a significant result, it must be based on one section only—namely, the surface of separation to which mention has been made previously (for instance, sections xy in Fig. 17(a) and 17(b)), and such surface must of necessity intersect both voids and solids which means that part of its area will be unaccessible to the interstitial water pressure.

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[30] "Experiments on Effective Uplift Area in Gravity Dams," by S. Leliavsky Bey, in "Uplift Pressure in and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 485, Fig. 27.

Another point of the author's argument in defense of his thesis (theoretical 100% uplift in dams) is derived from comparison with submerged "gravel, sand, and earth." Comparisons may or may not be significant, but in this case the argument goes rather against, than for, the thesis. As Mr. Harza states (under the heading, "Buoyancy"), granulated materials when submerged in still water "* * * are reduced in weight to the full extent of the displacement of individual particles thereof * * *." If the porosity is n, the displacement of the particles per unit volume is $1 - n$, which means that, should the same principle be applied by analogy to a masonry or a concrete structure, the uplift would be $(1 - n)\ \gamma\ H$, and not $\gamma\ H$, as assumed when calculating 100% uplift for gravity dams.

A fourth concept presented in the paper is illustrated in Figs. 4 and 19. The horizontal sectional area of a pore, say, $a_1, a_2, a_3, \cdots, a_n$ is projected upward until it meets another pore, say, $b_1, b_2, b_3, \cdots, b_n$. The vertical lines joining each pair of pores represent a set of imaginary cylinders, terminated at the top and bottom by pores. Each cylinder is assumed to be subject to the effect of two opposite hydraulic forces, F_n and f_n, the net resultant, for every cylinder, being equal to the difference $f_n - F_n$. This is offered as a theoretical demonstration of the 100% uplift action in dams.

In considering the significance of the latter demonstration, assume the cylinder used by the author as the basis of discussion, but, instead of attempting a conclusion from an isolated cylinder, extend the reasoning to its logical end, taking into account all the cylinders contained in the block above the surface of separation xy (Fig. 19(b)). In doing so, the investigator will follow a path somewhat similar to that which the author has used in Fig. 5; but, instead of assuming the base of the block to coincide with the foundation line, this base will be taken as the separation surface (Figs. 19(b) and 19(c)). The analysis will then yield both internal uplift and foundation uplift because they are indeed of the same physical nature (except, possibly, for the numerical values of the relevant constants, which may be affected by the difference in the materials above and below the separation surface).

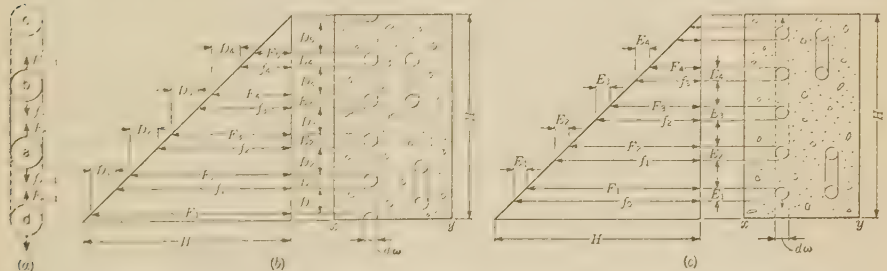

FIG. 19

By inspecting Fig. 19(a) it is obvious that, in addition to the two vertical forces, F_n and f_n, acting in every pair of pores that form an elementary cylinder, there are also two other vertical forces, f_{n+1} and F_{n-1}—that is, the upward force in the upper pore and the downward force in the lower pore. In order to take all these forces into account and to systematize the solution, it must be remembered that above pore a and below pore b, there are also many other pores. If the reasoning is to be continued to its logical end, these forces must all be taken into account. In other words, the elementary cylinder must be extended up, to the top of the concrete block, and down, to the surface of separation, xy. A column (Fig. 19(b)) is thus obtained having the area $d\omega$ as a base, and consisting of superposed voids and solids, in the average proportion n to $(1 - n)$, in which n is the physical porosity. Each of the solids represents the elementary cylinder visualized by the author. The resultant for all the cylinders in the column is the sum of the upward vectors, F, minus the sum of the downward vectors, f—that is,

$$\begin{aligned}\Sigma F - \Sigma f &= (F_1 + F_2 + F_3 + \cdots + F_n) - (f_1 + f_2 + f_3 + \cdots + f_n) \\ &= (F_1 - f_1) + (F_2 - f_2) + \cdots + (F_n - f_n) \\ &= D_1 + D_2 + D_3 + \cdots + D_n = \Sigma D \quad \text{..(15)}\end{aligned}$$

It is evident from Fig. 19(b) that the sum of the D-vectors can never attain the total hydrostatic uplift H. In fact, it is clear that ΣD bears the same relation to H as $1.0 - n$ bears to 1.0, which means that

$$\Sigma D = H \gamma (1 - n)\, d\omega \quad \text{..........................(16)}$$

in which γ is the specific weight of water.

Attention will next be called to the fact that the argument in Fig. 19(b) applies to that part of the area only, in which the plane xy intersects voids. According to Delesse's law this part is $n\,\omega$, and, therefore, the total upward force thus calculated is

$$\int_0^{n\omega} H \gamma (1 - n)\, d\omega = H \gamma (1 - n)\, n\, \omega \quad \text{.............(17)}$$

In addition to this force, one must also consider the part of the area (namely, $(1 - n)\,\omega$) in which the section xy intersects solid matter. The case is represented in Fig. 19(c). Following the same method shown in Fig. 19(a),

$$\begin{aligned}\Sigma F - \Sigma f &= (F_1 + F_2 + F_3 + \ldots + F_n) + - (f_0 + f_1 + f_2 + \cdots + f_n) \\ &= -(f_0 - F_1) + (f_1 - F_2) + (f_2 - F_3) + \cdots + (f_{n-1} - F_n) = -\Sigma E \quad \text{..(18)}\end{aligned}$$

The minus sign ($-$) shows that this is a downward vector. It bears the same relation to H as n bears to 1.00. Since the area to which this particular analysis applies is $(1 - n)\,\omega$, the total force is

$$\int_0^{(1-n)\omega} - H \gamma n\, d\omega = - H \gamma (1 - n)\, n\, \omega \quad \text{.............(19)}$$

Thus, the downward force is equal to the upward force, indicating that the resultant of all the interstitial actions above plane xy (which resultant is the uplift force for this plane) is equal to zero in this case.

This result was developed from other considerations about 1913. Various attempts have since been made to explain its apparent paradox. The surface suggested by Professor Terzaghi is the true explanation, because it gives, automatically, a superficial porosity, n_w, which is greater than the volumetric porosity, n. It follows that n_w is a basic factor in the study of the uplift, and since it can only be found by tests, the value $n_w - n$ is obviously an empirical parameter.

Thus far, the writer has used the cylinder proposed by the author as the starting point of the analysis, and has shown that this leads to results that differ basically from those derived in the paper. It must now be pointed out, furthermore, that conventional assumptions such as the author's cylinder should preferably be avoided entirely, because they may easily lead to misunderstandings.

To avoid the abstract reasoning that results from such assumptions, the uplift effect must be visualized as a physical problem, and dealt with, quantitatively, as such. For instance, the following solution is often adhered to. In a porous solid capable of resisting tension and subjected to the effect of a filtering liquid, the intensity of the interstitial pressure (*Stromungsdruck*) drops gradually (hydraulic gradient) from the entrance to, to the exit from, a seepage path. In addition, a heavy liquid, such as water, is subjected to the effect of its own weight (force of gravity) which produces an additional component of the variable pressure, depending on the relative levels of points in the microchannels (*hydrostatischer Druck*). As a result of the combined effect of these two causes, the hydraulic pressures acting on the opposite faces of an elementary pore (which constitutes an elementary length of a microchannel) are not the same.

If these infinitesimal differences are integrated for all the pores contained in a block of material, separated from the remainder by an imaginary surface of rupture, the total "force of uplift" is obtained, for that surface of rupture.

In specific applications the value of this integral can be computed as follows:[31] Consider the conditions of stability of the water contained in the pores of a solid block of material. This volume of water is subject to the effect of the following forces—(*a*) its weight and (*b*) the hydraulic pressures acting in the pores on the periphery of the block (including the external physical surfaces of the block and the imaginary surfaces of separation).

When these forces are not in equilibrium, their resultant is transmitted to the porous solid material through the interstitial pore action mentioned previously. Therefore, this resultant is the force of uplift, being equivalent to the integral of pore pressure differences. Since this reasoning is axiomatic and does not require to be substantiated by conventional assumptions about a hypothetical pore structure, informed opinion assumes as obvious, that the two methods (inductive and deductive) are equivalent.

For instance, Professor Terzaghi considers[29] that the water contained in the pores of block abcd in Fig. 18(*a*) is subject to the following forces:

(*a*) Its weight, $\gamma H A n$, in which A is the base area of the pier; H is the depth, below surface level, of the separation surface; and n denotes volumetric porosity;

(*b*) The pressures on the lateral vertical surfaces of the pier which balance each other and have no vertical component; and

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[31] "Die Wirksame Flächenporosität des Betons," by Karl Terzaghi, *Zeitschrift des Oesterreichischen Ingenieur-und-Architeken Vereins*, Vol. 86, 1934, pp. 1–4.

(*c*) The pressure in the pores intersected by the plane of separation ab in Fig. 18(*a*). According to Delesse's law the area of these pores is $n\,A$, and, as the pressure is $\gamma\,H$, it follows that the upward force acting on the water contents of the block is $\gamma\,H\,A\,n$.

Since the weight, defined in item (*a*), and the upward pressure, defined in item (*c*), are equal, the uplift will be zero in this case. This is a well-known result, demonstrated previously in this discussion, and quoted by other authors.

It must be realized, nevertheless, that a measurable uplift force would be found to exist in the section ab, should the separation plane be replaced by the Terzaghi surface, because, in this case, the superficial "effective" porosity, n_w, for item (*c*), would be greater than the volumetric porosity, n, for item (*a*). Therefore, the uplift would be equal to $\gamma\,H\,A\,(n_w - n)$.

Professor Terzaghi also analyzes the dynamic equilibrium conditions for the water contained in the pores of the elementary volume of Fig. 18(*b*). On its front and back faces (ab and cd in Fig. 18(*b*)) the pressure intensities in the pores are $H\,\gamma$ and $(H - i + \cos\alpha)\,\gamma$. Multiplying these quantities by their respective areas,

$$p_s = H\,\gamma\,n_w \qquad (20a)$$

and

$$p'_s = (H - i + \cos\alpha)\,\gamma\,n_w \qquad (20b)$$

and the net difference is therefore $(i - \cos\alpha)\,\gamma\,n_w$. In the same manner, the net difference between the lateral forces p_n and p'_n is equal to $-\,\gamma\,n_w \sin\alpha$. In addition to these two surface forces, the water content of the element is also subject to its own weight $\gamma\,n$. Resolving these three forces, one finds an upward vertical force,

$$\gamma_a = n_w\,\gamma - n\,\gamma = \gamma\,(n_w - n) \qquad (21a)$$

and another force, acting in the direction of the flow, which is equal to

$$\gamma_s = i\,\gamma\,n_w \qquad (21b)$$

The first of these two forces is similar to the uplift acting on the pier considered in the first example, and the second is characteristic of a liquid in steady motion.

Since the resultant of all the forces acting on a volume of water in such motion must of necessity be zero, the integral of all the pressure differences transmitted from the solid to the water must be equal and opposite to the forces γ_a and γ_s. This result indicates that the "uplift" or the total effect of the percolating water on the porous solid is equivalent to the forces γ_a and γ_s (*hydrostatischer Druck* and *Stromungsdruck*).

As a third instance, the same system of reasoning will be applied to the concrete cylinder AB, Fig. 20, which is placed in the container CD, and subjected, at the same time, to the effect of both (*a*) the water pressure, p; and (*b*) the mechanically applied vertical load, N. For mechanical reasons (to obtain the required watertightness of connections), the test cylinder AB is provided with the top and the bottom castings, E and F, which, however, do not affect the basic process in the critical section, mn.

The pressure in the container is raised gradually so that the water penetrates through the surfaces kL into the pores of the specimen, percolates upward (as shown by the arrows in Fig. 20), and is evacuated from the top of the specimen.

At the moment when the specimen fails, the water content in its upper part (above the surface of rupture mn) is subject to two forces—(*a*) its own weight, and (*b*) the superficial pressure acting in the plane mn. The latter force is equal to $n_w A_s p_m$, in which A_s is the surface area of the section and p_m is the average water pressure in the pores intersected by the section. Since the pressure on the top of the concrete is zero, it follows that $n_w A_s p_m$ is also the uplift force transmitted by the percolating water, through pore action, to the porous material in the top of the specimen (integral of pore pressure differences).

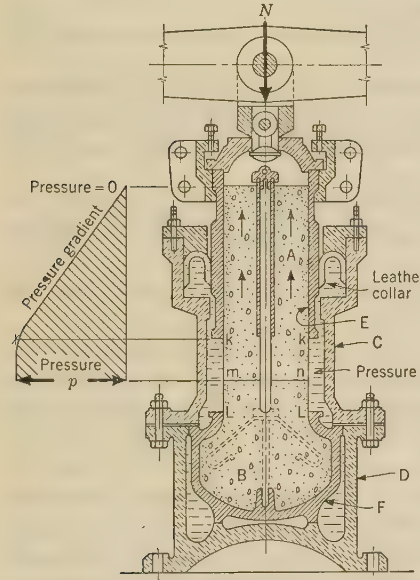

Fig. 20

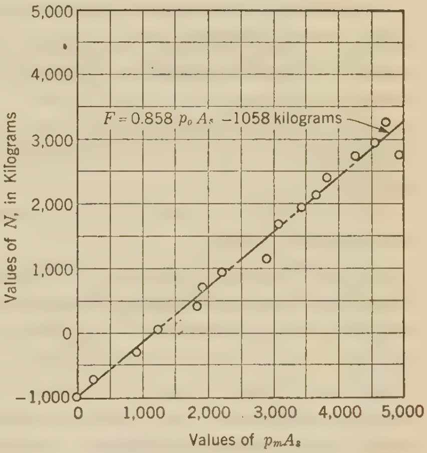

Fig. 21

In order that this force may be capable of causing failure, it must counterbalance (1) the resistance *z* of the concrete, and (2) the net value, *F*, of all the mechanically applied downward forces (inclusive of the weight) and the force *N* transmitted by the rocker. One thus obtains the simple formula:

$$n_w A_s p_m = F + z \qquad (22)$$

The same result can also be obtained from an elementary volume, such as abcd in Fig. 18(*b*).

For an elementary volume, in the upper part of the cylinder in Fig. 20, the force, $\gamma_s = i \gamma n_w$, is a vertical force directed upward, in which $i = dp/dh$. For the entire volume,

$$\int\int \frac{dp}{dh} n_w \, dh \, dA_s = n_w p_m A_s \qquad (23)$$

in which

$$p_m = \int \frac{p \, dA_s}{A_s} \qquad (24)$$

is the average pressure in section mn. Eq. 23 is the same formula as that derived by the deductive method. In this case the integral of the second force, $w \gamma (n_w - n)$, which is taken into account in calculating F, is a relatively very small value.

The author criticizes a set of experimental results obtained by the writer,[8] with the apparatus shown in Fig. 20, which disagree with his (the author's) initial conception and final conclusions.

The tests in question were conducted in such a manner that the value of the effective porosity, n_w, could be found experimentally.

The basic principle of these experiments is as follows: From Eq. 22

$$n_w = \frac{d(F + z)}{d(A_s\, p_m)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

If, therefore, a series of identical specimens (for which the average resistance, z, is constant) is tested under various values of F, and if these values are plotted against the corresponding, experimentally determined, values of $p_m\, A_s$, a diagram such as Fig. 21 is obtained.

The slope of the average line in this diagram is equal to n_w. From ninety-two experiments, n_w was found to be 0.915, rather than 1.00, as claimed by the author.

In the light of these findings, the author's challenge of this result does not seem to be justified.

For instance, Mr. Harza's reference to a wedging action in connection with these tests is not relevant, because such an action does not enter into the mechanical explanation of the experiment at all. In fact, there is no more wedging action in the writer's tests than there would be in sections xy of the beam and dam in Fig. 17. It was neither stated nor implied by the writer, and may indicate that the description of the elementary aspects of the tests should have been more detailed. to avoid misunderstanding.

Mr. Harza states, also, that the failure in section mn, Fig. 20, is due to tension and not to uplift—which begs the question. The fissure is indeed caused by tension, but the tension itself is the result and consequence of the uplift force transmitted through pore action to the concrete, in the block above the separation surface.

On the other hand, although the author states explicitly that unprotected concrete subjected to hydraulic pressure is noncompressible (under the heading, "Experimental Evidence"), he nevertheless attributes the measurable results obtained by the writer (clearly defined by the slope of the average line in Fig. 21) to Poisson's ratio. This does not seem consistent, since Poisson's effect is the result of elastic deformation, and, if such deformation is impossible, its effect cannot possibly exist.

In this connection attention is called to the fact that the writer himself has stated[8] that Poisson's effect (or more precisely, the three-dimensional elastic phenomena) may have caused a secondary effect in some of his tests, in the cases when an important difference of pressure existed between the periphery and the center of the cylinder. This contingency was thoroughly considered and investigated, and was proved to be nonexistent by a statistical examination of the records.

In addition to the author's views on the effective uplift area, his paper also contains (in the "Synopsis" and the "Conclusions") the philosophy embodied in the statement (under the heading, "Application to a Concrete Dam"):

> "Hydrostatic pressure is conventionally assumed to be exerted against the upstream or back of a dam. The reasoning presented herein has proved that it is exerted progressively throughout the route of seepage. If the hydraulic gradient is uniform, as in Fig. 9(*b*), then the application of the thrust is uniformly distributed along the route of seepage from back to face."

In the writer's opinion this is the only acceptable conclusion of the paper. It was discovered in 1913, and has since been the subject of heated controversies in German and French publications. In English, it was first stated by the writer.[8]

Although the author does not say so, the investigator is faced here with the same conditions as those in which water percolates through a granular foundation beneath a weir, which means that the methods of stress analysis applicable in one case are also valid for the other.

It follows that Fig. 9, which is intended to represent, in the paper, the trajectories of seepage water in dams, could be much improved by the application of the theory of potential—that is, in practice, by the method of flow-net diagrams. In fact flow-net diagrams for this particular case, embodying the theory in question, have been published by Professor Terzaghi and others.

There is also much interesting material in the paper concerning such characteristics as the size of the pores, water-cement percentages, and modern concrete procedures which, however, does not seem to be relevant to its main issues.

A. H. Davison,[32] M. ASCE.—For its emphasis on basic engineering principles in the discussion of hydrostatic uplift on concrete gravity dams, this paper constitutes a valuable addition to the literature on the subject. The determination of hydrostatic uplift at any critical plane of the structure under investigation involves two factors: (1) The distribution of uplift force, and (2) the magnitude of uplift force. The hydraulic gradient through the structure shows that hydrostatic uplift acts over the entire horizontal area of any plane at or below the tailwater surface (TWS). In the case of any plane above the tailwater surface elevation, hydrostatic uplift acts over part of the horizontal area of that plane. The magnitude of the uplift force applied to any plane can be computed by reference to the hydraulic gradient through the structure.

Fig. 22(*a*) shows a typical section of a concrete gravity dam on rock foundation with the hydraulic gradient through the structure indicated from headwater surface (HWS) to tailwater surface. The difference in elevation between HWS and the hydraulic gradient at any point is the friction head lost by water percolating through or under the dam to that point. The uplift pressure at any point on any plane through the dam is represented by the pressure head at that point, which is equal to the vertical distance from that point to the

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[32] Civ. Engr., Glens Falls, N. Y.

HWS elevation less the friction head. The computed uplift on the base of the dam is represented in Fig. 22(*c*).

Fig. 22(*a*) is considered more representative of practical construction than is Fig. 9. The hydraulic gradient through the structure is theoretically limited by the straight line from points E to D. Obviously, any horizontal plane above the TWS elevation is only partly exposed to hydrostatic uplift.

M. A. Selim, Assoc. M. ASCE, has expressed the Darcy law as

$$Q = K \frac{h\,A}{L} \qquad (26)$$

From continuity,

$$Q = A\,V \qquad (27)$$

and

$$L = K \frac{h}{V} \qquad (28)$$

in which Q is the discharge, in cubic feet per second; h is the head, in feet; A is the gross cross-sectional area, in square feet; K is a dimensional coefficient; V is the velocity, in feet per second; and L is the length of flow path, in feet.

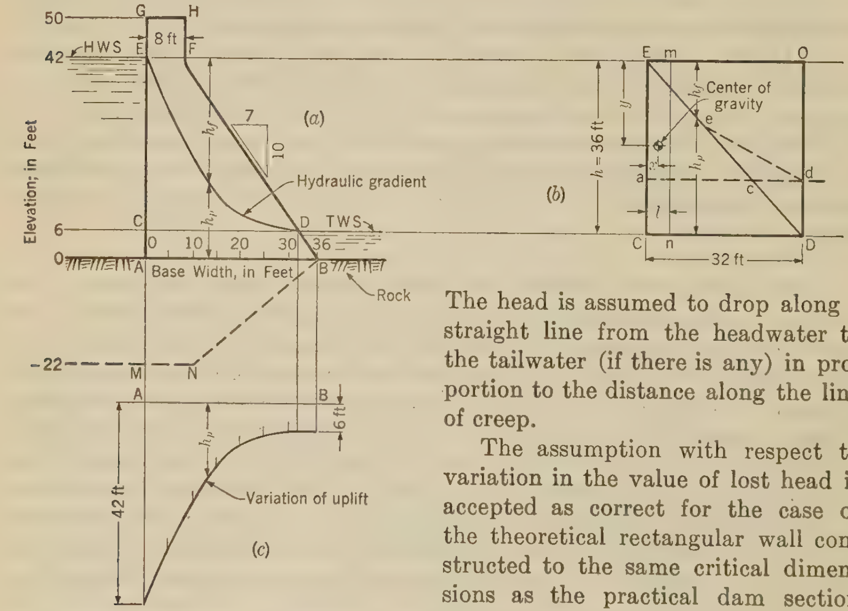
50
G
H
42
HWS
E
8 ft
F
7
10
(a)
h_f
h_p
Hydraulic gradient
6
C
D
TWS
0
10
20
30
36
0
A
Base Width, in Feet
B
Rock
Elevation, in Feet
-22
M
N
A
B
6 ft
h_p
Variation of uplift
42 ft
(c)
E
m
O
Center of gravity
e
y
h_f
h_p
h = 36 ft
(b)
a
d
c
l
C
n
D
32 ft

Fig. 22.—Typical Section of Concrete Gravity Dam on Rock Foundation

(*a*) Hydraulic Gradient
(*b*) Theoretical Rectangular Wall
(*c*) Variation of Uplift on Base

The head is assumed to drop along a straight line from the headwater to the tailwater (if there is any) in proportion to the distance along the line of creep.

The assumption with respect to variation in the value of lost head is accepted as correct for the case of the theoretical rectangular wall constructed to the same critical dimensions as the practical dam section under consideration. These critical dimensions are (1) total head lost, and (2) total width of section as measured at the TWS elevation. For other cross-sectional shapes than the theoretical rectangular wall the variation of the lost head (or friction head) factor cannot be expected to conform to the straight line, because of the notable differences in variation of basic measurable or computable factors. These factors are (1) the distribution of the weight of the structure across the width of the section, and (2) the variation in the position of the center of gravity of accumulated sectional area across the width of the section as denoted by coordinates measured from the source of hydrostatic pressure.

Natural foundation materials, materials of construction, and pore spaces in such materials, are compressed by superimposed weight. This compressibility factor affects the friction head lost by water percolating through the mass of the structure. The mass of the structure includes that part of the foundation which is reinforced by cutoffs or otherwise made part of the aboveground structure by construction methods. The compressibility of materials and pore spaces is uniform if the superimposed weight is distributed in a uniform manner, as in the case of the theoretical rectangular wall. Because of the variation in weight distribution in the case of the theoretical triangular dam, and in all practical dam sections, there is a greater proportionate loss of head through or under the upstream part of the dam section. The realistic hydraulic gradient through the structure drops from HWS elevation in a curve which is concave upward.

With respect to past, satisfactory performances of existing dams, the foregoing seems to indicate a better explanation than the author's conclusion No. 6 and is confirmed by tests for uplift on existing dams.

From a structural standpoint, the base of the dam is usually the critical plane to be investigated. The modern concrete dam has such homogeneity and integrity of construction that the designer feels justified in assuming that the resultant of forces can be allowed to intersect any plane of the concrete structure above the base at or about the downstream limit of the middle third of the width of that plane. However, the foundation under the dam may be of more doubtful character, and the designer may prefer to have the resultant of forces intersect the base of the dam (which bears on the foundation) at a point near the middle of the base.

The idea that the material of a properly constructed and maintained concrete dam necessarily becomes saturated in a manner similar to the material of an earth dam does not appeal to the writer. There are many examples of relatively thin concrete bulkhead walls, decks of hollow dams, and the like, which are watertight for all practical purposes. Even the existence of occasional pinhole leaks and seepage through such structures appears to have no necessary connection with the hydraulic gradient through the structure. Even though a concrete dam has been constructed or repaired with a facing, on the upstream side, of wrought iron plate to exclude percolation through the structure, it has a hydraulic gradient as evidenced by piezometer pipes to the foundation.

With respect to present practice in assuming uplift, as stated by the author in conclusion No. 1, as a design expedient an equivalent triangle or trapezoid is often substituted for the computed uplift diagram shown in Fig. 22. This expedient may perhaps be manipulated in a manner which would justify the author's statement. Common assumptions by authorities are (1) uplift of $\frac{2}{3}\,h$ at the heel decreasing to zero at the toe, and (2) uplift of $\frac{2}{3}\,h$ at the heel decreasing to $\frac{1}{3}\,h$ at the toe—h representing the net hydrostatic head.

Since determination of hydrostatic uplift is the principal issue, it appears that the most direct approach is to compute the hydraulic gradient and attendant friction head loss through the structure from consideration of the basic factors of (1) compressibility, (2) weight distribution, (3) location of the center of gravity for progressive strips across the width of the base, and (4) the Darcy law.

Fig. 22(*b*) shows the theoretical rectangular wall corresponding to the practical section with respect to the critical dimensions of total head loss for total width of section at the TWS elevation. The straight line ED is the hydraulic grade line for the theoretical section, and area ECD is the total uplift on plane CD. Should the TWS be considered to rise from elevation CD to elevation ad, area ecd is made equal to area cdD, and the hydraulic grade line becomes the broken line Eed. For a 1-ft length of homogeneous concrete gravity dam, l is the width and A is the area of progressive strips of the section, with x and y the corresponding coordinates of the center of gravity.

Friction head loss through the structure may be expressed by

$$h_f = K\,[w\,A\,(y + x)]^n \qquad (29)$$

in which h_f is the friction head, in feet; w is the weight of structure, in pounds per cubic foot; n is an exponent which varies with circumstances; and K is a dimensional coefficient which varies with circumstances. For a homogeneous structure or an equivalent one, like the concrete dam under discussion, the formula may become

$$h_f = K_1\,[A\,(y + x)]^n \qquad (30)$$

Modification of Eq. 30 may be required to apply it to a specific problem.

The writer's procedure includes tabulating the different elements for various values of l. The first step involves tabulation of the elements for Fig. 22(*b*), and satisfying the criterion that h_f must vary in a straight line from HWS to TWS. This means that computed values of the compound factor $[A\,(y + x)]^n$, or some modification thereof across the section must plot in a straight line. To do so, it is determined that the factor should be $\left[\dfrac{A\,(y + x)}{1{,}000} - 1\right]^{0.7}$ for the sections shown in Fig. 22.

The second step is to determine the value of coefficient K_1 in the formula. This value of K_1 for the specific problem is established by considering that part of Fig. 22(*a*) which is bounded by the limiting elevations HWS and TWS that determine the total lost head with which the specific problem is concerned. The formula is solved for K_1 by substituting the known value of 36 for h_f when $l = 32$. The value of K_1 is determined as 4.14, and the formula applicable to Fig. 22(*a*) becomes

$$h_f = 4.14\left[\frac{A\,(y + x)}{1{,}000} - 1\right]^{0.7} \qquad (31)$$

The remaining steps consist of applying Eq. 31 to the computation of the hydraulic gradient applicable to the total section area of Fig. 22(*a*), or any modification thereof, as long as the original ratio of total lost head to section width is maintained. For example, the total section area may include a grouted foundation area AMNB, which is an equivalent concrete area representing the effectiveness of grouted rock foundation through holes 20 ft deep under the heel. The equivalent concrete area is made 22 ft deep at the heel as shown in Fig. 22(*a*) because the specific gravity of rock foundation has been assumed 10% greater than concrete; for this total section the computed hydraulic gradient would drop from the HWS elevation somewhat more sharply than the hydraulic gradient curve drawn in Fig. 22(*a*) for the section above plane AB, before curving around to point D at the downstream face.

According to the writer's experience, computed hydraulic gradient curves are in practical agreement with results of current sound design practice and with reports by authorities of investigations of hydrostatic pressure occurring after construction in various dams of similar type in the United States and Europe. For gravity dams of earth, concrete, or masonry, 1 lin ft of dam is the unit length considered. For hollow concrete dams or cellular type retaining wall construction, the unit length is the average spacing of buttresses or counterforts. (An earth dam is not considered of homogeneous construction. The material in the middle of the dam is more compact and has a greater unit weight than the material which forms the outer shells, and the factor of saturated weights needs consideration.)

Apparently there is little need to fear for the safety of existing structures that have been soundly designed, constructed, and maintained.

R. E. BALLESTER,[33] ESQ.—With a precision not previously found in the technical literature the author undertakes to evaluate the uplift that may occur in the cross section of a dam—particularly in a gravity section. Many uncertainties confront the designer in his effort to analyze the theoretically ideal dimensions of a gravity dam—so many, in fact, that a prominent Spanish mathematician and engineer, Alfredo Peña Boeuf, has declared (the writer's translation from the Spanish): "Gravity dams are not subject to computation."[34]

The major uncertainty is the effective value of uplift—its magnitude and the area over which it acts. General practice in Europe is to assume that an uplift force acts over the entire horizontal area. The official 1925 regulations of the Italian Government[35] applying to the design of dams reflect this general practice clearly (the writer's free translation from the Italian adapting Mr. Harza's notation):

> "In the design computations, uplift shall be assumed to vary linearly from
>
> $$P_u = h\,\omega\,c \qquad (32)$$
>
> at the upstream face of a given horizontal section, to zero at the downstream face of that section."

In Eq. 32 P_u is the total uplift; h is the hydrostatic head acting on the given section; ω is the unit weight of water; and c is a reduction coefficient which is constant for the entire height of the dam but which varies, between different dam sections, from $\frac{1}{3}$ to 1. Inasmuch as uplift area does not enter the definition of *c*, it is implicit that 100% area is assumed except in so far as the intensity of uplift may differ between 100% and 33% for different conditions.

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[33] Prof. of Applied Hydraulics, Univ. of Buenos Aires, Buenos Aires, Argentina.

[34] "Grandes Estructuras y Presas," by Alfredo Peña Boeuf, Revista de Obras Publicas, Editor, Madrid, 1934, p. 65.

[35] "Annali delle Utilizzazioni delle Acque," Vol. III, No. 3–4, Rome, 1926, Article 21, p. 94.

The author discusses the characteristics of a hypothetical uplift force acting on a selected part of a horizontal cross section and, in progressive steps, undertakes to show that the entire area is affected since it is impossible to have a condition in which the concrete and the material in the foundation bed and in the bedrock are each entirely impermeable.

Referring to Fig 5(*b*), consider a cube with a specific weight less than water, or an empty crystal glass with a perfectly flat, smooth bottom. This cube or crystal glass will remain on the bottom as water is poured into the receptacle, if the fit between bottom surfaces of contact is so perfect as to exclude water seepage. This ideal condition is unattainable with mortar, concrete, and rock faces. Furthermore, the cube or crystal glass in Fig. 5(*b*) may be porous, in which case the water will seep through the pores and thus create a pressure on the bottom plane of contact. In practice, a dam section will have a buoyant force at the plane of the foundation and at every other plane or seepage path through the dam. This is a statement of the author's thesis, supported by text demonstrations.

According to Eq. 32, the magnitude of uplift is the product of hydrostatic pressure and the cross-sectional area, modified (for different dams) by a predetermined reduction coefficient. To which of the two quantities—pressure or area, or to the product of the two—should the reduction coefficient be applied? The writer believes that the logical factor to be modified is the pressure, which can actually be reduced by increasing friction values or by changing paths of percolation. Nothing ever changes the cross section of a dam as long as it is in service.

The conclusions in this paper establish extremely conservative rules for the design of large dams. Conclusion No. 6 offers five explanations to account for the satisfactory performance of dams built under less conservative assumptions. One of these (conclusion No. 6(*c*)) is: "The time required to saturate a large dam, even with constantly full reservoir." In support of this conclusion the writer can cite an inspection, in 1942, of the San Roque Dam in Córdoba, Argentina, which was built in 1888. The structure is no longer in service.

A tunnel was bored through the dam at an elevation about 46 ft below the spillway crest, which was the normal level of the reservoir. The center-line length of the tunnel (41 ft) represented the width of the dam section at that elevation. The downstream face of the dam was perfectly dry, showing no signs of percolation; but at distances of from 2 ft to 3 ft in from the downstream face the mortar was humid and, when first revealed, began to sweat small drops of water. The general appearance of the exterior gave the impression that the dam was entirely free from seepage and consequent uplift, completely belying the true condition of the interior.

It will behoove designers to give attentive consideration to the author's comment, beginning with Fig. 11, concerning the round-head buttress type of dam. It is an extraordinary fact that, although this design concept originated in the United States, it has never been built there. There will be stresses in the thin sections of concrete as the result of pore pressure, as well as danger from disintegration of concrete resulting from seepage flow. In the light of the author's comment concerning this rare type of dam, engineers who have had practical experience with it should consider it a duty to record their findings.

JOHN S. MCNOWN,[36] Assoc. M. ASCE.—Although the method for evaluating the magnitude of uplift forces acting on masonry dams is not yet completely developed, the author has determined one of the factors by demonstrating convincingly that the total base area is affected by uplift. Now attention should be focused on determining the pressure magnitude and on re-examining present design criteria in the light of these findings.

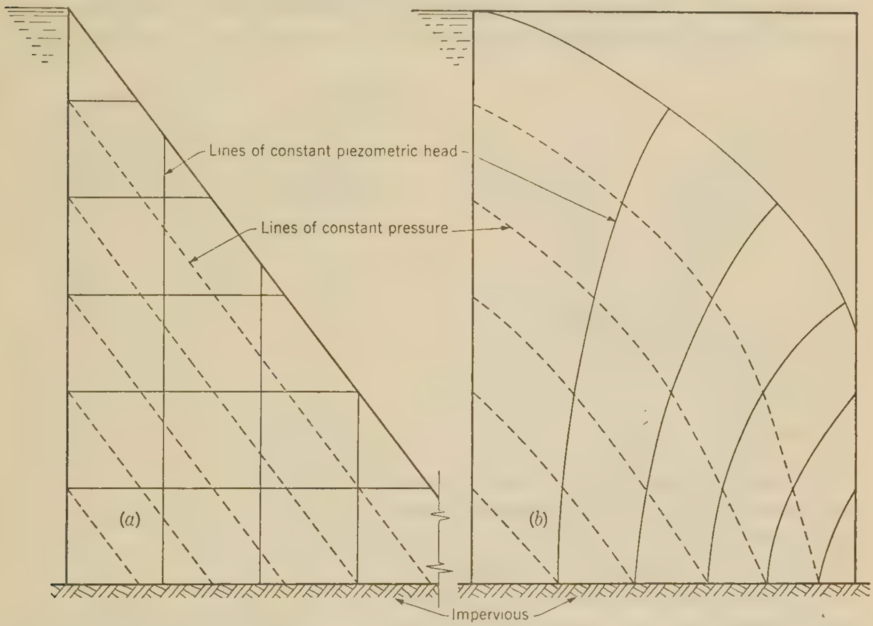
Lines of constant piezometric head
Lines of constant pressure
(a)
(b)
Impervious

FIG. 23.—PATTERN OF SEEPAGE FOR:
(a) Triangular Dam
(b) Rectangular Dam

The concept of a concrete dam as a porous structure introduces two factors which heretofore have not received consideration: (1) Uplift, as determined by the internal pressures, may act on the full area of any horizontal section, and (2) the hydrostatic force is distributed along the paths of seepage. Although both of the factors have been considered by the author, several points must be clarified. The impression is given (under the heading, "Application to Concrete Dam") that some uncertainty exists in determining the pattern of seepage through porous media. On the contrary, for any geometrical configuration a unique solution exists. Although exact analyses are often impossible, approximate solutions can be obtained graphically or by electrical analogy. In addition, direct methods of mathematical approximation have been developed in the past few years[37,38] which permit the solving of any problem of two-dimensional seepage, including determination of the free streamline.

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[36] Associate Prof., Dept. of Mechanics and Hydraulics, and Research Engr., Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

For the triangular dam shown in Figs. 9 and 23(*a*), the result is very simple, the streamlines and lines of constant piezometric head being, respectively, the horizontal and vertical lines shown in Fig. 23(*a*). Lines of constant pressure are shown as dotted lines, and the uplift pressure distribution has the simple triangular form indicated by the author. However, this solution is unique, and Fig. 9(*c*) and the related discussion are, therefore, irrelevant.

Usually the flow pattern is far more complex than that shown in Fig. 23(*a*) and the author's generalization to the rectangular cross section is somewhat oversimplified. The solution obtained by mathematical approximation, shown in Fig. 23(*b*), differs considerably from that indicated in Fig. 9(*d*). The distributions and centers of pressure are compared in Fig. 24; the moment of the uplift pressure about the toe is 10% greater for the correct distribution than for the triangular approximation. Fortunately, much of the increase in overturning moment is characteristic of the rectangular shape; for orthodox sections the increase would be relatively small.

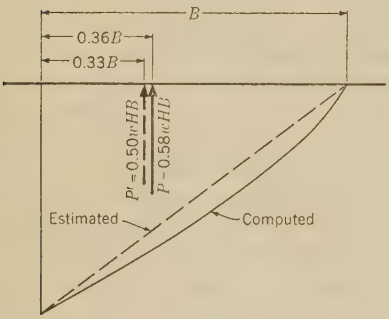

FIG. 24.—COMPARISON OF UPLIFT FORCES

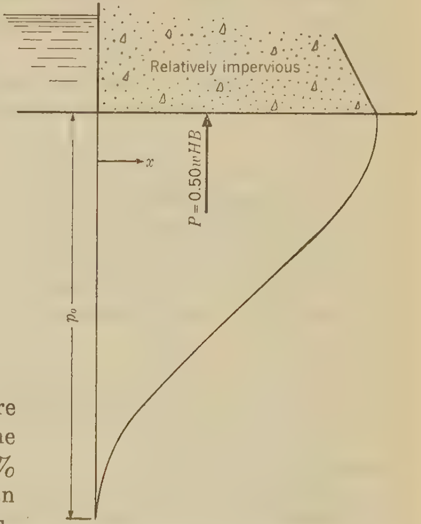

FIG. 25.—UPLIFT ON AN IMPERVIOUS DAM

The simplifying assumption of an impervious foundation material is at one extreme. Since the foundation porosity may equal or even greatly exceed that of the dam, the other extreme is also of interest. A direct solution is available for an impervious dam,[39] the result, of course, being independent of the shape of the dam. The pressure variation is defined by

$$\frac{p}{p_0} = \frac{1}{\pi} \cos^{-1}\left(\frac{2\,x}{B} - 1\right) \qquad (33)$$

in which p is the unit uplift pressure at any point along the base; p_0 is the

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[37] "Relaxation Methods in Engineering Science," by R. V. Southwell, Clarendon Press, Oxford, 1946, p. 191.

[38] "Photoelasticity," by M. M. Frocht, John Wiley & Sons, Inc., New York, N. Y., 1948, Chapter 8.

[39] "Higher Mathematics for Engineers and Physicists," by I. S. Sokolnikoff and E. S. Sokolnikoff, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, pp. 452–454.

unit uplift pressure at the upstream face of the dam; and x is the distance from the upstream face. The average pressure is $\frac{p_0}{2}$ and the center of pressure is three eighths of the distance from the heel of the dam to the toe as shown in Fig. 25; the moment about the toe is $0.312\, w\, B\, H$. The relative porosities will fix the result for given conditions somewhere between the two extremes.

The second effect of porosity—the distribution of the hydrostatic force along lines of seepage—is unimportant in the determination of external forces acting on a structure. However, the effect on internal forces may become important, particularly where the seepage lines are curved horizontally, as shown in Figs. 10 and 11. It is an easy matter to prove that incremental forces act everywhere in the direction of flow by considering the forces on the differential element shown in Fig. 26. If horizontal flow is assumed,

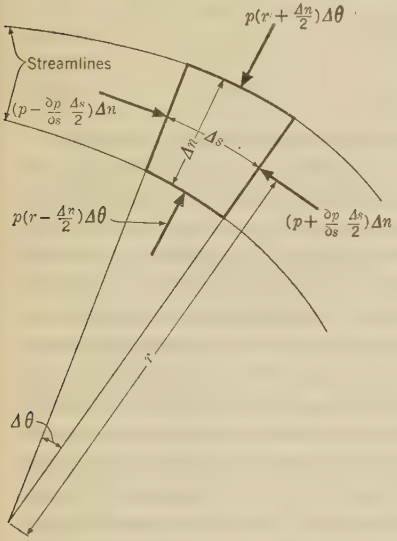
$p(r+\frac{\Delta n}{2})\Delta\theta$
Streamlines
$(p-\frac{\partial p}{\partial s}\frac{\Delta s}{2})\Delta n$
Δn
Δs
$p(r-\frac{\Delta n}{2})\Delta\theta$
$(p+\frac{\partial p}{\partial s}\frac{\Delta s}{2})\Delta n$
r
$\Delta\theta$

FIG. 26.—FORCES ON A DIFFERENTIAL ELEMENT

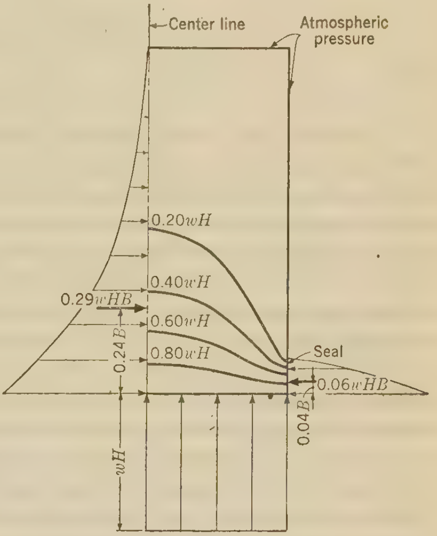
Center line
Atmospheric pressure
0.20wH
0.40wH
0.29wHB
0.60wH
0.24B
0.80wH
Seal
0.06wHB
0.04B
wH

FIG. 27.—PATTERN OF HORIZONTAL SEEPAGE IN A MONOLITH

$$dF_s = \left[\left(p - \frac{\partial p}{\partial s}\frac{\Delta s}{2}\right) - \left(p + \frac{\partial p}{\partial s}\frac{\Delta s}{2}\right)\right] \Delta n \cos\frac{\Delta\theta}{2} \ldots\ldots (34a)$$

nd

$$dF_n = \left[\left(r - \frac{\Delta n}{2}\right) - \left(r + \frac{\Delta n}{2}\right)\right] p\, \Delta\theta + \left[\left(p - \frac{\partial p}{\partial s}\frac{\Delta s}{2}\right) + \left(p + \frac{\partial p}{\partial s}\frac{\Delta s}{2}\right)\right] \Delta n \sin\frac{\Delta\theta}{2} \ldots (34b)$$

in which F_s and F_n are the forces along and at right angles to the streamline, respectively.

Inasmuch as the element under consideration is small, the approximations—

$$\cos \frac{\Delta\theta}{2} \approx 1 \quad \text{(35a)}$$

$$\sin \frac{\Delta\theta}{2} = \frac{\Delta\theta}{2} \quad \text{(35b)}$$

and

$$\Delta\theta = \frac{\Delta s}{r} \quad \text{(35c)}$$

—are permissible. After substitution and simplification,

$$dF_s = -\frac{\partial p}{\partial s} \Delta s \, \Delta n \quad \text{(36a)}$$

and

$$dF_n = 0 \quad \text{(36b)}$$

The resultant force at any point is, indeed, along the streamline. In a monolith of a gravity dam, drainage from the joints along the sides (see Fig. 10) could produce appreciable tensions in longitudinal sections. For an idealized case of two-dimensional flow in a horizontal section through such a structure (that is, neglecting the downward component of flow), the pattern of flow is as shown in Fig. 27. The total tension at the axis of symmetry is the difference in the total pressures for the conditions assumed. In this case, which is somewhat extreme, a force of 0.23 $w\ B\ H$ urges the two halves of the structure apart.

In clarifying one phase of uplift computation, the author has made a major technical contribution which may lead to modification of design criteria for masonry dams. However, uncertainties still exist, and no immediate changes are indicated. Evaluation of the porosity of concrete should permit determination of the time required for establishment of a steady seepage pattern. Field measurements may be required, because the effect of surface tension cannot be ignored; the dislodgment of air bubbles in small passageways can necessitate large forces. The solution of these and related problems should come much sooner as a result of the reorientation given to further investigation.

V. T. BOUGHTON,[40] M. ASCE.—The skilful way in which Mr. Harza leads his reader to his basic assumption serves to conceal its fundamental weakness. Neither observed conditions in well-built dams nor reasoning based on those observations gives support to that basic assumption or the conclusions drawn therefrom.

Mr. Harza's basic assumption (under the heading, "Application to a Concrete Dam") is:

> "* * * that concrete [in a dam with a full reservoir] is internally buoyant as truly as loose granular material, and that it obeys all the hydraulic and hydrostatic laws of loose granular material."

Stripped of all nonessential details and mathematics, what Mr. Harza wants the reader to believe, in order to give substance to his conclusion, is that the cement paste in concrete is so porous that water from the reservoir behind the dam moves through its pores just as it would through loose granular material, and that the pores are so numerous and interconnected as to place every particle of cement and sand and each piece of aggregate in the concrete in a state of buoyancy similar to that produced by free submergence in open water. This buoyancy, Mr. Harza states (also under the heading, "Application to a Concrete Dam"), is fully effective "* * * from back to face of dam, since each elementary vertical cylinder between pores would be buoyant."

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[40] Associate Editor, *Engineering News-Record*, New York, N. Y.

If this statement were true, then water would exude freely from the walls of inspection galleries in large dams and would run down the face of such dams when atmospheric humidity was at or close to 100%.

Mr. Harza would have the reader believe further that the pore pressure, which he considers as acting throughout the entire mass of the dam:

> "* * * is as applicable at any one horizontal section as another, instead of being confined to pour joints, base contact with rock, or other horizontal planes of greater permeability or weakness."

Also, because his basic assumption places every elemental unit of the dam below water level in the reservoir in a state of bouyancy, he holds that the mass of the dam that is effective in resisting overturning is reduced by the amount of the buoyancy of almost the entire section of the dam below water level.

To show that the condition of buoyancy envisioned by Mr. Harza cannot be "fully effective * * * from back to face of dam," even though it might be true of concrete in the upstream face of the dam, it is only necessary to point to the fact that it cannot possibly be true of the concrete exposed in the downstream face of the dam. There "each elementary vertical cylinder" of concrete cannot be in a state of submergence because one side is in open air. Any moisture that is in that concrete adds to its weight instead of detracting from it by buoyancy action. Therefore, even though Mr. Harza may be correct as to his assumption with respect to the elementary units of concrete in the upstream face of the dam, somewhere between the two faces there must be a reversal in the condition of the elementary units of concrete from a condition of submergence and buoyancy to one of nonsubmergence and no buoyancy.

As to where this reversal takes place, Mr. Harza's own basic assumption that the cement paste is so porous as to permit relatively free movement of water into the concrete would seem to require the further assumption that water can also move out of the concrete toward the downstream face with equal freedom. Hence, the more logical place to locate this reversal point under his theory of porosity would be somewhere near the middle of the dam, not at the downstream face as Mr. Harza does when he uses his internal buoyancy theory to demonstrate that uplift must be considered as acting over 100% of the horizontal area of the dam.

However, conditions in the inspection galleries of large dams, and the results of tests on the permeability of relatively thin sections of concrete of a density similar to that used in the face of modern dams, indicate quite clearly that the condition of buoyancy envisioned by Mr. Harza does not actually exist in the area between the midpoint and the upstream face.

In this connection, Mr. Harza's own statement in his discussion of uplift, under the heading, "Application to a Concrete Dam," seems to deny the logic of his reasoning that leads him to conclude that pore pressure is effective throughout the mass of the dam. He states:

> "Vertical joints (such as shrinkage cracks, construction joints, and monolith joints) often, if not usually, serve as drains to reduce internal pressure * * * whereas full uplift requires that all seepage must reach the downstream face before relief of pressure. Therefore, theoretical pressure probably seldom exists, except near the upstream surface as measurements verify and the theoretical is merely a mathematical limit of possibility. Such drainage reduces the magnitude of uplift pressures, but nothing can reduce the effective uplift area below 100% wherever pores are filled with water. Herein lies the fallacy of the usual assumption of one-half or two-thirds area for uplift, which it is the principal purpose of this paper to correct."

Mr. Harza states that measurements verify the fact that pressure seldom exists "except near the upstream face"; and he concludes further that it is effective, even though reduced in magnitude, over 100% of the horizontal cross-sectional area at any point in the dam provided that "the pores are filled with water." Nevertheless, a logical application of his basic assumption that all the hydraulic laws which apply to granular material must be applied to concrete makes it impossible for all the pores over 100% of a horizontal cross section of the dam to be filled with water.

If the concrete is so porous that it "obeys all the hydraulic and hydrostatic laws of loose granular material," then the water that comes in through the upstream face ultimately would drain out through the downstream face; and, since concrete dams have a narrow section in relation to their height (as compared to the earth dam shown in section in Fig. 8(*a*)), the line of saturation would be quite steep, not flat, as in an earth dam of uniform composition such as is shown in Fig. 8(*a*). Obviously, to maintain such a line of saturation would require that the concrete in the dam be much more porous than any concrete ever put in a well-built dam.

As it appears certain that the pores in any horizontal section through a dam of concrete, assumed to be so porous that the hydraulic laws governing loose granular material apply to it, cannot be filled with water under hydrostatic pressure from the reservoir behind the dam, the conclusion is inescapable that uplift could not act over 100% of any cross-sectional area.

Consequently, it would appear that Mr. Harza has not demonstrated that the normal assumption of uplift acting over only one half, or two thirds of the area of the base of a dam built of good quality concrete is unsafe, or illogical.

Ross M. Riegel,[41] M. ASCE.—The author of this interesting paper choses to rest his case upon deductive reasoning and seems to distrust experimental evidence, even when it appears to support his conclusions to an important degree. To the writer this presentation would be more convincing if the author were to welcome all experimental work, the conclusions of which would tend to support his argument. This would be especially true if the conclusions were reached by different methods of approach. The writer has reference specifically to the work of Karl Terzaghi,[7] Hon. M. ASCE, and of S. Leliavsky Bey,[8] M. ASCE.

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[41] Head Civ. Engr., Design Dept., TVA, Knoxville, Tenn.

There has been a wide divergence of thought among engineers on this point of effective area. As recently as 1945, a book on engineering for dams,[42] after summarizing the existing evidence on the subject, states:

> "The only conclusion that can be drawn from a study of available discussions on uplift is that the value of the area factor 'C' for masonry and rock is uncertain."

It is true that the book subsequently advocates the use of 100% area, but the writers of the book appear to be somewhat doubtful of the Terzaghi experiments, although the latter were published in 1934 and 1936.

In June, 1948, Douglas McHenry, M. ASCE, presented evidence pointing to a boundary porosity of 1.00, which conforms to Professor Terzaghi's conclusions.[43] In this case the investigation was made with the new triaxial testing machine of the Bureau of Reclamation, using high pressures.

To the writer the confirmation offered by the McHenry tests is very convincing so far as laboratory experimentation is concerned, and the author should welcome the support offered to his conclusion, even if the method of approach should be entirely different from his own.

In one other respect the argument of the author might perhaps be strengthened. In so far as the writer knows there has been no failure of a dam which has been traceable to pore pressure in the concrete. (There have been failures in foundations as the result of inadequate resistance to sliding, in which cases uplift was probably an important factor.) This suggests that concrete subject to internal pore pressure still has important properties of strength and is capable of resisting stresses in tension, compression, and shear. The author's analysis and illustrations create the impression that his ideal colums between pores are cylinders. If he were to state clearly that his ideal columns are prismatic in form and are everywhere (except where pores interrupt) in intimate contact (or continuous) with adjacent prisms, then he could also state that stresses in shear, tension, and compression may be transmitted through these contacts without disturbing the structural integrity of the concrete. The conception of continuity appears to be essential. This idea may be implied in the paper but the writer has failed to find a clear expression of it and can see no reason why it should not be stated. In the writer's view, the reasoning of the author would not be impaired, but would actually be strengthened.

This paper, as interpreted herein, together with the experimental evidence now available, offers considerable support to a design assumption of an effective area of 100%. In so far as this assumption is in excess of that used on so many structures which have functioned satisfactorily for so many years, it is well to recall the effectiveness of drainage (designed or accidental) upon intensities of pore pressure, and to recall also the effects of load transfer from loss of the heat of hydration and other effects[44] in modifying stress conditions established by conventional calculations.

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[42] "Engineering for Dams," Vol. II, by Julian Hinds, William P. Creager, and Joel D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1945, p. 267.

[43] "The Effect of Uplift Pressure Upon the Shearing Strength of Concrete," by Douglas McHenry, *Transactions*, 3d International Cong. on Large Dams, Stockholm, Sweden, 1948.

Douglas McHenry,[45] M. ASCE.—Questions regarding the effect of pore pressure on the strength and stability of concrete structures seem to present some elusive aspects which make answers by deductive reasoning particularly difficult. The writer has seen a lengthy interchange of correspondence that took place about 1920 to 1930 between two of the most eminent consulting engineers in the United States where both visualized concrete as a series of four-legged tables piled one above the other to form a tower. The stresses in the legs were computed with care, and then a man was placed on the top of each table and instructed to push upward with a certain force against the table above him. The effect on the strength and the stability of the tower was then discussed. These deliberations, however, led to two wholly conflicting views regarding the effect of pore pressure in hydraulic structures. Other discussions which have done little to clarify the problem have concentrated upon the forces resulting from frictional resistance and upon effects of differences in permeability of cement paste and aggregate.

The writer finds himself in the position of agreeing, for all practical purposes, with the author's conclusion regarding the area acted on by uplift pressure, but disagreeing with the reasoning behind that conclusion and disagreeing with his objections to the experimental method of determining the area. The writer believes that the only method of determining this area is by experiment.

The sense of the author's basic thought may be expressed briefly as follows: The entire solid volume of concrete can be divided into columns of irregular section and length, each terminating in a pore at the top and at the bottom. These pores will be under hydrostatic head differing by their differences in elevation, so that each column will be subjected to buoyancy. Because each column is buoyant, the entire internal structure is buoyant.

This contention appears to be adequately proved by the argument presented in the paper. However, "internal buoyancy" is a new concept in engineering, and the author has apparently assumed rather than investigated its effect on strength and stability. In this discussion, an attempt will be made to carry the reasoning a little further by the presentation of very elementary concepts, with no further apology than that of the author, who states that an elementary treatment seems to be necessary to clarify the subject.

Let one of the author's elementary columns be simulated by an enlarged model. Partly for convenience and partly to follow conventional assumptions, assure that the material has no tensile strength by building the column of blocks or laminae placed one above the other. The laminae fit so well together that if the column is immersed in water the joints will remain dry; but these joints, although transmitting no tension, have a known coefficient of sliding friction. It is desired, first, to determine the weight of the column in air. This may be done by placing the column in a scalepan and weighing it;

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[44] "Structural Features of Hydraulic Structures," by Ross M. Riegel, in "Design Developments—Structures of the Tennessee Valley Authority: A Symposium," *Transactions*, ASCE, Vol. 111, 1946, p. 1159.

[45] Head, Structural Research Section, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

or the force required to start sliding at any of the joints may be measured. Both methods are tried, and it is found that the unit weight is, say, for convenience, 62.5 lb per cu ft. Now the column is immersed in water and its immersed weight determined by the two methods. (To assure that no phenomena except buoyancy are involved, it may be preferable to immerse the column so that its top surface is flush with the water surface. Deeper immersion will not alter the argument, but will require certain corrections for the direct effects of the water pressure.) The first method (in the scalepan) gives a weight of zero; the sliding method gives 62.5 lb per cu ft. Continuing the experiment, a measurement is made of the force required to separate, by vertical pulls, the two laminae at, say, the midheight of the column. It is found that this requires the same force for the immersed column as for the column in air, although the lifting force for the immersed column reduces to zero as soon as separation occurs. Then the column is reunited, its lower half is fixed in a vertical position, and the force required to start an overturning action of the upper half is measured. The force is the same regardless of whether the column is in air, with no buoyancy, or whether it is in water with full buoyancy. Of course, for the immersed column the overturning force reduces to zero (practically) as soon as the overturning action has started. Thus, although buoyancy has reduced the scale weight of the column to zero, in all respects which have to do with its strength and stability it behaves in accordance with its true weight of 62.5 lb per cu ft.

If one of the joints in the column is so imperfect that water enters it and acts over practically its entire area it will, of course, be found that sliding and separation at this joint require no force, for they will depend upon the buoyant weight of the material above. This imperfect joint may be looked upon as simulating a pore in the concrete.

Thus, there is a seeming paradox: Concrete dams may be fully buoyant above the rough surface joining the lowest pores, but on any section (plane or irregular), this buoyancy reduces the strength and stability only where the section passes through a pore. Wherever the section passes through solid material, buoyancy may be neglected. Thus, to cause failure at a pore, it is necessary to overcome only the buoyant weight of the column of material above the pore; for failure between the pores, the strength of the material plus the full air weight of the material above must be overcome—not, as the author contends, the buoyant weight of the material. Actually, of course, there is no paradox; the appearance of one arises only if there is read into the expression "internal buoyancy," a meaning which is not there.

If the foregoing reasoning is correct, it may be concluded that uplift pressure (or pore pressure, or buoyancy) should be considered as acting only over that part of a surface which is in direct contact with the pressure fluid. It may be desirable to assume that the entire surface consists of pores, but this assumption is not in the sphere of deductive reasoning, for that question can be settled only by experiment.

Three experimental methods have been used and only one of them has, by any stretch of the imagination, anything to do with internal buoyancy. The most reliable data from the three methods are in reasonable agreement in indicating that pore pressure should be assumed, for practical purposes, to act over 100% of the internal area—in other words, that the boundary porosity is practically unity.

One of the methods employed by K. Terzaghi,[7] Hon. M. ASCE, consists of measuring the relative compressibilities of concrete cylinders subjected to a three-dimensional fluid pressure with and without encasing jackets—that is, with zero pore pressure and with a pore pressure equal to the applied fluid pressure. These tests yielded a value for the boundary porosity of practically 100%. Although the author apparently believes that experiments are unnecessary, and may even be misleading, he puts considerable faith in Professor Terzaghi's tests, and cites them as "complete and conclusive proof" of his own reasoning. However, inasmuch as pore pressures were equal in all parts of these specimens, and therefore buoyancy was not involved, there seems to be no connection between Professor Terzaghi's experiment and the author's reasoning.

The tests[8] by Serge Leliavsky Bey, M. ASCE, in which measurements were made of various combinations of internal pressure and axial load which produced tensile failure of concrete cylinders, lead to a value of 91%. The author's contention that Mr. Leliavsky misunderstood his own experiment seems unjustified. There is no indication in Mr. Leliavsky's paper that he attributed the failures to the "wedging action" of which the author writes. Inasmuch as a pressure gradient existed through the specimen in these tests, in contrast to the uniform pressure in Professor Terzaghi's tests, something akin to "internal buoyancy" may be involved.

The third method, which has been used in the research laboratories of the Bureau of Reclamation in Denver, Colo., involves determining the effect of pore pressure on the shearing strength of concrete. (A complete record of the tests and the theory of the method is contained elsewhere.[43]) Almost any consistent set of assumptions regarding the nature of the compressive failure of concrete might have been adopted, and would probably have led to about the same numerical result, but it has been assumed that the Mohr-Coulomb theory defines the condition under which compressive failure occurs.

The Coulomb equation is

$$Y = C + X \tan \phi \qquad (37)$$

in which Y is the unit shearing resistance on any given plane; X is the normal stress acting on the plane; C is usually called the unit cohesive strength; and ϕ is the angle of internal friction. Physically, Eq. 37 expresses the hypothesis that the shearing resistance along a plane is equal to a constant plus a linear function of the stress acting normal to the plane. The factors C and ϕ, which are functions of the material, can be evaluated best by triaxial compression tests in which cylinders of concrete encased in rubber jackets are loaded to failure under a circumferential pressure S_3 and an axial pressure S_1.

The fractional part of the area of the failure surface on which pore pressure is acting (called by Professor Terzaghi the boundary porosity n_b) may then be determined by testing additional specimens of the same material with pressure of known intensity acting in the pores. In the simplest testing procedure, the intensity of the pore pressure is made equal to S_3 simply by applying S_3 as a fluid pressure and permitting the fluid to penetrate into the specimen. The pore pressure, being equal in all directions, will not alter the shearing force, but it will reduce the normal stress, X, by the factor $S_3\, n_v$, so that Eq. 37 becomes

$$Y = C + (X - S_3\, n_b) \tan \phi \dots\dots\dots\dots\dots\dots (38)$$

Inasmuch as C and ϕ may be evaluated by Eq. 37, and X and Y may then be determined from the measured quantities S_1 and S_3, Eq. 38 may be solved for n_b.

In the Bureau of Reclamation laboratories, 6-in. by 12-in. concrete cylinders were tested in a triaxial testing machine. The specimens were oven-dried before testing, and nitrogen gas was used as the pressure fluid to assure the rapid development of uniform pore pressure. In all, 337 specimens were tested, representing fifteen different concrete mixes or test ages, with 56 specimens constituting the largest single group. Considerable dispersion was found in the test results, but the method is one in which high precision is very difficult to attain. The linear relationships assumed in the failure criterion appear to be valid within the limits of experimental error for the range of stresses covered by the tests (S_3 varied from 0 lb per sq in. to 1,800 lb per sq in.; S_1, from 3,270 lb per sq in. to 18,379 lb per sq in.). Values of n_b ranged from 0.78 to 1.18, with an average value of 1.02 with a standard deviation of ± 0.019, thus indicating that the pressure was effective over practically the entire area of the surface of failure. From the definition of boundary porosity, a value greater than unity is, of course, untenable; but in this method of determining n_b it is to be expected that the experimental values will range about equally above and below the true value. The group of 56 specimens gave a value of n_b equal to 1.07 with a standard deviation of ± 0.063.

Frederick L. Hotes,[46] Jun. ASCE.—Mr. Harza has attempted to establish a revised concept of pore pressures in hydraulic structures, particularly in concrete dams. Many of his comments are quite timely, even though perhaps somewhat controversial, but the writer wishes to discuss only that part of the paper dealing with the area of application of uplift forces to a horizontal section of a dam.

The author maintains (under the heading, "Experimental Evidence") that the area subject to uplift pressure in a submerged dam is "* * * exactly 100% of the area, and no less * * *." If only the author had used the words "substantially 100%" and omitted the "and no less" the writer would heartily agree for all practical purposes with his conclusion regarding this point. However, as the statement stands it can and should be challenged, and the writer submits the following as proof that the area subjected to uplift must be somewhat less than 100%.

Without debating the question as to whether or not all the pores in a concrete dam are connected, it will be assumed that they are. If so, a concrete mass submerged in water is analogous to a mass of sand or gravel, in that all pores are interconnected and filled with water and therefore each individual particle is subjected to buoyant forces. The writer must agree with the author at this point in his statement that the popular acceptance of the effect of buoyancy on such a submerged mass (Fig. 28) is that each particle is reduced in weight by the full extent of its displacement in water. However, popular acceptance is by no means proof.

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[46] Lecturer in Civ. Eng., Univ. of California, Berkeley, Calif.

If it be conceded that this is a very close approximation to the truth the resultant force acting on a submerged particle is a downward force equal to the weight of the particle less the weight of the volume of displaced water. Fig. 29 is a free-body sketch of an individual unloaded particle and shows that an upward reaction is needed for equilibrium to be maintained. Any reaction is merely the product of a stress times an area (R being the stress times the area on which it is applied). Consequently, there must be a state of stress and a finite area of the particle that is not exposed to water. That this stress intensity is relatively high and that the area is quite small should be realized, although the resulting extreme intensity of stress is of no particular significance in this problem. Inasmuch as a finite area does exist, the area of the particle subjected to uplift cannot be 100%, but must be something less, even though closely approaching 100%.

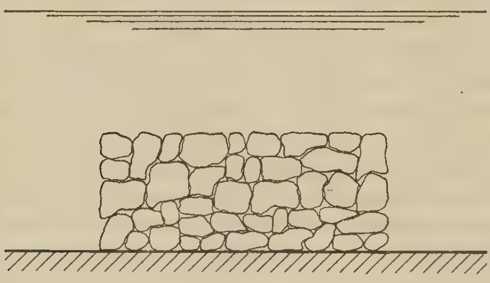

FIG. 28

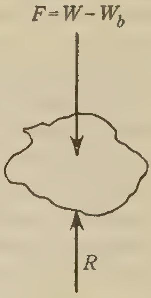

FIG. 29

Within the mass of a concrete dam additional compressive or tensile forces may act on a particle, and these forces themselves must be applied by means of a stress and an area. Certain parts of a horizontal cross section of a dam may have uplift pressures acting over the entire area—for example, where the forces act upward and the base is cracked—but, as the resultant of all forces acting on a well-designed dam is downward at any such section, the conditions of equilibrium can be satisfied only by many small areas of contact whose sum in itself will be quite small. In an earth dam this contact area is close to 0%, and in a concrete dam about 1%, of the entire base area. This latter value is based on the experiments of K. Terzaghi, Hon. M. ASCE, referred to by the author.[7]

Although exception has been taken to the "exactly 100%" theory from a technical standpoint, the writer believes that for practical purposes it is applicable to all cases where full buoyancy, or nearly full buoyancy, exists.

R. W. Carlson,[47] Assoc. M. ASCE, and Raymond E. Davis,[48] M. ASCE.—There appears to be sufficient proof for the author's principal contention that the uplift pressure acts over substantially 100% of the area. Not only do laboratory tests support this contention, but simple reasoning supports it also when the make-up of concrete is considered. Although a plane section cut through concrete reveals that the majority of the area is solid, a "billowy" section can be found near the plane section and around all the aggregate particles so that nearly all the section consists of voids. This discussion presents evidence that: (1) Pore pressure is likely never to develop to a serious intensity in continuous concrete, and (2) if pore pressure does develop in continuous concrete it is not a source of danger.

Mr. Harza's unintentional implications as to the nature of concrete tend to underrate its high degree of impermeability. His statement that "most of the pores in the cement paste will obviously be comparable in size with the cement grains" may be misleading, yet it certainly is true before the cement has hydrated. After hydration, however, part of each cement grain has been dissolved and reformed in all the water-filled interstices into a strong network of rigid "gel," with smaller amounts of solid crystalline by-products. In a cross section of hardened cement paste the gel appears solid, even under the best microscope. By indirect methods, it can be shown that the apparently solid gel is indeed porous but that the pores are from about 0.1 microinch to several microinches in size. These pores are very small as compared with cement grains, which average nearly 1,000 microinches in diameter. The effect of hydration in reducing the size of pores is so great that the permeability of concrete before hydration is about 100,000 times as great as after hydration. These relative permeabilities were measured by the Bureau of Reclamation.[3]

There is some evidence that, in a homogeneous concrete free from cracks, the pore pressure at any considerable distance from the upstream face of a dam may be negligible, at least under the conditions where the downstream face is subject to more or less continuous drying. This is indicated by the results of tests in progress in the Engineering Materials Laboratory of the University of California at Berkeley during the period from 1941 to 1948. The specimens were concrete cylinders, 8 ft long, cast in thin copper molds. After hardening, the cylinders were wound with steel wire under tension to produce a radial pressure of about 800 lb per sq in. between the copper mold and the concrete cylinder. Before winding, small capillary tubes were fixed at intervals along each cylinder in $\frac{1}{4}$-in. radial core holes of a depth equal to the radius of the specimen. Each of these capillary tubes was attached to a Bourdon tube, by which pressures were determined through observations with a micrometer microscope. Before the beginning of the test each unit, composed of a capillary tube, core hole, and Bourdon tube, was filled with water so that the Bourdon tube would immediately respond to changes in pore pressure within the concrete. The specimens were stored in a room of constant temperature (70° F) and constant relative humidity (50%), and a water pressure of 100 lb

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[47] Cons. Engr., Berkeley, Calif.

[48] Prof., Civ. Eng., and Director, Eng. Materials Laboratory, Univ. of California, Berkeley, Calif.

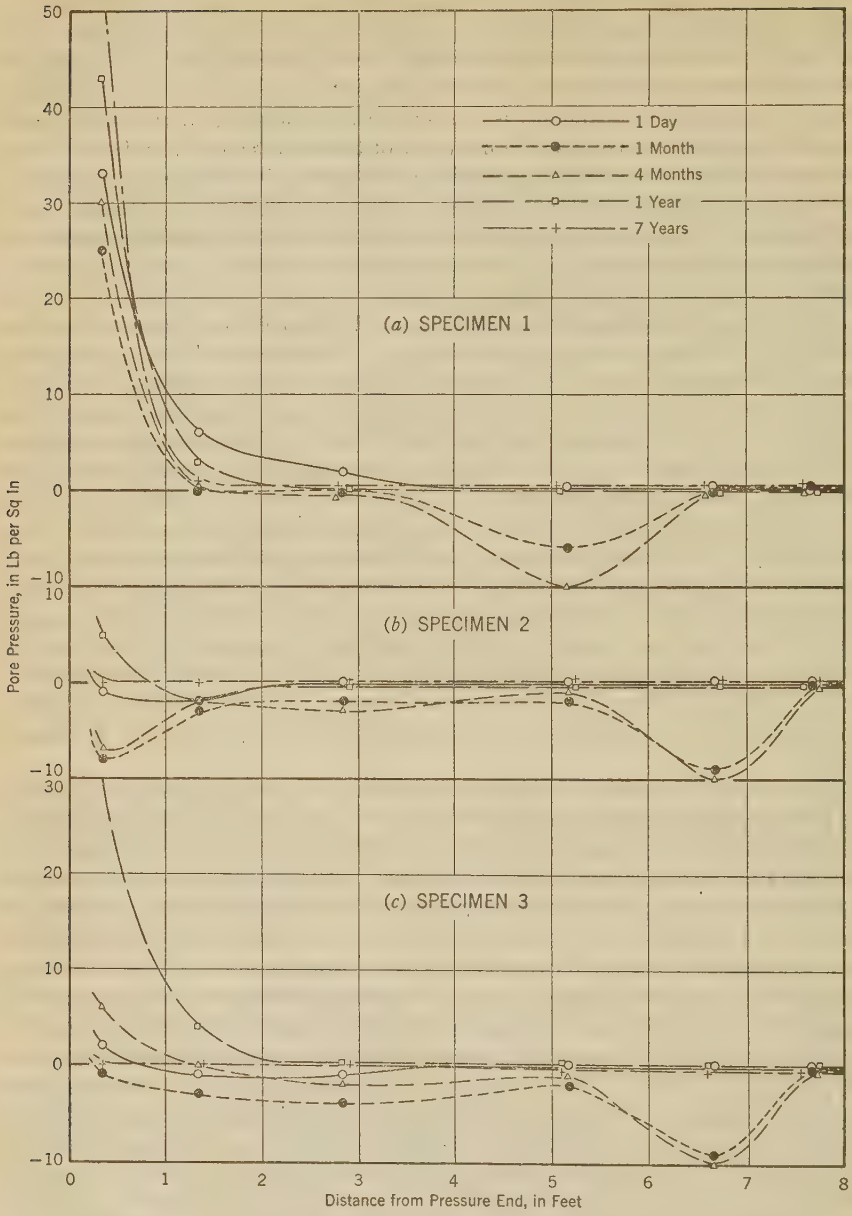

FIG. 30.—PORE PRESSURE IN 6-IN.-DIAMETER BY 8-FT-LONG CONCRETE CYLINDERS

per sq in. was continuously applied to one end of each specimen. The radial pressure between mold and cylinder prevented water from percolating along this surface. For specimen 1, the opposite end was enclosed in a tightly fitting, although not airtight, metal box; for specimens 2 and 3, opposite ends were exposed to the air of the room.

The pore pressures observed at various ages after the beginning of the test are shown in Fig. 30. It will be observed that at one month and four months, for all three specimens, negative pore pressures (produced by continued hydration of the cement) were indicated by most of the gages. For specimens 2 and 3, the maximum negative pressure was in the order of 9 lb per sq in.

At later ages these negative pressures were relieved; but, at distances in excess of 2 ft from the end to which the water pressure was applied, in no case was the positive pressure of appreciable magnitude, and at a distance of only 4 in. from the end to which the pressure was applied, the magnitude of the pore pressure did not in any way approach that of the external water pressure.

It is interesting to note that for specimen 1, even though the pore pressures at a point 4 in. from the pressure end increased with time, at the age of seven years the positive pore pressure at all other points along the specimen had decreased to zero. Evaporation from the downstream end of this specimen could not take place as freely as for the other two specimens because of its "metal box" enclosure.

For specimens 2 and 3, for which the downstream ends were exposed to air of 50% relative humidity, the maximum pore pressure 4 in. from the upstream end developed after about one year, and at this age the pore pressures as indicated by all other gages were substantially zero. For specimen 2, after the age of two years and four months, the observed pore pressures were zero for all gages, including the one 4 in. from the upstream face. After the age of four years, for specimen 3, the pore pressure was zero at all gages.

Although the scope of the tests was too meager to justify any sweeping conclusions as to what might take place within the uncracked mass of a concrete dam, the results seem to indicate that the pore pressure at an appreciable distance from the upstream face may be quite negligible in amount and that it may not increase with time, as has generally been supposed, but under certain conditions at the later ages it may actually decrease with time. An important detail of these tests was the rather high water-cement ratio, 0.80 by weight, which, in mass concrete where large aggregate is employed, roughly corresponds to a cement content of only about 0.7 bbl per cu yd.

Even if pore pressure of high intensity should develop in the interior of continuous concrete, the effect on stability would not be large. The author's principle (*e*), under the heading "Buoyancy," states in effect that the only difference between loose granular material and finished concrete is "* * * the reduction in the percentage and size of voids." This statement implies that the solidifying of concrete might actually conform to a common view that crystals or colloidal particles merely form in the interstices to change plastic concrete into hardened concrete. Actually, this view contradicts one of the most remarkable features of the hardening of concrete. The writers prefer to believe that the cement grains are dissolved partly and reformed into the rigid gel (instead of crystals), thus resulting in a strong framework around all the aggregate particles. In fact, the framework is so strong that the average tensile strength of a specimen is several hundred pounds per square inch, even though the framework makes up only a small part of the area. In general, the tensile strength of the skeleton within which the pore pressure acts is amply great as compared with the pore pressure.

Where the concrete is not continuous due to the occurrence of open joints or cracks subject to water pressure, it is obvious that 100% uplift will exist and that no reliance can be placed on tensile strength. This is also likely to be true of the surface of contact between dam and foundation rock. However, the treatment should be to drain such regions, including the foundation, rather than to design for 100% uplift.

The subject of drainage is interesting but is not under discussion. The principal contention of the writers is that the continuous sections of a concrete dam cannot be endangered by uplift.

W. G. Morrison,[49] M. ASCE.—Apparently, the author of this important contribution to the subject of uplift under dams expects to meet acute disagreement. It is the writer's opinion, however, that most engineers will agree cordially with practically all his arguments.

First, it would appear to be axiomatic that water pressure under a dam does apply to 100% of the area. The point that has been missed, or that certainly has not been stressed, is that when the water pressure is less than the total pressure over an area, it exists only as a contribution to the total pressure and its magnitude is of no consequence.

For example, consider a basement sunk 20 ft below ground-water level. The uplift on the base due to water pressure will be approximately 1,250 lb per sq ft. If the structure is light, or if it is assumed that the walls or footings transmit the load to a solid stratum, it may well be that the basement floor will have to be designed for 1,250 lb per sq ft less its own local weight. If, however, the basement floor is made to act as a raft for which the average load is greater than 1,250 lb per sq ft, it does not matter in the slightest whether or not full water pressure is assumed. Thus, if the average downward load is 3,750 lb per sq ft, the floor would reasonably be designed for a uniform reaction of 3,750 lb per sq ft. If the weight of the slab is 250 lb per sq ft, the uplift causing bending stresses in the slab will be 3,500 lb per sq ft, which is made up of soil reaction plus water pressure, and it does not concern the designer whether 1,250 lb or any other proportion of the reaction is due to water pressure.

Full hydrostatic head may be regarded as an inevitable component of downward pressure, and equally so of upward reaction. Provided it is less than total pressure it is not a factor causing instability, and it seems axiomatic that it should be applied to the whole surface whether caused by the water within the concrete or by the water surrounding the concrete. As in the basement illustration or in the case of a ship grounded on a mudbank on a falling tide, it is simply one component of the upward reaction.

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As applied to dams, Fig. 31(*a*) shows a dam with a base width of 0.846 *H*. For a 1-ft section of the dam, the total horizontal water pressure *P* compounded with the dam weight gives *R*, which intersects the base at point x. The horizontal component must be resisted by the shear *S*, whereas the vertical component *W′* must be resisted by upward reactions. Assuming straight line variation of base pressures, the intensity of upward reactions to maintain equilibrium must vary from 6.35 kips per sq ft at the heel to 8.69 kips per sq ft at the toe. Full hydrostatic uplift amounts to 6.25 kips per sq ft, so that if and where present it is only a component of the upward reaction. Although it may exist over 100% of the base, it is of no consequence statically.

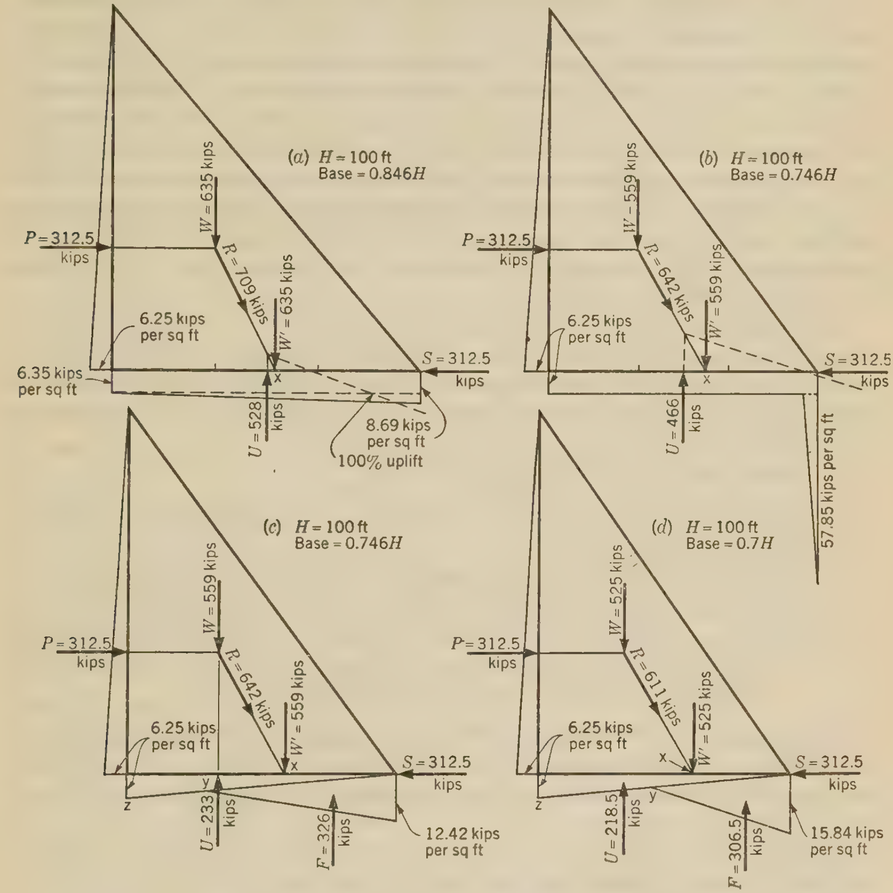
(a) H = 100 ft
Base = 0.846H
W = 635 kips
P = 312.5 kips
R = 709 kips
W′ = 635 kips
6.25 kips per sq ft
6.35 kips per sq ft
S = 312.5 kips
x
U = 528 kips
8.69 kips per sq ft
100% uplift
(b) H = 100 ft
Base = 0.746H
W = 559 kips
P = 312.5 kips
R = 642 kips
W′ = 559 kips
6.25 kips per sq ft
S = 312.5 kips
x
U = 466 kips
57.85 kips per sq ft
(c) H = 100 ft
Base = 0.746H
W = 559 kips
P = 312.5 kips
R = 642 kips
W′ = 559 kips
6.25 kips per sq ft
S = 312.5 kips
x
y
z
U = 233 kips
F = 326 kips
12.42 kips per sq ft
(d) H = 100 ft
Base = 0.7H
W = 525 kips
P = 312.5 kips
R = 611 kips
W′ = 525 kips
6.25 kips per sq ft
S = 312.5 kips
x
y
z
U = 218.5 kips
F = 306.5 kips
15.84 kips per sq ft

FIG. 31.—FORCES ACTING ON A 1-FT SECTION OF A 100-FT-HIGH TRIANGULAR DAM UNDER VARIOUS CONDITIONS

Fig. 31(*b*) shows a dam with a base width of 0.746 *H*. Again, *P*, is compounded with *W* to meet the base at point x. Now, if the uplift *U* is assumed to act with full intensity over the entire base, equilibrium can only be maintained by an upward force as shown at the toe, having a maximum intensity of 51.6

kips per sq ft over and above the water pressure, or a total of 57.85 kips per sq ft. This pressure is 25.8 long tons per sq ft on the rock, or 402 lb per sq in. on the concrete. In neither case are the pressures likely to cause failure. The dam is not unstable. Shearing stresses on the base would be too high, but an embedment of a few feet in rock at the toe would make a great difference to safety.

Fig. 31(*c*) shows the same dam as in Fig. 31(*b*) but one in which hydrostatic uplift, *U*, is assumed to vary from 100% at the heel to 0 at the toe. Again, the downward force at base level is *W′*, resisted by a known uplift *U* and a force *F*, which must be calculated. Assuming triangular distribution, *F* gives a pressure of 12.42 kips per sq ft at the toe, and is throughout additive to the water uplift pressure. Beyond point y it does not matter what contribution to the total base reaction is due to water pressure. The important point is that water pressure from point z to point y must be reduced to the value shown by grouting, drainage, or any other expedient. If water pressure in this region builds up to 100%, the dam need not fail, but it will progressively work back to the condition shown in Fig. 31(*b*).

Fig. 31(*d*) shows a dam with a base width of 0.7 *H*. The condition is as shown for uplift pressure reducing on a triangular basis from point z to point y. Again it does not matter what the water pressure beyond point y may be, as long as it does not exceed the total reaction. In this case, 100% water pressure on the whole base would create a condition of instability.

These examples are intended to show that:

1. The classical "middle third" theory cannot be applied as a necessary condition of stability;
2. Stability is possible when the reaction falls outside the middle third;
3. Water uplift is only of importance as part of a total reaction; and
4. If uplift can be reduced on a triangular basis for the upstream half of the dam, it does not matter what magnitude it is for the remainder of the base, as it is only part of the total reaction.

It is realized that the foregoing examples represent an oversimplification, and that the magnitudes of reactions under dams do not follow a straight line. However, it is certainly true that the elementary principles of static equilibrium must be observed in all cases.

In explanation of why some dams having bases narrower than 0.7 *H* have not failed, the writer would suggest these possibilities:

a. The reduction of the magnitude of the uplift pressure under the heel, as in Fig. 31(*d*), from point z to point y (this in accordance with the design, and the measures taken to achieve it);

b. The greater weight of the concrete near the water face because of the greater degree of saturation;

c. Friction between the blocks of the dam, with the more stable segments near the ends of the dam;

d. The fact that toe pressures much greater than those normally assumed may exist (Fig. 31(*b*)); and

e. For dams only occasionally filled there may even be "limpet" action due to air pressure on the back, and, through the medium of the water, on the front.

The resultant will be a contribution to stability unless and until the effect is communicated under the base by seepage, which will take time. In any case, the writer would discount tension between the dam and the base as most unlikely and very unreliable.

Raymond A. Hill,[50] M. ASCE.—No process of reasoning can alter the fact that a concrete dam or any other solid body composed of materials heavier than water cannot be supported entirely by water. On the contrary, if the author's plausible reasoning leads to a contrary conclusion, either his premise or his logic must be in error.

For purposes of illustration, a homogeneous triangular dam of the weight and porosity assumed by the author will be considered as being subjected to full buoyancy, "equivalent to a triangular uplift diagram with full water pressure at the back and zero at the face." Such concrete would weigh 145.75 lb per cu ft when dry and 153.25 lb per cu ft when saturated. The total weight of a triangular dam of this density, if saturated, would be $\frac{153.25\, H\, B}{2}$ lb per lin ft, in which H is the height in feet above any substantially horizontal surface and B is the base width at that level. The aggregate of all pore pressures acting upward at the same level (described by the author as a "rough surface joining the lowest pore in each elementary column") would be only $\frac{62.5\, H\, B}{2}$ lb per lin ft. Consequently, even if uplift were to be effective over the "entire gross area of the horizontal section of the dam," only a little more than 40% of the weight of the saturated dam could be supported by the water in the pores of the concrete; the other 60% of the weight would have to be supported by direct contact of solid matter. Incidentally, it can be demonstrated that for any given density and strength of concrete the proportion of the area that must be in direct contact to support the overlying weight is independent of the base width of any triangular dam.

The same necessity exists for direct contact between solid particles to resist the horizontal load of water against the upstream face of any dam. Such loads can be transmitted to the foundation only by shear, and shearing strength is not one of the attributes of water.

If the author had limited his claims to the potentiality of uplift over most of the area of any section, this could have been accepted because, in the case of dams of ordinary height, built of concrete of customary densities and strengths, there would not have to be as much as 10% of the area of any section in actual contact to transmit all loads to the foundation without failure of the concrete in either compression or shear. However, the author in effect claims that no area of direct contact is necessary when he categorically states that, "nothing can reduce the effective uplift area below 100% whenever pores are filled with water."

The author conceives of buoyancy as reducing the weight of a body, when as a matter of fact all it does is to reduce the weight to be supported by some other medium than water.

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[50] Cons. Engr. (Leeds, Hill and Jewett), Los Angeles, Calif.

Regardless of the area over which there may be pore pressure, the resultant of all forces above any horizontal section remains the same. The water pressure against the face is not changed; nor is the weight of the dam itself changed, except as such weight is increased by water in the concrete. Whenever water is not present along any section of a dam, the loads are transmitted directly through the particles comprising the concrete and the resultant reaction is equal and opposite to the resultant of all forces above that section. The presence of water under pressure in pores or open seams merely causes some of the load to be transmitted through water from one solid particle of concrete to another; but the total reaction is not altered thereby and the resultant of all forces below the same horizontal section must still be equal and opposite to the unchanged resultant of all forces above that level.

Percolation of water through the pores of concrete can have no adverse effect upon the factor of safety of a dam against overturning, as distinguished from its safety against sliding. On the contrary, a saturated dam would be heavier than a dry dam so that the resultant of all forces above any plane of reference would intersect the base farther upstream. Consequently, the real significance of pore pressures is the extent to which the overlying weight may be supported by water in the pores. To whatever extent this occurs, the resistance of a dam against sliding is reduced because this resistance is proportional to the load supported by direct contact between solid particles.

The concept of internal buoyancy is meaningless and leads to erroneous conclusions. If it were true that the weight of a concrete dam above any horizontal section would be reduced by internal buoyancy, a wet sponge would float.

JOHN V. SPIELMAN,[51] ASSOC. M. ASCE.—This paper has stimulated the writer to give considerable thought to the matter of uplift and buoyancy, leading to the conviction that the reasoning of the author is incorrect with respect to a new idea called "internal buoyancy," and that the conclusions drawn therefrom are not proved.

The author attempts to prove that hydrostatic uplift in a dam is effective over exactly 100% of the horizontal area by what he terms a "phase of the buoyancy phenomenon." In the development of his proof the author uses the term, "internal buoyancy" signifying that the concrete surrounding a pore gets a "lift" from the internal pressure in the pore. It seems to the writer that in a material having tensile and shear resistance, as long as the strength is not exceeded, the pore pressure is fully resisted by the stress within the material.

Furthermore, the author, near the beginning of his paper, takes pains to affirm that concrete is porous and that water can seep into all the voids, subjecting all pores to the hydrostatic pressure. There is no need to question this phenomenon as a possibility, but the writer doubts the probability of its occurring in many dams. However, because of the indirect method the author has chosen to prove his thesis, he needs to assume pore pressure throughout the mass; otherwise, by inference, he would not have uplift effective over the full area. The writer wishes to show by buoyancy relations that the opposite is true; that is, the buoyant force on a submerged mass of concrete can only be equivalent to the hydrostatic force on the full area, if the voids are empty. If there is water in the voids the effective uplift area (due to buoyancy) is always less than 100%.

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[51] Senior Engr., Design and Constr. of Dams, State Dept. of Public Works, Div. of Water Resources, State of California, Los Angeles, Calif.

Webster's Dictionary defines "buoyancy" as follows:

> "In physics, the upward pressure by a fluid on a body partly or wholly immersed therein; it is equal to the weight of the floating body, which in turn is equal to the weight of the fluid displaced."

Under the heading, "Buoyancy," the author shows the simple proof for the foregoing definition regarding an impervious body. If the body were porous, with interconnected pores, it would not displace its gross volume but only the volume of impervious matter, assuming, of course, that sufficient time is allowed for the full penetration of the fluid into the pores.

Assume a block of concrete of cross-sectional area A and height d, having a unit dry weight of w_c. Let the unit weight of water be w. Assume, that this block is suspended in a tank of water just to its own depth. If this block were covered with an impervious membrane the buoyancy will be expressed by

$$B = w\,d\,A \qquad (39a)$$

As the intensity of water pressure on the bottom of the block is $w\,d$, the effective area of uplift is 100%. This is the condition of buoyancy with the voids empty; but, if the surface of the block is not sealed with a membrane and the water is given time to penetrate into all the pores, then,

$$B = w\,d\,(1 - n)\,A \qquad (39b)$$

in which n is the porosity. Now, because the voids are filled, the effective area of uplift is $(1 - n)\,A$, which is something less than 100%.

Let the block be lowered so that it rests upon the bottom of the tank at the same time lowering the water surface to the top of the block (to simplify the equations). If some part of the block fits so closely to the bottom as to exclude water the magnitude of the uplift area may be further reduced. However, for the moment, assume that water reaches the entire bottom area as before. Then the buoyancy is unchanged. The block rests on the bottom with a resultant downward force:

$$F = w_c\,d\,A - w\,d\,(1 - n)\,A \qquad (40)$$

The second term in Eq. 40 is the uplift and, as before, the unit water pressure is $w\,d$, and the effective area is $(1 - n)\,A$. This would always be less than 100% of the gross area.

The downward force is of greater magnitude than the uplift (which is usually the case in dams). This leads the writer to believe that some impervious areas at the contact would exclude the water, thereby further reducing the uplift as full buoyancy would not be effective.

Mr. Harza gives data on the porosity of concrete which indicate that concrete of good quality has voids aggregating about 12% of the gross volume.

The writer has always assumed that by nature the Portland cement and the aggregate are crystalline substances and may be considered impervious. By the foregoing reasoning, concrete having a value of $n = 0.12$ would have an apparent effective uplift area of 88%, assuming full penetration of water in the voids and at least 88% of the bottom area subject to the hydrostatic pressure.

PAUL E. RAES.[52]—It seems obvious that the theory advocated by the author involves, not only a particular method for the calculation of concrete dams, but also an entirely new law of physics, applicable to porous solids. Therefore, it should be scrutinized severely before being accepted by the profession.

Internal Buoyancy.—To demonstrate his concept, Mr. Harza subdivides the material into small "columns" terminating in pores at both ends, and hence subjected to the difference of the hydrostatic pressure developed in those pores. This is called "internal buoyancy" and the author concludes (see heading, "Buoyancy"):

> "Likewise, since each column is buoyant, and the whole object is composed of such columns, then the entire internal structure is buoyant, and the effect of cementation is nullified."

In attempting to check the verity of this statement, one should remember that Archimedes' law has been demonstrated for a free body, completely submerged in still water, and not for a part of a solid limited by an imaginary surface, where elastic stresses must also be taken into account. Consider, for instance, the prism represented by Fig. 32(*a*), containing two spherical holes filled with a fluid under unit pressures p and p'.

The effect of the "internal forces" on the inner surface of the holes will be purely a change in the stress distribution; the equilibrium of the prism as a whole, and its resistance to sliding and to overturning, will remain unchanged. On the other hand, if column C is isolated, pressure p will indeed act downward on the horizontal section passing through the middle of the upper hole, and pressure p' will act upward on the corresponding section of the lower hole. If an equation of equilibrium is written by projection on a vertical axis, the shear stresses on the lateral surface of the column must be introduced.

Statics can determine the resultant of these stresses, but not their distribution; nevertheless it is likely that the pressures on the bases of the cylinder will be nullified by shear stresses transmitted to the parts of the lateral surface situated near both ends, in such a way that a slice at height A′ B′ (Fig.32(*a*)) will not be affected by the pore pressure. Passing to the extreme case, imagine that the fluid is enclosed in a strong casing touching the substance of the prism, but not pressing upon it, with the evident result that column C would bear no pressure at all, and could not possibly be called "buoyant." When weight is neglected, pore pressure can have no effect on the conditions existing in sections that do not intersect the pores under consideration, except through local stress variations that may occur without modifying the resulting force and couple of the stress system, in spots where pores are very near the section surface.

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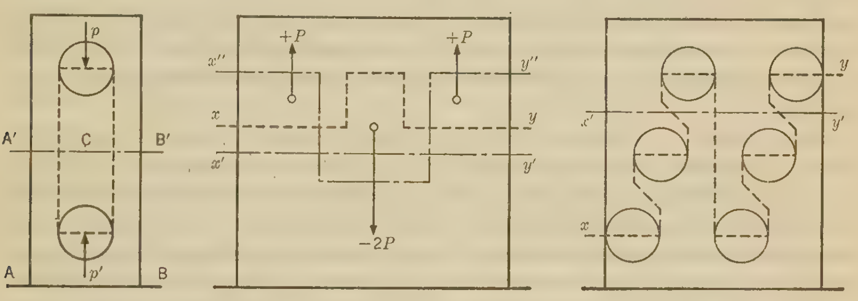
[52] Prof. of Soil Mechanics, Univ. of Ghent, Ghent, Belgium

Consequently, the writer believes that the concept of "internal buoyancy" is not only needless, but sometimes misleading, and should be avoided, although consideration of buoyant elementary columns gives correct results if the lateral stresses nullify themselves. Indeed, under the heading, "Bottom Uplift," the author uses the notion of internal buoyancy to compute the total load on the supporting surface of a submerged block, and reaches the conclusion that there is a resultant partial uplift.



(a) (b) (c)

FIG. 32

As the same calculation can be made for any other plane horizontal section cutting voids, the claim of full uplift is certainly not justified in all cases.

Uplift on a Given Surface.—In fact, full uplift is found on a very unusual surface, which has been visualized with much ingenuity by the author, and which the writer will call the "Harza surface" in this discussion. It is obtained by visualizing a block of columns terminating in pores. The lateral shear stresses disappear, of course, where two of these columns touch one another; but in the base of the block, constituting the Harza surface, certain lateral facets remain uncovered, and vertical shear stresses can still exist.

In 1943 the writer gave the name "pseudo-facets"[53] to small surface elements intersecting granular material and conforming as nearly as possible to a plane, but cutting no soil grains. The Harza surface can be considered as a generalization of those "pseudo-facets," completed by facets intersecting miscrosolids.

If different sections are considered, the forces applied on one side of the surface of separation may change, not only in the case of pore pressure, but in a general way, as will be noted in Fig. 32(*b*), where the sum of the forces is 2 P above section xy, zero above section $x'y'$, and $-2\ P$ above section $x''y''$. Indeed, one can trace the pseudo-facets with the intention of "catching" certain forces and "dodging" others. If, by doing so, the points of application of these forces are not far apart, the surfaces selected nevertheless can be very close to one another.

Thus, the author has succeeded in discovering a surface subjected to full uplift; and Fig. 32(*c*), where tubular voids are assumed, shows that it is even easy to draw a surface such as section xy, submitted to 150% uplift, or more. The latter surface is composed of vertical and sloping solid facets; but this is no obstacle, as long as stresses are ignored. However, on plane $x'y'$ the uplift will be zero. The Archemedian uplift still being exactly 100%, the foregoing examples demonstrate that the term "buoyancy" cannot successfully be applied to pore pressure, and that discussion about the true value of pore pressure is meaningless as long as the exact surface of separation is unknown.

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[53] "Théorie des Pseudo-Facettes en Mécanique du Sol," by P. E. Raes, *Annales des Travaux Publics de Belgique*, August, 1943.

Of course, since only the vertical components of pressures are reckoned with, the horizontal components have no importance; and, if all the facets inaccessible to the liquid are vertical (which is precisely the case for the Harza surface), the total uplift will be equal to the buoyancy effort. This case is exceptional and does not justify confusion between pore pressure uplift and buoyancy.

In the paper the ultimate aim is to compute normal stresses in a horizontal section, as established by Fig. 9 and by the fact that the theory of the "middle third" is considered. At first thought, it appears to be axiomatic that, for this purpose, only the geometrical horizontal plane will suffice. Thus, uplift would act on the fraction n of the area, if n is the ordinary porosity, according to Delesse's law; but it must be remembered, as Karl Terzaghi, Hon. M. ASCE, made clear, that the so-called "strength" of a porous material is nothing more than the resistance of a surface of failure, which is not a plane, but the weakest section that can be drawn through the existing (and unknown) pore structure.

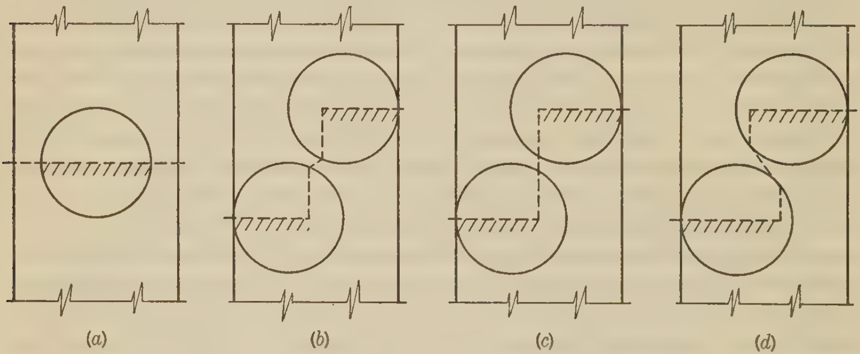
(a) (b) (c) (d)

FIG. 33

Hence, the "effective" uplift will be measured by the effective porosity n_w of the potential surface of rupture, just before it gives way, and this measurement can only be made experimentally. In this connection, the writer wishes to observe that the real surface of failure is not necessarily identical for a dry sample and for that same sample under pore pressure, because, by altering the section in the second case, the uplift may change.

Interpretation of Tests.—To determine n_w Professor Terzaghi and S. Leliavsky Bey, M. ASCE, performed clever experiments, and found, respectively, average values of 0.97 (série I), 0.97 (série II), and 0.998 (série III) in the Terzaghi compression tests,[29] and of 0.915 in the Leliavsky tension tests.[8]

From observation of Fig. 18(c), representing the Terzaghi surface, one understands why the experimenter assumes that the strength of the solid facets is only of a tensile nature, and is amazed by its magnitude. The structure is indeed visualized as an aggregate of grains, "welded" together in their contact points; and, in such a material, facets subjected to shear are unlikely in a section resembling AB.

On the contrary, if the material is assumed to be a continuous solid with holes in it, the separation surface may contain facets resisting through shear, and can even be constructed entirely of similar facets, like the Harza surface. Under such conditions it is evidently needless to imagine threads of astounding tensile strength when n_w approaches unity, and the value $n_w = 1$ is easily explained. However, this explanation does not demonstrate that the Harza surface is the true section of rupture.

In Fig. 33 different types of possible failure are suggested, and in each case the area of uplift is shaded. In Figs. 33(*a*) and 33(*b*), $n_w < 1$; in Fig. 33(*c*), $n_w = 1$; and, in Fig. 33(*d*), $n_w > 1$.

The last type, $n_w > 1$ (which is likely to occur only in compression failure), is interesting because it shows that n_w can become greater than unity, when such facets dominate. Some explanation of this kind had to be discovered, since values of 1, 1.07, and even 1.2 were found for n_w in the Terzaghi tests, and would be simply bewildering if only tensile strength was possible. The fact that the corresponding tests were based on compression, which makes ruptures of type (*d*) shown in Fig. 33 physically possible, supports the views suggested herein.

In the writer's opinion, this analysis confirms the masterful Terzaghi theory of "effective porosity," to be measured by experiments. Nevertheless, the writer is glad to state that the "Harza surface," conceived by the author, is very helpful in understanding the pore pressure problem.

L. F. Harza,[54] M. ASCE.—The writer appreciates very much the several contributions to completeness and clarity offered by the discussions, and also the revelation that his case had not been clearly presented in all respects. The discussions vary from almost unequivocal endorsement to the other extreme.

Messrs. Carlson and Davis have revealed the fact that the pores in cement paste (after setting) are much smaller than the writer had assumed; in fact they are of the general magnitude expressed in a few millionths of an inch instead of hundreths. The writer is especially indebted to Mr. Riegel for calling attention to something the writer considered obvious, but which apparently was not—namely, that the ideal columns are everywhere (except where pores interrupt) in intimate contact (or continuous) with adjacent cylinders so that stresses of shear, tension, and compression may be transmitted through these contacts without disturbing the structural integrity of the concrete. Of course, these cylinders are purely imaginary subdivisions of the material by imaginary lines and surfaces. They can have no more effect on its strength than can the imaginary meridians on the earth's surface make it fall apart like sections of an orange. Imaginary lines, for the purpose of discussion, obviously leave the concrete unchanged in tensile, shear, or compressive strength. Similar failure to distinguish between imaginary division of concrete into elementary units and actual physical dissection appears to form the basis of Mr. Meyer's statement that "* * * when the author applies his reasoning to concrete, he introduces a concrete lacking in all tensile strength." There is no other possible explanation for such an interpretation. Messrs. Leliavsky and Hill also are under a similar misapprehension that the writer, in making the statement that there are enough pores in each small differential in elevation in the concrete mass to form a full projected area of pores, actually moves these pores into one plane and thus destroys the strength of the concrete or (in the words of Mr. Hill): "* * * a concrete dam * * * composed of materials heavier than water cannot be supported entirely by water."

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[54] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

All these gentlemen greatly flatter the writer by endowing him with such occult powers as to change the physical nature of concrete from what it actually is by mere process of thought; but the better conclusion is that the writer has not made himself clear and that more explanation is needed and perhaps from a different viewpoint.

Messrs. Riegel and McHenry assert that the writer "seems to distrust experimental evidence" or that he has "objections to the experimental method." Nothing could be further from the truth. The writer is not antagonistic to experimental tests, but merely claims that they are only corroborative, after the investigator has first developed a complete understanding of the phenomenon.

The only exception taken to the Leliavsky tests was that, because of believed inherent inaccuracies in principle, they did not prove that the uplift area is not 100%. The McHenry tests[43] first came to the writer's attention in June, 1948, after publication of his paper, and obviously could not have been anticipated and properly "welcomed" in the paper. These tests, however, are now most welcome indeed.

Under the heading, "Experimental Evidence," the writer gives Professor Terzaghi full credit for the experimental determination of substantially 100% uplift area reported in his paper[7] and expresses surprise and regret that the profession in the ten years that elapsed had not generally accepted Professor Terzaghi's conclusions. Prior to the appearance of the Terzaghi paper in 1936, the writer himself had held the orthodox view that 100% uplift area could not be true because of the large proportion of the section that must be in contact to support the load.

The tests,[7] in which Professor Terzaghi was unable to compress a naked cylinder of concrete significantly by subjecting the liquid in which it was submerged to very high pressure, were the beginning and inspiration of the writer's paper in June, 1936, which was substantially completed that year. In 1936 the writer accepted the fact of practically 100% internal pressure area, equivalent to 100% uplift area, as thus demonstrated experimentally, and sought a conception of the structure of concrete which would explain this demonstrated but apparently anomalous fact. The reason for a proved fact can usually, and should always, if possible, be visualized as it aids in its acceptance and understanding. Thus the hen and egg sequence in this case began with Professor Terzaghi's research, and resulted in a theory in explanation of his results.

The effect of Mr. McHenry's discussion is to deny the writer his right to use these tests in support of his theory. The Terzaghi tests demonstrated, within the limits of experimental accuracy, that the internal pressure in the pores of a concrete cylinder was sufficient to oppose fully the external pressure on top and bottom. Thus, in order to resist external compressive forces it was necessary for the internal forces opposing them to be exerted over the same area as the external forces—that is, over 100% of the cross section. This justifies the use of the Terzaghi tests in support of the writer's theory for it is the basis of that theory. No ordinary section of the specimens, and no theory of minute contacts, could explain to the writer's satisfaction how this could be possible with 98%, 99%, or even 91% of pore area, leaving only from 1% to 9% of area for transmitting the load.

It has been shown that enough pores for a full projected area of pores occur in concrete at very close intervals which, by the revelation of Messrs. Carlson and Davis, would be at intervals of a few thousandths of an inch at most. To be sure, such a complete cross section of pores could not be incorporated in any plane section or in any smoothly undulated surface. The pores would obviously be staggered in position or elevation, and to incorporate such a complete pore section would require the surface to dodge vertically up and down through solid material in an erratic manner. That such a surface can logically be assumed to exist cannot be questioned, however; in fact, many such sections would exist at intervals of only a few thousandths of an inch.

In order to make the naked Terzaghi specimens practically incompressible by water pressure it was necessary that these many surfaces of 100% pores be under hydrostatic pressure, fully opposing each other, and the external pressure —with only solid, poreless material, subject to comparatively negligible compressive strain, intervening between these surfaces. Even with this assumption a small compressive strain reflecting the modulus of elasticity of the solid mineral particles or gel would be expected to occur; and this may explain the 99%. However, 99% is the same as 100% experimentally and requires the same explanation.

The Terzaghi experiments were conducted with high pressure and therefore practically with uniform internal pressure. However, if such a specimen were merely submerged in water with the top at the water surface (for simplicity), each of these numerous, internal, 100% area surfaces would be subjected to a pressure equal to its depth of submergence, instead of the nearly uniform high pressure imposed by Professor Terzaghi; and the intervening solid material would be in a buoyant state to which the writer applies the term "internal buoyancy" as self-explanatory. This is a logical, direct deduction from the Terzaghi tests. "Buoyancy" is defined by Webster as "The upward force exerted upon an immersed or floating body by a fluid." The definition does not require that the force be exerted only on the bottom. In the present application it would be defined as "The internal upward force exerted upon an immersed body or portions thereof by a fluid"; and "buoyant" would be defined as "subjected to the force of buoyancy."

Thus a submerged specimen of concrete resting on the bottom is not dependent on the penetration of water pressure along the bottom contact to produce buoyancy of the specimen; such a specimen is fully buoyant in its smaller subdivisions as the result of internal pressure (shown by Fig. 5), considered to be applied to elementary columns with a pore at the top and the bottom, or applied to the disk-like volumes between successive complete, projected areas of pores.

Messrs. Terzaghi[7] and McHenry[43] have proved, within experimental accuracy, that the uplift acts on 100% of the area and Mr. Leliavsky, in the writer's opinion, has not proved otherwise. Thus the reader is at liberty if he wishes to consider the writer's analysis as an explanation of a demonstrated fact (which in truth was the sequence of its evolution) rather than as a deductive proof. Then if he does not like this explanation he can seek another more to his liking. The writer has been unable to find a better or, in fact, any other explanation.

The writer's thesis is the end result of a process of thinking on the subject of uplift represented by Fig. 34, corresponding with the following stages of evolution:

(*a*) Not many years ago the existence of uplift was vigorously denied by eminent authorities (no illustration).

(*b*) The second stage of evolution was to concede uplift over the proportion of the area occupied by pores in a plane surface of intersection, as in Fig. 34(*b*). Thus, if concrete contained 12% voids, uplift was conceded over only 12% of the area.

(*c*) The third stage of evolution was to introduce a rough or undulating surface, as in Fig. 34(*c*), and speculate upon the percentage of pore section that could be incorporated in such a surface. This has resulted in the now orthodox assumption of uplift of over one-half and two-thirds area.

(*d*) The writer's thesis carries the evolution one stage further, as in Fig. 34(*d*).

The only new question introduced by the writer is how rough a rough surface is permitted to be in order to incorporate additional pore area without invalidating the theory that uplift could be exerted in the pores of such a surface. Who can say where the limit should be and by what theory or principle of judgment it would be determined? The only limit, in the writer's opinion, would seem to be nature's limit of roughness beyond which no more pore area could be incorporated in the assumed surface. It will be shown that, if it is rough enough, it can incorporate 100% of pore area as in Fig. 34(*d*).

This curve is platted from Fig. 34(*e*), which shows the result of rolling a seventy-four-ball "Bingo" game ten times, stopping each time that thirty-seven balls had been turned and platting their numbers represented by black dots in the corresponding squares of cross section paper containing seventy-four squares per line. Thus, each line is half-filled with black dots indicating pores chosen by chance, very crudely simulating, in two dimensions only, cement paste with 50% voids.

In Fig. 34(*f*) three lines have been shown through approximately adjacent 100% surfaces, the middle one being identical with Fig. 34(*d*). Starting from the median horizontal line there is platted in Fig. 34(*d*) the line of minimum departure from that line which will pick up a full projected area of black squares or voids. This procedure is adopted to show, with as little dependence on the imagination as possible, how rough a rough surface might need to be to incorporate 100% projected area of pores in cement. To do so, of course, requires the surface to dodge up and down through solid material between pores. Although this surface appears very rough, the actual dimensions are microscopic; such surfaces occur in close proximity in actual concrete and are separated in fact by only a few thousandths of an inch.

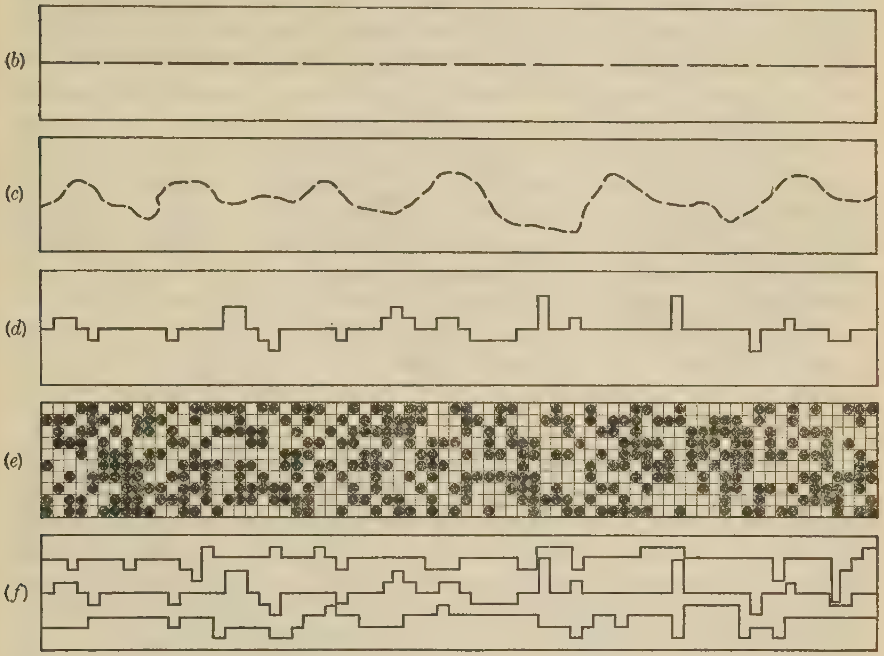

Fig. 34

To visualize more easily both the theory of a complete projected area of pores and the theory of elementary cylinders, Fig. 35 shows a small patch of a concrete section with contact areas between particles of great importance—not microcontacts by any means. This picture was made by a friend as his idea of the miscrostructure and without knowledge of its intended use. Therefore, it is fortuitous as far as the writer is concerned. The pore area is also much less than 50% of the total, as in cement, thus making the problem more difficult.

The big hurdle (which is difficult to clear in understanding this theory) is the belief that large areas of contact on the horizontal exposure of particles is necessary to support the load. This is true, but it is also true that these areas do not interfere with the buoyancy of the particles. It has previously been stated that a column erected through such a horizontal area of contact would eventually terminate in a pore at the top and the bottom and would thus be equivalent to ab in Fig. 1.

Fig. 35(*a*) has been subdivided by projecting all horizontal areas of contact upward and downward until they terminate in a pore at the top and the bottom or reach the limits of the sketch. The top of each such column is marked A and the bottom, B. At some places the limit of the sketch introduces an uncertainty as to how soon a pore would be encountered if the sketch were extended. Such points are marked A′ or B′ and in some places subdivision is not necessary. In fact, it is only necessary where horizontal contacts occur. One column, marked DE, encounters three horizontal contacts and no pores to the limits of the specimen; but it is reasonably assumed that, if the specimen extended further, pores would be encountered at the top and the bottom.

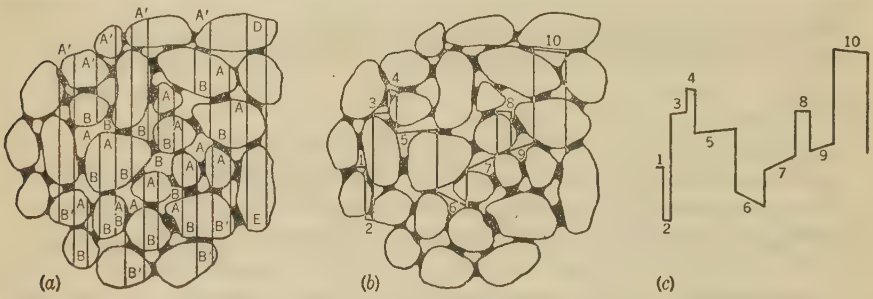

Fig. 35

A careful study of Fig. 35(*a*) will show that the entire area has been, or can be, so subdivided that these horizontal contacts are absorbed, so to speak, and become a part of a column with top and bottom pore, and that the entire area, ignoring uncertainties around the border, becomes a system of vertical cylinders or columns all in exactly the same position as column ab in Fig. 1, and therefore buoyant if submerged and the pores filled with water under pressure corresponding with the depth of each pore below the water surface.

More important than the foregoing is the fact that this pictured specimen is strong; it is a solid in the sense that any cement mortar or concrete is solid; the horizontal contacts that support the load are still there and yet they do not interfere with the buoyancy of the newly conceived cylindrical units. This is what the writer as been "harping on" for twelve years without perhaps having previously presented as clear a picture to help the imagination.

Likewise from the other standpoint of a full projected area of pores subject to buoyancy the writer has repeated Fig. 35(*a*) as Fig. 35(*b*) with the vertical cylinders omitted and with a staggered line resembling Fig. 34(*d*), chosen so that all the horizontal or sloping lines are within pores. Fig. 35(*c*) shows this staggered surface in detached position to indicate how closely it resembles the general form of Fig. 34(*d*). The horizontal and sloping lines are numbered to correspond with their respective pores.

In submerged concrete the elementary cylinders in Fig. 35(*a*), like ab in Fig. 1, are subjected to buoyancy by reason of the difference in depth of the top and the bottom below the water surface; or, if the reader prefers, the entire disk-like volume between two contiguous surfaces as in Fig. 34(*f*) or in Fig.

35(*c*) may be considered as collectively buoyant because each surface comprises a complete 100% projected area of voids.

To the foregoing argument some will state that Fig. 34(*d*), or Fig. 35(*c*), is an extreme and fantastic form of surface to assume. True enough, and it is easier, for the writer at least, to visualize the elementary cylinders; but it cannot easily be denied that such surfaces, incorporating complete projected areas of pores, actually exist at frequent intervals in the mass symbolically represented by the three surfaces in Fig. 34(*f*). If they exist the designer has a right to consider what the uplift force on such a surface would be. It would obviously be the hydrostatic pressure over 100% of the projected area of the specimen, and the specimen above this surface is not floating on water and without strength. In a submerged body of concrete the weight of that part above such an assumed surface would be reduced by buoyancy.

The conception, expressed by several discussers, that acceptance of 100% uplift area necessitates a weak concrete with no area left to support the load even to the point of requiring a structure "floating on water" seems to be the difficult "hurdle" to pass in an understanding of this problem; but the reverse is really the case as compared with other theories. Thus, the elementary cylinders visualized by the writer overlap each other with the pores at the top and the bottom of adjacent cylinders at different elevations and (substantially in the words of Mr. Riegel) are everywhere in intimate lateral contact with adjacent cylinders, permitting shear, tension, and compression to be transmitted through these contacts without disturbing the structural integrity of the concrete. (The word "cylinder" is substituted advisedly for Mr. Riegel's term "prism" because a cylinder is defined by Webster as "A surface generated by a line moving parallel to itself," and is therefore general, whereas a prism applies only to flat faces—a special case.)

These lateral contact areas are not limited by the writer's theory and may constitute in magnitude a large proportion of the section of the specimen. In fact, the original horizontal contact areas remain intact to support the load, which therefore does not presuppose weakness of the concrete.

On the contrary, any theory based on the belief that the load must be transmitted through minute points of contact and that the surface of uplift is an irregular surface which passes almost entirely through microvoids must necessarily assume a strength of these minute contacts far beyond any known possible strength of mineral substance. Thus an uplift area of 99% on this theory must assume that 1% of the area carries the load and that the strength of this contact is 100 times as great as the composite strength of concrete, thus requiring for 4,000-lb concrete a strength of minute contact points of 400,000 lb per sq in. Even the 91% of water contact and 9% of solid determined by Mr. Leliavsky would require a tensile strength of mineral particles of 9 × 4,000, or 36,000 lb, which is equal to the elastic limit of mild steel.

By the writer's theory it is therefore quite reasonable to believe in exactly 100% uplift area, whereas by the older theory it is very difficult if not impossible to accept even 91%, and definitely impossible to accept from 97% to 99%, of area. In fact, to be acceptable, these high areas of uplift require exactly the same explanation, in the writer's opinion, as 100%.

Professor Raes tells the story beautifully when he states:

> "Under such conditions [the writer's elementary cylinders] it is evidently needless to imagine threads of astounding tensile strength when n_w approaches unity, and the value $n_w = 1$ is easily explained."

On the other hand, Mr. Leliavsky declares that the assumed surface must be a potential surface of rupture "which is physically possible." This would seem to be a very reasonable condition to impose if actual rupture enters into the application of the phenomenon to a practical problem (which it does not, in uplift or buoyancy problems in concrete). In general, the buoyant effect is less than the weight, at least in concrete, and the question of fracture on such a surface does not enter.

Nevertheless, assume that such a surface does fracture. A tension rupture on such a ragged surface as shown in Fig. 34(*d*) or Fig. 35(*c*) is conceivable only with a similar tensile stretch of the imagination, and a shear failure along such a surface is inconceivable by any stretch of the imagination. It would appear then that such a surface cannot satisfy Mr. Leliavsky's criterion.

If an assumed failure section through the specimen were to cut across some of these high peaks or low valleys in the 100% line (and it is logical to believe that even a tension failure, and certainly a shear failure, would do so), the uplift from pore pressure on the resulting surface would, by common acceptance, be effective over only that part of the area of the section composed of pores.

Reference to the McHenry test[43] would seem to prove otherwise. These were triaxial shear tests from which the uplift area was determined to be 100% within the limits of experimental accuracy. These tests appear to be a very valuable contribution to the literature and it is unfortunate that they were not published in the United States where they could be more generally accessible and better understood and evaluated.

Although the writer is not familiar in detail with the tests beyond public information,[43] it is known that they are designed to determine the effect on the shearing strength of concrete resulting from variable pore pressure across the shearing surface obtained by nitrogen gas. One can be certain without inquiry that these shear failures did not break along a surface of 100% pore area, such as that in Fig. 34(*d*). In fact, one can be sure that almost poreless aggregate pieces were fractured in large proportion and that the pore area incorporated in the fracture was very much less than 100%; and yet the tests apparently indicate that the pore pressure had the effect, in the Coulomb equation, of weakening the specimens in shear to the same extent, by reduction in pressure across the surface of fracture, as would have been the case by pore pressure over 100% of the surface of fracture. Apparently irregular surfaces of 100% pore area, similar to those in Fig. 34(*f*), are so close together that the entire mass behaves under test merely like a lighter material as a whole, and therefore as one having 100% uplift over the entire surface of fracture even though much of this fracture is through solid material. Now that this has apparently been proved, it does not seem strange, although the writer would not have ventured such a prediction. (Incidentally, this is an important concession on the writer's part to the value of research.)

Thus it would appear that fracture follows the weakest section including aggregate pieces, where such pieces are weaker without pores than the cement paste with approximately 50% pores; but evidently the surface determining the effect of pore pressure on strength will be the surface incorporating the greatest pore area. Thus the two surfaces are determined by two different factors, one by physical weakness and one by pore area. Concrete therefore behaves with reference to shear fracture, when subjected to pore pressure, as if this pore pressure were exerted over 100% of the area. It would seem even more likely to do so under tensile stresses. The writer is greatly indebted to the Bureau of Reclamation and to Mr. McHenry for this proof.

One of the most common points raised in opposition to the writer's theory is that the concrete is too dense for the water pressure to penetrate except near the water surface. Consider only horizontal joints which are known as likely to develop seepage rapidly. In his paper the writer referred only briefly to the fact (in the fourth paragraph following Eq. 12) that only joint pressure needs to be considered; but Mr. Fucik has shown more clearly in his discussion that it is necessary only to consider joint seepage. His discussion requires some amplification. Referring to Fig. 9(*b*), assume that the elementary vertical cylinders extend from line AB to line DE (two successive construction joints), instead of extending only from pore to pore. The difference in pressure in the two joints is represented by line C′ B′ EF. From D to B′ the difference in pressure head is constant and equal to the height of the columns. From point B′ to point E the pressure in joint DE reduces to zero but so do the elementary columns which, in this section terminate in air at the face of the dam. The difference in pressure is still equal to the height of these columns. The principle of buoyancy of columns is still correct if each column terminates in a pore or pores in the construction joint at the top and bottom of a column. Since only microscopic departure from the nominal surface of the construction joint is necessary to incorporate a full projected area of pores, it would not seem questionable that pores close to the joint will reflect the pressure existing in the joint and therefore existing on the top or bottom of an elementary column. In fact, it is necessary only to assume two surfaces like those in Fig. 34(*d*), one in each construction joint, whereas it would be reasonable to expect many, as in Fig. 34(*f*).

Furthermore, it is necessary to consider that only one horizontal construction joint is subject to seepage such as AB, Fig. 9(*b*), instead of two. The joint pressure at point A is h and at point B is zero. A straight-line reduction between points A and B obviously results in full uplift of the assumed triangular dam if the area over which the pressure is effective is 100%. As before, it will be 100% on a rough surface incorporating a full projected area of pores.

Thus one may assume the entire body of the concrete as saturated, which is equivalent to assuming contiguous surfaces as in Fig. 34(*f*). On the other hand, one may assume elementary vertical cylinders between these contiguous surfaces, only the construction joints to be under pressure with like surfaces perhaps 5 ft apart or only one such surface in one construction joint or the foundation contact. The net results are the same in all cases—that is, uplift equivalent to buoyancy of the concrete. The entire theory herein presented is necessary to support the existence of 100% uplift at construction joints even if impervious concrete between joints were to be conceded.

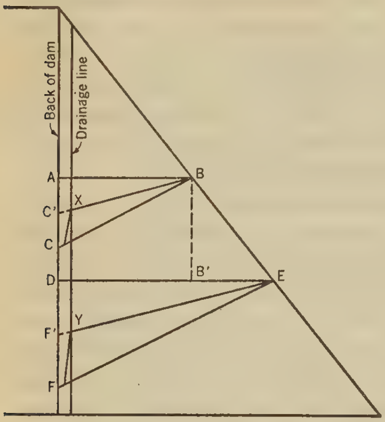

FIG. 36

Many of the points raised in discussion are believed to have been already answered herein, but a few remarks about individual discussions are in order and will be made in the order of appearance of the discussions in *Proceedings*, and only when not previously answered.

It is orthodox in these days to assume, as Mr. Creager does, that drainage near the back of a dam will reduce the uplift. The rule he gives is seemingly rational when considering any one horizontal section alone; but the important fact which deserves recording is that the reduction in magnitude of uplift pressure, if it occurs at all at adjacent joints, does not necessarily reduce the uplift force materially, if at all. This is because uplift is a differential force analogous to buoyancy, and the buoyancy of column ab, Fig. 1, would not be increased or decreased by a difference in the depth of submergence.

Thus refer to Fig. 36, which is similar to Fig. 9(*b*), with similar triangles ABC and DEF representing uplift at two elevations in the dam, precisely as in Fig. 9(*b*). It has been shown that the differential pressure between construction joints AB and DE is represented by the difference in the pressure lines CB and FE and is constant and equal to the difference in elevation of the sections or an uplift equal to full buoyancy.

Now suppose that drainage wells were to be provided near the back of the dam which would tend to reduce each pressure to one-half magnitude at the line of wells while remaining at full value at the back line of the dam as in lines CXB and FYE, much as recommended by Mr. Creager. Lines XB and YE would still be parallel, or substantially so. The differential pressure between elevations AB and DE would still be constant and equal to their difference in elevation; and therefore the lines indicate full buoyancy between the two elevations AB and DE to point B′ directly under B. From point B′ to point E it would be one-half buoyancy; but on 5-ft lifts this would be, perhaps, only 4 ft of a thickness of dam of the general magnitude up to 300 ft or 400 ft and thus would be negligible, disappearing entirely in considering only differential elevations.

The same conclusion applies for any other uniform, proportional reduction in uplift magnitude at two near-by elevations. Thus Mr. Creager's drainage analysis, theoretically, applies only if no more than one construction joint or rock contact is under pressure. In order to accomplish its purpose, drainage would have to be increasingly effective with increasing depth. For example, assume that drainage were to prove totally ineffective at elevation AB, thus maintaining the full static pressure line CB, and that it were to prove one half effective at elevation DE, thus reducing the pressure diagram to FYE, Fig. 36. Then the uplift diagram would be the area between line CB and line YE, indicating a reduction in buoyancy (uplift) near the back, where this result is most needed, gradually increasing to full buoyancy near the face. Therefore, drainage of the foundation, or drainage into the concrete near the base, is more effective than drainage for the entire height. Since drainage by wells accomplishes only partial relief, as in Fig. 36, its value, if any, is greatly overrated. To be fully effective, instead of wells, there should be a complete layer of drained, porous material near the back of the dam which will drain all the water entirely or there should be a sheet metal diaphragm to exclude it, as suggested by the writer in 1934.[2] The latter would preferably be a sheet rolled in a dovetailed section like some roof or floor metal now available, and of stainless steel or of other similar rust-resisting metal.

Messrs. Kendrick (Fig. 12) and McNown (Fig. 23(*a*)) have both shown flow nets for the simplified triangular dam with water at the apex, which the writer uses as the basis of analysis. These differ very greatly. Since there can be only one correct flow net, the writer would select that of Professor McNown, because Mr. Kendrick's net does not satisfy the important boundary condition that the drop in velocity potential (piezometric head) between adjacent lines of constant potential must be a constant. The pressure is atmospheric along the uppermost streamline, and therefore a constant increment in piezometric head can be obtained only if the increment of elevation is constant, a condition satisfied only by Professor McNown. This also requires correction of the uplift diagram ABU, Fig. 12, which then becomes triangular.

Mr. Albery admits full uplift on horizontal surfaces such as in a gravity dam, but limits lateral pressure on vertical surfaces parallel to lines of seepage flow to the percentage of porosity, say, 12%. In the ideal dam, with all seepage proceeding at right angles to the axis, the writer would exclude it entirely as the pressure would be equal on both sides of such a surface. Professor McNown has shown (Fig. 27) that it does exist where monolith joints divert seepage laterally. The pressure on vertical planes perpendicular to the lines of flow is the effect described by the writer as the progressive application of the water load through the structure instead of on only the back of the dam, each decrement in seepage pressure representing an increment in load. The writer cannot agree with Mr. Albery's Eq. 14 and maintains that Eq. 13 is the correct one for a bending moment about C due to pore pressure at section BC. However, it is not a complete solution of the bending moment about C since external forces and their moments about C would need to be included when considering the part of the wing above line BC as a free body. This was introduced merely to illustrate one prominent instance of false conventional reasoning.

The writer does not condemn this type of dam; in fact he considers it a very good one. He believes that none of the precautions indicated by Mr. Albery, nor the fears expressed by Professor Ballester, are justified. The writer once had occasion to examine such a dam nearing completion. The rapid external cooling had formed cracks along the cylindrical surfaces of the buttress heads and parallel to the axis of same, and a space of 1 in. existed between adjacent buttress heads, this space being provided with a copper seal. This situation was obviously very dangerous but was corrected merely by filling the 1-in. spaces. It was found that any spreading force due to pressure in the cracks or pores was thus resisted against the adjacent buttress heads, an ample precaution.

Mr. Cotton's report of the condition of the Manchurian dam is very interesting and his determination, by flow nets, of the seepage and uplift condition shows advanced thinking. The condition of this dam is very instructive especially as to the necessity of rigid field inspection during construction. It probably reflects the result of that major engineering mistake of divided responsibility, being designed perhaps by competent engineers who did not have control over construction.

Mr. Leliavsky's artful treatment of the writer's paper deserves revelation of its true significance. As a history and textbook on engineering it would qualify as covering the subject from Galileo to just prior to the writer's paper. According to him everything in the paper is wrong, basically erroneous, full of flaws, not clearly defined, neither standard nor rational, dependent upon unorthodox stratagems, imaginative, confusing, uninformed, etc. To prove his points he has subpoenaed as witnesses the great men of the past back to Galileo and, for some reason, has even invoked the German language. Yet, after all this colorful phrasing Mr. Leliavsky, with very minor exceptions, reaches exactly the same conclusions as those upon which the writer relies except those as to the nature of the uplift surface and the interpretation of his experiments.

For example, in Fig. 19 and Eqs. 15, 16, and 18, he arrives at precisely the same conclusions (except in different symbols and words) as does the writer from Fig. 5 and subsequent analysis, stating that:

> "* * * the effective weight of concrete above those elementary columns, and hence those areas, in such intimate contact with an impervious foundation as to exclude water pressure [column A, Fig. 5, or Fig. 19(*c*)], is the full dry weight of concrete plus pore water, or the saturated weight of concrete, whereas the effective weight above areas not in contact [column B, Fig. 5, or Fig. 19(*b*)] is only the buoyant weight of concrete."

However, by using different words, symbols, and drawings he would have the reader believe that he has proved something different. He also neglects to carry his analysis one step further to show that the column in Fig. 19(*c*) becomes buoyant as it does in Fig. 19(*b*) if the surface xy were a rough one passing through the lowest pore. In addition, he fails to admit, as shown by the writer, that no water-exclusive contact (which would have to be on glass, rubber, or metal) envisioned by column A, Fig. 5, or by Fig. 19(*c*), can possibly exist in natural foundations, which are always on porous material, and also that the condition in Fig. 19(*b*) is therefore the only applicable condition. Eq. 19, although interesting, is only of mathematical interest for it is based on a theoretical assumption that cannot exist in nature. Likewise most of the discussion is concerned with mathematical elaboration of no bearing upon the points of controversy and only tending to make a very simple problem seem complicated and confused.

Mr. Leliavsky deserves credit, however, for pointing out one fact not always clear in the writer's treatment of buoyancy. Although the writer is correct in stating that the effective weight is the buoyant weight of the concrete, yet (as Mr. Leliavsky points out) the volume of material above any datum to be made buoyant is not the gross volume but the volume reduced by voids or the net displacement. This is correctly stated in Eq. 6, but frequent references to "full buoyancy" must be interpreted as full buoyancy of the solid material of the concrete, not of its gross volume. Thus, 1 cu ft of concrete with a dry weight of 152.5 lb and 12% voids would have a buoyant weight, not of $152.5 - 62.5 = 90$ lb, but of $152.5 - 0.88 \times 62.5 = 97.5$ lb.

If Mr. Leliavsky were to concede the existence of, and the right to consider, a surface, no matter how rough, incorporating a full projected area of pores as in Fig. 34(*d*), he would thus exactly agree with the writer. This is the only argument between the two; since the writer has hereinbefore fully explained his viewpoint and since Mr. Leliavsky depends on his experimental determination of this area, it is next in order to examine these experiments.

The Leliavsky tests[30] involved a very clever principle (new to this problem) of graphically solving an equation containing two unknowns, z and n_w, by creating several simultaneous equations (for graphical solution) through the variation of the load N, Fig. 20. This principle is well described by reference to Mr. Leliavsky's paragraph preceding Eq. 22 and by reference to Fig. 20, except that previous experimenters omitted the load N. Herein lies the principal reason (together with the central hole) why he was able to obtain much more consistent results than previous experimenters.

These tests were conducted, very scientifically with utmost attention to every detail to secure a very high degree of accuracy. As such, they are precisely what they purport to be—namely, a vast improvement in technique over previous tests, almost if not quite to the degree of possible perfection as compared with Föppl, Woodward, Rudeloff, and Panzerbieter, but nevertheless predicated upon the same basic assumption that pore pressure at mn, Fig. 20, causes the fracture. Under these circumstances and with a feeling of admiration for the thorough experimental technique used by Mr. Leliavsky, it is with the utmost regret that the writer finds it unavoidable to differ with the fundamental conception upon which the tests of all the aforementioned gentlemen were based and which was inherited by Mr. Leliavsky from his predecessors.

Briefly, it is claimed very clearly in his original paper[30] that pore pressure at section mn breaks the specimen and that, considered as a free body, above the surface of fracture mn, at the moment of failure, the only force acting in an upward direction is the uplift force of the water pressure in the pores of the section mn, determined by the average hydrostatic pressure p_m applied to an unknown proportion n_w of the geometrical area of the plane section A_s—this value of n_w to be determined by the experiment. This uplift force is opposed by the tension z in the specimen and by the sum of all downward forces F, principally mechanical, or (see Eq. 22) $n_w A_s p_m = F + z$.

Mr. Leliavsky states (necessarily inadvertently), in the paragraph preceding Eq. 22, that:

> "* * * $n_w A_s p_m$ is also the uplift force transmitted by the percolating water, through pore action, to the porous material in the top of the specimen [seepage force above kk] * * *."

If this uplift force acts both at mn and above kk, Eq. 22 would necessarily read $2 n_w A_s p_m = F + z$. Mr. Leliavsky is not quite as clear and concise on this point in his discussion as he is in the original paper;[30] but it must be assumed that he still considers that the pore pressure at section mn breaks the specimen —otherwise there would be no relation between breaking strength and n_w at section mn which he claims to determine by the experimental strength.

Obviously this uplift force cannot act both at mn and above kk. The writer holds that there is absolutely no uplift force at section mn, and the only uplift force is the seepage force above kk. This need not have any relation to the "superficial effective porosity" n_w at section mn. It is related only to the mechanical strength of the specimen at mn. Considering the specimen above mn as a free body there can be no force resulting from pore pressure except as, and where, a differential pressure occurs—that is, above kk. A poreless bar of metal, or any other tension tie between kk and LL of the same tensile strength as the concrete specimen at mn, could be substituted for the specimen. If an imaginary material of the same cross section as the specimen, without pores but of the same tensile strength, could be substituted for the specimen (just far enough below line kk to permit entrance of sufficient water for upward seepage), the specimen would fail, nevertheless, at section mn through solid poreless material if that proved to be the weakest point in the specimen in mechanical tension. Thus the pressure in the pores at mn has nothing to do with the failure, and the failure data have no relation to (and cannot therefore be used to determine) the porosity characteristics of the surface of failure mn.

The following has partly been stated by the writer in the main paper just prior to Eq. 8: Pore pressure is uniform (in a vertical direction) between LL and kk (actually not quite true near kk, but sufficiently so for this purpose). Neither stresses nor strains can exist as the result of uniform pore pressure in any specimen; the condition between LL and kk is substantially as if the specimen between these points were merely submerged in the water. Any break in the specimen between sections LL and kk, therefore, is the result of tension applied above section kk, as if applied by a mechanical tension testing machine attached to the specimen above kk. The total amount of tension thus applied is the measure of the strength of the specimen when subjected to high hydrostatic pressure seeping radially toward the central drainage hole. It has no other significance and is not necessarily indicative of the strength of the concrete specimen in air. The only force of local origin tending to cause elongation of the specimen between sections LL and kk is that resulting from Poisson's ratio for the radially acting compressive force of the external water pressure on the cylinder. (The latter was plainly implied to be of no importance although Mr. Leliavsky magnifies this into a major point.) It exists only in the event of, and to the extent of, any difference between external water pressure and water pressure in the central hole.

Mr. Leliavsky apparently agrees with these basic principles (as witness the fifth paragraph after Eq. 25, and also the pressure diagram to the left of Fig. 20 which shows uniform pressure below kk), but he continues to maintain that his tests determine the "superficial effective porosity" n_w at section mn. The position that n_w, as determined by seepage forces above kk (Eq. 22), is equal to n_w at section mn is necessarily predicated upon assumption. The loads $F + z$ can measure only the forces that cause failure. The true equation (preferable to Eq. 8) should be

$$\text{Upward seepage force above section kk} = F + z \quad \ldots\ldots\ldots (41)$$

The "upward seepage force" is not a function of n_w at any one point, having no relation to mn and little, if any, to kk. It is difficult to visualize the exact physical significance of n_w as determined by these tests. It would appear to be a composite of the conditions existing from kk to the top of the specimen. It might imply the average effective proportion of area over which seepage forces act from kk upward but would not accurately reveal even that value because of inherent faults in apparatus and interpretation. For example, the flow lines necessarily bend sharply around the corner at k and the distribution of flow lines tends to crowd to the outer surface at first; and then, as the seepage progresses upward, the distribution tends to become uniform over the entire cross section, whereas, the assumption by Mr. Leliavsky,[30] in evaluating this distribution, is that it decreases radially on a logarithmic curve from the applied pressure at the outside to the reduced pressure at the central hole. This is true at mn, but nowhere is it true at or above kk, and this is further evidence that the rupture is considered by Mr. Leliavsky to be the result of pore pressure at mn. If the central tubular metal lining of the hole were to extend well below kk, it would seem to result in nearly uniform distribution at kk, equal to applied pressure, before the water begins to seep upward. Thus, the aforementioned sources of error might be nearly eliminated, permitting uniform distribution to be assumed regardless of the initial difference between outside and central pressures.

It is the writer's opinion that seepage forces also act over 100% of the area, as in the case of uplift forces, and as discussed with a reference to Figs. 3 and 4, but the aforementioned sources of error would be sufficient to obscure this result.

Another very important observation must not be overlooked; perhaps it is the most important of all. The tests are shown to measure seepage forces and these are horizontal (or nearly so) in a dam (Fig. 23(*a*)), whereas uplift forces in a dam are normal to seepage forces and would not be revealed by these tests.

For all the foregoing reasons the writer claims that the Leliavsky tests are incorrect in principle and do not disprove or weaken the 100% theory in any respect.

Mr. Leliavsky does concede one, and only one, "acceptable conclusion" (that the pressure against a dam is exerted progressively along the route of seepage); but he claims this to be an old story. This is quite possibly true but it was new to the writer. It has been promulgated by the writer since 1936 when this paper was prepared.

Mr. Davison, in Fig. 22, predicated his analysis on the basis of a hydraulic grade line dropping more steeply than a straight line to tailwater and thus concave upward. This is in very radical disagreement with Professor McNown's Fig. 23(*a*). The latter is believed to be more scientifically supported and the correct flow net for a triangular dam. The difference with respect to Fig. 23(*a*) would not be great when considering a dam with a normal top width. Fig. 23(*b*) is also believed to be a better picture of seepage through a rectangular dam than Fig. 22(*b*).

The writer does not believe that the compaction of sand, for normal loading, materially influences the permeability coefficient, K, as claimed; it certainly would not affect rock or concrete permeability. He is indebted to Professor McNown for correct seepage patterns in triangular and rectangular dams (Fig. 23), as well as for clearing up a number of other theoretical viewpoints, and eliminating Fig. 9(*c*) as incorrect for a triangular dam.

Mr. Boughton's forceful discussion would make the writer wince if it were not at fault. If he will refer to Fig. 9(*b*) and additional discussion by the writer in this closure he will note that full static pressure is not claimed to exist at the exposed face. The pressure reduces to zero at the face, which is not inconsistent with the fact that the face appears dry—nor is it inconsistent with the fact that the height of the column to be made buoyant by internal pressure also reduces to zero along with the pressure. Furthermore, only pressure, in the horizontal construction joints, is needed to support the writer's theory. As to the weight of water in the pores, adding instead of subtracting from the weight, he is referred to column B, Fig. 5 and related analysis following Eq. 4. This is also confirmed by Mr. Leliavsky's Fig. 19(*b*) and Eq. 15.

There is no reversal point as claimed, and there could not be, except by applying a vacuum to the face. It requires internal pressure all the way to the face to keep seepage moving and this pressure is commonly assumed to reduce along a straight line as in Fig. 9(*b*), at least in the construction joints.

Mr. Boughton misquotes the writer (by partial omission regarding internal pressure) as saying that "* * * pressure seldom exists 'except near the upstream face' * * *." The correct quotation is that "* * * theoretical pressure [meaning full straight line pressure as in Fig. 9(*b*)] probably seldom exists, except near the upstream surface * * *".

Mr. Boughton is referred to Professor McNown's Fig. 23(*a*) which shows that seepage lines are theoretically horizontal in a triangular dam instead of being "quite steep" as he claims. Moreover, contrary to his statement, the porosity has nothing to do with maintaining "such a line of saturation." The flow net for any homogeneous material in an earth or concrete dam of any cross section that obeys Boyle's law (and all granular materials substantially do) is identical for an identical shape of section no matter how porous it may be. All that the porosity does is to determine the quantity of seepage; it does not fix the steepness of the saturation line, or any other geometrical characteristics of the flow net.

The writer has nowhere made the claim contained in Mr. Boughton's last paragraph except as to the word "illogical." He does not maintain that existing design practice is necessarily unsafe but that it is illogical. He has listed mitigating circumstances to explain why dams thus designed have proved safe, and more could be added; but there is no reason why engineers should persist in one error just because there are other errors that may compensate. The right policy is to correct all errors, not just those with respect to uplift.

In reference to Mr. McHenry's discussion, the writer cannot visualize any relation between either loose concrete cylinders, or chairs piled one on top of one another, and the problems under discussion.

Mr. Hotes admits an uplift area of substantially 100% but claims exactly 100% to be physically impossible. He raises the old argument that 100% cannot be true because some area of contact—no matter how small—must remain to support the load, as quoted by the writer on the first page of his paper, and the refutation of which forms the one and only basic point of this entire paper. Obviously the writer's efforts have been wasted if he cannot get this one idea across to one of scientific mind. Practically, it does not matter whether the uplift acts over 100%, 99%, 97%, or even 91%, but, as a matter of principle, it does. Mr Hotes is again referred to Fig. 35 and the related discussion.

In a volume made up of cylinders as thus depicted, the uplift area is exactly 100% and yet the contact areas are large. The student needs to exert no stretch of the imagination to be convinced of the strength of this mass in tension, compression, or shear. No theory of microcontacts requiring tensile stresses of from 36,000 lb per sq in. to 400,000 lb per sq in. has to be invented to explain the large uplift area. The mass does not need to float on water, as indicated by Mr. Hill.

The writer is indebted to Messrs. Carlson and Davis for pointing out the small size of pores as compared with common conception concerning them. The experiments made at the University of California, in Berkeley, are interesting indeed. They would seem to indicate that in very dense, perfect concrete the hydration process absorbs initial seepage, and after a few years the pressure becomes zero a few feet from the entrance. It is puzzling, however, to note that in specimen 1, even after seven years, when the pressure reads zero from distances of 2 ft to 8 ft, there is still a steep gradient from the entrance to point 1.5 ft. The anomaly is that this gradient cannot exist without flow.

Unpublished experiments on seepage through concrete specimens are said to have been made at Zanesville, Ohio, during the construction of several dams in the Muskingum Valley. With a dry discharge surface these specimens refused to pass any water, but, when kept wet or submerged at the outlet end, free seepage resulted. These results apparently are related to the Berkeley tests. If capillary pressure resists or prevents seepage, then its effect might respond to rain or dry weather and also differentiate the part below tailwater from that above tailwater. This is a subject upon which the writer has no opinion; and it is not within the scope of this paper. It would make an excellent subject for field study. The apparent impervious character of these specimens, of course, has no bearing on the problem of seepage through joints and foundation contact.

Regarding the last paragraphs of the Carlson-Davis discussion referring to drainage, the reader is again referred to Fig. 36 and related discussion showing that drainage is not necessarily effective.

If the writer understands the Morrison discussion correctly uplift is considered to be equivalent to any other foundation reaction and is not a source of instability. This is true, of course, of Mr. Morrison's basement floor slab, provided that the downward load exceeds the upward load. It matters not whether the load of the building is supported entirely by concrete-to-earth compressive load or partly by uplift. However, this cannot be true of a dam where horizontal loading is involved. The shearing or sliding resistance is not a function of the total reaction including uplift as indicated by Mr. Morrison, but is a function only of the mechanical pressure, grain to grain, of concrete upon the foundation material. Uplift certainly reduces this pressure, and therefore the resistance to horizontal movement. The theory presented is a very dangerous one.

Nothing in the writer's theory, stated or remotely implied, justifies the weight of criticism in the first paragraph of Mr. Hill's discussion. The writer's theory certainly does not require any weight to be supported entirely by water, and yet Mr. Hill, in his sixth and seventh paragraphs, permits load to be carried by water in lieu of particle-to-particle contact of concrete, a very dangerous and incorrect theory and a direct reversal of the attitude expressed in his first paragraph.

Pore pressure definitely reduces the factor of safety against overturning as well as sliding, and Mr. Hill, not the writer, is the only one to deny it. Internal buoyancy is very real; a wet sponge sinks because the individual fibers are heavier than water—a very simple principle.

In the second paragraph of his discussion Mr. Spielman makes an interpretation directly contrary to the writer's statements. Also, he joins other discussers in criticizing the words "internal buoyancy." In no place does the criticism seem to be applicable to the principle, but only to the words adopted to describe it. The writer considers the words to be accurately descriptive. It merely means that the buoyant force is applied in voids and not entirely on the underside of the submerged body as is so often considered.

Mr. Spielman states:

> "If the body were porous, with interconnected pores, it would not displace its gross volume but only the volume of impervious matter, assuming, of course, that sufficient time is allowed for the full penetration of the fluid into the pores."

The foregoing is a perfect definition of "internal buoyancy" and shows that Mr. Spielman agrees exactly with the writer on this subject, except as to terminology; and he agrees exactly with Eq. 6 which is one of the most fundamental points of the entire paper. However, when Mr. Spielman declares, after Eq. 39*b*, "Now, because the voids are filled, the effective area of uplift is $(1 - n)A$, which is something less than 100%" he makes an unwarranted deduction. What he should state, to convey his true meaning (and also that of his statements following Eq. 40) is that, if the submerged object is to be considered as a whole using its gross volume and ignoring its porosity of 12% and ignoring the uplift exerted in the pores, then the uplift should be applied to only 88% of the bottom area to make it applicable to net displacement and thus compensate for ignored porosity. This is a very good reason why the writer uses the term "internal buoyancy"; it is applicable only to net solids and requires no such principle of compensating errors.

Professor Raes is also confused by the term "internal buoyancy," although he seems to confirm the phenomenon. Assume that the porous object in Fig. 35(*a*) replaces the solid object in Fig. 1. Then a vertical element as ab will intersect the pores. The buoyancy of the object as represented by the sum of many elements such as ab will no longer reflect the behavior of continuous elements, such as a to b, but of several short elements with intervening pores between and with total over-all length of a to b including pores. Each of these pores will be under pressure corresponding to its depth below the water surface and each part of the element ab will be individually buoyant because the pore pressure on the bottom will be greater than on the top. The buoyancy of the object as a whole then becomes the sum of the buoyancy of all the short vertical cylindrical elements. No one has suggested a better name than "internal buoyancy" to describe this situation. It is accurately and perfectly descriptive of what takes place. All the buoyant force is not applied at the bottom, but internally—hence the term. The usual conception of buoyancy is that all the uplift force must be applied at the bottom, which is correct for a poreless solid or a ship's hull, etc; but in the case of a porous submerged object it is obviously applied throughout the mass, if the pores are large enough, and if enough time elapses for the pressure to penetrate the pores. These conditions, of course, are assumed to exist.

Admittedly, internal buoyancy is the same as buoyancy of the gross volume with the bottom area multiplied by $(1 - n)$; but in establishing a principle it is not enough to define something that is merely equivalent.

Professor Raes seems to object principally on the grounds that the surface treated by the writer is not the surface of rupture. That this thesis is not concerned with the surface of separation or rupture has already been amply demonstrated. The weight of that part of the body above the rough surface, in Fig. 34(*d*) or Fig. 35(*c*), is reduced by buoyancy, and, unless the specific application requires and anticipates failure (which it seldom, if ever, does in uplift problems), then this surface is immaterial. Moreover, it seems to have been shown by the McHenry tests that failure, even in shear, is influenced by uplift over 100% of the area, although not along the surface conceived by the writer.

Some discussions would lead one to believe that modern dams are practically impervious even at horizontal construction joints. That is debatable, but it is not the question under debate in this paper. If true, why is uplift still considered? Allowance for uplift is still a part of design technique in even the most modern engineering agencies, usually assumed to act over two thirds, or like proportion, of the area. The theme of this paper is that it acts over 100% of the area. Any discussion of other matters is extraneous to the purpose of this paper.

The writer was once employed to place a new face, about 2 ft thick, on a gravity dam the surface of which was badly disintegrated. The new slab was certain to be much less pervious than the old concrete; and, although this new work was anchored back into the dam with sufficient bars to resist the entire static pressure, it was nevertheless desirable to drain known seepage through this new slab to prevent the building up of serious pressure behind it. The dam faced south and in daytime, with the surface moisture subject to evaporation, the surface of the dam seemed dry except in a few isolated places. To locate the areas of slow seepage it was necessary to inspect the face in the morning shortly after daylight. Many more seeping areas were then visible, including construction joints.

This situation is confirmed by Professor Ballester in connection with the driving of a tunnel into Dique San Roque in Córdoba, Argentina. An apparently dry face as the result of evaporation was found to be deceptive and the interior "weeped" when exposed. This also agrees with the writer's quotation from the paper by the late A. J. Wiley, M. ASCE.[9]

Undoubtedly very great improvement has been made in dam building technique such as in low water-cement ratio, aggregate mixtures requiring less cement, low heat cement, cooling of aggregates and water, use of ice, circulating cooling pipes, thorough cleaning by air and water of construction joints between pours, and grouting of contact areas; but the answer is that, even with this technique, bad contacts occur and consideration of uplift has not been abandoned. Therefore, the theory of uplift should be on a correct and realistic basis even if some other error compensates.

By common practice lifts are about 5 ft thick. One lift is allowed to cool from three days to five days before being covered with the next lift. In one of the most modernly constructed dams the writer observed frequent hair cracks on the surfaces of each lift before receiving the next, indicating shrinkage. The new lift is thus placed on top of a surface already partly cured, cooled, and shrunk. The new lift will tend to exert a shear stress at contacts that will require strong restraint to prevent creeping. The writer was told that actual lifting and opening of hair cracks around the edges of a pour at the construction joints had been observed at this dam due to warping of the new pour by more rapid cooling of the surface, much as a board curls up at the edges lying flat in the sun. The movement was very small and would substantially close again; but nevertheless it indicates the possibilities of imperfect contact. The only logical course to follow by those who claim that uplift can now be prevented is for them to show the courage of their convictions and omit it from consideration in dam design. The writer knows of no one who has this degree of courage and the writer is not one of those ready to concede omissions of uplift. The entire theory herein presented is necessary to support the existence of 100% uplift at construction joints even if impervious concrete between joints were to be conceded.

The writer makes no pretense of having solved all the problems of dams. His central theme is merely that uplift acts over 100% of the area. To prove this point it is not necessary to attack all the other numerous illusive problems. He has therefore adopted for illustration the most common orthodox assumption—that the magnitude of internal pressure in pores or joints varies from reservoir pressure at the back to zero or tailwater pressure at the face—well aware that this assumption is subject to modification. He also assumed a homogeneous dam without cracks or joints where all water seeks the face to escape (for example, the flow net in Fig. 23(*a*)). The writer is well aware that such a perfect condition does not exist in practice but the assumption aids and simplifies theoretical treatment. Adoption of these and other orthodox assumptions is not to be interpreted as endorsement but merely the avoidance of controversy not pertinent to the central theme of the paper and not within its proper scope. (However, these assumptions seem to have increased rather than avoided controversy.) For the same reason it does not seem within the proper scope to reply to all points brought out in discussions which are irrelevant to this central theme even though the writer was himself guilty of some such departure.

There are many puzzling problems regarding dams. When water was admitted against the Norris Dam of the Tennessee Valley Authority (TVA), it deflected upstream against the water pressure instead of downstream. Was this due to deformation of the reservoir bottom by the added water load, or to cooling of the back of the dam by water while the face remained exposed to the sun? Let the reader answer. Also, Mr. Riegel has shown[44] from observations on TVA dams that delayed central cooling apparently partly relieves central loading and transfers it toward heel and toe. This appears to be one of the most significant facts to come to light in recent years and may go far toward explaining the apparent stability of sections considered deficient. The Carlson-Davis discussion might imply that very great capillary forces exist in a dam capable of exerting a major influence in its behavior.

Likewise there is reason to believe that temperature, cooling, shrinkage, elastic deformation of dam and foundation, plus the foregoing enumerated effects, may even outweigh in importance the forces, stresses, and strains from water load, uplift, and foundation reactions which are the only ones considered in orthodox design practice.

The fact is that very little is known, scientifically, about the design of gravity dams and the engineer's confidence in practical rule-of-thumb methods is based largely upon the good behavior of dams thus designed. Little by little, one problem at a time may be fully studied and better understood until the art may become a science. The writer can only hope that he has contributed to a better understanding of one of the perplexing problems.

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THE RELATIONSHIP BETWEEN PILE FORMULAS AND LOAD TESTS

By Robert D. Chellis,[1] M. ASCE

With Discussion by Messrs. Donovan H. Lee, Jacob Feld, Richard F. Kafka, G. B. R. Pimm, George D. Hartley, W. Mack Angas, and Robert D. Chellis.

Synopsis

Pile-driving formulas are often used as the sole criterion of pile bearing capacity, without load tests. When load tests have been made, the results often have been at variance with the computed values. Consideration of the physical properties of soils and piles, and of the interpretations of terms in the formulas, indicates the reasons for such discrepancies, and also reliable means of avoiding them. Driving resistances bear definite relationship to static load resistances only in soils in which the predominating resistance is supplied by cohesionless materials. A study of forty-five cases in soils of this type, in which test load failure values and resistances computed from various formulas in use are compared, indicates the unsatisfactory spread of values obtained from the most common abbreviated formulas and the generally satisfactory results from the more comprehensive formulas. Use of a static formula for preliminary determination of pile lengths and as a check in all other cases appears advisable.

Introduction

Is there any relation between bearing capacity of a pile and its driving resistance? This question has long caused controversy. Scores of dynamic, static, and empirical formulas have been offered.

Dynamic formulas attempt to equate the kinetic energy of the hammer blow to the set (the amount of penetration of the pile tip under one blow of the hammer), or resistance of the soil to penetration. Static formulas equate the load-carrying capacity of the pile to the bearing and shearing values of the soil or to friction between pile and soil. Empirical formulas are usually based on the results of tests for limited conditions, or on the strength of the pile as a

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Note.—Published in May, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Structural Engr., Stone & Webster Eng. Corp., Boston, Mass.

post without relation to driving or soil conditions. The formulas in general use are of the dynamic type.

This paper does not purport to derive the various formulas, nor to analyze their mathematical merits. The formulas may be considered as theoretical or empirical in fact, if desired. It is the object of the paper to perform the service of comparing the results given by the most common formulas against those obtained from load tests. This will permit forming some opinion as to the spread of values which may be expected, and as to the actual factors of safety which may result.

DYNAMIC FORMULAS

The dynamic formula most commonly employed up to the present in the United States is the so-called "Engineering-News" formula. This formula received its name from the fact that it was proposed more than half a century ago by an editor of that magazine and appeared in its pages.[2] The Eytelwein formula often has been used with heavy piles. Modifications of the Engineering-News and the Eytelwein formulas, and the Navy-McKay formula have often been recommended for use with heavy piles. These modifications consist of increasing the last term in the denominator.

In England, the Hiley formula[3] is specified by the Institution of Structural Engineers, in London, and is in common use, although with the value of L considered as the full length of the pile.[4] It is also specified in slightly modified form in the Canadian National Building Code. Another modification of this formula is given in the Pacific Coast Uniform Building Code, and it is also specified in some form in other codes.

For convenience and comparison these formulas are now shown together. The symbols used are defined in the Appendix.

The Engineering-News formula, for drop hammers, is

$$R = \frac{2\,W_r\,H}{s + 1.0} \qquad \text{(1)}$$

for single-acting steam hammers,

$$R = \frac{2\,W_r\,H}{s + 0.1} \qquad \text{(2)}$$

and, for double-acting and differential-acting steam hammers,

$$R = \frac{2\,E_n}{s + 0.1} \qquad \text{(3)}$$

Eq. 3 has come to be used with double-acting hammers, not on the basis of reasoning, but because it was readily available and simple. Mathematically, Eqs. 1, 2, and 3 are based on inclusion of a safety factor of 6.

The Eytelwein formula is, for drop hammers,

$$R = \frac{2\,W_r\,H}{s\left(1 + \dfrac{W_p}{W_r}\right)} \qquad \text{(4)}$$

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[2] "Formulae for Safe Loads of Bearing Piles," by A. M. Wellington, *Engineering News*, December 29, 1888, pp. 509–512.

[3] "Pile Driving Calculations with Notes on Driving Forces and Ground Resistance," by A. Hiley, *The Structural Engineering*, July and August, 1930, pp. 246–259.

[4] "Specification for Concrete Pile-Driving," The Institution of Structural Engineers, London, 1936.

For single-acting steam hammers, Eq. 4 becomes

$$R = \frac{2 W_r H}{s + 0.1 \frac{W_p}{W_r}} \qquad (5)$$

Mathematically, Eqs. 4 and 5 also are based on inclusion of a safety factor of 6.

TABLE 1.—ALLOWANCE FOR TEMPORARY COMPRESSION FOR HEAD OF PILE AND PILE CAP, C_1, IN INCHES

Description	STRESS ON DRIVING CAP (LB PER SQ IN.)			
	500	1,000	1,500	2,000
Head of wood pile	0.05	0.10	0.15	0.20
Pre-cast concrete pile, 3-in. to 4-in. packing in cap	0.05[a] +0.07[b]	0.10[a] +0.15[b]	0.15[a] +0.22[b]	0.20[a] +0.30[b]
Pre-cast concrete pile, ½-in. to 1-in. pad on pile	0.025	0.05	0.075	0.10
Steel pile, wood packing with steel cover	0.04	0.08	0.12	0.16
Fiber disk $\frac{3}{16}$ in. thick between steel plates on steel pile	0.02	0.04	0.06	0.08
Head of steel pile	0	0	0	0

[a] Compression of cap and dolly above. [b] For packing below cap.

The Hiley formula, for drop hammers and single-acting steam hammers, is

$$R_u = \frac{e_f W_r h}{s + \frac{1}{2}(C_1 + C_2 + C_3)} \times \frac{W_r + e^2 W_p}{W_r + W_p} \qquad (6)$$

and, for double and differential-acting steam hammers,

$$R_u = \frac{12 e_f E_n}{s + \frac{1}{2}(C_1 + C_2 + C_3)} \times \frac{W_r + e^2 W_p}{W_r + W_p} \qquad (7)$$

in which

$$C_2 = \frac{R_u l}{A E} \qquad (8)$$

and values for C_1 and C_3 are given in Tables 1 and 2, respectively.

The Pacific Coast Uniform Building Code formula is

$$R_u = \frac{12 W_r H \frac{W_r + K W_p}{W_r + W_p}}{s + \frac{12 R_u L}{A E}} \qquad (9)$$

in which $K = 0.25$ for steel piles and 0.10 for other piles. A safety factor of 4 is to be applied to the result to obtain the safe working load.

TABLE 2.—TEMPORARY COMPRESSION ALLOWANCE[a] FOR QUAKE OF GROUND, C_3, IN INCHES

STRESS ON TIP[b] (LB PER SQ IN.)			
500	1,000	1,500	2,000
0 to 0.10	0.10 to 0.20	0.10 to 0.30	0.05 to 0.20

[a] These values may be double if the strata underlying the pile tips are very soft. [b] Cross section of bounded area for H-pile.

The Canadian National Building Code formula is

$$R = \frac{4 W_r H n}{s + (\frac{1}{2}) C} \qquad (10a)$$

in which, for friction piles,

$$n = \frac{W_r + e^2 W_p}{W_r + W_p} \quad \text{(10}b\text{)}$$

for refusal,

$$n = \frac{W_r + 0.5\, e^2 W_p}{W_r + W_p} \quad \text{(10}c\text{)}$$

and

$$C = \frac{3R}{A}\left(\frac{l}{E} + 0.0001\right) \quad \text{(10}d\text{)}$$

The resulting value is for drop hammers with triggers and should be multiplied by 0.9 for single-acting steam hammers, and 0.8 for drop hammers with winch drag. Eq. 10*a* gives safe working loads, predicated upon a safety factor of 3.

The modified Engineering-News formula, for single and double-acting hammers only, is

$$R = \frac{2 W_r H}{s + 0.3} \quad \text{(11)}$$

Mathematically, Eq. 11 is based on inclusion of a safety factor of 6.

The modified Eytelwein formula, for single and double-acting hammers only, is

$$R = \frac{2 W_r H}{s + 0.3 \dfrac{W_p}{W_r}} \quad \text{(12)}$$

Eq. 12 is based on inclusion of a safety factor of 6.

The Navy-McKay formula, also for single and double-acting hammers only, is

$$R = \frac{2 W_r H}{s\left(1 + 0.3 \dfrac{W_p}{W_r}\right)} \quad \text{(13)}$$

Eq. 13 is likewise based on inclusion of a safety factor of 6.

The Engineering-News formula was developed in the days when all piles were of timber and all were driven with a drop hammer. With the development of single-acting steam hammers an empirical modification was introduced, stated to take account of the soil lubricating action caused by more rapid strokes. It should be noted that this empirical change which affects the answer considerably was made in a purportedly dynamic formula. This latter form also came to be used with double-acting hammers when they were developed, although they are about as widely different from a single-acting hammer as the latter is from a drop hammer.

It has become customary to use the Engineering-News formula without consideration of the qualifications attached to it by its author in his original article.[5] Mr. Wellington accompanied the formula by the statement that it was intended

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[5] "Formulae for Safe Loads of Bearing Piles," by A. M. Wellington, *Engineering News*, December 29, 1888, pp. 511–512.

> "* * * for safe working loads for piles under all ordinary conditions, to be reduced under exceptional conditions (as notably with irregular penetration) but never exceeded unless the pile is known to rest on rock and act as a column * * * [it is] assumed to be sensible and at an approximately uniform rate [of driving]."

It seems to be common practice today to specify the average penetration under the last few blows. Reported values of *s* are not usually accompanied by any statement as to whether or not this figure represented fairly uniform driving over considerable distance, driving over the last few inches, or a sudden reduction in penetration per blow. In this paper it has been necessary to use the reported values, in line with common practice, without such distinctions as may have been intended by the author of the Engineering-News formula. However, this paper is intended to compare results obtained by everyday methods and practices.

The Hiley formula contains terms representing the temporary compression of the cap, pile, and ground, impact loss, efficiency, coefficient of restitution, and weight of pile—not appearing in the Engineering-News formula. The Hiley formula may be converted into the Engineering-News formula by disregarding the weight of the pile and by assuming the following: 100% efficiency; no impact loss; representation of half the sum of the temporary compressions of cap, pile, and ground by 1.0 in. if a drop hammer is used and 0.1 in. if a steam hammer is used, regardless of the length of the pile, its material, or whether the pile is friction or end bearing; and a safety factor of 6.

Any dynamic formula can give a true, permanent load-bearing value only if the soil resistance opposing the blow is the same as the resistance of the soil to continued uniform static load. In the case of cohesionless soils, such as sands and gravels, the force of the blow is resisted by the soil particles, because water in the voids is free to move elsewhere. Any formula of the dynamic type can be of value for load-carrying purposes only if the pile load is delivered into soils of this character. In clays the force of the blow is resisted by the water in the voids, because it cannot immediately flow elsewhere due to the relative impermeability of this type of soil. Water in the clay lubricates the sides of the pile during driving, since it cannot quickly escape. Friction between the pile and the clay will increase with time, and will approach or equal the shearing value of the undisturbed soil or the friction value between the undisturbed material and the pile face. However, general settlement under added load may take place at a slow rate, due to consolidation of the clay as the increased pressure on the pore water in the clay relieves itself by slow percolation of the entrained water.

Comparisons of values computed by the Hiley and other dynamic formulas with load test results have been published. These comparisons were intended to show that none of the formula results could be relied upon to agree with the actual ground resistance.[6,7] There appear, however, to have been two

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[6] Discussion by Karl Terzaghi of "Pile-Driving Formulas: Progress Report of the Committee on Bearing Value of Pile Foundations," *Proceedings*, ASCE, February, 1942, pp. 311–323.

[7] Discussion by G. G. Greulich of "Pile-Driving Formulas: Progress Report of the Committee on Bearing Value of Pile Foundations," *ibid.*, September, 1941, pp. 1391–1396.

basic errors in application of the formulas—namely, use of dynamic formulas in cohesive soils and, in the case of the Hiley formula, consideration of the value of l as the full length of the pile regardless of the location of the center of driving resistance. Use of the incorrect value of l also results in use of the wrong average cross-sectional area of a tapered pile. Of the thirty tests tabulated and plotted, only three were in cohesionless soils. Good agreement between dynamic formula results and test loads could not be expected under these circumstances, where most of the soils were of a type in which no dynamic formula is applicable. However, the results for the three piles in cohesionless soils, as computed by an approximation of the Hiley formula, fell within practically the same satisfactory range as did a number of other tests in cohesionless soils reported herein. The data indicate that no formulas give results in good agreement with load tests when driving in cohesive soils, which finding agrees with the theories of soil mechanics. However, the results would not seem either to prove or to disprove the value of the Hiley formula, properly applied, under conditions where a dynamic formula is applicable.

Although dynamic formulas applied in cohesive soils will give the temporary resistance to the blow, care should be exercised to avoid considering this resistance as a permanent load-carrying capacity. By using a formula of the Hiley type under cohesive soil conditions, however, it is possible to compute the fiber stress in the pile during driving.[8]

SEPARATION OF STUDIES OF VARIABLES AND FACTOR OF SAFETY

A study of reliability of dynamic driving formula results as one measure for bearing capacity is best approached by separating it into two phases. In the first, it is well to ascertain whether or not a dynamic formula can be found which will give for an answer the same value of the ultimate driving resistance, R_u, for all types of piles driven with all types, sizes, and speeds of hammers to the same depth in the same soils. This reconciles all variables in pile characteristics and driving equipment. The second phase consists of studying the relationship between R_u and the ultimate bearing values as determined by test loads.

A separation of these phases enables the problem to be studied intelligently, and avoids confusing the effects of pile and hammer variables with the factor of safety. After a method of obtaining uniform values of R_u has been found, selection of the safe working value, R, by adoption of a proper factor of safety for the conditions of the particular structure is simple.

RECONCILIATION OF VARIABLES IN DRIVING CONDITIONS

Data on the first phase of study of the value of pile-driving formulas, reconciling driving results when using various types of piles and sizes of hammers, have been presented in tabular form elsewhere.[9,10] In these tests, the following combinations were investigated:

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[8] "Field Determination of Damaging Stresses During Driving," by Robert D. Chellis, *Engineering News-Record*, May 30, 1946, pp. 863-865.

[9] Discussion by Robert D. Chellis of "Pile-Driving Formulas: Progress Report of the Committee on Bearing Value of Pile Foundations," *Proceedings*, ASCE, October, 1941, pp. 1517-1537.

[10] "Pile-Driving Handbook," by Robert D. Chellis, Pitman Pub. Corp., New York, N. Y., 1944.

1. Different types of piles (thin mandrel-driven corrugated shells, fluted steel shells, and pre-cast concrete) were driven with the same hammer to the same depth.
2. Different types of piles (fluted steel shells, and thin mandrel-driven corrugated shells) were driven with the same hammer to the same capacity by the Engineering-News formula.
3. The same type of H-pile was driven with different types of hammers (double-acting and differential-acting).
4. The same type of pile (wood) was driven with different types of hammers (drop and double-acting).

TABLE 3.—DATA ON PILES, DRIVING, LOAD

Test No.	Location	PILE CHARACTERISTICS				
		Type[a]	Length (ft)	Embedment (ft)	Butt size[b] (in.)	Tip size[b] (in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
						(a) TIMBER
40	San Francisco Harbor, Calif.	Douglas fir, creosoted	92	73	16	8
30	Lavaca Bay Bridge, Port Lavaca, Tex.	Upper 32 ft creosoted (24 lb)	84	63	12[g]	9[h]
37	Marathon, Ont., Canada	Douglas fir, green	68.8	64.8	15	10¼
44	Norfolk, Va., waterfront	Long leaf yellow pine	60	57.7	13½	8
45			70	67	17	7
33	Marathon, Ont., Canada	Douglas fir, green	63	59.5	14	7½
34			61	59.5	14	8
2	Pee Pee Creek Bridge, Pike County, Ohio	Wood, green	45	42	14	6
32	Marathon, Ont., Canada	Douglas fir, green	63	59.5	13½	8
31			75	71	13½	8
8	Kokasing River Bridge, Morrow County, Ohio	Oak, green	30	29	12	7
1	Crooked Creek Bridge, Pike County, Ohio	Wood	39	25	13	8
35	Marathon, Ont., Canada	Douglas fir, green	63	59.5	14	7½
						(b) COMPOSITE PIPE
36	Marathon, Ont., Canada	Douglas fir[p]	40	25	12½	10
						(c) FLUTED
7	Scioto River Bridge, Pike County, Ohio	11-gage	25	25	18	8
6		11-gage, straight upper section	45	26	18	8
4	Pee Pee Creek Bridge, Pike County, Ohio	11-gage	25	19	18	8
						(d) PIPE
3	Brush Creek Bridge, Adams County, Ohio	12¾ in. in diameter, ⅜-in. wall	50	47	12¾	12¾
43	Cuyahoga River Turning Basin, Cleveland, Ohio	12¾ in. in diameter, ½-in. wall	80	78	12¾	12¾
42			80	70	12¾	12¾

These data appear to indicate that the Hiley formula will reconcile with variables of different hammers and pile types, and give closely the same computed driving resistance in any particular soil conditions at the same depth. This is as it should be, since the types of piles driven into it and subjected to load make no difference to the soil, provided the dimensions of the piles are comparable.

COMPARISON OF DRIVING AND TEST LOAD VALUES

The second phase of study of the value of pile-driving formulas can now be investigated with more surety, on the basis that the pile and hammer variables can be reconciled within reasonable practical limits. This permits direct comparison of driving resistance values with the failure points of load tests.

TESTS, AND ULTIMATE CAPACITY BY FORMULAS

Set, s, (in.)	HAMMER			Pile cap weight (lb)	Pile weight,^{d} W_p (lb)	Failure test load (tons)	ULTIMATE CAPACITY BY FORMULA (TONS)							
	Type	Energy per blow^{c}	Blows per minute				Hiley	Pacific^{e}	Canadian^{f}	Engineering-News^{g}	Eytelwein^{g}	Modified Engineering-News^{g}	Modified Eytelwein^{g}	Navy-McKay^{g}
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
PILES														
3.0	Drop	2,900 ×240	....	....	3,700	45	45	47	45	88	52	....	....	....
2.45	Drop	3,290 ×60^{i}	....	1,080^{j}	3,130	28	17	24.5	21.3	28.5	24.6	....	....	....
2.10	S.A.^{k}	5,000 ×36	....	....	2,750	25	18	23	22	40	42	38	39	35
1.0	S.A.	5,000 ×36	....	65	3,025	44^{l}	37.5	47	49	81	85	69	76	76
0.86	S.A.	5,000 ×36	....	65	3,115	40^{m}	43.5	52.5	57	94	98	77	85	88
0.6	D.A.^{n}	6,790	128^{o}	....	2,200	40	23	22.5	22	58	54	45	39	46
0.54	D.A.	7,240	132^{o}	....	2,200	40	23	26	25.5	68	63	53	45	56
0.48	S.A.	5,000 ×35^{o}	....	100	1,400	71	68	71	75	150	148	134	188	204
0.46	D.A.	7,240	132^{o}	....	2,200	40	28	27	27	78	73	57	47	65
0.4	D.A.	8,200	140^{o}	....	1,820	43	34.5	32	34.5	98	95	70	65	90
0.33	S.A.	3,000 ×25^{o}	....	40	780	37	45	48	49	87	105	60	92	104
0.3	S.A.	3,000 ×29^{o}	....	100	1,300	45	50	51	53	109	144	72	102	129
0.19	D.A.	8,200	140^{o}	....	2,100	40	44.5	39	43.5	170	147	100	76	176
AND WOOD PILES														
2.00	S.A.	7,500 ×38^{o}	....	....	1,800	26	32	45.5	32	75	78	68	75	68
STEEL SHELL PILES														
0.43	S.A.	5,000 ×35^{o}	....	670	1,100	88	94	100	50	166	206	108	192	198
0.5	S.A.	5,000 ×35^{o}	....	670	1,500	78	72	76	46	146	174	108	174	168
0.50	S.A.	5,000 ×35^{o}	....	670	1,100	88	83	93	48	146	177	108	168	170
PILES														
0.12	S.A.	3,000 ×29^{o}	....	600	3,100	46	38	79	53	194	164	102	115	276
0.10	S.A.	5,000 ×36^{o}	....	1,080	3,250	120^{q}	85	107	74	450	626	225	390	798
0.09	S.A.	5,000 ×36^{o}	....	690	3,250	150^{r}	86	108	74	474	556	230	300	822

TABLE 3.—

Test No.	Location	Pile Characteristics				
		Type[a]	Length (ft)	Embedment (ft)	Butt size[b] (in.)	Tip size[b] (in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
						(*e*) STEEL
16	Wharf, Bremen, Germany	10-in., 60 lb per ft	39.3	35.4	10	10
19			39.3			
11			39.3			
12			39.3			
13			39.3			
15			39.3			
17			39.3			
21			29.3	25.4		
22			29.3	25.4		
20			39.3	35.4		
26	Bay Bridge Falsework, San Francisco, Calif.	12 in., 65 lb per ft	99.2	43.2	12	12
14	Wharf, Bremen, Germany	10 in., 60 lb per ft[e]	27	23	10	10
18		10 in., 60 lb per ft	39.3	35.4	10	10
39	Cowen Park Bridge, Seattle, Wash.	10 in., 42 lb per ft	46	32	10	10
41	Cuyahoga River Turning Basin, Cleveland, Ohio	10 in., 54 lb per ft	80	78	10	10
24	Bay Bridge Falsework, San Francisco, Calif.	12 in., 65 lb per ft	98.8	40.5	12	12
27			99.5	44.7	12	12
10	Alameda Creek Bridge, Niles, Calif.	10 in., 42 lb per ft	38	35	10	10
25	Bay Bridge Falsework, San Francisco, Calif.	12 in., 65 lb per ft	99	44.4	12	12
28			99.6	40.4	12	12
29			100.1	41.4	12	12
9	Strip Mill Foundations, Lackawanna, N. Y.	10 in., 57 lb per ft	30.8	24.7	10	10
23			40	30	10	10
						(*f*) PRE-CAST
5	Scioto River Bridge, Pickaway County, Ohio	12 in. by 17 in.	14	11	12 × 17	12 × 17
38	Naval Base, San Diego, Calif.	20 in. by 20 in.	53	....	20	20

[a] Wood piles untreated unless specifically noted. [b] For round piles, diameter; for square piles, one side; for
[d] Includes cap weight. [e] Pacific Coast Uniform Code. [f] Working load × 3. [g] Working load × 6. [h] Upper
[j] Wood dolly in steel cap, rope coil on pile. [k] Single acting. [l] Day of driving. [m] 30-day duration. [n] Double
in diameter by ⅜ in. thick by 50 ft long. [q] Compression test 7 days after driving; 140 tons pulling test 17 days
sets of projecting fins tilted from horizontal. [t] Compression test 9 days after driving; 85 tons pulling test 19

Basic data for forty-five test piles meeting resistance in cohesionless soils are given in Table 3. Also shown are the test values and ultimate driving resistances computed by dynamic formulas, as well as data on the hammers used in driving the piles. Data on the borings at the selected sites are given in Table 4.

The scattering of the ratios between the ultimate driving resistances computed by the various dynamic formulas in most common use and the failure points of test loads of the piles in Table 3 are shown in Fig. 1. Because the Engineering-News, Eytelwein, and Navy-McKay formulas are intended to give working loads with a safety factor of 6, the results computed by their use have been multipled by 6 to obtain ultimate resistances. Also, since the

(Continued)

Set, *s*, (in.) (8)	HAMMER			Pile cap weight (lb) (12)	Pile weight,[d] W_p (lb) (13)	Failure test load (tons) (14)	ULTIMATE CAPACITY BY FORMULA (TONS)							
	Type (9)	Energy per blow[e] (10)	Blows per minute (11)				Hiley (15)	Pacific[e] (16)	Canadian[f] (17)	Engineering-News[g] (18)	Eytelwein[e] (19)	Modified Engineering-News[g] (20)	Modified Eytelwein[e] (21)	Navy–McKay[e] (22)
H-PILES														
0.76	....	10,850	....	....	....	56	42.5	48	38	76	78	61	64	70
0.76						44.5	42.5	48	38	76	78	61	65	70
0.66						45	50	54	40	86	88	68	72	80
0.63						55	55	57	42	90	92	70	75	84
0.55						67.5	54	61	52	100	104	77	82	96
0.47						45	60	71	50	114	118	84	92	113
0.47						79	62	71	50	114	118	84	93	113
0.45						45	61	71	50	114	118	84	93	113
0.45						67	61	71	50	114	118	84	93	113
0.36						89	72.5	84	48	141	154	99	109	144
0.36	D.A.	18,920	110[o]	....	7,200	88	65	78	57	244	210	171	126	204
0.30	....	10,850	....	....	....	78.8	80	100	57	162	171	109	120	177
0.30	....	10,850	....	....	....	112+	80	100	57	162	171	109	120	177
0.27	S.A.	5,000×36	....	750	2,680	70+	92	152	84	243	291	300	780	∞
0.17	S.A.	5,000×36[o]	....	660	4,900	75[i]	79	100	66	336	353	192	210	426
0.14	D.A.	18,920	110[o]	....	7,200	90	83	100	67	472	360	258	168	534
0.13	D.A.	18,920	110[o]	....	7,200	80	84	104	70	492	372	264	172	564
0.12	S.A.	3,000×29	....	....	2,300	99	75	72	60	194	244	102	123	300
0.08	D.A.	18,920	110[o]	....	7,200	140	89	108	69	636	444	258	170	528
0.06	D.A.	18,920	110[o]	....	7,200	150	92	110	72	729	496	318	194	1,332
0.05	D.A.	18,920	110[o]	....	7,200	140	93	114	75	755	504	324	195	1,494
0	S.A.	5,000×36	....	....	2,500	190	175	210	144	875	1,800	300	360	∞
0	S.A.	5,000×36	....	....	3,000	128	157	248	90	900	1,920	300	654	∞
CONCRETE PILES														
0.26	S.A.	5,000×28[o]	....	1,480	4,460	70	75	153	42	192	218	124	158	225
0	S.A.	7,500×34[o]	....	....	22,550	150+	180	214	82	1,380	523	520	174	∞

rectangular piles, both dimensions. [e] Ram weight (lb) times stroke (in.); or manufacturer's rated energy, in ft-lb. sections only; lower section 9 in. to 13 in. butt diameters, 8 in. to 9 in. tip diameters. [i] Cable drag on drum. acting. [o] Measured. [p] Driven through casing extending to lower sand stratum, using pipe mandrel $10\frac{3}{4}$ in. after driving. [r] Compression test 10 days after driving; 80 tons pulling test 16 days after driving. [s] With 4 days after driving.

Canadian National Building Code formula contains a safety factor of 3, the results obtained thereby have been multiplied by 3 to secure ultimate resistances. The Hiley and Pacific Coast Uniform Building Code formulas give ultimate resistance directly.

The best and safest range of values in Fig. 1 appears to be obtained from the Hiley formula, falling between approximately 55% and 125%, with an average of 92%. No dangerously high percentages appear. The Pacific Coast formula shows a somewhat wider range—between approximately 55% and 220%, with an average of 112%. The Canadian formula range falls into somewhat lower figures, from 55% to 140%, with an average of 80%. The range of the Engineering-News formula is from 100% to 700%, with a 289%

average, and that of the Eytelwein formula, from 90% to 1,800%, with an average of 292%. The range of values by the modified Engineering-News formula is from 98% to 430% (182% average); that of the modified Eytelwein formula is from 98% to 508% (202% average); and that of the Navy-McKay formula is from 99% to ∞.

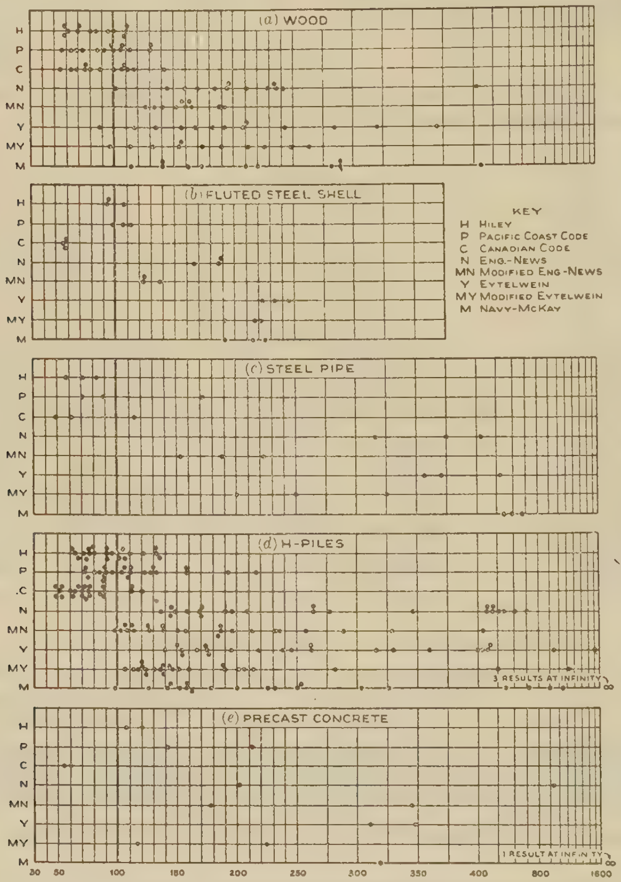

Fig. 1.—Scattering of Ratios Between Resistances Computed by Various Pile Formulas and Observed Failure Test Loads

It may be observed in Fig. 1 that the results of the Hiley, Pacific Coast, and Canadian formulas are grouped in some proximity to the 100% test line, so that, with the factors of safety assumed, none would be actually unsafe, and none very wasteful. On the other hand, the scattering of results from the Engineering-News, Eytelwein, and Navy-McKay formulas is marked and too wide to be comprehended economically within any one factor of safety. Even a factor of 6 would not be adequate for some results.

Curves showing the relationship between test loads and driving resistances computed from the Hiley and the Engineering-News formulas—both placed on a working load basis by the application of factors of safety—are given in Fig. 2. The Engineering-News formula purports to include a factor of safety so that results are plotted directly. A safety factor of 2½ has been assumed sufficient for use with test loads and the Hiley formula. The values in Fig. 2 have been grouped according to the type of pile and have been placed in descending order of set values within each group. The increasing divergence of the Engineering-News values as sets became smaller should be noted. There are many other variables involved, but this trend is apparent. The divergences appear to start at larger sets with the heavier piles. Curves of the Pacific Coast and Canadian formulas have not been shown in Fig. 2 since they would be confusing and since the values group well with the Hiley values, as may be seen in Fig. 1. The similarity in ranges between the Eytelwein, Engineering-News, and Navy-McKay formulas, including the modified forms, have likewise resulted in the omission of curves for these formulas.

TABLE 4.—SOIL CONDITIONS FOR PILE TESTS LISTED IN TABLE 3

Test No.	SOIL CONDITIONS
1, 2, 5, and 6	Sand and gravel
3	14 ft silt, clay, and fine sand; 10 ft clay; gravel, fine sand, and broken stone
4	12 ft silt; sand and small gravel
7	25 ft silt, sand, and gravel; sand and gravel
8	12 ft silt, fine sand, and gravel; sand and gravel
9	6 ft clay; 2 ft sand and gravel; 13 ft soft clay; 4.7 ft sand and gravel to rock
10	Gravel and small boulders
11	28 ft sand; 0.5 ft hard clay; 2 ft clay with sand; 5 ft fine sand
12, 13	23–24 ft sand; 1.5–2 ft clay; 0.5 ft peat; 0.5–1 ft clay; 8–10 ft sand
14, 39	Fine sand
15, 16	13 ft sand; 1–2 ft loam; 10 ft fine sand; 1–1.5 ft clay; 3–6 ft clay and sand; 3–7.5 ft fine sand
17, 18	23–25 ft sand; 2 ft clay; 0.5 ft peat; 1 ft clay; 7–8 ft clayey sand
19	22.5 ft sand; 1.5 ft clay; 1 ft sand and clay; 9.5 ft sand
20	23 ft sand; 1 ft clay; 1 ft sand; 3 ft sandy clay; 6.5 ft sand
21	23 ft fine sand; 1 ft clay; 1 ft peat; 0.5 ft sandy clay
22	18 ft fine sand; 4 ft sand and silt; 2 ft clay; 1.5 ft sand
23	7.5 ft clay; 3 ft sand; 10.5 ft soft clay; 6 ft sand and gravel to rock
24 to 29	40 ft water; 13 ft soft mud; 20 ft clayey sand; 18 ft sandy silty clay; 37 ft silty clay
30	Silt with some shell strata
31	10 ft sand; 5 ft silt; 2 ft sand; 38.5 ft plastic clay; 8 ft siltv sand; 8 ft fine sand[a]
32 to 35	5 ft sand; 39.5 ft plastic clay; 3 ft plastic silt; 12 ft fine sand[a]
36	14 ft sand; 32 ft plastic clay; 60 ft fine sand[a]
37	5.5 ft sand; 37 ft plastic clay; 44 ft silty sand[a]
38	Water, mud, and sandy clay to firm sand
40	Harbor mud
41, 42, 43	7.5 ft sand and loam; 26.5 ft fine sand; 1 ft gravel; 59 ft silty sand and stiff dry clay
44, 45	9 ft sandy silt; 12 ft soft silt; 6 ft sand and silt; 18 ft soft clay, some sand; 4 ft silt, some peat; 4 ft silty sand; 75 ft fine sand, shells, and some clay

[a] Under hydrostatic pressure.

The reasonable coincidence of the curves in Fig. 2 for wood piles, for the easier drivings, indicates that the actual factor of safety provided by the Engineering-News formula is often nearer 2½ than 6. There would usually be no objection to this, and such a result would be desirable, but reliance should not be placed on the existence of a safety factor having a value of six times the working load. Fluted steel shell piles are lightweight, and appear to have many of the driving formula characteristics of wood piles. A similarity between the curves is, therefore, to be expected, as may be observed in Fig. 2.

As piles become heavier, the Engineering-News, Eytelwein, and Navy-McKay formulas, including the modified forms, become more dangerous to use with small sets. For the tests plotted, it would not be safe to use them with sets of less than $\frac{1}{4}$ in., although there are so many possible variables involved that this should not be considered a rule.

The lighter the hammer, the smaller the set will be and, consequently, the larger the computed carrying capacity by the Engineering-News formula. According to common specifications in which no limit is set on the weight of the pile or the smallness of the hammer, this formula could result, when using a very small hammer, in tiny sets or refusal at short penetrations. Thus, it is seen that the length of pile driven would depend on the size of hammer or weight of pile, instead of on reaching some definite stratum capable of supporting the load safely with a proper factor of safety. At refusal the Navy-McKay formula gives infinity as the ultimate bearing, which makes the results doubtful when sets are so small as to approach refusal.

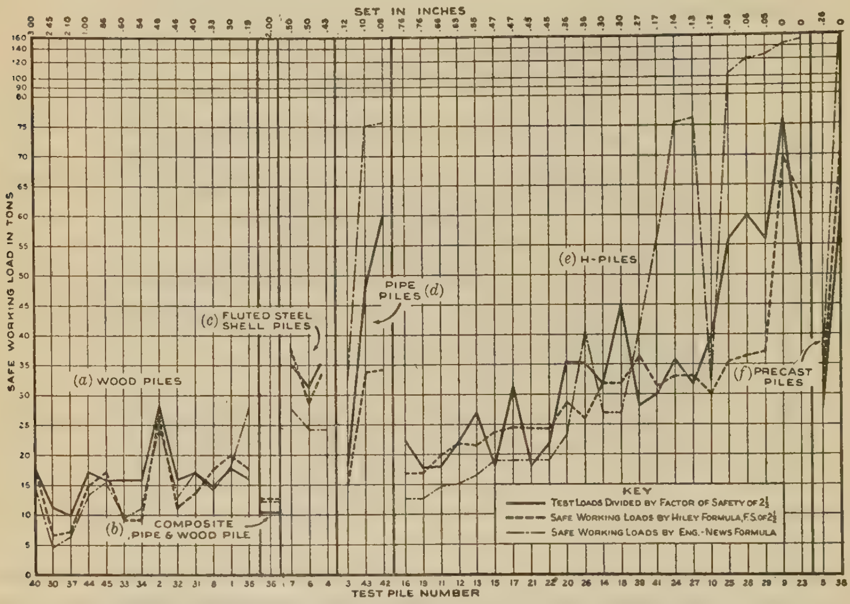

Fig. 2.—Relation Between Pile Formula and Test Load Results

The reason for the danger in the use of the Engineering-News and the Eytelwein formulas with small sets may be understood from consideration of the relative values of the terms in the denominators. The last term of 1.0 for drop hammers or 0.1 for other hammers in the Engineering-News formula, and the last term of $0.1\ W_p/W_r$ for single-acting or double-acting hammers with the Eytelwein formula, are intended to represent the average elastic compressions. If the average elastic compressions, which are half of those observed at the head, are reasonably close to these figures, the results might not vary too widely. Nevertheless, under hard driving the elastic compressions may actually be many times these values and may exert a far greater reduction on the result than the set. As sets decrease, elastic compressions always increase. The relationship between set and compression would be different for each type of pile. At any particular driving resistance there will occur specific values of set and compression, but because these values are always changing in inverse ratio no such formula (the Engineering-News or the Eytelwein), which has a fixed value in the denominator, could be expected to apply to all conditions. Sets and elastic compressions are equally important in a formula, and it is no more logical to assume a constant value for the temporary compression than it is to assume it for the set. The Engineering-News formula was intended for use with wood piles, but is not suited to hard driven piles, or to heavy piles.

Failure Point in Load Tests

The point at which failure is considered to occur in a load test provides the basic value against which the pile-driving formula results are compared. Load is always plotted against settlement in load test curves. Settlement of the head is composed of elastic shortening of the pile and ground, and of movement of the pile relative to the ground or plastic deformation of the ground. Rebounds measured after temporarily removing the load before reapplying it with the next load increment will enable these two forms of settlement to be separated for consideration. The graph of total head movement can then be plotted as two separate elastic and plastic settlement curves.[11] It is only permanent movement which actually governs the failure point.

If the theoretical elastic pile deformation under the loads, computed on a length from butt to estimated center of soil resistance, is plotted on the load settlement curve, the curve of elastic deformation of the pile may be compared with it.[11] Until the elastic deformation curve drops as low as the theoretical elastic line, the center of resistance is higher than assumed, and the upper strata are being tested instead of the lower strata.

Certain strata are often selected as providing the bearing resistance, and the upper part of the pile is intended to by-pass the overlying strata. In such cases, the value of the temporary load-carrying capacity of these upper strata should be subtracted. The load test should be carried to such a point that a satisfactory amount of load reaches the lower strata to be tested. Many load tests are not of sufficient magnitude relative to the working load. Building codes often require test loads to be 1.5 or 2.0 times the working load. It would be advisable in many instances for the engineers to carry the test to a greater value, and preferably to failure so that the factor of safety may be known.

After its separation from the total head settlement curve, the curve of permanent movement should be inspected for the failure point. Failure is usually considered to occur where the rate of movement begins to increase sharply in proportion to the increase in load. It is an obvious point in some tests, whereas in others the curve appears to suffer a gradual increase in the rate of change of slope, thereby making selection of the failure point difficult.

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[11] "Making and Interpreting Pile Load Tests," by Robert D. Chellis, *Engineering News-Record*, June 13 1946, pp. 914–919.

Judgment and experience have resulted in many rules to aid in selecting the failure point. Some of these are contained in building codes, some in textbooks and articles, and others are matters of practice among various engineers. A partial summary has been published.[11] Sometimes when difficulty is experienced in applying certain rules, replotting the curve to a different scale of settlements, or plotting it on semilogarithmic paper, will bring out the points more clearly. On a recent test, application of fourteen of these rules gave working loads ranging from 11 to 18 tons, with an average of 14½ tons.[11] If difficulty is experienced in applying one of the rules, several or all these and others may be applied and the results averaged. The discrepancy between the result and the true value would not seem likely to be larger than could be covered by the usual factors of safety.

A load test may reveal that there has been an increase in bearing value of the soil over that computed, and, on the other hand, it may reveal a decrease. Cases have been reported where coarse-grained, saturated pervious soils have lost up to 40% of the indicated bearing value in 24 hours after driving.[12]

Very loose fine-grained sands and silts are occasionally encountered, and if these are submerged and below critical density, excess water temporarily forced away from the pile during driving may cause a momentary "quick" condition, resulting in low resistance readings, although static load tests would show much higher values.[13]

Factor of Safety

It has often been said that the pile foundations have generally been satisfactory, and that this justifies continued reliance on the Engineering-News formula. Most foundations have not failed. Only those would fail, presumably, in which the actual factor of safety fell below 1.0.

In this connection, it should be borne in mind that full design live load is generally not applied. Careful studies have shown that only a small percentage of the usual office building live loads actually occurs. The same would seem to be true of some other structures. Furthermore, some live loads are transient, whereas others are of insufficient duration to cause squeezing out of the water and settlement in upper cohesive strata, which thus reduce the load on those sections of piles in the strata capable of providing permanent resistance.

There is, however, an impressive list of failures, even though these are not generally publicized; and many instances occur in which the factor of safety is very low—far below that of the balance of the structure. Excess footage of piles frequently has been driven. Piles have been overdriven and broken, as found when they have been pulled or excavated (Fig. 3). When using piles of different types, such as light shells or pre-cast concrete, widely different tip grades have resulted, one of which must surely be wrong. The same result occurs when using different types or sizes of hammers on piles of the same kind.

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[12] "Soil Reactions in Relation to Foundations on Piles," by R. M. Miller, *Transactions*, ASCE, Vol. 103, 1938, pp. 1193–1216.

[13] "Application of Soil Mechanics in Designing Building Foundations," by A. Casagrande and R. E. Fadum, *ibid.*, Vol. 109, 1944, p. 383.

Fig. 1 reveals that the Engineering-News, Eytelwein, or Navy-McKay formulas, or their modified forms, cannot be expected reliably to keep the results within a spread which can be encompassed by any reasonable factor of safety. Even the purported value of 6 used is not adequate for this purpose. The comprehensive formulas fall within a reasonably narrow spread, the Hiley formula giving the smallest scattering of values.

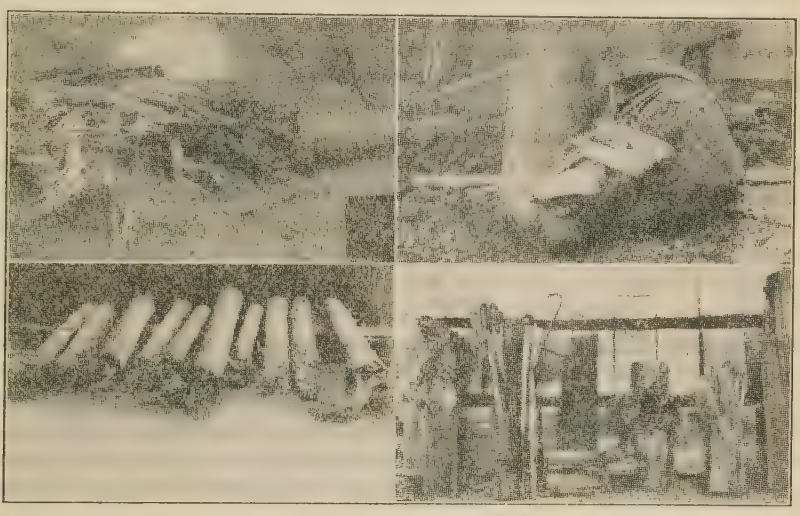

Fig. 3.—Examples of Overdriven and Broken Piles

Comparison of results obtained by the Engineering-News formula with those obtained by the Hiley formula have shown a relative variation in the factor of safety from as low as $\frac{1}{2}$ to as high as 16. In this particular series of tests comparison of results obtained by the Engineering-News formula with load test results showed an actual range in the safety factor of from $\frac{2}{3}$ to 6.

The spread of computed results by the Hiley formula, compared with load tests, is such that it can be used with a safety factor of $2\frac{1}{2}$ or 3. This compares favorably with the safety factors used in structural steel and concrete designs. The uncertainties in interpretation of failure values from load test curves, and of selection of hammer efficiency, center of driving resistance, and cap and ground quake values in the pile formula seem to be hardly more broad than the uncertainties and neglect of conditions of end restraint, for instance, or the theory of concrete design, used in the superstructure. The safety factors used in superstructure designs are intended to cover differences between theory and actual conditions, as well as provide a true "factor of safety." For this reason, it would be unfair to expect a higher degree of accuracy in the design procedures for substructure, and not to permit the so-called factor of safety some latitude in covering discrepancies between theory and actuality.

Reasonable reconciliation of driving results by the Hiley formula with load test results permits the use of any selected factor of safety. This may be chosen with regard to the economic importance of the structure, its expected life, the reliability of data based on completeness of boring, driving and test information, the uniformity of soil conditions, and other variables.

STATIC FORMULA

It is recommended as good practice to use a static formula in preliminary considerations of pile problems and selection of test pile lengths. It is also advisable to use a static formula to check values computed by a driving formula, particularly if a load test is not made.

The static formula recommended merely consists of multiplying the bounding area of the pile by the square-foot friction values estimated for the various soil strata, plus an amount for end bearing. The friction values may be judged from boring records, shear tests, loading and pulling tests, and experience. A table of such values for various types of soils, determined from load tests, has been published.[10] This is a rough guide as to the possible range when more accurate data are lacking.

LIMITATIONS AFFECTING VALUE OF PILES

In considering the value of either a computed resistance or a load test on a single pile, two points should be borne in mind. First, the load-carrying capacity of a group of piles is often less than that given by multiplying the number of piles in the group by the value of a pile tested or driven singly, due to overlapping of zones of influence. Second, no pile can improve poor strata below its tip, and if the character of the underlying soils is such that they will consolidate under the load of the structure, this will not be prevented by driving piles. Reduction due to group action is not a fully understood subject. It is likely that it affects piles driven in cohesive soils much more than in cohesionless ones,[10,14,15] but it may be of considerable magnitude. For the theory of consolidation of cohesive strata any textbook on soil mechanics may be consulted.

CONCLUSIONS

It would appear that the Hiley, Canadian National Building Code, and Pacific Coast Uniform Building Code formulas are practical tools for use in pile foundation design (within the limits of applicability of individual pile values and underlying soil conditions) in soils in which the predominant driving resistance is supplied by cohesionless materials. It further appears that these formulas would not result in uneconomical foundations, and that they would provide ultimate driving resistance values in sufficiently good agreement with test values to permit their use with factors of safety comparable to those in use for other phases of structural engineering. These formulas are also useful in cohesive strata for determining the temporary resistance of the pile under the blow, as a means of computing the fiber stress during driving.

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[14] "Timber Friction Pile Foundations," by F. M. Masters, *Transactions*, ASCE, Vol. 108, 1943, pp. 115–173.

[15] "The Efficiency of Piles in Groups," by J. F. Seiler and W. D. Keeney, *Wood Preserving News*, November, 1944, pp. 109–118.

For each pile the length and weight used in the Hiley formula were the effective length to the center of driving resistance and the total weight of the pile plus driving cap. In the results obtained by the Canadian and the Pacific Coast formulas, the pile lengths were taken as full lengths, and pile weights did not include cap weights, to agree with these terms as defined by the authorities sponsoring the two formulas. Had these terms been applied as recommended and used for the Hiley formula, some variations would have resulted, but the scattering would not have been as small as obtained by that method.

Consideration of the assumptions made in the Engineering-News formula makes it evident that the formula cannot be of universal value in these days of such wide ranges of piles and equipment, even though there are certain combinations of values in which it will give good results. It would seem from the Mr. Wellington's original qualifications that this formula was intended for friction piles rather than for piles which resist driving somewhat suddenly and become, in effect, end-bearing piles or columns. Many of the unduly high results from this formula would be avoided if such cases of fairly sudden small sets were eliminated. However, to be sure that it is within the safe range, it is necessary to run a loading test or compare with results by the Hiley formula, unless full reliance is placed upon a static formula. There would not seem to be much point in continued use of the Engineering-News formula, except as a matter of interest in comparing it to results of more modern methods and becoming aware by personal experience of its limitations. Many building codes still retain it, but the engineer should not rely upon it in such cases, and for reasons of safety, uniformity, and economy should apply more accurate methods as well. Most code authorities will accept load test results modified by a reasonable factor of safety, if code formulas appear to cause uneconomical pile lengths.

Although use of the modified forms of the Engineering-News and Eytelwein formulas with heavy piles appears to reduce the range of results compared to load tests, these tests do not indicate that sufficient improvement is obtained to justify their use instead of a formula of the Hiley type. Neither do these tests indicate justification for use of the Navy-McKay formula.

On many small jobs load tests are not practicable. On most projects it is desired to obtain an idea of the carrying capacity or length of piles required before the time necessary to make an adequate load test elapses. If the borings show soil of a nature in which a dynamic driving formula is usable, it appears that sufficiently reliable results for these purposes may be computed. On large projects these values should be confirmed by a load test.

ACKNOWLEDGMENTS

Thanks are due to W. H. Rabe of the Bureau of Bridges, State of Ohio, for the data on tests 1 to 8, inclusive; to E. M. Cummings, M. ASCE, of the Bethlehem Steel Company, Bethlehem, Pa., for the data on tests 9 to 29, inclusive, and on test 39; and to G. B. Sowers, M. ASCE, Director of Public Works of the State of Ohio, Columbus, Ohio, for the data on tests 41, 42, and 43. Some of the data for tests 11 to 22, inclusive, were published by A. Agatz.[16]

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[16] "Der Rammstahlpfahl fuer Pfahlvostbauwerke," by A. Agatz, *Die Bautechnik*, February 2, 1934, pp. 56–60 and February 9, 1934, pp. 68–71.

Some of the photographs of damage to overdriven piles were provided by the courtesy of the Raymond Concrete Pile Company, New York, N. Y. The data for tests 30, 38, and 40 are available elsewhere.[17,18,19]

APPENDIX. NOTATION

The following letter symbols have been adopted for use in this paper:

A = average of cross-sectional area of driven parts of pile at butt and center of resistance to driving, in square inches;

C = temporary compression allowance, in inches (when without subscript is defined by Eq. 10d):
  C_1 = for pile head and cap;
  C_2 = of pile;
  C_3 = of ground (quake);

E = modulus of elasticity of pile material;

E_n = rated energy of blow, in foot-pounds, as published by manufacturers for various speeds of hammer;

e = coefficient of restitution: 0.5 for ram striking steel anvil on steel or pre-cast concrete piles; 0.4 for ram striking steel anvil on wood piles, also for steel helmet containing wood and driving steel piles; 0.32 for ram striking on steel plate cover of wood cap of steel piles; 0.25 for ram striking well-conditioned wood caps on pre-cast concrete piles, also driving directly on wood piles; and 0 for deteriorated condition of heads of wood piles or of wood caps, and for excess packing in cap;

e_f = efficiency: Suggested as 100% for trigger released drop hammers; 75% for friction winch drop hammers, differential-acting steam hammers, and single-acting steam hammers; and 85% for double-acting steam hammers—frequently from 5% to 10% greater values (or values under 100%) are obtained under favorable conditions, sometimes less under unfavorable; use of listed values suggested for computing bearing values, and of higher values for computing stresses in piles;

H = height of free fall of ram, in feet;

h = height of free fall of ram, in inches;

L = length of pile, in feet, measured from head to center of resistance to driving (defined as full length of pile in Pacific Coast Uniform Building Code);

l = length of pile, in inches, measured from head to center of resistance to driving (defined as full length of pile in Canadian National Building Code);

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[17] "Pile Tests Justify Increased Loading," by W. A. King, *Engineering News-Record*, June 28, 1945, p. 73.

[18] "Driving and Loading of Concrete Test Piles at the Naval Supply Depot, Naval Operating Base, San Diego, Calif.," *Bulletin No. 36*, Public Works of the Navy, Washington, D. C., October, 1927.

[19] Correspondence on Reinforced-Concrete Piles Driven in a Gravel Fill at Vancouver, B. C., by H. E. Squire, *Minutes of Proceedings*, Inst. C. E., Vol. 226, 1929, pp. 103, 108, and 109.

R = safe carrying capacity of pile, after applying factor of safety, in pounds;

R_u = ultimate carrying capacity of pile (considered as ultimate resistance to driving, in pounds, before applying factor of safety);

s = final set per blow, in inches;

W_p = weight of pile, in pounds, including shoes and driving cap and also of follower, or driving cores and mandrels, if used; shoes and caps not specified to be included in weight by Pacific Coast and Canadian building codes; and

W_r = weight of falling mass, in pounds.

DISCUSSION

Donovan H. Lee,[20] M. ASCE.—In a clear manner, the author has compared the results obtained by the variety of impact formulas selected, and by loading tests.

It is not difficult to demonstrate that the Hiley formula and the two adaptations of it are the only formulas that have any claim to reasonably accurate forecasts of the ultimate supporting value of piles. This position has been taken for granted, in the United Kingdom, at least, for the last ten years, although it must be admitted that there are still a few engineers in England who are reluctant to give up the Dutch (or Eytelwein) formula. Similarly, in the United States the Engineering-News formula "dies hard."

The author states that he purposely avoids the comparison of results by the impact formula and the test loads in the case of cohesive soils. In the writer's opinion this is particularly to be commended as there is little or no direct relationship between the resistance to penetration of piles in cohesive soils and their ultimate supporting value.

Actually, the author does include among his comparisons a number of tests in which clay strata are involved, but presumably has set aside tests in which the tip of the pile is bearing in clay. Nevertheless (as the author would probably agree), the methods of comparison used in Figs. 1 and 2 leave much to be desired; too much attention has been directed to showing the inaccuracy of formulas which are noted to be entirely unsuitable for the purposes to which they have been applied and too little to ascertaining in what respects and for what reasons the Hiley formula is not in close accordance with the test results.

The Hiley formula is intended to give the ultimate resistance to penetration and, although it is generally hoped that this is closely related to the ultimate supporting value of noncohesive soils, those cases for which the Hiley formula seems to give seriously incorrect results might be explained by the manner in which the loading tests were made, or the cohesive properties of some of the soils penetrated—rather than by some fundamental deficiency in the formula itself to allow properly for the subsoil conditions encountered.

The paper seems essentially to reinforce the already well-established belief that the Hiley formula is at present the best available. Notwithstanding this fact, the writer does not believe that the Hiley formula is theoretically correct; he is of the opinion that in a few years certain anomalies will be explained.

A study of the research by W. H. Glanville, Geoffrey Grime, and W. W. Davies[21] and of the paper by A. E. Cummings,[22] M. ASCE, published in 1940 supports this view.

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[20] Cons. Engr., London, England.

[21] "The Behaviour of Reinforced-Concrete Piles During Driving," by W. H. Glanville, Geoffrey Grime, and W. W. Davies, *Journal*, Inst. C. E., Vol. 1, 1935-1936, p. 150.

[22] "Dynamic Pile Driving Formulas," by A. E. Cummings, *Journal*, Boston Soc. of Civ. Engrs., Vol. 27, 1940, No. 1, p. 6.

Jacob Feld,[23] M. ASCE.—This excellent critical analysis of the correlation between the ultimate load value of piles as given by load tests and the theoretical ultimate value as given by the several pile-driving formulas is based on the assumption that the load tests give true results. Failure of a pile in cohesionless soil under the slowly increasing load normally applied in a structure, if it is carried to the ultimate value, probably occurs under a greater load than is determined from a pile load test extending over one day, or at the most, two days. Load tests on piles in cohesive soils, however, often indicate safe values which prove excessive in the actual structures. Even in saturated silty soils, 18-in. pipe piles which passed a load test of 200 tons, after being driven by a No. 1 Vulcan hammer to a "safe" value of more than 100 tons as determined by the Engineering-News formula, were found to have a driving resistance of much smaller values one month later.

The author convincingly proves the superiority of the Hiley type of formula over the older types, and very properly explains that Arthur M. Wellington, M. ASCE, never intended that his formula be employed for the type and length of piles now being driven, or for the large impact energies of modern hammers. It is to be noted that the maximum safe carrying capacity of piles in tons which can be determined by the application of the Engineering-News formulas is 0.001 times the energy of the blow in foot-pounds for drop and single-acting hammers, and 0.01 times the rated energy in foot-pounds for double-acting hammers. In these cases, the set per blow is zero. This means that a double-acting hammer of 10,000-ft-lb rated energy cannot produce a pile value of more than 100 tons, and a drop hammer weighing 3,000 lb and dropping 5 ft cannot produce a pile value of more than 15 tons. Many more inconsistencies can be shown in these formulas, yet they served well and for many years. Certainly, their application should be limited, as is customary among the engineers practicing in the New York (N. Y.) area, to pile loads not more than 40 tons (some limit further to 30 tons) and in soils which are predominantly granular.

The more serious problem is to find a true guide for values of piles driven in plastic or viscous soils. No energy resistance formula gives reliable values; load tests also disregard the important factors of time and loading rate. The author's suggestion that surface shear resistance be computed as a guide and check on load tests is probably as good a method as the present state of knowledge permits. The surface shear resistance determined at failure is smaller than the latent shear resistance when no motion occurs. However, a long time load test will show a creep settlement and there are no available data on the relationship between the amount of creep (settlement), the percentage of maximum shear resistance being balanced by the load, and the time. Another serious gap in necessary values is the reliability of simultaneous action of end bearing and surface friction resistance. Many of these missing data must be determined before a complete solution of the pile-driving formula becomes possible.

Richard F. Kafka.[24]—It gave the writer great satisfaction to read this paper, as it reminded him in some respects of his own studies on this subject, around 1908. The determination of the loading capacity of driven piles was rather a dark field then and it is still not entirely clarified as is apparent from Mr. Chellis' paper.

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[23] Cons. Engr., New York, N. Y.

[24] Editor, *Engineering Index, Inc.*, New York, N. Y.

First, it should be stressed that a structural member which carries the loads to rock or to a stratum that acts like rock is not a pile from the point of view of statics, even if it has been driven like a pile. It is, rather, a column. The only difference between a genuine column and a pile-like column or end-bearing pile is that the latter is surrounded by earth, which prevents it from buckling and thus increases the load capacity. If the rock at and below the tip of the pile actually exists and the pile is capable of transmitting the load to this stratum, all conditions required for an effective foundation are fulfilled as far as statics is concerned. In such cases, no driving formula need be applied to determine the loading capacity.

Thus, excluding column-like piles from the discussion and considering only genuine piles, the writer agrees with Mr. Chellis that loading tests are the most reliable method of determining the ultimate and the permissible load of a pile. However, engineers have need of a quick and simple method of determining the loading capacity and thus have been led automatically, so to speak, to the application of driving formulas or dynamic formulas.

Among the numerous driving formulas commonly used, those must be disregarded which are based on impossible assumptions (for example, on perfectly elastic or perfectly inelastic pile and hammer) or which neglect the elasticity (if any) of the soil. Driving formulas can be expected to give true load-bearing values only if applied to piles driven into cohesionless soil, as mentioned by the author, under the heading "Dynamic Formulas." In developing a driving formula published elsewhere,[25] the writer was well aware of this fact, as well as that approximate assumptions of the elastic properties, both of the pile and the soil, constitute the weak point of all driving formulas. The only possibility of overcoming conditions which can make even the most scientific driving formulas valueless is the measurement of the elastic deformation caused by the hammer blow. This method, which was first suggested by M. Genel and M. Konrad, has been treated extensively, and an apparatus has been described[25] by which the measurement can be accomplished. The measurement has the advantage that all variables of pile, hammer, and soil, including friction, air resistance, and other factors, which affect the driving results are taken into consideration automatically. Mr. Chellis has stated (under the heading, "Comparison of Driving and Test Load Values") that the relationship between set and elastic compression changes not only with each pile, but also with each hammer blow; hence, no formula can be expected to apply to all conditions. The measurement of the sets and the elastic compression also solves this problem and there is no reason to exclude the application of driving formulas to cohesive soils.

G. B. R. PIMM.[26]—In discussing dynamic formulas the author states, under the heading, "Dynamic Formulas," that it is a basic error to consider the value of l as the full length of the pile regardless of the location of the center of driving resistance. In using the Hiley formula the value of C, unless actually measured, should therefore be calculated from l as defined by the author. The same point is made, under the heading, "Failure Point in Load Tests," in considering the elastic deformation under static loading. However, it is difficult to ascertain the position of the center of driving resistance in the absence of full information as to the proportions of the total resistance caused by frictional resistance and toe resistance, respectively.

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[25] "Die Theorie der Pfahlgruendungen," by Richard F. Kafka, J. Springer, Berlin, 1912.

[26] Central Chambers, Ealing, London, England.

There is another aspect of the question of frictional support that is of even greater importance than its effect on the elastic strain. When a pile is partly supported by friction, there may be an elastic movement of the ground material throughout the frictionally supported length. The weight set in motion by each hammer blow is then not merely W_p but $W_p + W_g$, in which W_g is the weight of an unknown quantity of ground material which moves in sympathy with the pile and absorbs an unknown proportion of the energy of the blow. The omission of this factor is a basic defect in all formulas containing the ratio $\frac{W_p}{W_r}$ or any modification thereof, such as the Hiley, Eytelwein, Pacific Coast Uniform Building Code, Canadian National Building Code, and many other formulas. In the Hiley formula (Eqs. 6 and 7), for example, the efficiency is not $\frac{W_r + e^2 W_p}{W_r + W_p}$, but $\frac{W_r + e^2 (W_p + W_g)}{W_r + (W_p + W_g)}$.

The errors due to (1) taking the full length instead of the effective length, and (2) neglecting the energy expended in moving the weight W_g are of opposite sign, and therefore tend to cancel out, but in any given case in which one or the other predominates, the result may be seriously affected. In both cases the error has its origin in the frictional support, which must be eliminated before the toe resistance can be arrived at by an impact formula. To achieve this, recent British practice has been to carry out preliminary tests by the prepiling method, in which the toe resistance is recorded independently of the frictional resistance at all material depths, and is interpreted by the Hiley formula. The apparatus consists of an outer tube and an inner mandrel, the latter being driven separately (thus eliminating skin resistance) at those points in depth at which it is desired to ascertain the true toe resistance. Between these points the assemblage is driven as a whole, like an ordinary pile, against the total resistance; but, since the calculation of the latter, from the driving record, would entail the reintroduction of the error referred to previously, the frictional resistance is found separately, by dead loading or by jacking.

In this way, the two elements of the resistance are accurately ascertained, each by an appropriate method. Experience has shown, however, that the frictional resistance is a very capricious factor—an additional reason for eliminating it from the calculation of the more dependable toe resistance.

George D. Hartley,[27] Assoc. M. ASCE.—By drawing on a wealth of experience obtained from a wide variety of pile-driving records, Mr. Chellis has shown the lack of positive correlation between pile capacities derived by the use of a standard formula and pile capacities as obtained by test loadings. On a recent project capacities of piles driven in Norfolk, Va., were based on the results of test loadings applied to two piles. Although the soil was not composed of truly cohesionless materials, but showed intense stratification, it is thought that the results obtained may be of interest.

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[27] Supt. of Constr., Doyle and Russell, Gen. Contractors, Richmond, Va.

TABLE 5.—NUMBER OF BLOWS REQUIRED FOR A PENETRATION OF 1 FT

Depth of pile below ground (ft)	PILE NO.								
	219	220	221	222	223	224	225	239[a]	240
5	15	15	10	19	....	19	20	33	15
6	....	....	15	15	16	....	20	40	16
7	....	....	19	15	....	....	21	40	16
8	....	....	15	19	....	....	20	44	21
9	....	....	16	20	....	....	20	60	25
10	33	21	17	23	20	25	29	58	30
11	....	....	16	19	....	....	37	59	30
12	....	....	14	21	....	....	40	60	30
13	....	....	12	18	....	....	36	60	29
14	....	....	13	14	....	....	23	56	22
15	41	25	11	13	21	25	17	56	19
16	....	....	10	10	....	....	15	33	12
17	....	....	9	10	....	....	14	34	12
18	....	....	7	10	....	....	13	34	12
19	....	....	7	7	....	....	13	33	11
20	18	17	8	8	13	16	13	35	11
21	....	....	7	8	....	....	12	....	10
22	....	....	8	9	....	....	13	....	10
23	....	....	7	9	....	....	14	....	11
24	....	....	7	10	....	....	15	....	12
25	15	16	7	11	16	19	18	30	14
26	....	....	8	11	....	...	17	25	15
27	....	....	9	13	....	....	18	33	17
28	18	18	13	14	14	20	17	32	20
29	18	17	15	16	15	18	21	30	....
30	23	20	15	....	17	20	25	29	....

[a] Pile No. 239 was 50 ft long, but only values to 30 ft have been shown for comparative purposes.

The building code (1948) of the City of Norfolk provides that piles be driven in accordance with the Engineering-News formula. As shown in Table 5, which is the driving log for a number of piles, erratic driving conditions were encountered, as illustrated by the fact that pile No. 240 required approximately only one half of the number of blows required by pile No. 239 to obtain the same penetration although the two piles were driven only 2 ft 6 in. apart. This condition could not be traced to any unusual conclusions based on pile-driving operations, as there was a time lapse of one month between the driving of these piles, pile No. 239 being driven first. As a further indication of comparative driving resistance it may be interesting to note that piles Nos. 219 to 225, inclusive, were driven within a 6-ft radius from load test pile No. 222.

In view of this experience with erratic driving conditions, it was decided to make loading tests on two of the piles driven—Nos. 222 and 240. These piles were selected because they showed the most variance on the basis of allowable load from the design load of 15 tons based on the Engineering-News formula. The load tests were conducted in strict accordance with the City of Norfolk Building Code (1948), which required that the following conditions be satisfied:

"* * * Safe load carrying capacities may be determined by means of loading tests performed on not less than two typical piles of an installation. The allowable load shall in no case exceed one-half of that load which causes a net settlement (total settlement less rebound) of 1/200 of an inch per ton of test load or more than one-half inch, nor shall it exceed one-half of that load under which, during a 48 hour period of continuous constant load application, no appreciable additional settlement takes place."

TABLE 6.—LOAD TEST DATA FOR PILES NO. 222 AND NO. 240

Time (min) (1)	Load (tons) (2)	Settlement (in.) (3)	Time (min) (1)	Load (tons) (2)	Settlement (in.) (3)
(a) PILE NO. 222			(b) PILE NO. 240		
0	0	0	0	0	0
50	6	0	23	6	0
73	12	0	52	12	$\frac{1}{32}$
86	15	0	73	15	$\frac{1}{32}$
96	18	0	81	18	$\frac{1}{32}$
99	20	0	89	20	$\frac{1}{32}$
117	24	$\frac{1}{32}$	103	24	$\frac{1}{16}$
124	25.5	$\frac{1}{16}$	118	27	$\frac{3}{32}$
142	30	$\frac{1}{8}$	133	30	$\frac{1}{8}$
48[a]	30	$\frac{1}{8}$	48[a]	30	$\frac{5}{32}$
90[a]	0	$\frac{1}{16}$[b]	90[a]	0	$\frac{1}{16}$[c]

[a] Hours. [b] Rebound $\frac{1}{16}$ in. [c] Rebound $\frac{3}{32}$ in.

As shown in Table 6 the load test on these piles easily satisfied the test load requirements of the building code. The allowable load capacity as derived from the Engineering-News formula is 8.54 tons for pile No. 222, and 12.1 tons for pile No. 240. The ultimate capacities based on a safety factor of 6 would indicate values of 51.24 tons and 72.6 tons for piles Nos. 222 and 240, respectively. Design loads for the piles under consideration were based on an allowable load of 15 tons, which was indicated as permissible as a result of the load tests. The test piles were not loaded to failure and the actual safety factor applicable to the results obtained by the load test is not known. It may possibly be that, in dealing with two separate provisions of a building code (pile capacity based on formula and pile capacity based on the results of load tests), there are involved two variable safety factors each separate and distinct from the other.

As piles were driven and it was noticed that the erratic driving conditions continued, a check was made on all variables which might account for the discrepancies. However, no positive correlation between these factors and the penetration per blow could be determined. The factors observed were: Size of tip and butt, steam pressure, rate of blows, and order of driving. Piles were driven with a single-acting steam hammer.

Relatively speaking this was a small job, but the cost of the load tests was a justified expense that was more than offset by the saving in pile costs.

W. MACK ANGAS,[28] M. ASCE.—An excellent case is made in this valuable paper for the superiority of the Hiley formula over the formulas with which it is compared. It has already been stated that the Engineering-News formula, its variants, and older formulas "die hard." Nevertheless, the engineering profession should recognize and accept the fact that statistical comparisons of the accuracy of older formulas and the Hiley formula clearly indicate the superiority of the latter.

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[28] Rear-Admiral, CEC, U. S. Navy, New York, N. Y.

The author is to be congratulated on the manner in which he emphasizes the necessity of limiting the application of dynamic pile-driving formulas to piles driven into noncohesive soils. It is unfortunate, perhaps, that he adds to his warning against the application of dynamic formulas to cohesive soil conditions the statement that by using a formula of the Hiley type it is possible to compute the fiber stress in a pile during driving, even under cohesive soil conditions.

It is doubtful that the stresses in a pile during driving may be determined with reasonable accuracy by the Hiley formula or any of its widely used predecessors, irrespective of the type of soil into which the pile is driven. This doubt is based on pile-driving experiments conducted at San Diego, Calif., early in 1927, the results [18] of which have been referred to by the author. These experiments included the driving and loading of a 20-in. by 20-in. precast reinforced concrete test pile to determine the value of a dynamic pile-driving formula proposed by the Bureau of Yards and Docks, and the subsequent driving of a 20-in. by 20-in. reinforced concrete test pile 65 ft long. The latter pile was not loaded. Two 20-in. by 20-in. Douglas fir test piles and round Douglas fir test piles, were also driven to secure comparative data on the behavior under the hammer of concrete and wood piles. All test piles were driven with a single-acting hammer. In an attempt to determine the stresses occurring in the concrete test pile during driving, a crude homemade strain gage, consisting essentially of a steel tape, three hardwood clamps, and incidentals as later described, was attached to the piles. The lower end of the tape was securely attached to the pile by a hardwood clamp set well above the mud line by a diver. The upper end was attached to a second clamp near the top of the pile by heavy elastic bands cut from an automobile inner tube. The third clamp, to which a small sheet of drawing paper was attached, was then put on the pile near the upper end of the tape. A hole punched in the tape served as a guide for the point of a pencil, which recorded on the drawing paper the deformations in the length of the pile between the point of attachment of the lower end of the tape and the pencil. The device was first used during the driving of the 53-ft concrete pile with a tape 26.8 ft long, the recording paper and the top of the tape being about 11 ft below the top of the pile. The recorded deformation, or the shortening under the hammer blow, was 0.09 in. in 26.8 ft, the corresponding value of F_c being 640 lb per sq in. A repetition of the experiment using a 12-ft tape length, with its upper end as close as possible to the driving cap, yielded a mean deformation of 0.066 in. and a corresponding F_c of 1,054 lb per sq in. This experiment was repeated a third time during the driving of the second concrete test pile, which was 65 ft long. This time a 21-ft tape length was used, the top being again as close as possible to the driving cap. Records were taken from the first stages of driving up to the time the pile was at virtual refusal. The recorded deformations did not increase when increase in resistance developed during driving, but remained practically constant at 0.09 in. in 21 ft throughout the entire driving operation. The mean of nine determinations was 0.093 in., the corresponding value of F_c being 840 lb per sq in.

The method used in these determinations was, of course, crude and subject to error. Nevertheless, the experiment indicates that the stresses produced in a concrete pile by driving are materially higher in the upper part of the pile than in the lower part, and that they are largely independent of the resistance to driving.

In conclusion, the writer believes that there are theoretical flaws in the Hiley formula, and he is also skeptical of the theoretical accuracy of any of the widely used dynamic formulas. Again, this skepticism is based on discrepancies between the observed behavior of piles under the hammer and the behavior predicted by dynamic formulas. A widely used dynamic formula[22] indicates that under identical driving conditions, a reinforced concrete pile should penetrate farther than a wood pile under the force of a given blow, and that, accordingly, a given penetration indicates that a concrete pile will carry a heavier load than a wood pile. Actually, the San Diego experiment showed the reverse to be the case. The penetration per blow of a concrete pile of the same shape and dimensions as a wooden test pile, driven close to it with the same hammer and the same type of cushion, was only a fraction of the penetration of the wood pile when the tips of the piles were at the same elevation.

There is still much to be learned about the mechanics of pile driving, although whether further experimental work can produce dynamic formulas of greater practical value than the Hiley formula is open to question, unless standard forms of cushion can be developed for use with pre-cast reinforced concrete piles. The question of stresses incident to driving is, however, of great importance, and it is believed well worthwhile to consider the experimental determination of these stresses by SR-4 strain gages of the type successfully used in such tasks as the determination of the stresses in airplane parts (even airplane propellers) during flight.

ROBERT D. CHELLIS,[29] M. ASCE.—Valuable points that should be kept in mind when arriving at decisions regarding pile capacities have been presented in the discussion of this paper.

Mr. Lee calls attention to tests involving clay strata. Because of the greatly restricted use of formulas, if soils with some clay or silt strata were excluded, a number of cases having mixed but predominantly granular soils were included. In some such cases the ratios of formula to test loads were above 100% and in other cases below, with no trend definitely apparent. By consideration of foot-by-foot driving records and borings, the engineer should judge the effects of the clay strata and make such allowance as he deems proper for this condition.

Mr. Lee states that it has been recognized in England for the last 10 years that the Hiley type of formula is the only one having any reasonable claims to fairly accurate forecasts of the ultimate supporting value of piles. The writer is glad to have this fact again called to the attention of engineers in the United States. Mr. Lee feels that too much attention has been directed to showing the inaccuracy of formulas that are known to be entirely unsuitable for the purposes to which they have been applied. This is true in England, but in the United States the Engineering-News formula is still in practically universal use and is contained in most building codes. It was published again in the influential new building code for New York, N. Y., thoroughly revised and adopted on November 17, 1948. To shake this trust, actual evidence is needed, such as is produced in the paper. Admiral Angas' discussion bears witness to the fact that the Engineering-News formula and its variants "die hard" in the United States.

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[29] Structural Engr. Stone & Webster Eng. Corp., Boston, Mass.

There are two aspects to the problem of why the Hiley type of formula may not have given even better agreement with test results. One is the question of theoretical accuracy of the formulas and measurement of values for use in the formulas, and the other, mentioned both by Mr. Lee and Mr. Feld, is the question of the accuracy and interpretation of load test results. Investigation of the theoretical correctness of the Hiley type formulas was considered beyond the scope of this paper. The writer's concern was with results. Regarding load tests, the graphical results plotted in Fig. 1 and Fig. 2 are based on the values reported for the test loads. The tests were performed under a variety of conditions, by different engineers, and were of various durations. Failure points were picked by various persons who doubtless used different rules for interpreting these values. Data were not available to put all test results on a common basis, and no doubt if they had been some differences in results would have appeared. Nevertheless, the results do represent the general type of information now obtained in the United States and, because of the number of the tests, should give general trends for the various types of formulas, piles, and hammers. It is hoped that the use of accurate driving readings and load tests to failure will become universal, and that better data will be published and become available for study.

Mr. Feld indicates the time factor in settlement and questions the accuracy of load test results. This factor has a considerable effect. In tests performed by the writer, he endeavors to allow each increment of load to come to rest without settlement for a day, thus obtaining a more accurate load settlement curve than is usual. Most of the tests reported were made by others, however, and the time factor was frequently not reported. There are many rules in use for selecting the failure point for test loads, giving quite a difference in results. A number of such rules were published for comparative study in 1946.[11]

Mr. Feld and Mr. Pimm mention the distribution of resistance between end bearing and surface friction. It is regretted that the methods commonly used in England for driving a mandrel in a shell arranged to drive the shell and tip together—or, alternately, to obtain total driving resistance and end resistance separately—are not yet in general use in the United States, where very few contractors are equipped to perform this operation.

The writer read with interest Mr. Kafka's article[25] proposing a driving formula containing terms for elastic losses. With regard to the application of driving formulas to cohesive soils, however, it would seem that the apparent elastic properties of such soils during resistance to a dynamic blow might not be the same as the elastic properties present over a long time. If clay were like rubber, it would possess one set of definite elastic properties. Pile driving in cohesive soils, however, often meets with high resistance to quick displacement of the contained water, and also creates a zone of temporarily remolded material and a layer of water around the pile that temporarily cannot be forced into the soil. This water forms a lubricating film next to the pile so that neither the transfer of stress value between the pile and soil, nor the bearing value of the soil, acting during the blow, are values that have a known relationship to the pile-bearing value. Both Mr. Kafka's formula and the Hiley type of formulas have terms for elastic rebound of the soil and thus assign an expenditure of a proportion of the driving energy to the soil. This deduction from energy is desirable, but would seem to be more accurate in the case of cohesionless soils than in the case of cohesive soils. There is the further consideration, mentioned by Mr. Feld, that load tests of cohesive soils often indicate safe values that prove excessive in the actual structures. It is to be supposed that indicated dynamic resistances on piles would be subject to the same consideration.

Mr. Pimm suggests that the weight of ground moved downward by a pile during driving be included in the term for pile weight in the formulas. This condition probably occurs to some extent with all piles, but particularly with H-piles for which the writer previously has advocated the inclusion of the weight of soil between the flanges.[10] This soil has elastic properties different from those of the pile; but probably the modulus of elasticity, even if it were known, is small relative to that of the pile, and may be ignored if an elastic loss is computed. When the value of rebound is measured in the field, this value is taken into account.

Mr. Pimm states that experience has shown that frictional resistance is a very capricious factor. The test results presented by Mr. Hartley illustrate this statement. Driving conditions similar to those given by Mr. Hartley have just been noted by the writer on a friction pile project on the west bank of the Hudson River opposite New York City.

Admiral Angas doubts that stresses in a pile during driving may be determined with reasonable accuracy by the Hiley formula. The writer was able to observe the driving of 566 pine piles with varying tip diameters and cross-sectional areas to various sets, and to measure rebounds for use in the Hiley formula.[8] The small percentage of breakage was reasonably constant for the specified minimum sets for the various tip diameters, as determined by computed stresses, rising quickly when the assumed limiting value of fiber stress was exceeded. This method of determining driving limits for wood piles extending through water or soft material to short penetration in a bearing stratum has been used by the writer on other projects and has allowed driving with practical avoidance of breakage. The piles with the larger tips can withstand hard driving compared to those with smaller tips, the problem being the determination of the stress and avoidance of stresses exceeding the yield point. With this type of bearing and wood piles, it is the stress near the lower end that becomes critical, because of the small diameter at this point.

The writer has also had opportunity to compute stresses in H-piles and fluted steel shell piles by the Hiley formula, and has observed failure of the metal at the resistances giving a computed fiber stress equal to the yield point.[10]

The writer's experience agrees with the observations of Admiral Angas in the San Diego experiment, with heavier piles requiring smaller sets to reach a given tip elevation.

No method of computation of pile capacities, whether by dynamic formulas or measurement or computation of stresses at various points in the pile, would seem to be correct unless the same load-carrying capacity were indicated for piles of the same dimensions regardless of weight and material of their composition, when driven by the same hammer in the same soil to the same tip elevations. The capacity of the soil to support load would not seem to be affected by the material of which the pile is composed inside its bounding dimensions.

The writer thanks all discussers of the paper, and hopes that much further material on the correlation of driving and test results will be made available to the profession by others, so that more final conclusions can ultimately be reached and be accepted as working tools.

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Paper No. 2370

UNUSUAL DESIGN PROBLEMS, HARLAN COUNTY DAM

BY LOUIS G. FEIL,[1] ESQ.

SYNOPSIS

Details and results of extensive analyses of stilling basin, spillway, and embankment designs for a major unit in the proposed flood control plan for a major tributary of the Missouri River are presented in this paper. Among the difficulties overcome in the final design are such foundation conditions as weak shaly chalk, badly faulted and fractured, and interbedded with numerous near-horizontal seams of bentonitic clay, together with almost certain differential settlement of an overlying loess and loess-silt mantle.

LOCATION AND STRUCTURES

The Harlan County Dam with its multipurpose reservoir, scheduled for completion in 1951, is one of the major units proposed by the so-called "Pick-Sloan Plan" (the comprehensive plan for flood control and other purposes in the Missouri River Basin, contained in *House Document No. 475,* and *Senate Document No. 247,* Seventy-eighth Congress 2d Session, approved by the Flood Control Act, December 22, 1944) for the control of floods in the Kansas River Basin, at Kansas City (in both Missouri and Kansas), and along the Missouri River downstream from Kansas City. It was designed by the Kansas City District, Corps of Engineers, Department of the Army, which is cooperating with the Bureau of Reclamation, the Fish and Wildlife Service, the National Park Service—all of the United States Department of Interior—the State of Nebraska, and other agencies to assure that optimum use will be made of this project for flood control, irrigation, fish and wildlife conservation, recreation development, and other public services.

The dam site is located on the Republican River in south central Nebraska, approximately 236 miles above the point at which the Republican and Smoky Hill rivers join to form the Kansas River. The reservoir impounded by the dam has a gross capacity of 850,000 acre-ft, of which 500,000 acre-ft is allo-

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NOTE.—This paper was published in September, 1948, *Proceedings.* Position and title given are those in effect when paper was received for publication.

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cated to flood control, 150,000 acre-ft to irrigation, and 200,000 acre-ft to conservation and silting. The reservoir controls the runoff from 20,750 sq miles, or about 83% of the total area of the Republican River Basin. Storage, sufficient to permit control of all floods of record in the Republican River, affords direct protection to 138,000 acres along the Republican River and, in addition, lessens the flood hazards for 150,000 acres and 583,000 acres along the Kansas and Missouri rivers, respectively. Storage allocated to irrigation provides water to irrigate approximately 86,700 acres downstream from the dam. At maximum operating pool level the reservoir has a 22,800-acre area, a 15-mile length, and an 118-mile shore line. The conservation pool, extending upstream slightly less than 12 miles, has a 58-mile shore line and a 13,700-acre surface area.

The Republican River Valley at the project site is about 2 miles wide; the river channel, located near the middle of the valley, is approximately 250 ft wide. The flood plain (from El. 1885 to El. 1900) occupies the south half of the valley, lying mostly on the right bank of the river. The left or north side of the valley is occupied by a broad terrace, generally from 35 ft to 60 ft above the flood plain. Bedrock beneath the left terrace is approximately at stream bed elevation (El. 1877), affording a favorable site for an overflow type of spillway structure with relatively short approach and outlet channels. Preliminary studies indicated conclusively that the most economical plan of development would consist of rolled fill embankment sections on the flood plain, the terrace, and the river channel, a concrete overflow spillway section and stilling basin located on the terrace adjacent to the river channel, and adjoining concrete bulkheads tying to the rolled fill embankment sections.

Embankment.—The embankment has a total length of approximately 10,600 ft, a maximum height of approximately 110 ft in the channel section, and average heights in the terrace and flood plain sections of 85 ft and 100 ft, respectively. On the 30-ft-wide top of the embankment, at El. 1982, a 22-ft paved roadway is provided. Embankment side slopes vary from 1 on 2½ to 1 on 4. Berms constructed of waste material (principally from the spillway excavation) have been added at the toes of the upstream and downstream slopes on the right bank and channel sections. A section of pervious fill is incorporated in the upstream slope immediately beneath the riprap to minimize sloughing during drawdown.

Concrete Structure.—The concrete structure, including the overflow spillway and bulkhead sections, is approximately 1,400 ft long with a maximum height of 147 ft from bottom of concrete to top of dam. The spillway (Fig. 1) is an ogee type overflow structure controlled by eighteen Tainter gates, 40 ft wide by 30 ft high, which will pass the spillway design flood of 457,000 cu ft per sec with a corresponding head of 31.1 ft above the spillway crest. Inspection, operation, and grouting galleries are provided in the spillway. A highway bridge with a 22-ft roadway and a sidewalk on each side is supported on the gate piers. Although the bridge serves solely as an access roadway during the initial development of the project, it was designed in accordance with highway standards to allow future incorporation in the highway system.

Flood control storage will be released through nine sluices, 5 ft wide by 8 ft high, and uniformly sloped so that their discharge inverts are tangent to the bucket at the toe of the spillway. To minimize cavitation the height of the sluices is reduced 15% at the bucket. Each sluice is controlled by one service gate and one emergency, hydraulically operated slide gate with vented frames. Total sluice capacity varies from 19,500 cu ft per sec with full flood control pool to 14,300 cu ft per sec with depleted irrigation pool.

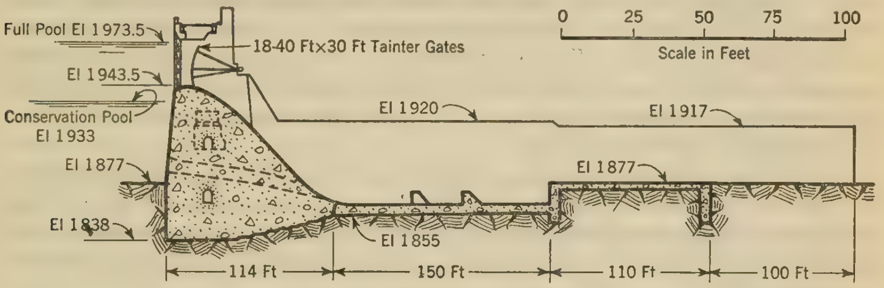


FIG. 1.—ADOPTED DESIGN OF SPILLWAY AND STILLING BASIN

Irrigation releases of approximately 2,000 cu ft per sec will be made through a combination consisting of the flood control slucies, one circular sluice 2.5 ft in diameter in the right bulkhead section, and one circular sluice 5 ft in diameter in the left bulkhead section. The energy of the water discharged by the spillway is dissipated by a 150-ft-long, concrete-lined stilling basin with two rows of 8-ft-high baffle piers, and a control sill 15 ft high and 110 ft wide.

Construction of the embankment requires approximately 12,000,000 cu yd of fill and 260,000 cu yd of riprap and spalls for slope protection. Concrete structures require approximately 3,200,000 cu yd of excavation and 435,000 cu yd of concrete. Total cost of the Harlan County Dam and reservoir is estimated to be more than $25,000,000.

POOR FOUNDATION CONDITIONS

Construction of dams in the Missouri River Basin continually presents unusual problems of interest to the designer. The foundation conditions, varying from dolomite and limestone in the lower sections of the basin to the soft clay shales in the upper reaches of the basin, dictate radically different designs. In the lower areas sites may usually be selected that lend themselves to economical development of all concrete structures; however, the poorer foundations, together with the scarcity of local sources for the production of satisfactory concrete aggregates and the broader river valleys, generally require long, earth fill dams in the upper reaches of the basin, with the concrete structures limited to spillways and outlet works. Although the Harlan County site is of the latter type, local foundation conditions materially affected the designs of both the spillway and embankment structures. Generally the top of the bedrock underlying the site has the same configuration as the existing ground surface. Bedrock formations are chiefly Niobrara shaly chalk and Pierre shale, but in the right bluffs approximately 40 ft of semiconsolidated Ogallala sand rests unconformably on the Pierre shale. Unconsolidated formations, chiefly loess-silts and sand, cover the bedrock to depths of from 40 ft to 70 ft, or more, in both abutments and in the broad terrace.

The Niobrara shaly chalk is the principal bedrock formation, underlying the entire dam site and the surrounding country for a distance of many miles. It is composed chiefly of minute calcareous shells and, as a rock, is relatively soft. The Niobrara, which is uniform, relatively impervious, and a firm, fairly heavily bedded formation where it is unweathered, is the most competent foundation material present. There are faults and many fracture planes, generally at angles of 45° or larger with the horizontal, as well as numerous near-horizontal seams of impure bentonitic clay varying from a small fraction of an inch to more than 3 in. in thickness.

The Pierre shale is a dark gray, compact clay shale, which disintegrates into a mass of small flakes or crumblike fragments when exposed to weathering. Numerous faults of small displacement cut the Pierre formation, dividing it into blocks of variable dips. It is present in the right abutment and also in an area under the left terrace upstream from the dam.

The Ogallala formation consists of semiconsolidated sands and silts, considerable amounts of volcanic ash, and local lenses of hard indurated quartzite.

Formations covering the bedrock in both abutment ridges consist of a sequence of gravelly sands at the base, overlain by compact and clayey silt of apparent alluvial origin and a surface mantle of loose loess-silt. An alluvial sequence of about 15 ft of gravelly sand overlain by about 10 ft of sandy silt occurs over the bedrock of the terrace. This sequence was subsequently covered by about 25 ft of loess-silt, probably reworked but not readily distinguishable from the loess of the abutments. The alluvium of the flood plain consists of from 5 ft to 10 ft of sandy silt overlying from 5 ft to 35 ft of sand and gravelly sand.

Foundation explorations in the spillway area revealed differences in elevation of the thicker bentonitic seams suggesting vertical displacements approximating 25 ft, attributed to faults in the foundation. One of these faults passes under the spillway structure slightly to the right of the center; the other two pass close to the right and left sides of the main spillway structure. The aggregate thickness of the bentonitic seams in this area is about 1.5 ft in the 100-ft depth of chalk explored. Compression tests of cylinders of sound chalk indicated that the chalk, if free from fracture planes or bentonitic seams, is an adequate foundation. However, the faults and attendant fractures, together with the closely spaced interbeds of bentonitic material, are all indications of the low resistance of the foundation to horizontal forces. Design of an adequate concrete section 147 ft high on such a foundation necessitated thorough investigation.

Special Sampling Procedure

In addition to the usual core drill samples, the presence of the faults and bentonitic seams made it highly desirable to inspect the foundation "in situ" and to obtain relatively large undisturbed samples of the bentonitic material for laboratory testing. Results of shear and consolidation tests were required in order to evaluate properly the resistance of the foundation to vertical and horizontal forces. Two 36-in. calyx holes drilled for "in situ" inspection were carefully logged and continuous photographic records prepared. Bentonitic seams encountered in one of these holes are shown in Fig. 2. A procedure was devised which allowed recovery of cylindrical samples 33 in. in diameter and of varying heights, containing a bentonitic seam—undisturbed except for the rebound due to the removal of superimposed load.

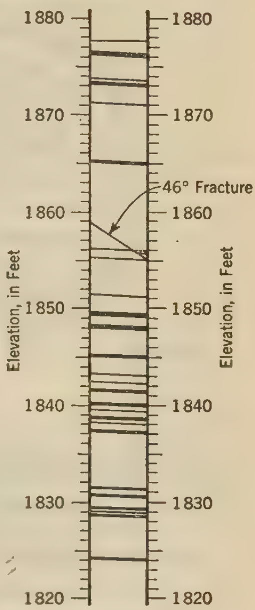
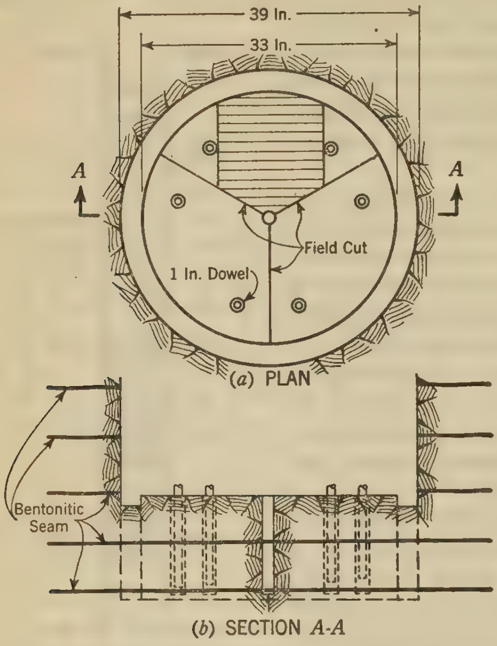


Fig. 2.—Typical Log of 36-In. Calyx Hole Showing Bentonitic Seams

To facilitate removal of the cylinder it was necessary to utilize other bentonitic seams as cleavage planes, which resulted in the use of three seams to recover one for testing. The seams desired for testing were carefully selected from the logs of the two calyx holes, and additional calyx holes were drilled immediately adjacent to those holes. In obtaining the sample every precaution was taken to eliminate rotation or jarring of the sample. Drilling was stopped at the seam at the top of the desired sample, and the following procedure was utilized:

Seven 1½-in. holes were drilled to the approximate depth of the sample. Hardwood pins 1 in. in diameter were "grouted" in six of the seven holes with plaster of Paris, the center hole being utilized to pick up the sample. The calyx hole was then drilled to the seam below the desired sample, as shown by the dotted lines in Fig. 3, and the sample was lifted free by a rod and wedge pickup without barring or blasting. The cylinders were trimmed to approximately 12 in. in height and were cut into three equal segments to facilitate handling in the laboratory. Use of hardwood dowels and plaster of Paris allowed all cutting and trimming to be done with a crosscut saw. This method of sampling permitted the laboratory to test the complete segment with the exception of the small corners lost by removal of the dowels. The three samples obtained from each seam permitted testing of each seam under three different vertical loads. The size of the sample required construction of a large shear box and the improvisation of loading and recording devices.

Results of laboratory tests on the chalk varied over a wide range, the compressive breaking strength of sound samples without bentonitic seams averaging approximately 1,000 lb per sq in. Shear tests of the bentonitic seams gave fairly uniform results averaging approximately 0.20 for the coefficient of internal friction, and 0.25 tons per sq ft for cohesion. The consolidation tests indicated that the bentonitic material would consolidate approximately 15% of its original thickness under loads comparable to those imposed by the concrete structure. Design values adopted for the Niobrara shaly chalk were: Bearing, 200 lb per sq in.; shear, 100 lb per sq in.; and modulus of elasticity, 110,000 lb per sq in. For the bentonite, assuming a safety factor of 1.5, the coefficient of internal friction was chosen as $\frac{0.20}{1.5} = 0.13$, and the cohesion as $\frac{0.25}{1.5} = 0.17$ tons per sq ft. Test values for the bentonitic material conclusively indicate the inherent weakness of the foundation to resist horizontal forces and that the design of any structure could not be considered adequate until the possibility of foundation failure by sliding along a bentonitic seam had been thoroughly investigated. Other important design considerations are: The probability of differential settlement due to the varying thicknesses of bentonite underlying monoliths having identical loadings; the probability of progressive shearing failure due to local overstressing of the laminated foundation and consolidation of the bentonitic material; and the faulted zones in the foundation indicating the probable existence of numerous joint planes in the vicinity of these zones that would affect the ability of the chalk to resist horizontal movement of the structure.



(a) PLAN

(b) SECTION A-A

FIG. 3.—METHOD OF RECOVERING UNDISTURBED SAMPLES FROM 36-IN. CALYX HOLE

STILLING BASIN DESIGN STUDIES

In the design of projects of this type one condition in nature is usually of paramount importance and materially affects the design of any major structure. At the Harlan County Dam site this one condition was the low resistance of the foundation to horizontal forces. The problem then was to devise an economical section that would transmit all forces to the foundation in such a manner as to mobilize the greatest possible resistance to sliding, differential settlement, and progressive failure. The low resistance of the foundation to horizontal forces precludes the possibility of the bottom of the stilling basin slab being at a lower elevation than the base of the overflow spillway section; therefore, the over-all height and cost of the overflow section is directly dependent on the design of the stilling basin. To arrive at an economical design for the spillway and stilling basin, designs of five stilling basins were thoroughly tested in 1-to-80 scale models by the United States Waterways Experiment Station at Vicksburg, Miss. In all these tests the efficiency of energy dissipation, the bottom velocities of the basin and outlet channel, the scour in the outlet channel, and the water surface profiles were carefully analyzed.

The first design consisted of a sloping apron approximately 365 ft long divided into primary and secondary basins by a subdam 40 ft high located 180 ft downstream from the toe of the spillway. The bottom of the apron was set at El. 1856.5. Two rows of 10-ft-high baffle piers and a 2-ft-high stepped end sill were included in the secondary basin. The performance of this design was very satisfactory in the dissipation of the spillway flow before it entered the outlet channel. The model studies indicated that the subdam height could be reduced 5 ft without impairing the efficiency of the basin.

In order to reduce the length of the concrete apron and that of the massive training walls flanking the stilling basin, a second design, consisting of a horizontal apron 122.5 ft long having two rows of 8-ft-high baffle piers and a 10-ft-high stepped end sill, was tested. The bottom of the apron was set at El. 1848. In this design it was assumed that the tailwater depth could be reduced 15%, approximately 7 ft, because of the baffles. The performance of this design was not satisfactory. Excessive impact on the baffle piers indicated that the piers should be moved farther downstream and the apron lengthened and lowered approximately 5 ft.

The modifications indicated by the tests of the second type of basin were incorporated in a third design; they consisted of a sloping apron 186 ft long with two rows of 11-ft-high baffle piers and a 3-ft-high stepped end sill. The apron bottom was set at El. 1840. Performance of this design was generally satisfactory; however, estimated cost of the spillway and stilling basin for this design was approximately $800,000 more than the first design. Also, this third design definitely indicated that deeper tailwater should be obtained by raising the end sill, rather than by lowering the apron.

A fourth design, which consisted of a horizontal apron 136 ft long with two rows of 8-ft-high baffle piers, a 10-ft-high stepped end sill, and a control sill 5 ft above the top of the end sill extending downstream 105 ft and terminating in a cutoff wall, was tested. The apron bottom was set at El. 1855. Performance of this design was not as satisfactory as that of the first and third types in that the piers were subjected to severe impact and the erosion of the outlet channel would cause a secondary jump to form immediately downstream from the control sill. Elimination of the baffle piers resulted in the jump being swept from the apron.

A fifth design, similar to the fourth type except that the stilling basin was lengthened to 150 ft and the stepped end sill was replaced with a 15-ft-high vertical end sill, was also tested. Performance of this design, although still not quite as efficient as that for the first and third types for maximum discharge conditions, satisfactorily overcame the difficulties of the fourth design. It was found that the training walls flanking the spillway could be reduced from a total length of 460 ft for the first design to 360 ft for the fifth design without an appreciable effect on erosion downstream from the control sill.

The fifth type of stilling basin design was adopted, thus establishing El. 1855 as the top of the key for any spillway structure. It is estimated that the adoption of this design in lieu of the first design reduced the cost of the spillway and stilling basin by approximately $500,000, showing conclusively the value of the hydraulic laboratory as a tool for the designer.

Spillway Design Studies

Four concrete overflow sections were studied in considerable detail. The main features of each were essentially the same, varying only in the details required to adapt them to the differing design requirements of the individual structures. The first of the four gravity type spillway structures studied mobilized horizontal resistance to sliding by a conventional tension key at the heel. In the analysis of this section it was considered that failure would occur by sliding along a bentonite seam at the bottom of the key and by shear of the chalk at the downstream toe along a plane inclined at 45° to the horizontal and extending from the bentonite seam to the bottom of the stilling basin slab. The vertical base pressures followed the normal pattern for a gravity section with the typical fluctuation from low to high toe pressure in conformity with the change from empty to full reservoir conditions. Vertical pressures and the magnitude of the pressure fluctuation could be somewhat reduced by sloping the upstream face of the section, and the horizontal pressures could be reduced by deepening the key. Heavy reinforcement would be required, however, to withstand the horizontal shear at the top of the key and the tension in the concrete downstream from the key.

In order to eliminate the tension in the concrete and the necessity of heavy shear reinforcement, a second gravity type structure was studied. In this design the base was sloped and the horizontal forces were transmitted to the foundation by the toe, which was at a lower elevation than the bottom of the stilling basin slab—thus placing the concrete in compression. In the analysis of this section it was assumed that failure would occur by shear of the chalk along a plane inclined at 45° to the horizontal and running from the lowest point of the toe to the bottom of the stilling basin slab. The distribution of foundation pressures would be essentially the same as for the section having the tension heel. The compression toe does not, however, mobilize the weight of the material immediately below the spillway structure, nor does it tend to distribute any of the horizontal forces to the foundation upstream from the toe. Although computations indicated that this structure would be stable, it would aggravate any tendency of progressive failure; in cross section, it has the appearance of attempting to chisel out the material immediatley downstream from the toe and slide into the stilling basin.

The third structure studied was of the round-head buttress type in which horizontal resistance to sliding was mobilized by a bulb at the toe of each monolith. Some of the advantages normally attributed to the buttress type of structure include: (1) The flexibility attained by articulation of the upstream and the downstream slabs, thus minimizing the detrimental effects of differential settlement (in the round-head design, however, a great deal of the flexibility is lost due to the mass of the round heads); (2) the more even distribution of base pressures than for a gravity section (the unit pressures, however, are usually somewhat larger than for a gravity section of the same height); (3) the minimized uplift pressures due to the smaller base areas; (4) the ease of periodically inspecting and grouting the foundation remaining exposed between buttresses; and (5) the reduction in concrete quantities, which in some instances would be sufficient to offset the increased costs due to the increase in reinforcement and formed areas. The last item would hold particularly in localities where labor is cheap and material costs are high.

The round-head type of structure, although not common, has been successfully utilized for several important projects, among them the Don Martin Dam in Mexico and the Rio Negro Dam in Uruguay. However, in these instances special provisions were not required to transfer horizontal forces to the foundation. The round head, which is the upstream water supporting member, is, in plan, a segment of a circle flaring from the buttress and thus allowing transmission of stresses induced by the water pressures to the buttress by direct compression. Advantages claimed for this type of water supporting member are:

1. The member is in compression under full load.
2. Reinforcement is minimized, as bending and diagonal tension stresses for all practicable purposes are eliminated.
3. A reduction in costs may be effected due to the mass concrete construction of the round head.

Detailed studies indicated that for the foundation conditions at the Harlan County site the buttress type of structure has the following undesirable characteristics:

(*a*) As no upstream cutoff is provided, full hydrostatic head could develop in joints and seams, possibly resulting in a foundation blowout between the buttresses.

(*b*) The weight of the structure would not assist in mobilization of horizontal resistance.

(*c*) The chalk lying between the buttresses and above the bottom of the bulb cannot be mobilized to resist horizontal forces except by friction along the buttress faces.

(*d*) The concentration of stresses at the bulb would aggravate any tendency toward progressive failure.

(*e*) Lack of continuity of the toe, under adverse and differing jointing conditions of the foundation at individual monoliths, would result in greatly reducing and varying the resistance to horizontal movement afforded these monoliths.

(*f*) Computed tensions occur in the bottom of the buttresses under certain loading conditions.

With a laminated foundation, such as presented by the chalk with interbeds of bentonitic material, it is difficult to design a buttress dam and a gravity dam that would have exactly comparable safety factors. Unquestionably the gravity structure will mobilize an area equal to the full depth of the key and the width of the monolith, regardless of the occurrence of fracture planes. Under perfect conditions and in a homogeneous material bulbs could be designed to mobilize horizontal resistance to the same degree and in the same manner as the key of the gravity section. However, the chalk is not homogeneous but is laminated by bentonitic seams and broken by fracture planes, making it very susceptible to progressive failure occasioned by local overstresses. Because the fracture planes in the foundation might be so arranged that overstresses in contact bearing or shear could not follow a normal distribution pattern, it was assumed: Failure would occur along a plane inclined at 45° and limited in width to the width of the bulb, and the width and depth of the bulb would be determined by the allowable contact bearing of 200 lb per sq in.

Consideration of the apparent advantages of the gravity section for this particular site led to the study of a modification of the section having a tension heel. The large step originally included in the heel was eliminated, the bottom of the heel was widened, and the remainder of the base was sloped upward at an angle approximately normal to the resultant of the applied forces. The final design cross section is shown in Fig. 1. The upstream face of the section was battered to assure positive base pressures at the heel for all conditions of loading. With this cross section the weight of the chalk below the structure is mobilized and tension in the concrete is kept within allowable limits. In the analysis it was assumed that failure would occur by sliding along the bentonite seam on a level with the base of the heel and shear along an inclined plane from that level at the toe to the bottom of the stilling basin slab, as outlined for the section with the tension key.

Preliminary estimates indicated that the cost of the gravity section would be somewhat less than that of the round-head buttress. Since there appeared to be very real benefits attributable to the selection of the gravity design for the Harlan County foundation, and since no economy would result from the construction of the round-head buttress, the modified gravity section was adopted for construction.

Embankment Design

Foundation problems are not limited to the concrete structure. Laboratory tests indicate that the loose loess-silts of the terrace and abutments, with an average dry weight in their natural state of 80 lb per cu ft, consolidate in a very erratic manner under varying conditions of loading and saturation. The unpredictable nature of the consolidation of saturated loess and the extreme differential settlement which would occur in an earth structure having an over-all height of 85 ft, and founded partly on chalk and partly on 35 ft of loose loess-silts, led to the adoption of a very conservative treatment in the initial stages of the design (Fig. 4). The design contemplates the removal of the loess under the embankment and the construction of an impervious cutoff through the underlying sand and gravel, having sufficient contact with the chalk to assure that the amount of underseepage would be negligible. In the left abutment, however, a section of the embankment must of necessity rest on undisturbed loess and is subject to some axial and differential settlements. It is planned to strip the loess to a slope of 1 on 3½ and flare the embankment slopes so as to limit the estimated differential settlement of the embankment to a maximum of 1 ft in 100 ft along the axis. This solution is not economical for the main sections of the embankment because the very flat side slopes require an excessive yardage of compacted fill.

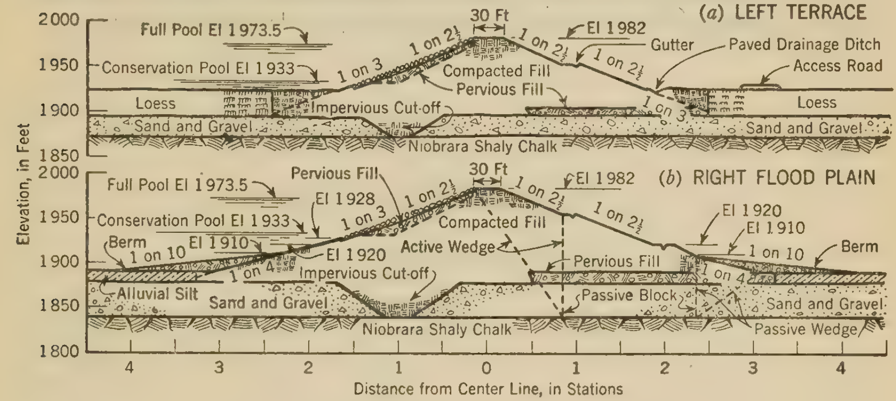

(a) LEFT TERRACE
Full Pool El 1973.5
Conservation Pool El 1933
30 Ft
El 1982
1 on 2½
1 on 2½
1 on 2½
1 on 3
1 on 3
Gutter
Paved Drainage Ditch
Access Road
Compacted Fill
Pervious Fill
Impervious Cut-off
Loess
Loess
Sand and Gravel
Sand and Gravel
Niobrara Shaly Chalk
(b) RIGHT FLOOD PLAIN
Full Pool El 1973.5
Pervious Fill
El 1928
30 Ft
El 1982
1 on 2½
1 on 2½
1 on 2½
1 on 3
Conservation Pool El 1933
Compacted Fill
Berm
1 on 10
El 1910
El 1920
Active Wedge
El 1920
El 1910
1 on 10
Berm
1 on 4
1 on 4
Impervious Cut-off
Pervious Fill
Alluvial Silt
Sand and Gravel
Passive Block
Sand and Gravel
Niobrara Shaly Chalk
Passive Wedge
Elevation, in Feet
2000
1950
1900
1850
2000
1950
1900
1850
1800
4
3
2
1
0
1
2
3
4
Distance from Center Line, in Stations

Fig. 4.—Typical Valley Embankment Cross Sections

The possibility of reducing the removal of loess was a subject of laboratory investigation. It was thought probable that suitable provisions could be made to extend the period of time required for the loess downstream from the cutoff to become saturated, or that positive methods could be devised to prevent this loess from becoming saturated. This, accompanied by partial removal of the loess, would tend to minimize any differential settlement. If such settlement could be limited so as to present only a minor maintenance problem, a material reduction in excavation on the terrace could be safely effected. Such reduced excavation lines in the terrace section approximate a line from the top of the gravelly sand at the downstream side of the impervious cutoff to the ground surface at the downstream toe.

Particular attention was required for the stability of the embankment and foundation as a whole, because of the numerous bentonitic seams which form approximately horizontal shear planes in the chalk. Based on the assumption of sliding on such a seam immediately below the surface of the chalk a wedge type analysis, in which the thrust of the active wedge was equated to the resistance of the passive block and passive wedge, indicated the need for berms in the river channel and flood plain sections. The loess in the right abutment underlain by the faulted Pierre shale required treatment similar to that for the left abutment. In addition, the sand and Ogallala formations require blanketing upstream and drainage downstream from the dam.

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INDUSTRIAL WASTES
A SYMPOSIUM

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With Discussion by Messrs. S. Leary Jones, and A. M. Quattlebaum.

Note.—Published in September, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

EFFECT ON STREAM POLLUTION LEGISLATION AND CONTROL

By Don E. Bloodgood,[1] Assoc. M. ASCE

When attempting to show the effect of one thing on another, it is sometimes possible to correlate completely unrelated items so as to make it appear that one is directly the result of the other. Such an unjustified conclusion may be drawn if it is attempted to show the individual effect that industrial wastes have had on bringing about legislation for the control of stream pollution.

Historical Notes

In reviewing the past history of stream pollution it must be recalled that the early complaints resulted from nuisances caused by discharge of sewage and other wastes into streams. People downstream objected to the obnoxious odors which changed their pure country air into the "foul depressing atmosphere" described in many court cases. Whether the elimination of a nuisance is more important than the removal of a positive health hazard is not a difficult question for the sanitary engineer, but to the lay public the more obvious a situation is, the more attention it is given. Whether industrial wastes have been more responsible for the passage of stream pollution control legislation is largely a matter of conjecture. R. Hollowell, Jr.,[2] uses, as an illustration of the earliest objection to this nuisance, in Indiana, the case in 1856 when one Sloan was convicted of causing a nuisance by pouring the slops from the Bates House in Indianapolis into the street, where it made offensive pools along the west side of Illinois Street. Mr. Hollowell also states that the first case of stream pollution in Indiana to come to the attention of the courts occurred in 1868, when a Mr. Taylor was indicted for depositing domestic waste in a spring from which travelers on a near-by highway were accustomed to drink. This indicates that the first recorded difficulties involving wastes were not those of an industrial type, unless for the sake of argument, someone contends that the slops from the Bates House were those from an industrial establishment.

One of the earliest records of complaint of discharge of industrial waste into a watercourse is found in 1886 when a Mr. Mergentheim was indicted, under the nuisance statute of Indiana, "for discharging the water from his woolen mills into the canal at Peru, Indiana, to the discomfiture of those in the vicinity, particularly when the water in the canal was low." In 1893 the Indianapolis Water Company obtained an injunction against the American Strawboard Company which at least registered that the waste from the mill was causing sufficient difficulty to warrant action in the courts. It can be seen from these instances that there were court cases involving industrial stream pollution as early as the 1880's. It is presumed, however, that there were about as many complaints over domestic sewage as there were over industrial wastes. In many cities where stream pollution is or has been a problem it is very difficult to state definitely whether the municipal or industrial wastes caused the stream pollution and which of the two brought about the legislation curbing the pollution.

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[1] Associate Prof., San. Eng., Purdue Univ., West Lafayette, Ind.

[2] "History and Legal Background of Stream Pollution Control in Indiana," by Robert Hollowell, Jr., *Proceedings*, 1st Industrial Waste Utilization Conference, Purdue Univ., West Lafayette, Ind., 1944, pp. 31–42.

AMOUNT OF INDUSTRIAL WASTES

Industrial wastes discharged into streams in the United States amount to an equivalent population of 60,000,000 and the domestic sewage discharged into these same streams is contributed by from 55,000,000 to 60,000,000, according to one authority.[3] Another states that, roughly, the raw sewage of 47,000,000 persons is emptied into the receiving streams of the United States and that there is an additional equivalent of from 55,000,000 to 60,000,000 persons added by untreated and uncontrolled industrial wastes.[4] These data substantiate each other and indicate that industrial wastes are responsible for at least half of the contamination of the streams today.

In another report[5] it is stated that industrial wastes have a pollutional effect on the streams of Tennessee equivalent to a domestic sewage population of about 1,268,000, which is approximately 1.35 times the population discharging domestic sewage into the streams. The equivalent population does not take into account some of the objectional inorganic compounds from industry that are deterimental to the streams and make some of them unsuitable as sources of domestic water supplies or as habitats for desirable aquatic life. The report of the Ohio River Pollution Survey[6] shows that wastes from 1,800 industrial plants were investigated and that the wastes from these plants represented an estimated equivalent population of 10,000,000. This certainly represents a sizable problem, which must be solved by hard work spurred on by fair and just legislation.

It is not always the magnitude of the industrial waste problem that calls it to the attention of the public. In past years Indianapolis packing plants were cited as the cause of pollution of the White River. In the 1934 report of a survey,[7] it is shown that industrial wastes in Indianapolis contributed only an equivalent population load of 267,728 as against a total equivalent population for the whole city of 600,728. In many cities it may be the additional load brought on by industry which makes it impossible for the receiving stream to purify itself naturally. It is usually true, however, that where there is an industrial waste load there is usually a domestic sewage load of some proportion, and separation of the two may be all but impossible. In some cities industries have contributed quantities of wastes which far exceeded the domestic wastes. In Decatur and Peoria, in Illinois, discharges of industrial wastes have been so great, in comparison with the domestic sewage load, that they have been used as outstanding examples in other states to show the need for strict and strong legislation for industrial waste control. Concentration of some industries, such as the pulp mills in Maine, has brought about difficulties that have attracted legislative attention. These examples indicate that every effort must be made to see that future legislation is adequate to cover all situations involving industrial wastes.

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[3] "Industrial Wastes Equivalent to 60 Million Population," by J. K. Hoskins, *Sewage Works Engineering*, October, 1945, p. 493.

[4] "Responsibility for Industrial and Municipal Wastes," by Abel Wolman, *Journal*, A.W.W.A., July, 1946, p. 883.

[5] "Stream Pollution in Tennessee," *ibid.*, April, 1945, p. 350.

[6] "The Ohio River Pollution Survey, 1942," Final Rept, the Ohio River Committee, "Industrial Waste Guides," Supplement D, Federal Security Agency, U. S. Public Health Service, Cincinnati, Ohio.

[7] "The Population Equivalent of Certain Industrial Wastes," by C. K. Calvert and Elliott H. Parks, *Sewage Works Journal*, November, 1934, p. 1159.

It is of interest to note that some laws must be changed by acts of legislature because they are too specific. In 1945, the Pennsylvania Legislature amended its act of 1937, particularly with respect to industrial wastes, because the 1937 act specifically exempted acid mine drainage and coal mine silt from its provisions until practical methods for treating these wastes became known. The 1945 act removed the exemption from silt, and anthracite operators prepared plans and instituted work on a large scale to comply with the law. These changes indicate the trend in legislation and show that even the difficult tasks are being undertaken on a cooperative basis.

Federal Legislation

No federal stream pollution control law exists, although almost one hundred bills have been submitted to Congress since November, 1897. It is believed that the main reason for this lack is that most of the bills extended federal control.[8] It is interesting to note the indication[8,9] that the failure to pass a federal law governing stream pollution control cannot be blamed on industry. In general, industry has shown a decided interest in assisting the stream pollution abatement program with certain groups sponsoring research projects in their particular fields. On the other hand, industry cannot be expected to show progress in stream pollution abatement when disagreement exists as to which type of federal law is needed.

State Legislation

Legislation provided by the states was strengthened during 1945 and 1946, and is a trend that may be expected to continue. Many of the states have found that their legislation is far from being effective in the proper control of stream pollution because the laws do not permit control measures to be enforced until damage has been done. In preparing added or modified legislation there is danger in making the law too long and cumbersome. Because law is no better than its administrators, however, there is but little danger when administrative power is vested in a board of control.

Indiana legislation requires that pollution of a stream or body of water must be stopped, but does not specify the degree of waste treatment required. This is as it should be, and yet it becomes necessary, sooner or later, to establish permissible standards. Establishment of these standards, which must be founded on scientific investigation, can certainly be helped by the federal services. A question such as, "What quantity of dissolved zinc can be permitted in a stream?" bothers industry more than do federal and state laws.

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[8] Report of the Committee on Sewage Disposal, Am. Public Health Assn., Public Health Eng. Section, presented in Cleveland, Ohio, November 12, 1946.

[9] "The Pollution Parade Passes," by Morris M. Cohn, *Sewage Works Engineering*, October, 1946, p. 513.

Some industries have tried to assist in gathering information that is or will be helpful in establishing the needed standards. Progress has been made in determining the toxicity of some of the wastes of the canning industry.[10] W. B. Hart has also contributed data on toxicity studies for the petroleum industry.[11]

Establishment of standards and development of new processes for industrial wastes treatment require study and research. It seems entirely reasonable that this work be carried out by federal and state agencies with funds appropriated for the specific purpose.

CONCLUSIONS

(1) Federal legislation can serve the citizens of the United States to the greatest extent by coördinating the activities of state stream pollution control units.

(2) State legislation should authorize a board of control having broad powers so that the board can establish the necessary rules and regulations, which must change with the development of the times.

(3) Industry has no objection to well-written, reasonable laws on stream pollution control.

(4) Development of suitable and improved industrial wastes disposal methods can be brought about by legislation providing for the necessary studies.

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[10] "Lethal Effect of Chemicals on Fresh Water Fish," by N. H. Sanborn, *Food Packer*, July, 1945, pp. 40–44.

[11] "The Evaluation of Industrial Wastes, Chemicals and Other Substances to Fresh-Water Fishes" by W. B. Hart, P. Doudoroff, and J. Greenbank, Atlantic Refining Co., 1945.

POSSIBILITIES OF RECOVERY AND UTILIZATION

By F. W. Mohlman,[12] Esq.

Introduction

The course of industrial practice is characterized by the progressive recovery, as salvage, of products once discharged to sewers as worthless wastes. The line of distinction between salvage and wastes is steadily moving toward recovery of products within the factory, thus relieving the pollution load in most industries. How this is planned and accomplished is the subject of this paper, which is based largely on data accumulated in Chicago, Ill., during a twelve-year study of industrial wastes, in which more than three hundred plants were inspected, wastes measured, sampled, and analyzed over at least a two-week period, and results computed in pounds of losses per day. During this survey, unit losses were computed, specific processes investigated, excessive losses noted and called to the attention of the plant managers, new salvage processes studied, and the general principles of loss prevention noted. Studies made for industries not in Chicago have also led to knowledge of waste problems of the various chemical, pharmaceutical, metallurgical, and packinghouse industries.

The practice in Chicago has been to measure flow and collect samples every fifteen minutes, to spend at least two weeks at each plant, and to compute data for a number of plants to obtain basic units for each type of industry. Such basic data are needed as the criterion of what is good or bad practice in each industry, but obviously the basic records must not be too limited to permit establishment of norms significant of good practice in operations.

Such norms have been established for packinghouses, breweries, and laundries, as well as for the corn products, by-products, coke, paper, and some types of canning industries. Information is available on salvage practices in the paper pulp, chemical, alcohol, petroleum refining, pickling, plating, and metallurgical industries. Although these results can be mentioned only briefly, data are presented for each industry and discussed as to practices which recover specific substances of value for sale or re-use.

Packinghouse

Wastes in the packinghouse industry originate in the various departments—stockyards, killing floor, carcass washing, washing and cleaning entrails, paunch manure, blood drainage, tankage, evaporators, hide drainage, hog hair removal, pickling, cooking, rendering, and general cleanup. The larger and more complex the packinghouse, the more effort is made to recover by-products.

In order to show the unit losses, survey data from fourteen Chicago packinghouses are given in Table 1. These results include hog, cattle, and mixed kills, and tonnages varying from 51 to 680 per day. Although individual houses vary rather widely from the average shown, and several surveys have been excluded because losses were known to be excessive and were in process of being reduced, the data are considered to be representative of average practice in the industry. If losses are greater than these averages, search should be made for the causes which usually can be found.

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[12] Director of Laboratories, Eng. Dept., San. Dist., Chicago, Ill.

Separation of the losses per ton of cattle as against the losses per ton of hogs was considered of interest, with data from four surveys of each type available.

TABLE 1.—UNIT PACKINGHOUSE LOSSES, CHICAGO, ILL.

Plant No. (1)	Kill: Type (2)	Kill: Tons per day (3)	Liquid loss (gal per ton) (4)	Pollutional Losses (Lb per Ton): B.O.D. (5)	Suspended solids (6)	Nitrogen (7)	Grease (8)
1	Hogs	51	5,500	36.0	24.0	5.34	1.80
2	Hogs	114	2,200	30.1	18.2	2.58	4.61
3	Mixed	127	2,600	25.4	9.1	4.05	2.88
4	Hogs	139	4,900	26.2	19.6	2.50	5.66
5	Cattle	148	1,900	41.5	29.6	4.47	1.35
6	Cattle	165	2,800	31.4	29.5	4.02	3.58
7	Cattle	168	2,000	21.0	19.9	2.04	2.01
8	Mixed	185	2,800	39.4	18.8	5.18	1.20
9	Hogs	190	4,500	19.5	14.3	2.93	0.54
10	Mixed	303	4,600	33.4	29.9	4.37	4.00
11	Cattle	397	6,200	20.0	22.0	2.16	1.10
12	Mixed	429	6,500	29.4	26.5	3.41	3.00
13	Mixed	661	2,600	13.0	12.5	1.58	1.00
14	Mixed	680	8,700	38.5	44.3	4.20	4.20
Average	....	....	4,130	28.9	22.7	3.49	2.64

Such averages did not show very significant differences except, as shown in Table 2, wherein the hog kill shows relatively more biochemical oxygen demand (B.O.D.) and grease per ton. Losses are reduced by efficient skimming to recover grease, increased evaporation to recover tankage, screening to recover paunch manure, and dry rendering to recover grease and tankage.

Skimming can be made more efficient if interplant grease basins are from 2 ft to 4 ft deep, are not too wide, provide from $\frac{1}{2}$-hr detention to 1-hr detention, maintain a velocity of from 1 ft to 2 ft per min, and have arrangements for easy removal of greases by hand skimming. Mechanical rakes or flights, and possibly vacuators or "pressurators," may be helpful, but are not essential. E. N. Mortenson[13] states that efficient operation should remove enough floating grease so that the effluent contains not more than 0.2 lb per ton kill, as determined by an ingenious procedure originated by him.

TABLE 2.—COMPARISON OF AVERAGE UNIT LOSSES IN CATTLE AND HOG KILLING OPERATIONS

Type of kill	Liquid loss (gal per ton)	Pollutional Losses (Lb per Ton): B.O.D.	Suspended solids	Nitrogen	Grease
Cattle	3,225	28.5	25.3	3.17	2.01
Hogs	4,300	31.4	20.9	3.15	3.29

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[13] "Grease Recovery in the Meat Packing Industry," by E. N. Mortenson, *Bulletin No. 60*, 2d Industrial Waste Conference, Purdue Univ., Lafayette, Ind., January, 1946, p. 28.

A rather crude method of hand skimming has been used for several years at the combined packinghouse flow at the Racine Avenue pumping station of the Sanitary District of Chicago. During the first nine months, from January to September, 1944, the income was $24,730 for 1,124 tons of inedible grease. Since then the income has fallen off, in part, because of the difficult conditions of removal. Because the grease must be mixed in low percentage with other grease or distilled in order to be salable, the limited number of processors who can handle it reduces its field of sale.

One of the most troublesome features of packinghouse wastes is their high nitrogen content. This is largely due to the discharge of tank water, entrainment of foaming in hard-pressed evaporators, and actual loss of blood. Evaporators should be of ample capacity, say, 3,000 gal per hr for a 600-ton house, and so operated that no feed water must be dumped, each day's production is handled immediately, and no tankage is held over. The recovered tankage is valuable and one of the best salvage products in the industry. Screens recover a good proportion of paunch manure, but are not particularly attractive from the standpoint of salvage. Dry rendering is one of the most helpful advances in packinghouse operation for prevention of losses, but the demand for "prime steam lard" hampers its extension in existing packinghouses. However, this helpful procedure is one of the most important advances in the industry.

TABLE 3.—TOTAL UNITED STATES PACKINGHOUSE KILL, DAILY TONNAGES, AND POPULATION EQUIVALENT OF WASTES

Year	Annual kill (million lb)	Average kill (tons per day)	Waste population equivalent (millions)
1930	30,164	50,300	8.70
1935	27,358	45,600	7.89
1940	35,669	59,500	10.29
1941	36,775	61,300	10.60
1942	40,676	67,800	11.73
1943	45,632	76,100	13.16
1944	47,395	79,000	13.67
1945	43,618	72,700	12.58

An over-all estimate of the B.O.D. (computed to population equivalent) of the total United States kill is shown in Table 3. This estimate indicates a maximum loss of 13,670,000 population equivalent in 1944 for the total animal kill. The kill under government inspection was only about 65% of this.

WOOL SCOURING

A recovery worth noting in a field allied to packinghouse practice is the recovery of grease from wool scouring at Bradford, England. It is estimated that about one half of the world's wool supply is cleaned at Bradford and that half the sewage volume is wool scouring liquors. The grease content of the sewage is 890 ppm and the alkalinity 815 ppm. By adding sulfuric acid to an acidity of 100 ppm, wool grease is precipitated with the sludge, which is again acidified and the grease separated through filter presses and drawn off from the water. More than $800,000 worth of products have been sold in one year. This is rather good salvage for a city of 263,000 population.

BREWERY

Brewing is not often recognized as the source of a great deal of industrial waste, although many cities of 20,000 have local breweries, whereas the Chicago survey, for example, included twenty-eight breweries with production of from 120 bbl to 1,280 bbl of 31 gal each per day, averaging 400 bbl. Most breweries do not include a malting plant and none of those in Chicago produced their own malt. In brewing, malt and rice mashes are boiled separately, mixed, and the wort drawn off to large copper kettles of 450-bbl capacity. The residual grain is usually recovered by pressing in expellers, followed by steam drying. Dried grains amount to about 25% of the original grain used in the process, or about 10 lb per bbl of beer. More recovery would be possible if a more efficient dewatering process were used. In the Chicago studies, the rather large losses from the expellers constituted only 7.5% of the total flow of wastes, but accounted for 37% of the B.O.D. and 31% of the suspended solids. Hops are added to the strained wort and the mixture is boiled, after which the hops are strained out and sometimes burned or dried with the grains. They are not a large proportion of wastes, amounting only to from 0.3 lb to 0.5 lb dry solids per bbl. Their salvage value is low.

After the boiled wort is cooled, yeast is added and fermentation proceeds for about ten days. The beer then is filtered, stored for three months or more, clarified, and bottled. Yeast increases 4 to 1 during fermentation, thus giving a surplus of 3 lb wet yeast, or 0.5 lb dry yeast, per bbl. Formerly, this yeast was wasted, but it is realized that the wastage is a valuable food source, and more and more effort is made to recover the yeast. For example, one Chicago company handles 500,000 lb of yeast (dry basis) per yr, collecting wet yeast from practically all Chicago breweries and refrigerating the wet yeast until ready for processing. The yeast is washed and "de-bittered," then digested with enzymes, filtered, and the filtrate dried and sold at fancy prices as a very rich source of vitamin B complex. The filter cake is dried and used as feed. Other extraction products are also made.

TABLE 4.—COMPARATIVE COST OF TYPICAL PROTEIN FOODS

Food			Protein (%)	Required weight[a] (g)	Cost per day
Type	Unit	Unit cost			
(1)	(2)	(3)	(4)	(5)	(6)
Dried yeast..	Lb	$0.50	45	155	$0.17
Meat........	Lb	0.50	17.5	400	0.44
Beef liver....	Lb	0.50	19.7	355	0.39
Whole milk...	Qt	0.15	3.5	2,000	0.33
Eggs.........	Dozen	0.50	11.4	614	0.51

[a] To yield 70 g of protein.

Some large brewers dry their entire surplus yeast. The protein value is higher than most foods, as indicated in Table 4. In addition, the vitamin content is especially rich, and the dried yeast can be added to other food products to augment protein or vitamin B deficiencies. Growing realization of the food value of dried brewers' yeast has induced many brewers to keep this product out of the sewer. In Indiana eight of thirteen breweries are equipped to dry their yeast.

The Chicago brewery tests, made in 1937 when only some of the grains were recovered and practically none of the yeast, resulted in unit losses, per barrel, as follows: Gallons of waste, 320; B.O.D., 3.1 lb per day; suspended solids, 1.68 lb per day; and nitrogen, 0.134 lb per day. Based on annual production and with no recoveries, the unit B.O.D. loss of 18.6 population equivalent per bbl would result, throughout the United States, in the losses shown in Table 5. These data indicate a maximum brewery waste population equivalent of 5,430,000, as compared with 13,670,000, for packinghouse and slaughterhouse waste. However, the brewing industry is aware of the salvage possibilities of grains and yeast and will undoubtedly reduce the over-all losses materially, inasmuch as the potential recovery of brewers' yeast is 43,000,000 lb per yr, a sizable item. During World War II, the Siebel Institute of Chicago, a leading brewers' organization, circulated a 68-page brochure containing data on the value of brewers' yeast as a "victory food." Postwar recoveries confirm the claims of this report on basic research.

TABLE 5.—TOTAL LOSSES FROM UNITED STATES BREWERIES

Year	Quantity (million bbl)	Population equivalent (millions)
1936	56.055	3.47
1938	53.513	3.31
1940	53.675	3.33
1941	59.990	3.72
1942	67.700	4.20
1943	75.042	4.65
1944	85.469	5.30
1945	87.662	5.43

DISTILLERY AND ALCOHOL

Still residues from alcohol manufacture were probably the largest source of pollution of any industrial waste during World War II because of the tremendous program of alcohol manufacture for use in the synthetic rubber industry. It was estimated that without any recovery of still "slop" the population equivalent of such wastes might reach 27,000,000 in 1944, but a substantial proportion was recovered by dewatering and drying in multiple-effect evaporators. At one distillery in Peoria, Ill., the stillage is screened and the effluent centrifuged and finally dried in triple-effect vacuum pans to produce a feed worth from $30 to $35 per ton (in 1943). There appear to be several ways to increase the degree to which vacuum pans will function, among them the preliminary centrifugal treatment. However, all such methods reduce the cost of drying and lead to recovery by production of dried feed. It was estimated[14] that some 1,000,000 tons would be dried in the United States in 1945. This recovery was probably the largest salvage problem undertaken by industry.

CORN PRODUCTS

The story of recoveries of feeds in the corn products industry is rather well known. Losses dropped from a population equivalent of 5.6 persons per bu, before any recoveries were made, to 4.0 by settling of gluten wastes, then to 1.0 when solubles were concentrated by recirculation and dried, and finally to a minimum of 0.45 in 1936. Since then the losses have leveled off at an approximate population equivalent of from 0.6 to 0.8 persons per bu, except for a period of a few months in 1945 when wheat was ground and losses increased again to 1.5 population equivalent per bu (Table 6). During this time the recovered solids were mixed with gluten and hulls and dried, to be sold as feed at $35 or $40 per ton.

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[14] "Feed By-Products from Grain Alcohol and Whiskey Stillage," by C. S. Baruff and L. P. Weiner, *Proceedings*, 1st Industrial Waste Utilization Conference, Purdue Univ., Lafayette, Ind., November, 1944, p. 87.

PULP AND PAPER

Pulping processes include sulfate, soda, and sulfite, of which the first two are, almost completely, "bottled up" by recovery of chemicals. The sulfite process is the concern of all sanitary officials, however, because of its well-known polluting character. Half of the wood goes into the waste liquor, largely made up of calcium salts of lignin, sulfonates, sulfites, wood sugars, and lime. The calcium salts are not recoverable as they reappear as calcium sulfate if calcined. Nevertheless, it is possible to use magnesium hydroxide as the basic chemical, add sulfur dioxide, and recover the hydroxide as the same salt by burning off the organic matter and calcining the magnesium sulfite to oxide. This process appears feasible, but is too costly for reconversion of existing plants using the calcium salt. However, new plants can probably use the magnesium salt and thus solve the waste problem just as it has been solved in the sulfate and soda processes.

TABLE 6.—UNIT LOSSES IN CORN PRODUCTS WASTES

Year	Grind (bu per day)	TOTAL POPULATION EQUIVALENT (THOUSANDS)			Loss reduction (%)	Unit population equivalent (persons per bu)
		No recovery[a]	Measured loss	Recovery		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1922	65,500	367	367	0	0	5.6
1923	62,400	348	270	78	22.4	4.3
1924	63,100	352	282	70	19.9	4.5
1925	65,700	368	257	111	30.2	3.9
1926	66,200	371	156	215	58.0	2.4
1927	72,700	407	77	330	81.0	1.05
1928	74,200	416	56	360	86.5	0.75
1929	77,200	432	71	361	83.6	0.9
1930	75,500	422	56	366	86.7	0.75
1931	67,400	378	78	300	79.5	1.15
1932	68,000	382	64	318	83.3	0.95
1933	69.100	388	59	329	85.0	0.85
1934	68,400	383	91	292	76.2	1.3
1935	68,000	382	60	322	84.3	0.9
1936	69,250	389	30	359	92.3	0.45
1937	71,800	404	41	363	89.8	0.55
1938	73,100	411	40	371	90.2	0.55
1939	69,700	391	49	342	87.5	0.7
1940	71,200	400	51	349	87.3	0.7
1941	85,500	480	56	424	88.0	0.65
1942	88,400	496	82	414	83.5	0.95
1943	88,800	498	71	427	85.7	0.8
1944	86,600	495	72	423	85.5	0.85
1945	82,000	467	125	342	73.3	1.5

[a] Proportional to grind.

About all that can be done with sulfite wastes of the calcium type is to investigate the possibilities of fermenting the wood sugars (about 12% of the solids) to alcohol, or recovering fodder yeast by similar processes. Alcohol recovery has been operated at Thorold, Ont., Canada, and Bellingham, Wash., on a large scale, with investment of more than a million dollars. It appears possible to produce alcohol at a price somewhat more than that of alcohol made by chemical synthesis. Prior to World War II about 80% of German sulfite mills recovered alcohol by the fermentation process.

Yeast has been produced from pulp wastes in Europe for some years and the processes are under investigation in Oregon. A committee of the industry in late 1946 studied data available at that time. It appears possible to produce 70,000 tons of high-protein feed per yr in the northwest at less than $0.08 per lb. The economics of this procedure are under investigation, including feeding experiments. Yeast plants are operating at Bellingham and Sumner, Wash. Research is also being directed by the Sulfite Paper Manufacturers' Committee on Waste Disposal in Madison, Wis., in cooperation with the Institute of Paper Chemistry at Appleton, Wis.

The alcohol and yeast processes, however, use only the wood sugars, leaving the lignin and inorganic salts. One process recovers these as calcium sulfolignate, which can be burned or used for making various chemicals. This type of lignin disposal has not had development outside a paper mill at Rothschild, Wis., and hence a more attractive process for disposal of lignin appears desirable.

For paper mill wastes, various degrees of recovery of fiber appear possible, depending on the type of "save-all" adopted. Losses should not exceed 1% of the product. Recirculation of liquor, using chlorine or some other germicide to prevent slimes, appears essential. The National Council for Stream Improvement (of the Pulp, Paper, and Paperboard Industry) is actively studying these and other salvage procedures, as well as final treatment of unrecoverable solids, and may be expected to point the way toward solution of the pulp and paper waste problem.

BY-PRODUCTS COKE

Salvage of phenol from ammonia liquor produced in by-products coke plants has been accomplished for many years by the solvent extraction process or the steaming-out process. Of the four by-products coke plants in Chicago, three have no phenol recovery, whereas the fourth includes a light-oil extraction plant. The data for surveys of these plants are shown in Table 7. These results show that about 1,500 lb of phenol can be recovered daily by a 1,750-ton plant, with a phenol recovery efficiency of 98%. However, the reduction of

TABLE 7.—UNIT LOSSES IN BY-PRODUCTS COKE WASTES

Plant No. (1)	Coal use (tons per day) (2)	Polluting Wastes (Lb per Ton of Coal) Phenol (3)	5-day B.O.D. (4)	Remarks
1	816	7.62	2.85	No phenol recovery
2	1,526	0.73	1.52	....
3	1,526	0.41	2.09	....
4	1,549	0.77	1.72	....
5	2,075	0.68	1.25	....
6	2,915	1.25	1.97	....
Average	1,735	0.91	1.90	....
7	1,788	0.023[a]	0.44[b]	Phenol recovered

[a] Phenol reduction 97.5%. [b] 5-day B.O.D. reduction 76.8%.

B.O.D. is only 77% by such removal, leaving a population equivalent of 4,600 for a 1,750-ton plant. This residue should be handled in a sewage treatment works, if possible. In Germany the phenol recovered in the Ruhr Valley supplied the needs for phenol-formaldehyde resins and it can be used for such purposes in the United States. Other solvents are possible, especially a wartime product consisting of a mixture of esters of aliphatic alcohols.

Chemicals, Plating, and Pickling

Recovery of products in a chemical plant is usually unattractive financially, as acids or alkalies may be too dilute to warrant concentration, and organic residues may be too dilute or unstable to be recovered. Some chemicals are being recovered, however, notably from 5 tons to 10 tons per day of sulfur from dye wastes containing sodium thiosulfate. Other recoveries include anilin, phthalic acid, and solvents.

Plating solutions are not dumped deliberately, but the metals are usually recovered and the cyanides destroyed. Tin, nickel, chromium, silver, and gold are usually recovered, but not always zinc, lead, or aluminum.

Pickling of copper and brass produces cuprous solutions which may be treated with iron filings or turnings for "cementation" of the copper and discharge of ferrous sulfate. This method of recovery usually is not very expensive and is sometimes profitable. In some cases, base exchange by means of resins may be used for complete recovery of the copper, but the expense is likely to be greater than the return.

Pickling of iron and steel still remains a source of industrial wastes for which no process that will operate profitably has been discovered. Fortunately, pickle liquors are not highly objectionable as stream pollutants, as the acidity is usually neutralized and the iron precipitated, in the stream, with some absorption of dissolved oxygen to oxidize the iron. In some cases the black ferrous sulfide is objectionable in streams, but does not produce odors or menace health. However, there are numerous methods of recovery of ferrous sulfate proposed in patents and with government subsidies, some of which were operated in Germany. Also, one or two plants are operating in England, recovery of sintered iron has been operated in the United States, and the search for a process with more attractive possibilities of return goes on—led by the work of the American Iron and Steel Institute at Mellon Institute in Pittsburgh, Pa. Richard D. Hoak, who is in charge of this work, states:

> "The steel industry has no expectation of making a profit from pickle liquor treatment; but, in accordance with sound business principles, it has adopted temporary disposal expedients to serve in a partially satisfactory manner until research and technology provide the most practical and economical processes for permanent installation."

This is a frank and honest statement of the position of iron and steel manufacturers.

Oil Refineries

Oil-skimming basins of adequate (American Petroleum Institute) design are essential in modern refinery practice and they usually recover enough oil to pay for their operation. Skimming basins in large refineries near Chicago, of inadequate capacity, still collect several thousand barrels of oil daily. Acid sludge can be handled by various procedures. Alkaline wash waters are still mainly a problem of disposal rather than of salvage.

TANNERY, RUBBER, AND TEXTILES

Tannery operation does not lend itself to salvage operations, although there are many patents on recovery of chromium salts. Leather scrap, hair, and fleshings may be saved and treated with acid to produce fertilizer, seldom, however, is the procedure profitable. Rubber regeneration is practiced as a salvage operation at a number of plants in the United States, but the liquid wastes from the process are purely a waste treatment problem of considerably increased magnitude, and no salvage appears possible. Textile wastes also offer little opportunity for salvage.

LAUNDRIES AND DAIRIES

Laundries and dairies offer little opportunity for salvage. In the former, heat may be conserved by countercurrent flow of hot and cold water. Laundry wastes surveys in Chicago disclosed the unit losses shown in Table 8. When unit losses were applied to all the laundries in the Chicago Sanitary District, the laundry wastes amounted to 7 tons suspended solids per day and a population equivalent of 294,000, based on B.O.D.

TABLE 8.—UNIT LOSSES IN LAUNDRY WASTES

Plant No. (1)	Service volume (tons per day) (2)	Waste volume (gal per ton) (3)	Polluting Wastes (Lb per Ton)			
			5-day B.O.D. (4)	Suspended solids (5)	Total nitrogen (6)	Grease (7)
1	3.2	15,000	95	26	2.0	....
2	3.5	13,000	75	91	1.3	32
3	5.0	14,000	101	39	2.3	30
4	5.3	17,000	168	47	1.6	....
5	7.0	21,000	110	52	2.0	....
6	7.0	13,000	56	35	0.8	28
7	7.5	13,000	80	58	1.1	37
8	9.0	15,000	94	61	2.4	41
9	9.5	17,000	116	53	2.2	42
10	10.0	24,000	129	58	2.4	....
11	13.0	12,000	67	44	0.7	31
12	18.5	22,000	141	34	2.2	....
13	22.0	13,000	75	91	1.3	32
Average	9.3	16,000	100	53	1.7	34

In dairies, drippings and whey may be recovered, but the low value does not appear to pay for the cost of recovery. Milk bottling wastes in Chicago surveys totaled 6 tons suspended solids and a population equivalent of 145,000.

CANNING, BEET SUGAR, AND CANDY

The drying of leaky residues from various canneries is under investigation and may lead to recovery and sale of such residues at a profit. Cannery wastes are unsuitable for recovery and must be treated at the least possible cost. Beet sugar plants produce a waste that is a source of betain and other salvage products. Food processing offers little opportunity for salvage. Candy plants can lose considerable quantities of sugar if not operated carefully.

SUMMARY

This rather brief review of possibilities of recovery and utilization of products from industrial wastes indicates that there are good prospects of salvage in numerous industries and that many former wastes are now recovered and sold at a profit. Even though no profit is realized the procedures discussed are those which recover salable materials and thus reduce the cost of final treatment. A recapitulation of average losses per ton of raw material or product, in selected types of industries, is shown in Table 9.

No illusions should be held as to the possibilities of profitable recovery. A newcomer envisions great prospects for industry by rather obvious methods of recovery, but by the time all costs are included the process usually works out at a loss. The loss, however, is not so great as the cost of final disposal of the materials by sewage treatment methods, and hence is preferable.

TABLE 9.—LOSSES PER TON OF RAW MATERIAL OR PRODUCT

Type of industry (1)	Raw material or product (2)	Losses per Ton: B.O.D. (lb) (3)	Losses per Ton: Population equivalent (4)
Packinghouse	Live weight	28.9	173
Brewery	Beer	23.6	142
Corn products	Corn	4.8	29
By-products coke	Coal	1.9	11
Laundry	Dirty clothes	100	600

Many recoveries are profitable, such as grease and tankage from packinghouse wastes, wool grease from wool scouring, grains and yeast from breweries, feed and possibly yeast from distilleries, feed from corn products, possibly alcohol or yeast from sulfite liquors, phenol from by-products coke, sulfur from chemical wastes, copper from pickling wastes, oil from refineries, and whey from cheese plants. All these recoveries are in use. However, manufacturers in time come to regard salvage as part of factory operations, and one-time losses such as gluten, feed, grease, yeast, and other products are no longer classed as by-products or wastes, but as primary products of plant operation and profits. This is not a discouraging attitude, but makes the future prospect for profitable recoveries more bleak, although still an attractive field for the conservationist.

DISCUSSION

S. Leary Jones,[15] Assoc. M. ASCE.—The necessity of establishing, sooner or later, permissible standards of pollution of surface waters was commented on by Professor Bloodgood. From an administrative viewpoint it is desirable to have such standards, but at present exact standards for most types of pollution are unknown. Considerable research and scientific investigation are certainly indicated and undoubtedly would be welcomed by all control agencies. In addition to helping to establish these standards, federal services can aid by assembling available data concerning the effects of various types of pollutants on streams.

The Tennessee Stream Pollution Control Board adopted the "General Criteria for the Definition and Control of Pollution in the Waters of Tennessee" in November, 1946, as a guide in determining the permissible conditions of waters with respect to pollution and also the preventive or corrective measures required to control pollution in various waters or in different sections of the same waters. The criteria recognize that the uses of waters and their capacities for sewage or waste disposal vary, depending on the volume of water, local needs, and other factors. The relative weight assigned to each type of use varies for different waters, and therefore the rigid application of uniform criteria of water quality is not considered desirable or reasonable.

Minimum criteria of water quality are established for the following: Solids, floating materials, and deposits; temperature; turbidity or color; taste or odor; acidity or alkalinity; hardness or mineral compounds; dissolved oxygen; bacteria; toxic substances; and other pollutants, including anything that may have been omitted in the other nine criteria. Interpretation is based upon the effect on the stream and the damage to reasonable and necessary uses. Numerical limits are included for only dissolved oxygen, bacteria, alkalinity, and pH. Such criteria are flexible and well suited to the type of cooperative program approved by the board. The criteria are not suited to a cookbook type of control program which would eventually become unreasonable.

Stream pollution control in Tennessee is on a cooperative basis. No distinction is made between industrial wastes and sewage in so far as recommendations for correction are concerned. Each source of pollution is considered in relation to its effect on the receiving waters. No exceptions are provided in the law for either sewage or wastes, and the Tennessee Stream Pollution Control Board has not favored the classification of any stream as hopeless.

The Stream Pollution Control Act provides for a unified approach in that the membership of the board consists of two representatives from municipalities. two from industries, and the commissioners of the State Departments of Public Health, Conservation, and Agriculture. The commissioner of health is the chairman, and the Department of Public Health is the administrative agent for the board with the director, Division of Sanitary Engineering, serving as technical secretary.

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[15] Prin. San. Engr., Stream Pollution Control, Tennessee Dept. of Public Health, Nashville, Tenn.

A. M. QUATTLEBAUM,[16] ASSOC. M. ASCE.—Because of the rapidly increasing industrialization of the southern states the problem of industrial wastes is becoming more and more important to the peoples of every state, and the lawmakers of individual states are becoming increasingly conscious of this important problem.

Professor Bloodgood has presented the problem in a concise and interesting manner, and it is significant that conclusions 2, 3, and 4 of his paper have been included in a bill proposed to the General Assembly in South Carolina in February, 1949.

The General Assembly of 1948 created a committee of three members for the purpose of "investigating and determining the proper and effective use of the fresh water resources of the state." This committee was composed of one representative of the state board of health, one representative of industry, and one representative of the South Carolina Wildlife Federation. In the letter of transmittal of a proposed bill to the General Assembly the committee stated that they had worked harmoniously together and as a result of the investigation had reached unanimous agreement that the major factors to be considered in the proper utilization of the state's waters were:

1. The promotion and maintenance of public health;
2. The conservation of the state's valuable wild life resources; and
3. The development and maintenance of industries in the state.

In the investigation South Carolina was divided into eleven areas representing the major drainage areas. The total population discharging sewage into these streams is 1,208,000 with a sewage population equivalent to 701,000. The industries of the state discharge a sewage population equivalent to 1,614,000. This shows that at the present time industrial wastes amount to approximately 2.3 times the domestic sewage.

The proposed bill for the control of the waters of South Carolina creates a water control authority consisting of five members—namely, one member from each of the following: The Cotton Manufacturer's Association of South Carolina, the pulp and paper industries, the South Carolina Wildlife Federation, and the Municipal Association of South Carolina (the fifth member being at all times the State Health Officer of South Carolina who is to be chairman of the group).

The authority is charged with the control of the waters of the state and is instructed to classify all rivers as to usage and to establish the standards of purity to be maintained in all cases. It shall be allowed to classify certain streams as industrial streams and in such cases the industries using said streams shall be ordered to take only such precautions as are pertinent to the health of the public. From this it can be seen that the authority is to have far-reaching powers and this section of the bill is certain to cause considerable debate.

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[16] Chf. Engr., Harllee Quattlebaum Constr. Co., Florence, S. C.

The writer considers the bill as drawn to be far from perfect; however, he does consider it very significant that persons of such diversified interests have worked harmoniously together and it is believed that such cooperation could do much toward a better understanding and a more profitable solution to industrial waste problems everywhere.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2372

EXPERIENCES WITH PREDETERMINING PILE LENGTHS

By William W. Moore,[1] M. ASCE

With Discussion by Messrs. A. W. Earl, Hamilton Gray, Jacob Feld, George F. Sowers, H. Cambefort, and William W. Moore.

Synopsis

In the design of pile foundations there are two problems for which no adequate solutions have been found in common engineering practice. One of these problems is to estimate the required lengths and probable behavior of different types of piling when it is not feasible or economical to perform full-scale driving and loading tests previous to preparation of complete construction plans. The other is to evaluate the significance of various factors that affect the behavior of piles both during driving and under test loads. The methods described for predetermining pile lengths and bearing capacities are not offered as a final answer to all problems in the design of pile foundations, but rather are intended to complement other methods of determining the probable safety of such foundations. These methods can be used for evaluating the bearing capacity of single piles—however, determination of the probable settlements and safe loadings for large groups of piles requires other and additional analyses. Incompleteness of knowledge and imperfectness of materials impose such wide limits of accuracy on pile foundation analyses that most conclusions must, to some degree, be empirically based upon observations and performance records. Typical foundation designs employing various types of piling are presented to illustrate the methods described.

Introduction

There have been attempts by some investigators to evaluate the supporting capacity of piling in terms of the skin friction which the soil would offer on the embedded surface area of the pile. All these schemes have followed the line of observing the results of field loading tests on piles and dividing the tested capacity by the gross surface area of the pile, and then trying to apply the information to some other site where the soils, by visual examination, appeared similar.

Note.—Published in November, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Partner, Dames & Moore, San Francisco, Calif.

The methods outlined herein result from an attempt to arrive at a more rational design for pile foundations which would be more consistent with the improvements in structural analysis than are the older and more or less generally accepted dynamic pile driving formulas. This approach must be combined with good judgment and the lessons of experience in practical foundation construction, and, when so used, may help to avoid some pitfalls and failures.

From experience records a few typical cases have been selected to illustrate the design of foundations employing wood piling, cast-in-place concrete piling, pre-cast concrete piling, and steel H-piling.

WOOD PILING

Design of a wood pile foundation is typified by one for support of the Submarine Spare Parts Building, a wood frame industrial warehouse situated on reclaimed marshlands within the United States Navy Yard, Mare Island, California. Explorations were made with wash boring equipment, and undisturbed core samples of the soils were extracted by means of a driven tubular sampler for laboratory inspection and testing. Typical soil conditions underlying this construction area are given in Table 1. Physical properties of the soils as determined by laboratory tests using a ring-type double-shear apparatus for determining shear and friction values are given in Table 2. The tests were generally

TABLE 1.—SOILS AT SITE OF SUBMARINE SPARE PARTS BUILDING, UNITED STATES NAVY YARD, MARE ISLAND, CALIFORNIA

Depth below surface (ft)	Type	Color	Remarks
0 to 7..	Sandy loam, rocks	Brown	Fill
7 to 15..	Silty loam, decayed vegetation	Black	Soft bay mud
15 to 26..	Silty loam	Dark gray	
26 to 40..	Peaty clay	Brown	
40 to 47..	Peaty clay	Dark gray	
47 to 54..	Clay	Blue-green	Firm clayey soils
54 to 65..	Clay and shells	Light brown	
65 to 70..	Clay	Dark gray	
70 to 80..	Clay	Blue-green	
80 to 90..	Clay	Light brown	

TABLE 2.—PHYSICAL PROPERTIES OF SOILS IN TABLE 1

Depth[a] below surface (ft) (1)	Surcharge pressure (lb per sq ft) (2)	Moisture content (%)[b] (3)	Dry density (lb per cu ft) (4)	Shearing strength (lb per sq ft) (5)	Frictional resistance on wood (lb per sq ft) (6)	Yield point bearing (lb per sq ft) (7)
1..........	100	17.0	106.9	6,400 +	....	20,000 +[c]
8..........	950	94.0	48.5	160	160	500
20..........	1,450	107.6	42.2	240	....	750
30..........	1,950	131.0	35.2	360	320	1,125
40..........	2,550	69.1	56.6	400	....	1,250
50..........	500	24.8	100.5	950	....	3,000
50..........	3,300	24.9	102.1	1,100	1,100	3,500
60..........	3,900	42.5	79.2	1,430	1,110	4,500
70..........	4,500	44.5	75.5	1,190	1,030	3,750
80..........	4,100	41.1	77.7	1,590	....	5,000
89..........	4,750	36.3	82.7	2.380	1,590	7,500

[a] Point at which undisturbed sample was taken. [b] Percentage of dry weight. [c] Rock fragments in shear planes; not a representative test.

made with a confining surcharge pressure that approximately equaled the weight of the field overburden after compensation for the buoyant effect of the ground water. Col. 5, Table 2, indicates the yield point shearing strengths. For the purpose of assisting a general appreciation of the qualities of the foundation soils, the equivalent yield point bearing values[2] are given in Col. 7. The contrast between the soft bay mud and the firmer clayey soils is evident by the differences in strengths, moisture contents, and densities. The frictional values of the soils in contact with wood (Col. 6), representing wood piling, also indicate the marked difference between the soft mud and the firm clays.

A summary of the results of the shear and friction tests performed on the soil samples extracted from the two borings on this site is shown in Fig. 1. This summary was used to select representative values of the shearing strengths of the soft bay deposits and the firmer clayey soils (the two principal soil types) and representative values of the frictional resistances offered by these materials to wood piling.

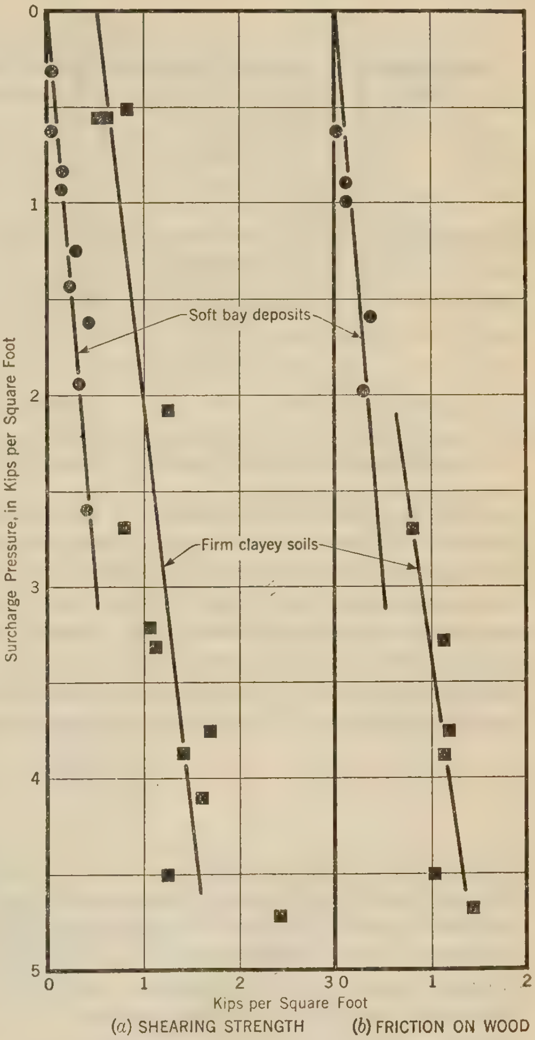

Fig. 1.—Summary of Soil Test Data from Two Borings at Mare Island (California) Site

The nature of the forces considered to be exerted on the pile by the soil are shown by vectors in Fig. 2. These forces consist of horizontal compressive forces perpendicular to the surface of the pile, and vertical shearing (or frictional) forces parallel to the axis of the pile. It is assumed that the force perpendicular to the surface of the pile at any point is a function of the weight of the soil overburden above that point and may be greater than the weight of overburden by a factor which depends upon the shearing resistance of the soil—that is, the driving of the pile into the ground will displace the soil so that an increased lateral pressure (similar to "passive resistance") is created on the surface of the pile. The actual value of this lateral pressure is believed to be dependent not only on the shearing strength of the soil, but also, in some unknown degree, on the compressibility of the soil and the volume of soil displaced by the pile. Thus,

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[2] "Practical Shear Tests for Foundation Design," by Trent R. Dames, *Civil Engineering*, December, 1940, pp. 784–785.

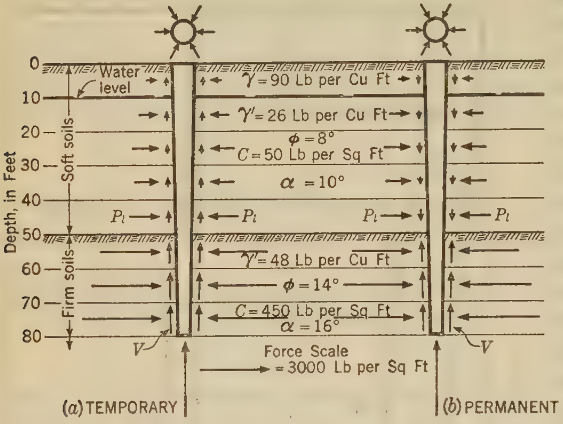

0
10
20
30
40
50
60
70
80
Depth, in Feet
Soft soils
Firm soils
Water level
$\gamma = 90$ Lb per Cu Ft
$\gamma' = 26$ Lb per Cu Ft
$\phi = 8°$
$C = 50$ Lb per Sq Ft
$\alpha = 10°$
P_l
P_l
P_l
P_l
$\gamma' = 48$ Lb per Cu Ft
$\phi = 14°$
$C = 450$ Lb per Sq Ft
$\alpha = 16°$
V
V
Force Scale = 3000 Lb per Sq Ft
(a) TEMPORARY
(b) PERMANENT

Fig. 2.—Diagrams of Lateral and Supporting Forces on Wood Piling Having 8-In. Tip and Tapering 1 In. in 10 Ft

$$P_l = K \sum \gamma h + f(S) \quad \text{..(1)}$$

in which P_l is the lateral pressure; γ is the unit weight of the soil; h is the depth to the point being considered; and $f(S)$ is a function of the shearing strength of the soil. The increased pressure created by displacement of the soils during driving is believed to be, generally, on the order of from three to five times the shearing strength, although for jetted piles or piles of small displacement there may be no appreciable increase in the lateral pressure. The potential supporting forces parallel to the axis of the pile are assumed to be limited either by the friction which would be developed by the estimated lateral pressures, or by the shearing strengths of the soils surrounding the pile, whichever is less. Thus,

$$V \leqq P_l \tan \alpha \quad \text{......................(2)}$$

or

$$V \leqq S \quad \text{..........................(3)}$$

in which V is the supporting force parallel to the axis of the pile; α is the angle of friction; and S is the shearing strength of the soil. The method of evaluating the total capacity of the pile to support external loads consists of an arithmetic summation of the potential shearing or frictional forces in each of the soil strata as shown by the computation in Table 3. In Table 3, the values in Cols. 3, 4, 5, and 6 are calculated from

$$S = C + P \tan \phi \quad \text{......................(4)}$$

$$F = P \tan \alpha \quad \text{..........................(5)}$$

$$F_m = F \frac{\pi S + P}{P} \quad \text{........................(6)}$$

$$B = \pi S \left(\frac{\pi d^2}{4} \right) \quad \text{......................(7)}$$

respectively, in which C is the cohesion of the soil; P is the surcharge pressure; ϕ is the angle of internal friction; F is the average friction; F_m is the friction modified; B is the end bearing of the pile; and d is the diameter of the pile at its tip.

TABLE 3.—COMPUTATION OF TEMPORARY AND PERMANENT CAPACITIES OF WOOD PILE SHOWN IN FIG. 2

Depth below surface (ft)	Surcharge pressure, P (lb per sq ft)	Average shear, S (lb per sq ft)[a]	Average friction, F (lb per sq ft)	Modified friction, F_m (lb per sq ft)	End bearing, B (lb)	SUPPORT OR DRAG (IN KIPS) FOR PENETRATION INTO FIRM CLAY (FT)				
						0	10	20	30	40
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(*a*) TEMPORARY SUPPORT OR POTENTIAL DRAG										
0										
	450	113	79	141		3.70	3.99	4.29	4.58	4.88
10	900									
	1,030	195	182	290		5.87	6.38	6.89	7.40	7.91
20	1,160									
	1,290	231	227	354		6.35	6.95	7.56	8.16	8.77
30	1,420									
	1,550	268	273	421		6.67	7.38	8.07	8.77	9.47
40	1,680									
	1,810	304	319	487		6.76	7.56	8.37	9.15	9.95
Total						29.35	32.26	35.18	38.06	40.98
(*b*) PERMANENT SUPPORT										
50	1,940	933			1,026	1.03				
	2,180	993	625	1,518		30.38	22.10	24.70	27.30	29.9
60	2,420	1,053			2,662		2.66			
	2,660	1,113	762	1,763			57.02	24.77	27.68	30.60
70	2,900	1,173			3,190			3.19		
	3,140	1,232	900	2,009				87.84	27.41	30.64
80	3,380	1,293			3,718				3.72	
	3,620	1,352	1,038	2,255					124.17	30.09
90	3,860	1,412			4,246					
										162.21
Temporary pile capacities, in tons						15.19	28.51	43.92	62.08	81.10
Permanent pile capacities, in tons						−14.16	−3.75	8.74	24.02	40.12

[a] Controlling values.

The results of the pile capacity analyses may be shown graphically, as in Fig. 3, from which the desired pile lengths may be taken directly. The pile capacities are considered in two parts. The first part of the analysis considers the condition after the piling is driven and before sufficient time has elapsed to permit subsidence of the fill and soft bay mud. Under this condition, all the frictional forces would tend to support the piling; capacities for this condition are indicated by curve (*a*), Fig. 3. The second part of the analysis considers the condition when sufficient subsidence of the fill and compression of the soft bay mud has occurred to develop the frictional forces which will tend to drag the pile downward. Under this condition, the calculations indicate that a penetration of approximately 15 ft into the firm clayey soils would be required to resist the forces which would be applied by the subsiding fill and bay mud. Any additional penetration into the firm clayey soils would provide a potential resistance which might be utilized to support the structural loads—for example, a penetration of 34 ft into the firm clayey soils would provide adequate capacity to support a 30-ton total design load. These capacities are indicated by curve (*b*). In connection with these curves it should be noted that, for the particular structure considered, it was decided that the recommended maximum dead load should not exceed two thirds of the permanent pile capacity.

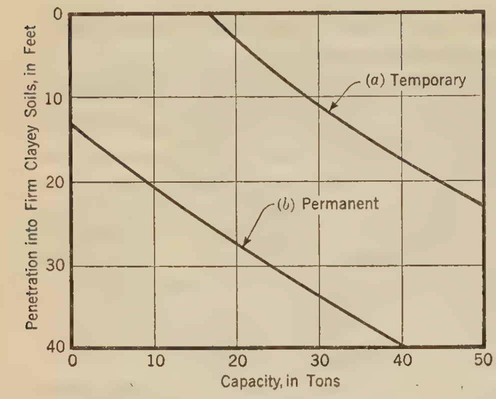


Fig. 3.—Computed Capacity of Single Wood Piles, or Each Pile of Small Groups, at Mare Island (California) Site

Some variation in the surface elevations of the firm clayey soils was disclosed over the site. Data obtained by borings and driven test rods enabled construction of surface contours of the firm clayey soils. Use of these contours in conjunction with the calculated penetrations into the firm soils required to support the proposed loads enabled selection of the lengths of piling required for the various parts of the proposed structure.

After calculating the pile capacities, based on the field explorations and laboratory tests, it was desired that some full-scale confirmation of the computed capacities be obtained. Therefore, field loading tests were conducted on wood piles driven on a site adjacent to that for which the explorations already described were made. The soil conditions were very similar to those illustrated, but differed in the thickness of fill and depth of soft bay mud. The pile loading test setup is illustrated in Fig. 4. The 67-ft-long test pile was between two anchor piles, and a reaction girder was framed between the two anchor piles so that load could be applied to the test pile by gear-driven screw jacks, using calibrated beam gages to measure the applied loads. Settlements of the test pile were recorded by micrometer dial gages reading to 0.001 in., the dial gages being supported on a reference beam independent of the loaded elements. The dial gage, which indicates settlements of the test pile, may be noted in front of the left jack in Fig. 4.

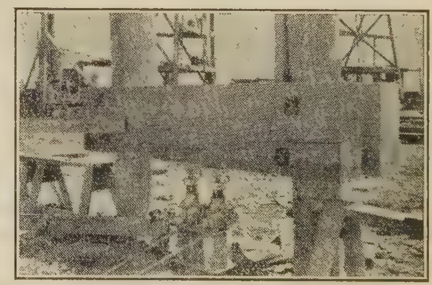


Fig. 4.—Setup for Wood Pile Loading Test

Loads were applied to the test pile in increments, each load being maintained constant until settlement observations indicated that the rate of subsidence had substantially ceased. Each load was usually held until movement of less than 0.001 in. was observed in a 15-min. period. At certain loadings, the load on the test pile was removed and then reapplied in order to observe the effect of repeated loading in producing progressive settlements. Results of the loading test are shown in Fig. 5. Since the loading test was conducted only a few days after the pile was driven, it was expected that the frictional forces of all the soils surrounding the pile, including the fill and soft bay mud, would assist in supporting the pile during the test. Therefore, it was expected that the test would be in substantial agreement with the computed temporary pile capacity. Computed pile capacities were determined by the method previously described, adjustments being made to conform with the depths of fill and bay mud at the location of the pile test. The relationship between the computed temporary pile capacity and the settlement of the pile under load is shown in Fig. 5. Yielding of the pile under load occurred in the vicinity of 50 tons as shown by a rather definite break in the slope of the curve and the increasing permanent settlements observed under repeated loadings. The computed permanent pile capacity, with due allowance for the frictional forces which would be imposed by subsidence of the fill and compression of the soft bay mud, is also indicated in Fig. 5. Loads on the test pile were increased until ultimate failure was observed under a load of approximately 100 tons.

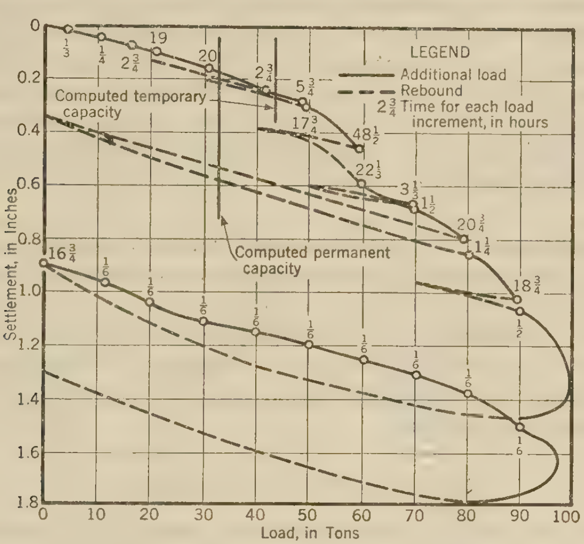


Fig. 5.—Settlement of Wood Test Pile Under Load

Since the results of the loading test were in substantial agreement with the computed pile capacity, the lengths of piling to be driven for the proposed building were selected as previously outlined, and the final lengths of the piling driven agreed quite closely with the predetermined lengths, the extreme variations being from 4 ft shorter to 2 ft longer than those computed. Variations between driven lengths and computed lengths were due to variations in lengths of available stock, except for a few cases where the piles were driven to refusal above the computed tip elevations.

Although the driving resistances, as computed by the *Engineering News* formula, varied from approximately 9 tons to 35 tons because of such factors as variations in straightness, grain of the wood, squareness of the blow on the pile, and variations in the soils over the site, all the piles were capable of supporting the 30-ton total design load. Since completion of the building in the fall of 1942, observations indicate that the floor, which was not supported on piling, has settled slightly more than 1 ft. Although no continuous instrumental observations of the elevations of the pile caps are available since erection of the building, such observations as have been made indicate that if there has been any settlement of the piling it is on the order of small fractions of an inch.

A near-by structure, of a very heavy and rigid type of reinforced concrete construction, is also supported on piling driven in substantially the same soils as those described, and the lengths of piling driven were determined in the same fashion. Observations of this building also indicate that no measurable settlements have occurred—in this type of structure the occurrence of any appreciable differential settlements would certainly be evidenced by severe cracking of the walls and floors.

CAST-IN-PLACE CONCRETE PILING

Use of the method in a design involving cast-in-place concrete piles is illustrated by a foundation for the support of the fluid catalytic cracking unit in the Avon Refinery of the Tide Water Associated Oil Company at Associated, Calif. Typical soil conditions are given in Table 4, and the physical properties of the soils in Table 5. It will be noticed that the soils were moderately firm to firm clays interspersed with strata of fine sand. Design values of the principal soil properties selected for use in calculating capacities of steel-shelled cast-in-place concrete piling and wood piling for various lengths of embedment in these soils are given in Fig. 6. In selecting representative design values for the shearing strengths of the soils, rather conservative interpretations of the data have been made.

TABLE 4.—SOILS AT OIL REFINERY SITE AT ASSOCIATED, CALIF.

Depth below surface (ft)	Type	Color
0 to 4½....	Clay	Dark brown
4½ to 11....	Clay	Light brown
11 to 18....	Fine sand	Brown
18 to 30....	Clay	Light brown
30 to 40....	Fine sand	Brown

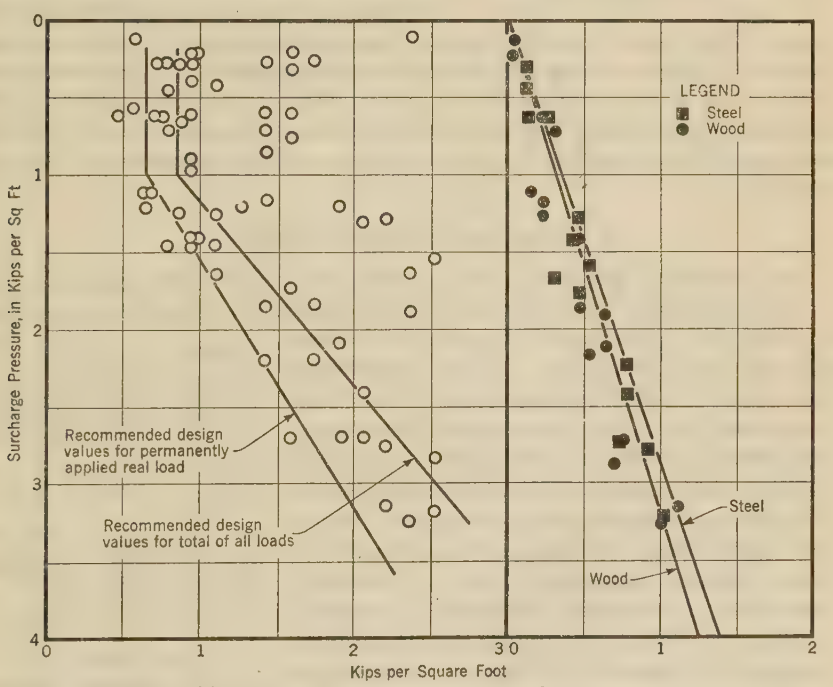
Because of the lack of continuity of the sand strata beneath the proposed structure, the analyses were made so that the pile capacities would be adequate to support the design loads where the soils were predominately of clayey nature; and, where sand strata of considerable extent were encountered, an additional factor of safety would be realized. Results of the analyses of pile capacities are shown in Fig. 7.

TABLE 5.—PHYSICAL PROPERTIES OF SOILS IN TABLE 4

Depth[a] below surface (ft) (1)	Surcharge pressure (lb per sq ft) (2)	Moisture content (%)[b] (3)	Dry density (lb per cu ft) (4)	Shearing strength (lb per sq ft) (5)	Frictional resistance on wood (lb per sq ft) (6)	Friction on steel (lb per sq ft) (7)	Yield point bearing (lb per sq ft) (8)
1.7......	200	29.5	91.3	1,590	40	...	5,000
5........	600	21.0	107.2	950	...	160	3,000
9........	1,100	29.8	91.9	640	160	...	2,000
12........	1,400	24.7	101.0	1,000	...	440	3,130
19.5......	1,850	21.1	105.9	1,750	480	...	5,500
25........	2,200	20.6	107.5	1,750	...	790	5,500
32........	2,700	21.0	106.2	2,070	790	...	6,500
39.5......	3,200	20.1	108.2	2,540	...	1,030	8,000

[a] Point at which undisturbed sample was taken. [b] Percentage of dry weight.

It was expected that considerable variation in the driving resistance of the piles would be observed because of discontinuity of the sand strata. However, as indicated by the pile capacity analysis, an adequate capacity to support the design load of 40 tons per pile was assured at all locations on the site by cast-in-place concrete piles penetrating to approximately 30 ft below the ground surface, with the pile cap placed at a 5-ft depth. Experience during the driving



(a) SHEARING STRENGTH (b) FRICTION ON STEEL AND WOOD

FIG. 6.—SUMMARY OF SOIL TEST DATA AT REFINERY SITE

of the piles indicated that, where the lower layer of sand was not encountered, use of the *Engineering News* formula would have required an additional 10 ft or more of penetration to support the design loads. According to the *Engineering News* formula, bearing capacities generally on the order of 20 tons and higher were developed. The cost of unnecessary increased lengths of the piling of course would have been a considerable item.

The fluid catalytic cracking unit, completed late in 1944, imposes a dead load of approximately 8,700 tons, on 352 step-tapered, cast-in-place, concrete piles 25 ft long, or slightly more than 24 tons of dead load per pile.

Observations of the settlement of the structure since its erection have been recorded by the engineering forces of the owner. Plottings of these readings indicate some apparently erratic readings, which are believed to represent the approximate accuracy of the observations—that is, it is impracticable to measure, with an ordinary engineer's level, settlements of the order of those that have taken place. In fact, the greatest settlement thus measured during the first year after completion of the unit was 0.012 ft.

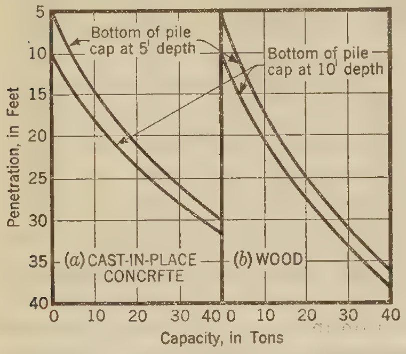


Fig. 7.—Computed Capacity of Single Piles, or Each Pile of Small Groups, at Oil Refinery Site

Pre-Cast Concrete Piling

Soil conditions into which the pre-cast concrete piling to support a reinforced concrete pier at a shipyard in Richmond, Calif., was driven are given in Table 6. The physical properties of these soils are given in Table 7. All the soils below the dredged depth, approximately 32 ft below mean lower low water, are moderately firm clays. The summary of shear test results (Fig. 8(*a*)) indicates that there is no dependable increase in the shearing strength of the clay with depth. The design values of shearing strength and frictional resistance indicated in Figs. 8(*a*) and 8(*b*) were used as previously described to compute the support offered to pre-cast concrete piling embedded in these soils. Results of these pile capacity analyses are shown in Fig. 9.

TABLE 6.—Soils at Shipyard Site at Richmond, Calif.

Depth below MLLW[a] (ft)	Type	Color
0 to 32.5	Water	....
32.5 to 39	Clay	Greenish gray
39 to 53.5	Clay	Brown
53.5 to 62.5	Clay	Gray
62.5 to 87.5	Clay	Dark gray

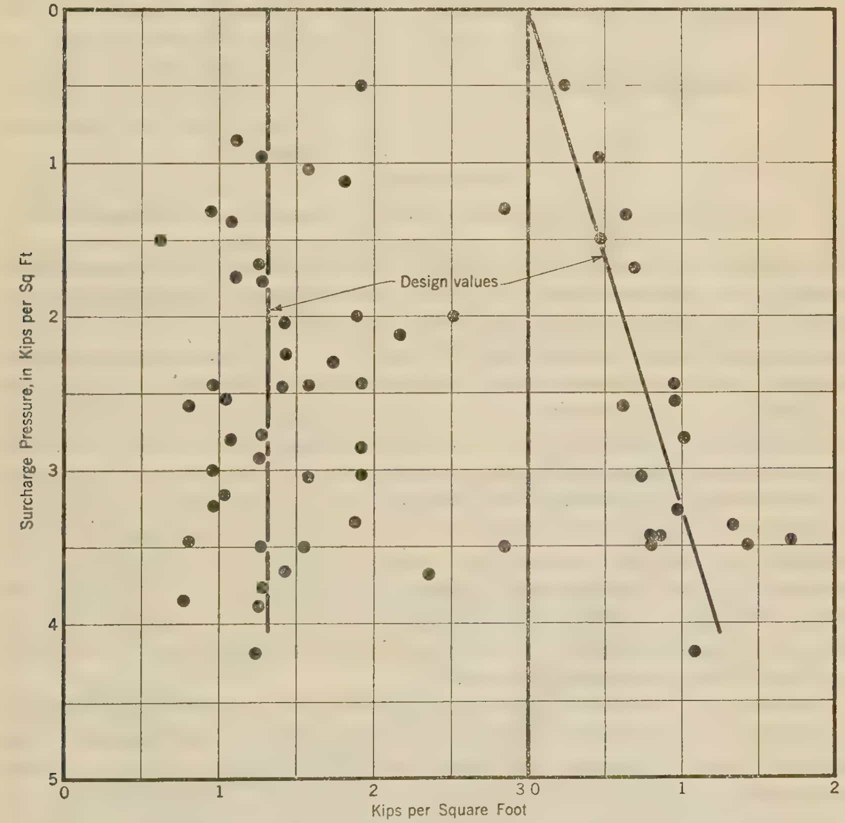
[a] Mean lower low water.

A load test was made on a pile typical of those to be used in the structure. Elements of the loading test setup were similar to the setup previously described for the wood pile test except that at the test location, which was in the bay, it was impracticable to support direct recording deflection instruments to record the settlements, and consequently settlement measurements were made

TABLE 7.—PHYSICAL PROPERTIES OF SOILS IN TABLE 6

Depth below MLLW[a] (ft) (1)	Surcharge pressure (lb per sq ft) (2)	Moisture content (%)[b] (3)	Dry density (lb per sq cu ft) (4)	Shearing strength (lb per sq ft) (5)	FRICTIONAL RESISTANCE (LB PER SQ FT): On wood (6)	On steel (7)	On concrete (8)	Yield point bearing (lb per sq ft) (9)
45.5......	850	28.3	97.0	1,110	400	...	...	3,500
54........	1,300	35.2	86.8	960	...	480	...	3,000
58.5......	1,500	68.7	60.1	640	...	...	480	2,000
71.5......	2,050	46.5	74.7	1,430	640	...	...	4,500
80........	2,450	33.4	89.0	1,910	...	880	...	6,000
86.5......	2,800	31.6	91.5	1,110	...	...	1,040	3,500

[a] Depth below mean lower low water at which undisturbed sample was taken. [b] Percentage of dry weight.



(*a*) SHEARING STRENGTH (*b*) FRICTION ON CONCRETE

FIG. 8.—SUMMARY OF SOIL TEST DATA AT SHIPYARD SITE

with an engineer's level securely mounted on a three-pile dolphin having no structural connection with the loaded elements. Considerable difference was observed between the driving resistances for the test pile and those for the anchor piles, but these are attributed to differences in the shapes of the piles rather than to any important difference in the strengths of the soils penetrated. Settlements of the test pile under load are indicated in Fig. 10, in which is also shown the relationship of the yielding of the pile under load to the computed capacity. The load-uplift relations for the anchor piles are shown in Fig. 11.

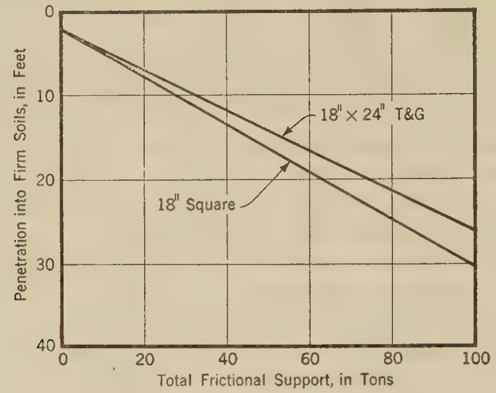


FIG. 9.—COMPUTED PERMANENT CAPACITY OF PRE-CAST CONCRETE PILES AT SHIPYARD SITE

In addition to determining capacities of the individual piles, it was desired to evaluate the probable settlement of the structure that would result from compression of the soils supporting the entire pile foundation. Stress conditions in the principal supporting soils are shown in Fig. 12. The analysis was made assuming that the settlement would be the same as though the structure were supported on a spread foundation at the elevation of the pile tips. The width of the equivalent spread foundation was assumed equal to the width of the pile group plus an additional width equal to the depth of penetration of the piles into the firm soils. Consolidation tests on the firm clays were used in conjunction with the indicated stresses to evaluate the order of magnitude of settlement which would be expected during the life of this structure. The magnitude of these settlements is indicated by Fig. 13. This range of expected settlements is intended to represent the order of accuracy of the analysis—that is, the actual settlement should fall within the range shown if the design loads were permanently applied.

In driving the permanent piles for support of the pier, it was specified that they be driven to the tip elevations indicated by the analyses, provided that this depth could practicably be reached without damaging the piles during driving and that the final driving resistance did not fall below nine blows per foot with an OR Vulcan hammer. This procedure was adopted to disclose any unexpectedly soft soils. The pre-cast concrete piles were of two types: 18-in.-square and 18-in. by 24-in. tongue and groove sheet piling. Except for a few piles in locations subjected to special loading conditions in which the piles were driven to elevations as deep as −72 ft, the 18-in.-square piles were generally driven to predetermined tip elevations of from −62 ft to −64 ft, or approximately 32 ft into the moderately firm clays. The *Engineering News* formula indicated safe loads ranging from 27 tons to 65 tons, whereas the computed capacity for these piles was approximately 100 tons. The 18-in. by 24-in. tongue and groove sheet piles were generally driven to a tip elevation of −57 ft and, being of slightly larger size, were computed to develop approximately the same ultimate bearing capacity by skin friction as the 18-in.-square piles. The driving resistances of these piles, however, varied from 40 tons to 105 tons as indicated by the *Engineering News* formula. All the piles were driven with the same hammer, delivering a blow of 24,450 ft-lb with an 8,000-lb ram. Very few piles were cut off at all. Such cutoffs as were made generally resulted from the use of piles cast longer than required for a given location. The dead load imposed by the structure on the piles varies from 18 tons to 28 tons, and the maximum dead and live loads from about 60 tons to 66 tons, all of which was regarded as potential real load.

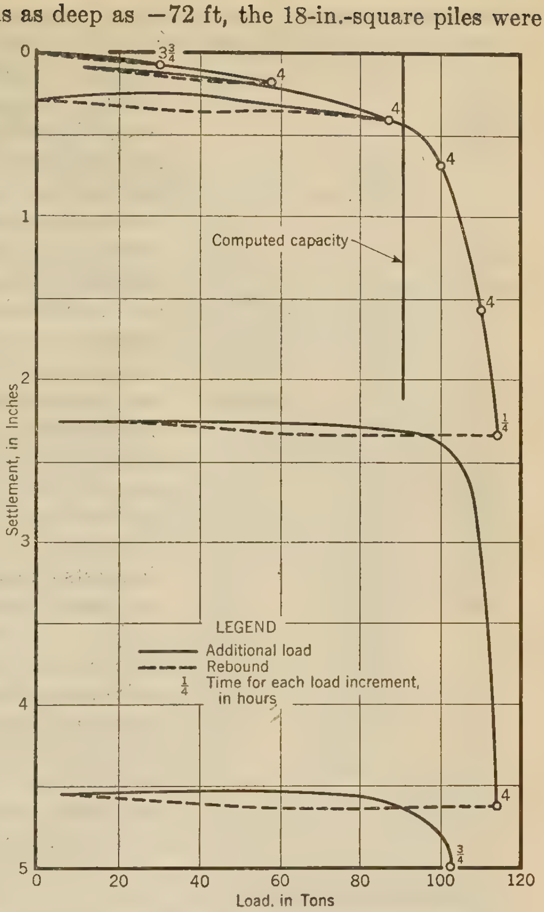


FIG. 10.—SETTLEMENT UNDER LOAD OF 18-IN. BY 24-IN. TONGUE AND GROOVE PRE-CAST CONCRETE TEST PILE AT SHIPYARD SITE

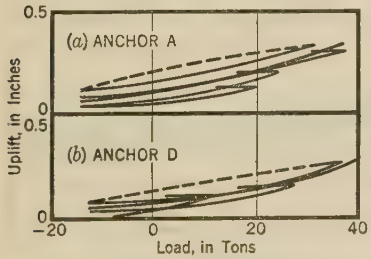


FIG. 11.—LOAD-UPLIFT RELATION FOR ANCHOR PILES A AND D, SHIPYARD SITE

During the interval since completion of the pier, it has been subjected to crane loads and deck loads in some cases exceeding those in the design. At one time a deck load of about 1,300 lb per sq ft was applied for a period of several days. Level readings on the crane rails indicate deflections varying from an indicated uplift of 0.02 ft to an indicated settlement of 0.02 ft at different points along the pier.

Another example of the capacity of pre-cast concrete piling which may be of interest is that of a loading test made for the California Point Ammunition Loading Project which the United States Engineer Office was contemplating at one time. The data from one of these pile loading tests are shown in Figs. 14 and 15. The test pile was placed into soft bay mud, the surface of which was approximately 30 ft below mean lower low water. The driving resistance of the pile was practically zero—that is, the pile was placed to its final tip elevation without a blow from the hammer, having penetrated more than 50 ft under its own weight, and another 10 ft under the additional weight of the hammer. When test loads were applied thirteen days after the pile had been driven, the data indicated by Figs. 14 and 15 were obtained. Calculations of the pile capacity based on shear and friction tests on core samples of the soil indicated that the pile should be capable of supporting loads of about 42 tons. As may be observed from Fig. 14, the load actually carried before yielding is in much closer agreement with the computed capacity than it is with the capacity indicated by a dynamic formula. This is, of course, one of the most extreme cases that could be selected, but nevertheless it is a very practical one.

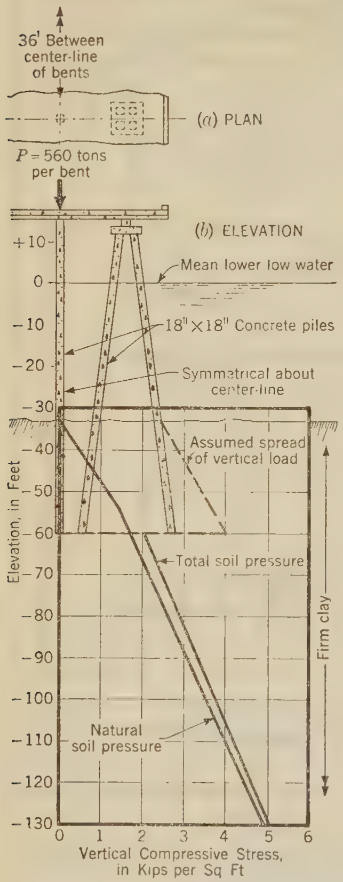

36' Between center-line of bents
(a) PLAN
P = 560 tons per bent
(b) ELEVATION
Mean lower low water
18"×18" Concrete piles
Symmetrical about center-line
Assumed spread of vertical load
Total soil pressure
Natural soil pressure
Firm clay
Elevation, in Feet
+10
0
−10
−20
−30
−40
−50
−60
−70
−80
−90
−100
−110
−120
−130
0
1
2
3
4
5
6
Vertical Compressive Stress, in Kips per Sq Ft

Fig. 12.—Soil Pressures Under Pre-Cast Concrete Pile Foundation at Shipyard Site

Steel H-Piling

The last case selected to illustrate this method of predetermining pile lengths is that of the new Ras Tanura Refinery in Saudi Arabia, for the Arabian American Oil Company. The site is on a narrow sandy peninsula between the Persian Gulf and a small bay. Investigations by undisturbed sampling and laboratory testing disclosed that the site was covered by various thicknesses of sands and sandy loam soils, sometimes interspersed with very soft compressible strata. At depths ranging from 5 ft to as much as 20 ft, there was found an extensive bed of rather weak sandstone, which comprised the

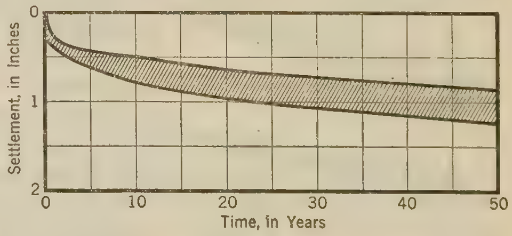


Fig. 13.—Range of Expected Settlement of Shipyard Pier

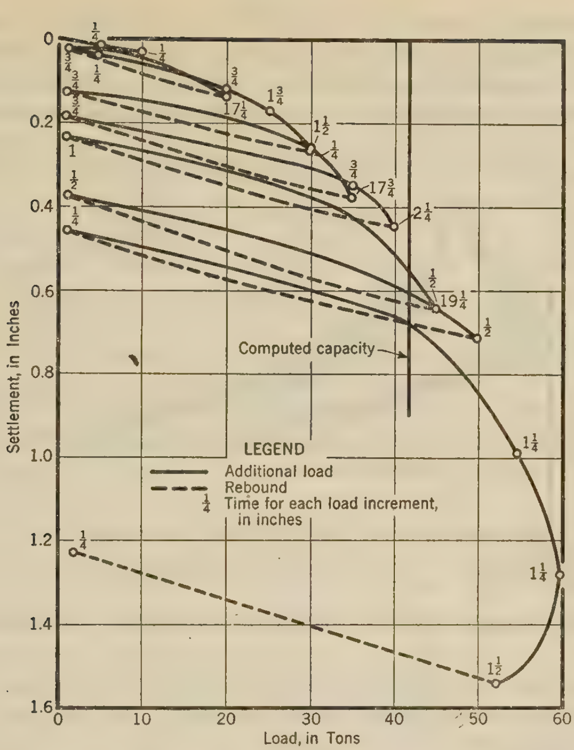


Fig. 14.—Settlement Under Load of Pre-Cast Concrete Test Pile at California Point Site

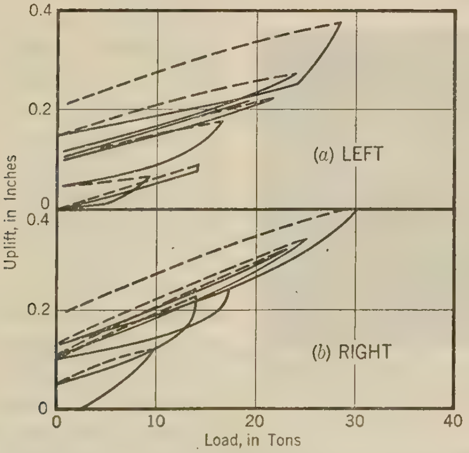


Fig. 15.—Load-Uplift Relation for Anchor Piles, California Point Site

principal supporting stratum for the refinery foundations. A representation of the typical sequence of soils from the surface down to the sandstone is presented in Fig. 16, which also gives the results of computations of bearing capacities of steel H-piling having various penetrations into the soils on this site.

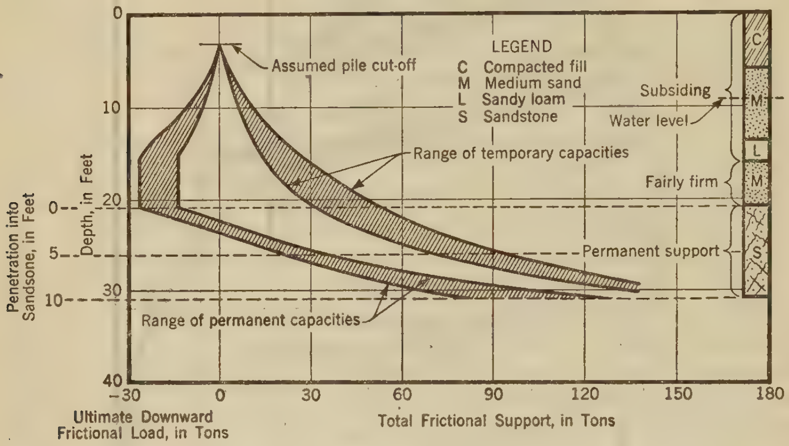


FIG. 16.—COMPUTED CAPACITY OF STEEL H-PILES AT ARABIAN OIL REFINERY SITE

A loading test was made early in the construction program within a few hundred feet of the area covered in this analysis. The test pile was a 12H53 shape driven with a Vulcan No. 1 hammer to a depth of 17 ft below the ground surface, which resulted in approximately 9 ft of penetration into the sandstone. The dynamic driving resistance reached approximately 0.5 in. per blow—thirteen blows for the last 6 in. Under the application of test loads up to 65 tons, a total settlement of about 0.08 in. was observed without any indication of yield being developed. Unfortunately, the reaction frame for this test was not strong enough to permit carrying the test to ultimate failure, but the results did add considerably to the knowledge of the bearing capacities of piling for this project.

After erection of some of the heavier refinery vessels, the units were filled with water for testing. The water load, although applied for a period of only a few hours, imposed more severe forces on the foundations than would be expected at any other time during the life of the structures. Level readings taken during the application of this water load on one of the heaviest units showed maximum settlements on the order of $\frac{1}{4}$ in.

FIG. 17.—SETTLEMENT AWAY FROM STRUCTURE OF WOOD PILE DRIVEN THROUGH ROCK FILL AND SOFT MUD INTO FIRMER SOILS

Conclusion

The methods of analysis employed for predetermining pile lengths are in a formative stage, and it is hoped they will be improved as knowledge increases. This approach to pile foundation design should be of considerable assistance in avoiding situations such as those shown in Fig. 17, which represents a condition disclosed while investigating the settlement of a waterfront structure where piling had been driven through rocky fill material into firmer soils. Driving resistances were ample according to dynamic pile driving formulas; but the force imposed on the pile by subsidence of the fill and compression of the soft bay deposits was too great, and the pile had settled away from the structure it was intended to support. The approximate magnitudes of the forces tending to support the pile and those pushing the pile downward are shown in Fig. 18. According to the log of a boring drilled about 10 ft away from the pile to investigate this failure, the pile tip was driven to the surface of the somewhat decomposed bedrock and probably was crushed against the rock formation. A proper appreciation of the order of magnitude of the potential forces on piling would, in some cases, lead to the selection of a different type of piling or a different type of foundation, whereas, in other cases, it might lead to a decision that the piling should penetrate into firmer strata for a greater distance than would be required by use of a dynamic driving formula.

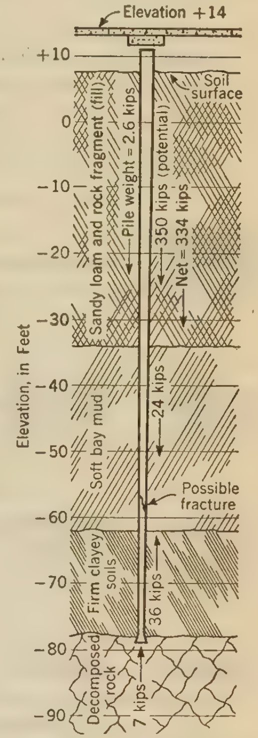


Fig. 18.—Frictional Forces on Pile Shown in Fig. 17; Driving Resistance by *Engineering News* Formula Equaled 27 Tons

Pile foundations have been installed on several scores of projects utilizing information of the nature described. Unfortunately, on many of these projects, it has been impracticable or uneconomical to obtain complete soil data, pile loading test data, driving records, and settlement data. There are, however, a number of projects which furnish fairly complete data on the strengths of the soils and also on full-scale loading tests on typical foundation piling. Table 8 shows comparisons between the capacities of piling as given by the *Engineering News* formula, as indicated by direct full-scale loading tests, and as computed in the fashion described in this paper utilizing data from tests on the soils.

In selecting a suitable type of foundation to support a given structure, consideration must be given to the problems of construction as well as to those of design. If it is not feasible to develop sufficient embedment to provide adequate support for the structure with a given type of piling and driving equipment, the foundation design should be revised, perhaps using a different type of piling, in order to assure adequate and satisfactory support for the proposed structure. The methods described have been used for several years with satisfactory performance, and better balanced and more economically completed structures have been realized in many cases than would have been achieved through the use of customary dynamic formula control methods.

TABLE 8.—COMPARISONS OF PILE CAPACITY DETERMINATIONS BY DIFFERENT METHODS

Location[a] (1)	Type of pile (2)	Pile length (ft) (3)	Driving hammer (4)	CAPACITY (TONS)		
				Engineering News formula (5)	Load test yield point (6)	Computed temporary (7)
(*a*) WOOD						
Mare Island	8-in. tip, 1 in. in 10-ft taper	60	Vulcan No. 1	7	45	40
Mare Island		70		14	50	44
Richmond		75		11	80	59
Mare Island		80	McKiernan-Terry 10–B–3	22	25	20
Mare Island		80		25	20	21
(*b*) CAST-IN-PLACE CONCRETE						
San Diego	Fluted metal	60	Vulcan No. 1	43	160	132
San Diego	Step-tapered	60	Raymond special	41	160	148
Compton	Step-tapered	32	Vulcan No. 1	10	33	32
(*c*) PRE-CAST CONCRETE						
Benicia	20 in. square	95	Vulcan OR	21	100	100
Richmond	18 in. × 24 in. tongue and groove	65[b]	Vulcan No. 1	138	90	89
California Point	18 in. square	101	Vulcan No. 0	0	35	42
California Point	18 in. square	101	Vulcan No. 0	24	50	54
(*d*) STEEL H-PILES						
San Diego	14WF87	60	Vulcan No. 1	33	140	128
San Diego	14WF87	60	Vulcan No. 1	37	154	165

[a] All locations in California. [b] Penetration into soil 23 ft.

ACKNOWLEDGMENTS

The procedure presented herein for predetermining pile lengths is protected by United States patent No. 2,296,466 and Dominion of Canada patent No. 413,675 issued in the names of the author and Trent R. Dames, M. ASCE. They, however, offer the results of their experience with the method, together with their patent rights, for the free use of the profession. It is expected, nevertheless, that suitable acknowledgment or recognition will be made by those who use the method.

The writer is most grateful for the generous cooperation of those clients for whom the data presented have been gathered and for their permission to use the data in this paper.

DISCUSSION

A. W. Earl,[3] M. ASCE.—The method for determining the proper lengths for foundation piles as developed by the author and his associate, Trent R. Dames, M. ASCE, has been used with success on several jobs with which the writer is familiar, to the extent of several thousand piles. From acquaintance thus gained, the method is believed to be the best and most practical so far proposed in that it provides piles no longer than necessary for safety and takes into account the effect of settlement in the upper layers of the soil.

In the absence of any better procedure for the past fifty or sixty years it has been usual to drive to a bearing capacity as determined by the *Engineering News* formula, or some formula of similar type. In spite of the fact that such formulas leave out many important factors and are supposedly quite limited in their application, the results, on the whole, have been satisfactory in so far as safety is concerned, and the percentage of failures has been exceedingly small. However, although piles driven to a bearing capacity determined by the *Engineering News* formula are generally of safe length, it is probable that in most cases they are considerably too long. In these days of high prices, even a small saving in length per pile on a large job means a large saving in money. Another serious drawback to the use of a driving formula is that it gives no indication of the ultimate capacity of the pile after settlement has taken place. Engineers realizing these and other inadequacies have attempted more rational methods of procedure, but so far as is known, none of these methods, aside from the one described by Mr. Moore, has been very successful.

The method of Messrs. Dames and Moore for determining pile lengths takes into account, more or less rigorously, the various factors that make up the bearing capacity of a pile and, at the same time, makes possible an evaluation of the loads that subsidence of upper strata will impose on the pile. It therefore does the important things that such a method should do—and it does them very well. However, anyone who has delved into soil mechanics literature or has followed the discussions on the subject in the *Proceedings* of the Society can readily imagine that a number of very erudite objections might be raised to such a simple, straightforward method of attack.

One such objection might be that no account has been taken of deflections, both of the pile with reference to the soil and of the various strata with regard to each other, and how these deflections affect the distribution of skin friction on the pile. A question might also be raised as to whether the firmer strata might not have to supply more than their share of resistance because the deflection of the pile might not be enough to mobilize the full resisting forces in the softer strata. However, in view of the differences in subsurface conditions usually found from place to place at a site, too theoretical a viewpoint might lead to a result no closer to the facts than the simpler approach, and the fact remains that any method for determining lengths of piles must stand or fall on its service record and the results of tests in the field. The Dames and Moore method is no exception, and this paper is therefore of particular interest in that it shows the correspondence of calculated values and test values for a variety of soils and types of piles.

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[3] Cons. Civ. Engr., San Francisco, Calif.

It is outside the scope of the author's paper to discuss the reduction in load capacity per pile for piles in groups, but it may be well to suggest that it is something that should be taken into account—even more carefully with the Dames and Moore method than when driving to bearing capacity as determined by the *Engineering News* formula, because in general the piles will be shorter and their surplus capacity will be less.

Pile driving can never be reduced to exact rules and the knowledge gained from the logs of test holes is extremely useful in making such adjustments in driving as are frequently necessary. It should be stressed that one of the great virtues of the method is that it necessitates taking borings. This virtue should be cherished. In addition, the number of borings should be generous, otherwise the contours of penetration are mere guesses that lead to endless trouble in the field.

With no background of service records or tests, it required a certain amount of courage on the part of Messrs. Dames and Moore to sponsor this new method and assume the responsibility such sponsorship entailed. That the method has stood the test, as evidenced by the data presented, must be very gratifying to them.

HAMILTON GRAY,[4] Assoc. M. ASCE.—Foundation engineers have long known from experience that the supporting power of piles embedded in cohesive soils cannot be predicted with consistent reliability on the basis of observations made during driving operations. Hence, the use of dynamic pile formulas, when resistance to penetration is derived from cohesive soils, properly meets with scant favor at the present time. Static field loading tests appear to furnish far more reliable information in such cases and, in spite of their expense, are unquestionably justified whenever the cost of the foundation exceeds a modest sum.

Prior to the performance of such tests, a relatively inexpensive attempt may be made to anticipate the test results. Logically, such an attempt involves the static resistance, furnished by the surrounding soil, to relative displacement of the embedded pile, and therefore requires a knowledge of the pertinent physical properties of the surrounding earth. This problem is discussed briefly elsewhere,[5] where the shortcomings of the static approach are described. These shortcomings appear fully applicable to the method described by Mr. Moore, whose analysis is, however, more acceptable in that it requires the determination of the physical properties of the soils actually underlying the building site, rather than the use of "handbook" or average values. Nevertheless, the uncertainties involved should not be minimized. It is also generally accepted, by virtue of experience, as well as by theoretical indications, that the behavior of a group of loaded piles driven into any but the most compact materials will be quantitatively unlike the behavior of a single loaded pile.

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[4] Prof. of Civ. Eng., Univ. of Maine, Orono, Me.

[5] "Pile Foundations and Pile Structures," *Manual of Engineering Practice No. 27*, ASCE, 1946, pp. 71–72.

The analysis of the behavior of a loaded pile embedded in cohesive soils would appear to require two main considerations: First, the displacement or slipping of the pile relative to the surrounding earth, in which category must also be classed the slipping of a roughly cylindrical mass of stiffened soil, which may adhere to, and move with, the pile, relative to the weaker surrounding soil; second, the displacement, due to volume change or consolidation, induced by the compressive stresses created in the soil by the loaded piles. The analysis of these two modes of behavior is fraught with uncertainty.

When a pile is driven into cohesive soils, these are violently displaced with a consequent alteration of the grain structure or configuration. Many soils are amazingly weakened by this distortion, the ratio between the unconfined compressive strengths of "undisturbed" and "molded" samples of some clays reaching values well in excess of 20. Temporarily, therefore, the displaced material immediately surrounding the driven piles may be considered weaker than the original deposit, although clays appear to exist which present exceptions to this statement. In time, however, consolidation of this reworked material occurs, as a result of which it generally acquires a greater strength than the remainder of the stratum. Consequently, it is to be expected that the slipping tendency of a pile is greatest immediately after driving, and diminishes with the passage of time. The rate of increase of resistance depends on the drainage characteristics of the soil. Thus, cohesionless soils possess such large permeabilities that the gain in resistance is usually so rapid that for practical purposes it may be considered instantaneous. The time increase in driving resistance offered by a pile embedded in cohesive soil, and the mass of cohesive earth that often adheres to piles pulled after driving, appear to substantiate the foregoing concept.

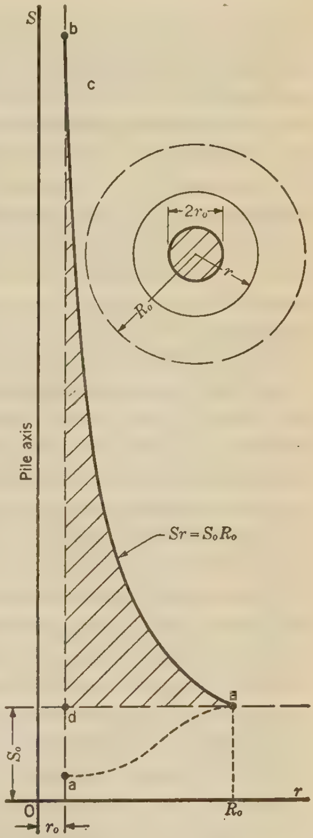

FIG. 19—POSSIBLE VARIATION IN STRENGTH OF SOIL SURROUNDING A PILE

Behavior of the earth immediately surrounding a pile may be visualized by a consideration of Fig. 19, which pertains to an isolated pile driven into a clay deposit. Here, the behavior of the soil after driving of the pile is represented graphically. Assuming for simplicity that the clay stratum penetrated by the pile is homogeneous, its shearing strength at a given depth prior to the driving of the pile may be represented by S_o, the value of which is essentially a function of the void ratio, or water content of the soil, and of the particular soil type. As the pile is driven, the molding effort is felt appreciably within a radial distance denoted by R_o. In all probability, the degree of soil disturbance and the degree of structural weakening increase as r diminishes. Immediately after driving, the strength may vary according to some such relation as indicated by the curve a-a. However, by virtue of subsequent consolidation of the molded material, the strength of the soil within the distance R_o increases until, upon pulling the pile, the soil fails along some quasi-cylindrical surface not larger than R_o. If F denotes the pulling force per foot of pile length, the average shear stress along a cylindrical surface of radius, r, must be

$$S = \frac{F}{2\pi r} \qquad (8)$$

Should the slipping actually occur along the cylindrical surface with radius R_o, the shear stress intensity on smaller concentric cylindrical surfaces would be represented by the ordinates of curve ab in Fig. 19. The strength of the soil would be represented by a curve with ordinates greater than those of ab—for example, ac. Should slipping occur on a surface with diameter less than $2\,R_o$, the strength of the soil must be less than the ordinates of curve ab at some value of $r < R_o$. In any event, the strength when $r_o < r < R_o$ must, if soil adheres to a pulled pile, lie within the limits of the crosshatched zone, abd, in Fig. 19. It must not be assumed that slipping would actually occur along a true cylindrical surface, the use of such a surface being merely for convenience in analysis.

On the other hand, the strength of a soil which is not sensitive to molding would remain essentially at the value S_o. Since the author apparently used the value S_o as a conservative value, it would seem that the actual factor of safety against slipping would vary from locality to locality—depending on the sensitivity of the various clays to disturbance. The same considerations would apply to groups of closely spaced piles.

Having determined by conservative methods that the piles are safe from slipping, the question of subsidence of the supporting medium must be faced. The manner in which the piles transmit their load to the soil, and the stress distribution around and beneath the piles, are matters about which no quantitative information appears to exist. Therefore, only crude guesses can be made as to the expected subsidence. In the examples cited by the author, no consequential subsidence developed; this apparently was due to the stiffness of the firm clays into which the lower ends of the piles penetrated, a consistency acquired perhaps from some large prehistoric overburden. It is interesting to speculate on the possibility that a clay firm enough to carry a loaded pile without slipping can also carry its load without undergoing undue volume change. Certainly the observations and predictions cited in this paper indicate this.

The author states that "yield point shear strengths" were determined. This term requires definition since in testing a structural material "yield point" is defined on the basis of the slope of the stress-strain diagram, and "yield strength," by the shape of the stress-strain diagram beyond the proportional limit. Presumably, the tests were performed at high rates of stress increase to avoid consolidation during the testing.

No novel theory appears involved in the method, and engineers will appreciate the logical nature of this approach to the problem—combining as it does ordinary statics and the shearing properties of the surrounding soil. However, it would be well to bear in mind (*a*) the uncertainties involved in estimating the actual shearing strength of the soil about the pile, as well as (*b*) the assumptions regarding the mode of failure of the pile. General acceptance should be preceded by satisfactory experience with the method in various types of clays, and, to this end, added factual data on the effect of disturbance, shearing properties, and Atterberg limits of the various soils described in the paper would be valuable.

JACOB FELD,[6] M. ASCE.—The director of the Patent Policy Survey of the National Research Council, Archie M. Palmer,[7] in a report favorably inclined to the patenting of scientific and technical discoveries, recently stated:

> "Discoveries or inventions possessing commercial application that are merely published and are thus made available to everybody equally are seldom adopted."

Karl T. Compton,[8] President of the Massachusetts Institute of Technology at Cambridge, is also quoted, from an annual report, as being in agreement with such advice:

> "Responsibility does not always end with the mere publication of patentable scientific discovery or invention; the public benefits derivable from the patent laws and contemplated by the framers of those laws should not be lost through a failure to solicit patent protection."

Even more to the point, Elihu Thompson[9] in 1920 advised in an address before graduating students at the Massachusetts Institute of Technology:

> "Publish an invention freely, and it will almost surely die from lack of interest in its development. It will not be developed and the world will not be benefited. Patent it, and if valuable, it will be taken up and developed into a business."

Perhaps it was such thoughts that induced the author and his associates to apply for and accept patent rights for the method described in this paper, even if, as the author states in the "Conclusion":

> "The methods of analysis employed for predetermining pile lengths are in a formative stage, and it is hoped they will be improved as knowledge increases."

These quotations agree in assuming that the discovery to be patented is correct as well as commercially applicable. No doubt the construction industry would welcome a new, correct method of predetermining pile lengths and would gladly pay for the privilege of profiting by the savings through such an invention. However, the method described in this paper is neither new nor correct.

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[6] Cons. Engr., New York, N. Y.

[7] "Patents and University Research," by A. M. Palmer, *Scientific Monthly*, February, 1948, p. 149.

[8] *Technology Review*, December, 1932, p. 101.

Electrical World, Vol. 75, 1920, p. 1505.

In 1932 the writer outlined the method described in this paper and also showed what uncertainties and errors were involved.[10] However, he makes no claim to having discovered a new method at that time, for exactly a century earlier many of the concepts were published by M. Moreau.[11]

There are a number of basic errors in the method described in the paper:

1. Eq. 1 assumes that the lateral pressure against the face of a pile at any depth is the sum of two items. The first is the same as the "equivalent liquid pressure" found in many discussions of lateral earth pressure and is given as a constant, *K*, times the total weight of the overlying soil. It has been shown that the lateral pressure against a vertical face of narrow dimension, such as a pit or a pile, is theoretically not a linear function of the depth.[12] Actually, at certain depths, there is no lateral pressure, depending on the ratio of width to depth and on the soil characteristics. This theoretical consideration is corroborated by the well-known experience with pressures to be resisted in sheathed pits[13] and by the fact that cored holes and dug wells are possible in most soils. Even in the general case of earth pressure against unlimited wall faces, the factor *K* is not a constant unless the soil is uniform, and even in that case there is some doubt as to whether depth is not an influencing variable.

2. The last term of Eq. 1 is a function of the shearing strength of the soil. The yield point bearing of the soil in Table 2, Col. 7, seems to be derived from the assumption that, since the dilatancy of elastic bodies undergoing elastic strain is equal to the sum of strains along the three rectangular axes, the bearing strength is equal to three times the linear shearing strength. This may be approximately true for isotropic bodies in which the elastic limit is not exceeded, if the higher powers of the infinitesimal strain values are disregarded, but such a condition does not describe the shoving away of the soil by the introduction of a pile.

3. The assumption that frictional resistance of soil along the face of a moving body can be described in terms of the laws of static friction, as first postulated by Coulomb, is not consistent with the action of pipes jacked through soil, of caissons sunk vertically, of shield driven tunnels, or of piles undergoing loading and pull-up tests. The frictional resistance in all these cases can be explained only by the laws of viscous flow, with proper consideration for adhesion and tackiness.[14,15]

4. The average friction along the surface of the piles cannot be based on a coefficient of friction greater than that of the soil itself, as is shown in Fig. 2 by the values of α being greater than those of ϕ. Actually, in Table 2, Col. 6, values of frictional resistances on wood are smaller than the soil shearing strength at the corresponding depths in Col. 5. It can be easily seen that, where the frictional resistance exceeds the soil shearing strength, failure will occur in the soil immediately outside the contact surface of the pile.

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[10] "Point Resistance of Piles Analyzed," by Jacob Feld, *Civil Engineering*, December, 1932, p. 779.

[11] "Notice sur une Nouvelle Manière de Fonder en Mauvais Terrain," by M. Moreau, *Memorial de l'Officier du Génie*, No. 11, Paris, 1832, pp. 11–40.

[12] *Transactions*, ASCE, Vol. LXXXVI, 1923, p. 1579.

[13] "Precast Concrete Sheathing," by Jacob Feld, *Journal*, A. C. I., Vol. 16, 1945, p. 441.

[14] "Soil Resistance to Moving Pipes and Shafts," by Jacob Feld, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, Holland, 1948, Vol. VII.

[15] "Jacked in Place Pipe Drainage," by Jacob Feld, presented at the annual meeting of the Highway Research Board, National Research Council, Washington, D. C., 1947.

5. The method of evaluating the total pile capacity, shown in Table 3, is an arithmetic summation of resistances computed in the various soil layers. This method is exactly equivalent to assuming that the strength of a laminated beam consisting of nonequal planks laid flat on each other is the summation of the individual strength of the planks. Unless the strains are equal at the resistances considered, such an arithmetical summation has no meaning and is a serious error.

The comparison of pile capacity determinations listed in Table 8 indicates that the *Engineering News* formula used does not apply to the driving conditions, especially when the heavy Vulcan No. 1 and Vulcan No. 0 hammers were employed. The author does not state which form of the formula was used, but it is well established that the original formula, as well as modifications introduced for applying steam hammer impact, does not indicate true bearing values of piles when such large hammers are used.

George F. Sowers,[16] Jun. ASCE.—Methods such as advocated by the author in this significant contribution to foundation engineering not only aid in predetermining pile lengths, but also point out the significant factors that make up the capacity of a pile. The usual methods of pile "design" to support a given load can be summarized under the headings: (1) Guess or estimate based on experience; (2) Driving of piles to required resistance to penetration as determined by one of nearly a hundred formulas; and (3) Driving of several piles to different depths and load testing each to determine those which are satisfactory.

The frequent failures of pile foundations to meet expectations when designed on the basis of experience alone, or on the basis of formulas, indicate the shortcomings of the first two methods. The test pile method is now required on most major jobs to insure adequate foundation capacity, but it furnishes information on only the characteristics of similar piles driven in the same manner at the same location as the test piles. The test results tell nothing about how pile capacity might vary with different piles at different locations. The rational method of pile analysis proposed by the author not only gives the capacity of a pile, but also indicates how that capacity might change with variations in length or cross section, and, if adequate soil borings were made, how capacity might change throughout the building site.

The writer had available boring records, pile test data, and some meager information on the soil characteristics at two sites in the Cuyahoga River valley at Cleveland, Ohio. The borings and pile tests were made under the direction of G. B. Sowers, M. ASCE, and detailed descriptions of the foundation conditions encountered are available elsewhere.[17,18] The Cuyahoga River valley in Cleveland lies to one side of a deep preglacial valley filled with deposits of fine sand, silt, and clay. The two sites are about 1 mile apart, and although similar types of soils are encountered at each site, the soil profiles are different. At the first site, the Republic Steel plant built in 1943, the soil is of medium consistency, gray silty clay of medium plasticity with thin seams of silt and occasional thick strata of stiff silty clay (Fig. 20(*a*)). At the turning basin, built in 1941 as part of the Cuyahoga River Improvement, fine sand overlies the clay deposits to depths of 50 ft (Fig. 21(*a*)).

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[16] Asst. Prof., Civ. Eng., Georgia School of Technology, Atlanta, Ga.

[17] "Through 130 Ft of Plastic Clay to Rock," by George B. Sowers, *Engineering News-Record*, November 18, 1943, p. 107.

[18] "The Cuyahoga River Improvement 1938–1942," by G. B. Sowers, thesis presented to the Case School of Applied Science at Cleveland, Ohio, in December, 1942, in partial fulfilment of the requirements for the degree of Civil Engineer.

Soil tests for shear and for friction on steel were made in the laboratory at Case Institute of Technology (formerly Case School of Applied Science) in Cleveland, on samples from the turning basin site.[19] Unfortunately only index classification tests were made on the soils on the Republic site;[18] however, the shear and friction characteristics were estimated from the penetration resistance and by comparison with the other site. Although the soil data are inadequate, it is believed that the strength and friction profiles in Figs. 20 and 21 are a fair representation of the true soil characteristics.

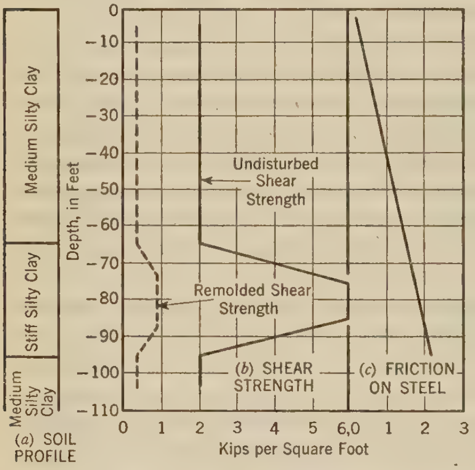

FIG. 20.—SOIL DATA FROM REPUBLIC STEEL SITE

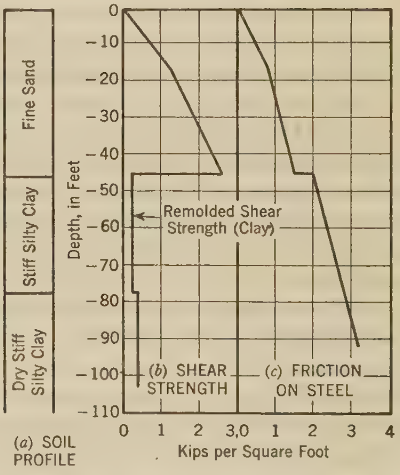

FIG. 21.—SOIL DATA FROM TURNING BASIN SITE

Nearly one hundred pile tests in compression were made at the Republic site and five tests in both tension and compression at the other. Four of the Republic piles were analyzed for bearing as described by the author. The first computation used undisturbed shear characteristics of the soil; a second computation was made using the remolded shear characteristics. Two piles at the turning basin site were analyzed using the remolded shear strength of the soil. Lateral earth pressure in both cases was assumed to be equal to the vertical pressure.

The results of computing pile capacities at the Republic site from the undisturbed soil characteristics indicate a serious error, for computed capacities are more than double the test capacities (Table 9). When the remolded shear strength was used, the computed capacities were comparable to the pile test results, the greatest error being about 14%. This is excellent agreement considering the inadequacy of the soil information. At the turning basin site the computed capacities (based on the remolded shear strength of the clay strata) were higher than the test pile results, but again the agreement was well within the accuracy of the soil information (Table 10). In this case the best agreement was for the steel WF pile (assuming the soil between the flanges to act as part of the pile).

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[19] "A Comparison of a Laboratory Determination of the Tension Value of Steel Piling with Field Tests," by E. Schmidt, thesis presented to the Case School of Applied Science at Cleveland, Ohio, in June, 1941, in partial fulfilment of the requirements for the degree of Bachelor of Science.

TABLE 9.—SUMMARY OF RESULTS AT REPUBLIC STEEL SITE (CLEVELAND, OHIO)

Test pile No. (1)	Size[a] (2)	Length (ft) (3)	Test load (tons)[b] (4)	COMPUTED LOAD (TONS)	
				Undisturbed (5)	Remolded (6)
4	10¾ in.	78	45	104	47
5	10¾ in.	81	50	109	52
6	10¾ in.	83	45	114	50
7	16 in.	89	95	172	89
8	16 in.	69	70	144	60

[a] Steel pipe, outside diameter. [b] Compression.

The results of these computations led the writer to check in detail the assumptions of the author's method in order to suggest improvements in the procedure. As the author states, the method is in the formative stage and further analysis is needed.

TABLE 10.—SUMMARY OF RESULTS AT TURNING BASIN SITE

Test pile No. (1)	Type (2)	Length (ft) (3)	TEST LOAD (TONS)		COMPUTED LOAD (TONS)	
			Tension (4)	Compression (5)	Tension (6)	Compression (7)
5	10 WF 54	76	90	90	97	98
6	12¾-in. steel pipe[a]	78	85	100	91	100

[a] Outside diameter.

Experienced construction men recognize that entirely different characteristics are exhibited by silts, sand, and gravel, on the one hand, and clay soils on the other. Any method for analysis of piles should recognize this difference when determining lateral pressure along the pile, friction on the pile surface, and soil shear strength.

In granular soils, test borings and laboratory investigations furnish information to determine the vertical pressure in the soil deposit, $\Sigma \gamma\, h$; void ratio and relative density; the angle of internal friction of the soil, ϕ; and the coefficient of friction of the soil on the pile, $\tan \alpha$. The angle of internal friction of granular soils varies with relative density. Pile driving tends to compact such soils to their densest condition, and, therefore, ϕ, for the densest state of granular soils, should be used. The greatest difficulty lies in computing the lateral soil pressure. In natural sand and gravel deposits the lateral pressure P_l ranges from 0.4 times to 0.6 times the vertical pressure P_v. If the author's formula for friction on the pile (Eq. 6), which he calls modified friction in order to account for increase in lateral pressure due to pile displacement, is rearranged,

$$F_m = (P_v + \pi S) \tan \alpha \qquad (9)$$

or, in other words,

$$P_l = P_v + \pi S \qquad (10)$$

Substituting in Eq. 10 the expression for shear strength of granular soils,

$$S = P \tan \phi \qquad (11)$$

Where $\tan \phi$ ranges in value from 0.6 to 1.0 for typical sands and gravels,

$$P_l = 3 \text{ to } 4\, P_v \qquad (12)$$

This is an increase in P_l of about 600% over that of the natural deposit. The writer can see no logical justification for this tremendous increase in pressure. In loose sands the soil structure breaks down with the driving, and in passing from the loose to the dense state the pressure may increase only slightly. Application of the "modified friction equation" (Eq. 6) to the turning basin piles gives results far too high. In dense sands such great difficulty is experienced in driving that jetting is used to sink the piles, and the resulting lateral pressure is probably close to that of the natural deposit. At any rate, much study is needed on this question.

The end bearing of a pile in sand or gravel is probably too low when computed by Eq. 7. The basis of Eq. 7 is the expression derived by L. Prandtl and quoted by A. Nádai:[20]

$$q = \pi S \qquad (13)$$

in which q is the bearing capacity per square foot. Eq. 13, however, was based on the assumptions of: (*a*) Cohesive, weightless soil; (*b*) Continuous footing; (*c*) frictionless base of footing; (*d*) footing resting at the surface of the soil; and (*e*) failure occurring as soon as plastic equilibrium is established at any point.

None of these conditions are met when the formula is used to estimate end bearing of piles in sand. Probably the worst discrepancy between theory and reality results from assumptions (*a*) and (*d*). K. Terzaghi,[21] Hon. M. ASCE, gives the unit bearing capacity of a round footing on a cohesionless soil to be approximately

$$q = \gamma D_f N'_q + 0.6 \gamma R N'_\gamma \qquad (14)$$

in which D_f is the depth of footing below the ground surface; R is the pile radius; and N'_q and N'_γ are factors for local shear failure that depend on the angle of internal friction, ϕ. For an 80-ft pile with a point diameter of 1 ft driven in an average saturated sand or gravel ($\phi = 35°$ and $\gamma_s = 60$ lb per cu ft), the end bearing capacity computed by Eq. 13 would be 67,000 lb per sq ft, as compared with 11,300 lb per sq ft when computed by the author's formulas. Although Eq. 13 applies to footings where D_f is small compared to R, it illustrates the possible magnitude of error in applying Eq. 7 to cohesionless soils.

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[20] "Plasticity," by A. Nádai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

[21] "Theoretical Soil Mechanics," by K. Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943.

In clay soils the phenomena that result in pile bearing capacity are more complex. The vertical pressure and void ratio are determined by laboratory tests. In natural deposits the clay does not necessarily follow the commonly accepted law of increase in shear strength with pressure expressed by Eq. 4, but may actually decrease in shear strength with depth. This necessitates shear tests (unconfined compression tests are adequate and simple) on samples taken throughout the deposit. A chart showing shear versus depth can be made, such as Figs. 20(*b*) or 21(*b*). In addition, the variation of shear strength with remolding and subsequent consolidation must be considered.

There is some question in the writer's mind whether true friction exists at all between a clay soil and a pile. The few clay friction tests he has conducted or observed indicate that the resistance to sliding along a steel surface (be it friction or adhesion) is considerably more than the remolded shear strength of the clay. This results in a smear of clay remaining on the steel surface as movement takes place. Observation of piles pulled from clay with such smears of clay adhering to the surface indicates that the surface phenomenon of sliding resistance is not as important as remolded shear, but more study is needed to clarify this point.

A most important fact brought out by the author, and in another manner by the writer's analysis, is that pile bearing in clay varies with time. The author ably discusses the changing effect of soft strata along the upper sections of a pile (from support of the pile to drag on the pile). The change in the shear characteristics of clay and its effect on pile support, however, are not discussed.

A. Casagrande,[22] M. ASCE, has pointed out that, as a pile penetrates clay, a zone of shear failure and plastic flow is created by the point. This remolded zone surrounds the pile throughout its length and has an unknown thickness of from perhaps a few inches up to one and one-half times the pile diameter. The surface of the pile is therefore supported by remolded clay, but the stressed zone at the point reaches into the undisturbed clay. The computations summarized in Tables 9 and 10 indicate that this was the case for the glacial clays at Cleveland, where the remolded shear strength is about 15% of the undisturbed shear values.

As time progresses the remolded soils surrounding the pile consolidate because of the breakdown of their structure, and, at the same time, some of the disturbed colloidal gels tend to resolidify. This zone, then, gains in shear strength with time, and in some soils may even exceed the strength of the original undisturbed soil. What happens to the friction or adhesion along the pile surface is unknown, but it probably increases too. In other words, the soft disturbed zone eventually becomes a hard zone, attached to the pile more or less securely. Part of this increase in strength of the disturbed zone occurs rapidly, as is illustrated by the increase in driving resistance after a short rest has occurred. How long it will take for complete consolidation and gain in strength to take place depends largely on the permeability, void ratio, and thickness of the soil. Old piles pulled from clay often bring with them a 1-in. coating or a 2-in. coating of hardened clay.

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[22] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, in "Contributions to Soil Mechanics—1925–1940," Boston Soc. of Civ. Engrs., Boston, Mass., 1941, p. 91.

Another factor not discussed by the author is the relative strain required to mobilize the full shear or friction resistance of each stratum. Slipping or full failure in a hard stratum may occur before much resistance is developed in a soft stratum; and, if the shear or friction resistance of the soil after failure is less than that at failure, the pile will fail progressively at a lower loading than the computations might indicate. However, this is probably of minor importance and could be neglected inasmuch as the softer layers contribute only a small part of the total pile capacity.

Of course it must be recognized that two questions are left unanswered by this analysis:

1. How much load will a pile group carry?
2. How much settlement will occur if the piles can carry the load without slipping into the ground?

The author's analysis, by increasing our knowledge of the factors that contribute to the capacity of a single pile, opens the way toward rational analysis of pile groups and pile settlement.

Probably many so-called practical engineers will scoff at the detail involved in a rational analysis of a pile foundation. Engineers who go to great lengths to design a building superstructure to the last rivet often "design" foundations by guesswork or handbook formulas. Foundation engineering will find its place with other branches of civil engineering when as much time and effort are put into it as the relative cost of the foundation warrants.

A similar method of pile analysis based on shear and friction in soils was suggested by G. B. Sowers as a thesis project for students of Case Institute of Technology in 1940. One such study[19] was made, but World War II interfered with further development.

H. CAMBEFORT.[23]—Although the various examples given by Mr. Moore seem to confirm the design method in the paper, the writer is not completely convinced that this is so. It would have been helpful (especially for the European reader) if the theoretical basis for this method had been outlined. Apparently, the calculations depend on an assumption that there is passive earth pressure from the surrounding soil.

In the simple case of a wall, however, this pressure will not come into action before an appreciable displacement has occurred, and it is probable that the problem is more complex in case of a pile. Only experiments can supply an answer to this question; but it is necessary that these experiments be made with a certain objective insight, and some extra care. The usual test does not furnish serviceable information, giving only the penetrations as functions of the loads. This applies to most cases of piles driven in nonhomogeneous soil.

It is advisable for the engineer to examine the soil strata carefully, not only the layers which the pile must penetrate, but also those some distance below the pile tip. In general, a soft layer penetrated by a pile will eliminate the support that the upper and more rigid layers might give the pile. The upper layers, being subjected to the often important frictional forces along the pile shaft, cannot find support in the soft layer. The writer made a test on a pile that had been equipped with extensometers at the level of the boundaries of every soil layer. The stress in the pile immediately below the soft layer corresponded to nearly the total load applied at the pile top, whereas only one sixth of the total load was transmitted to the lower boundary of the upper layer (in the present problem). This distribution illustrates clearly the ambiguity of the theoretical case presented by Mr. Moore. Although the negative friction of all the layers seldom exceeds the resistance of the bearing layers, Mr. Moore takes this friction into consideration consistently, without showing the consolidation diagram to justify it.

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[23] Civ. Engr., École Nationale des Ponts et Chaussées, and Head, Dept. for Studies of Constr., Entreprise de Fondations et Travaux Hydrauliques, Paris, France.

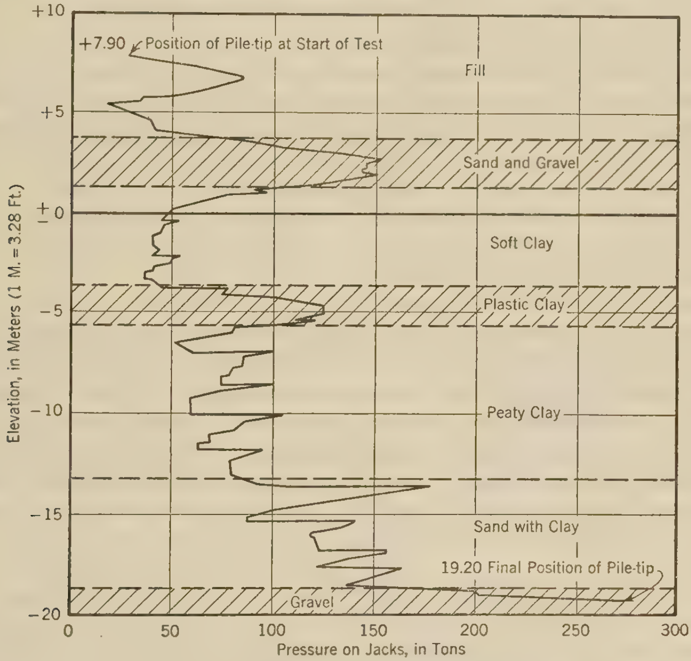

FIG. 22.—TEST OF A PRE-CAST CONCRETE PILE TO DETERMINE VARIATIONS IN ITS RESISTANCE TO PENETRATION

Furthermore, when it can be predicted that the upper layers will settle more than the tip of the pile (thus loading it as the result of the negative friction) the easiest procedure would be to eliminate this friction by lubricating the skin surface of the pile shaft. Thus, one should not try to drive the pile inordinately deep. Indeed, it is not always possible to do so.

The soft layers above the resisting layer (the stratum in which the pile is supported) are also of great importance. If the pile tip is too close to the lower boundary of this resisting layer, and if this layer rests on soft soil, the pile will penetrate it—that is, will fail. On the other hand, if the pile had been shorter, the penetration would not have occurred and the pile would have met sufficient resistance. In such a case, however, settlement must be expected.

This phenomenon occurred in the first foundation for the Trans-Atlantic Railroad Terminal at Le Havre, France.[24] The piles, supported in the sand and gravel between El. +1.50 m and +4 m (from 5 ft to 13 ft), had sufficient bearing capacity, but the building settled nevertheless.

The foundations were repaired by jacking down pre-cast piles to El. −19 m (62.4 ft). The load-penetration diagram of one of these piles, as shown in Fig. 22, is a good illustration of the varying resistance.[25] When the tip reached the lower part of the firm layers (sand and gravels, and plastic clay) just above the soft layers, the bearing capacity decreased very rapidly; and, when the layer was penetrated, the bearing capacity was very nearly the same as it was above this layer. In other words, pressure of the soft strata has precisely the same effect as if the firm layers were not there.

Considering again the penetration of the firm layers, if the maximum capacity indicated in the diagram is created entirely by end bearing resistance, the bearing capacity of the pile will remain constant from the point where the pile tip reaches the soft clay. This is not the case. Before it stabilizes, the bearing capacity decreases through the first decimeters of this soft layer. It appears, then, that the lateral friction varies in the same manner as the end bearing resistance. This phenomenon was recognized in the test described herein. The pile tip was protected by a hydraulic jack, which enabled the investigator to measure the resistance.

Fig. 23 shows[26] that in this case (loading cycle 10), the lateral friction is much less prominent than in the case where the end bearing was active (loading cycles 3, 4, and 6). This curve shows also, that the lateral friction does not develop completely, except for a certain penetration of the pile (loading cycles 2 and 3). Furthermore, the frictional resistance to uplift is not comparable to the lateral friction generated by sinking the pile. However, it was approaching this value (corresponding to the settlement) when the end bearing resistance was omitted (loading cycles 7 and 10). Lifting the tip jack by inflation produces a stronger friction than lifting the pile by its head (loading cycles 9 and 7).

Thus, present computation methods are incapable of explaining all the phenomena. Therefore, it is necessary to collect experimental data. As a contribution to this end the writer submits the following: A rod, 130 mm in outside diameter, is driven into a very dense sand and gravel formation. When the rod reaches 5 m or 6 m, a crater is formed at the surface, about 50 cm in diameter (1 m is equal to 39.37 in. or 3.281 ft). The failure takes place along a vertical cylinder and the settlement reaches from 10 cm to 15 cm.

A tube 560 mm in diameter, driven in the same soils, produced failure within a cylinder 2.2 m (7 ft) in diameter. The ratio between the diameter of the disrupted soil prism and the diameter of the rod or tube was very nearly 4. Could

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[24] "Étude Expérimentale de la Force Portante d'un Pieu Foré," by H. Cambefort, Editions Science et Industrie, Paris, France, 1947.

[25] *Ibid.*, p. 15, Fig. 17.

[26] *Ibid.*, p. 10, Fig. 12.

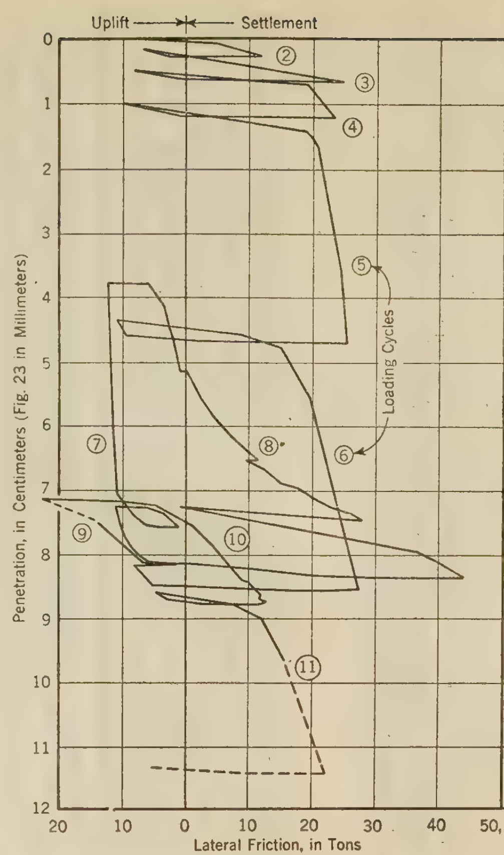

FIG. 23.—VARIATION IN LATERAL FRICTION

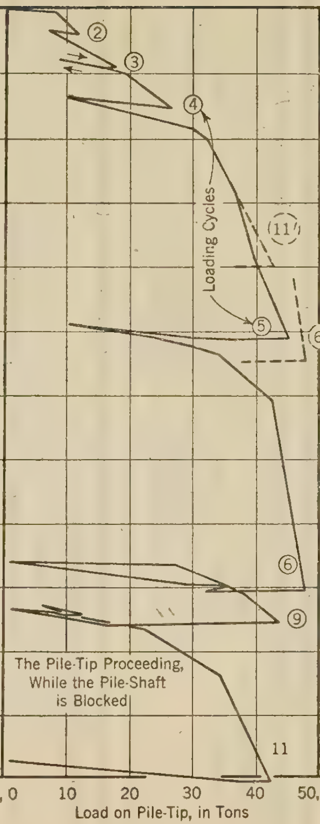

FIG. 24.—PENETRATION OF THE PILE TIP

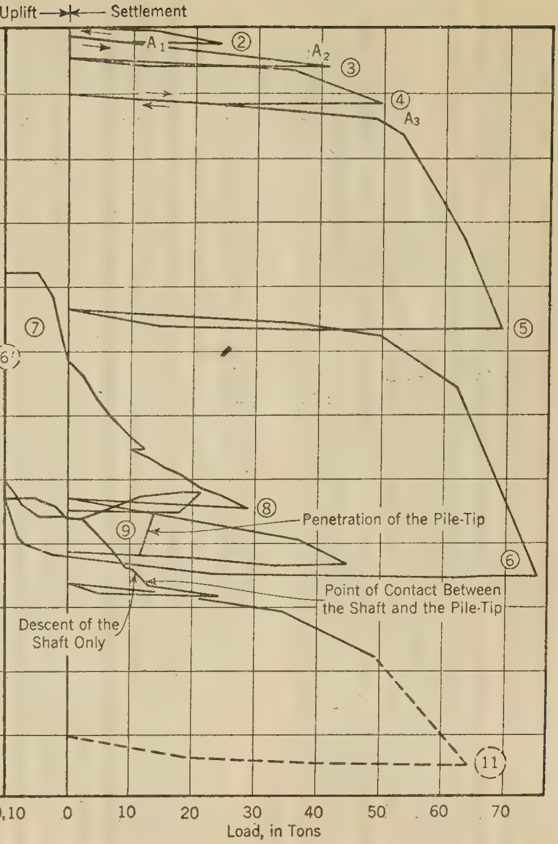

FIG. 25.—POINT OF CONTACT BETWEEN THE SHAFT AND THE PILE TIP

this be regarded as an effect of the energy input at the tip? In a way, the actual pile was transformed into a hypothetical pile of much larger diameter, the proportion between the diameters being a function of the angle of internal friction of the soil. In the present case the proportion is 4, and the angle of internal friction is estimated at from 37° to 40°.

When the end bearing was zero, the lateral friction did not affect parts other than the actual pile shaft. In that case it would be four times smaller and, in any case, the small-scale tests made by J. Lehuérou-Kérisel[27] show that the frictional resistance F, to a pile driven into pure sand is given by the formula:

$$F = \frac{c\,s\,h}{2} \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) e^{(1.77+\phi)\tan\phi} \sin\phi \ldots\ldots\ldots\ldots (15)$$

in which, in addition to the notation of the paper, c is the circumference of the pile and s is the vertical stress at a depth h. The accuracy of Eq. 15 was checked by tests on a pile of 420-mm diameter, the penetration being from 5 mm to 8 mm.

Eq. 15 should not be applied to cohesive soil. However, a sufficiently accurate approach may be made by simply multiplying the cohesion by the lateral contact area. The penetration of the tip is illustrated by a broken line in Fig. 24, where the changes of the slope correspond to the loads P given by the formula of J. Ferrandon:

$$P_1 = \frac{s\,A}{\frac{1}{2}\frac{1+\sin\phi}{\sin\phi}\sqrt{\frac{1+\sin\phi}{3-\sin\phi}} - 1} \ldots\ldots\ldots\ldots (16a)$$

which is an extension of the formulas for piles devised by O. K. Froehlich[28] and by the formula of A. Caquot:[29]

$$P_2 = s\,A \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) e^{\pi\tan\phi + 0.2\tan\phi} \ldots\ldots\ldots\ldots (16b)$$

In Eqs. 16, A is the cross-sectional area of the pile tip. Unfortunately, the method of computing the corresponding penetrations t is not known. Possibly, the Boussinesq formula indicates a solution:

$$t = \frac{2}{\pi}\frac{s\,d}{a_v} \ldots\ldots\ldots\ldots (17)$$

in which s is the stress on the tip of diameter d and a_v is a coefficient, depending on the soil. The analysis of this test shows that the penetration curve for the pile top (Fig. 25) presents[30] points of changing slope, corresponding to the following phenomena: A_1 is the limit of the elastic zone, the point defined by Messrs. Ferrandon and Froehlich; A_2 is the lateral friction now fully developed;

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[27] "La Force Portante des Pieux," by J. Lehuérou-Kérisel, *Annales des Ponts et Chaussées*, May, 1939, p. 602.

[28] "Druckverteilung im Baugrunde," by O. K. Froehlich, Springer, Wien, 1934, p. 142.

[29] "La Force Portante des Pieux," by J. Lehuérou-Kérisel, *Annales des Ponts et Chaussées*, May, 1939, p. 583.

[30] "Étude Expérimentale de la Force Portante d'un Pieu Foré," by H. Cambefort, Editions Science et Industrie, Paris, France, 1947, p. 6, Fig. 3.

and A_3 is the penetration and failure, the point defined by Mr. Caquot. Point A_2, Fig. 25, commonly referred to as the point of failure, corresponds to nothing more than the final development of the lateral friction.

These curves apply to a cast-in-place pile (drilled and cast in the soil), and it is possible that the conditions are somewhat different in the case of a driven pile. In particular, point A_1, Fig. 25, should disappear in the case of a driven pile. In so far as the designer does not consider a pile as an hyperstatic system, in which the resistance depends primarily on the displacements, he may conclude that the bearing capacity can never be calculated no matter what formula is applied.

It seems to the writer that, in the case of cohesionless sands, investigators are "on the right track" with the Kérisel, the Ferrandon, and the Caquot formulas; but these formulas must be supplemented by a study of the displacements—a primary requirement for exploring the behavior of a pile. Next the two coordinates should be obtained for each of the points A_1, A_2, A_3, in Fig. 25. The same considerations may apply to the end bearing resistance in cohesive soils; but nothing is known concerning the lateral friction in such soils.

Mr. Caquot's "theorem of corresponding stages" reduces this case to one of a cohesionless sand, but it is only valid for a series of stabilized conditions of the soil, which does not at all correspond to the case involving a pile. In the same way, the theory of consolidation defined by Karl Terzaghi, Hon.M. ASCE, cannot apply to the first instant that a pile is loaded, because the phenomena occur much too rapidly at that stage. It can only apply when data are available as a result of a study of the life of the foundation.

The structure of clay resembles that of sand, but it is likely, that their physical properties are more like those of solids or liquids of high viscosity, than those of sands. It seems more convenient, however, to abandon consolidation and shear tests, and to devise others, with sufficiently reasonable similarity to the actual conditions of a pile (or of the failure of a soil).

In order to do this, it may be useful to treat the soils as perfectly plastic media, or even as viscous liquids, and then to study their "seepage" and their "yield value." The experience of the glassmaker should not be forgotten. It may be interesting, from another point of view, to determine the apparent coefficient of elasticity of soils more frequently. Such studies, in connection with tests, will undoubtedly help to clarify the problem of determining the bearing capacity of piles.

William W. Moore,[31] M. ASCE.—The comments and suggestions offered by the several discussers and informal suggestions received from interested members of the engineering profession are sincerely appreciated.

The comments offered by Mr. Earl are of particular interest and value because of his experiences both with the method suggested by the writer and other more conventional methods of controlling the installation of pile foundations. Mr. Earl's comment that the method suggested enables taking into account the effect of settlement in upper layers of soil, such as filled-in areas, is believed to be particularly important because the need for pile foundations most frequently arises in such areas. Most other methods for evaluating the behavior of pile foundations offer no opportunity for even approximating the influence of upper subsiding strata.

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As suggested by Mr. Earl, the obtaining of accurate information on the subsurface conditions by borings is of the greatest importance. When a sufficient number of borings is available, the important features of the foundation soil structure usually become apparent; however, minor variations in the strengths of individual strata at various locations on the site may be such that it is futile to attempt extreme precision in determining the lengths of piles required. The problem, then, becomes one of determining a safe and practicable length of piling for use on a site or a particular part of a site. The important decisions are to be certain that the lengths selected are adequate but not excessive and that these pile lengths can be installed practicably with available construction methods and equipment.

The comment by Mr. Feld that the basic concepts of the method of analysis presented by the writer have been previously suggested by various investigators only suggests that sufficient study must be directed to improve modern knowledge to a degree which will restrict the uncertainties and errors so that practicable results can be assured. The comments of Mr. Sowers are most interesting in describing the development by entirely separate investigators of a method of analysis which appears to be parallel to that suggested by the writer. Mr. Feld's comparison of the strength of a laminated beam consisting of nonequal planks laid flat on each other to the strength of a pile penetrating several soil layers is applicable; if the planks were to be imagined as having the property of continued yielding without severely reduced strength, it would be true that the ultimate strength of the beam would be approximately indicated by arithmetic summation of the strengths of the several planks. It may be suggested that, where the stiffnesses or strengths of the several soil strata do not differ by a wide order of magnitude, the strengths of all the strata may be almost fully mobilized before any actual failure or substantial slippage of the pile relative to the soil can occur. Within the range of working loads, it is quite possible that the stiffer strata may carry much more than their estimated proportion of the total imposed load. However, if no failure can occur without developing resistance in each of the strata, it would appear that the method suggested offers an opportunity for safely evaluating what may be termed the yield point bearing capacity for the pile. The computations can be readily modified to include only the supporting value of firm strata, in cases where certain soil strata may be too weak or compressible to contribute to the safe supporting capacity of the piles.

The writer has fully recognized, as suggested by Messrs. Feld and Sowers, that precise determination of the lateral pressure of the soil against the side of a pile is a quite complex and indeterminate problem. However, he is unable to accept the thesis that the lateral pressure against the side of a driven pile can be zero, or even that it can be less than the vertical weight of overburden above any given depth. Such conditions of low lateral pressure have been observed, as noted by Mr. Feld, in the walls of drilled or excavated shafts where the arching of the material can permit internal soil stresses to develop which result in very low or zero lateral pressures against the casing or bracing within the shaft. Since these circumstances appear to be quite different from those surrounding a pile forced into the soil with accompanying lateral displacement and compression of the surrounding material, it appears reasonable to presume that the lateral pressure on the side of a pile will be greater than the weight of adjacent overburden; and, furthermore, that the intensity of the pressure will be related to the shearing resistance and probably also to the compressibility of the material displaced during the driving of the pile. The equation used by the writer for evaluating the lateral pressures on the surface of the pile (Eq. 1) is noted to be largely empirical and has been reached through analyses of the behavior of piles during static loading tests. The equation suggested is not intended to provide a rigorous theoretical solution for the potential lateral pressures. However, it has generally been found that, if the increments of lateral pressures in excess of the weight of overburden are assumed to lie within a range on the order of three to five times the shearing strengths of the materials penetrated, the resulting calculations have been reasonably consistent with the observed results of static loading tests on full-sized piling. The relatively higher lateral pressures are usually indicated for quite sandy or gravelly soils. Some studies made in 1948 and 1949 by the writer and his associates have involved the installation of strain gages in full-sized piling under test conditions, and it is believed that analyses of these observations will throw some light on the distribution of frictional forces along the length of the piles[32] and in turn may indicate the intensities of the lateral pressures developed. In this connection, it may be noted that a report[33] by Armand Mayer, M. ASCE, published in 1947 describes a concept of analysis almost identical with that suggested by the writer, except that Mr. Mayer's analysis results in a substantially higher evaluation of the end bearing factor when piles are driven into granular deposits. Recent experiences have indicated that the computations suggested by Mr. Mayer may result in an improved evaluation of the field behavior of piling in granular deposits, although some questions still remain as to the permanent dependability of very high end bearing resistances and the behavior of piles so supported when subjected to vibrating loads.

During recent years, the writer and his associates have developed, in collaboration with the late Ralph Byrne, Assoc. M. ASCE, the following equation as perhaps offering a somewhat improved evaluation of the end resistance of piling in sand and gravel deposits:

$$P = \frac{\pi C + p\,[1 + (\pi/2 + \phi) \tan \phi]}{1 - (\pi/2 - \phi) \tan \phi} \qquad (18)$$

in which P is the limiting bearing pressure; p is the surcharge of weight of overburden adjacent to footing; ϕ is the angle of internal friction of soil; and C is the cohesive shearing strength of soil.

The numerical values of potential skin frictional support for piles in sand, indicated by the formula presented by Mr. Cambefort (Eq. 15), result in quite high values which do not conform with the field loading test data available to the writer. Although the evaluations of frictional resistance and end bearing indicated by Eqs. 5, 6, and 7 suggested by the writer are known to result in rather low values for some sandy soils, it is his opinion that the possible magnitude of safe increases of these values depends on factors which at present are not completely understood. Some of these interrelated factors are the density and the compressibility of the deposits penetrated, the volume of the pile, and the weight and the energy of the driving mechanism used. It is the writer's belief, therefore, that the values should be increased above those suggested in the paper only where quite complete local data are available, including loading tests made on the specific site with pile sizes and driving conditions comparable to those of the ultimate structure.

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[32] "Electrical Resistance Strain Gages for Determining the Transfer of Load from Driven Piling to Soil," by L. LeRoy Crandall, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, June 21 to 30, 1948, Vol. IV, p. 122.

[33] "Some Recent Tests on the Bearing Capacity of Piles," by Armand Mayer, *Pamphlet No. 34*, Series 1, Technical Inst. of Constr. and Public Works, Paris, March 31, 1947.

As suggested by Professor Gray, the use of static field loading tests is often a most valuable aid in determining safe and adequate piling installations. Although the writer has used the proposed method of computing capacities on many projects without the assistance of static field loading tests, it is preferable to have such loading test data whenever the economic factors involved in the project will permit. The experience of the writer has indicated, however, that field bearing tests on piling or driving records of test piling are frequently of little value and perhaps may even be seriously misleading unless knowledge of the character of the soils into which the piles are driven is reasonably complete. It has been found that the expense involved in making the soil tests and analyses of pile lengths is not usually a serious burden, since this cost seldom exceeds approximately one third to one half of the cost of making borings, taking samples, and studying purely visual descriptions of the subsurface materials.

The pile loading test described by Mr. Cambefort illustrates a very desirable type of test in which the skin friction on the lateral surfaces of the pile can be distinguished quantitatively from the end resistance on the bottom of the pile. Data of this character are certainly of great assistance in understanding the factors influencing the supporting capacity of piling. The circumstances described wherein the test pile penetrated alternately firm and very soft strata illustrate the need for evaluating the compressibility of the several strata, either purely by engineering judgment or preferably by some definite test data, in order to estimate the dependable supporting capacity of each of the soil layers. It has been the writer's practice to reach a decision as to which strata will be depended on for the primary support of the piling by consideration of the texture, firmness, moisture content, density, and sometimes consolidation characteristics. In numerous cases, it has been decided that certain strata which overlie weak and very compressible layers will be either completely neglected in computing the permanent potential support or considered as possibly imposing downward frictional forces as a result of expected future subsidence of these upper strata.

The discussion by Professor Gray concerning the disturbance of the soils immediately surrounding a driven pile and the possible variations in soil strength with time is quite pertinent and correct. The suggested thesis that the strength of the undisturbed soil may be regained after a time appears to be substantiated by the investigations reported in 1948 by A. E. Cummings, M.

ASCE, G. O. Kerkhoff, and R. B. Peck,[34] Assoc. M. ASCE. In view of these studies, as well as the experiences of the writer and his associates, it is believed that the bearing capacities determined by the method of analysis presented are generally conservative, and under certain conditions may be appreciably below the ultimate bearing capacity of the piling.

Although the examples cited by the writer did not include cases where any large settlements had developed, it seems unduly optimistic, as mentioned by Professor Gray, to presume that a clay firm enough to carry the design load on one pile without slipping would in all cases carry the imposed load without appreciable volume consolidation. The settlement resulting from volume consolidation would depend largely on the intensity of stress and the area loaded by the pile groups. In instances where large pile groups may impose fairly heavy loads over an appreciably large area, it is believed that studies of the possible volume consolidation of the soils beneath the pile tips are essential.

Although the behavior of pile groups was not included within the scope of the paper, the behavior of such groups may be, and has been, evaluated within practicable ranges of accuracy by considering the stresses in the body of soil bounding the pile group and directly beneath the pile group. For example, the sum of the potential shearing resistance around the periphery of the pile group, plus the bearing resistance of the material directly beneath the pile group, may be evaluated as if the group were a single large foundation, such as a caisson. Evaluation of the probable factor of safety against failure of the pile group as a whole may be made in this fashion, as well as evaluation of the probable settlements due to compression of the soil materials on which the piles themselves are dependent for support.

Professor Gray's suggestion of the need for definition of the yield point shearing strength may perhaps be answered by illustrating the results of typical shear tests of two materials with the following characteristics:

Description	Sample 1	Sample 2
Material	Clay	Medium sand, with some small gravel
Dry density, in pounds per cubic foot	91.4	104.4
Moisture content, in percentages of dry weight	38.9	21.6
Surcharge, in pounds per square foot	1,650	3,100

These are presented graphically in Fig. 26. In general, the shear tests used by the writer and his associates have been made by applying increments of loading at 1-min intervals of time in a direct ring-type shearing apparatus and measuring the resulting deflections.[35] The stress-strain graph is examined and the yield point selected as the load above which rapidly disproportionate increments of deflection are noted for equal increments of loading. There is evidence that other time rates of testing (perhaps in some cases very much more rapidly, and in some cases very much more slowly, than the tests generally made previously) may offer a method of approximating the applicable field conditions more closely and thereby enabling better evaluations of the behavior of piling on a particular site. Mr. Feld's comment that the frictional resistance for various rates of movement may be important in the resistance of piling is most interesting and pertinent. It is believed that the use of an extremely wide range of rates of deflection for testing may be of assistance in evaluating the ratio of the static bearing capacity to the dynamic driving resistance; also, such data may assist in estimating the dependable static supporting capacity of jacked-in piles in relation to their resistance while being jacked into place.

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[34] "Effect of Driving Piles Into Soft Clay," by A. E. Cummings, G. O. Kerkhoff, and R. B. Peck, *Proceedings*, ASCE, December, 1948, p. 1553.

[35] "Practical Shear Tests for Foundation Design," by Trent R. Dames, *Civil Engineering*, December, 1940, p. 783.

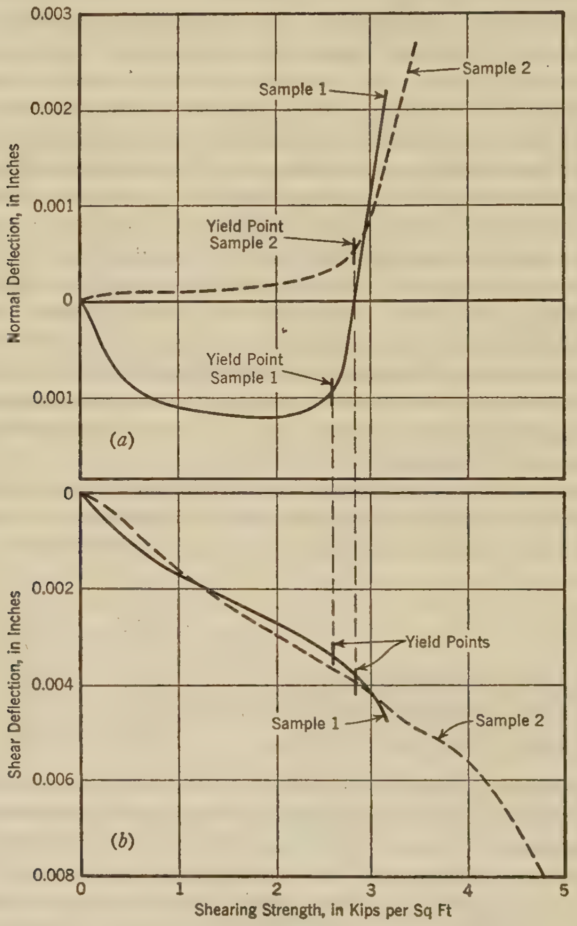

Fig. 26.—Stress-Strain Curves from Shear Tests

In paragraph 4 of Mr. Feld's discussion, it is noted that the average friction along the surface of the pile cannot be based on a coefficient of friction greater than that of the soil itself. The numerical values of soil properties used in the sample calculation presented in Fig. 2 of the paper indicate that the shearing resistance of the soil includes a cohesive resistance of 450 lb per sq ft and an angle of internal friction of 14°. The shearing resistance for the lower ranges of surcharge pressure, when both the cohesive strength and internal friction are included, will exceed the frictional resistance between the pile and the soil based on a coefficient of friction of 16°.

It has generally been the writer's experience, as suggested by Professor Sowers, that clayey soils develop an adhesion to the pile so that the actual failure takes place in the soil close to the pile surface rather than with a strictly frictional movement between the pile surface and the soil. Since it is nearly always difficult to classify materials as purely cohesive or purely granular, and since a very large number of soil deposits combine some of the properties of the cohesive, and some of the properties of the granular, materials, the writer prefers to use test methods that include the influence of both the cohesive and the frictional factors. With these methods, the test results will place an appropriate relative importance on the different properties of cohesion and internal friction for a specific soil condition.

Professor Sowers' experiences with the use of unconfined compression tests do not appear to be fully confirmed by the writer's experiences. In many clayey soils encountered during investigations, it has been found that neglecting the influence of confining pressures during the tests may result in serious errors and sometimes excessively low indications of strengths for the available soil deposits. Frequently, it is difficult to distinguish, by visual examination, between purely cohesive clays and those containing significant proportions of very fine granular particles. It is the writer's belief, therefore, that, until experience has fully confirmed the reliability of the unconfined compression test for a particular local soil condition, a considerable number of tests should be made which evaluate the influence of confining pressures on the strengths of the soils.

The studies made by Professor Sowers using the results of remolded shear tests are especially interesting. The writer believes that such remolded tests should certainly be made when clays having a characteristically severe loss in strength on remolding are encountered. However, the long-time supporting capacities of piling driven into such clays may be less seriously affected than would at first be indicated by considerations of the remolded strengths. Further research, and perhaps pile loading tests made some time after driving, may produce valuable information on this problem. The stress conditions suggested in Professor Gray's discussion and the data presented by Messrs. Cummings, Kerkhoff, and Peck[34] are indications of the direction which these investigations may take.

As suggested by Professor Sowers, the use of jetting to sink piling into sand and gravel deposits interposes an additional and frequently highly indeterminate factor—probably influencing both the lateral pressures and, in some instances, the end resistances of piling placed in sand and gravel deposits.

As Professor Gray suggests, the results of computations utilizing the suggested method should be accepted cautiously and with much study of local experience records. In many instances, a careful study of local experience records together with analyses and perhaps field loading tests will indicate how appreciable economies in foundation construction may be realized with full safety for the structures involved. Also, the type of studies described may enable the design of foundations capable of much more desirable performance than would be achieved with more conventional design methods, and frequently at an equal or lower construction cost. However, where the lengths or loadings of piling indicated by the method of analyses suggested result in marked differences from the current local practice, thoroughly adequate investigations and studies should be made to explain fully the circumstances and the reasons for these differences in order that the satisfactory behavior of any new installation may be assured. As commented by several of the discussers, observation of high driving resistance is not reliable proof that the driven piles are capable of supporting the design loads. On the other hand, a relatively low driving resistance under certain conditions may not in any degree warrant a conclusion that the piles are inadequate to support the planned loads. In many cases, large increases in pile lengths required to realize conventional dynamic driving resistances constitute a completely unnecessary and severe economic waste.

Mr. Feld's comment that the *Engineering News* formula $\left(\text{which was used in the paper in the form of } P = \frac{2\,w\,H}{s + 0.1}\right)$ is not applicable for heavy hammers, such as a Vulcan No. 1 and Vulcan No. 0, makes it the more imperative that the profession develop a practical and reliable method of evaluating the behavior of piling because the present-day trend is increasingly toward the use of heavier driving equipment and higher design loads per pile.

Summary.—The consensus of the discussers' contributions indicates the basic concepts of the method of analysis presented have been suggested at various times in the past by leading investigators and will eventually be developed by the profession to a point where such a method will be of substantial assistance in the design of safe and adequate pile foundations. The use of exploratory borings and of adequate sampling of the soils is of primary importance in applying the method of analysis suggested and in understanding the subsurface conditions and the behavior of the foundation soils on any site. The use of pile driving tests and of pile loading tests may be of great assistance in evaluating construction problems and in confirming analyses of the supporting capacities of individual piling. However, such data can be utilized with assurance of reliability only when there is adequate knowledge of the structure and the properties of the subsurface soil materials. Consideration of the stresses produced in the foundation soils surrounding and below clusters of piling is essential when structures involving any very heavy loads are to be supported, or when large pile groups are to be erected, on sites where soils of appreciable compressibility extend to depths below the pile tips.

The results of the method of analysis suggested should be compared carefully and studied in relation to local experience and practice; and, where apparent differences between the results of the analyses and local practice appear, the reasons for such differences should be analyzed and fully explained. After studying the soil properties and perhaps results of pile loading tests for any specific project, it will be feasible and desirable to establish some criteria for driving resistance (not necessarily related to any dynamic formula) as a method of field control of the pile driving during the actual construction so that any important and unexpected differences in soil conditions would be indicated by the pile driving and appropriate measures could be taken to assure adequate and satisfactory support.

Additional study and research concerning the intensities of lateral pressures created when driving piling into various soil types are needed; and, similarly, laboratory research and field measurement of the end resistances, particularly in granular soils, will doubtless result in improvement and revision of the detailed methods of calculation. Future field loading tests and research programs should include measurements of the stress transmitted from the pile to the soil at various elevations and determinations of lateral pressures on the sides of the piles together with studies of the changes in these effects over periods extending for many months after the piling is driven.

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Paper No. 2373

ADJUSTMENT OF THE "SHORAN" TRIANGULATION

By CHUNG-JUI CHU[1]

WITH DISCUSSION BY MESSRS. ROBERT C. SHELDON, H. W. HEMPLE, AND CHUNG-JUI CHU.

SYNOPSIS

Application of "shoran," a variation of radar, to triangulation necessitates a new method of adjustment. In this paper the various phases of the problem and the several alternatives as to the actual steps to be taken in making the adjustment are discussed. The method employed is first to determine the approximate position of the triangulation stations, and then to find the corrections by the method of least squares. The formulas employed in this method have been simplified by use of the chord distance.

INTRODUCTION

In addition to many other peacetime uses, shoran has found a possible application in triangulation. Instead of measuring the horizontal angles, the distances between the triangulation stations are measured by shoran equipment. This new method will solve the problem of projecting triangulation across fairly wide waters and over high mountains, upon which depends the project of reducing geodetic work throughout the world to the same geodetic datum. A word of caution might be added that at this stage in its development the shoran method cannot completely replace the old method since the new method of measuring short distances does not give the required accuracy.

The chief advantages of the shoran method are:

1. Field operations are simpler than those of the conventional method and can be completed more rapidly. Weather has no great effect on the field work.
2. Rugged terrain presents no problem in the choice of strong triangulation figures.
3. As compared with the old method, formation of the observation and condition equations is much easier, as will be shown hereinafter.
4. An entire country, for example, China, can be covered with no more than fifty pentagons, each with sides about 500 miles in length. Simultaneous

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NOTE.—Published in September, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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adjustment of the whole work is possible without too much labor. The solid network of triangulation thus adjusted should be much stronger than the ordinary arc triangulation of the same order of accuracy.

In general, there are fewer extra equations for the shoran method than for the old method. The number of extra observation equations for a polygon with n sides and a central station, wherein the distances between all the stations except the two fixed ones are measured, is

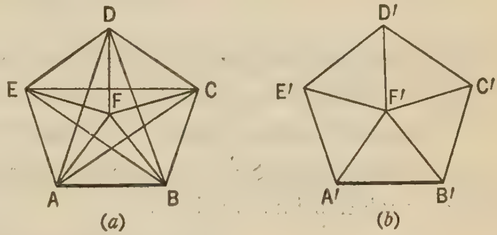
$$O_e = ({}_{n+1}C_2 - 1) - 2\,(n - 1) = {}_{n+1}C_2 - (2\,n - 1) \ldots\ldots\ldots (1a)$$

in which

$$ {}_{n+1}C_2 = \frac{\underline{|n+1}}{\underline{|(n+1)-2}\,\underline{|2}} = \frac{(n+1)\,n}{2} \ldots\ldots\ldots\ldots\ldots (1b)$$

For example, for the pentagon in Fig. 1(a) in which A and B are fixed stations, $O_e = {}_6C_2 - 9 = 15 - 9 = 6$.

Fig. 1(b) is a similar pentagon showing only the directions usually observed by the old method. In this case there are six condition equations, consisting of five angle equations and one side equation. In making this comparison it must be remembered that in Fig. 1(b) the directions, A′ D′, A′ C′, B′ E′, B′ D′, C′ A′, and C′ E′, are not observed. The comparison is not made, therefore, on the same basis, and is intended only to show how a net can be strengthened by measuring all distances between stations. Hereinafter is developed a method of adjusting a solid network of triangulation starting from two fixed stations.



(a) (b)

FIG. 1

DETERMINATION OF THE APPROXIMATE POSITIONS

In determining the approximate positions of the triangulation stations the earth is considered to be a sphere. Methods suitable for this determination include: (a) Solution of the spherical triangles; (b) solution of the auxiliary plane triangles; and (c) graphical solution of the spherical triangles by a map projection. The first two methods are too well known to require comment, but it is deemed desirable to explain briefly the graphical method (see "Appendix I").

Stereographic projection is particularly suited to the graphical solution because it has the useful property that all circles on a sphere, great or small, are represented by circles on the projection if the straight lines are considered to be circles of infinite radius. In connection with the methods of determining the preliminary positions, it has been suggested that the astronomic positions might be used instead, for which the zenith camera developed by the United States Army will serve very well indeed.

CORRECTIONS TO THE APPROXIMATE POSITIONS

In deriving the formulas for corrections to the approximate positions, the chord distance is used, and the mathematical surface taken to represent the earth is the ellipsoid of revolution. The chord distance, c_{12}, between two stations, S_1 (x_1, y_1, z_1) and S_2 (x_2, y_2, z_2), is given by

$$c_{12} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \tag{2}$$

Differentiating Eq. 2,

$$dc_{12} = X_{12}\,(dx_1 - dx_2) + Y_{12}\,(dy_1 - dy_2) + Z_{12}\,(dz_1 - dz_2) \tag{3a}$$

in which

$$X_{12} = \frac{x_1 - x_2}{c_{12}} \tag{3b}$$

$$Y_{12} = \frac{y_1 - y_2}{c_{12}} \tag{3c}$$

and

$$Z_{12} = \frac{z_1 - z_2}{c_{12}} \tag{3d}$$

If the increments Δx_1, Δy_1, Δz_1, Δx_2, Δy_2, Δz_2, and Δc_{12} are small enough, Eq. 3*a* may be written

$$\Delta c_{12} = X_{12}\,(\Delta x_1 - \Delta x_2) + Y_{12}\,(\Delta y_1 - \Delta y_2) + Z_{12}\,(\Delta z_1 - \Delta z_2) \tag{3e}$$

If either station is fixed, Δx, Δy, and Δz are, of course, equal to zero. From the geometric relationships involved,

$$x_1 = N_1 \cos\phi \cos\lambda_1 \tag{4a}$$

$$y_1 = N_1 \cos\phi_1 \sin\lambda_1 \tag{4b}$$

$$z_1 = N_1\,(1 - e^2) \sin\phi_1 \tag{4c}$$

$$x_2 = N_2 \cos\phi_2 \cos\lambda_2 \tag{4d}$$

$$y_2 = N_2 \cos\phi_2 \sin\lambda_2 \tag{4e}$$

and

$$z_2 = N_2\,(1 - e^2) \sin\phi_2 \tag{4f}$$

in which N_1 and N_2 are the normals to the meridians through the two stations and terminating at the axis of revolution; e is the eccentricity of the ellipsoid of revolution; and λ and ϕ are the longitude and geodetic latitude, respectively, of the station designated by the subscript. Hence,

$$\Delta x_1 = -N_1 \sin\phi_1 \cos\lambda_1\,\Delta\phi_1 - N_1 \cos\phi_1 \sin\lambda_1\,\Delta\lambda_1 \tag{5a}$$

$$\Delta y_1 = -N_1 \sin\phi_1 \sin\lambda_1\,\Delta\phi_1 + N_1 \cos\phi_1 \cos\lambda_1\,\Delta\lambda_1 \tag{5b}$$

$$\Delta z_1 = N_1\,(1 - e^2) \cos\phi_1\,\Delta\phi_1 \tag{5c}$$

$$\Delta x_2 = -N_2 \sin\phi_2 \cos\lambda_2\,\Delta\phi_2 - N_2 \cos\phi_2 \sin\lambda_2\,\Delta\lambda_2 \tag{5d}$$

$$\Delta y_2 = -N_2 \sin\phi_2 \sin\lambda_2\,\Delta\phi_2 + N_2 \cos\phi_2 \cos\lambda_2\,\Delta\lambda_2 \tag{5e}$$

and

$$\Delta z_2 = N_2 (1 - e^2) \cos \phi_2 \, \Delta\phi_2 \ldots\ldots (5f)$$

in which N_1 and N_2 may be regarded as the constants. Substitution of Eqs. 5 in Eq. 3*e* gives

$$\begin{aligned}\Delta c_{12} = {} & N_1 [- X_{12} \sin \phi_1 \cos \lambda_1 - Y_{12} \sin \phi_1 \sin \lambda_1 + Z_{12} (1 - e^2) \cos \phi_1] \Delta\phi_1 \\ & + N_2 [X_{12} \sin \phi_2 \cos \lambda_2 + Y_{12} \sin \phi_2 \sin \lambda_2 - Z_{12} (1 - e^2) \cos \phi_2] \Delta\phi_2 \\ & + N_1 (- X_{12} \cos \phi_1 \sin \lambda_1 + Y_{12} \cos \phi_1 \cos \lambda_1) \Delta\lambda_1 + N_2 (X_{12} \cos \phi_2 \\ & \sin \lambda_2 - Y_{12} \cos \phi_2 \cos \lambda_2) \Delta\lambda_2 \ldots\ldots (6)\end{aligned}$$

With the approximate station positions known, the coefficients in Eq. 6 can be computed, and Δc_{12}, the difference between the measured and computed chord distances, determined. The computed chord distance is

$$c_c = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \ldots\ldots (7)$$

There are as many relationships of the type of Eq. 6 as there are measured chord distances. Solution of these observation equations, together with any condition equations, by the method of least squares gives the corrections to the approximate positions.

A slightly different approach to this problem first obtains the corrections to the approximate Cartesian coordinates and then converts them to the corresponding corrections for the approximate geographical coordinates. A variation of this method determines the geographical coordinates directly from the corrected Cartesian coordinates. In addition to the equations, similar to Eq. 3*e*, there should also be included the condition equations like

$$x (1 - e^2) \Delta x + y (1 - e^2) \Delta y + z \Delta z = 0 \ldots\ldots (8)$$

in order to have the adjusted points also on the ellipsoid of revolution.

A question naturally arises as to how accurate the preliminary positions should be. The error introduced by use of expressions like Eq. 3*e* should be smaller than Δm, the error in the measured distance, in miles. On this basis the longitude should be accurate to within $\frac{1}{\cos \phi \sin (1 \text{ sec})} \sqrt{\frac{\Delta m}{2 c_a}}$ sec and the latitude to within $\frac{1}{\sin (1 \text{ sec})} \sqrt{\frac{\Delta m}{2 c_a}}$ sec. In these criteria, ϕ is the approximate latitude of the station in question, and c_a is the approximate chord distance between the two stations. Derivations of these criteria are given in "Appendix II."

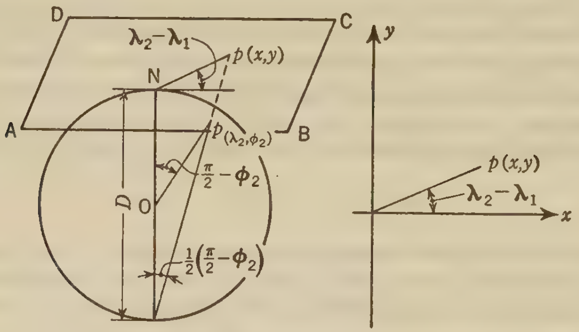
CONCLUSION

On an ellipsoid of revolution the chord between two points is unique, but the arc is not. The complicated expression for the change in length of a particular arc due to small variations of the end points is too difficult to solve readily without making approximations. The chord triangle is, therefore, superior to the spheroidal triangle as far as the simplicity of the formulas is concerned.

Judgment must be exercised as to the actual steps to be followed when dealing with a particular problem. It has been suggested that in some regions the vertical deflections of the plumb lines might be so great as to make the astronomic positions unsuitable for the prupose of this method. In this case, or under other circumstances when the preliminary positions, computed or otherwise, are not accurate enough, the individual polygons must be adjusted before adjusting the whole network. Should it be proved necessary to introduce the azimuth equations to strengthen the net, they may be treated as the condition equations.

APPENDIX I. APPROXIMATE POSITIONS BY POLAR STEREOGRAPHIC PROJECTION

Because of the ease with which it can be constructed and also because the projected circle representing all points equidistant from a station on a sphere is symmetrical with respect to the projected meridian through that station, the polar stereographic projection is preferred to any other projection. In



(a) (b)

FIG. 2

Fig. 2(*a*) ABCD represents the projection plane tangent to the sphere at the north pole, N, and the circle is the meridian of longitude λ_1, whose projection is taken as the x-axis of the coordinate system shown in Fig. 2(*b*). With the origin at the projection of the north pole the coordinates of the projection of any point P (λ_2, ϕ_2) are

$$x = D \tan \frac{1}{2}\left(\frac{\pi}{2} - \phi_2\right) \cos (\lambda_2 - \lambda_1) \quad \text{(9a)}$$

and

$$y = D \tan \frac{1}{2}\left(\frac{\pi}{2} - \phi_2\right) \sin (\lambda_2 - \lambda_1) \quad \text{(9b)}$$

in which D represents the diameter of the earth and the sign of the longitude is plus or minus depending if it is east or west.

If σ_1 and σ_2 denote, respectively, the angles subtended at the center of the earth by the arcs from an unknown station, S_x (λ_x, ϕ_x), to two known stations, S_1 (λ_1, ϕ_1) and S_2 (λ_2, ϕ_2), the plane coordinates of two points on the meridian of longitude λ_2 and at angular distance σ_2 from S_1 are

$$x_1 = D \tan \frac{1}{2}\left[\frac{\pi}{2} - (\phi_2 + \sigma_2)\right] \cos (\lambda_2 - \lambda_1) \qquad \text{(10}a\text{)}$$

$$y_1 = D \tan \frac{1}{2}\left[\frac{\phi}{2} - (\phi_2 + \sigma_2)\right] \sin (\lambda_2 - \lambda_1) \qquad \text{(10}b\text{)}$$

$$x_2 = D \tan \frac{1}{2}\left[\frac{\pi}{2} - (\phi_2 - \sigma_2)\right] \cos (\lambda_2 - \lambda_1) \qquad \text{(10}c\text{)}$$

and

$$y_2 = D \tan \frac{1}{2}\left[\frac{\phi}{2} - (\phi_2 - \sigma_2)\right] \sin (\lambda_2 - \lambda_1) \qquad \text{(10}d\text{)}$$

The projected locus of a point at angular distance σ_2 from station S_2 is the circle which passes through the coordinates given by Eqs. 10 and has its center at the midpoint of the line joining them. There is a corresponding circle for S_1. One of the intersections of the two circles gives the position of the unknown station. If a projection showing the meridians and the parallels of latitude has been constructed, the position will be given directly in geographical coordinates (λ_x, ϕ_x). If the projection does not include meridians and parallels, the geographical coordinates can be computed from the scaled plane coordinates from the expressions,

$$\tan^2 \frac{1}{2}\left(\frac{\pi}{2} - \phi_x\right) = \frac{x^2 + y^2}{D^2} \qquad \text{(11}a\text{)}$$

and

$$\sin^2 (\lambda_x - \lambda_1) = \frac{y^2}{x^2 + y^2} \qquad \text{(11}b\text{)}$$

APPENDIX II. DERIVATION OF THE CRITERIA OF ERROR

To obtain an expression for the residual discrepancy between the recomputed and the measured chord distances the errors introduced in translating Eq. 3*e* into Eq. 6 by use of Eqs. 4 and 5 are neglected. Since these errors are small as compared with Δx, Δy, and Δz, this omission is justified.

If Δx_1, Δy_1, and Δz_1 denote the required increments of the coordinates of one point so that the computed chord distance between the new point and the fixed one equals the measured chord distance c_1, then,

$$(x_1 + \Delta x_1 - a)^2 + (y_1 + \Delta y_1 - b)^2 + (z_1 + \Delta z_1 - c)^2 = c^2_1 \qquad \text{(12)}$$

in which (x_1, y_1, z_1) are the coordinates of the unknown point, and (a, b, c) are those of the fixed point. Substituting

$$c^2_a = (x_1 - a)^2 + (y_1 - b)^2 + (z_1 - c)^2 \qquad \text{(13}a\text{)}$$

and

$$\Delta c_1 = c_1 - c_a \qquad (13b)$$

in Eq. 12 gives

$$2(x_1 - a)\,\Delta x_1 + 2(y_1 - b)\,\Delta y_1 + 2(z - c)\,\Delta z_1 = c_a^2 + 2c_a\,\Delta c_1 + (\Delta c_1)^2 - [(x_1 - a)^2 + (y_1 - b)^2 + (z_1 - c)^2] - [(\Delta x_1)^2 + (\Delta y_1)^2 + (\Delta z_1)^2] \qquad (14a)$$

or

$$\frac{x_1 - a}{c_a}\Delta x_1 + \frac{y_1 - b}{c_a}\Delta y_1 + \frac{z_1 - c}{c_a}\Delta z_1 = \Delta c_1 + \frac{(\Delta c_1)^2 - [(\Delta x_1)^2 + (\Delta y_1)^2 + (\Delta z_1)^2]}{2\,c_a} \qquad (14b)$$

Since the last term on the right side of Eq. 14*b* has been omitted in writing Eq. 3*e*, the recomputed distance would fail to equal c_1 by an amount equal to $\dfrac{(\Delta c_1)^2 - [(\Delta x_1)^2 + (\Delta y_1)^2 + (\Delta z_1)^2]}{2\,c_a}$.

In the following discussion the earth is considered to be a sphere. It is also assumed that chords AB and A′B′ (Fig. 3(*a*)) lie in the same plane, Fig. 3(*a*) shows a cross section of the earth.

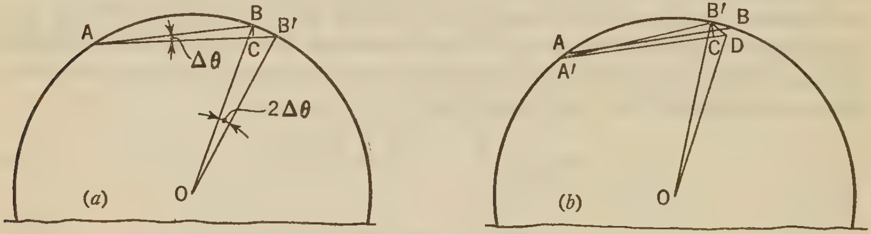


FIG. 3

It is evident that

$$[(\Delta x_1)^2 + (\Delta y_1)^2 + (\Delta z_1)^2] - (\Delta c_1)^2 \doteq (\overline{BB'})^2 - (\overline{CB'})^2 = (\overline{BC})^2 \qquad (15)$$

However,

$$4{,}000 \times 2\,\Delta\theta = \overline{BB'} \doteq \overline{CB'} \doteq \Delta c_1 \qquad (16)$$

and

$$\overline{AB}\,\Delta\theta \doteq \overline{BC} \qquad (17)$$

thus,

$$\overline{BC} = \overline{AB}\,\frac{\Delta c_1}{8{,}000} = \Delta c_1\,\frac{c_a}{8{,}000} \qquad (18)$$

It is required that the error introduced be smaller than Δm, the error in the measured distance in miles. Therefore,

$$\frac{\left(\Delta c_1\,\dfrac{c_a}{8{,}000}\right)^2}{2\,c_a} < \Delta m \qquad (19a)$$

or

$$\Delta c_1 < 8{,}000 \sqrt{\frac{2\,\Delta m}{c_a}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

If both points are allowed to vary as shown in Fig. 3(*b*), in which $\overline{\text{AB}}$ and $\overline{\text{A}'\text{D}}$ are drawn to be equal and parallel, the error is

$$\frac{(\Delta c_1)^2 - [(\Delta x_1 - \Delta x_2)^2 + (\Delta y_1 - \Delta y_2)^2 + (\Delta z_1 - \Delta z_2)^2]}{2\,c_a}$$

$$\doteq \frac{(\overline{\text{CD}})^2 - (\overline{\text{B}'\text{D}})^2}{2\,c_a} \doteq -\frac{(\overline{\text{B}'\text{C}})^2}{2\,c_a} \doteq -\frac{\left(\Delta c_1 \dfrac{c_a}{8{,}000}\right)^2}{2\,c_a} \ldots\ldots\ldots (20)$$

Hence, the limit of Δc_1 is the same; that is, Eq. 19*b* still governs. It is safe to allow a limit $\frac{\Delta c_1}{2}$ at each point as the error falls within the set limit, provided that neither the difference along the meridian, in miles, between the approximate position and the final position, nor that along the parallel of latitude, is greater than $2{,}000 \sqrt{\frac{2\,\Delta m}{c_a}} \left(= \frac{1}{2} \times \frac{1}{2}\, 8{,}000 \sqrt{\frac{2\,\Delta m}{c_a}} \right)$. The limits of the difference in longitude and in latitude are given by

$$\Delta\lambda \text{ (in seconds)} < \frac{1}{\cos\phi \sin (1 \text{ sec})} \sqrt{\frac{\Delta m}{2\,c_a}} \ldots\ldots\ldots\ldots (21a)$$

and

$$\Delta\phi \text{ (in seconds)} < \frac{1}{\sin (1 \text{ sec})} \sqrt{\frac{\Delta m}{2\,c_a}} \ldots\ldots\ldots\ldots (21b)$$

DISCUSSION

Robert C. Sheldon,[2] **Assoc. M. ASCE.**—A method of adjustment that is to be used with "shoran" measurements is presented in this paper. These measurements differ from conventional triangulation measurements by giving field data in lengths without direction in place of the usual directions without lengths. The lengths of the courses in any given figure do not provide as complete control of the figure, for adjustment purposes, as the several angles. The author proposes a system of pentagons with distances between all stations measured. This will tend to provide strength that is lost by not observing the angles of a more simple figure.

The proposed method consists of obtaining approximate positions for the several new stations in any manner. The author has suggested: (1) Computations using the field data, (2) use of the field data graphically on a stereographic projection, and (3) the possible use of astronomic positions. These approximate positions are then to be corrected by the method of least squares with the simultaneous solution of equations formed from computed and field data of the several lines.

In deriving the formulas by which these comparisons are made the author first shows that any change (in either length or direction) of any line in space is related to the changes in the space coordinates of its extremities by the relations shown in Eq. 3*e*. The direct relation between the space coordinates of the extremities of this line in space and the latitude and longitude of the same points on an ellipsoid of revolution is then shown by Eqs. 4. Eqs. 5 indicate that any changes in latitude and longitude on this ellipsoid are related to the changes in their corresponding space coordinates by the relations given. The substitution of Eqs. 5 in Eq. 3*e*, to form Eq. 6, thus gives the effect of any change in the line in space on the latitude and longitude of its extremities on an ellipsoid of revolution. The change in the line of space is the amount of correction necessary to bring the measured length of the chord in agreement with the computed length between the approximate stations.

For further simplification, Eq. 6 can be written

$$\Delta c_{12} = KI\,\Delta\phi_1 + KII\,\Delta\phi_2 + KIII\,\Delta\lambda_1 + KIV\,\Delta\lambda_2 \ldots\ldots\ldots (22)$$

in which Δc_{12}, KI, KII, KIII, and KIV are numerical values for any given line. Eq. 22 with the numerical values will be used in the least squares adjustment of the net. It is in the determination of the numerical values for the constant Δc_{12} and the coefficients KI, KII, KIII, and KIV that the proposed method becomes laborious.

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[2] Civ. Engr., Marietta, Ga.

The constant Δc_{12} would be determined first, as the length of the chord c_{12} used to find Δc_{12} is also used in determining the coefficients KI, KII, KIII, and KIV:

$$c_e = \sqrt{(x' - x^2)^2 + (y' - y^2)^2 + (z' - z^2)^2}$$

$$= \sqrt{(N_1 \cos \phi_1 \cos \lambda_1 - N_2 \cos \phi_2 \cos \lambda_2)^2}$$

$$\overline{+ (N_1 \cos \phi_1 \sin \lambda_1 - N_2 \cos \phi_2 \sin \lambda_2)^2}$$

$$\overline{+ (N_1 (1 - e^2) \sin \phi_1 - N_2 (1 - e^2) \sin \phi_2)^2} \quad \text{..(23)}$$

Tables could be prepared for the N-values in Eq. 23 and for the term $N(1 - e^2) \times \sin \phi$; but tables for other values would be prohibited by the fact that they contain both latitude and longitude. It is apparent that the solution for the numerical value for c_e would require eight separate logarithmic computations and four arithmetic summations. Having found a value for c_e it can be used for c_{12} in finding the coefficients for Eq. 22.

$$\mathrm{KI}_{12} = N_1 \left\{ \left[\left(\frac{N_1 \cos \phi_1 \cos \lambda_1 - N_2 \cos \phi_2 \cos \lambda_2}{c_{12}} \right) \sin \phi_1 \cos \lambda_1 \right] \right.$$

$$- \left[\left(\frac{N_1 \cos \phi_1 \sin \lambda_1 - N_2 \cos \phi_2 \sin \lambda_2}{c_{12}} \right) \sin \phi_1 \cos \lambda_1 \right]$$

$$\left. + \left[\left(\frac{N_1 (1 - e^2) \sin \phi_1 - N_2 (1 - e^2) \sin \phi_2}{c_{12}} \right) (1 - e^2) \cos \phi_1 \right] \right\} \quad \text{...(24)}$$

Since the numerical values for several terms in this equation have already been determined, the solution requires only four logarithmic summations and one arithmetic summation. Coefficients KII_{12}, KIII_{12}, and KIV_{12}, would require similar computations.

To explain further how fixed locations of points are obtained from the method, the simple case of one point E located from two fixed points A and B in Fig. 4 can be shown. Although Fig. 4 is a plane drawing it can be assumed to represent a part of the surface of an ellipsoid of revolution. The three points are held to this surface by the relations expressed in Eq. 6. By substituting in Eq. 22,

$$\Delta c_{BE} = \mathrm{KII}_{BE}\, \Delta \phi_E + \mathrm{KIV}_{BE}\, \Delta \lambda_E \quad \text{..............(25a)}$$

and

$$\Delta c_{AE} = \mathrm{KII}_{AE}\, \Delta \phi_E + \mathrm{KIV}_{AE}\, \Delta \lambda_E \quad \text{..............(25b)}$$

As A and B are fixed points, the values of $\Delta \phi_A$, $\Delta \lambda_A$, $\Delta \phi_B$, and $\Delta \lambda_B$ are all zero and the effect of the coefficients KI and KIII is lost. Chord corrections Δc_{BE}

and Δc_{AE} are constants that can be determined and numerical values can be computed for KII_{BE}, KIV_{BE}, KII_{AE}, and KIV_{AE}, thus leaving two equations from which the two unknowns can be obtained. In this simple case the two measured chord lengths govern the position of point E without resort to any adjustment. Fig. 4 shows only a part of the pentagon in Fig. 1(*a*), on which it is proposed to use the method. It is apparent then, that the effects of all lines coming into point E from the several points of the pentagon, when adjusted by a least squares solution, will give the strongest probable position of point E obtainable from the measured lines.

The corrections finally found by the solution of the several equations will be in linear values (as can be determined by inspection of Eq. 6), corresponding to the unit of measurement used. The computation of the small angular values necessary for any precise determination of the locations of the points from these linear values will be difficult, using existing tables.

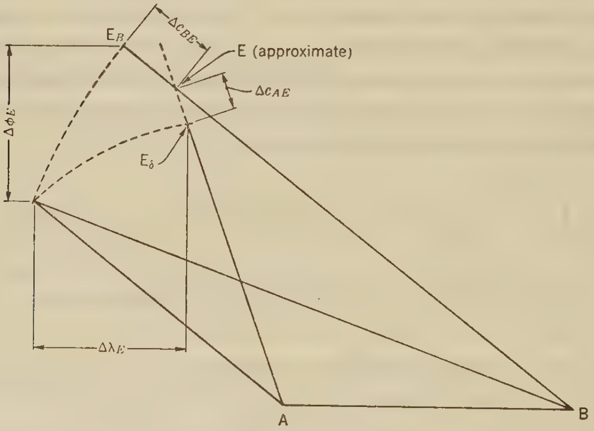

Fig. 4

Although the method is called, "adjustment of 'shoran' triangulation," and although it does require adjustment in its processes, it can be said that it is primarily a method for computation of geodetic positions. Adjustment and triangulation enter only as minor factors. For this reason comparison must be made with the conventional method of computation of geodetic positions rather than with methods of adjustment or triangulation.

The method of computation in use in the United States (presented with tables elsewhere[3]) is limited to lines of 100 miles or less. By the introduction of the fourth term of the expansion of the Taylor theorem, as developed by H. M. Doolittle,[4] this limit is increased to several hundred miles. With somewhat smaller figures than those proposed by the author, precise adjusted positions can be obtained by the use of this conventional method (and tables) by computing the several triangles in the figure by their measured sides, and following the usual methods of adjustments. For the accuracies obtained by "shoran" the conventional method of computation would be satisfactory on the sides of 500 miles in length.[5] In any comparison between old and new methods the advantages is with the old, in that the tables and processes have been standardized, and that the work is routine. With the new method the preparation of tables, the arranging of forms for computation, and the actual calculations, are large projects.

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[3] "Formulae and Tables for the Computation of Geodetic Positions," *Special Publication No. 8*, U. S. Coast and Geodetic Survey, U. S. Govt. Printing Office, Washington, D. C., 6th Ed., 1919.

[4] *Ibid.*, p. 96, footnote (*e*).

"Shoran" is limited to long lines by the fact that the results have an error of accuracy fixed between definite limits, not dependent on the length of the line, rather than errors that are proportional to the length, as in the usual triangulation methods.[5] As "shoran" is used on lines long enough to have proportional errors smaller than the conventional methods, it becomes a very useful tool for geodetic surveys requiring long lines. It would be especially useful in carrying existing datums across large bodies of water to place isolated islands and continents on a common coordinate system. The solution of any problem of this type would be a special project for which ways and means would be devised to fit the particular case. It is the writer's opinion that the direct method of computation,[6] would be the proper approach in these cases.

The determination of the coordinated positions of geodetic monuments is but a step in the ultimate aim of fully coordinated maps and surveys. In 1948 it is beginning to be realized that one of the principal reasons for the reluctance of engineers and surveyors, in the United States, to use geodetic survey control in their surveys is the wide spacing of the control points. The U. S. Coast and Geodetic Survey is placing additional points, in certain locations, to bring the spacing of the national net from 25 miles to 12.5 miles. The State of Pennsylvania has adopted a maximum spacing of 5 miles in the state control system. Other organizations are supplementing the national net for practical use. The writer therefore believes that for surveys on land a system of points as suggested by the author (these points being without direction control unless such control is obtained by astronomic observation) would be of limited usefulness.

For the purpose for which it is recommended the author has presented a unique solution. It can be stated that the author has made a great step toward answering the challenge to geodesy implied in the statement made by Flecher G. Watson in 1946.[5]

H. W. Hemple,[7] M. ASCE.—One of the significant scientific advances resulting from World War II was the development of electronic distance measuring methods for use in surveying and mapping. These methods were originally used for positioning airplanes engaged on bombing missions to assist in locating targets, and for precise navigational purposes. Of the various electronic distance locating methods developed, the "shoran" method gives the best promise of attaining the accuracy necessary for geodetic survey purposes.

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[5] "Radar Shows Promise in Mapping," *Civil Engineering*, July, 1946, p. 294.

[6] "Formulae and Tables for the Computation of Geodetic Positions," *Special Publication No. 8*, U. S. Coast and Geodetic Survey, U. S. Govt. Printing Office, Washington, D. C., 6th Ed., 1919, p. 93.

[7] Capt., Chf., Div. of Geodesy, U. S. Coast and Geodetic Survey, Washington, D. C.

When it is used for distance determinations in geodetic surveying, the shoran equipment is mounted in an airplane. This airplane flies approximately normal to a line connecting two ground stations between which the distance is desired. At each of these stations, radio receiving and transmitting instruments are established. Radio impulses are transmitted from the airplane to the ground stations, amplified, and retransmitted back to the receiving device on the plane. There are two dials on the receiving equipment in the plane—each of which indicates the distance in miles from the plane to a corresponding ground station. The altitude of the plane is also determined, usually with an electronic altimeter. When the plane is on the line between the two ground stations, the sum of the distances from the plane to the ground stations will be a minimum. These distances are then reduced to the ground distance between the two stations.

The basic physical requirements for shoran distance determinations are a knowledge of the velocity of radio waves, and a method of measuring the very minute time intervals involved in the travel of the radio waves from the plane to the ground stations and back. Electronic clocks which are used determine the time in microseconds. The clock dials on the shoran equipment are graduated to read in statute miles for convenience.

To date shoran work in triangulation has been accomplished with the instrumental equipment originally used for bombing missions with modifications made to effect improvements from time to time. No shoran equipment specifically designed for geodetic purposes has yet been manufactured. It is hoped that eventually electronic engineers can build an instrument which will give a timing accuracy of 10 ft; the fundamental principle of shoran is such that development to this high precision may be anticipated. If this accuracy can be obtained, then the resulting instrument will be capable of work comparable to first order triangulation.

In the computation and adjustment of shoran triangulation new procedures are required. Since the distances involved may be of the order of several hundred miles, the commonly used geodetic formulas for position, distance, and azimuth no longer apply. In addition, a new method of adjustment is required because the observed data comprise distances rather than horizontal directions.

Professor Chu presents one method for accomplishing the necessary computations and adjustment. His method differs from one developed by the United States Coast and Geodetic Survey, primarily because he uses Cartesian coordinates in space, rather than the more conventional latitude and longitude coordinates.

The theory of long-line geodetic computation is exhaustively treated in works of F. W. Bessel, F. R. Helmert, A. R. Clarke, and other geodosists of the nineteenth century. Their rigorous formulas have been utilized by the Coast and Geodetic Survey for shoran computation, after adaptation to modern calculating methods. The resulting computation system has proved convenient and has the advantage of expressing the data in the same terms as those used in conventional triangulation. This consideration is important in cases where the old and new types of triangulation are intermixed.

The adjustment procedure is essentially the same for both systems, being based on the principle of the variation of coordinates. Observation equations are written, relating a small change in length of a line to corresponding small displacements of the stations joined by that line. Professor Chu expresses these displacements in terms of space coordinates whereas the Coast and Geodetic Survey method expresses them, with equal simplicity, in terms of latitude and longitude.

Since the shoran points must still lie on the reference ellipsoid after adjustment, the Chu method requires additional equations of the condition type. Although the general form of this type is illustrated by Eq. 8, there is no mention made as to how observation equations (see Eq. 6) and condition equations (see Eq. 8) are combined in a least-squares adjustment. A comprehensive treatment of this subject was presented by Joseph L. Stearn and Norman F. Braaten in 1948.[8]

The ultimate accuracy of the shoran method will depend on further experiments to determine the basic value for the velocity of the radio propagation wave and of radio path characteristics. Variations due to signal intensity are also troublesome and their control must be studied further. Additional research and development for electronic methods of distance determination are needed, and it is hoped ultimately to develop instruments and procedures which will produce results as accurate as those obtained in basic control networks by conventional optical methods.

CHUNG-JUI CHU.[9]—Several valuable ideas have been added to the subject of the paper by Mr. Sheldon. However, it should be pointed out that the solution of equations of the type of Eq. 6 will give the angular values of the corrections expressed in radians to the approximate positions, not the linear values as mentioned by Mr. Sheldon. It is a simple matter to express the corrections in seconds by multiplying the right-hand member of Eq. 6 by a constant. Furthermore, the writer does not agree with Mr. Sheldon in the statement that the method is suitable primarily for computation of geodetic positions. To use the direct method of computing the geodetic positions the computer must first determine the adjusted values of the measured distances. Such a method is similar to the conventional method, but is very laborious. On the contrary, by the writer's method the geodetic positions are obtained by adding the corrections to the approximate positions, which may be just astronomic positions.

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[8] "A Method of Simultaneous Solution of a System of Observation and Condition Equations," by Joseph L. Stearn and Norman F. Braaten, *Transactions*, Am. Geophysical Union, April, 1948, pp. 157–162.

[9] Prof. of Surveying, National Tsing Hua Univ., Peiping, China.

Captain Hemple has given more valuable information on this new technique of measuring the distances and has stated that in the near future the "shoran" method may be used for first order triangulation.

The shoran method should find wider application in the field of surveying. It may yet be applied to the location of the positions of the airplane in aerial surveying, the location of the traverse stations, and the finding of a suitable world-wide geodetic datum. When that time comes, geodetic engineers will have seen great changes in the field operations and the office computations of surveying.

In closing, the writer wishes to express his gratitude to Mr. Sheldon and Captain Hemple for their contributions to the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2374

LATERAL EARTH PRESSURES ON FLEXIBLE RETAINING WALLS

A SYMPOSIUM

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NOTE.—Published in January, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

INTRODUCTION

By William H. Smith,[1] M. ASCE

The Bureau of Yards and Docks, United States Navy Department, has always been deeply involved in the problems of heavy earthwork. Much of the Navy's shore establishment is located on existing harbors and waterways, where the engineer must accept foundation and subsoil conditions as he finds them. Almost any harbor improvement project involves filling out beyond the natural shore line, building a quay wall or bulkhead, and dredging outside it. In many cases the engineering problems of the Navy are rendered more difficult because of the heavy surcharges to be provided for. The weights of guns, barbettes, turrets, armor, and other components of capital ships, the generally heavy character of equipment, and the tendency to do more and more prefabrication and preassembly—all lead to heavier types of shore construction.

On the other hand, rising costs and shortages of materials lead to progressive development of designs to utilize modern theories, materials, and methods to the best possible advantage. The days of massive, granite-faced gravity walls, for example, seem to have passed. These developments impose greater and greater responsibilities on engineers to accomplish economical results without reducing the factor of safety inherent in their structures.

The best way to improve engineering design is to improve knowledge of the forces to be resisted and of the mechanics of the structures in resisting them, so that every pound of material is working at the fullest possible efficiency and yet is never overstressed. The test program undertaken in 1943 at Princeton University (Princeton, N. J.) under the sponsorship of the Bureau of Yards and Docks has this specific objective in view. By careful laboratory studies of lateral earth pressures, and, particularly, studies of methods to reduce the lateral fluid pressure of hydraulically placed backfills, engineers can insure against failures of bulkheads and can arrive at more economical designs that will be safe and reliable. By further study engineers can learn even more about the behavior of flexible bulkheads, and the effect of bulkhead displacement on soil pressures, so that by modifying designs or methods of construction or both, they can build better bulkheads at lesser cost.

Briefly, the broad topics studied under this program can be divided into four main categories. First, investigation was launched into the determination of lateral pressures exerted on flexible steel bulkheads by backfills of clay, a sand-clay mixture, and a clean sand. This phase of the problem was important in order to obtain the basic information as to the lateral pressures to be expected with backfills of these types. The second phase of the problem was to determine methods for reducing these pressures. Tests were conducted to determine the effectiveness of sand dikes, placed on a normal one-on-two slope behind the bulkhead, in reducing lateral pressures. The effectiveness of the sand dike and of other methods attempted will be covered in the first Symposium paper. As the third phase of the investigation, and as a corollary to the sand-dike experiment, tests were made to determine the effectiveness of a sand blanket of varying widths placed back of the bulkhead in reducing lateral pressures. A fourth investigation was launched into the development of a method of accelerated consolidation of clay backfills designed to reduce quickly the excess pore pressures in the backfill and, correspondingly, to reduce the lateral pressure on the bulkhead. Finally, as a result of these studies, data of basic importance have been obtained for the analysis of the phenomena related to the problems of lateral pressures of soils, and many avenues of approach for further research and study in this field have been revealed. The present status of the tests, the preliminary findings and conclusions, and the contemplated future course of action on the completion of the test program are fully discussed in the papers.

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[1] Rear Admiral, CEC, U. S. Navy, Washington, D. C.

Several aspects of the Symposium deserve emphasis:

(1) It is not a final statement of results. It is rather a progress report on what has been accomplished, a discussion of the tentative conclusions and interpretations of the test results, and an indication of the direction that further tests will take.

(2) Although there have been free and intensive discussions among Professor Tschebotarioff, his associates at Princeton, and the designing engineers of the Bureau of Yards and Docks, the Navy has not attempted to force any unanimity of conclusions or opinions. It is only by the application of basic principles of free speech—by the full and free presentation of divergent points of view—that a meeting of minds and a constructive, intelligent, and sound set of principles can be obtained.

(3) It is neither the intention nor the hope of the sponsors of this investigation to resolve, once and for all, the many open questions in the field of soil mechanics. In one who does not attempt to qualify as an expert specialist in this science, there frequently arises a feeling of confusion and futility when one tries to reconcile the deep and apparently fundamental disagreements between different authorities on the interpretation and application of the theories of soil mechanics.

The general practitioner in civil engineering must make allowances for the relative youth of this branch of engineering as a science. He must recognize the complexity and variability of the materials with which these experts deal, and must have patience with them in their efforts to bring law and order into an involved and intractable situation.

Medical doctors have been struggling for years to solve the mystery of the common cold. Perhaps, in comparison, the engineering profession should applaud soils engineers for the great progress they have made in clarifying and delineating the behavior of mother earth, rather than look askance at their achievements because they are occasionally clouded by controversy. These active differences must be recognized rather, as proof in themselves of the virility, the vigor, and the dynamic character of this young science.

NOTATION

The relatively few letter symbols in this Symposium conform essentially to "Soil Mechanics Nomenclature," ASCE *Manual of Engineering Practice No. 22.* Discussers are requested to adapt their symbols to this list:

A = reaction at level A;
A' = the slope of the semilogarithmic plot of a void-ratio curve (Eq. 30); $A = A'/2.306$;
B = reaction at level B; also B = a constant in Eq. 30;
C = reaction at level C;
C' = a constant defined by Eq. 9;
c = cohesion per unit area:
D = diameter; average diameter of a pile;
E = modulus of elasticity in bending and tension; E_G = modulus of elasticity in shear;
e = void ratio:
  e_{cr} = critical void ratio;
  e_s = void ratio at the surface, $h = 0$;
  e_t = void ratio corresponding to $p_s = 0.1$ ton per sq ft;
$\mathbf{e}$ = base of Naperian logarithms;
G = specific gravity of solids;
H = height of bulkhead:
  H'_c = critical depth, or the depth at which a vertical cut will stand unsupported;
  H_g = depth of ground water level below the ground surface;
h = vertical distance from the top of the bulkhead to any point along its vertical height:
  h_i = the depth where $(\gamma_a - \gamma_w)h_i$ is 0.1 ton per sq ft;
  h_o = distance h to any point along the vertical height of a bulkhead;
I = rectangular moment of inertia;
I_p = plasticity index;
K = hydrostatic pressure ratio; ratio of horizontal pressure to vertical pressure; a consolidated pressure coefficient:
  K_A = ratio for the reaction A at the anchor level A;
  K_a = ratio for active pressure;
  K_B = ratio for the reaction B at the anchor level B;
  K_h = ratio of pressures at depth h;
  K_p = ratio for passive pressure;
  K_R = ratio for the total pressure R;
  K_s = ratio for the buoyed weight of solids;
  K_{s+w} = ratio for the nonbuoyed combined weight of soil and water;
  K_ϵ = ratio for the bending strain, ϵ_o;
  K_1 = active pressure coefficient for a triangular sand dike in Fig. 40;
  K_2 = active pressure coefficient for a level sandbank;

L = span length; vertical distance between the A-level and the B-level of the model bulkhead; L_p = length of pile;
M = bending moment;
m = a factor defined by Eq. 29;
N = a ratio of K-values; N_o = the value of N for any height h_o;
n = tangent of the angle of wall friction;
P = pressure force exerted on a bulkhead (pounds per linear foot):
  P_a = total active force exerted on a bulkhead (pounds per linear foot);
  P_{af} = total active force exerted on a bulkhead, considering wall friction;
  P_{ax} = total active force exerted on a bulkhead for a variable angle, X, of rupture plane;
  P_{cr} = critical load on a pile;
  P_p = total passive force exerted on a bulkhead (pounds per linear foot);
  P_1 = total horizontal pressure due to weight W_1 (Figs. 43 and 45);
  P_2 = total horizontal pressure due to weight W_2 (Figs. 43 and 45);
  P_3 = total force from fluid clay (Figs. 43 and 45);
p = unit pressure (pounds per unit area):
  p_a = unit active pressure;
  p_c = capillary pressure;
  p_d = difference in unit pressures;
  p_h = unit lateral pressures;
  p_n = normal pressure;
  p_s = consolidation pressure;
  p_1 = the major principal stress; unit pressure for a triangular dike;
  p_2 = the minor principal stress; unit pressure for liquid clay;
  p_3 = unit pressure for a triangular dike;
  p_4 = unit pressure for liquid clay;
Q = ratio of friction angles n/μ;
q = unit compressive strength of unconfined specimens; q_e = unit strength of specimens consolidated under pressure corresponding to effective weight of overburden;
R = resultant force; total lateral pressure; horizontal reaction; subscripts indicate point of application;
S = ratio of pressures, P_{af}/P_a;
s = spacing distance; as a subscript, s denotes "solids"
t = a numerical factor less than 1.0 (berm width); ratio of top width to height of trapezoidal dike;
V = shear force, in pounds;
v = shearing resistance, shear force per unit area;
W = total value of a distributed weight w:
  W_1 = buoyed weight of sand dike above rupture plane (Figs. 43 and 45);
  W_2 = buoyed weight of fluid clay above rupture plane (Figs. 43 and 45);

w = a variable distributed load or weight; as a subscript, w denotes "water";

X = angle between the assumed rupture plane and the horizontal;

z = distance from the top of the bulkhead to the point of maximum bending moment on the bulkhead; z_o = distance from the top of the bulkhead to the point of theoretical zero pressure on the upper part of the bulkhead;

α = tangent of the angle X:

- α_a = tan X for active pressure;
- α_p = tan X for passive pressure;
- α_x = tan X;

β = tangent of the angle θ;

γ = weight per unit volume:

- γ_a = unit weight of soil in air;
- γ_e = effective unit weight of soil, as transmitted from grain to grain; equal to buoyed weight, when soil is submerged;
- γ_w = unit weight of water;
- γ_1 = buoyed weight per unit volume for sand;
- γ_2 = buoyed weight per unit volume for clay;

ϵ = bending unit strain; ϵ_o = maximum strain of an equivalent beam freely supported at levels A and B;

μ = tangent of the angle ϕ;

ϕ = angle of internal friction; ϕ_q = slope of a line relating unconfined compressive strength to consolidation pressure; and

θ = surcharge angle (positive above level bank).

General Acknowledgment

The Bureau of Yards and Docks pays special tribute to the efforts that have been made, by Professor Tschebotarioff and his associates, in the preparation of this Symposium. Officers and designing engineers in the Department of Planning and Design, Bureau of Yards and Docks, also deserve hearty commendation. First, the duties imposed by World War II and subsequently the reduction of funds and personnel have added burdens on all of them. During the transition from a wartime to a peacetime footing it has not been possible to assign any Navy personnel to this task full time and all have had to do their share while handling many other assigned tasks.

LARGE-SCALE MODEL EARTH PRESSURE TESTS ON FLEXIBLE BULKHEADS

BY GREGORY P. TSCHEBOTARIOFF,[2] M. ASCE

SYNOPSIS

Large-scale model tests of the earth pressure on flexible bulkheads are described in this paper. The main design features of the testing installation at Princeton University (Princeton, N. J.) are outlined. A description is given of tests performed with submerged sand backfills, with clay and with sand-clay mixture backfills placed in a fluid condition, and with sand dikes and with vertical sand blankets of varying thickness backfilled with fluid clay.

Measurements are reported of the pressures exerted by the backfilled model flexible bulkhead, against its supports, of the bending strains and of the deflections of the bulkhead itself, and of pore pressures in the backfill during and after its consolidation. Results of laboratory soil classification tests and of shearing strength determinations are given. Critical comparisons are made between the model and the laboratory test results and the results of various known methods of analysis of the problems studied.

Findings are stated which should be considered when designing anchored flexible bulkheads. A method of approach to the problem of determining lateral earth pressures for bulkhead design is presented. Points requiring further clarification are outlined, as well as the tests that are in progress.

INTRODUCTION

In presenting this interpretation of findings at Princeton, the writer will need to make comparisons with previously accepted or published views. In the course of such comparisons he will refer to some relevant published statements by Karl Terzaghi, M. ASCE, not all of which have been fully substantiated by the tests described. No criticism of Professor Terzaghi is implied by such comparisons; these references serve as the most authoritative and most progressive available standard by which a comparison can be made of advances in the field of soil mechanics. The tests are concerned with problems about which little or no factual data were available. For that reason most previous recommendations concerning these particular problems necessarily were in the nature of guesses. The writer was impressed when he discovered how often Professor Terzaghi was right in extrapolating his recommendations beyond the realm of available data. The cases in which his judgment of incomplete or inadequate data was not vindicated only emphasize the importance of large-scale experimental research in the field of applied soil mechanics, a point often stressed by Professor Terzaghi himself.[3,4]

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[2] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[3] "Anchored Bulkheads," by K. Terzaghi, *Proceedings*, Purdue Conference on Soil Mechanics and Its Applications, Lafayette, Ind., 1940, p. 270.

[4] "General Wedge Theory of Earth Pressure," by Karl Terzaghi, *Transactions*, ASCE, Vol. 106, 1941, p. 79.

Findings reported in this paper, as compared to previously held views, have shown the dangers of taking too much for granted where soils are concerned. Every effort is made, therefore, to indicate the possible limitations of established facts and to differentiate between these facts and the writer's personal interpretation of the significance of recorded data. Whenever an interpretation different from that of the writer has been advanced by one of the other Symposium authors, this fact will be mentioned.

Problems to which no answer has been obtained are stated and the measures taken for their solution are outlined. In this respect, the paper has the character of a progress report, since testing is continuing.

PART I. THE PROGRAM OF FIRST TEST SERIES

A. General Objectives of First Series of Eleven Tests

(1) The original objective set for the Princeton project in February, 1943, was to determine, by large-scale model tests, the quantity of sand that would have to be interposed between the sheet piling of an anchored flexible bulkhead and a hydraulic backfill with high clay content, in order to decrease, effectively, the fluid lateral pressures of the unconsolidated backfill against the sheet piling. Three tests of this type were performed in the succeeding four years.

The study of objective (1) involved combined operations with two limit types of soil backfills—with clean sands and with fluid clays. The installation that would have to be designed and built for this purpose could well be used for the determination of some practically important aspects of lateral pressures exerted by either type of backfill alone. The following two further major objectives were added to the program:

(2) Study of lateral pressures exerted against flexible bulkheads by fluid clay backfills and by fluid sand-clay mixtures before, during, and after their consolidation—the study to include procedures for accelerating consolidation; and

(3) Verification of the alleged[5] considerable decrease of bending moments in sheet piling resulting from the vertical arching of sand backfills between the anchor level and the soil below the level of the dredged line.

By the end of 1946 four tests each had been performed in the study of objectives (2) and (3), completing the test series with the first type of testing installation.

B. Selected Method of Approach to the Problem

The three objectives defined in Section A cover a wide range. The Bureau of Yards and Docks was naturally anxious to obtain quickly results which could be applied directly in the field. At the same time it was not possible to obtain results without exploring some of the most complicated theoretical aspects of the effective shearing strength of soils and other similar, only partly explored, branches of soil mechanics.

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[5] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, pp. 217–219.

For the foregoing reasons no attempt was made to reproduce all possible combinations of relevant factors. Instead, tests were developed which would permit the estimation of the relative importance of factors likely to be relevant, and a study of the most important ones. To that end the program was frequently modified as the work progressed.

In the design of the installation emphasis was placed on tests with fluid clay backfills and special attention was paid to model similarity with steel sheet piling, in so far as the magnitude of bulkhead deflections was concerned.

C. DESIGN FEATURES OF TESTING INSTALLATION FOR FIRST SERIES OF ELEVEN TESTS

The testing installation (see Fig. 1), under Palmer Stadium, at Princeton University, is believed to be the most extensive of its kind in existence. Nevertheless, as completed in 1944, with all measuring equipment and other accessories, it cost 10% less than the largest comparable setup[6] which was built by Professor Terzaghi at the Massachusetts Institute of Technology at Cambridge, in the late 1920's, when building costs were much lower. The present investigation benefited from the experiences of Professor Terzaghi.

The reinforced concrete testing tank (see Fig. 1) is housed in a timber frame enclosure. Its capacity is 70 cu yd. A monorail is suspended from the roof of the stadium and extends from the interior of the enclosure over soil storage bins (see Fig. 2) and a truck ramp. An electrically operated 3-ton hoist with a ¾-yd clamshell bucket runs on the monorail and is used for filling the testing tank with sand backfills and for emptying it of all types of backfills. The testing tank was filled with fluid clay and sand-clay backfills (simulating hydraulically placed backfills) by "mudjacks."

Screened silty clay soil material was stored in bins from which it could be transported when needed along the monorail by the crane and the clamshell bucket to a specially constructed platform shown in Fig. 3. From the platform the dry screened soil is shoveled by hand into the mudjack, is mixed in it with water, and then is pumped into a half-yard dump bucket. The consistency of the pumped mixture is made quite liquid, corresponding to approximately two or three blows on the liquid limit device. The crane transports the filled dump buckets into the enclosure where they are emptied into the tank.

Essentially, the tank (see Fig. 4) consists of a massive buttressed reinforced concrete box which encloses (on three sides only) a space 9 ft deep, 13 ft wide, and 18 ft long. The fourth side is closed by the model flexible bulkhead which is made in three entirely separate vertical sections to minimize the effect of friction between the backfill and the side walls of the tank on the center bulkhead. All conclusions are based on measurements on the center bulkhead. All joints between the three bulkheads, the side walls of the tank, the movable bottom plate, and the concrete floor of the tank are sealed by very flexible rubber loops so that no load transfer can occur through any of these joints.

Each of the three bulkheads receives independent support at three elevations, referred to as the A-level, the B-level, and C-level. The *A*-reaction

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[6] "Record Earth-Pressure Testing Machine," by K. Terzaghi, *Engineering News-Record*, September 29, 1932, p. 369.

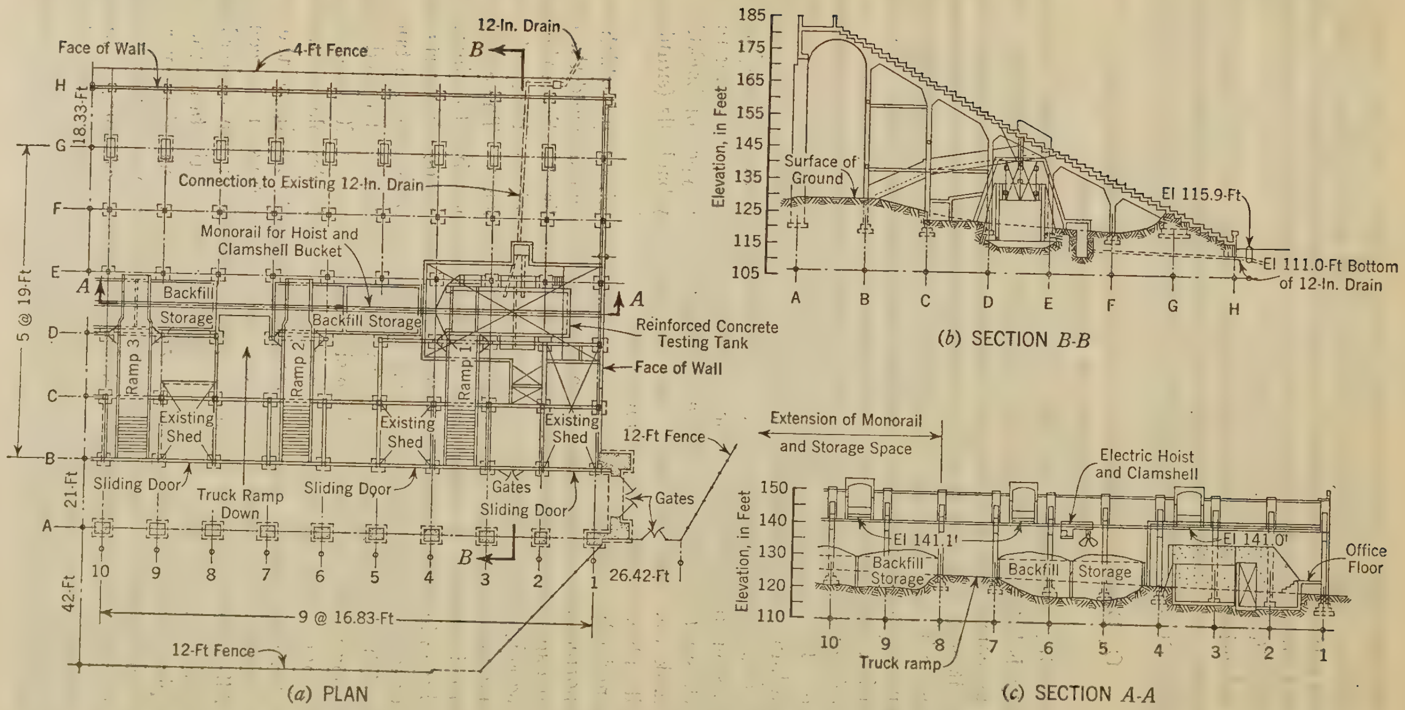

Face of Wall
4-Ft Fence
12-In. Drain
B
H
G
F
E
D
C
B
A
18.33-Ft
5 @ 19-Ft
21-Ft
42-Ft
Connection to Existing 12-In. Drain
Monorail for Hoist and Clamshell Bucket
Backfill Storage
Backfill Storage
Ramp 3
Ramp 2
Ramp 1
Existing Shed
Existing Shed
Existing Shed
Reinforced Concrete Testing Tank
Face of Wall
A
Sliding Door
Truck Ramp Down
Sliding Door
Gates
Sliding Door
Gates
12-Ft Fence
10 9 8 7 6 5 4 3 2 1
26.42-Ft
9 @ 16.83-Ft
12-Ft Fence
(a) PLAN
185 175 165 155 145 135 125 115 105
Elevation, in Feet
Surface of Ground
El 115.9-Ft
El 111.0-Ft Bottom of 12-In. Drain
A B C D E F G H
(b) SECTION B-B
Extension of Monorail and Storage Space
Electric Hoist and Clamshell
150 140 130 120 110
Elevation, in Feet
El 141.1'
El 141.0'
Backfill Storage
Backfill Storage
Office Floor
Truck ramp
10 9 8 7 6 5 4 3 2 1
(c) SECTION A-A

Fig. 1.—General Layout of Testing Installation

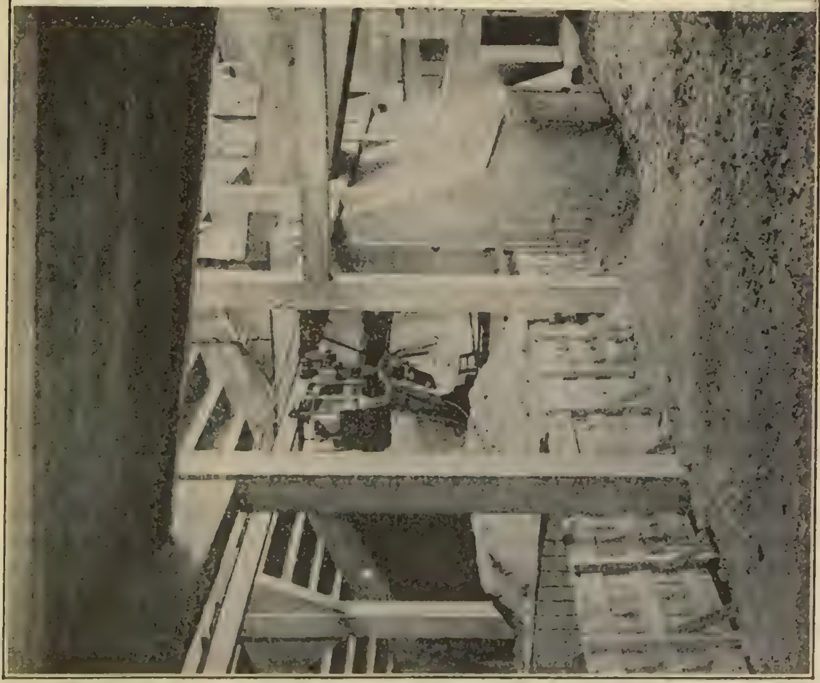


FIG. 2.—MONORAIL, TESTING TANK ENCLOSURE, AND STORAGE BINS

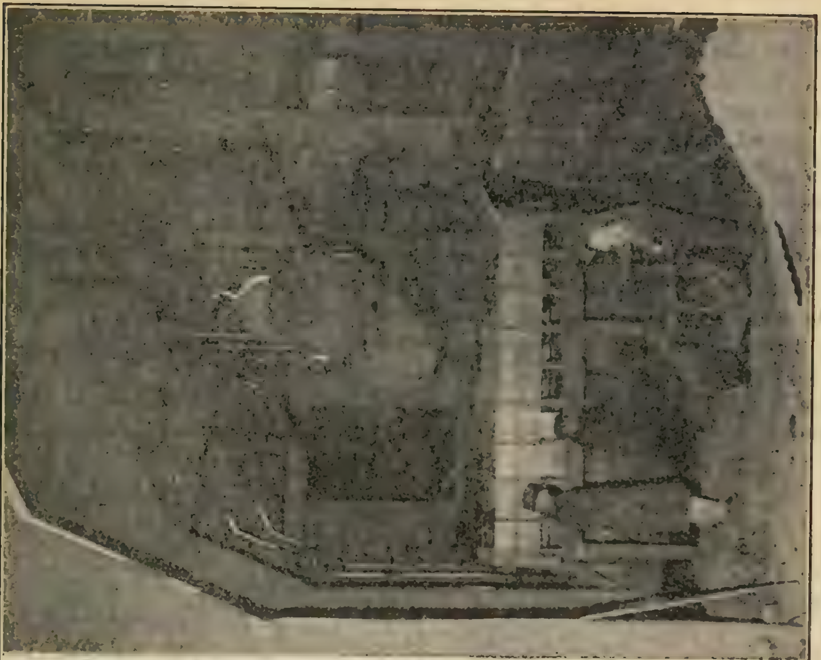


FIG. 3.—OPERATIONS AT THE MUDJACK LOADING PLATFORM

corresponds to the anchor pull and the *B*-reaction and the *C*-reaction are intended to simulate (of course, to a limited extent only) the conditions of support and of varying restraint of the embedded part of a flexible anchored sheet pile bulkhead. To that end, the model bulkheads have been so designed that they cannot undergo any lateral displacement at the C-level but are free to rotate around their vertical support at that level. The *A*-reaction and the *B*-reaction are so designed that controlled lateral displacement of their supports is possible. The bottom plate covering 4 ft of the floor of the tank can be made to yield vertically next to the B-support, when the latter is released—to simulate soil displacements in the field. Such displacements of a backfill in the field are induced when passive resistance of the soil at the toe of the bulkhead is developed after deformation. The free span between the A-level and the B-level is 6 ft, which corresponds to a model scale of approximately 1:5. At the points P (see Fig. 4), six pre-stressing rods are provided (two for each bulkhead) for purposes of calibration and to induce additional deflections.

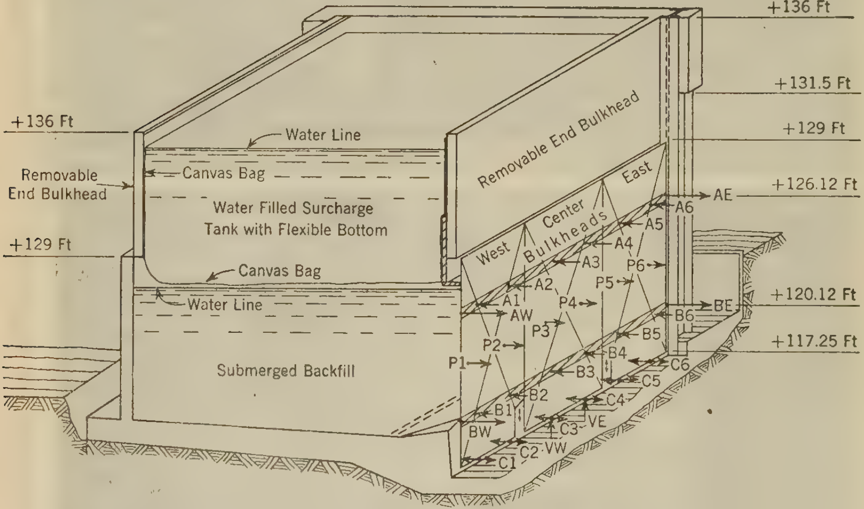
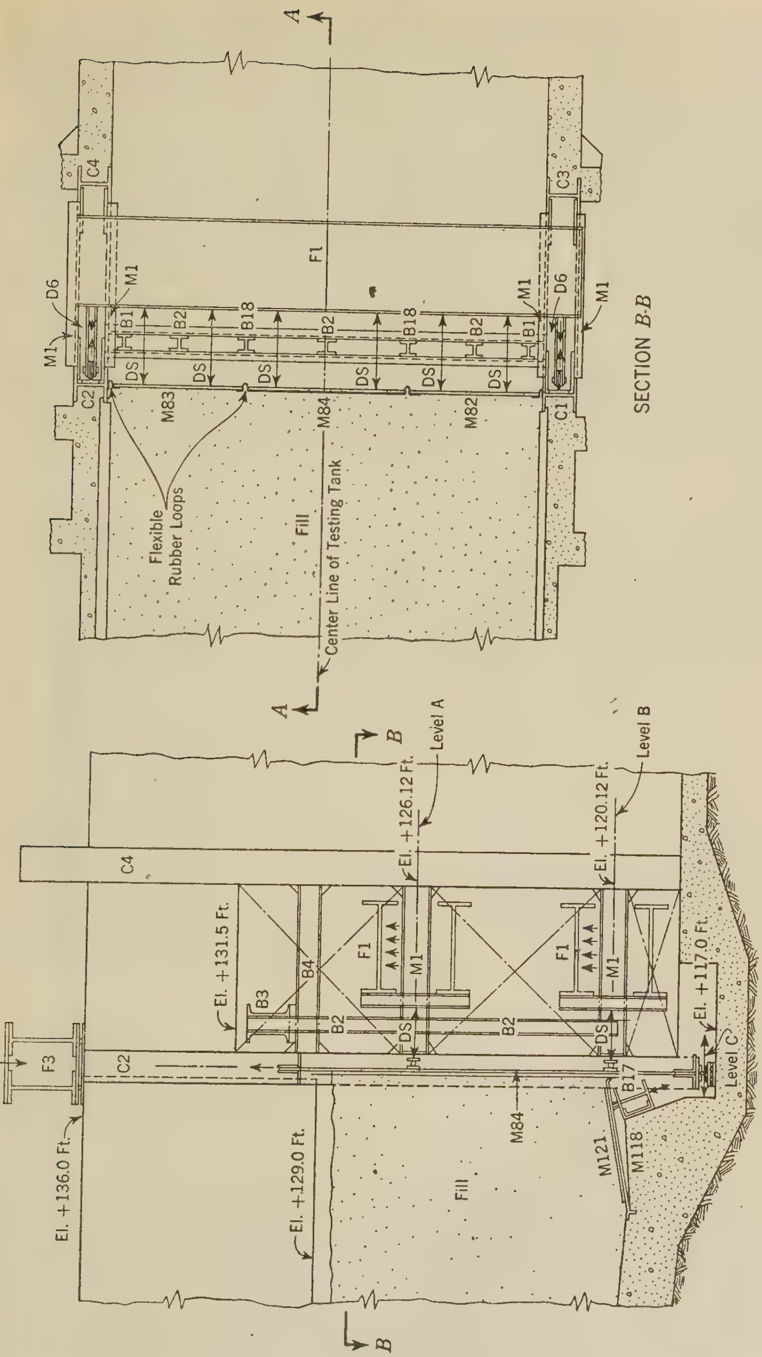

+136 Ft
+131.5 Ft
+129 Ft
+126.12 Ft
+120.12 Ft
+117.25 Ft
+136 Ft
Removable End Bulkhead
+129 Ft
Water Line
Canvas Bag
Water Filled Surcharge Tank with Flexible Bottom
Canvas Bag
Water Line
Submerged Backfill
Removable End Bulkhead
West
Center
Bulkheads
East
AE
A6
A5
A4
A3
A2
A1
AW
P6
P5
P4
P3
P2
P1
BE
B6
B5
B4
B3
B2
B1
BW
C6
C5
C4
C3
C2
C1
VE
VW

Fig. 4.—Section Through the North End of the Testing Assembly

The upper part of the testing tank consists of two parallel longitudinal reinforced concrete walls 7 ft high. The space between these walls can be closed at both ends by removable steel bulkheads easily transported along the monorail by the overhead electric crane. A canvas bag fits into the space thus formed, which can be filled with as much as 6 ft of water thus providing a uniformly distributed surcharge over the entire surface of the backfill.

A schematic drawing of the general assembly is presented in Fig. 5. The structural members C1, C2, C3, C4, and M1 form part of the finished testing tank. Places designated DS indicate the center lines of the six supports of the three bulkhead sections at two different elevations. These supports are combined with dynamometers and transmit the fill pressure to the two


A
C4
C3
F1
D6
M1
M1
B1
B2
B18
B2
B18
B2
B1
M1
D6
M1
DS
DS
DS
DS
DS
DS
C2
C1
M83
M84
M82
Flexible Rubber Loops
Fill
Center Line of Testing Tank
A
B
Level A
Level B
El. +126.12 Ft.
El. +120.12 Ft.
C4
El. +131.5 Ft.
B4
B3
F1
M1
F1
M1
B2
B2
DS
DS
El. +117.0 Ft.
Level C
F3
C2
B17
M84
M121
M118
El. +136.0 Ft.
El. +129.0 Ft.
Fill
B

SECTION B-B

SECTION A-A

FIG. 5.—GENERAL ASSEMBLY OF STEEL BULKHEAD AND THE SUPPORTING FRAME

main supporting frames marked F1. Designations M82, M83, and M84 indicate the three bulkhead sections; and B17, M118, and M121, the movable base that supports the fill. Designations B1, B2, B3, B4, and B18 constitute the supporting frame for measuring deflections, which was provided with supports independent of those designed to take care of the bulkhead reactions.

The vertical reactions were measured on the central bulkhead with two Carlson pressure cells embedded below the C-level in the concrete floor of the testing tank. Overhead beams were used to suspend the bulkheads only during their erection. Bulkhead deflections and displacements were measured all along the vertical center lines of each bulkhead at elevation intervals of $4\frac{1}{8}$ in. by mechanical dial extensometers reading to 0.001 in.

Electric resistivity strain gages (SR–4) were used in the construction of the specially designed dynamometers[7] which measured the reactions at the bulkhead supports. The same type of gage was used to measure the bending strains across the entire width of the center bulkhead and at every $4\frac{1}{8}$-in. interval of elevation.

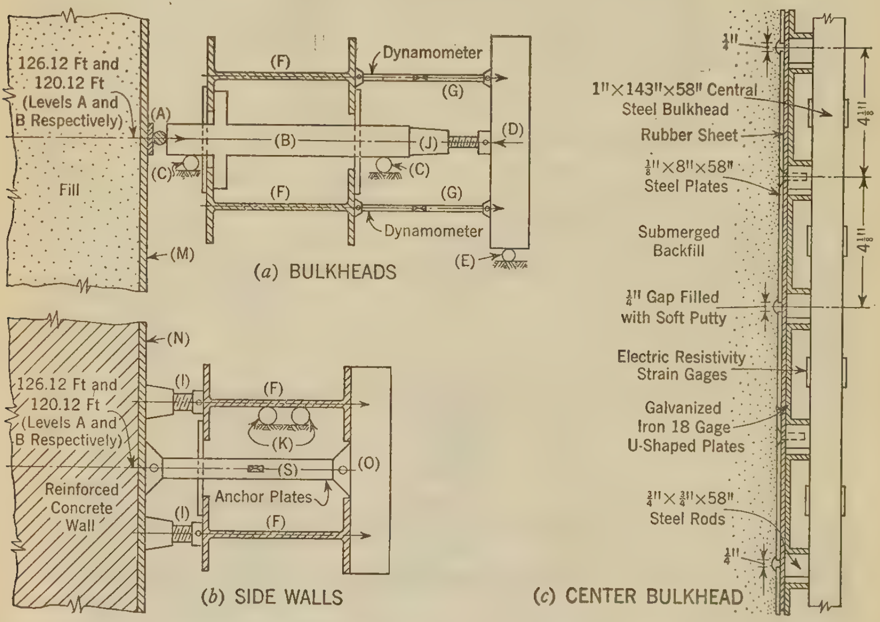

126.12 Ft and 120.12 Ft (Levels A and B Respectively)
(A)
(F)
Dynamometer
(G)
(D)
(B)
(J)
(C)
(C)
Fill
(F)
(G)
Dynamometer
(M)
(E)
(a) BULKHEADS
(N)
126.12 Ft and 120.12 Ft (Levels A and B Respectively)
(I)
(F)
(K)
(S)
(O)
Reinforced Concrete Wall
Anchor Plates
(I)
(F)
(b) SIDE WALLS
1/4"
1"×143"×58" Central Steel Bulkhead
Rubber Sheet
1/8"×8"×58" Steel Plates
4 1/8"
4 1/8"
Submerged Backfill
1/4" Gap Filled with Soft Putty
Electric Resistivity Strain Gages
Galvanized Iron 18 Gage U-Shaped Plates
3/4"×3/4"×58" Steel Rods
1/4"
(c) CENTER BULKHEAD

Fig. 6.—Vertical Sections Through Bulkheads and Side Walls, Showing Instrument Setup

Figs. 6(*a*) and 6(*b*) are diagrams indicating how apparatus was arranged at levels A and B, for measuring reactions, compensating for temperature variation, pre-stressing members, and controlling the yielding of supports. The main horizontal supporting steel girders (F) are anchored back to the web (N) of the steel column forming an integral part of the reinforced concrete tank. This web is located in the same plane with the steel bulkheads (M). As a result of this arrangement, the length of all compression members of the supports is equal to the length of all tension members. Since they are all supported on rollers, temperature deformations of these supports will permit them to move without producing any motion of the bulkhead itself. This phenomenon is important since otherwise active and passive earth pressure would be brought into play alternately. The jack (J) shown in Fig. 6(*a*) is used to release the support when so desired; the jacks (I) in Fig. 6(*b*) are used for pre-stressing and balancing purposes only. After the erection was completed, it was found that no frictional or other resistances to lateral motion could be measured in the entire setup.

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[7] "Use of Electric Resistivity Strain Gages Over Long Periods of Time," by Gregory P. Tschebotarioff *Proceedings*, Soc. for Experimental Stress Analysis, Vol. III, 1946, No. 2, p. 47.

The special arrangement illustrated in Fig. 6(*c*) permitted the fixation of strain gages on both faces of the central steel bulkhead including the face exposed to the submerged backfill. The result of this design was that the earth and water pressures were transmitted to the main steel bulkhead (1 in. thick) as concentrated line loads 4½ in. apart in a vertical direction. In so far as the bending moments were concerned, for all practical purposes this combination was equivalent to distributed loading, since the span between supports A and B was 72 in. The steel plates (⅛ in. thick by 8 in. wide) in direct contact with the backfill were screwed to the main bulkhead along their horizontal center line only and were free to slide at their ends on the Koroseal rubber sheet they covered. It was surmised that this arrangement would have practically no effect on the general rigidity of the bulkhead so that the moment of inertia of the entire system would not be noticeably different from that of a solid steel plate 1 in. thick. This assumption proved to be correct as shown by calibration measurements.

The interior of the tank, during repairs to the insulation of the gages on the center bulkhead, is shown in Fig. 7. Five pairs of strain gages were spaced vertically on 4½-in. centers, on the center bulkhead. The readings of these gages were averaged electrically in two and three pairs, respectively.

The control room is shown in Fig. 8. The south upper removable end bulkhead, the reference system for deflection measurements, and the A-level bulkhead supporting construction are shown in the background.

D. Soils Used for the Tests

The grain-size distribution curves of the sand, the clay, and the sand-clay mixture used as backfill, as determined by the standards of the American Society for Testing Materials (ASTM), are presented in Fig. 9. The liquid limit (w_L), of this silty clay was 30%; the plastic limit (w_P), 23%; and the plasticity index (I_P), 7%. Hans F. Winterkorn, Assoc. M. ASCE, made a further mineralogical and physicochemical study of the red-colored clay in his Soil Physics Laboratory at Princeton University. The results showed that the clay mineral was almost entirely kaolinite. Professor Winterkorn further stated that this New Jersey red clay soil was: "* * * well leached, contains few, if any, exchangeable bases, representing mainly an H-clay, and consequently is microbially quite sterile and chemically quite stable." The clay is a product of shale weathering, and water absorption data indicated a

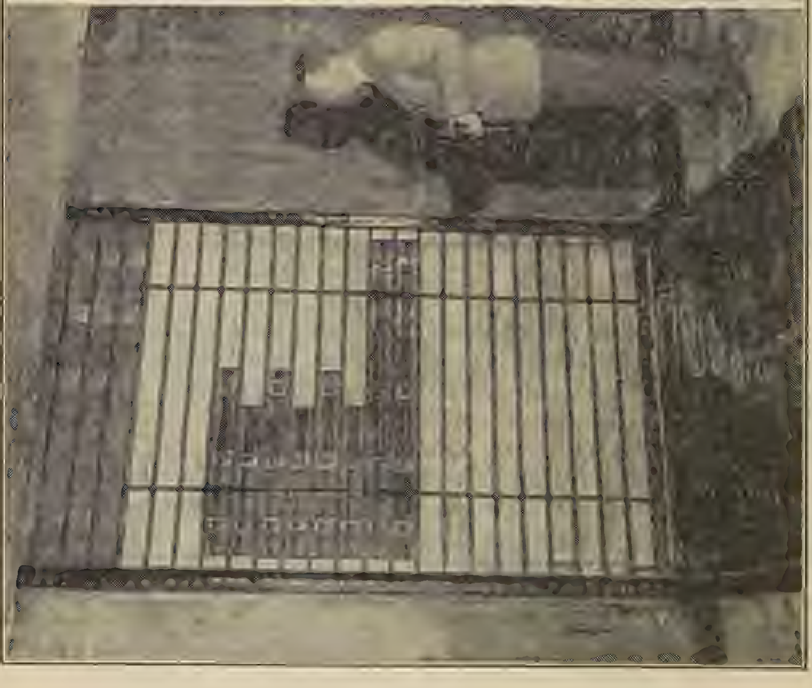

FIG. 7.—STRAIN GAGE INSTALLATION

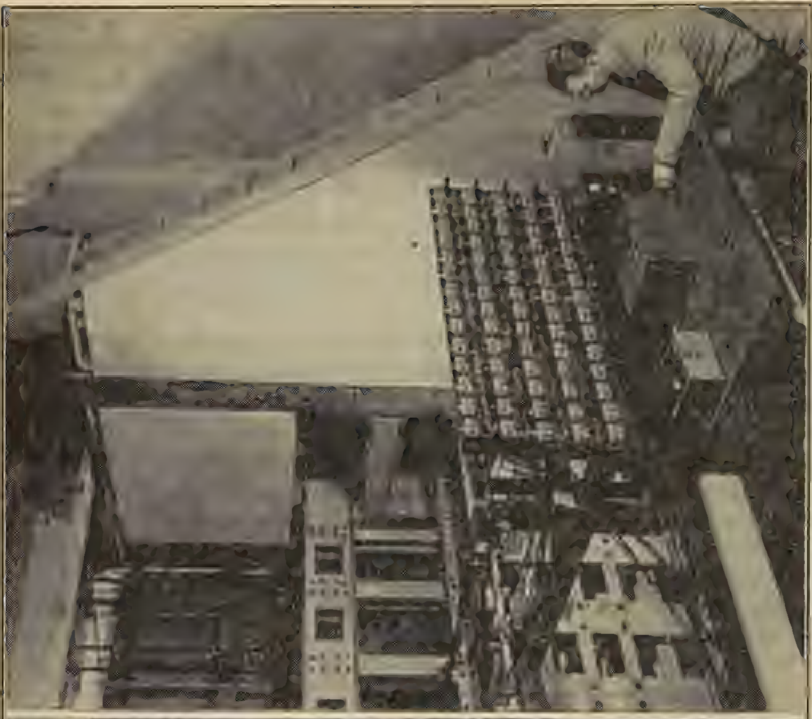

FIG. 8.—CONTROL ROOM

clay content of about 30%—that is, higher than that obtained by standard ASTM sedimentation tests.

Direct and triaxial shear and unconfined compressive strength tests were performed in the writer's Soil Mechanics Laboratory at Princeton University.

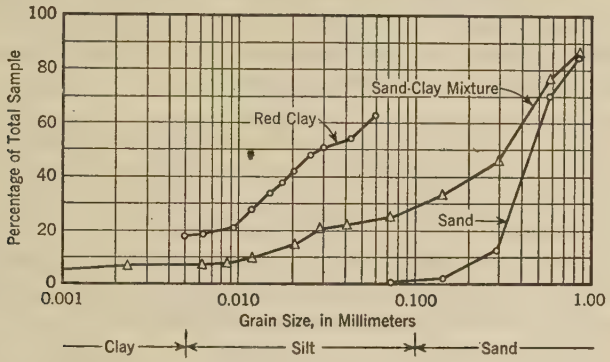

FIG. 9.—SOILS USED FOR BACKFILLING PURPOSES DURING THE MODEL TESTS

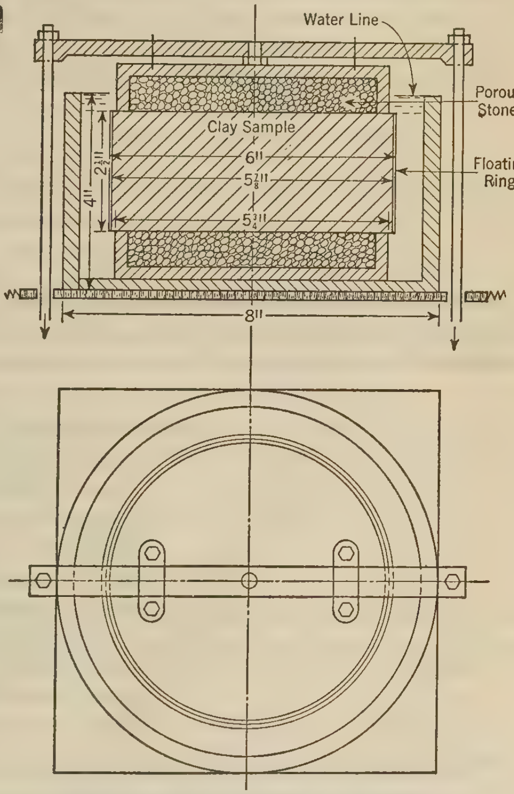

FIG. 10.—OVERSIZE FLOATING RING CONSOLIDOMETER

Both direct and triaxial shear tests indicated that the sand in a loose condition corresponding to the densities prevailing in the backfill during the tests (from 92 to 96 lb per cu ft dry weight) had an angle of internal friction varying from 32° to 36°. Consolidated rapid shear tests performed on the clay placed in the laboratory testing devices in an originally fluid condition gave an angle of internal friction of approximately 17°. The corresponding value for the sand-clay mixture was approximately 32°. Further shear tests were performed on undisturbed samples from the backfill.

In addition, it was attempted to duplicate in the laboratory the consolidation processes that occurred in the backfill of the testing tank. For this purpose two oversized floating ring-type "consolidometers," 6 in. in diameter, were designed by the writer (see Fig. 10). By use of this device the original fluid backfill material can be consolidated in the laboratory under any desired pressure. The consolidated product can then be cut into 1-in. by 1-in. by 2-in. specimens for the unconfined compressive strength tests. A few tests were also made with still larger floating ring consolidometers (12 in. in diameter) which permitted cutting from the consolidated clay samples 2 in. by 2 in. by 4 in. for unconfined compressive strength tests.

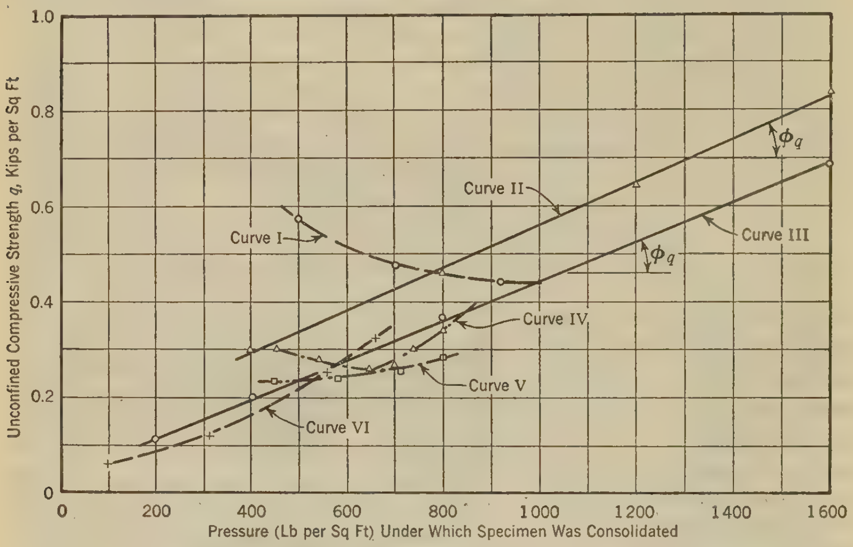

Fig. 11.—Tests for Compressive Strength of Clay and Sand-Clay Mixtures

The unconfined compressive strengths of both clay and sand-clay mixtures are plotted against the consolidation pressure, in Fig. 11. Curves I and II represent the mixtures, and other descriptions are:

Curve	Description
	Sand-Clay Mixture, Consolidated in—
I	Testing tank; sample under its own weight, plus a surcharge of 375 lb per sq ft
II	Oversized floating ring consolidometer (laboratory sample)
	Clay Only, Consolidated in—
III	Oversized floating ring consolidometer (laboratory sample)
IV and V	Testing tank; sample under its own weight, plus a surcharge of 375 lb per sq ft
VI	Testing tank; sample under its own weight

The results obtained from tests on laboratory specimens (curves II and III) give straight lines with identical slopes (the angle of which is referred to as ϕ_q). The undisturbed samples from the backfill, however, do not follow a similar pattern even though their unconfined compressive strengths were plotted in Fig. 11 against consolidation pressures smaller than the actual weight of the overburden, some allowance having been made for effects of friction against the walls of the tank. During the tests with the clay and with the sand-clay mixtures, and after their consolidation was completed under 6 ft of water surcharge, the unconfined compressive strength and the water contents of both types of backfill were found to have an almost constant value throughout the entire depth. In the case of the sand-clay mixture the unconfined compressive strength was even higher in the upper than in the lower layers. The condition observed in the clay backfill may have been caused during a stage of drainage by some evaporation through the 8-in.-thick sand mat overlying the clay. In the test with the sand-clay mixture a similar result may have been obtained because some segregation appears to have occurred during the placing of the fluid backfill so that the upper layers had a higher content of fines and, correspondingly, a somewhat higher unconfined compressive strength at the same water content.

Check tests of long duration (5 months) were performed in consolidometers 15 in. in diameter, to determine whether any of the observed phenomena could be attributed in part to the increased structural strength that results from "aging" of the clay. With this type of clay no effects were observed of the kind described by Professor Terzaghi in 1946[8] in reporting on laboratory developments of his earlier related studies of natural deposits.[9]

E. INVESTIGATION TECHNIQUES

Two independent calibrations of the testing apparatus were performed. The first calibration was made with the pre-stressing rods P (Fig. 4) which induced bending strains in the bulkhead both before and after the flexible rubber loops and the outer steel plates ($\frac{1}{8}$ in. thick by 8 in. wide) were mounted on the central bulkhead. The results of this calibration showed that the bending-strain readings were identical both before and after these plates were mounted, thus proving that the stiffness of the central bulkhead had not been noticeably increased by the special construction illustrated in Fig. 6(*c*).

The second type of calibration was performed by filling the testing tank with water. Fig. 12 shows the bending strains measured on the central bulkhead during the water calibration. The readings were sufficiently accurate to permit determination of the shears V by the formula:

$$V = \frac{dM}{dx} \qquad (1)$$

in which M denotes "bending moment." A smooth curve was passed through the points obtained and this in turn permitted the determination of the pressures p by use of the formula:

$$p = \frac{dV}{dx} \qquad (2)$$

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[8] "Pitfalls of Soil Mechanics," by Karl Terzaghi, *The News*, Philadelphia Section, ASCE, Philadelphia, Pa., May, 1946.

[9] "Undisturbed Clay Samples and Undisturbed Clays," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., July, 1941, p. 211.

A fairly reasonable agreement was obtained between the actual and the computed pressure distribution. This method of pressure computation, however, was possible only when strain readings were taken over relatively short periods of time not exceeding a few days, since otherwise too great a scattering of the bending-strain readings occurred to permit the double differentiation procedure. Furthermore, reasonably accurate values could be obtained only for the central part of the span between the A-support and the B-support. Within a foot or so from these supports, where the slope of the bending curve changes rapidly, this procedure was not reliable. Nevertheless, in some cases (for instance, for the determination of lateral pressure variation due to increments of surcharge loading), this procedure of lateral pressure computation from observed bending strains of the bulkhead disclosed certain important facts which otherwise could not have been ascertained (see subsequently in Part II, Section D).

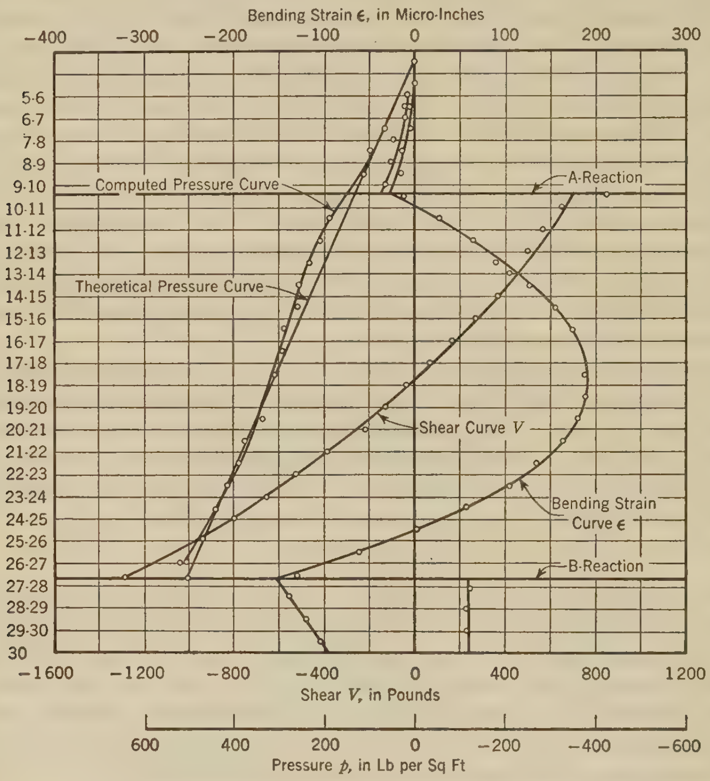

FIG. 12.—CALIBRATION CURVE (8 FT OF WATER)

The calibration studies permitted the determination of the accuracy of readings taken with various measuring units of the testing assembly. As a result of the calibration it can be stated that the measured reactions on the central bulkhead could be determined with an accuracy of ±10% or better, the A-level reactions having a slight tendency to read somewhat too high and the B-level reactions having a slight tendency to read too low.

The most reliable type of readings were the bending-strain and the deflection readings. This follows from the nature of this type of measurement, from the calibration tests, and from routine checks of the bending strains and of the deflections against each other. Such checks were performed for all important test stages. Deflections were computed from the measured bending strains by the elastic weight method. The computed deflections compare very favorably to the measured deflections, as a rule not deviating by more than approximately 5%. This is an additional indication of the reliability of the bending-strain readings recorded in this investigation.

The electric resistivity strain gages used in this study permitted measurements of a type which otherwise would have been impossible. Strains of a millionth of an inch per inch (for example, stresses of 30 lb per sq in. in steel) can be detected under favorable conditions. Nevertheless, these gages are extremely delicate and require special techniques of handling. The difficulties encountered and the methods developed to overcome them, have been described elsewhere.[7] As a final result, these gages were adapted to the purposes of the investigation quite satisfactorily. A calibration check was made after each test to permit discarding the readings of gages that had developed signs of instability. It was found advisable to install at least 50% more gages than needed, since approximately one third of them were likely to go out of order during prolonged tests. Tests longer than 6 months appear inadvisable.

The two pressure cells for the measurement of vertical reactions under the center bulkhead performed satisfactorily. However, at low pressures, their readings could be used only as an indication of trends—not of precise values—because the cells had a somewhat higher load capacity than actually proved necessary.

The Carlson pore pressure cells were calibrated and checked for continued stability of readings in a water filled standpipe, 14 ft high. Only cells that performed well were embedded in the backfill. Some trouble developed at first because the cells built up to pressure in the backfill, but did not subsequently show any decrease when the pressure was relieved. This condition was traced to the clogging of the too fine porous stones. After the stones were replaced by a coarser variety the pore pressure cells performed very well and provided valuable data.

Test results were compared mainly by the use of the following four values (see Fig. 13):

(1) The upper reaction A, per foot of width, at the anchor level A;
(2) The lower reaction B, per foot of width, at level B;
(3) The total pressure per foot of width of the bulkhead:

$$R = A + B - C \qquad (3)$$

(4) The maximum bending strain ϵ_o of an equivalent beam freely supported at levels A and B.

The significance of the value ϵ_o is illustrated by Fig. 13. The maximum positive bulkhead bending strains that occur approximately midway between the A-support and the B-support and the maximum negative bending strains at the B-support are very strongly affected by any displacement of these supports. For this reason neither one of these values, taken by itself, can be used for the estimation of pressure intensity and distribution along the span. An accurate control of the displacements of the supports is not practicable; nor is it possible to reproduce "field" conditions by such control since innumerable variations of the conditions of restraint of the lower embedded parts of bulkheads are possible in the field. For the foregoing reasons the writer decided to introduce the value ϵ_o for his analysis. It represents the maximum bending strain of an equivalent beam freely supported at levels A and B. Therefore the value of ϵ_o is not affected by any displacements of the supports and depends only on the magnitude of the total pressure acting against the span A–B of the bulkhead and on the distribution of pressure along that span.

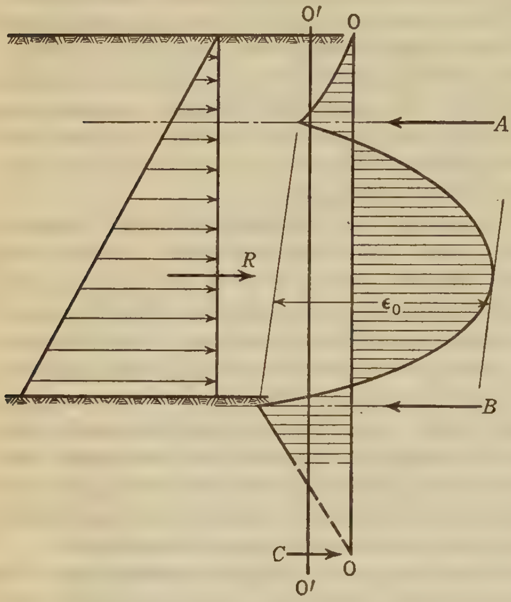

FIG. 13.—SIGNIFICANCE OF THE TERM ϵ_0

In order to compare the test results further the so-called hydrostatic pressure ratios K were computed for each of the preceding four values (A, B, R, and ϵ_o) for all stages of the tests. The corresponding K-values are referred to as the K_A-value, the K_B-value, the K_R-value, and the K_ϵ-value, respectively. As shown in Fig. 13 the K_A-value is equal to the ratio between the actually measured value of the reaction A and the value of A computed for a completely frictionless fluid of unit weight equal to that of the backfill. The K_B-value, the K_R-value, and the K_ϵ-value are computed in a similar manner. All the K-values thus obtained are compared to the theoretical K-values computed on the strength of different types of laboratory tests and of different formulas and assumptions concerning the shearing strength which is effective in reducing lateral pressures.

Possible arching effects were estimated by comparing the measured K_ϵ-values to the conventional K_ϵ-values. Additional information on this point was obtained by comparing the measured K_ϵ-values to the measured K_R-values since arching produces only a redistribution of pressure which therefore should affect the bending moments only and not the total pressure. The actual measured values of the negative bending strains at the A-support were also considered in this connection.

The K_A-coefficient and the K_B-coefficients were further used for comparison of lateral pressures exerted by the backfill in its upper (conventionally assumed as tensile) zone and in its lower (compressed) zone limited by a rigid boundary below it.

PART II. THE PRINCIPAL FINDINGS OF FIRST TEST SERIES

A. Structural Effect of Sand Dike

Tests were performed with a sand dike provided with a berm 1 ft wide along line 6–6, Fig. 14, with a sand blanket of a thickness equal to the height of the bulkhead (line 8–8, Fig. 14), and with a sand blanket only one half as thick (line 9–9, Fig. 14). The lateral pressure of the sand dike alone produced bending strains ϵ_o equal to approximately 60% of the values corresponding to a backfill composed entirely of sand.

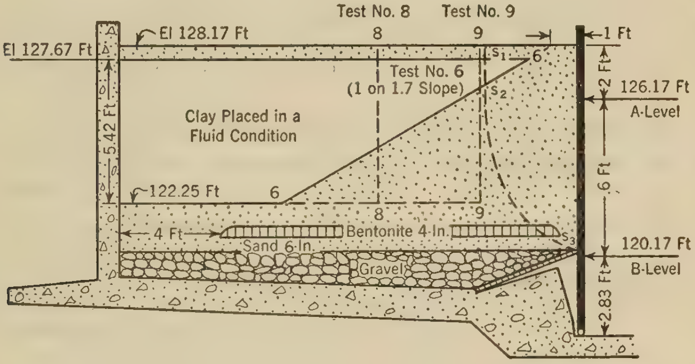

Fig. 14.—Structural Effect of a Sand Dike

The use of a sand dike sloping away from the bulkhead on a natural (1:1.7) slope was found to be fully effective in reducing the fluid lateral pressures transmitted to the bulkhead from the unconsolidated fluid clay backfill behind the dike. The pressures against the bulkhead were no greater than those exerted by a backfill composed entirely of sand.

The interposition between the fluid clay backfill and the bulkhead of a vertical sand blanket with a width equal to the bulkhead height was found to be just as effective as the interposition of a sand dike. When the width of the blanket was equal to one half of the bulkhead height, it was only approximately one half as effective, and it was completely ineffective when its width was equal to one tenth of the bulkhead height. In that latter case the lateral pressures transmitted to the bulkhead from the unconsolidated clay backfill were no smaller than those of a fluid.

The foregoing statements should be taken only as indications of order of dimension, accurate to approximately ±10%.

Consolidation of the clay did not further decrease lateral pressures in the case of the sand dike. Fig. 14 illustrates that only a small and the least effective part of the likely logarithmic spiral slip surface (S_1–S_2, Fig. 14) lies within the clay. The same observation concerning lateral pressures was made in the case of the wide sand blanket (8–8, Fig. 14). In that case the entire probable slip plane lies within the sand mass; this could not happen when the thinner sand blanket (9–9, Fig. 14) was used, and therefore its action was less effective at first. Subsequent consolidation of the clay backfill correspondingly decreased the lateral pressures. The foregoing analysis is somewhat different from the one advanced by the author of the fourth Symposium paper.

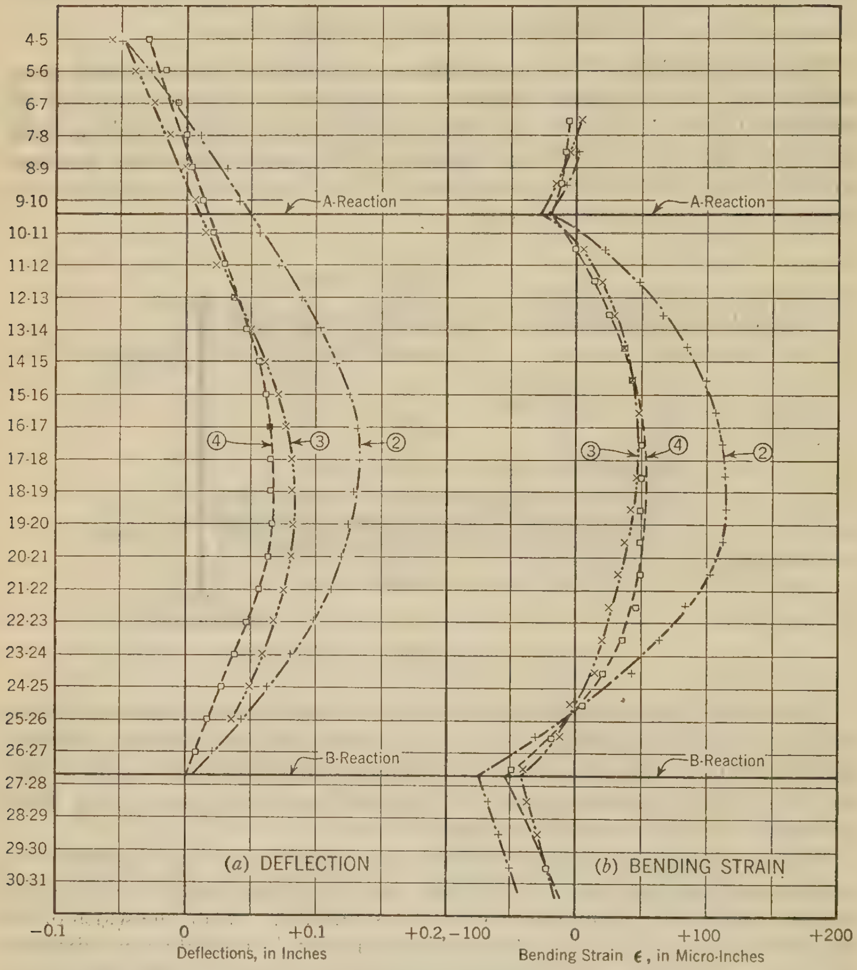

Fig. 15.—Composite Curves for the Completely Fluid Clay Backfill Condition, Minus 8 Ft of Water

It should be noted that a bentonite layer 4 in. thick had been placed over the sand and gravel drainage layer during a preceding test. Although originally placed at a water content well above its liquid limit, the layer was rapidly consolidated so that it had an unconfined compressive strength of approximately 200 lb per sq ft. As a result, its presence does not seem to have affected the action of the sand dike.

During a preceding test with a backfill entirely composed of sand, the presence of a bentonite layer (placed as shown in Fig. 14) had the effect of reducing the lateral pressures somewhat, in comparison with those in another test in which no bentonite layer was present. This observation may be explained by the lateral expansion of the sand backfill as a result of the consolidation of the bentonite, which expansion may have reduced the lateral pressures.

Curve 2, Fig. 15(*b*), shows the component of the bending strains produced by the buoyed weight of the solid fraction of the fluid clay backfill during test No. 3 when the entire backfill was clay; curve 3 shows the same values for test No. 5 when the backfill was composed entirely of sand; and curve 4 gives almost identical values obtained during test No. 6—when the sand dike was backfilled with fluid clay. The effectiveness of the sand-dike action is clearly shown.

This finding is confirmed by a similar relationship of the deflection curves of the center bulkhead during the three tests just mentioned (see Fig. 15(*a*)).

B. Lateral Pressures of Consolidating Clay Backfills

The use of a sand dike can effectively reduce the lateral pressures of a fluid clay backfill, but it cannot speed up the consolidation of the backfill itself and thereby increase its power to support vertical loads.

(*1*) *Accelerated Consolidation of Fluid Clay Backfills by Use of Prefabricated Horizontal Sand Drains.*—The consolidation of originally fluid clay backfills can be accelerated effectively by the use of a system of prefabricated horizontal sand drains (for example, burlap cylinders filled with sand) connected to well points. This method was successful during one of the tests.

Vertical sand drains have been used successfully for some time; but,[10] in so far as lateral pressures of backfills are concerned, they have some disadvantages. First, they cannot be installed until most of the backfilling is completed. Second, each drain must be pumped out or bailed out individually if uplift pressures are to be neutralized temporarily within the entire mass of the backfill in order to increase its effective weight, and thus to accelerate consolidation still further.

An interconnected system of horizontal prefabricated sand drains does not have the aforementioned disadvantages and has some further merits as compared to horizontal sand blankets—the continuity of which is likely to be destroyed by mud wave formation during backfilling operations. The installation of such a system of prefabricated drains in the field, however, is likely to present some difficulties and to be expensive. Fig. 16 shows the writer's conception of a possible construction procedure.

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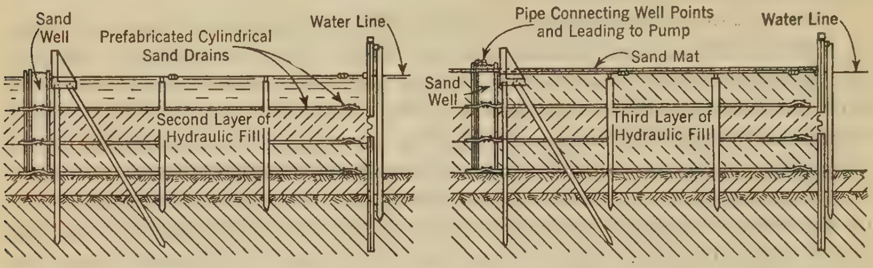
[10] "Studies of Fill Construction Over Mud Flats Including a Description of Experimental Construction Using Vertical Sand Drains to Hasten Stabilization," by O. J. Porter, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., 1938, Pt. II, pp. 129–141.

(*2*) *"At Rest" or "Consolidated Equilibrium" Lateral Pressures.*—During the consolidation (in a submerged state) of an originally fluid clay backfill, investigators observed that, in the backfill, there was a decrease of pressures, and that, in the bulkhead itself, there was a simultaneous decrease in bending strains, deflections, and horizontal reaction pressures. These decreases continued until the consolidation was completed and an "at rest" or (to use a possibly more appropriate term) a "consolidated equilibrium" condition was reached.


Sand Well
Prefabricated Cylindrical Sand Drains
Water Line
Second Layer of Hydraulic Fill
Pipe Connecting Well Points and Leading to Pump
Water Line
Sand Mat
Sand Well
Third Layer of Hydraulic Fill

STAGE 3 STAGE 4

FIG. 16.—INTERCONNECTED LAYER SYSTEM OF PREFABRICATED HORIZONTAL DRAINS

The decrease of bending strains during consolidation of the clay backfill is illustrated by Fig. 17(*a*). The progress of the test is recorded by stages, as follows:

Stage	Date (1944 and 1945)	Description
1	December 13	Initial position of bulkhead
3	December 21	Placing backfill plus sand mat completed to El. 128.17; flooded
4	December 27	Last reading before drainage
8	January 29	First reading of the first 6-ft water surcharge
10	February 20	Last reading of the first 6-ft water surcharge

Test stages 3 to 4, Fig. 17, correspond to backfill consolidation under its own weight and test stages 8 to 10, Fig. 17, correspond to the consolidation of the backfill under a surcharge of 375 lb per sq ft. Fig. 17(*b*) shows a similar decrease of deflections during the same test stages.

Conventional stress-strain equations have no meaning in this case, since the increment of strain has a negative value, whereas both the increment of stress that is effective in reducing lateral pressures and the modulus of elasticity have positive values.

According to one current popular concept,[11,12] outward motions of more than 5% of the height of the bulkhead are needed to mobilize fully the shearing

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[11] "A Fundamental Fallacy in Earth Pressure Computations," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., April, 1936, p. 87.

[12] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 96.

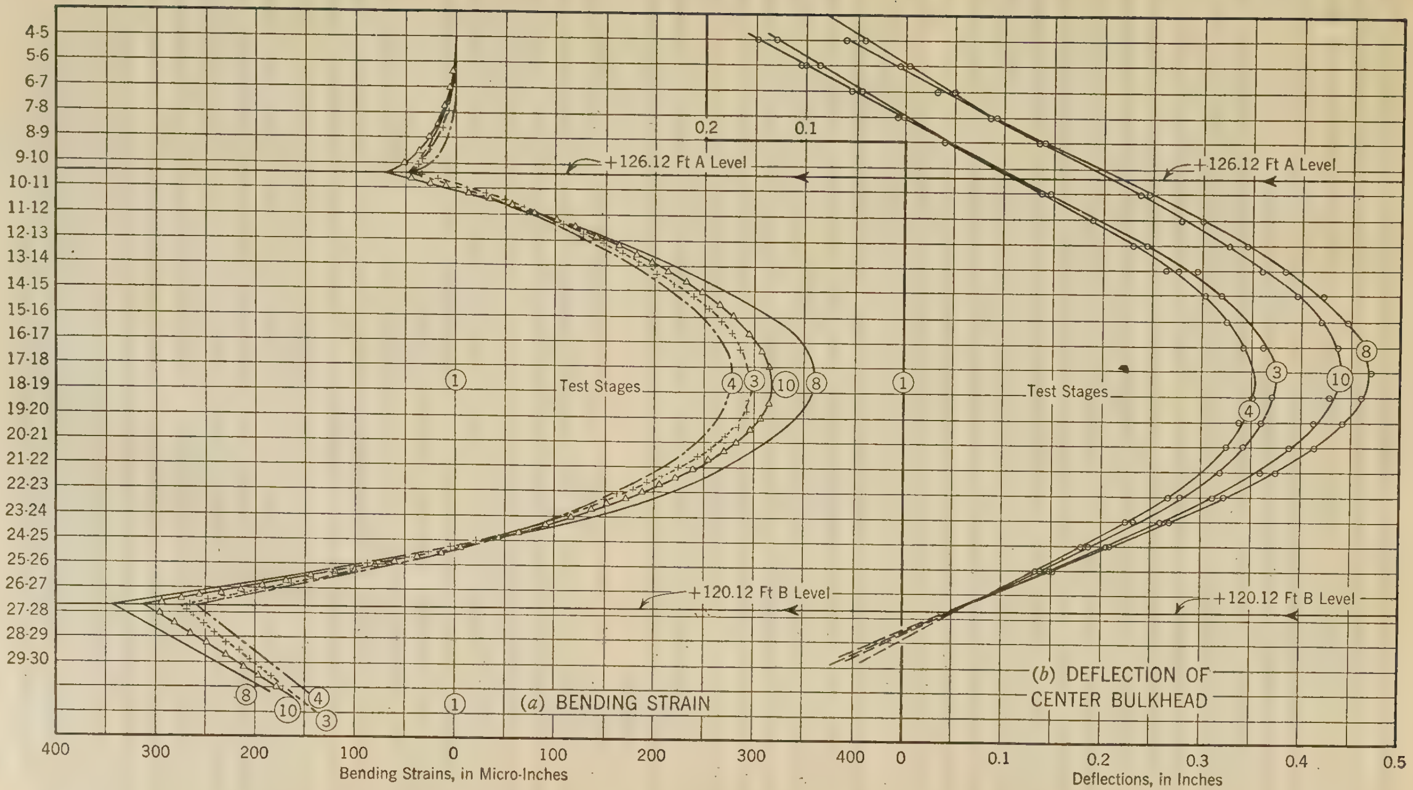

Fig. 17.—Decrease in Bending Strains and Deflection During Consolidation (Test No. 3)

strength of a clay soil which is effective in reducing lateral pressures. The Princeton tests show that this concept applies neither to plastic backfills behind flexible bulkheads nor to their "consolidated equilibrium" pressures. The latter are smaller than fluid pressures; and no induced outward motion of the bulkhead is needed to achieve the corresponding reduction.

Conventional laboratory methods of determining shearing strength coefficients of soils are based on the strength of soil specimens at failure. There is no reason to assume that the values of the "shearing strength at failure," thus determined in the laboratory, are necessarily identical to the values of the soil shearing strength which are effective in reducing fluid lateral pressures of unconsolidated clay backfills against flexible bulkheads to a "consolidated equilibrium" condition. This pressure reduction occurs without any outward motion of the bulkhead and therefore without the development in the backfill of deformations of the type necessary to produce failure in laboratory specimens. Obviously, this condition does not involve complete utilization of all the latent shearing resistance of the consolidated clay.

During the Princeton tests the over-all K_R-values (see Part I, Section E)—for the "consolidated equilibrium" condition of submerged clay backfills—were found to lie between the limits of 0.70 and 0.80. These coefficients are the K_{s+w}-values defined subsequently in Part II, Section E. They include the weights of both solids and water. The K-coefficients are usually intended to represent the so-called hydrostatic pressure ratio, or the ratio of actual lateral pressure to the fluid lateral pressure of a liquid of a unit weight equal to that of the soil. For naturally consolidated clays the K-values "at rest" have not been known; but they have been estimated by Professor Terzaghi, apparently on the strength of intuitive reasoning, as probably ranging between 0.66 and 0.88.[13]

W. S. Housel,[14] M. ASCE, also has reported measurements on tunnel linings, from which the final value of the ratio of measured lateral pressure to the computed weight of the overburden at the same elevation can be computed. It is approximately 0.66. The driving of the tunnel must have produced appreciable disturbance of the clay in its vicinity; hence this value, similar to the Princeton data, corresponds to a "consolidated equilibrium" condition and not to an "at rest" state of a natural deposit.

(3) *Lateral Pressures in Upper Unrestrained Zone of Backfill.*—During the tests an induced outward tilting motion, as much as 1.5% of the height of the bulkhead, did not continue to decrease the "consolidated equilibrium" pressures in the upper zones of either a clay backfill or a sand-clay mixture. This condition was found to be true even when the backfill was overconsolidated by the application of a surcharge and appreciably increased its cohesion (as much as four times) as evidenced in the laboratory by unconfined compressive strength tests on undisturbed samples taken from the backfill. (The unconfined compressive strength was increased from approximately 70 lb per sq ft to approximately 300 lb per sq ft for the clay backfill and from an unknown value to 570 lb per sq ft for the sand-clay mixture backfill.) Since the shearing stress

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[13] "Liner-Plate Tunnels on the Chicago (Ill.) Subway," by Karl Terzaghi, in "Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium," *Transactions*, ASCE, Vol. 108, 1943, p. 989.

"Earth Pressure on Tunnels," by W. S. Housel, *ibid.*, p. 1045, Fig. 41.

and shearing strain of plastic clays have a parabolic relationship, and since no evidence was found of an increasingly effective shearing strength at 1.5% outward motion of the bulkhead, the writer considers that no further appreciable utilization of strength could be encountered in the upper backfill zone at larger induced displacements—say, at an outward motion of 5% of the height of the bulkhead. In addition, such large motions could not be normally induced in the field and therefore were not included in the Princeton tests.

Since these findings and conclusions were presented at the January, 1947, Annual Meeting of the Society, the writer has learned of confined triaxial tests performed in 1934 in Holland (from a letter dated June 24, 1947, from E. C. W. A. Geuze, chief engineer, Research Department, State Laboratory of Soil Mechanics, Delft, Holland). A decrease of lateral pressure was observed then during the consolidation of undisturbed clays which was accompanied by a decrease of the diameter of the specimen. The tentative conclusion was also arrived at there that the active pressure of clays should not differ greatly from their "at rest" pressure. These findings were presented in a report to the 1936 "Congrès de l'Urbanisme et du Sous-Sol" in Paris. The report has not yet been published.

Since definite values of lateral pressures were actually measured against the upper part of the bulkhead, this finding shows the fallacy (or, at best, the limitation) of a popular concept illustrated by Fig. 18(*a*), according to which there will be no lateral pressure of a plastic clay backfill to a so-called "critical depth H'_c"—that is, to the depth at which a vertical cut will stand unsupported.[15] Since plastic deformations and outward bulging always occur in such unsupported cuts, any anchors or struts that prevent bulging should naturally receive some pressure even if the "critical depth" of excavation, or the "critical height" of backfilling, is not exceeded. A quantitatively exaggerated example of this bulging has been presented by Dale T. Harroun,[16] Assoc. M. ASCE. If the critical depth is exceeded, lateral pressures in the upper zone can no longer

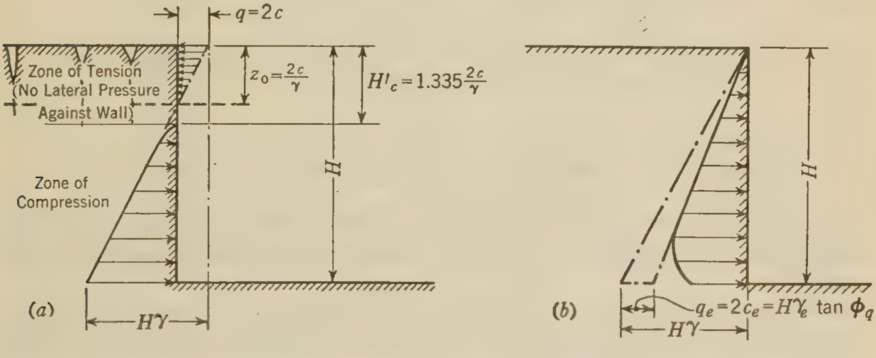
q=2c
Zone of Tension (No Lateral Pressure Against Wall)
$z_0=\frac{2c}{\gamma}$
$H'_c=1.335\frac{2c}{\gamma}$
H
Zone of Compression
(a)
Hγ
H
(b)
$q_e=2c_e=H\gamma_e \tan\phi_q$
Hγ

Fig. 18.—Comparison of Concepts Concerning Lateral Backfill Pressures of Cohesive Soils

be governed by the preceding conditions and will depend on the deformation of the entire plastic mass, including that which occurs below the "critical height." After any outward induced bulkhead motion, the unrestrained upper zone of the plastic clay will readjust itself to conditions corresponding to the original consolidated equilibrium and will restore the corresponding pressure.

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[15] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 97.

[16] "Stability of Vertical Embankments," by Dale T. Harroun, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., 1940, p. 754, Fig. 6.

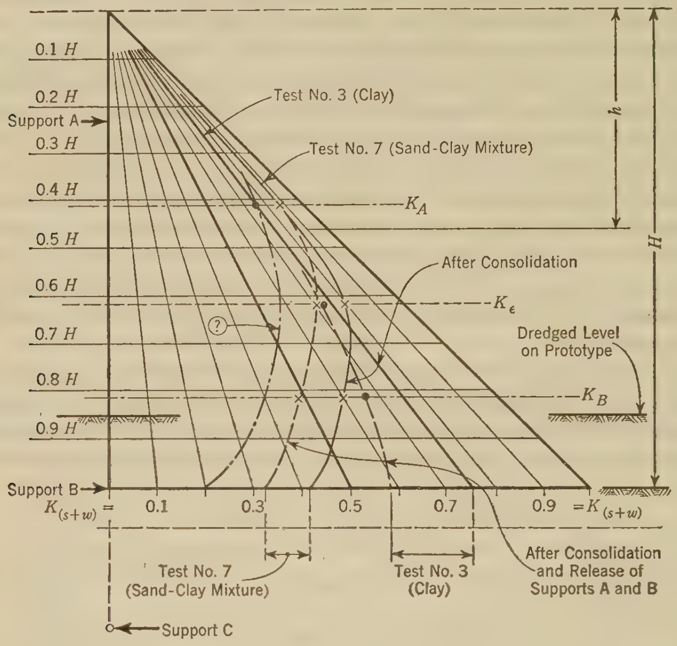

FIG. 19.—ESTIMATED PRESSURE DISTRIBUTION OF CLAY AND SAND-CLAY BACKFILLS

The concept illustrated by Fig. 18(*a*) is apparently intended for use with stiffly plastic clays above water level; but it has not been definitely limited to such conditions.

The writer is not aware of any one having measured, in plastic soils of any consistency, an absence of lateral pressures against the upper "critical height" part of retaining structures with a height greater than the "critical"—that is, of the kind illustrated by Fig. 18(*a*).

The general trend of pressure distribution established during the Princeton tests is illustrated by Fig. 18(*b*). It applies to plastic clay backfills behind flexible bulkheads; and it is valid for material that has been fully consolidated under its own weight as well as for material that has been overconsolidated along the entire length of a bulkhead. The significance of the term q_e will be discussed subsequently in Section B(5).

(*4*) *Lateral Pressures in Lower Restrained Zone of Backfill.*—Induced outward motion of the bulkhead did decrease the "consolidated equilibrium" pressures in the lower zone of the backfill which is restrained by the presence of an underlying rigid boundary. This was particularly pronounced in the case of a more "rigid" backfill consisting of a sand-clay mixture, as illustrated by Fig. 19, which gives the approximate distribution, with depth, of the K-values determined from the measured K_A-value, K_B-value, and the K_c-value. Since

$$K_h = \frac{p_h}{\gamma\, h} \qquad\qquad (4)$$

it also gives an idea of the distribution with the depth h of the lateral pressures p_h of a soil of unit weight γ. The dash-dot curve in Fig. 19, marked "?," represents an hypothetical pressure distribution which may occur in the field if the relative dredged level in the prototype is higher than that in the model.

(5) *Measured Lateral Pressures of Plastic Cohesive, and of Semicohesive, Backfills Compared to the Results of Laboratory Tests.*—No generally valid relationship could be established between the shearing strength of clays and of sand-clay mixtures at failure (as determined in the laboratory from unconfined compressive strength, and direct shear and triaxial shear tests) and the measured lateral pressures. The conventional formula[17] for total lateral pressure, R, based on the Rankine-Résal theories is

$$R = \frac{\gamma\, H^2}{2} \tan^2 \left(45^\circ - \frac{\phi}{2} \right) - 2\, c\, H \tan \left(45^\circ - \frac{\phi}{2} \right) \qquad (5a)$$

and the corresponding formula for unit lateral pressures p_h is

$$p_h = \gamma\, h \tan^2 \left(45^\circ - \frac{\phi}{2} \right) - 2\, c \tan \left(45^\circ - \frac{\phi}{2} \right) \qquad (5b)$$

Eqs. 5 refer to fully consolidated soils and state that fluid lateral pressures are decreased as a result of internal friction (expressed by its angle ϕ) as well as by the cohesion c.

No generally accepted procedure has been devised for determining the values ϕ and c for practical use in Eqs. 5. For the same soil the values of ϕ and c may vary greatly depending on the procedure used to determine them in the laboratory.

Studies by Professor Terzaghi[18] and Ralph B. Peck,[17] Assoc. M. ASCE, have resulted in the wide acceptance of a trend to consider that friction (as expressed by laboratory values of ϕ) is inactive in clays, and to take into consideration only the cohesion c, which is assumed equal to one half of the unconfined compressive strength q:

$$c = \frac{q}{2} \qquad\qquad (6)$$

Other procedures, nevertheless, are still used occasionally. The writer strove to approach the matter with a completely open mind—to compare results obtained on the strength of different laboratory testing procedures with actual measurements of lateral pressures and to let the facts speak for themselves. These facts were as follows:

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[17] "Earth-Pressure Measurements in Open Cuts, Chicago (Ill.) Subway," by Ralph B. Peck, in "Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium," *Transactions*, ASCE, Vol. 108, 1943, pp. 1008–1036.

[18] "Liner-Plate Tunnels on the Chicago (Ill.) Subway," by Karl Terzaghi, *ibid.*, pp. 970–1007.

a. Consolidated rapid shear tests on undisturbed samples extracted by means of Shelby tube samplers, 3 in. in diameter, from lined bore holes, 4 in. in diameter, in the backfill gave much too high values of ϕ and c for use in Eqs. 5. Lateral pressures computed with these values were too low even if either ϕ or c were neglected.

b. Results of rapid shear tests on originally fluid samples of clay consolidated in the laboratory, when plotted in the conventional manner, gave values of $c = 0$ and of $\phi = 17°$ which, when used in Eqs. 5, agreed approximately with measurements on the model. In the case of the sand-clay mixture the values of ϕ (32°) thus obtained were much too high (approximately twice as high).

c. Unconfined compressive strength tests on the clay samples gave values of c determined from Eq. 6 which, when used in Eqs. 5, were in reasonable agreement with observed pressures as long as the clay was not overconsolidated. Otherwise the c-values were too high. In the case of sand-clay mixtures the c-values thus obtained were too high under all circumstances.

The following expression, developed by the writer, happens to correlate laboratory test results and the measured values of lateral pressure both of consolidated clays and of consolidated sand-clay mixtures (see Fig. 18(*b*)):

$$p_h = \gamma h \left(1 - \frac{q_e}{\gamma h} \right) = \gamma h \left(1 - \frac{\gamma_e}{\gamma} \tan \phi_q \right) \ldots\ldots\ldots\ldots (7)$$

in which γ_e represents the effective unit weight of the soil transmitted from grain to grain. In the case of a submerged soil it represents its buoyed dry unit weight.

The significance of the angle ϕ_q can be seen from Fig. 11. Its value is almost identical both for the clay and for the sand-clay mixture. A further discussion of this point appears premature.

It should be emphasized (see Fig. 19) that the consolidated clay and the consolidated sand-clay mixture exerted very similar lateral pressures, although these two types of soil had completely different characteristics when tested to failure by different conventional laboratory procedures (see Fig. 11 and Part I, Section D). The greatest discrepancies between observations and results of laboratory strength tests have been recorded for the sand-clay mixture. This type of material is frequently used for hydraulic backfilling operations.

(*6*) *Possible Use of the Theories of Elasticity and of Plasticity.*—The preceding findings make it appear advisable to examine possible methods of determining lateral pressures which do not relate them to their strength at failure, since under normal field conditions plastic clay and sand-clay backfills cannot be deformed to the point of failure. In the past, suggestions have been made by Lazarus White, E. A. Prentis, and the late George Paaswell,[19] Members, ASCE, that the theory of elasticity be used for the determination of lateral pressures of all soils. However, the knowledge of the value of the Poisson ratio of soils, at least, is necessary for the purpose. There is no known laboratory method of determining this value; and usually it is estimated in reverse, from actual measurements.

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[19] "Cofferdams," by Lazarus White and Edmund Astley Prentis, Columbia Univ. Press, New York, N. Y., 1940, pp. 75–78.

For the time being, therefore, a more direct experimentally justified approach to design problems appears advisable, as recommended subsequently under Section E. Nevertheless, the adaptation of the theories of elasticity and plasticity to soil problems, so as to permit the development of practically usable methods for the determination of lateral pressure distribution of different types of soils under varying conditions of boundary restraint, should not be considered permanently impossible. It poses a challenge both to theoreticians and to experimenters. New attempts are continuously being made. For instance, W. H. Weiskopf,[20] M. ASCE, has offered a modification of one of the conventional assumptions. He starts with a varying ratio between the modulus of shear E_G and of the modulus of elasticity E, instead of the conventional constant ratio.

C. Phenomena of Arching in Sands

(*1*) *Findings of the Princeton Model Tests.*—With reference to normally placed backfills, no evidence was found of any pressure relief ascribable to vertical arching between the anchor level and the level of the soil below the dredged line. No decrease of bulkhead bending moments was observed corresponding to such arching action, as is sometimes assumed. This finding refers to normally placed sand backfill—material dumped through water so that it slopes away from the bulkhead at a uniform height across the entire width of the tank, at all stages of the backfilling.

Indications were that there was arching under such conditions when the sand is in place at both end supports of a yielding structural unit before its final deflection. For instance, arching in a horizontal direction occurred when the sand was not uniformly dumped across the entire width of the tank. Relatively greater pressures were developed on the bulkhead section against which the sand was placed first, which then could serve as an "abutment" for the horizontal arch in the sand which relieved pressures against the remaining bulkhead sections that were backfilled last.

Arching in a vertical direction did develop after completed backfilling and an additional induced deflection between the supports. This corresponded to the small-scale tests reported by J. P. R. N. Stroyer.[21]

Such arching (in a vertical direction) as did occur was found to be unstable, so that it should not be relied on for design purposes where sand backfills are concerned. A slight release of the bottom plate of the tank near the B-support immediately and strongly increased lateral pressures at all levels.

Indications were obtained, however, of possibly smaller lateral pressures of sand than are usually assumed below the natural ground level against bulkheads.

(*2*) *Other Conceptions and Their Analysis.*—In the case of clean sand it has been held that[22] the actual bending moment in a sheet pile bulkhead, as produced by a pressure distribution corresponding to line $as_1t_1b_1$, Fig. 20, will not exceed about one half of the maximum bending moment computed on the basis of hydrostatic pressure distribution (line atb_1, Fig. 20). The writer, among others, shared this view. The total pressure represented by both lines is approximately the same. Therefore, a decrease of positive bending moments below the anchor level can be produced only by a redistribution of pressures over the span beneath the anchor level; or by an increase of pressure above the anchor level with a resulting increase of negative bending moments at that level.

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[20] "Stresses in Soils Under a Foundation," by Walter H. Weiskopf, *Journal of the Franklin Institute*, June, 1945, pp. 445–465.

[21] "Earth-Pressure on Flexible Walls," by Jens Peter Rudolf Nielsen Stroyer, *Journal*, Inst. C. E., London, England, Vol. 1, 1935–1936, pp. 94–139 and 550–557.

[22] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, pp. 217–219.

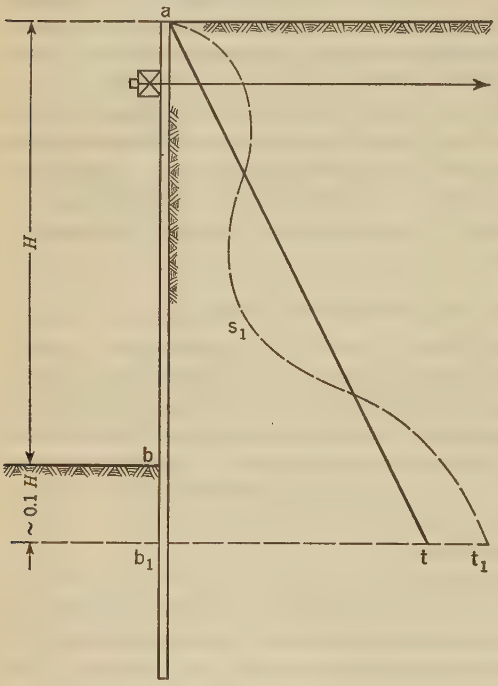

Fig. 20

Fig. 21 illustrates the changes that occur in the maximum bending moments of a freely supported beam, depending on six possible limit cases of load distribution, the total load remaining the same. It shows that a redistribution of pressures over a span of the type given by Fig. 20 cannot alone produce a 50% reduction in the bending moments; at best it can produce a reduction of from 20% to 25%. The Princeton tests show that passive pressures above the anchor level could not have developed a 50% total reduction. Such pressures are not actually developed, presumably because of the progressive yielding of the anchor support, which occurs simultaneously with the deflection of the bulkhead itself (see Figs. 15(*a*) and 17(*b*)). The support did yield in spite of the fact that the A-support was somewhat more unyielding than a real anchor would have been.

Current opinions concerning a strong effect of arching on the decrease of bending moments in the sheet piling have been strongly influenced by the writings of R. Stroyer,[21,23,24] in England. In suggesting the use of his graphs for actual design, the fact has been often obscured[25] that some of his recommendations were purely speculative and that others were based on small-scale model tests which did not reproduce actual backfilling conditions.

In their turn, Mr. Stroyer's conclusions have been influenced by recommendations of the Danish Society of Engineers, first issued in 1926[23] and then amplified in 1937.[26] As far as the writer could ascertain, these recommendations represented an empirical attempt to take account of the fact that some bulkheads had stood up, whereas according to conventional ideas they should have collapsed.

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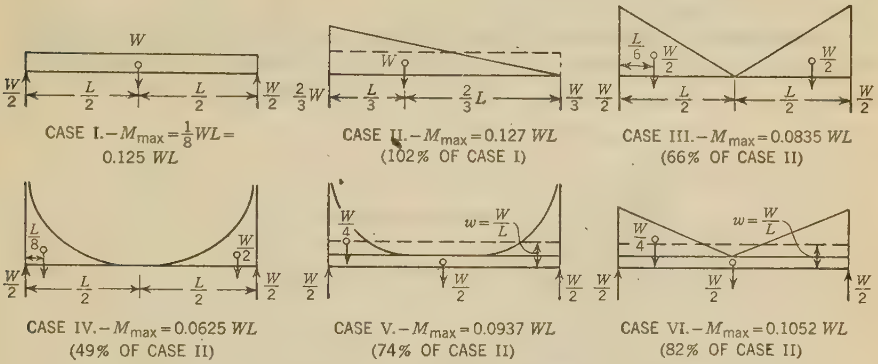
[23] "Earth-Pressure on Flexible Walls," by Rudolf Nielsen Stroyer, *Minutes of Proceedings*, Inst. C. E., London, England, Vol. 226, 1929, pp. 116–134.

[24] "Concrete Structures in Marine Work," by R. Stroyer, London, England.

[25] "Sheet Piling, Cofferdams, and Caissons," by Donovan H. Lee, Concrete Publications, Ltd., London, England, 1945, p. 35.



CASE I.—$M_{max} = \frac{1}{8}WL =$ 0.125 WL

CASE II.—$M_{max} = 0.127\ WL$ (102% OF CASE I)

CASE III.—$M_{max} = 0.0835\ WL$ (66% OF CASE II)

CASE IV.—$M_{max} = 0.0625\ WL$ (49% OF CASE II)

CASE V.—$M_{max} = 0.0937\ WL$ (74% OF CASE II)

CASE VI.—$M_{max} = 0.1052\ WL$ (82% OF CASE II)

FIG. 21.—POSSIBLE CHANGES IN MAXIMUM BENDING MOMENTS, THE TOTAL LOAD REMAINING UNCHANGED

The distribution in Fig. 22 produces approximately a 40% reduction of bending moments as compared to conventional diagrams. However, only an approximately 18% reduction is caused by a redistribution of pressures that accompany arching and the remaining approximately 22% reduction results from an assumed decrease of the total pressure. The anchor pulls are not decreased by the Danish distribution, almost the entire pressure reduction occurring along the lower half of the span below the anchor level. This is in general agreement with the Princeton findings, although the shape of the pressure distribution curve in Fig. 22 does not agree.

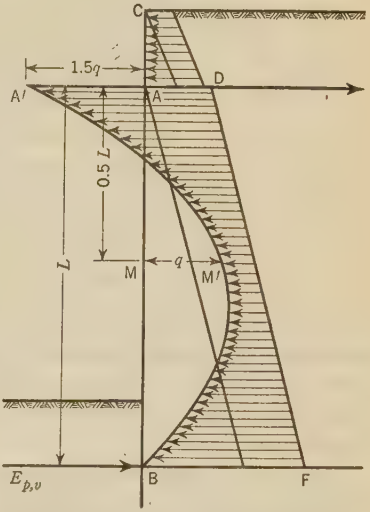


FIG. 22.—PRESSURE DISTRIBUTION RECOMMENDED BY THE DANISH SOCIETY OF ENGINEERS

Fig. 23 shows the limits of the K-value variation along the depth of the bulkhead; and therefore it also shows the limits of the pressure distribution (according to Eq. 4), as established with sand during the Princeton tests. The shape of the Princeton curves is quite different from the shape of the curve corresponding to the Danish recommendations. Again, as in the case of the sand-clay mixture (see Fig. 19), only in the lower zone do the sand tests show a departure from conventionally assumed hydrostatic pressure distribution.

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[26] "Normer for Vandbygnings-Konstruktioner," Udgivet af Dansk Ingeniøforening, København, 1937

D. Restraining Effects at Lower Boundary

On the basis of all the available evidence (subject to further checks) the writer has formed an opinion that the dominant factor in any actually possible reduction of pressures may be the influence of an underlying rigid boundary in relation to the rigidity of the backfill.

Under actual prototype conditions the lower boundary (that is, the dredged line) will be located as shown in Fig. 23 well above the level of the resultant of passive earth pressure (B-level, Fig. 23). Accordingly, the effects of this boundary should extend further so that a distribution of pressures may result as shown by a dotted line and a question mark in Fig. 23. As in Fig. 19, the curve marked "?" represents an hypothetical pressure distribution that may occur in the field if the relative dredged level in the prototype is higher than that in the model.

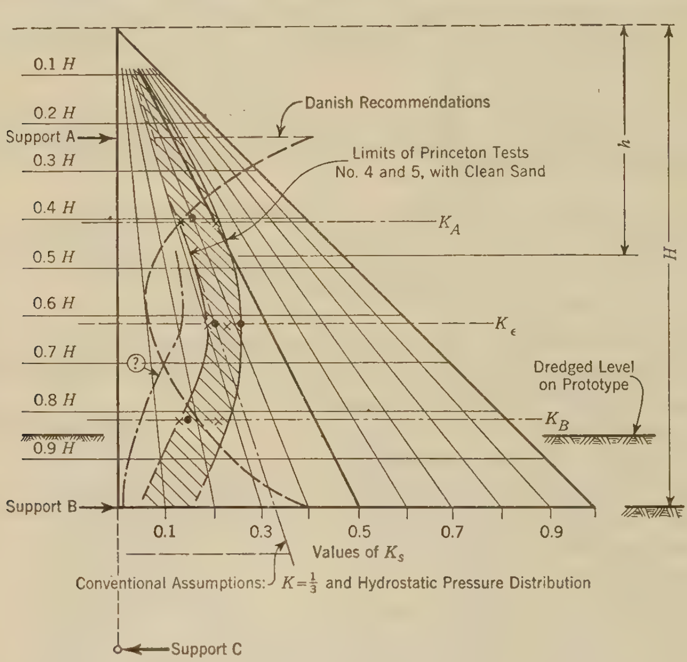

Fig. 23.—Estimated Pressure Distribution of Sand Backfills

The great effect that the nature of a boundary may have on pressure distribution, on its relation to the stiffness of the soil, and on its relation to the intensity of vertical pressures acting on it is illustrated in Fig. 24. Fig 24(*a*) shows the increment of bending strains recorded right after the application of a surcharge of 125 lb per sq ft to a clay backfill consolidated under its own weight. The shears and lateral pressures, computed by the use of Eqs. 1 and 2, as well as the measured increments of pore pressure are also shown. A sandbank, sloping upward and away from the toe of the bulkhead, was in place during that test. As a result of the outward yield of the bulkhead caused by lateral pressure against the upper three fourths of its height, lateral pressures actually decreased against the lower bulkhead section. This result not only can be noted from the curve of computed pressures in Fig. 24(*a*), but it also follows from the clearly reversed curvature of the directly measured bending strains. Furthermore, the pore pressure cells did not register any increase of pressure in that lower backfill zone. The unconfined compressive strength of that part of the clay backfill at that test stage should have been somewhat higher than the surcharge applied.

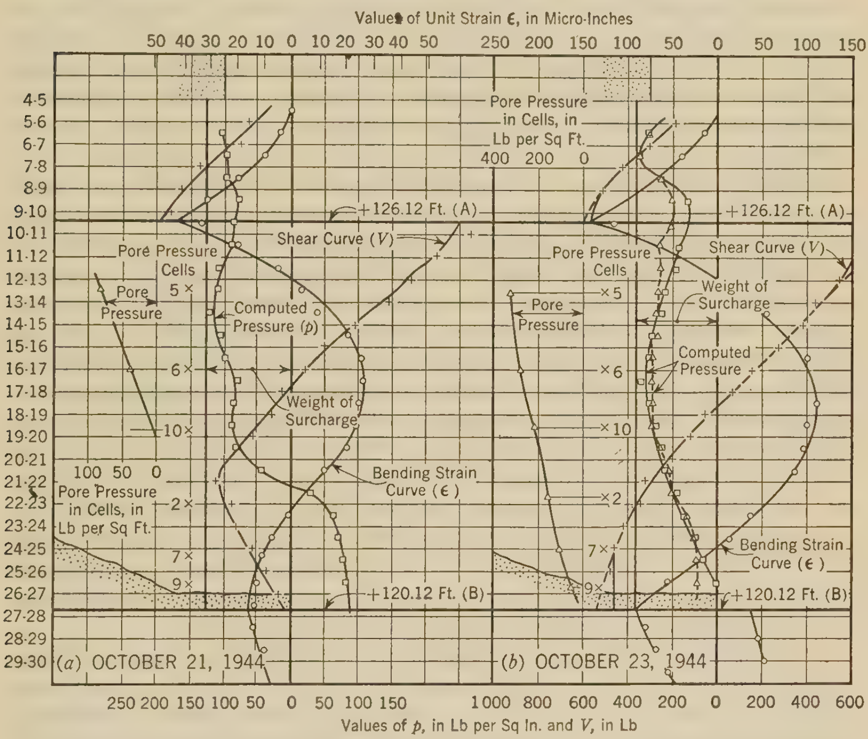

Fig. 24.—Increment of Bending Strains and of Pressures as a Result of Surcharge

(*a*) Surcharge, 2 Ft of Water
(*b*) Surcharge, 6 Ft of Water

As the surcharge was increased to 375 lb per sq ft, or to approximately twice the value of the unconfined compressive strength of the clay backfill, a positive increment of lateral pressure and of pore pressure was registered in the lower zone of the backfill (see Fig. 24(*b*)). Nevertheless these pressure increments decreased along the depth of the bulkhead.

In connection with the entire problem of boundary effects, small-scale experiments by Ek-Khoo Tan, Jun. ASCE, showed how the slip plane was affected by the boundary location and how it could develop only at some distance from it.[27]

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[27] "Stability of Soil Slopes," by Ek-Khoo Tan, *Proceedings*, ASCE, January, 1947, p. 19.

E. Points to be Considered When Designing Anchored Flexible Bulkheads

It does not appear advisable to rely on the arching effect in sand backfills for a reduction of bending moments. On the contrary, construction procedures should take into account the possibility of local pressure increases resulting from arching in a horizontal direction, unless backfilling is advanced more or less uniformly.

The use of a sand dike offers a practical method of decreasing fluid lateral pressures of unconsolidated backfills with high clay content. In addition, if a rapid increase of the supporting power of such backfills is desired, an attempt can be made to achieve this end by using either conventional type vertical sand drains, or a more effective but more complicated procedure involving an interlaced layered system of horizontal prefabricated drains connected to well points.

If a fluid backfill is used which is composed entirely of a cohesive or a semicohesive soil (such as a sand-clay mixture) and if such a backfill is placed rapidly, then bulkhead design should be based entirely on fluid pressures. If time permits placing such a backfill in layers and accelerating their consolidation by drains of any type, then lateral pressures smaller than fluid ones can be used. However, it does not appear possible to relate the values of these smaller lateral pressures to the shearing strength of the plastic backfills as determined from laboratory tests carried to failure; nor does it appear rational to attempt to do so. Neither the theory of elasticity nor the theory of plasticity is practical for analyzing all phases of the problem. The necessary soil constants cannot yet be evaluated in the laboratory. For that reason it appears advisable to determine lateral pressures for use in bulkhead design from empirical charts of the type illustrated by Figs. 19, 23, and 25, based on actual measurements on models and in the field. Laboratory tests would still be necessary, but would be used mainly for soil classification and identification purposes. Two types of K-coefficients would have to be used.

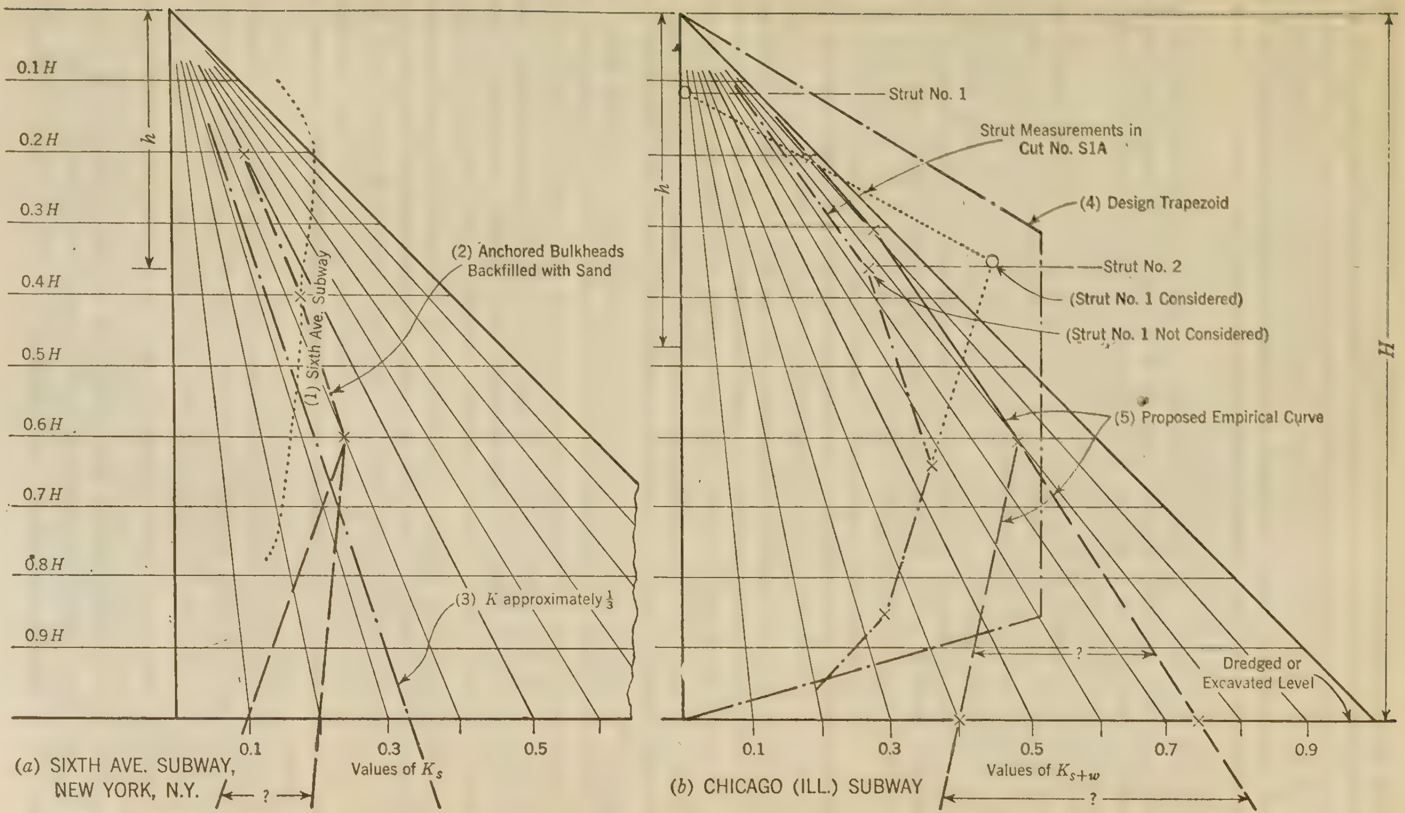
The K_s-values would be used for granular backfills, since these coefficients are related to the weight of the solids only (see Fig. 25). The K_{s+w}-coefficients are related to the combined weights of solids and water under conditions of complete saturation, and are suggested for use in connection with cohesive and semicohesive soils (see Figs. 19 and 25(*b*)). The two coefficients can be related to each other by the formula:

$$K_s = \frac{K_{s+w} - C'}{1 - C'} \quad \text{......................} (8)$$

in which

$$C' = \frac{1 + e}{G + e} \quad \text{......................} (9)$$

In Eq. 9, e is the void ratio; and G is the specific gravity of solids. In Eq. 8, K_s refers to the buoyed weight of the solids and K_{s+w} refers to the nonbuoyed combined weight of soil and water.



(*a*) SIXTH AVE. SUBWAY, NEW YORK, N.Y.

(*b*) CHICAGO (ILL.) SUBWAY

FIG. 25.—OBSERVED PRESSURE DISTRIBUTION IN FULL-SCALE CUTS COMPARED TO DESIGN RECOMMENDATIONS FOR CUTS AND FOR ANCHORED BULKHEADS

Fig. 25(*a*) shows the type of K_s-chart suggested for use with granular backfills and anchored sheet pile bulkheads. The curve obtained from the first series of Princeton bulkhead tests with submerged backfill (curve (2), Fig. 25(*a*)), compared to conventional assumptions, showed a similar but less pronounced trend than did the curve obtained from strut measurements performed in New York on the Sixth Avenue Subway (curve (1), Fig. 25(*a*)).[28] The soil there was wet, but free draining, sand.

In 1941, Professor Terzaghi[29] suggested a method of determining lateral pressures, which method was also used by him and by Professor Peck on the Chicago Subway.[30] It consisted of a pressure trapezoid based on K-values related to laboratory strength tests (curve (4), Fig. 25(*b*)).

The writer's suggested procedure (curve (5), Fig. 25(*b*)) is based instead only on empirical K-values determined from actual measurement. It abandons the trapezoidal shape which, apparently, was recommended mainly for the purpose of providing for possible pressure variations between individual differently wedged braces in cuts.[31] Variations in the sequence and in the timing of the bracing in relation to the progress of the excavation were taken care of by the trapezoidal shape of the pressure diagram, as shown in Fig. 25(*b*), for cut No. S1A.[32] During construction strut No. 1 was placed after the excavation had reached the level of strut No. 2. Similar erratic variations cannot occur in the case of an anchored bulkhead, and the writer's suggested K-values or pressure variation curves are triangular in the upper one half to two thirds of the height of the bulkhead, with a parabolic decrease thereafter.

Fig. 25(*b*) gives a K_{s+w}-chart proposed for use with plastic cohesive and semicohesive soils—that is, with clays and with mixtures of sand and clay. The term "plastic" refers to soils that require approximately 20% strain in order to fail when subjected to an unconfined compressive strength test. A curve of the K-values or pressure variation measured on the Chicago Subway and computed by the writer from strut loads given by Professor Peck[32] for cut No. S1A (curve (5), Fig. 25(*b*)) has a shape similar to the one determined from the Princeton model tests with consolidated fluid backfills, except for a more pronounced effect of the lower boundary, in accordance with the semirigid structure of the natural deposit of Chicago clay. The lateral pressures are also correspondingly smaller. The corresponding "design trapezoid" recommended for use on the Chicago Subway cuts by Professor Peck[30] is shown as curve (4), Fig. 25(*b*). Its application to anchored bulkhead problems would produce much too high anchor pull values, as compared to the method proposed by the writer.

Fig. 25 is intended to illustrate only the method proposed. The values on these diagrams are based on the first series of tests only. They are conservative and are subject to revision, especially in the lower backfill zones, as further test and observation data become available.

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[28] "Cofferdams," by Lazarus White and Edmund Astley Prentis, Columbia Univ. Press, New York, N. Y., 1940, p. 79.

[29] "General Wedge Theory of Earth Pressure," by Karl Terzaghi, *Transactions*, ASCE, Vol. 106, 1941, pp. 77–78.

[30] "Earth-Pressure Measurements in Open Cuts, Chicago (Ill.) Subway," by Ralph B. Peck, in "Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium," *ibid.*, Vol. 108, 1943, p. 1032.

[31] "General Wedge Theory of Earth Pressure," by Karl Terzaghi, *ibid.*, Vol. 106, 1941, p. 77.

[32] "Earth-Pressure Measurements in Open Cuts, Chicago (Ill.) Subway," by Ralph B. Peck, in "Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium," *ibid.*, Vol. 108, 1943, p. 1027, Table 3.

PART III. THE SECOND AND THE THIRD SERIES OF TESTS

A. Points that Required Further Clarification After Completion of First Series of Tests

The testing setup described in Part I permitted the controlled motion of the model bulkhead supports and the study of the effects of various types of such motion on lateral pressures. A separate study of this variable would not have been possible with a model bulkhead actually embedded in soil.

The results described in Part II clarified a number of important points, especially in respect to lateral pressures in the upper backfill zone and to anchor pulls. The model bulkhead was designed and was operated to have full model similarity with prototypes in respect to its flexibility. The similarity of behavior of noncohesive soils in models and in prototypes is generally known.[6,29] However, as explained in Part II, there remained some questions concerning the validity of the results obtained in the lower backfill zones. Specifically, these questions were related to:

(1) The effect on backfill pressures and on bulkhead and on anchor stresses of the rigidity of soils underlying the dredged level;

(2) The values of lateral pressures of cohesive soils exerted under prototype intensities of vertical pressure;

(3) The performance of soils other than the types tested; and

(4) The effect of wall rigidity, as compared to the "spring action" of deflected bulkheads, on the lateral pressures of consolidating clay backfills.

Two additional series of tests were initiated for the purpose of clarifying these four points.

The second series of tests was started with the "lateral pressure meter." This apparatus is illustrated subsequently in Fig. 29 and is described in the third paper of the Symposium. This second series of tests is intended to clarify questions (2), (3), and (4).

A third series of tests was started for the purpose of clarifying question (1). A new model bulkhead setup for combined active and passive earth pressure tests was used. It is described in the third paper of the Symposium.

B. First Results of Second Series of Tests

Several tests have been performed with the "lateral pressure meter" (illustrated subsequently in Fig. 29) under intensities of vertical pressure equal to 0.25 ton per sq ft, 0.50 ton per sq ft, and 1.00 ton per sq ft. These vertical pressures correspond to the buoyed weight of approximately 8 ft, 17 ft, and 33 ft of submerged soil.

Originally fluid red clay (see Fig. 9) was used at a consistency of approximately three blows on the liquid limit device. In the upper unrestrained zone of the apparatus the initial ratio of lateral to vertical pressure was found to be approximately $K = 1.00$. After completed consolidation this value decreased to approximately $K_s = 0.50$ for all three previously mentioned vertical pressure intensities. According to Eq. 8 the corresponding value for a nonbuoyed but saturated soil should be $K_{s+w} = 0.75$. During the first series of model bulkhead tests this value was found to lie between 0.70 and 0.80. The similarity between model and prototype can therefore be considered proved in respect to question (2), Section A, Part III.

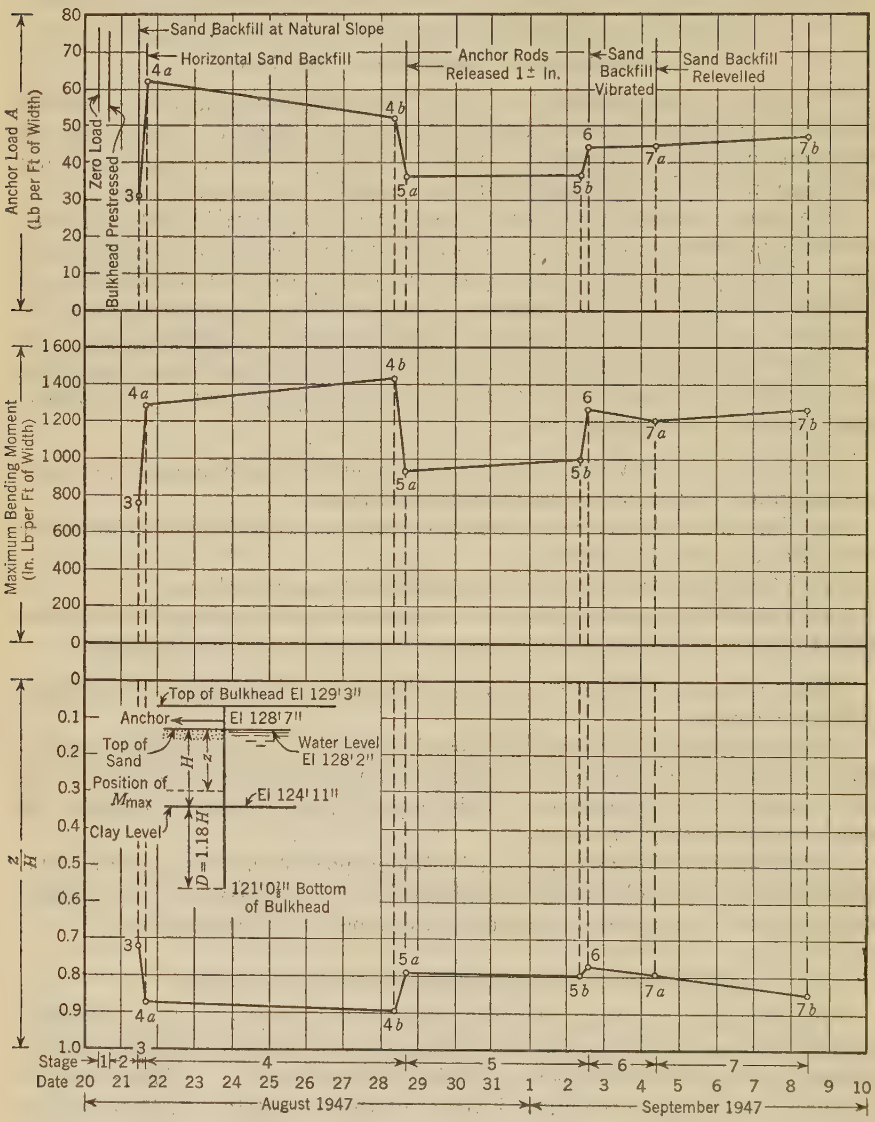

FIG. 26.—RESULTS OF FIRST TEST WITH NEW MODEL BULKHEAD SETUP FOR COMBINED ACTIVE AND PASSIVE EARTH PRESSURE TESTS (TEST 11A)

These tests were performed with almost perfectly rigid supports, the maximum yield of which was approximately 0.003 in. (or 0.00025 of the sample diameter). The elastic recovery of the supports after completed consolidation was approximately one third of the foregoing values. It would appear, therefore, that question (4), Section A, Part III, can be answered in the sense that the wall rigidity has little effect on the decrease of lateral pressure during the consolidation of the clay backfills.

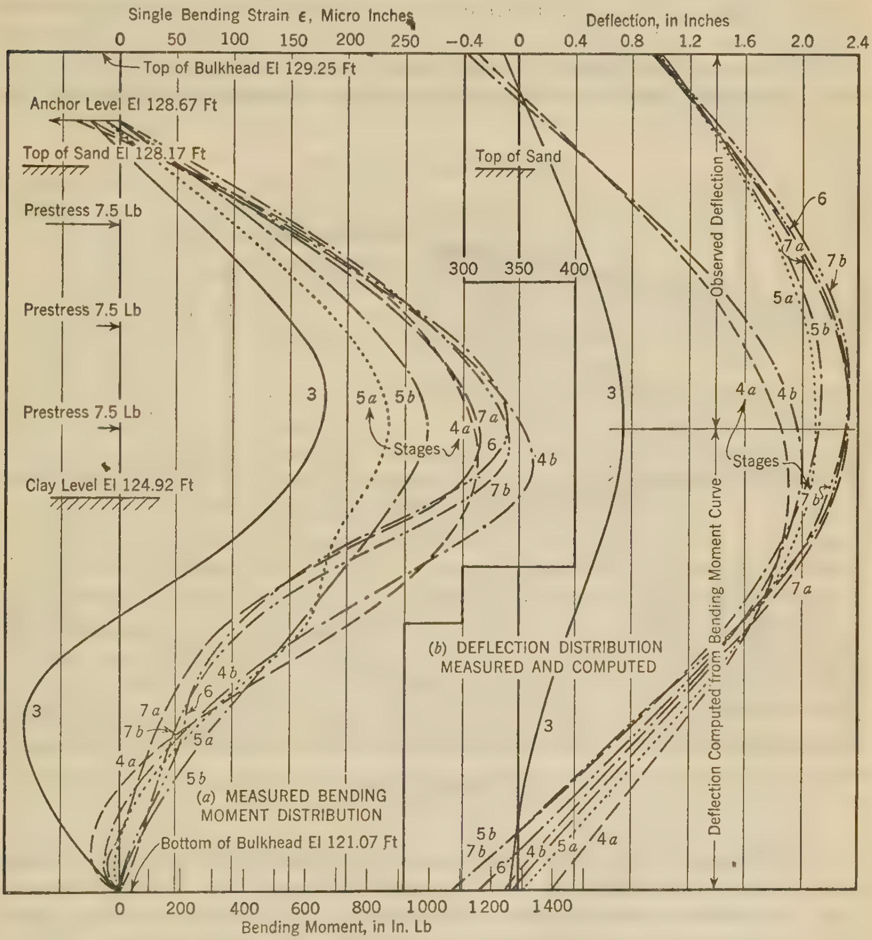

Fig. 27.—Variation in Test Results for Different Test Stages Measured

After completed consolidation a release of the ring supports equal to 0.3 in. or to 2.5% of the sample diameter did not decrease the lateral pressures exerted against the upper rings any further.

These findings confirm the corresponding conclusions reported under Section B, Part II.

C. First Results of Third Series of Tests

The new setup for combined active and passive earth pressure tests (see the third paper of the Symposium) has been carefully calibrated and was found to work satisfactorily. The results of the first test performed with it are shown by Figs. 26, 27, and 28.

The model bulkheads were embedded to a depth of 46 in. in softly plastic red clay (Fig. 9) with a consistency of thirty-nine blows on the liquid limit device. This consistency corresponds to an unconfined compressive strength of approximately 300 lb per sq ft in the material. A depth of sand backfill $H = 39$ in. was then placed up to the water level 5 in. below the anchor rods. The average dry density of the sand after the end of the test was 93 lb per cu ft.

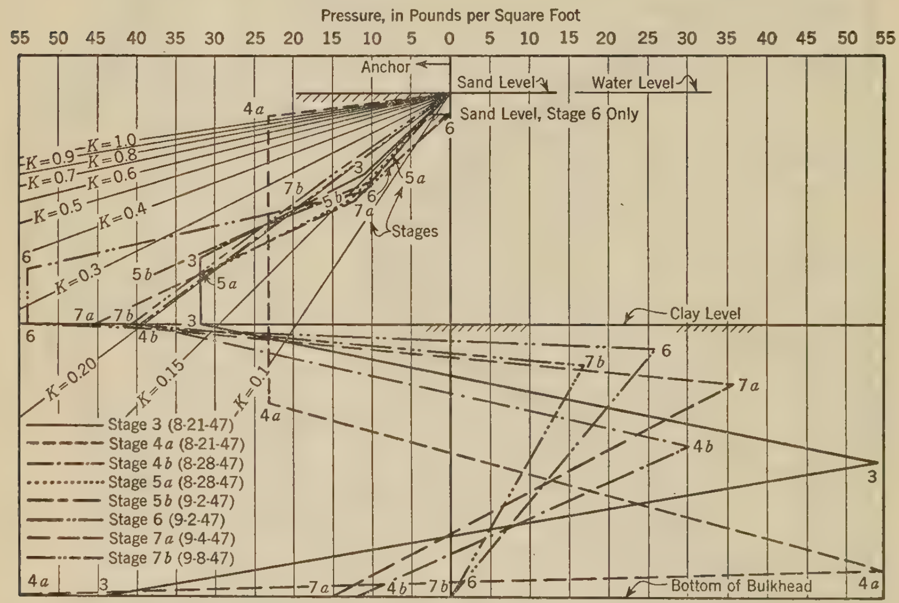

Fig. 28.—Lateral Pressure Distribution (Computed)

Fig. 26 shows the variation with time of the anchor pull A, the variation with time of the maximum bending moment $M_{\max}$, and of the location of $M_{\max}$ in terms of the ratio z/H. All recorded variations are entirely logical and are in agreement with each other and with the testing sequence. It can be seen that the location of the maximum bending moment is just as important as the distribution of active pressures.

The variation for different test stages of the bending moment distribution is shown by Fig. 27(a); of the deflection distribution by Fig. 27(b); and of the estimated pressure distribution by Fig. 28. The progress of the test, recorded by stages, is as follows:

Stage	Date (1947)	Description
3	August 21	Backfill completed on natural slope
4*a*	August 21	Backfill leveled off
4*b*	August 28	Same condition, 7 days later
5*a*	August 28	Anchors released 1 in.
5*b*	September 2	Same condition, 5 days later
6	September 2	Sand backfill compacted by vibration
7*a*	September 4	Sand added to previous level
7*b*	September 8	Same condition 4 days later

The unit pressures were first computed down to the depth z corresponding to the location of the measured maximum bending moment and hence of zero shear. Their distribution to that level was estimated from the two equations:

$$A = \Sigma(p_h\, dz) \qquad (10a)$$

and

$$A\,(z + 5 \text{ in.}) - (\Sigma p_h\, dz)\, z_o = M_{\max} \qquad (10b)$$

This distribution was then similarly checked by means of the measured bending moment at the depth $z/2$ and by means of the procedure governed by Eqs. 1 and 2. The pressure distribution below depth z was estimated in a similar manner.

Test stages 4*b* and 7*b* represent relatively stable conditions and therefore appear most suited for consideration as a basis for design recommendations. Such recommendations can be made in final form only after completion of the entire test program. At present, however, it can be stated that the knowledge of three factors is sufficient for the design of an anchored bulkhead. These three factors are: (1) The minimum safe depth of embedment of the bulkhead; (2) the anchor pull value; and (3) the value of the maximum bending moment.

The first factor, as well as the possible variations of the value z, is the subject of further studies within the third series of tests. Once a safe depth of embedment is selected for a given set of conditions, the knowledge of the depth z at which $M_{\max}$ is located below the backfill surface in conjunction with active earth pressure diagrams of the type shown by Figs. 19, 23, 25(*a*), 25(*b*), and 28 permits the rapid determination of the anchor pull A and of the maximum bending moment. Fig. 28 shows that for the test stages 4*b* and 7*b* the active K_s-value does not exceed 0.22. This substantiates the earlier statement that the tentative recommendations shown by Fig. 25(*a*) are of a conservative nature, as was intended prior to the completion of the test program.

ACKNOWLEDGMENTS

As stated in the foreword by Admiral Smith, the project reported in this paper was sponsored by the Bureau of Yards and Docks of the United States Navy, under Rear-Admiral J. J. Manning, M. ASCE, as chief of the Bureau. Commander L. C. Coxe, as officer in charge of the project for the Navy, devoted considerable time and effort to the organization of this Symposium. Admiral Ben Moreell, Hon. M. ASCE, and Rear-Admirals L. B. Combs and W. H. Smith, and Capt. E. H. Praeger, Members, ASCE, were instrumental

in initiating the project. Lieutenants S. C. Nickerson and R. Z. T. Anthony also took an active part in the early planning stages of the project. Harris Epstein and L. A. Palmer participated in discussions of the test results.

At Princeton University it was Philip Kissam, M. ASCE, who initiated the project. The writer, as engineer in charge of the project, is indebted to Elmer K. Timby, M. ASCE, chairman of the Department of Civil Engineering, and to Kenneth H. Condit, dean of the School of Engineering, for their continued support. Edwin L. Kimble, and Ek-Khoo Tan, as research associates, helped the writer greatly through the early and most difficult stages of its development. The latter, with Harris Epstein, should receive credit for suggesting the sand dike between the bulkhead and the fluid clay backfill. Students, under the supervision of Professor Timby, performed check computations of deflections from measured bending moments. For providing the necessary space, which resulted in an appreciable reduction in cost of the installation, the project is indebted to the Princeton Athletic Association.

The entire design was made at Princeton; and the writer is particularly indebted to N. J. Sollenberger and David H. Crater, Assoc. Members, ASCE, for help in this connection. A total of thirty-six design drawings were prepared for official Navy approval.

The authors of the third paper of the Symposium, as well as J. D. Welch, performed the tests which provided the data reported in Part III.

The mudjacks mentioned in Part I, Section C, were rented from the New Jersey State Highway Department through an arrangement authorized by the courtesy of Spencer Miller, State Highway Commissioner. Other members of the department—Alex W. Muir, F. D. Woodruff, M. A. Lanzara, and C. H. Weller—were materially helpful in this phase of the work.

Roy W. Carlson, Assoc. M. ASCE, designed for the project specially sensitive pore pressure cells which were used to measure the variation of neutral stresses within the backfill in which they were embedded.

Over all, the writer is indebted to the friendly interest of Professor Terzaghi, in acquainting the writer with the potentialities of electric resistivity gages before they became readily available.[7]

SPECIAL FEATURES ON LARGE-SCALE EARTH PRESSURE TESTS

By Edward R. Ward,[33] Assoc. M. ASCE, John R. Bayliss,[34] Esq., and Philip P. Brown,[35] Jun. ASCE

Introduction

The second and the third series of the Princeton tests outlined in Part III of Professor Tschebotarioff's paper required the development of new equipment, the special features of which are described in this paper.

PART I. THE LATERAL EARTH PRESSURE METER

The second series of the Princeton tests is mainly concerned with the determination of lateral pressures under vertical pressure intensities corresponding to values reached on the prototype. The comparative effects of immovable and of elastically and plastically yielding supports on changes of lateral pressure of different types of soils represent another important question studied. In an attempt to isolate and study some of these factors individually, it was decided to construct a device which has been called a "lateral pressure meter." This device is capable of testing a cylinder of soil 1 ft in diameter and 1.5 ft high, with a normal load at the top, and of measuring lateral pressures and lateral displacements at small increments throughout its depth.

Its general shape (see Fig. 29) is that of a thick-walled cylinder cut in half vertically. The left half (M) is solid from top to bottom; but the right half is sliced horizontally into twelve half-circular rings (N), Fig. 29. These rings are mounted on ball bearings so that each one is free to roll out away from the solid half. The end of each ring is connected to the solid half by a dynamometer (R) on which electric strain gages (S) are mounted, thus permitting the total lateral pressure on any ring to be determined. Outward motions of the rings are measured by mechanical dials (V).

Normal vertical pressure is applied to the top of the soil sample by air pressure at point E which acts through a thin rubber membrane (B) that is free to adjust its shape to conform to the contour of the soil surface. This procedure insures that no shearing forces can be transferred to the upper boundary of the soil surface when the vertical pressure is applied. Vertical settlement is measured by a series of indicator rods (F), resting on the rubber membrane in contact with the soil surface, which activate the mechanical dials.

The entire device is surrounded by a watertight jacket (U), Fig. 29, permitting tests on submerged soils. The inner faces of the soil chamber are lined with porous stone (X) to permit lateral drainage.

At the start of a test the soil sample (A) may be consolidated for a given length of time at any desired pressure up to 3 tons per sq ft. During this period of consolidation the variations in lateral pressure at different depths may be noted from the change in the dynamometer readings. The twelve movable rings (N) may be supported in at least three different ways, depending on how the dynamometers holding them are fastened onto the fixed half cylinder: (*a*) Rigidly—bolted down; (*b*) elastically—pulling against a coiled spring (Y); or (*c*) in such a manner that a controlled release is permitted to remain as permanent "set."

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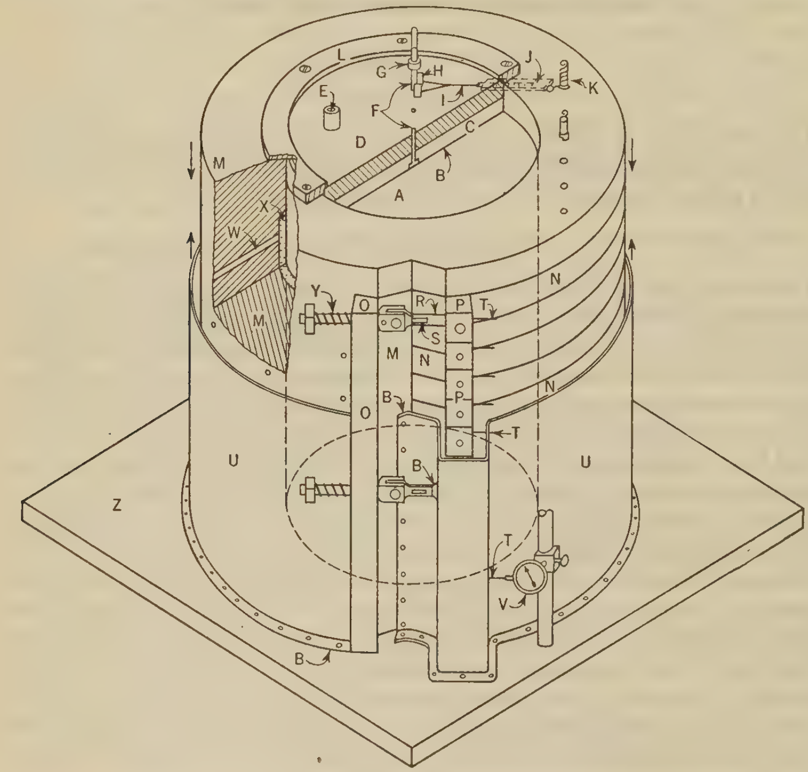

Fig. 29.—Lateral Earth Pressure Meter

The laboratory of the United States Engineer Department at Seattle, Wash., uses a device somewhat similar to Fig. 29 for measuring the lateral pressures of coarsely graded materials only. This machine, however, uses a rigid plunger to apply the vertical load, and the lateral deformations and pressures are measured together as a unit when the observer notes the deformation of the calibrated springs holding its five movable rings in place.

Quite a number of variable testing procedures are afforded by the device described in this paper, depending on the soil used, extent of consolidation, applied pressure, method of supporting the rings, and other factors, for which provisions have been made in the design, such as pore pressure measure-

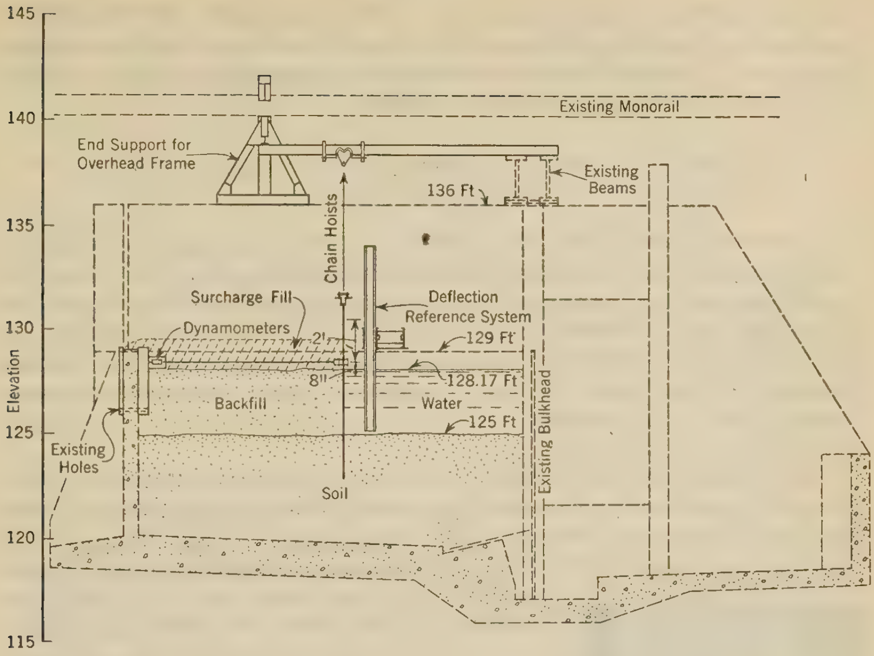
145
140
135
130
125
120
115
Elevation
Existing Monorail
End Support for
Overhead Frame
Existing
Beams
136 Ft
Chain Hoists
Surcharge Fill
Deflection
Reference System
Dynamometers
2'
129 Ft
8"
128.17 Ft
Backfill
Water
125 Ft
Existing
Holes
Soil
Existing Bulkhead

FIG. 30.—MODEL BULKHEAD FOR COMBINED ACTIVE AND PASSIVE EARTH PRESSURE TESTS

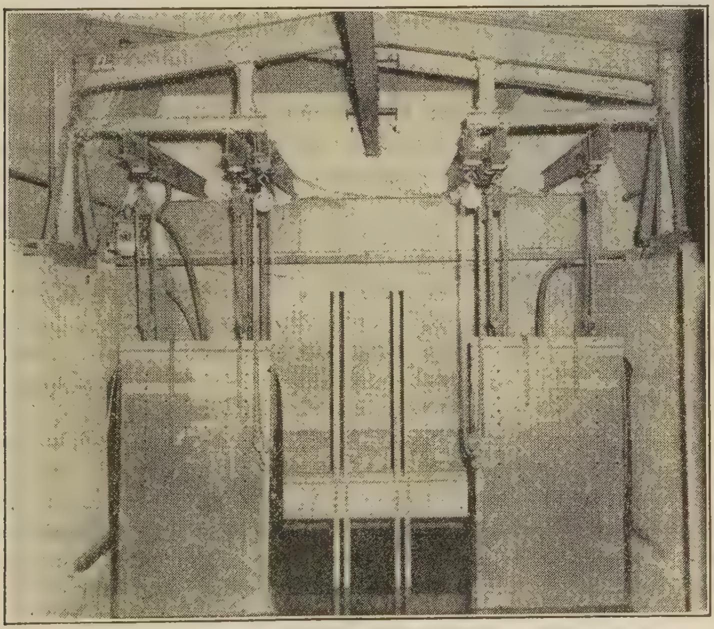

FIG. 31.—VIEW OF THE TWO SIDE BULKHEADS DURING CONSTRUCTION, DECEMBER, 1946

ments and electroosmotic phenomena. Initial preliminary calibrations were completed satisfactorily, using a rubber bag filled with air under pressure in place of soil. Results obtained from tests with actual soil are given in Section B, Part III, of the second Symposium paper.

PART II. COMBINED ACTIVE AND PASSIVE EARTH PRESSURE TESTS

A. General Layout.—The third series of the Princeton tests was concerned with the model study of flexible bulkheads embedded in soil at their lower end. A setup for combined active and passive earth pressure tests was designed for the purpose and is illustrated by Fig. 30. Fig. 31, a view taken from the anchorage end of the bulkhead during construction, shows two side bulkhead sections. At this stage, the center bulkhead section (on which reaction, bending strain, and deflection measurements are taken) had not been placed in position.

The overhead steel structure and chain hoists permit the adjustment of the bulkhead in vertical and longitudinal positions for various test conditions. The vertical channels attached to the rigid horizontal frame shown in the center of Fig. 31 are provided as a support for gages designed to measure the deflection of the center bulkhead. The direct measurement of the deflection of the bulkhead is desired as a check on deflections computed from the bending strain measurements, the latter being obtained by means of electric resistivity strain gages attached to the center bulkhead and waterproofed.

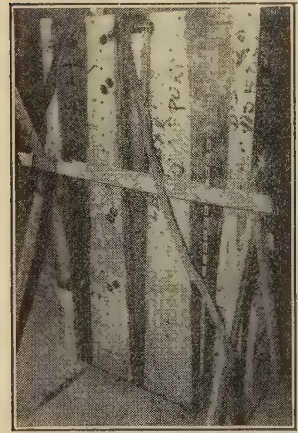
FIG. 32.—VIEW OF THE CENTER BULKHEAD DURING MOUNTING OF THE GAGES, DECEMBER, 1946

B. Underwater Measurements of Bending Strains.—In order to test the model bulkhead for combined active and passive earth pressure, some effective method had to be devised to waterproof the electric resistivity strain gages for use under water. The method, as originally[36] developed, consisted in baking the gages on the plate until proper electrical resistance between gage and plate was obtained. Then a sheet of "Koroseal" was applied over the gages and cemented firmly to the plate with Koroseal cement. Hot vaseline was poured into the semicylindrical space between the gages and the Koroseal.

In Fig. 32, the model bulkhead (5 ft wide) is held in an upright position by a wooden frame. On the right-hand side is an uncovered strip of several gages which have not been waterproofed. The strip on the left shows the manner in which the lead wires were arranged to facilitate application of the Koroseal. At the bottom the Koroseal strip is cemented over the gages. Into this space between the flexible rubber sealing strip and the plate, hot

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[36] "A Method of Effecting SR-4 Strain Gage Operation Under Water," by Edwin L. Kimble, *Proceedings*, Soc. for Experimental Stress Analysis, Vol. III, No. 2, 1946, pp. 53–54.

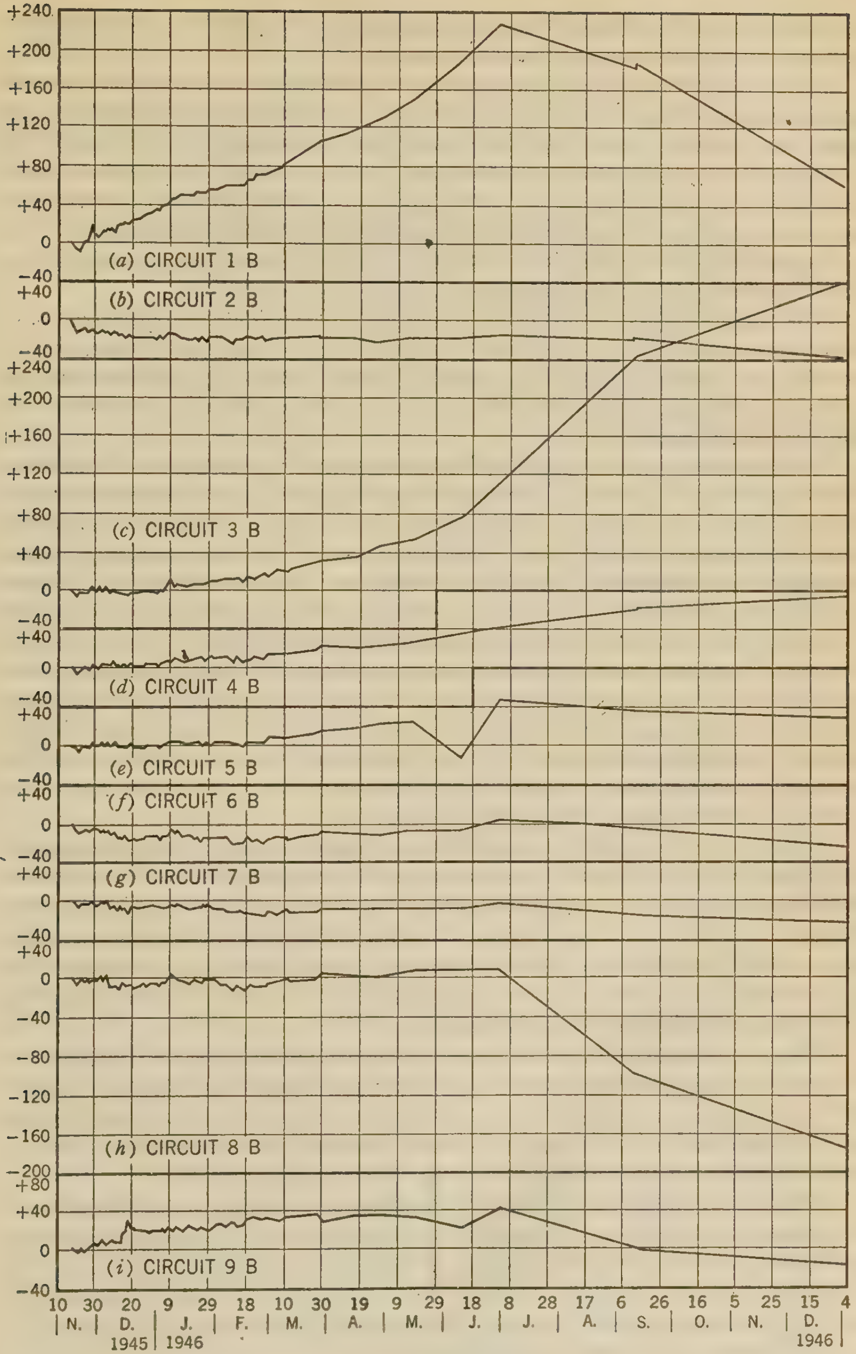

FIG. 33.—RECORD OF SUBMERGED STRAIN GAGE READINGS

vaseline is poured in lifts sufficiently small so as not to burst the bond between Koroseal and plate with hydrostatic pressure.

In the center there is a completed strip—the Koroseal having been covered with a copper strip merely for protection. Two strips are applied at one time—one on either side of the bulkhead. By proceeding in this manner the copper strips can be bolted on immediately after cementing the Koroseal strip and before pouring the vaseline. The copper gives added support to the Koroseal strip and permits pouring molten vaseline in lifts of greater depth. The arched section of the copper is slotted at 1.5-in. intervals to prevent its increasing the rigidity of the bulkhead.

The stability tests of gages waterproofed in this manner were begun in November, 1945. The gages were cemented to an unstressed plate, waterproofed, and then submerged in a tank of water. Direct and reverse readings were taken throughout to eliminate any error resulting from a "zero shift" of the reading instrument.

The behavior of nine gage circuits throughout the period from November, 1945, to December, 1946, is illustrated by Fig. 33. The ordinates indicate variations from the initial readings of the gages. Over a period of 6 months from November, 1945, to May, 1946, only two of the gage readings varied more than approximately 20 micro-inches from the initial readings. These two circuits (Figs. 33(*a*) and 33(*c*)) showed general instability. Since the observers were able to obtain an accuracy of approximately 10 micro-inches during all previous tests with this instrument, after discarding approximately one third of all gage readings subsequent to recalibration, this variation is not excessive. However, beyond this 6-month period three more gage circuits showed variations considerably greater than would be acceptable as reliable readings. Consequently, beyond the 6-month duration, readings cannot be relied on without intermediate calibrations.

Since the tests on the new bulkhead, designed for active and passive pressure research, are planned to run over a maximum period of 2 or 3 months for each test, the stability of gages waterproofed in this manner is entirely satisfactory. It had been hoped that, with this method of waterproofing, strain gages could be applied to full-scale structures in order that strain readings could be recorded over a period of years. However, further research in waterproofing is necessary if this is to be accomplished.

Three independent strips of gages have been placed on the bulkhead at intervals of 2 in. over the lower 3 ft and at intervals of 3 in. over the upper 4 ft. Consequently, three independent gage readings are made at each elevation on the bulkhead. If a gage becomes unreliable, for any reason, there are still two independent readings to average at the same elevation.

With this number and arrangement of gages a bending-moment curve for the stressed bulkhead is possible which occasionally is sufficiently accurate to determine, by direct differentiation, a pressure distribution curve over the active and passive pressure areas. The estimation of lateral pressures is possible, however, in all cases by the procedure outlined under Section C, Part III, of the second Symposium paper.

C. Underwater Deflection Measurements.—To measure deflections it was first found necessary to solve the problem of making underwater measurements with a desired accuracy of 0.001 in. and a range of $1\frac{1}{2}$ in. After much investigation, it was decided that this could be accomplished by the use of the linear variable differential transformer shown in Fig. 34. This is a sketch of the transformer deflection gage as built to meet the requirements of the test. It consists of a primary coil 1 in. long wound on a bakelite spool approximately $\frac{3}{4}$ in. in diameter and carries a constant voltage. On the same spool 2 in. away a secondary coil, 3 in. long, is wound. The voltage of this coil is determined by the position of a relay steel core ($6\frac{1}{2}$ in. long by $\frac{7}{16}$ in. in diameter) as it moves along the axis of the coils. The change in voltage created by the change in position of the core may be measured with a highly sensitive voltmeter. Then, using a previous calibration of the voltage readings against the movement of the core, the extent of this movement can be determined at any time by reading the change of voltage.

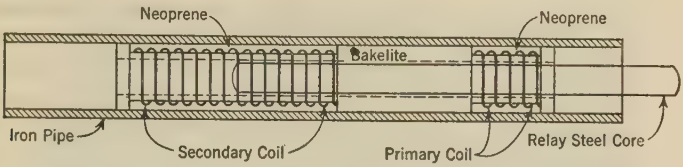

Fig. 34.—Linear Variable Differential Transformer

To ascertain its stability under water, a transformer gage, with its core fixed in a definite position, was submerged in water for about 3 months, no precautions being taken to waterproof it. Periodic readings of the voltage across the secondary coil were taken and were found to vary within about 3%, ignoring errors from power line voltage fluctuations. As an added precaution, however, in the construction of the gages for use on the bulkhead, the coils were waterproofed with "neoprene." The steel cores also were painted with neoprene to prevent them from rusting.

It was decided to mount six of these gages on the vertical channels shown in Fig. 31. These channels serve as fixed reference supports. The gages are placed at three elevations below water level at points where deflection readings are desired, with two gages 20 in. apart at each elevation to measure any twisting of the bulkhead. The cores of these gages are attached to the center bulkhead as shown in Fig. 35.

In addition to the electrical measurement, it was thought advisable to provide some method of a fairly accurate mechanical measurement of the same points both as a check and as a precaution for some unforeseen failure of the transformer gage. A wire, 0.01 in. in diameter, is attached to the end of the core of the deflection gage away from the bulkhead, passing under a pulley, up through the water into the air, over a second pulley and down to, and attached to, the top of a mechanical dial gage graduated to 0.001 in. The spindle of the dial is weighted with a 4-lb weight to keep the wire taut. Any movement of the core of the gage is transferred to the mechanical dial gage where it can be easily read. This procedure also permits the measurement of deflections in excess of the range provided for by the transformer gages.

Instead of using a voltmeter calibrated against the movement of the core of the transformer gage to read deflections, it was thought advisable to balance a seventh gage at point A, Fig. 36, using a null balance indicator (C) against each of the gages (B) attached to the bulkhead, and a method was provided to read the deflections directly. By exercising care in manufacture, seven gages were produced which, when tested, showed a total error of only 0.003 in. in the entire range of 1.5 in. The manufacturer built these gages so that they all have the same electrical characteristics in order that an equal movement of the cores (D) in any of the gages gives the same voltage change in any of the secondary coils.

A micrometer screw (E) was attached to one of the gages and was mounted on the null balance indicator. With a selector switch (F), Fig. 36, this gage can be connected electrically in a circuit with any of the other six gages which are attached to the bulkhead at points where the deflections are measured.

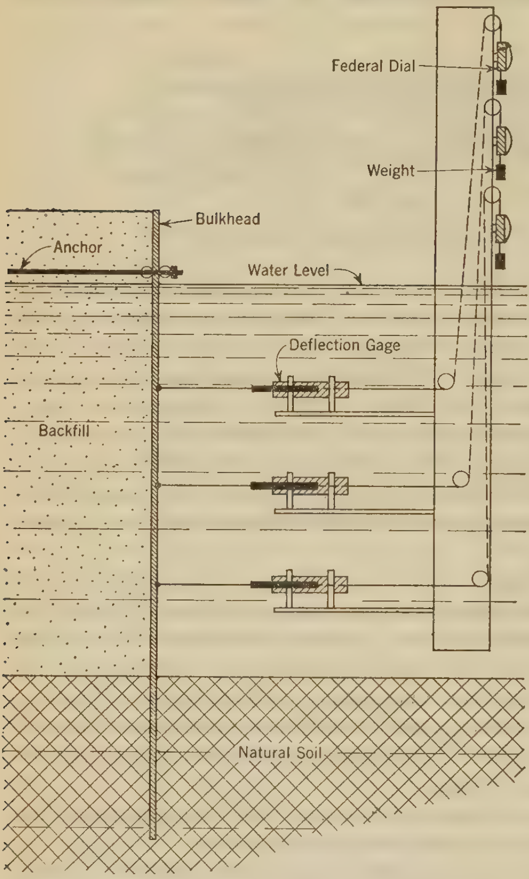

Fig. 35.—Underwater Deflection Measurements

The null balance indicator was placed in the circuit in such a way that, if there is any movement of the core of the gage at the bulkhead, the pointer on the indicator moves off the zero point. The core of the instrument gage is then moved in or out until the indicator again reads zero. The difference in the readings on the micrometer screw before and after moving equals the movement of the core of the bulkhead gage. The circuit was designed so that any change of voltage in the primary coil from 95 v to 135 v, well within the possible voltage fluctuation of the power supply, has no effect on the relative movement of the cores.

Since it was also found necessary to heat up the gages before readings are taken, a second selector switch (G), Fig. 36, was placed in the circuit, thus permitting all the gages to be heated while recordings are being made. With this transformer gage, a 0.0001-in. movement of the core of the bulkhead gage can easily be detected, with an accuracy of 0.001 in.

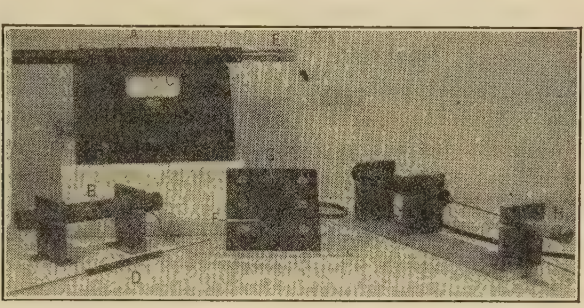

Fig. 36

Because it was desirable to have some method of checking the instrument, a second micrometer screw (H) was provided that can be attached in turn to any of the bulkhead gages. The core of the bulkhead gage can thus be moved in or out any desired distance, and the instrument can then be balanced with the instrument micrometer screw. The movement of the instrument screw is read; and, if any discrepancy exists between this reading and that of the movement of the core of the bulkhead gage, an adjustment can be made with a potentiometer shunted across the primary coil of the bulkhead gage.

Preliminary tests with these gages proved satisfactory. However, during the first bulkhead tests, displacements were found to exceed the range of the gages (see Figs. 26, 27, and 28) so that the mechanical deflection measuring device had to be relied upon.

ACKNOWLEDGMENT

The setup illustrated by Fig. 30 became possible as the result of a procedure developed by Edwin L. Kimble,[36] which permitted the use of the electric resistivity strain gages under water. Professor Tschebotarioff adapted this procedure to the model testing problems at Princeton. The conception and program of study with the lateral pressure meter were developed by Professor Tschebotarioff. C. E. Kjetsaa, instructor in mechanical engineering, supervised machine work and made many helpful design suggestions.

APPLICATION OF TEST RESULTS TO QUAY WALL DESIGN

By Harris Epstein,[37] Esq.

Economic Considerations

The problem of reducing the cost of building quay walls and bulkheads is quite old and antedates the science of soil mechanics. Even in "prehistoric" times—when no great distinction was made between clays and sand, when clays were considered ideal foundation materials, before any such thing as pore pressure had been discovered to plague the conscience of economically minded designers, when the subway walls of all the subway systems in New York, N. Y., and Philadelphia, Pa., were designed for a flat K-value of 1/3—even in those distant days the Bureau of Yards and Docks of the United States Navy Department had been grappling with the problem of the excessive cost of quay walls and how to reduce them. The main cost of quay wall construction is invested in two items—(1) the cost of the sheet piling, and (2) the cost of the backfill. To be significant, any saving must be effected in the cost of these two items. The better the backfill material, the shorter will be the length, and the lighter the weight, of sheet piles required for a given dredged depth, and the less will be the cost.

Good filling material, such as sand and other noncohesive soil, is not always available at the site where it can be pumped in hydraulically at a relatively small cost. When good filling material must be brought in from a distance and handled by barges, its cost is greatly increased and, if the quantity of the backfill so handled is large, its increase in cost will more than offset any saving that might be effected in the sheet piling. The same applies to the case in which the original ground underlying the quay wall site is considerably above the required dredged depth. When the original material into which the sheet piles are driven is more or less of a soft, cohesive, unconsolidated nature, the active pressure on the wall becomes quite large especially under the surcharge of heavy live loads. At the same time the resisting passive pressure below the dredged bottom is greatly reduced—resulting in greatly increased length and weight of the sheet piling with considerably increased cost of the quay wall. In fact, where the dredged depth is greater than 25 ft, the moment caused by pressures of unconsolidated backfill is increased to such a magnitude as to preclude the use of commercially rolled steel pile sections. A conventional installation, utilizing the actual materials found in place, is shown in Fig. 37. Again, the evident remedy is to dredge off the soft material and substitute a good granular fill. In this case, however, the cost of filling must also include the cost of dredging and disposing of the original soft material, and this must be balanced against the saving effected in the steel sheet piling.

The unit cost and the quantity of the required borrowed backfill are the two factors that determine whether the use of this borrowed fill construction method is economical. It is not necessary, however, to have the entire fill made of imported granular material; enough good material may be placed to form a sand dike behind the wall, and the remainder of the backfill may be pumped in hydraulically from the local inferior material.

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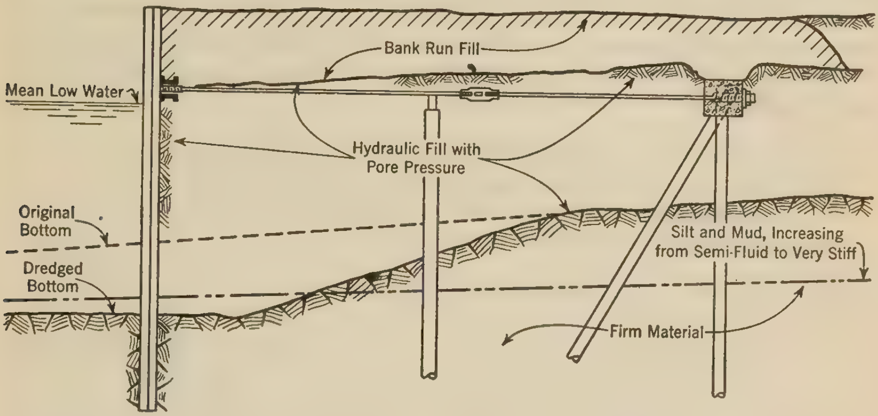

Bank Run Fill
Mean Low Water
Hydraulic Fill with Pore Pressure
Original Bottom
Dredged Bottom
Silt and Mud, Increasing from Semi-Fluid to Very Stiff
Firm Material

Fig. 37.—Conventional Construction of Quay Wall

Construction Methods

The Bureau of Yards and Docks has made use of the borrowed fill method of quay wall construction in a number of instances. Representative examples are the quay walls at Key West and Jacksonville, Fla. At Key West, a quay wall of steel sheet piling (MZ32) was designed for a 22-ft dredged depth to take an hydraulically pumped fill behind a light coral rock dike. At Jacksonville, the use of a relieving platform and a dike of oyster shells made it possible for a steel sheet pile wall (MZ38) to resist 48.0 ft of hydraulically pumped fill of local fine sand. In these cases the relief in lateral pressures is attributed to the high angle of repose and the light weight of the dike materials.

In adopting this construction procedure the reasoning was based mainly on common sense analogy with earth dams, where an earth fill is depended on to keep back the full water pressure. In the case of a quay wall, the pressure on the wall is considerably less than that on a dam of comparable height, since the water pressure acts on both sides of the wall and, therefore, is equalized. The natural slope of the downstream side of the earth dam is replaced in the quay wall construction by the sheet pile wall. The width of the top berm in earth dams is determined primarily by requirements of imperviousness to water seepage and practical construction considerations with respect to the use of land equipment, in addition to the resistance requirements for the full horizontal water pressure. The necessity of a top berm and its size for sand dikes in the construction of quay walls cannot be determined by analogy with earth dams since, in this case, there is no need for imperviousness. There is also no need for construction roads, when floating equipment is used, and the horizontal pressure is also smaller than in the case of dams. Consequently a small top berm has been used by the Bureau for all dikes in the construction of quay walls.

The successful use of this construction method did not extend to hydraulic clay fills; it did not establish the minimum dimensions of the dike required to be fully effective in reducing the horizontal pressure of the hydraulically placed fluid fill; nor did the installations lead to any clues by which to judge how, and in what manner, these reductions in the fluid pressure are effected.

Reduction in pressure can be conceived as resulting from the three following causes:

a. The structural strength or shearing resistance of the sand dike;
b. The continuous drainage course afforded by the sand dike; or
c. The increased friction of the sand against the sheet piles as compared to the friction of the fluid clay.

Therefore it was important to determine by test:

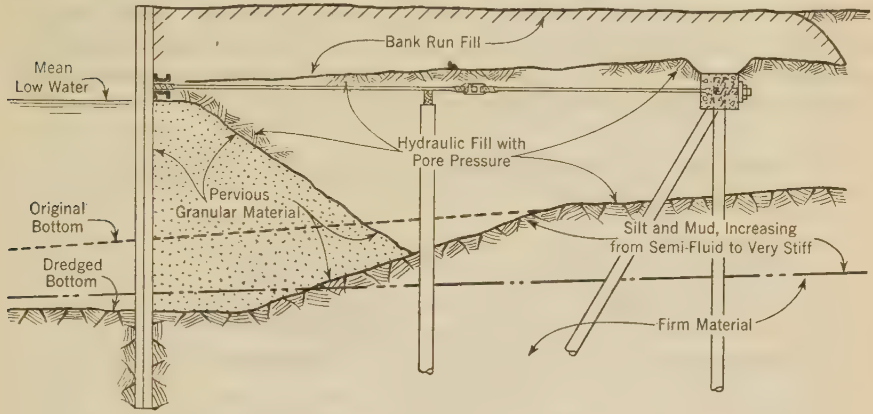
Case 1. The minimum dimensions of the sand dike to make it fully effective against an hydraulic clay fill (the sand dike may be of rectangular, triangular, or trapezoidal shape); and
Case 2. The variation of the effectiveness of the dike, with its size and dimensions, which would also furnish an indication as to how the reduction in the fluid pressure is accomplished.

The full effectiveness (case 1) of a sand dike is said to be realized, when the resultant pressure against the wall is the same as would have obtained had the entire fill behind the wall been made of the dike material. These were the two original objectives of the Princeton tests. The conclusions reached, based on the tests, are given in Professor Tschebotarioff's paper. One states that the full effectiveness of a sand dike is attained either when the dike is placed on the natural sand slope (1:1.73) or when a uniform vertical sand blanket of a thickness equal to height of the wall is interposed between the sheet pile wall and the fluid backfill.

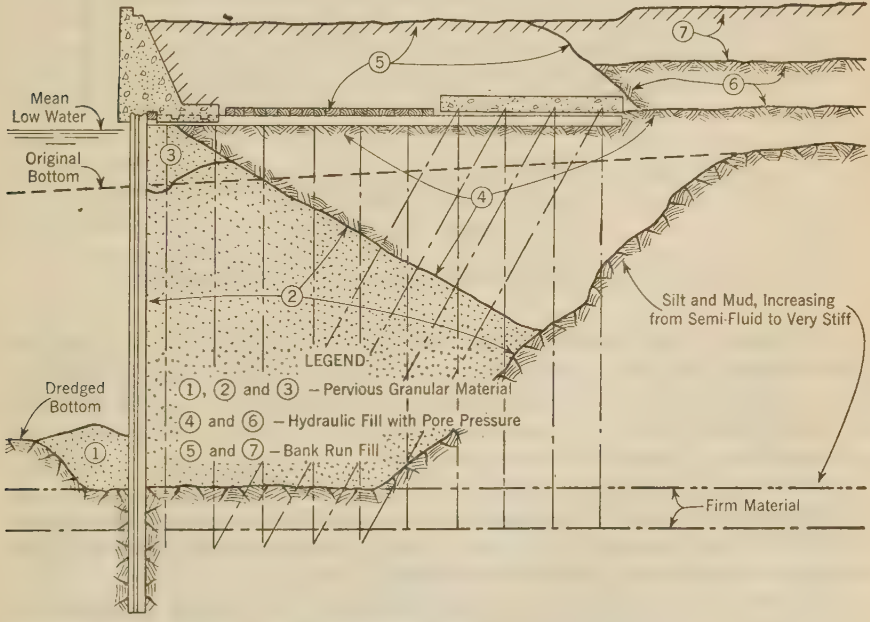
The sloping dike was provided with a 1-ft berm on the top. In practice the sloping dike fill is the one to be used, since it can be placed more economically and also since the quantity of the backfill is less than the quantity required in the vertical blanket as long as the berm width is less than 0.135 times the height of the wall. Proposed typical design and construction procedures are shown in Fig. 38 for the two site conditions discussed at the beginning of this paper.

Another conclusion from the tests as given by Professor Tschebotarioff is that, if a vertical sand blanket of a thickness equal to one half of the bulkhead height is used, its effectiveness in decreasing the fluid clay pressure against the wall is reduced by one half and the effectiveness of the sand blanket is zero when its thickness is one tenth of the bulkhead height. The manner in which the effectiveness of the dike varies with the nature of the dike material —for instance, when gravel is used in the dike—was not determined. The present tests do not furnish enough data to find the variation of the effectiveness of a vertical (rectangular shaped) sand blanket with its thickness. It is expected that the future tests will supply the missing data.

There are sufficient data, however, to indicate that the drainage has very little influence on the reduction of the fluid pressure against the wall. The drainage effect is evidently the same whether the thickness is 0.1 or 0.5 of the wall height H; and yet a thickness of 0.1 H was entirely ineffective in reducing the fluid pressure of the clay fill. Although there can be no doubt that the


Bank Run Fill
Mean
Low Water
Hydraulic Fill with
Pore Pressure
Pervious
Granular Material
Original
Bottom
Silt and Mud, Increasing
from Semi-Fluid to Very Stiff
Dredged
Bottom
Firm Material

(*a*) WITHOUT RELIEVING PLATFORM


7
5
6
Mean
Low Water
3
Original
Bottom
4
2
Silt and Mud, Increasing
from Semi-Fluid to Very Stiff
LEGEND
①, ② and ③ – Pervious Granular Material
④ and ⑥ – Hydraulic Fill with Pore Pressure
⑤ and ⑦ – Bank Run Fill
Dredged
Bottom
1
Firm Material

(*b*) WITH RELIEVING PLATFORM

Fig. 38.—Proposed Method of Building Quay Wall

drainage factor must be operative, its effect is evidently too slow to make itself felt under the driving force of the material's own weight.

The action of the drainage course under a surcharge loading is not known. The influence of surcharge on the dimensions and the effectiveness of the sand dike is very important in the design of quay walls. No tests have been performed to date (October, 1947) to determine this influence. It is strongly suspected that a definite width of berm at the top of the sand dike will be required in this case for full effectiveness. This is one of the problems for which a solution will be sought in the continuing tests.

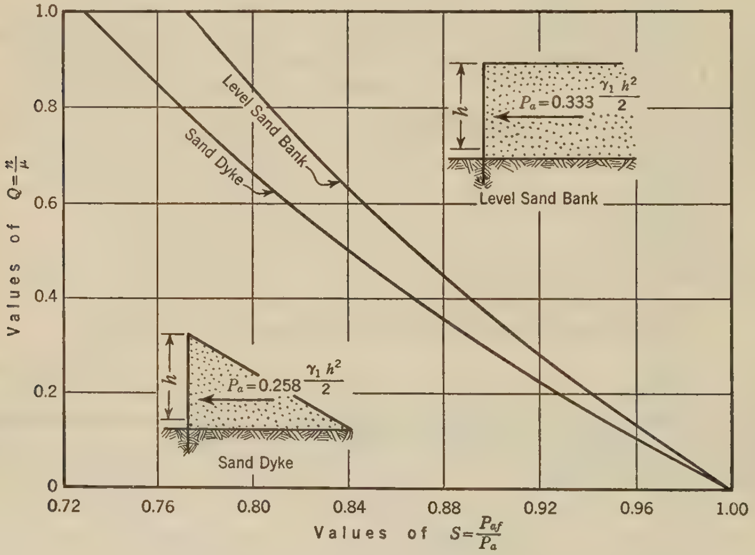


Fig. 39.—Influence of Wall Friction on Active Sand Pressures

The factor of wall friction, on the other hand, may play a more significant role in making the dike an effective barrier. The beneficial effect of wall friction is based on the transfer of a portion of the weight of the sliding wedge to the sheet piling, thereby reducing the weight that causes sliding along the rupture plane. Undoubtedly the interposition of a sand blanket of thickness 0.1 H induces a potential friction force at the wall, and this was actually confirmed by the tests. The reason that no reduction in the pressure applied to the wall was recorded in this case is that a sand thickness of 0.1 H is evidently not sufficient to render the wall friction effective.

Fig. 39 shows the influence of wall friction on the active pressure of a sand backfill, both for a level bank and a triangular dike. In Fig. 39, n denotes the tangent of the angle of wall friction; μ denotes the tangent of the angle of internal friction of the backfill material (0.577 for sand); γ_1 denotes the unit weight of sand; P_{af} denotes the maximum horizontal pressure per linear foot of wall at any height, h, and any value of n greater than zero; P_a denotes the corresponding value of the horizontal pressure for $n = 0$; Q denotes $\frac{n}{\mu}$; and S denotes $\frac{P_{af}}{P_a}$. The capacity of any vertical section of the sliding wedge to transmit a portion of the weight of the wedge to the next adjoining section is measured by the shearing resistance of the section, namely $P_{af}\,\mu$. In no case will it be possible to mobilize an effective friction force, between the sand and the sheet piling, of magnitude greater than the shearing resistance of the backfill material adjacent to the wall. Since the available shearing resistance along a vertical section at the boundary between the sand dike and the fluid clay is zero, it follows that the reduction in the force that causes sliding can be made effective only for that portion of the wedge lying between the wall and the sand boundary—that is, for the portion of the wedge composed of sand.

Thus the maximum possible weight which can be transferred to the piling by wall friction is realized for the case where the entire sliding wedge is composed of sand—that is, for a vertical sand dike of a thickness equal to the width of the sliding wedge, or 0.577 H. However, even with this width of dike, the maximum possible reduction in the active pressure against the wall is comparatively small as can be seen from Fig. 39. The values of S for $Q = 1.0$ (that is, when the wall friction is equal to the internal friction of the sand) are 0.77 for level bank and 0.73 for a triangular dike, thus indicating values of 23% and 27%, respectively, for the maximum reduction in horizontal pressure resulting from wall friction. A reduction of this magnitude cannot account for the full effectiveness of a sand dike; the principal source of the dike's capacity to relieve the sheet piling must come from the third factor—namely, its structural strength.

THE CONCEPTION OF STRUCTURAL STRENGTH OF THE TRIANGULAR SAND DIKE

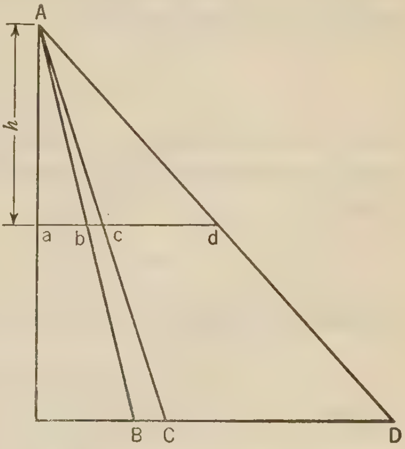


FIG. 40.—COMPARISON OF PRESSURE INTENSITIES

Line AB, Fig. 40, is the original pressure against the wall exerted by the sand dike itself. The intensity of pressure at any distance h from the top is

$$p_s = \overline{\mathrm{ab}} = K_1 \gamma_1 h \ldots\ldots (11a)$$

in which γ_1 is the buoyed weight of sand and K_1 is the active pressure coefficient for the triangular sand dike. Line AC, Fig. 40, is the pressure against the wall after the liquid fill is pumped in behind the sand dike. The intensity of pressure now, by reason of the full effectiveness of the dike, is

$$p_a = \overline{\mathrm{ac}} = K_2 \gamma_1 h \ldots\ldots\ldots\ldots (11b)$$

in which K_2 is the active pressure coefficient for a level sandbank. The increase in the intensity is

$$p = p_a - p_3 = \overline{\mathrm{ac}} - \overline{\mathrm{ab}} = \overline{\mathrm{bc}} = \gamma_1 h (K_2 - K_1) \ldots\ldots\ldots (11c)$$

Line AD, Fig. 40, represents the pressure of the liquid clay against the sand dike, which is the same as for a wall with no interfering dike, with intensity of pressure at the same depth h of magnitude:

$$p_4 = \overline{\mathrm{ad}} = \gamma_2 h \ldots\ldots\ldots (11d)$$

in which γ_2 is the buoyed weight of clay, the lateral pressure being the same as the vertical pressure. The difference between the pressure intensity against the wall and the fluid clay pressure intensity against the dike is

$$p_d = \overline{\mathrm{ad}} - \overline{\mathrm{ac}} = \overline{\mathrm{cd}} = h (\gamma_2 - K_2 \gamma_1) \ldots\ldots\ldots (11e)$$

Thus, passing through the dike, the intensity of pressure dropped by the amount $\overline{\mathrm{cd}}$, Fig. 40, because of the resistance of the dike, similar to a loss in potential across an electrical resistance, or reduction in head in a water pipe due to friction.

The resisting capacity of the dike may perhaps be better studied if, instead of intensities, total pressures are considered. The total active pressure, P_a, of a soil material with any positive or negative angle of surcharge, $\pm \theta$, and any angle of internal friction, ϕ, is

$$P_a = 0.5 K_a \gamma_e h^2 \ldots\ldots\ldots (12)$$

in which γ_e is the effective weight of soil and K_a is the active pressure coefficient determined from the relation:

$$K_a = \frac{\alpha_a - \mu}{(\alpha_a - \beta)(1 + \alpha_a \mu)} \ldots\ldots\ldots (13)$$

In Eq. 13, α_a is the tangent of the angle X defining the plane of rupture and β is the tangent of the angle θ defining the surcharge. The tangent α_a is defined as

$$\alpha_a = \sqrt{(1 + \mu^2)\left(1 - \frac{\beta}{\mu}\right)} + \mu \ldots\ldots\ldots (14)$$

Similarly, the passive pressure coefficient is

$$K_p = \frac{\alpha_p + \mu}{(\alpha_p - \beta)(1 - \alpha_p \mu)} \ldots\ldots\ldots (15)$$

in which

$$\alpha_p = \sqrt{(1 + \mu^2)\left(1 + \frac{\beta}{\mu}\right)} - \mu \ldots\ldots\ldots (16)$$

The solution of Eq. 13 for representative values of $\mu = \tan \phi$ is presented in Fig. 41 and the solution of Eq. 15 is presented in Fig. 42.

In the sand dike case $\theta = -\phi = -30°$; therefore, $\frac{\beta}{\mu} = \frac{\tan \theta}{\tan \phi} = -1.0$ and K_a from Fig. 41 equals 0.258. Substituting for K_a in Eq. 12 gives the total active pressure at height h. This procedure presupposes that the full shearing resistance of the sand dike has been mobilized. Eqs. 13 and 15 are ordinarily used in design as a standard procedure by the profession at large. The shearing strength of the sand dike is increased after the placing of the liquid clay by the increased normal pressure on the sloping dike surface produced by the buoyed weight of the clay fill; but the gain is small compared with the additional lateral pressure exerted by the liquid clay.

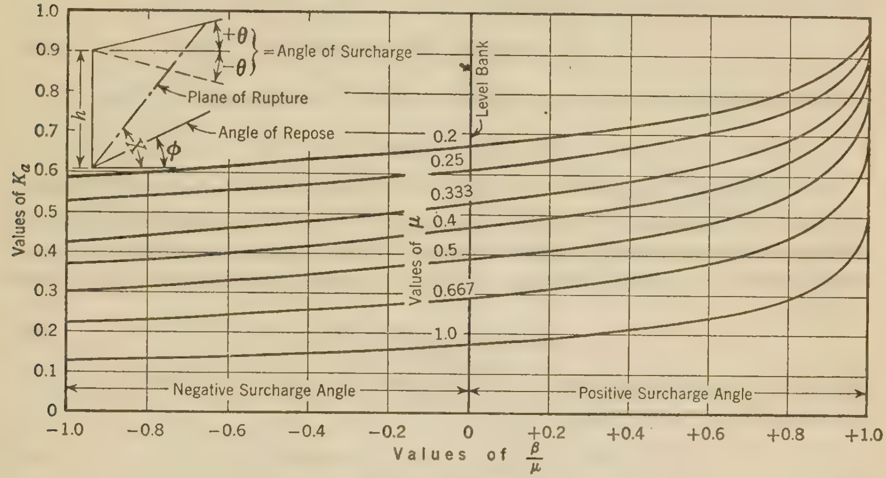


FIG. 41.—VALUES OF ACTIVE PRESSURE COEFFICIENTS K_a

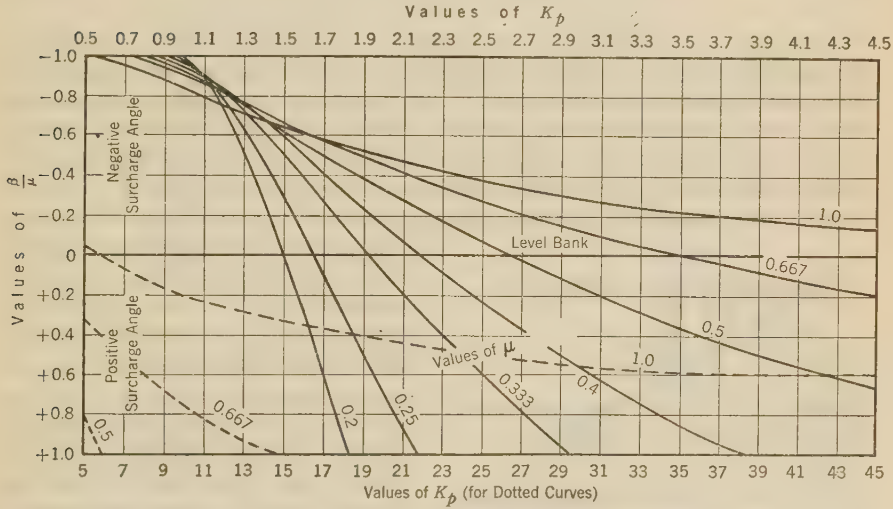


FIG. 42.—VALUES OF PASSIVE PRESSURE COEFFICIENTS K_p

Consider the equilibrium of the trapezoidal prism ABCD, shown in Fig. 43, above an assumed rupture plane BC making an angle X with the horizontal.

The weight of the upper triangle ABC of the sand dike is

$$W_1 = \gamma_1 h^2/2\,(\alpha_x + \mu) \qquad (17)$$

in which α_x denotes the tangent of angle X. The weight of triangle ACD of the clay fill on the rupture plane is

$$W_2 = \gamma_2 h^2 \mu/2\,(\alpha_x + \mu)^2 \qquad (18)$$

The horizontal force of the clay fill on side CD of triangle ACD is

$$P_3 = W_2 \mu \qquad (19)$$

The tendency of the prism to move down along the rupture plane toward the sheet pile wall results from the body forces W_1 and W_2 and from the force P_3. Resistance to movement is offered by the friction along the surface BC and by the wall's horizontal reaction, which is equal in magnitude to the active pressure

$$P_{ax} = P_1 + P_2 + P_3 \qquad (20)$$

in which P_1 and P_2 are the horizontal pressures due to weights W_1 and W_2 and are given by formulas:

$$P_1 = W_1\,(\alpha_x - \mu)/(1 + \alpha_x\,\mu) \qquad (21a)$$

and

$$P_2 = W_2\,(\alpha_x - \mu)/(1 + \alpha_x\,\mu) \qquad (21b)$$

The formulas are based on the assumption that the full shearing strength of the sand dike has been mobilized in each case. The active pressure (P_{ax}-curve) of the wall, at any depth h is plotted in Fig. 44 for various slopes of the rupture plane and for values of γ_1 and γ_2 of 60 lb per cu ft and 56 lb per cu ft, respectively. According to the conclusion obtained from the tests, the pressure should be the same as for a level sandbank, which, by Eq. 12, with $K_a = 0.333$ (Fig. 41 for $\mu = \tan 30°$) and $\gamma_e = 60$ lb per cu ft is $P_a = 0.5\,K_a\,\gamma_e\,h^2 = 10\,h^2$. The pressure is represented by the P_a-curve in Fig. 44, which is an arc of a circle with radius $10\,h^2$. For rupture planes in which the P_{ax}-curve falls within the P_a-curve, there is reserve strength in the wall and no additional shearing resistance is required in the sand dike. However, in the region from $X = 41°$ to $X = 68°$, the P_{ax}-curve falls outside the P_a-curve, showing that additional shearing resistance must be provided in the sand dike in order that the force against the wall should not exceed $P_a = 10\,h^2$.

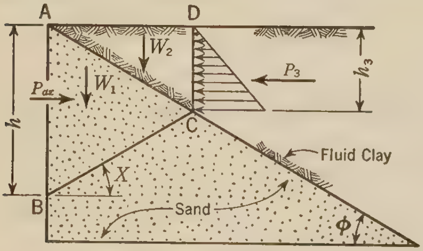


FIG. 43.—FORCES ACTING ON A TRIANGULAR DIKE

This strength may be provided in the form of a horizontal berm at the top of the sand dike. The inset diagram in Fig. 45 shows such a dike having a berm of width $t\,H$, t being a numerical factor less than 1.0. The weight of quadrilateral ABCD of the sand dike above the rupture plane BC is

$$W_1 = \gamma_1 [1 + \mu\, t\, (2 + t\, \alpha_x)\, H^2/2\, (\alpha_x + \mu) \qquad (22)$$

The weight of triangle CDE of the clay fill on the rupture plane is

$$W_2 = \mu\, \gamma_2\, (1 - t\, \alpha_x)^2\, H^2/2\, (\alpha_x + \mu)^2 \qquad (23)$$

The horizontal force of the fluid clay on triangle CDE is

$$P_3 = W_2\, \mu \qquad (24)$$

The tendency of the prism to move downward along the rupture plane toward the wall results from the forces W_1, W_2, and P_3. Resistance to movement is offered by the friction along the surface BC and by the horizontal reaction P_{ax} of the wall which is equal in magnitude to the active pressure on the wall and is given by the formula:

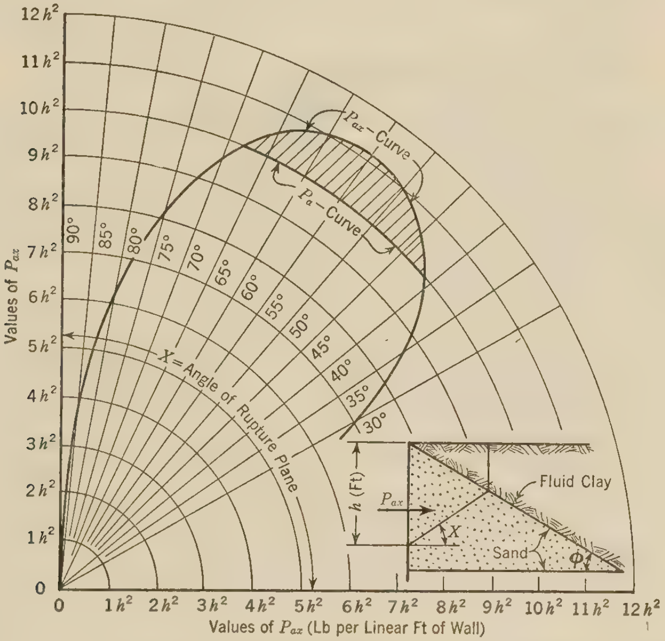


FIG. 44.—ACTIVE PRESSURES FOR TRIANGULAR DIKES

$$P_{ax} = P_1 + P_2 + P_3 \qquad (25)$$

in which P_1 and P_2 are the horizontal pressures due to weights W_1 and W_2 and are given by formulas:

$$P_1 = W_1\, (\alpha_x - \mu)/(1 + \alpha_x\, \mu) \qquad (26a)$$

and

$$P_2 = W_2\, (\alpha_x - \mu)/(1 + \alpha_x\, \mu) \qquad (26b)$$

The active pressures (P_{ax}-curve) at the total depth H is plotted in Fig. 45 for various slopes of the rupture plane and for three different values of $t = 0.1$, $t = 0.2$, and $t = 0.4$ and for values of $\gamma_1 = 60$ lb per cu ft and $\gamma_2 = 56$ lb per cu ft. The curve $P_a = 10\ H^2$ corresponding to the total pressure of a level bank is also plotted in Fig. 45. An analysis of these curves shows that the only P_{ax}-curve that falls entirely with the P_a-curve is the curve for $t = 0.4$—indicating that a sand dike to be fully effective must have a berm on top equal to 0.4 of the height of the wall. The berm provided in the test is considerably smaller. The discrepancy may be explained by the probable error in measurement readings, estimated to be approximately 5%. Errors of this magnitude are sufficiently large to account for the difference. Thus the maximum total pressure for the dike with berm width $t = 0.2$ is $10.5\ H^2$, whereas that for a level sandbank is $10.0\ H^2$—a 5% difference. Another possible explanation is that the fluid clay in the lower regions may have consolidated partly and did not exert full liquid pressure against the dike.

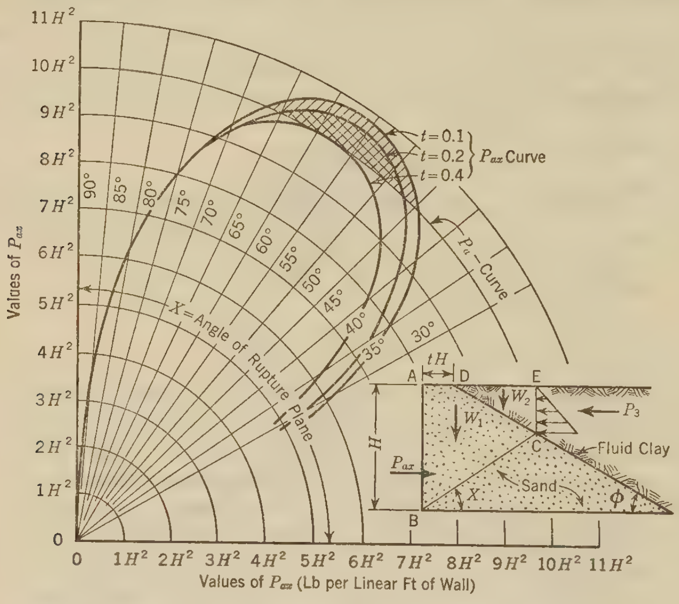


Fig. 45.—Active Pressures for Trapezoidal Dikes

As the tests progressed, they branched out in various directions, and the scope of the objectives was extended to include studies of such problems as the determination of the K-value for liquid clay, the presence of arching in cohesive materials, and other related questions.

VERTICAL ARCHING

The Bureau of Yards and Docks has never taken advantage of arching in design, for the reason that arching is not considered a permanent phenomenon, although the presence of arching in granular material has never been questioned. The arching problem was added to the objectives of this study, for the purpose of determining whether arching is also effective in cohesive material. The findings of Professor Tschebotarioff relative to this question, therefore, were in the nature of a big surprise. According to these findings, there is no arching even in granular material when the backfill is built up against the wall in steps. However, after a little thought, the absence of arching in this case seems to be reasonable, since the layers of sand during deposit are not restrained on the surface at each filling stage and the condition is somewhat similar to the cantilever wall where no arching is possible. Equally reasonable is the conclusion that the vertical arching, where present, is of an unstable character, resulting from the effect of compaction and vibration. This is the main reason why the Bureau always neglected the vertical arching effect in all designs of flexible quay walls.

DESIGN CONSIDERATIONS AND RECOMMENDATIONS

The fact that the stability of a bulkhead depends on two components of different materials—soil and the structural members—raises an economical consideration that is of even greater importance to the structural engineer than it is to soil scientists. It is a known fact that there are no quay wall failures on record that can be attributed to failures of sheet piles; most of those recorded are the result of the movement of the upper anchors and, to a limited extent, of insufficient penetration of piles at the bottom. Thus, the failures of bulkheads are largely the fault of the soil and not the structural members. In other words, sheet piles do not fail in bending and tie rods do not fail in tension. The reason for this can be mainly ascribed to the fact that the structural members are designed for a much higher factor of safety than that usually allowed for the soil. In any design, if the full values for the angle of internal friction, cohesion, and wall friction (as found in the laboratory from boring samples) are used—the usual and accepted procedure—a factor of safety is entirely lacking in the bulkhead in so far as the soil resistance is concerned.

It has been customary to neglect the cohesion factor for all soils having an angle of internal friction greater than about 17° in computing both the active and passive pressures according to the formulas (for active pressures):

$$P_a = \gamma_e h \tan^2\left(45° - \frac{\phi}{2}\right) - 2c\tan\left(45° - \frac{\phi}{2}\right) \ldots\ldots\ldots (27a)$$

and (for passive pressures)

$$P_p = \gamma_e \tan^2\left(45° + \frac{\phi}{2}\right) + 2c\tan\left(45° + \frac{\phi}{2}\right) \ldots\ldots\ldots (27b)$$

in which c is the cohesion of the soil per unit area. This procedure has been followed purposely to provide the missing factor of safety for the soil. This factor of safety may be only imaginary, however, and cannot be depended on. In the first place, friction and cohesion may not act simultaneously, the resistance of the two being somewhat similar to the resistance offered by a joint partly welded and partly riveted. The frictional resistance of the soil is mobilized almost at once whereas the mobilization of the cohesive resistance requires appreciable movement, and it is doubtful whether the deflection of the bulkhead allows enough movement in the soil to mobilize the full cohesion. The omission of any beneficial effect from arching, even in cases where arching exists and could be depended on, affords a greater factor of safety for sheet piles; but it does not help the soil situation. The only reliable factor of safety realized in conventional designs is the omission of the beneficial effect of wall friction in computing the active and passive pressures. However, the magnitude of the factor of safety afforded by this omission is not considerable.

Fig. 39 shows the variation of the active pressure with the angle of wall friction for a sand fill for either a level bank or a dike. As noted from the curve, the maximum possible reduction in the active pressure, when the angle of the wall friction is equal to the angle of internal friction, is only about 23% for a level bank, which is equivalent to a factor of safety of 1.3. The omission of the wall friction for the passive pressure will probably raise the factor of safety to about 1.75. When computing the resistance of the wall to a Swedish slide, most designers consider themselves lucky when they obtain a factor of safety of 1.50. Moreover, the factor of safety of steel is based on the elastic limit which ordinarily means a factor of safety of about 4, on the ultimate strength, whereas the factor of safety of the soil will seldom, if ever, reach a value of 2.0. The disparity between the factors of safety of the two materials will not be affected by any increase in the active, or reduction in the passive, pressures, since the sizes of the structural members will be increased by this procedure in the same proportion. Since the strength of a chain is determined by its weakest link, there is evidently an undue waste of money and material resulting from the use of present design methods. A recommendation to increase the allowable unit stress in the structural parts to approximately the elastic limit of the material would seem to be in order. As to the factor of safety of the soil, it would seem logical to adopt the following procedure: For the active pressure, (*a*) use the full values for the angle of internal friction, cohesion, and wall friction as found from tests, (*b*) apply a factor of safety of 1.5 to the theoretical values of the passive pressures in determining the size and dimensions of the upper anchor reactions, and (*c*) omit the effect of wall friction in figuring passive pressures.

MODEL TEST REQUIREMENTS

To derive more or less rational formulas from model tests, the model must include all the variables that may have an influence on the problem to be solved. Then, and only then, can a rational interpretation of the model results be made and a proper application to the prototype be assured. The problem of pressure distribution on flexible walls involves the interaction of two different materials—the soil and the sheet piles. It should be studied, therefore, not only from the points of view of the elasticity and the plasticity of the soil, but also from the points of view of the elasticity and the stiffness of the sheet piling. The magnitude and the distribution of pressures on the bulkhead must be such that the corresponding deformations of the soil, obeying its elastic and plastic laws, must coincide with the elastic deflection curve produced by these pressures in the sheet piles according to the laws governing the piles, their modulus of elasticity, and their stiffness—that is, the magnitude and the distribution of pressures is controlled by the boundary condition between the soil and the sheet piles.

The problem is similar to a foundation on an elastic medium. The Boussinesq law of pressure distribution is not applicable when the load is transferred to the soil through foreign mediums such as concrete slabs and beams until the contact pressure distribution between slab and soil has been determined. In such cases, the moment of inertia and the modulus of elasticity of the transferring medium, as well as the modulus of the soil reaction, determine the distribution of the soil pressures. The variation in the wall stiffness may not be as important in the determination of the total pressure on the wall; it is of great importance, however, in the determination of pressure distribution, which seems to baffle all experimenters. It would seem that, in the future, model tests should include the wall stiffness variable in the hope that the results may lead to a rational pressure distribution formula. It is felt that the tests reported in this Symposium have advanced the knowledge of the behavior of flexible bulkheads considerably, but much remains to be learned. It is hoped that the exceptional facilities of this model, and the new improved testing methods used, will encourage and spur on further research in the indicated direction. Such a study will require considerable effort both in resources and patient work, but the task is well worth doing.

EXPERIENCES WITH SOIL TYPES IN NAVAL CONSTRUCTION

By L. A. Palmer,[38] Esq.

Synopsis

The principles of soil mechanics are demonstrated by two examples in this paper. One concerns pile foundations in plastic clay, and the other concerns the stability of an extensive mass of sand deposited as a foundation material for a cellular pier, comprised of two rows of cells of interlocking sheet piling. A rather unusual condition in an homogeneous clay deposit, 90 ft or more deep, of apparently sedimentary origin is described. The density and moisture content of the clay are apparently unrelated as to depth. Consolidation and shear curves are shown. Two expressions are derived, one for the case of capillary saturation and the other for the case of complete submergence, which show that the in-place consolidating pressure of fully consolidated soil (homogeneous as to type and without structural load) is an exponential function of depth.

A critical density study of the sand used both to fill and to support the steel cells of the cellular piers is described. The purpose of this study was to indicate the possible decrease in stability of the sand mass that occurs as the result of severe earthquake shock.

Introduction

During the decade before World War II, there were many advances on a broad front in the field of theoretical and applied soil mechanics. Prior to this time a splendid groundwork had already been laid by Professor Terzaghi and his associates and the many researches that later followed this pioneering made a wealth of valuable material available for utilization in the large wartime construction program of the United States Navy.

During this time the Bureau of Yards and Docks, like the Corps of Engineers of the United States Army, was early confronted with the job of converting poor foundation soils into good soils, a procedure known as "soil control." This procedure, once frequently referred to as "soil stabilization" in highway construction, had its place in the subgrade preparation for paving Naval and Marine Corps air stations, in the stiffening and consolidation of mud deposits by vertical sand drains, in the compaction of sand by vibroflotation, and in the substitution of sand dikes for mud as backfill material.

This paper is concerned with a few of the physical properties of two very distinct types of soils, plastic clay and sand, as they were encountered on two

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Naval construction projects. Both projects involved principles of soil mechanics that are typical of those that have been receiving considerable study.

PLASTIC CLAY

In connection with the design of two rigid frame hangars for blimps at Houma, La., the foundation soil exploration included deep borings, soil sampling and testing, plate loading tests in open pits from 7 ft to 9 ft deep, and static load tests on timber piles. The low bearing values obtained by plate loading tests precluded any notion of carrying the A-frame loads on spread footings. Beginning at a depth of from 10 ft to 20 ft below the ground surface and extending to a total depth of 90 ft or more, the soil was composed of a mixture of silt and clay. At least 90% of all samples could be washed through the No. 270 sieve. The clay fraction contained 20% or more of colloids. The shapes of the consolidation curves indicated no definite relationship between the depth of overburden and the preconsolidation pressure (vertical intergranular pressure) of the clay soil in place. The bulk densities of samples from one of the borings, taken at depth intervals of 10 ft, did not vary more than 2 lb per cu ft. The unit cohesion of the clay obtained by unconfined compression tests was somewhat variable, ranging from 450 lb per sq ft to 750 lb per sq ft, independently of the depth. The computed unit skin friction of the test piles varied from 500 lb per sq ft to 750 lb per sq ft, on the average being somewhat more than the average unit cohesion of the clay. If it is assumed that the clay sheared to form, with the pile, a cylinder of diameter equal to that of the pile butt, then the average computed skin friction is about the same as the average unit cohesion of the clay.

TABLE 1.—OBSERVATIONS ON TEST PILES AT HANGAR A, UNITED STATES NAVAL AIR STATION, HOUMA, LA. (HAMMER NO. IV)

Pile No.	DIAMETER (IN.)		Blows per ft[a]	Penetration (ft)	Contact area (sq ft)	SETTLEMENT, IN FEET, AT:		Total load (tons)	UNIT SKIN FRICTION (LB/FT²)		Rebound (ft)[c]
	Butt	Tip				30 tons	Maximum load		Observed	Computed[b]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1–60 . . .	15	10	6¾	51	175.0	0.007	0.027	57.2	654	571	. . . .
1–80 . . .	20	10	11⅔	71	256.0	0.007	0.029	60.0	. . . .[d]	. . . .[d]	0.016
2–80 . . .	20	10	11⅔	. . . .	256.0	. . . .	0.039	90.0	704	486	. . . .
2–60 . . .	18	10	8	51	186.0	0.008	0.028	66.45	714	552	. . . .
2–80 . . .	17	9	7	71	236.0	0.011	0.035	80.0	680	506	. . . .
5–60 . . .	15	10	7⅓	51	167.0	0.007	0.018	60.0	721	600	. . . .
5–80 . . .	19	11	15½	71	280.0	0.012	0.036	75.0	535	424	0.017
6–60 . . .	15	8	5	51	149.2	0.008	0.017	55.4	740	550	. . . .
6–80 . . .	21	9	16	71	254.5	0.011	0.038	89.1	700	457	. . . .

[a] Determined by computing the average of the blows in the last 3 ft of penetration. [b] The values of skin friction (pounds per square foot) in Col. 11 are as computed, for piles of constant diameters equal to the butt diameters, for the entire lengths. The average of these values is about the same as the average unit cohesion of the clay. By this method, average skin friction is equal to 518 lb per sq ft, whereas the average cohesion is equal to 535 lb per sq ft. [c] Except as indicated for pile Nos. 1–80 and 5–80, each pile was driven to failure. For the piles that failed, the computed average pile friction was 701 lb per sq ft (see Col. 10). The rebound of pile No. 6–80 could not be determined. [d] The load was removed after loading to 60 tons. Upon reloading, this pile failed under a load of 90 tons.

Table 1 contains pile load test data at the site of one of the two hangars. Fig. 46 shows plotted triaxial test data. No significant effective angle of internal friction ϕ was evident from shear tests made by either the direct method (quick shear) or the triaxial device in which the tests were also made quickly and without loss of moisture during the test.

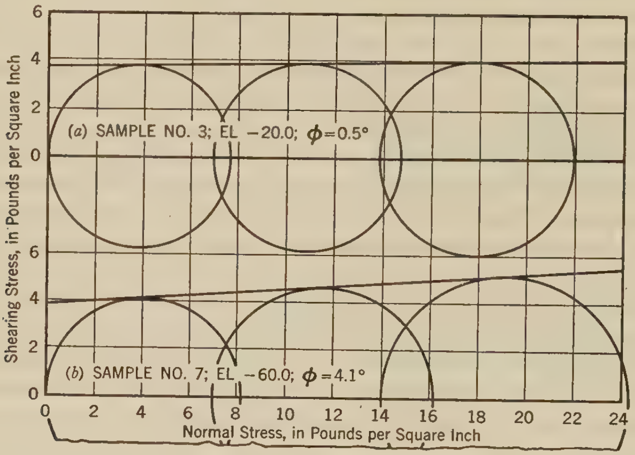

FIG. 46.—TRIAXIAL SHEAR TEST DATA; PLASTIC CLAY; HANGAR NO. 2, HOUMA, LA.; $c = 560$ LB PER SQ FT

It was concluded that settlement by slow consolidation of the clay, under the load of the piles, would not be realized for the reason that there could be little possibility of relief of pore pressure developed by axial loads on piles. The few small local sand lenses, at considerable depths below the estimated required depth of penetration of the piles, could not act as drainage courses for egress of pore water. Furthermore, the computed pore pressure delivered to surrounding soil was practically zero for the top several feet below the ground surface so that there was no driving force to carry the flow of water entirely to the surface. No appreciable settlements were reported during the 4 years following construction.

The moisture content of this clay varied from 30% to 75% (dry weight basis) with most of the values ranging from 35% to 50%. The moisture content was entirely independent of the depth.

This clay belongs in the category of those described by Karl Terzaghi,[18] M. ASCE, Ralph B. Peck,[17] Assoc. M. ASCE, and W. S. Housel,[39] M. ASCE, with one notable exception—in the laboratory shearing tests the measured angle of internal friction in the present case did not exceed 5° whereas the laboratory value reported by Professor Peck was $\phi = 17°$.

The possibility of buckling of timber piles driven to a total penetration of 80 ft, the lower 2 ft being in sand lenses, might be considered for the reason that the clay tended to deform noticeably at stresses very much less than those corresponding to failure. Thus, in plate loading tests at 9 ft or more below grade, a unit load of 1 ton per sq ft produced a settlement of 1 in. although the bearing value was theoretically at least $(\pi + 2)\,c$, or 3,100 lb per sq ft, with a unit cohesion (c) of 600 lb per sq ft which is characteristic of the clay under the loading plate. Using the expression:[40]

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[39] "Earth Pressure on Tunnels," by W. S. Housel, in "Earth Pressure and Shearing Resistance to Plastic Clay: A Symposium," *Transactions*, ASCE, Vol. 108, 1943, pp. 1037–1058.

$$P_{cr} = \frac{\pi^2 E I}{L_p^2}\left(m^2 + \frac{D K L_p^4}{m^2 \pi^4 E I}\right) \quad \text{.................} (28)$$

in which the factor m can be evaluated by the expression:

$$m^2 (m + 1)^2 = \frac{D K L^4}{\pi^4 E I} \quad \text{....................} (29)$$

Let the length of the pile $L_p = 80$ ft; $E = 216{,}000{,}000$ lb per sq ft; $I = \pi/64$; the average diameter of the pile, $D = 1$ ft; and $K = 24{,}000$ lb per cu ft. Assuming that the factor m is 5 (Eq. 29 solved to the nearest whole number), Eq. 28 yields $P_{cr} = 1{,}031{,}000$ lb. This value is in excess of 500 tons, which is three times the compressive strength of the cypress piles.

The Relation of Pressure to Depth in Natural Soil Deposits

The equation of the straight line part of a typical consolidation curve, with e, the void ratio, plotted against $\log_{10} p_s$, the consolidation pressure, is

$$e = B - A' \log_{10} p_s \quad \text{.....................} (30)$$

in which A' is the slope of the semilogarithmic plot of the curve; and B is the intercept of the straight line part of the curve at the ordinate 1 ton per sq ft.

Converting $\log_{10} p_s$ to $\log_{\mathrm{e}} p_s$, Eq. 30 becomes

$$e = B - \frac{1}{2.3026} A' \log_{\mathrm{e}} p_s \quad \text{..................} (31a)$$

or

$$e = B - A \log_{\mathrm{e}} p_s \quad \text{....................} (31b)$$

in which $A = \frac{1}{2.3026} A'$ and e is the Naperian base of logarithms.

Now consider a homogeneous clay type soil of indefinite depth and extent and assume the following conditions:

1. The ground water level exists at a depth H_g below the surface;
2. Water has risen by capillarity to the ground surface and all voids are filled with water;
3. The constants, A and B, are characteristic of the entire soil mass; and
4. The soil is completely consolidated by the in-place vertical load at all depths.

The soil at the surface is subjected to capillary pressure, p_c,

$$p_c = \gamma_w H_g \text{ (at ground surface)} \quad \text{................} (32)$$

γ_w being the unit weight of water.

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[40] "The Stability of Foundation Piles Against Buckling Under Axial Load," by A. E. Cummings, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 18, 1938, Pt. II, pp. 112–119.

At any depth, h_o, less than H_g, the vertical intergranular pressure, or the consolidation pressure, p_s, is

$$p_s = \gamma_w (H_g - h_o) + \gamma_a h_o \ldots\ldots\ldots\ldots (33)$$

γ_a being the unit weight in air of the saturated soil. Then at the depth $h_o + dh_o$, the consolidation pressure is $p_s + dp_s$. The average γ_a-value of the soil within dh_o is $\dfrac{\gamma_a + (\gamma_a + d\gamma_a)}{2}$ and

$$dp_s = \left(\gamma_a + \frac{d\gamma_a}{2}\right) dh_o = \gamma_a \, dh_o + \tfrac{1}{2} \, d\gamma_a \, dh_o = \gamma_a \, dh_o \ldots\ldots (34)$$

Rewriting Eq. 31*b*,

$$p_s = \mathrm{e}^{(B-e)/A} \ldots\ldots\ldots\ldots (35)$$

from which

$$dp_s = -\frac{\mathrm{e}^{(B-e)/A}}{A} de \ldots\ldots\ldots\ldots (36)$$

by substitution of Eq. 36 in Eq. 34,

$$\gamma_a \, dh_o = -\frac{\mathrm{e}^{(B-e)/A}}{A} de \ldots\ldots\ldots\ldots (37)$$

also

$$\gamma_a = \frac{G + e}{1 + e} \gamma_w \ldots\ldots\ldots\ldots (38)$$

G being the specific gravity of the soil solids. From Eq. 33,

$$p_s = \gamma_w H_g + h_o (\gamma_a - \gamma_w) = \gamma_w H_g + h_o \left(\frac{G + e}{1 + e} - 1\right) \gamma_w \ldots (39a)$$

Therefore,

$$\gamma_a \, dh_o = \gamma_w \left(\frac{G - 1}{1 + e}\right) dh_o \ldots\ldots\ldots\ldots (39b)$$

Substituting Eq. 39*b* in Eq. 37:

$$\gamma_w \left(\frac{G - 1}{1 + e}\right) dh_o = -\frac{\mathrm{e}^{(B-e)/A}}{A} de \ldots\ldots\ldots\ldots (40)$$

Eq. 40 is now integrated but not between limits. The constant of integration is

$$\text{Constant} = -\frac{\mathrm{e}^{(B-e_s)/A} (1 + e_s + A)}{\gamma_w (G - 1)} \ldots\ldots\ldots\ldots (41)$$

in which e_s is the void ratio at the surface, $h_o = 0$. That is,

$$e_s = B - A \log_e \gamma_w H_g \ldots\ldots\ldots\ldots (42)$$

so that, finally,

$$h_o = \frac{p_s [1 + B + A (1 - \log_e p_s)] - \gamma_w H_g [1 + B + A (1 - \log_e \gamma_w H_g)]}{\gamma_w (G - 1)} \ldots (43)$$

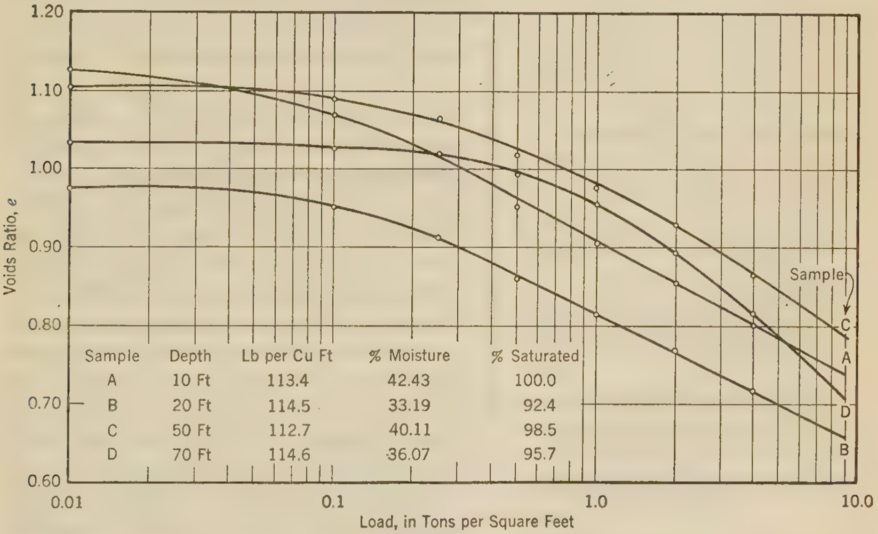
In Eq. 43, the constants, A and B, and the value, G, are laboratory values and H_g is known from boring data. If an arbitrary value is assigned to p_s, the corresponding value for h is obtained from Eq. 43. Thus, the relation between depth and intergranular vertical pressure is exponential and not linear, as often assumed.

If the ground water is constantly at the surface, the consolidation pressure is zero at the surface. The consolidation pressure at any depth h below the ground surface is $(\gamma_a - \gamma_w)\,h$ in this case. The relation between h and e is again

$$h_o = \frac{e^{(B-e)/A}\,(1 + e + A)}{\gamma_w\,(G-1)} + \text{constant} \quad \text{.............} \quad (44)$$

but the constant of integration in Eq. 44 differs from that obtained in the case of capillary saturation.

It may be considered that the soil is at its liquid limit at the ground surface. Since the condition of complete consolidation is assumed as existing at all depths, Eq. 31a should be applicable at all finite depths that correspond to values of p_s that are within the range of values, for this variable, included in the straight line part of the laboratory-determined consolidation curve. F. A. Robeson[41] has shown, however, that for certain conditions, the straight line part of the consolidation curve applies only for the values of p_s equal to or greater than 0.1 ton per sq ft. It may then be assumed that from $h = 0$ to a depth h_i where $(\gamma_a - \gamma_w)\,h_i = 0.1$ ton per sq ft, the soil mass is at its liquid limit and that below h_i the linear relation between e and $\log_e p_s$ expressed by Eq. 31a obtains. Then, from Eq. 44,



Sample	Depth	Lb per Cu Ft	% Moisture	% Saturated
A	10 Ft	113.4	42.43	100.0
B	20 Ft	114.5	33.19	92.4
C	50 Ft	112.7	40.11	98.5
D	70 Ft	114.6	36.07	95.7

FIG. 47.—TYPICAL CONSOLIDATION TEST DATA, PLASTIC CLAY; HANGAR NO. 2, HOUMA, LA.

$$\text{Constant} = h_i - \frac{e^{(B-e)/A}\,(1 + e_i + A)}{\gamma_w\,(G-1)} \quad \text{.............} \quad (45)$$

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[41] "A Method of Predicting Settlement of Fills Placed on Muck Beds," by F. A. Robeson, *Public Roads*, February, 1936, p. 249.

in which e_t is the void ratio corresponding to $p_s = 0.1$ ton per sq ft; and, hence,

$$h_o - h_i = \frac{\mathrm{e}^{(B-e)/A}\,(1 + e + A) - \mathrm{e}^{(B-e_t)/A}\,(1 + e_t + A)}{\gamma_w\,(G - 1)} \qquad \text{......(46)}$$

but, as before, $\mathrm{e}^{(B-e)/A} = p_s$ and $1 + e + A = 1 + B + A\,(1 - \log_e p_s)$. Also, $\mathrm{e}^{(B-e_t)/A} = 0.1$ ton per sq ft and $1 + e_t + A = 1 + B + A\,(1 - \log_e 0.1)$. Then, by substitution in Eq. 46,

$$h_o - h_i = \frac{p_s\,[1 + B + A\,(1 - \log_e p_s)] - 0.1\,[1 + B + A\,(1 - \log_e 0.1)]}{\gamma_w\,(G - 1)} \qquad \text{..(47)}$$

Eq. 47 again shows (and this time for the case of complete submergence) that the change in consolidation pressure, p_s, with depth, in soil that is completely consolidated by the existing vertical pressure is an exponential relationship. It is obvious from consolidation curves that the consolidation pressure is an exponential function of the void ratio, but it is not obvious that p_s is also an exponential function of depth in a homogeneous soil type that is completely consolidated under weight of overburden plus any existing capillary pressure.

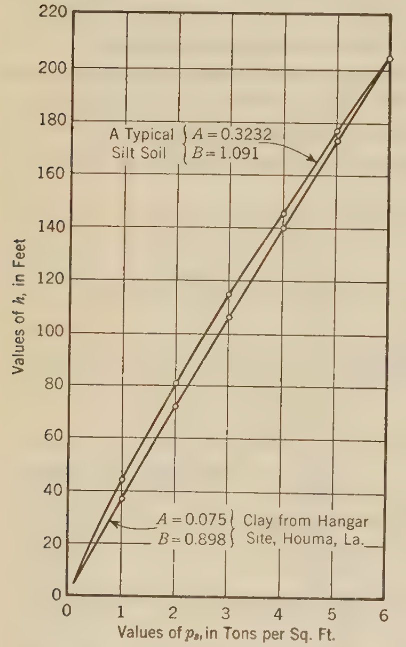

Fig. 48.—Curves of Eq. 34, For Two Typical Soils

In the case of the plastic clay at Houma, the preconsolidation pressures, estimated according to the procedure described by A. Casagrande, M. ASCE, and R. E. Fadum,[42] Assoc. M. ASCE, varied from 0.5 kips per sq ft to 3.0 kips per sq ft, throughout depths ranging from 10 ft to 70 ft and there seemed to be no good relation between these values and the depths at which the samples were taken. In this instance the shapes of the (e versus $\log_e p_s$)-curves did not lend themselves to the analysis, since there seemed to be no well-defined points of sharpest curvature. The depth to ground water, at the time of sampling was 4 ft. The estimated preconsolidation pressures were generally somewhat less than the estimated overburden loads, indicating incomplete consolidation of the deposit.

Fig. 47 shows four typical consolidation curves for samples of this clay taken at depths of 10, 20, 50, and 70 ft. The bulk density and moisture content (dry weight basis) of the same samples, as received, are given in Table 2. Fig. 48

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[42] "Notes on Soil Testing for Engineering Purposes," by A. Casagrande and R. E. Fadum, *Soil Mechanics Series No. 8*, Harvard Univ., Cambridge, Mass., January, 1940.

shows two graphs of Eq. 47 for two soils, one for the plastic clay at Houma and the other for a typical silt soil from another source. The greater the increase in unit weight produced by progressively increasing consolidation pressures, the more the graphs of Eqs. 43 and 47 depart from a linear relationship between depth and consolidating pressure. For a constant unit weight, unchanging with depth, the relation would be strictly linear for a homogeneous soil. The graph for the plastic clay, Fig. 48, is almost linear. This fact, in itself, indicates small changes in density with depth even for a condition of complete consolidation of the clay in place.

TABLE 2.—TEST DATA, PLASTIC CLAY SAMPLES IN FIG. 47

Sample	Depth (ft)	Unit weight (lb per cu ft)	Moisture content (%)	Percentage saturation
A........	10	113.4	42.43	100.0
B........	20	114.5	33.19	92.4
C........	50	112.7	40.11	98.5
D.......	70	114.6	36.07	95.7

SAND

Fig. 49 shows a typical section through a cellular pier, approximately 400 ft wide with berth lengths along the two sides of 1,700 ft and 1,000 ft, respectively. This project was on the west coast at Hunters Point, California. The periphery of the pier is bounded by a continuous series of steel sheet pile cells driven into a deep sand plug, deposited for this purpose. Those cells in which steel H-piles were driven for the crane tower foundations are not shown in Fig. 49. It was considered that the driving of the H-piles would tend to increase the density of the sand in and below these cells to such an extent that there would be no serious question as to sudden loss of stability by tectonic forces.

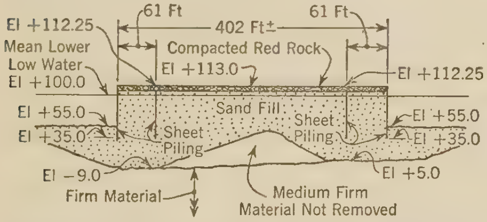

FIG. 49.—TYPICAL SECTION THROUGH CELLULAR PIER; HUNTERS POINT, CALIFORNIA

The sand was obtained from various sources on this project, yet it was surprisingly uniform in gradation. In the main, it was a poorly graded, medium sand, with from 30% to 60% passing the No. 40 sieve and retained on the No. 60 sieve, with practically no material retained on the No. 10 sieve and with very little material passing the No. 200 sieve.

Triaxial and direct shear tests were made with the sand under variable conditions with respect to void ratio and density of the sand. The direct and triaxial shear tests were made according to the procedures described by Professors Casagrande and Fadum.[42] Fig. 50 shows the relationship between the initial void ratio, e, the minor principal stress, p_2, maintained constant during an individual triaxial test, and the indicated percentage change in volume that accompanies a change from the initial to the critical void ratio. It may be noted from Fig. 50 that a definite critical void ratio, e_{cr}, corresponds to each separate value of minor principal stress, p_2. This method of analysis has been treated in detail by D. W. Taylor,[43] and by Professor Casagrande and J. D. Watson.[44]

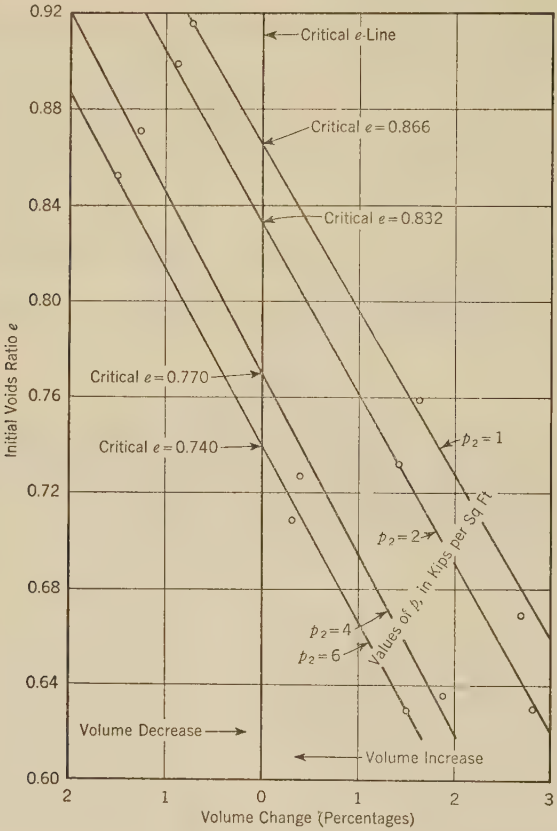

FIG. 50

Beginning with the well-known Rankine relation between the principal stresses p_1 (major) and p_2 (minor), Fig. 51 reveals that

$$v = p_n \tan \phi \qquad (48)$$

Also,

$$p_n = \overline{OB} = \overline{OC} - \overline{BC} = \frac{p_1 + p_2}{2} - \frac{p_1 - p_2}{2} \sin \phi$$

$$= \frac{p_1 (1 - \sin \phi) + p_2 (1 + \sin \phi)}{2} \qquad (49)$$

---

[43] "A Comparison of Direct Shear and Cylindrical Compression Tests," by Donald W. Taylor, in "Symposium on Shear Testing of Soils," *Proceedings*, ASTM, Vol. 39, 1939, p. 1058.

[44] "Compaction Tests and Critical Density Investigation of Cohesionless Materials for Franklin Falls Dam, Merrimack Flood Control," by A. Casagrande and J. D. Watson, Corps of Engrs., U. S. Army, Boston, Mass., April 1, 1938, Appendix BII, pp. 1–38.

and, since,

$$p_1 (1 - \sin \phi) = p_2 (1 + \sin \phi) \quad \dots\dots (50)$$

it follows that

$$p_n = p_2 (1 + \sin \phi) \quad \dots\dots (51)$$

Consequently,

$$v = \text{shearing resistance} = p_2 (1 + \sin \phi) \tan \phi \quad \dots\dots (52)$$

It is apparent from Fig. 50 that with a relatively high minor principal stress the reduction in volume of the sand from its loose state to the critical density (at which it shears at constant volume) may be several times the volume reduction accompanying the transition from loose to critical state with a relatively low minor principal stress. The appreciable decrease in volume of the submerged sand requires considerable egress of water from the large mass of sand. Since no sand is permeable enough to render the escape of water instantaneous, a very sizable pore pressure would tend to be developed and maintained for a finite time if there were a sudden and severe earthquake. Under these conditions the combination of static overburden load and vibration would tend to effect a sudden diminution in volume of the sand mass, which in the lower reaches would trap the water and convert a considerable part of the intergranular stress to pore pressure. In the absence of any vibration, the sand mass, even in the loose state, is capable of developing the full shearing resistance as expressed in Eq. 52.

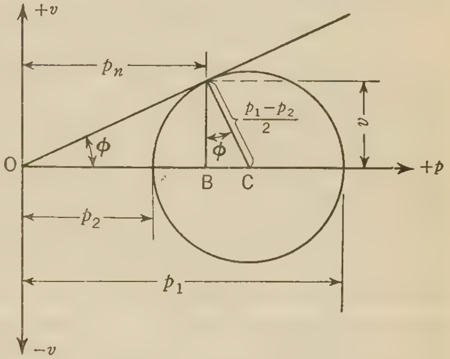

Fig. 51.—Shearing Resistance of Cohesionless Soil at Failure

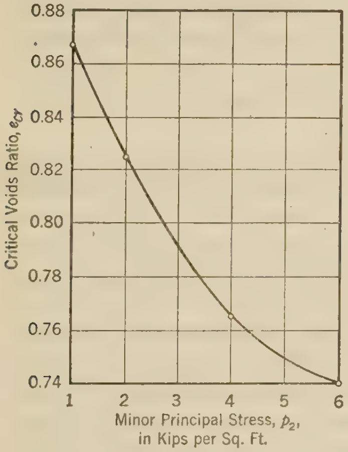

Fig. 52

Thus, at El. + 35 (Fig. 49) and in the mole back of the cells, if p_1 is taken as approximately equal to the weight of overburden (5.00 kips per sq ft); and, with ϕ equal to 28°, p_2 by Eq. 50 equals $\frac{0.53}{1.47} \times 5 = 1.80$ kips per sq ft and the shearing resistance v, by Eq. 52, is $1.8 \times 1.47 \times 0.532 = 1.41$ kips per sq ft.

In Fig. 52 the critical void ratios are plotted against the corresponding p_2-values of the triaxial tests. Using the data of Fig. 50, the critical void ratio for p_2 = 1.8 kips per sq ft is 0.834. If the void ratio at El. + 35 and between the rows of cells (see Fig. 49) does not exceed 0.834, there is an indicated minimum shearing resistance of 1.41 kips per sq ft. However, if the void ratio should be at 0.866, or slightly above this value, then, from Fig. 50, the effective intergranular value of p_2 could not exceed 1 kip per sq ft; and, for this condition, Eq. 52 yields $v = 1 \times 1.47 \times 0.532 = 0.78$ kips per sq ft. Thus, a sudden disturbance would tend to reduce the shearing resistance to approximately one half of its value under conditions of static load. The intergranular value of p_1, with disturbance, would become, by Eq. 40, $\frac{1 + \sin 28°}{1 - \sin 28°} \times 1 = 2.78$ kips per sq ft. Consequently the temporary pore pressure developed would be $5.00 - 2.78 = 2.22$ kips per sq ft.

At the outboard side or toe of the cells, because of the rotating moment, p_1 is considerably greater than the weight of overburden (5 kips per sq ft) at the lower ends of the sheet piling. Without going into details of the computations of the vertical pressure distribution at this level within the cells, it may be stated that the computed maximum vertical pressure at the toe is 7.5 kips per sq ft. It is recognized that the major principal stress at this point very likely exceeds this value but a complete and satisfactory stress analysis, applicable to this region, is nonexistent to the best of the writer's knowledge. For an assumed or estimated value of 7.5 kips per sq ft for p_1, the corresponding value for p_2 by Rankine's relation would be 2.7 kips per sq ft, and the computed shearing resistance from Eq. 52 is 2.10 kips per sq ft. From Fig. 52, for $p_2 = 2.7$, the critical void ratio is 0.80; and, within this region, with e equal to, or less than 0.80, any sudden disturbance would not reduce the unit shearing resistance below 2.1 kips per sq ft. Consequently, no momentary pore pressure would result as long as the quake does not increase p_1 beyond its initial value of 7.5 kips per sq ft. However, if the void ratio is 0.832, or slightly more, then the effective intergranular value of p_2 would be 2.00 kips per sq ft, the unit shearing resistance would be 1.55 kips per sq ft, and there would be a developed pore pressure of $7.5 - 5.5 = 2.0$ kips per sq ft. This amount of suddenly developed pore pressure at the toes of the cells would cause piping of the sand between El. + 35 and El. + 55 outside the cells.

In the direct or box shear tests, the angle of internal friction of the submerged sand varied from 28.3° to 38.3°, with corresponding initial void ratios of 0.85 and 0.62, respectively. Thus, a constant value of $\phi = 28°$ in the foregoing computations is not strictly accurate; but in view of the fact that, at best, the density in place of the sand can only be estimated, no refinements in such computations are warranted.

With reference to Fig. 49, the sand below El. + 55 in the area of the cells was deposited from dump barges and as many as 200 tons of sand were dumped in the mass at one time. The sand filling the cells was deposited hydraulically and in a manner to minimize both segregation and unbalanced lateral pressure. The backfill behind the sheet piling cells consisted of both sand and bank-run fill, both being predominantly stable granular materials. Option was given to deposit the backfill either hydraulically or by dumping.

Laboratory studies indicated that, by dumping, a void ratio not exceeding 0.80 could reasonably be expected. This in turn indicates that no suddenly developed pore pressure or other instability would be realized with sudden disturbance for values of p_1 not appreciably exceeding 8 kips per sq ft. In those cells in which steel H-piles were driven for the crane tower foundations there can be no doubt that densification of sand, both in and below the cells, occurred to an extent such that there can be no likelihood of instability of the sand mass under earth shock. In all the cells timber templet piles were driven, nine in each cell, prior to driving the sheet piling and when the top elevation of dumped sand was at El. + 55. Then the sheet piles were driven around the templet piles. These operations tended to further increase the density of the sand below El. + 55. The cells were then filled hydraulically before backfilling. With the sand level within the cells at El. + 107, the lateral movement of sand within the cells was quite appreciable, tending to increase the density of the sand in the cells.

The use of sand in this project, and in the manner described, resulted in a very considerable reduction of cost below that estimated for filling by broken rock and other locally available materials which were considered to have greater stability. Earthquakes of minor intensity can serve only to increase the sand density and therefore also the structural stability. Such minor disturbances have already been realized. In the event of a major disturbance, approaching that of 1906 in intensity, it is problematical if extensive damage can occur; and, with the passage of time, the likelihood of any such damage is progressively lessened.

This project is of special interest for the reason that it invites consideration of possible control procedures in earthwork of this type and on this scale. The basic difficulty is that of obtaining undisturbed samples of sand on a scale required for a reliable survey of conditions. Perhaps the problem could be studied to better advantage by large-scale model tests. The results of research should have wide application and they would be especially applicable in the case of bulkheads with sand backfills.

CAVE-INS OF SANDY BACKFILLS

By J. C. Gebhard,[45] M. ASCE

Synopsis

The other papers in this program have dealt principally with various laboratory and design aspects of lateral earth pressures against bulkheads. In applying experimental and theoretical knowledge to the building of actual structures, the engineer is never quite sure just how conservative his designs and construction procedures have been unless there is some visible sign of distress or actual failure. Since such occurrences help to build up the fund of practical judgment, Commander Coxe and the writer reasoned that it would be of interest to discuss a few jobs that gave disappointing results. This paper concerns a case involving a nonplastic sandy backfill whereas the last Symposium paper by Commander Coxe describes a job that involved plastic soils.

Description

During World War II, it was necessary to increase the land area at one of the United States Naval stations by filling an indentation in the harbor with hydraulically placed sandy material. It was also necessary to construct a 3,000-ft quay wall across the outer end of the bight to retain the fill and to provide a wharf with a 40-ft depth of water alongside.

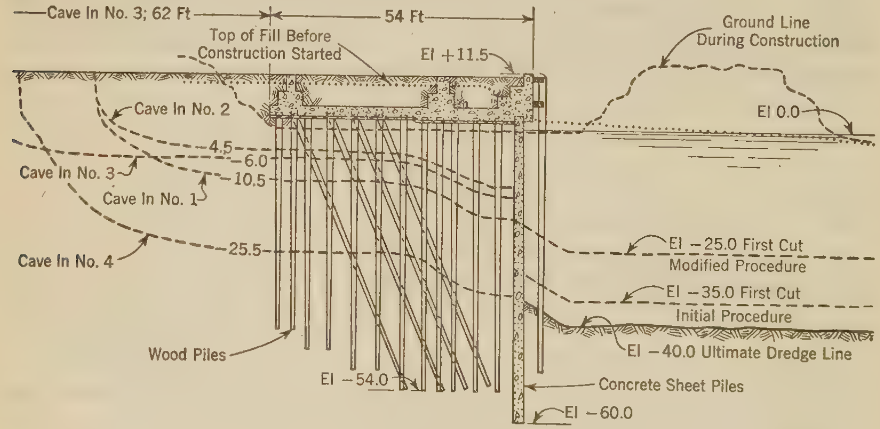
Cave In No. 3; 62 Ft
54 Ft
Top of Fill Before Construction Started
El +11.5
Ground Line During Construction
El 0.0
Cave In No. 2
4.5
6.0
10.5
Cave In No. 3
Cave In No. 1
25.5
Cave In No. 4
El −25.0 First Cut Modified Procedure
El −35.0 First Cut Initial Procedure
El −40.0 Ultimate Dredge Line
Wood Piles
El −54.0
Concrete Sheet Piles
El −60.0

Fig. 53.—Typical Cross Section Through Quay Wall

The quay wall (see Fig. 53) is of the relieving platform type with concrete sheet piles on the outer face. A flat slab concrete platform, about 53 ft wide, with its bottom surface about 1.5 ft above low water is supported on a mat of closely spaced untreated wooden piles (see Fig. 54). The concrete sheet piles are anchored to the platform and the lateral thrusts are taken by wooden batter piles. A gravity wall extending to El. 11.5 forms the outer edge of the wharf and retains the surface fill.

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[45] Capt. (CEC), U. S. Navy, Public Works Officer, 9th Naval Dist., Great Lakes, Ill.

The depth to the original sandy bottom varied considerably but was taken as El. −20 for purposes of design. Before starting work on the quay wall, a wide sand dike was constructed on the line of the quay wall to about El. + 11. The dike material was placed by a slotted dredge discharge pipe and it is reported that most of the thin overburden of muck or silt on the bottom of the bay was washed out in the process. Wooden test piles indicated that all piles would have to be jetted and that bearing values greater than 40 tons per pile would be reached at a point elevation of about El. −30. The fill is a fine nonplastic sand weighing about 110 lb per cu ft. All of it passes a No. 8 screen; about 90% passes a 50-mesh screen; 20% passes a 100-mesh screen; and only 4% passes a 200-mesh screen. The outboard beach of the dike was on a slope of about 1 on 10 prior to the beginning of the work on the quay wall.

CONSTRUCTION PROCEDURE

The quay wall contractor first excavated a wide trench in the dike to about El. 1. He then jetted and drove the mat of wooden piles to a point elevation of about El. − 54. After this, he leveled off the top of the fill, placed a 4-in. layer of crushed stone, and cast all but the outer 9 ft of the concrete relieving platform. He then jetted and drove the concrete sheet piles to a point elevation of El. − 60 and anchored them back to the relieving platform. A general view of the construction is shown in Fig. 55.

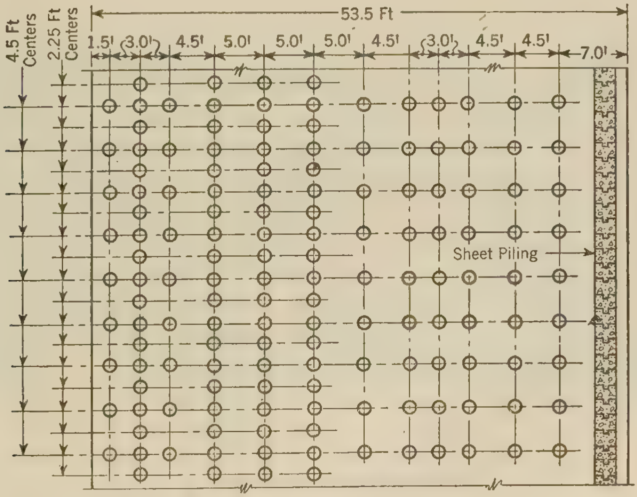


FIG. 54.—TYPICAL PILE PLAN

At first it was planned to use steel sheet piles along the outer face of the wharf to retain the fill but, because of wartime conditions, they could not be obtained. Accordingly, concrete sheet piles, 24 in. deep and 19 in. wide, were used. The tongues projected 2¾ in. and the grooves were 5¼ in. deep. Thus, vertical holes were left between the piles, measuring about 2½ in. by 7½ in. in plan as shown in Fig. 56. It was recognized that tight joints between the

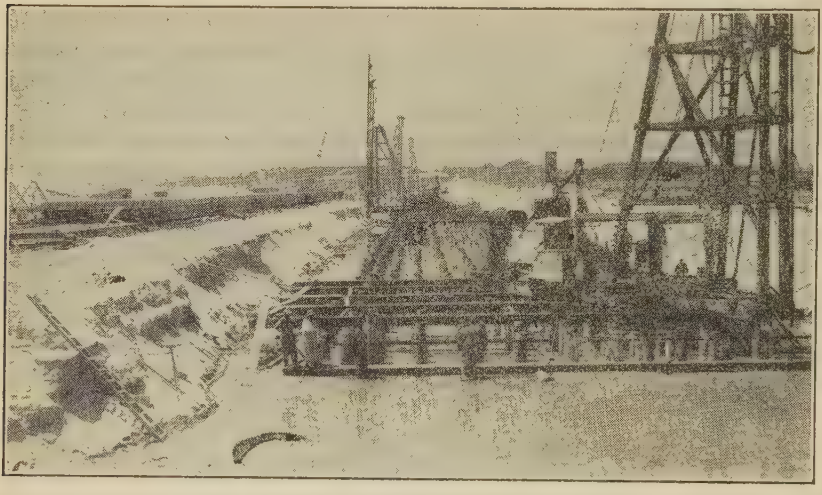
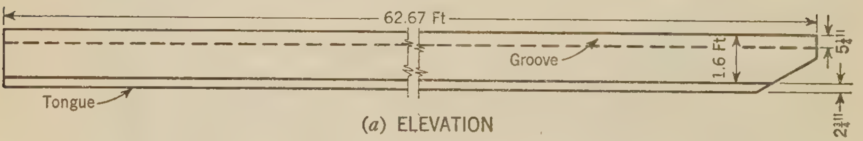
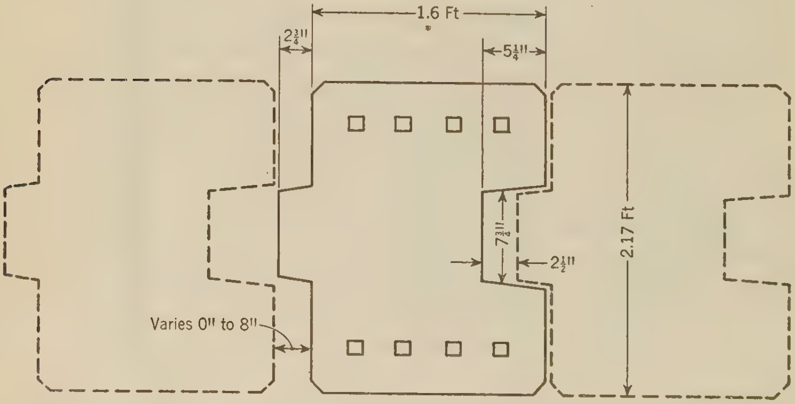


Fig. 55.—Quay Wall Under Construction


62.67 Ft
Groove
Tongue
1.6 Ft
5¼"
2¾"

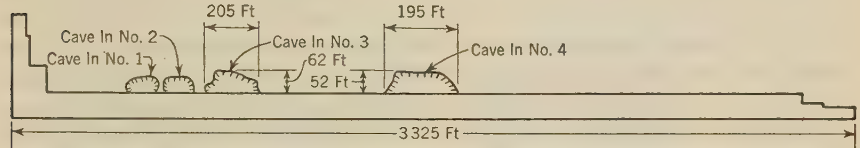
(*a*) ELEVATION


1.6 Ft
2¾"
5¼"
7¾"
2½"
2.17 Ft
Varies 0" to 8"

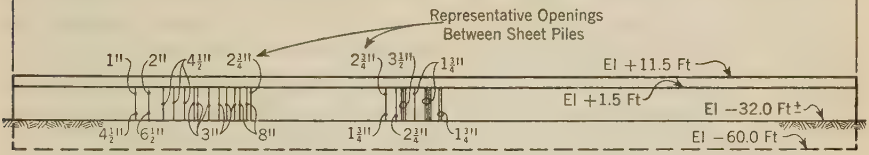
(*b*) SECTION

Fig. 56.—Dimensions of Concrete Sheet Pile

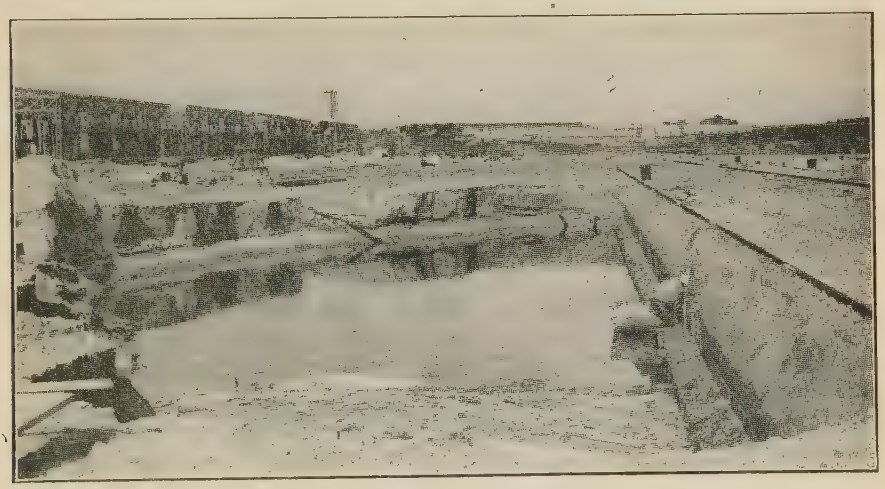
sheet piles could not be expected in practice. The specifications, therefore, required that all joints be cleared of sand and silt, by a water jet, to a depth of 3 ft below the future dredge line—namely, to El. −43. The holes and spaces between the piles were then to be filled with cement grout pumped into place from El. −43 to the tops of the piles. To test the effectiveness of arrangement, two 30-ft sheet piles were driven so that their lower ends were about 10 in. apart with their tops in contact. Neat cement was forced into the groove through a pipe whose lower end was raised as the grout rose around it. When the piles were pulled, it was found that the grout had effectively filled the gap between the piles. Therefore, it was decided to proceed with the use of neat cement grouting. Because the entire lengths of the sheet piles were in the ground, it was not possible to inspect the results of driving or grouting until after the material on the outside had been dredged.


205 Ft
195 Ft
Cave In No. 2
Cave In No. 1
Cave In No. 3
62 Ft
52 Ft
Cave In No. 4
3325 Ft

(*a*) GENERAL PLAN


Representative Openings Between Sheet Piles
1"
2"
4½"
2¾"
2¾"
3½"
1¾"
El +11.5 Ft
El +1.5 Ft
El −32.0 Ft±
4½"
6½"
3"
8"
1¾"
2¾"
1¾"
El −60.0 Ft

(*b*) OUTBOARD ELEVATION



(*c*) VIEW OF CAVE-IN NO. 3

Fig. 57.—Location of Cave-Ins Behind Quay Wall

To speed his work, the dredging contractor began by dredging to El. −35 in one cut right to the sheet piles, using a 30-in. suction dredge. He then intended to make a second cut to reach the required depth of 40 ft. As the dredging progressed, four large cave-ins occurred suddenly soon after the outside material was removed. The location of these failures, and a view of cave-in No. 3 are shown in Fig. 57. These cave-ins extended as much as 60 ft beyond the inner edge of the relieving platform, or more than 100 ft from the sheet piles. They varied in depth to 35 ft and in length to about 200 ft. The largest single cave-in involved about 8,000 cu yd. It is impossible to determine the rate at which the material was lost since it is not known how long the loss may have been in progress under the relieving platform before the final cave-ins occurred. It appears however that as much as ±7,000 cu yd were lost in 24 hours, or an average of about 300 cu yd per hr. The peak rate is not known.

Inspections by divers showed that in about three hundred of the two thousand joints, the adjacent faces of the sheet piles did not meet properly by amounts varying from less than 1 in. to a maximum of about 8 in., measured at El. −30. These conditions were scattered along the entire length of the wall and there were numbers of openings extending through the wall where grout had failed to plug the joints effectively. In some cases, loose cylinders of grout the size of the grout pipe were found where the character of the bottom material had prevented proper intrusion of the grout.

The officer-in-charge was concerned about exposing the untreated wooden piles under the relieving platform to the action of marine borers and rot. Accordingly, he made a detailed exploration of the settlements that occurred under the entire length of the platform in locations other than the cave-ins and found that the fill had settled over wide areas from little or nothing to as much as 30 in. There was no evidence of any lateral translation of the relieving platform or of any part of the sheet piles. Mounds of sand found by divers outside the sheet piles had slopes as steep as 2 on 1. However, divers did not report seeing sand pouring through the openings. In one case, a hole to El. −44 was found outside the sheet piling in a region where the level of adjacent dredging was about 10 ft higher. It is not known whether this resulted from accidental overdredging or from a "blowout" under the ungrouted gaps between the piles below El. −43, or from both.

The dredging procedure in the second half of the job was then changed so that the first cut was made to El. −25, the second to El. −35, and the final cleanup cut to El. −40. Also, a small berm was left at the foot of the sheet piling to guard against overdredging, and the cutter head was kept at least 25 ft from the wall. The −25-ft level was used as a working platform from which divers grouted all suspicious looking joints. Wide joints that appeared dangerous were grouted practically to the tips of the piles before the dredging was extended to lower levels. With this revised procedure no further cave-ins occurred. The contractor finally pumped concrete sand under the slab so that at times the pavement back of the wall was heaved without, however, starting any further loss of material.

DISCUSSION OF FAILURE

It is not uncommon for a structurally adequate wall to fail to retain the backfill because of leaks through the sheet piling. Ordinarily, without the relieving platform, leaks can be detected early by the formation of small holes in the surface just behind the sheet piles. With the platform, there was no chance to see such holes if they formed. The cave-ins occurred suddenly and covered rather large areas. (In Fig. 53 the cave-in lines beneath the platform were assumed.)

The obvious answer, of course, is to make the sheet piles tight. Since this may not always be entirely practicable it is desirable to understand how material is lost through small openings, so that precautionary steps may be taken to provide appropriate safeguards. This appears to be especially important where sand dikes are relied on to reduce the lateral pressures of clay backfills, since a loss of dike material might then subject the bulkhead to dangerous lateral loads.

In this case a fine sand fill was placed hydraulically to its full height before any of the piles were driven. Large storehouses were built on spread footings on this type of fill with no noticeable settlement. Although the pile jetting operations probably loosened the sand, it is likely that the final driving of the last few feet of each pile, without jetting, recompacted it to some extent. No data are available to show what densities were actually obtained. Some of the sand that escaped was virgin bottom, not recent fill.

A number of explanations have been considered. A sudden jar caused by construction operations, or a sudden outward movement of the sheet piling, might cause a "quick" condition in the fine sand and make it behave temporarily as a heavy liquid mass. There was a suspicion that the cutterhead of the dredge struck the piling in one case. However, there is no suspicion of this in the other cases and no movements of the piles were observed. Therefore, it is not likely that the cutterhead of the dredge struck the piling since such an occurrence would not explain the fact that the loss of material continued for 24 hours. Another explanation considered was that, as sand slowly washed out between the sheet piles, cavities were formed behind the piles and these holes were maintained by the arching action of the sand. As the cavities enlarged and the loads become too great, the arches collapsed successively and built up pore pressures which liquefied the material and permitted it to flow out rapidly. The extent to which arched cavities could form would depend on the cohesion of the material in the backfill since ordinary cohesionless sand will not arch in this manner under water. This also is not a likely explanation even though such action may have occurred locally and contributed in a minor way to the loss of some material; nor is it likely that the phenomenon of internal erosion was involved—that is, the gradual formation of underground channels caused by percolating water escaping into a free opening through which material can be carried.

Where the grouting extended only a few feet below the dredged level, and the spaces between the piles were large enough, it is possible that the passive pressure on the outside was not great enough to resist the inside, active pressure.

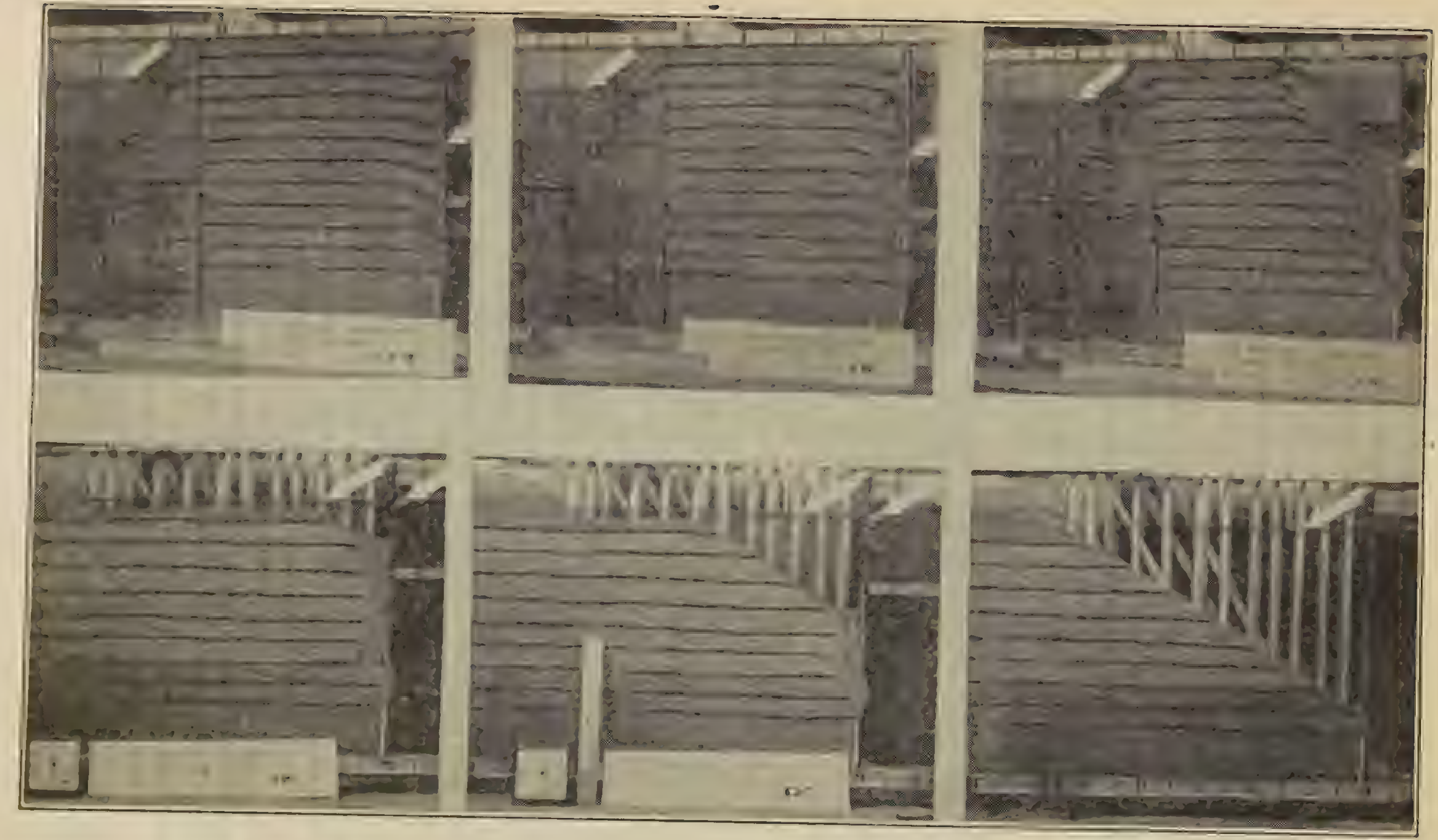


FIG. 58.—LABORATORY TESTS ON FINE DAYTONA BEACH SAND

This may have caused blowouts which then created quick conditions behind the bulkhead.

It is concluded, however, that most of the loss was started by the simple escape of cohesionless sand through openings between the piles above the dredged bottom. In an effort to visualize what might have happened, Professor Tschebotarioff ran a few laboratory demonstrations with a fine Daytona Beach sand. The tests were made in a glass sided tank 60 in. long by 24 in. high by 8 in. thick. A semicircular hole, 2 in. in diameter, was cut in the bulkhead against the glass side about 16 in. from the top.

In the first case the water was kept at the same level on both sides of the bulkhead and above the level of the sand. When the plug was removed, the cohesionless sand ran out of the hole by gravity. This quickly formed a vertical "pipe" of moving "quick" sand falling through water. The hole became enlarged within a few minutes. In the second case, the water was kept above the sand but was lowered on the outside. The action was the same but accelerated.

In the third case (see upper row, Fig. 58), water was kept slightly below the sand surface in back of the bulkhead and was kept somewhat lower on the outside. The action was similar to the other cases. In still another case (see lower row, Fig. 58), dowels were installed to simulate piles; again, the action was similar. The piles had little effect in retarding the rate of loss of the sand. It is possible that the reduction of the static pressure in the moving mass, as it acquires velocity, accelerates the loss of material from the banks, since the pressure in the banks would then be relatively greater. The escaping material was in a quick condition and the action would be expected to continue until the sand reached its natural angle of repose. In the prototype, it is entirely possible that chunks of material having some cohesion might have tumbled off the banks and caused a successive spread of localized quick conditions. Also, the wood piles could retard the loss of material having some cohesion and aid in building up higher and steeper banks than would otherwise be the case. This condition would tend to build up larger cavities and pressures and to produce sudden slides. The basic initial cause, however, appears to have been the simple flow of noncohesive material seeking its natural angle of repose under water.

The foregoing suggests that sandy materials used in backfills not only should have as great an angle of repose as practicable but also should be selected and placed so as to minimize the danger of loss from possible leaks in the bulkhead. The studies indicate the desirability of using densely graded coarse angular materials against the bulkhead to stop the loss of fines and to develop arching action across small openings. Of course, the thicker the sand dike behind the wall, the greater will be the factor of safety against damage resulting from unexpected leakage of the backfill.

Summary

Although there is nothing particularly new in this case, it emphasizes certain points—that is: (*a*) The importance of having tight walls sufficiently deep; (*b*) the need for guarding against conditions that can result in temporarily induced pore pressures which, in turn, cause a loss of materials or excessive lateral pressures not contemplated in the design; (*c*) the fact that a dense mass of piles behind the bulkhead cannot be relied on to retard the loss of materials or to reduce lateral pressures especially if a quick condition is initiated; and (*d*) the material that is placed immediately in back of the bulkhead should be carefully selected and its placement controlled. Where a dike of selected material is relied on in the design to reduce the lateral pressures of other softer backfills, it is especially important to guard against its loss and to provide means for easy inspection and replacement of lost material. The Bureau of Yards and Docks now provides vertical 12-in. pipes between bents to facilitate inspection and replacing material under relieving platforms and pavements.

FAILURE OF QUAY WALL AT MARE ISLAND, CALIFORNIA

By L. C. Coxe,[46] Esq.

The failure of the south quay wall at the Navy Yard, Mare Island, California, and subsequent investigations, conclusions, and remedial measures, are described in this paper.

The south quay wall, approximately 850 ft long, was part of a project to provide berthing and servicing facilities for the fleet. As shown in Fig. 59, the entire project included a north quay wall, a south quay wall, a west quay wall, and three finger piers. The south quay wall was constructed during the latter part of 1941 and the early months of 1942. Speed of construction was essential, and the rapid completion of all these waterfront facilities was the desperate need of the day.

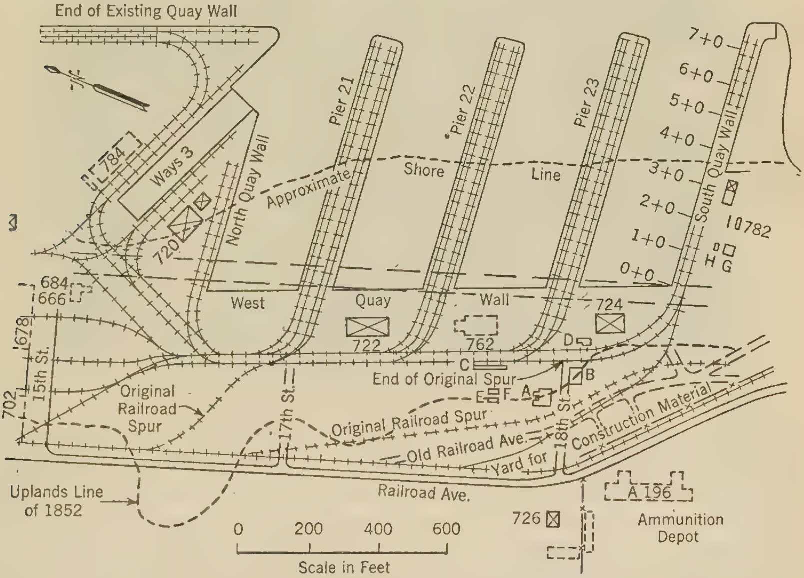

End of Existing Quay Wall
Ways 3
784
720
North Quay Wall
Approximate
Shore
Line
Pier 21
Pier 22
Pier 23
South Quay Wall
7+0
6+0
5+0
4+0
3+0
2+0
1+0
0+0
10782
H
G
684
666
678
702
15th St.
West
Quay
Wall
722
762
724
D
C
B
End of Original Spur
E
F
A
Original Railroad Spur
17th St.
Original Railroad Spur
Old Railroad Ave.
18th St.
Construction Material
Yard for
Railroad Ave.
Uplands Line of 1852
0
200
400
600
Scale in Feet
726
A 196
Ammunition Depot

Fig. 59.—Facilities for Berthing and Servicing Ships

The south quay wall is typical of many such structures constructed by the Civil Engineer Corps of the United States Navy. Reinforced concrete sheet piling 2 ft thick was used because of the shortage of steel. Behind the sheet piling, Fig. 60 shows a timber relieving platform supported on bents of twelve vertical wood piles and four wood batter piles. These bents, spaced at intervals of 5 ft, also support the heavy concrete foundations for the crane rails. A concrete head wall or quay wall, supported by the concrete sheet piling and the timber platform, completes the structure. Selected backfill was then placed on the relieving platform up to finish grade. Concrete anchors, supported by wood vertical and batter piles and connected to the quay walls by steel tie rods, 2 in. and 2.5 in. in diameter, were not included in the original design but were installed after failure of the wall. Fig. 61 is a longitudinal view of the quay wall during construction, indicating the size and weight of the crane railway girders and foundations.

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[46] Comdr. (CEC), U. S. Navy, Design Mgr., Bureau of Yards & Docks, Navy Dept., Washington, D. C.

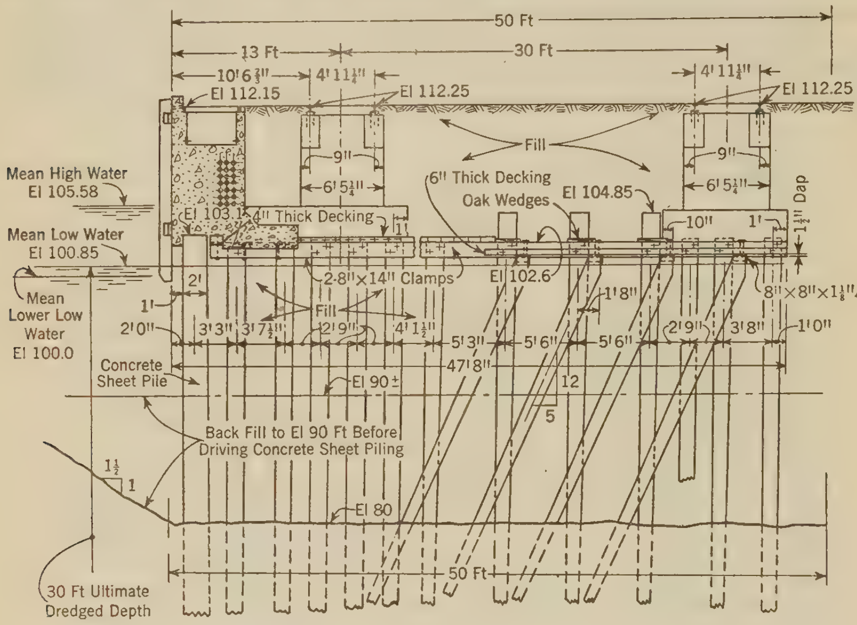

50 Ft
13 Ft
30 Ft
10'6¾"
4'11¼"
4'11¼"
El 112.15
El 112.25
El 112.25
Fill
9"
9"
6'5¼"
6'5¼"
Mean High Water El 105.58
6" Thick Decking
Oak Wedges
El 104.85
El 103.1
4" Thick Decking
1'
10"
1'
1½" Dap
Mean Low Water El 100.85
Mean Lower Low Water El 100.0
2'
1'
2'0"
2-8"×14" Clamps
El 102.6
Fill
1'8"
8"×8"×1⅛"∠
3'3"
3'7½"
2'9"
4'1½"
5'3"
5'6"
5'6"
2'9"
3'8"
1'0"
47'8"
Concrete Sheet Pile
El 90±
12
5
Back Fill to El 90 Ft Before Driving Concrete Sheet Piling
1½
1
El 80
50 Ft
30 Ft Ultimate Dredged Depth

Fig. 60.—Typical Section of the South Quay Wall

Construction was started by using a suction dredge to clear the site of the quay wall to El. 70, or 30 ft below mean lower low water. For design purposes, the elevation of the mean lower low water was taken as El. 100. After dredging was completed, 20 ft of fill material were deposited in the quay wall area by bottom dump barges. Then the wood piling was driven for the relieving platform and framed with timber at the platform elevation (El. 102). The concrete sheet piling was driven from a derrick barge using a Vulcan No. 0 hammer. The concrete quay wall was then constructed to anchor the top of the sheet piling, and backfill was placed to the underside of the relieving platform deck. Finally, the relieving platform was constructed, craneway footings were poured, and the relieving platform was backfilled to El. 112.

Cracks began to appear in the face of the concrete quay wall in January, 1942, shortly after it had been poured (see Fig. 62). The relieving platform in the vicinity of the early cracks had not been loaded with backfill. In an attempt to stop further outward motion of the wall, the relieving platform was immediately unloaded and 300 tons of "one-man" stone were deposited in front of the quay wall to stabilize the toe of the sheet piling. Construction of heavy pile-supported concrete anchorages was begun at Station 5 + 00 and at Station 7 + 25 (Fig. 59). By February, 1942, the anchorages were completed but not connected to the wall. On March 1, 1942, without the knowledge of the engineers in charge of the project, a sinking Navy tug was brought alongside the quay wall at Station 6 + 50 near its outer end and tied up pending salvage operations. The weight of this tug was further added to the lateral forces at work on the quay wall.

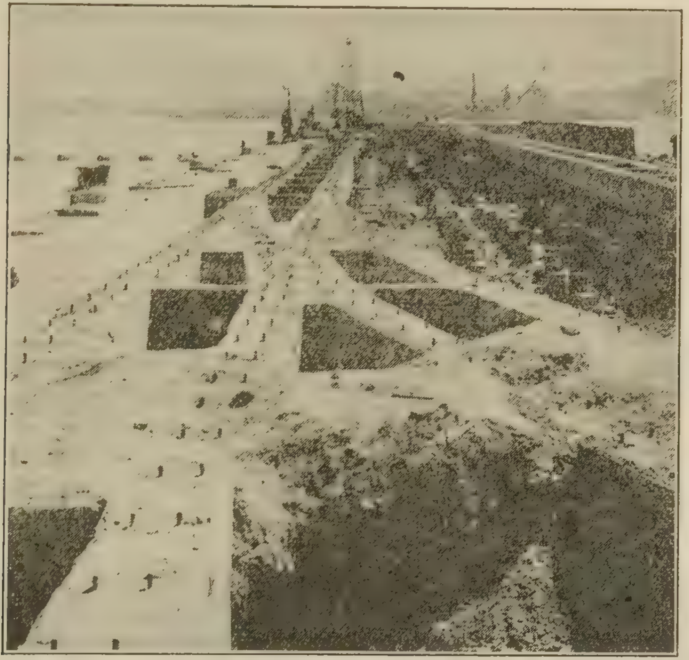


FIG. 61.—CONSTRUCTION VIEW OF QUAY WALL

A survey made from January through May, 1942, revealed that, at Station 6 + 75, where the sinking tug was tied up, the wall had deflected outward a total distance of 6.4 ft, while settling vertically approximately 1.9 ft. Most of the vertical settlement and lateral movement of the wall occurred during January and February. Anchor rods were connected to the outboard anchor on February 24, 1942, and to the inboard anchor on March 4, 1942; yet despite these tie backs there was still an appreciable movement. Indeed, on later investigation, it was found that five of the tie rods, 2.5 in. in diameter, of the outboard anchorage had failed in tension. The failure of these tie rods was attributed to the additional load placed on the wall when the sinking tug was moored alongside.

The outward motion of the bulkhead caused five major cracks more than 1 in. wide and six minor cracks of less than 1 in. Fig. 63(*a*) shows an 8-in. crack at Station 6 + 69, at the location of the sunken tug and Fig. 63(*b*) shows two large cracks at the outboard end of the quay wall. Fig. 63(*c*) is a longitudinal view of the quay wall showing a crack and the irregularities in the face of the wall.

In July, 1942, O. J. Porter, M.ASCE, was called in to investigate the causes of the failure and to make recommendations on methods of arresting further movement of the quay wall. Soil borings were taken which revealed very soft materials to El. 17, or 83 ft below mean lower low water (see Fig. 64). Below this elevation, a stiff greenish sandy-clay stratum was encountered which contained considerable glauconite. This greenish glauconite clay is a marine deposit formed by the action of organic matter during a period when there was little or no filling. Beneath this glauconitic layer was a stratum of brown clay formed by oxidation and weathering of older shales. Shale bedrock, probably cretaceous, was encountered beneath the brown clay at approximately El. 8.

Above the glauconitic clay, the material consisted of relatively recent marine deposits of clay, peat, and mud. The fill material under the relieving platform was found to be very soft and unstable. This condition probably was the result of the slaking of the clay-shale material used in the filling operation. The material under the fill had a unit wet weight ranging from 87 lb per cu ft to 102 lb per cu ft and a moisture content ranging from 52% to 118% of the dry weight of the soil particles. All indications were that the excessive lateral pressure acting on the bulkhead was caused by pore pressure, both in the backfill and in the fine-grained soil underlying the backfill. The pore pressure was caused by the weight of the 20 ft to 30 ft of fill placed during construction and

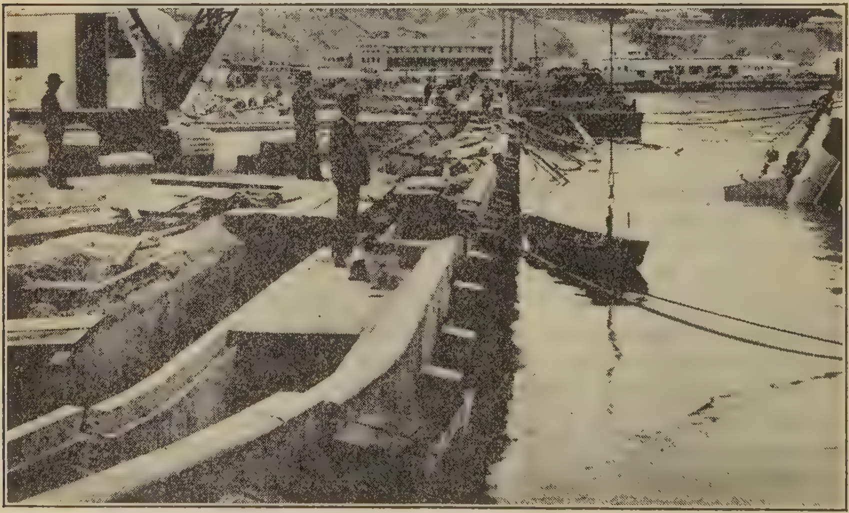


FIG. 62.—CONSTRUCTION PROGRESS, FEBRUARY, 1942; SUNKEN TUG IN RIGHT CENTER

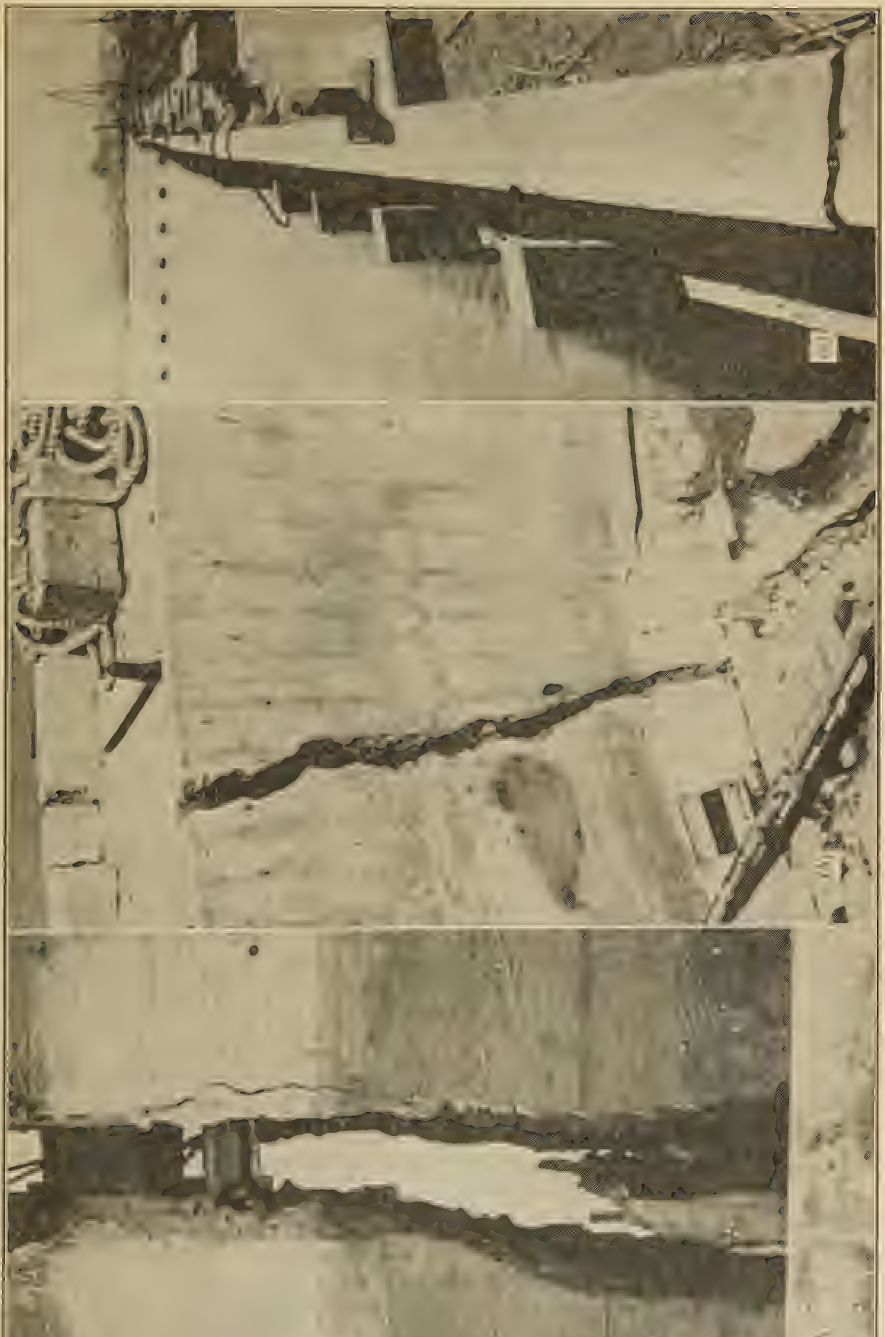


Fig. 63.—Major Cracks in the Bulkhead

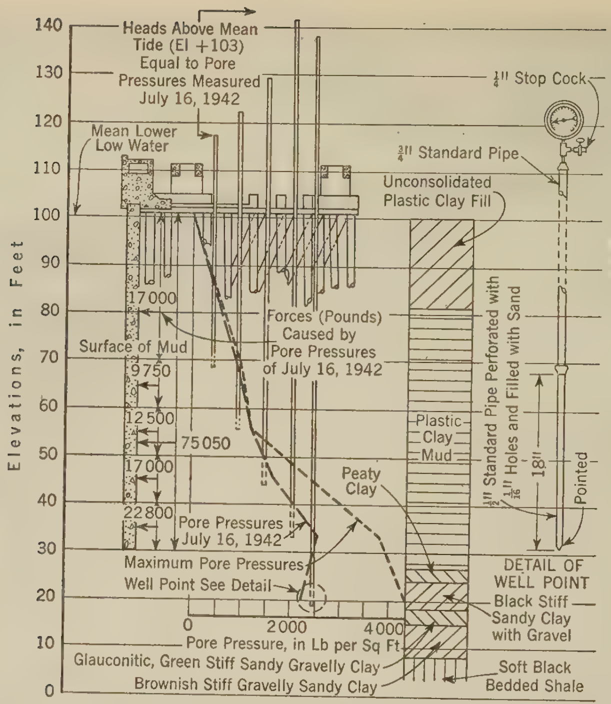

Elevations, in Feet
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
Heads Above Mean Tide (El +103) Equal to Pore Pressures Measured July 16, 1942
Mean Lower Low Water
¼" Stop Cock
¾" Standard Pipe
Unconsolidated Plastic Clay Fill
17000
Forces (Pounds) Caused by Pore Pressures of July 16, 1942
Surface of Mud
9750
12500
75050
17000
22800
Plastic Clay Mud
½" Standard Pipe Perforated with 1/16" Holes and Filled with Sand
18"
Pointed
Peaty Clay
Pore Pressures July 16, 1942
Maximum Pore Pressures
Well Point See Detail
DETAIL OF WELL POINT
Black Stiff Sandy Clay with Gravel
0
2000
4000
Pore Pressure, in Lb per Sq Ft
Glauconitic, Green Stiff Sandy Gravelly Clay
Brownish Stiff Gravelly Sandy Clay
Soft Black Bedded Shale

FIG. 64.—PORE PRESSURES

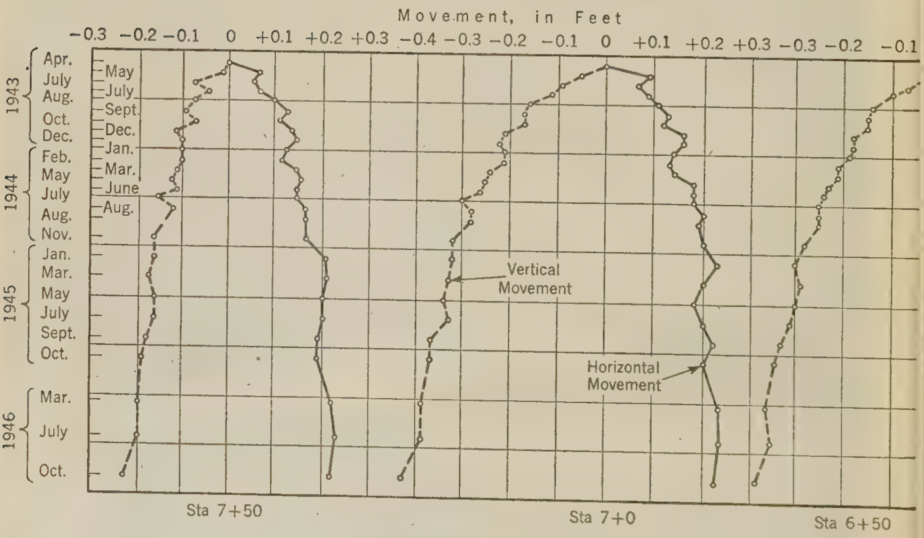
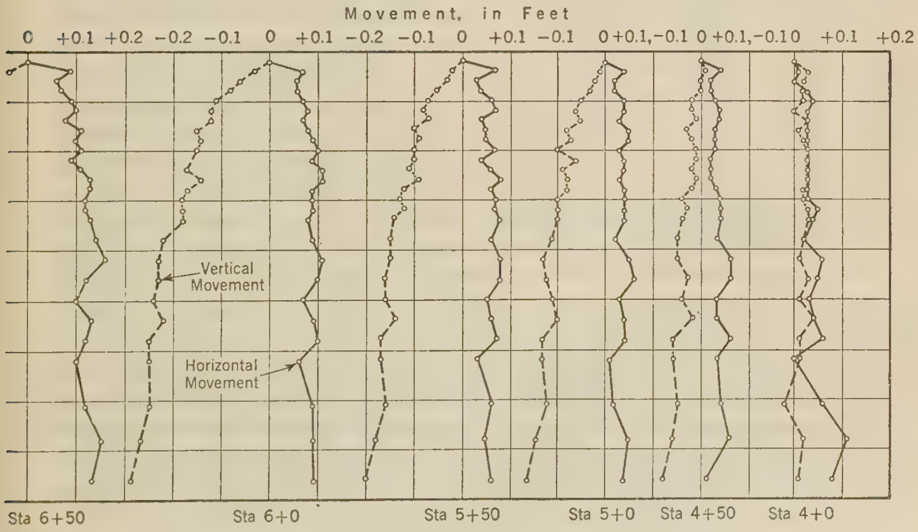


FIG. 65.—HORIZONTAL AND

also by that part of the weight of the quay wall structure which was transmitted as skin friction from the piles to the mud. To determine the extent and variation of pore pressure with depth, pore pressure measurements were made at El. 71, El. 56, El. 45, El. 33, and El. 20. A 1-in. soil sampler, devised by Mr. Porter, with a 0.5-in. perforated pipe extension was driven into the soil to the desired depth; and, after filling the pipe with water, pressures were measured by a gage attached to the upper end of the pipe. Readings were taken over a period of days and recorded as pounds per square inch after first correcting for the weight of the column of water above the tide level existing at the time the readings were taken. These readings indicated pore pressures ranging from zero at the surface to a maximum value of approximately 4,500 lb per sq ft 80 ft below the surface. The effect of pore pressure was to increase the lateral forces on the bulkhead by 75,000 lb per lin ft, over and above that which would have existed had pore pressure not been present. Pore pressure alone increased the lateral pressure 40% or more over that which might be calculated by assuming fully mobilized frictional and cohesive resistance in the soil.

To relieve this pore pressure quickly, work was begun in August, 1942, based on Mr. Porter's recommendation, to install vertical sand drains to a depth of 80 ft below mean lower low water. These drains were spaced 8 ft on centers in one direction and 10 ft on centers in the other, in the area under the relieving platform; and those in back of the relieving platform were spaced 12 ft by 13 ft. A total of three hundred and thirty-six drains were driven. The construction procedure called for the removal of sections of the relieving platform, cleaning out the mud and fill to a depth of 3 ft below the relieving platform, and backfilling with clean, coarse, concrete sand, or pea gravel. The sand drains were driven using a steel pipe, 12 in. in diameter, with a collapsible cap on the end. The mandrel was slowly withdrawn as the sand filled the driven hole. The



VERTICAL MOVEMENT

main difficulty was the refusal of the sand to stay in place when the mandrel was pulled. This difficulty was finally overcome by placing an airtight cap on the top of the mandrel after it was filled with sand and applying air pressure. The air pressure forced the sand out of the mandrel as the mandrel was withdrawn. The driving of the sand drains was completed in October, 1942.

In addition to the sand drains, and to hasten the reduction of pore pressure, surcharge in the form of sand blankets 5 ft or more thick was placed at various locations on the fill. When the pore pressures were reduced and the quay wall stabilized, the sand blankets were removed.

The action of the sand drains and surcharge in accelerating consolidation and pore pressure reduction was very effective although no actual data as to the reduction of pore pressures are available. This method of stabilization, combined with the anchored tie rods, completely checked any further motion of the quay wall and effective stabilization was realized in December, 1942—approximately two months after the sand drain installation was completed.

Inasmuch as the movement of the wall had stopped, it was decided in January, 1943, to dredge alongside the quay wall to the full 30-ft depth—necessitating the removal of 20 ft of clay from the outside face of the wall. The dredging was completed in February. Structural repairs to the pier were completed in May, 1943, and the quay wall was placed in service. Fig. 65 shows the results of the stabilization procedures. Elevations and alinement variations for the stations shown cover the 42-month period from April, 1943, to October, 1946. The minor changes of elevation and alinement occuring since May, 1943, were undoubtedly caused by the live load on the pier and the additional dredging. The changes in alinement for the 42-month period from April, 1943, to October, 1946, are on the order of only one twentieth of the magnitude of the changes during the 4-month period from January to May, 1942, prior to the installation of sand drains and anchors (see Table 3). The changes in elevation were similarly reduced. As of January 1, 1947, no further movement of the quay wall was observed and the backfill is now considered completely stabilized.

TABLE 3.—HORIZONTAL AND VERTICAL MOVEMENT, IN FEET

Station (see Fig. 57)	OUTWARD DEFLECTION		CHANGE IN ELEVATION	
	4-month period[a]	42-month period[b]	4-month period[a]	42-month period[b]
7+50...	4.0	0.22	−1.12	−0.30
7+00...	5.8	0.22	−1.96	−0.43
6+50...	6.2	0.13	−1.97	−0.38
6+00...	4.9	0.09	−1.45	−0.29
5+50...	4.5	0.06	−1.37	−0.20
5+00...	4.2	0.04	−1.18	−0.16
4+50...	3.8	0.01	−0.75	−0.08
4+00...	3.1	0.08	−0.30	+0.01

[a] 4 months from January, 1942, to May, 1942.
[b] 42 months from April, 1913, to October, 1946.

The account of this quay wall failure further emphasizes the many soils problems encountered by the Bureau of Yards and Docks in waterfront construction throughout the world, and explains the great interest that this Bureau has in the development of test procedures for determining, accurately, characteristics of soils of all types, methods of design, and construction procedures that will insure against future failures.

DISCUSSION

Ralph B. Peck,[47] **Assoc. M. ASCE.**—The following discussion is concerned only with those papers of the Symposium dealing with the tests at Princeton University (Princeton, N. J.). Professor Tschebotarioff has stated three objectives for the first series of tests. The writer believes that the first objective was successfully reached in large measure, and that the results of tests to determine the influence of sand backfill in reducing the lateral earth pressures of cohesive soils are of interest and value. The results of experiments to determine the lateral pressure exerted by fluid cohesive mixtures (the second objective) are of interest, but the interpretation of the results and their explanation seem to be unnecessarily complicated.

The studies of arching in sand backfills (the third objective) appear to have been planned and executed without adequate consideration of the vital factors involved. It is the writer's opinion that no valid conclusions concerning arching can be drawn from the results of the experiments as yet.

Distribution of the lateral pressure exerted by sand against a retaining structure depends primarily on the manner in which the retaining structure can deflect. Many statements by Professor Tschebotarioff indicate that he is conscious of this fact. Nevertheless, the writer believes that the deformation conditions associated with the experimental bulkhead were fundamentally so different from those associated with most real bulkheads that the results of the sand experiments are devoid of practical value in connection with bulkhead design.

It would seem advisable to precede an extensive series of experiments on bulkheads by field observations concerning the deflections of actual structures; this apparently has not been done. However, the type of deformation can be inferred from a general knowledge of the stress-deformation characteristics of somewhat similar structures. Commonly, a bulkhead is made by driving sheet piles in shallow water. The anchorage is constructed, and the sheet piles are connected to it by tie rods. Up to this point no significant deformations of the soil mass are experienced.

The next step is likely to be the backfilling of the sheet piles. While this occurs, the anchor rods are stressed, whereupon they stretch and also tend to pull the anchorage system toward the channel. If the anchorage system is properly designed, these movements are relatively small, on the order of 1 in. or 2 in. At the same time, however, the weight of the backfill causes shearing stresses in the soil in which the lower ends of the sheet piling are buried. Under the influence of these stresses, together with the lateral pressure of the fill against the lower part of the sheeting, the sheet piles below the anchor also tend to move toward the channel. This movement is likely to be at least as large as that at anchor level.

The final stage of construction involves dredging the channel to the desired depth. During this stage the middle part of the sheet piles, and even their lower ends, are likely to move out energetically unless the underlying material is very stiff. This outward movement may be several times as great as that at anchor level.

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[47] Research Asst. Prof., Soil Mechanics, Univ. of Illinois, Urbana, Ill.

Since bulkheads are commonly built where the subsoil conditions are relatively poor, it is the rule rather than the exception that the sheet piles should move outward near the dredge line, a distance considerably more than at anchor level. This constitutes the most important deformation condition. Nevertheless, the apparatus at Princeton was so constructed that movements of this type could not be simulated. Although the support at level B could be moved outward by some distance (the papers do not indicate how much), the support at C was not capable of translation. This fact must have appreciably restricted the outward movements possible in the vicinity of the bottom of the bulkhead. Even in spite of this restriction, the distribution of pressures inferred from the measurements indicated a moderate degree of arching because the center of pressure of the curves shown in Fig. 23 is at approximately 0.38 of the distance from the dredge line to the top of the bulkhead.

Apparently the restrictions on outward movement were recognized and the third series of tests was performed to simulate more nearly the conditions in the field. The results of bending moment and deflection observations for one test are shown in Fig. 27; computed lateral pressures are shown in Fig. 28. These pressures, however, were obtained by a method so crude as to eliminate them from serious consideration at the present time. The process of double differentiation to obtain pressure from measured bending moments is subject to very large errors, especially if the distribution of pressure is as irregular as that shown in Fig. 28.

The procedure described by Professor Tschebotarioff was reasonably accurate in connection with the bulkhead arrangement for the first tests because the reactions were measured and the laws of statics could be applied to provide a check. However, even under these relatively favorable conditions, errors in pressure of as much as 20% are indicated in the calibration curve (Fig. 12) for the bulkhead acted upon by water. The writer would have no confidence whatsoever in the results of any pressure distribution curves obtained by a process of double differentiation without at least a method for checking the computed pressures by the measurement of reactions.

An additional factor tending to reduce any tendency for arching in the experiments pertaining to Fig. 28 is the lack of backfill at and above anchor level. Hence, the writer does not believe that either the accuracy of the computations or the test arrangements justify Professor Tschebotarioff's statements that arching is unlikely to occur in sand behind real bulkheads. On the other hand, the writer is inclined to be somewhat more impressed with the observations made by Danish engineers of some bulkheads that had stood up, whereas, according to conventional ideas, they should have collapsed. Where the results of field observations and experience with full-sized structures appear to be in contradiction to those of somewhat questionable experiments, the writer would consider it unwise to place much reliance upon the experiments.

Concerning the deformation conditions—the writer would like to point out that the bending of the sheet piles is likely to be a matter of small consequence in the redistribution of pressure. Probably of far greater importance is the movement of the anchor and the outward movement of the sheeting near the dredge line. As nearly as can be judged, the experimental arrangement for the third series of tests permits appreciable outward movement of the sheet piles at the dredge line. The actual stiffness of the piles is not stated. It is of interest to note that the deflection gages, provided to observe the movements of the buried part of these piles, could not extend far enough to measure the actual movements. This suggests the extent to which such movements were underestimated and serves to explain some of the shortcomings of the original apparatus.

Furthermore, as the apparatus now exists there is no satisfactory way of determining the lateral pressures. It would seem necessary to find some method of determining the actual shears in the piling near the dredge line. A similar necessity with measurements in open cuts led to the expedient of cutting the piling at the bottom of the cuts and of measuring the reactions necessary to hold the piling in its existing position. Such measurements are not convenient or highly accurate, but they are far more accurate than the results of estimates based on the measurements of bending moments.

In Part II, Section B, Professor Tschebotarioff discusses at some length various methods for computing the earth pressure of soft cohesive soils against the model bulkhead. Much of the discussion is devoted to classical earth pressure theories and to the results of measurements against the bracing of open cuts in clay. The writer believes that this discussion is largely irrelevant.

All theories of this type are based on the assumption that the backfill has reached a state of failure either throughout a considerable region, or at least in the neighborhood of some surface of sliding. These conditions may be realized even in soft natural clay deposits when an open cut is excavated because a moderate deformation is likely to be sufficient to develop the shearing resistance of the soil. Furthermore, the deformations of a natural clay stratum during excavation are fully effective in producing shearing strains and developing shearing resistance in the soil mass.

In cases of bulkheads backfilled with almost fluid materials, this condition may not be realized. At the time of placement, the shearing resistance of the backfill is largely of a viscous nature. It seems unlikely that permanent shearing stresses can exist during the first days after construction. However, under the weight and lateral pressure of the backfill, large movements of the sheet piles can (and do) occur. Because of the viscous nature of the backfill, the pressures exerted by the soil mass remain largely hydrostatic and a definite shearing resistance along a potential surface of sliding cannot be developed.

Hence, at first the lateral pressures are likely to be almost equal to those of a fluid. As time passes, the backfill material may either consolidate or harden. If hardening occurs, it is likely to be associated with a shrinkage of the clay and possibly a backward movement of the bulkhead. Consolidation would occur not only vertically, but also laterally, under the influence of the small but nevertheless real forces of restitution exerted by the deflected bulkhead. These forces would lead to the observed backward movement of the model bulkhead.

The final condition would correspond to what the author terms the consolidated equilibrium condition. Since further outward deformations would be necessary to establish a state of failure in any region of the backfill, it seems obvious that the lateral pressures observed against the model bulkhead could not be computed successfully by any earth pressure theory based on the shearing resistance of the soil. Hence, the writer would consider the behavior of the clay backfills quite normal and sees no reason for attempting to invoke earth pressure theories. There does not appear to be any valid analogy between a bulkhead retaining a hydraulically placed backfill of clay-like material and an open cut in a natural soft clay deposit, and there seems to be no point in discussing design procedures developed for open cuts.

In conclusion, the writer would state that the Princeton tests represented a very ambitious program. They have undoubtedly served a very valuable practical purpose in providing information about sand dikes during the national emergency when such information was highly important. On the other hand, the writer believes that the so-called fundamental information derived thus far from the tests is disappointing and, in the case of sand backfills, misleading. It would be unfortunate if more satisfactory information were not obtained by modifying the test procedure so as to take proper account of deformation conditions, and to provide reliable means for estimating the distribution of lateral pressure.

Howard C. Roberts,[48] Esq.—In one of the early paragraphs Professor Tschebotarioff describes the paper as a progress report. It is in this vein that the writer wishes to offer a number of comments and suggestions.

The test program described is an ambitious one, and will undoubtedly serve as an example to other investigators, and for other installations. It is apparent because of this, and also because of the small amount of such information available in the literature, that any additions or emendations which will serve to inform the reader better will be worthwhile.

One most serious criticism to be offered by the writer has to do with the manner in which the paper is written. The experimental techniques used in this project are certainly as interesting as the reported conclusions (at least to the writer). To a person expecting soon to engage in similar test work, they might well be far more interesting; and to a person interested only in the quality and nature of the conclusions, a detailed account of techniques employed is useful in providing criteria for judging the results. The descriptions of the apparatus and the techniques in this paper are not sufficiently detailed, nor are they specific enough in the details which are presented.

Under the heading, "Part I. The Program of First Test Series" of Professor Tschebotarioff's paper, there are two points where additional information is desirable. First, what were the experimental circumstances which caused or contributed to the scatter in the bending-strain values for the bulkhead? The wire type resistance strain gages (apparently) used are subject to many sources of error, but most of these errors are systematic—not random. The scattering of points suggests mechanical difficulties—varying restraints or the like—but the description does not contain information enabling the reader to form an opinion. Second, the Carlson pressure cells used for measurement of end reactions are stated to have caused trouble by clogging. This implies (and it is a circumstance to be expected of the usual form of Carlson cell) that the movement of the cell diaphragm was large enough not to be negligible. What, exactly, are the conditions of this measurement, and would not a null method pressure measuring device be preferable?

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[48] Special Research Associate Prof. of Civ. Eng., Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

Since the data obtained from these gages were used in a computation involving double differentiation, small errors in the observed data might easily lead to far larger ones in the final result. It is the writer's opinion that on the basis of the published information this method should be considered subject to question.

Under the heading, "Part I. The Lateral Earth Pressure Meter" in the paper by Messrs. Ward, Bayliss, and Brown, there is described a device called a "lateral earth pressure meter." In this device, the lateral pressure resulting from a vertical load applied to a confined soil mass is measured in terms of the deflections of a series of split rings forming the walls of the device. The condition of the sample is described as one of "almost perfectly rigid support." Whereas it is probably true that for most samples the condition could be considered as equivalent to a perfectly rigid support, it has the appearance of a hybrid—representing an approximation the exact degree of which is not known. There is too little restraint to be called complete, but far too great to be comparable to that near most sheet piling bulkheads. In this case, too, it would seem that a null method is much to be preferred. Such a system can be set up with little more difficulty than that herein described (this statement is based on the writer's own experience), and it may be worth doing if for no better reason than the effect on the engineer's peace of mind.

In "Part II. Combined Active and Passive Earth Pressure Tests," there is a brief description of a transformer type deflection gage (a variable mutual inductance gage) and its installation. Gages of this kind have had wide applications for many years, yet are virtually unknown in general engineering circles. A detailed description with at least approximate specifications for construction should be included. In this connection, it may be as well to state that the writer's experience includes work with magnetic gages which required heating before use; but as far as his memory serves, the effect was always traceable to the selection of an unsuitable magnetic core material or an improper design of the core.

The writer wishes finally to point out that he desires to offer constructive criticism of a paper describing a project of great promise.

Armand Mayer,[49] M. ASCE.—It is most gratifying to see that, after having computed the pressure on retaining walls or bulkheads so many times, the validity of the assumptions forming the basis of these calculations can be checked. The papers of Karl Terzaghi,[50] Hon. M. ASCE, in 1936 had already made clear the mechanism of the pressure on a retaining wall, and had shown that all the classical methods of computation supposed a relatively important displacement of the wall, a condition not necessarily reached with modern construction methods.

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[49] Inspecteur Général des Mines, Ministère de l'Industrie et du Commerce, Paris, France.

[50] "Large Retaining-Wall Tests," by Karl Terzaghi, *Engineering News-Record*, February 1, 1934, pp. 136–140.

Professor Tschebotarioff based his experiments on somewhat different assumptions—a flexible bulkhead, rigidly fixed at its base and at a definite level by an anchorage. However, like Professor Terzaghi, he shows that the classical assumptions would be verified only if there were a shift of the bulkhead (undesirable by modern standards), so that the calculation should be made with a distribution of pressures very different from that generally accepted. Therefore, he should be thanked, but also encouraged to take up or to continue the tests so as to give, for all cases, the basis of computation that will lead to practical results. It would be regretted that, after having brought together the setup seen in Princeton N. J., by the writer, Professor Tschebotarioff would limit himself to the study of anchored bulkheads, in which the flexibility of the bulkheads brings in a factor almost impossible to take into account.

The writer would like to see model tests on the whole question of retaining walls—that is, measurement of the pressures at different levels for different natures of soil, with walls first rigid, then tilting within very small limits, as in actual construction. This should lead to experimental results that would replace the tables of Jean Résal[51] or Hans Krey,[52] of which the limits of validity are well known, but whose use is continued because others are not available.

This would certainly be a very extensive program; but Professor Tschebotarioff has the tools at hand to accomplish it. He is to be congratulated for what he has already done and should be encouraged to continue in the same line and to give the profession these needed additional data.

S. Packshaw,[53] M. ASCE, and J. Owen Lake,[54] Assoc. M. ASCE.—The tests carried out by Professor Tschebotarioff and his assistants are the most elaborate that have ever been undertaken. His objective is finding the answer to problems that have an important bearing on the design of sheet pile retaining walls, and he has therefore rendered a service to the engineering profession. The results, however, have shown very little evidence of arching in the sand backfill. This finding is so much at variance with accepted theories, and also with practical tests, such as those of J. P. R. N. Stroyer,[21,23] that one feels compelled to question whether the conditions in the model correspond with those that develop in the case of full-scale retaining walls backed by cohesionless material.

The design of the apparatus is not likely to have a great influence on the results of testing very soft clay backfills with a consistency approaching the liquid limit. For firmer cohesive materials, however, it seems doubtful whether the author's deductions about the relation of lateral pressure to shearing strength are justified.

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[51] "Poussée des Terres," by Jean Résal, C. Béranger, Paris, France, 1910.

[52] "Erddruck, Erdwiderstand, und tragfähigkeit des baugrundes," by Hans Krey, W. Ernst und Sohn, 1st Ed., Berlin, 1912.

[53] Director, The British Steel Piling Co., Ltd., London, England.

[54] Civ. Engr., The British Steel Piling Co., Ltd., London, England.

In Part II, Section B (under the heading, "(2) 'At Rest' or 'Consolidated Equilibrium' Lateral Pressures"), it is stated that:

> "According to one current popular concept,[11,12] outward motions of more than 5% of the height of the bulkhead are needed to mobilize fully the shearing strength of a clay soil which is effective in reducing lateral pressures."

Professor Tschebotarioff disagrees with this assumption and states that the tests at Princeton University (Princeton, N. J.) show that such a concept applies neither to plastic backfills behind flexible bulkheads nor to the "consolidated equilibrium" pressures. The writers have always assumed, however, that the value given by Karl Terzaghi, Hon. M. ASCE, of a 5% movement is intended to indicate the approximate magnitude of the yield that must occur in a bulkhead or in the timbering of a trench in order to reduce the "at rest" pressures in a cohesive backing to the "active pressure," assuming that no change in water content takes place. The hypothesis that a reduction of lateral pressure takes place because of forward movement of the wall is based on a completely separate phenomenon from the one in which the reduction in lateral pressure is due to the consolidation of a plastic backfill under its own weight.

It is suggested that the reason why no apparent reduction in lateral pressure with increased outward motion of the bulkhead was observed can be attributed to the movement being limited to only 1.5%. Even more important is the fact that the clay possessed a very low shearing strength. Although the forward movements of a retaining wall backed by very soft clay do not result in an appreciable reduction of lateral pressure, it is erroneous to reason that similar results are to be expected with cohesive backfills of higher shear strength; in other words, in the initial stages of wall movement the behavior of a very soft clay backfill, as compared with that of a stiff clay, will more nearly exhibit the characteristics of an ideal soil whose shearing resistance is independent of the degree of deformation. Furthermore, as the shearing resistance of a very soft clay is very small, the deformation resulting from wall movement is almost in the form of viscous flow, rather than plastic flow, which occurs with stiff clays.

It should be pointed out that very deep excavations have been made through clay within sheet pile cofferdams which have been braced internally to resist pressures computed on the assumption that the active pressure approximates that shown in Fig. 18(*a*). These designs have been justified by subsequent performance. Furthermore, the measurements made by Ralph B. Peck,[17] Assoc. M. ASCE, on the Chicago (Ill.) subway were in reasonable agreement with the theoretical total pressure based on the conception indicated by Fig. 18(*a*). Professor Tschebotarioff's statement that: "The concept illustrated by Fig. 18(*a*) * * * has not been definitely limited to such conditions [stiffly plastic clays]," is therefore justified.

The angle ϕ_q as given in Fig. 11 is stated to be almost identical for clay and sand-clay mixtures. Such a result is only to be expected, as both materials were initially of an almost fluid consistency. The coarser grains of the sand-clay mixture would be in a loose and random distribution, whereas the clay fraction would be in a honeycombed, and therefore extremely compressible, state. This is confirmed by the statement in Part I (under the heading, "D. Soils Used for the Tests") that:

> "* * * the unconfined compressive strength and the water contents of both types of backfill were found to have an almost constant value throughout the entire depth."

Under such conditions, the shearing characteristic of these materials is likely to be almost identical during the early stages of consolidation, as the shear strength is then almost entirely dependent on the properties of the clay matrix. Therefore, it was not surprising that the lateral pressures exerted by these two materials were very similar.

Referring again to the previously quoted paragraph from Part II, Section B, the phenomena of decreased bending strains, deflections, and horizontal reaction pressures observed during the natural process of consolidation of the fluid backfill can be explained by the action of excess hydrostatic pressures which occur during the consolidation process. Immediately after the clay backfill is deposited the "effective stress" is equal to zero because the process of consolidation has not commenced. During the subsequent process of consolidation the excess hydrostatic head gradually reduces to zero, and the "effective stress" correspondingly increases from zero to its final "consolidated equilibrium value." Thus, there is a noticeable reduction of total lateral pressure which was observed by the investigators.

The expression which Professor Tschebotarioff has developed to relate the laboratory test results with the measured values of lateral pressure can be more simply written as

$$p_h = \gamma\, h - 2\, c_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

From Eq. 53 it can now be seen that, if ϕ is assumed to be zero and c_e is the apparent cohesion obtained due to the overburden pressure, there is fundamentally no difference between Eq. 7 and the usual equation for the pressure of cohesive soils. For instance, the value of c_e for the fluid clay backfill at the surface is essentially zero in Fig. 18(*a*) as well as in Fig. 18(*b*).

As regards the phenomena of arching in sands (Part II, Section C), the sand was deposited through water, and under such conditions it is logical to expect only small percentage reductions in total pressure, because the water pressure had already caused the largest proportion of the deflection. The addition of the sand would scarcely cause an increase in deflection amounting to more than a third of that caused by the water pressure. Consequently, the additional deflection induced by the sand fill may not always be sufficient to reduce the lateral pressures appreciably by arch action.

Referring to Fig. 21—which shows six possible limit cases of load distribution, the total load remaining the same—the author infers that redistribution cannot possibly be arranged to yield a reduction greater than 50%. A reasonable diagram can be drawn, however, which gives an even greater reduction, especially if the anchor is located slightly below the top of the wall.

In Part II, Section D, Professor Tschebotarioff states: "* * * that the dominant factor in any actually possible reduction of pressures may be the influence of an underlying rigid boundary * * *." The writers agree completely that such an influence is always present, but they feel that the author has been misled into thinking that arching cannot occur to any appreciable extent simply because the Princeton tests gave little or no indication of this phenomenon. It is the writers' belief that the absence of any noticeable arching was due to those conditions in the model not analogous to conditions usually present in a full-scale bulkhead in the field. For instance, the 4-ft-wide sloping steel plate immediately behind the bulkhead at the level of support B may have caused misleading results. The presence of this plate forced the slip plane to start from the level of support B, whereas in an actual retaining wall the slip plane would probably extend somewhat deeper.

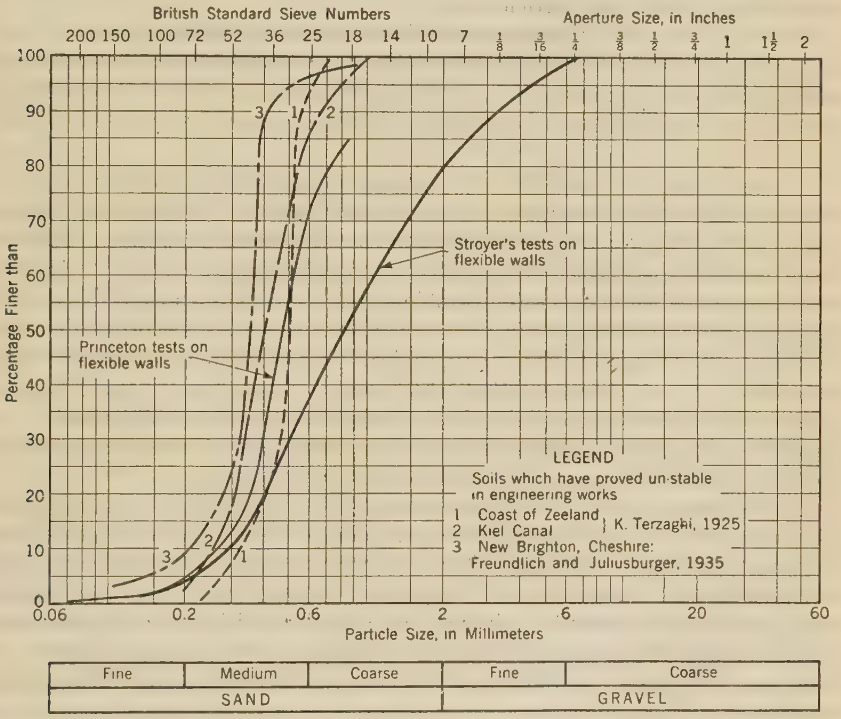

FIG. 66.—COMPARATIVE GRAIN-SIZE DISTRIBUTION CURVES

Furthermore, the tests showed that, if the sloping plate were released, arching would be immediately destroyed. This is only to be expected as a state of flux is then created which destroys the arch action; unless the plate was very rigid, it might possibly have deflected sufficiently under the weight of the backfill to interfere with arching and thus to cause the lack of stability which was observed in the test.

Another factor, considerably influencing the results, is that the sand used in the Princeton test was a particularly uniform, medium sand with a uniformity coefficient of about 2 and a voids ratio during testing of about $e = 0.8$.

A material possessing these characteristics is likely to prove unstable when subjected to changes in stress conditions; the changes would inevitably occur during flexing of the bulkhead and it may be that this factor explains, at least in part, the absence of any noticeable reduction in lateral pressure due to arch action. It is to be noted that, in the comprehensive range of tests conducted by Mr. Stroyer, the sand of the Thames River in England was found on analysis to be well graded and was considered to be a fairly representative sample of the material commonly used for backfill of retaining walls. Fig. 66 shows the grain-size distribution curve of the sand used in Mr. Stroyer's tests. Fig. 66 also gives the distribution curves for three types of sand which, as quoted by R. Glossop and A. W. Skempton,[55] have proved unstable because of their uniformity and particle size.

The results of the Princeton tests do not appear, therefore, to prove that no arching takes place behind flexible walls. The writers hope, however, that further tests will be undertaken by Professor Tschebotarioff, and that the apparatus will be modified in so far as it is necessary to reproduce field conditions.

JACOB FELD,[56] M. ASCE.—No amount of theoretical analysis based on arbitrary assumptions can produce even a small part of the advance in knowledge resulting from such experimental work as is reported by Professor Tschebotarioff and his associates. The writer can appreciate the extent of time and energy expended in this research problem, based on his own experience in the program on the determination of lateral earth pressure on rigid walls at the University of Cincinnati (Cincinnati, Ohio).[57] In addition, Professor Tschebotarioff had a time schedule within which usable results were required, and he must be complimented not only on the performance of the task assigned, but also for the concentration necessary to design, build, and calibrate the apparatus and also to perform the tests in such a short period of time.

The apparatus used in the first series of tests (Fig. 5) has some objections, from the point of view of similarity to the actual conditions encountered in an anchored sheet pile bulkhead. These objections are chiefly concerned with the existence of a pin-connected reaction, C, at the bottom of the sheeting, which cannot be expected to simulate an actual condition of embedded sheet piles. In the revised apparatus, used in the later series of tests, these objections are eliminated and the results must therefore be taken with greater confidence.

Incidentally, a similar division of the bulkhead into three sections, with pressures measured on only the center section, was used by R. O. Comer in determining the clay soil pressure for the design of the locks for the Inner Harbor Navigation Canal at New Orleans (La) in 1921. A summary of the results and a general description of the apparatus were included in a discussion by the late Henry Goldmark,[58] M. ASCE. The pressures were determined by successive differentiation of the measured deflections of a steel plate 8 in. wide and

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[55] "Particle-Size in Silts and Sands," by R. Glossop and A. W. Skempton, *Journal*, Inst. C. E., London, England, Vol. 25–26, 1945–1946, p. 81.

[56] Cons. Engr., New York, N. Y.

[57] *Transactions*, ASCE, Vol. LXXXVI, 1923, p. 1448.

[58] *Ibid.*, p. 1555.

6 ft high. The adjacent strips were free to move and deflect relative to the test area and to the sides of the bin. In general, Mr. Comer's results indicate a liquid pressure for freshly placed clay fills, reducing considerably with age, as the clay consolidated—sometimes as much as 18 in. in the height of 6 ft.

Supporting the test plate at three points introduces indeterminate conditions so that the results do not give sufficient data to solve all the equations and obtain the amount, direction, and distribution of the soil pressures. The measuring devices used are the most accurate apparatus yet employed for these purposes and require infinitesimal movements or strains to determine the stresses, from which the loads are computed. However, since the values of the individual reactions, *A*, *B*, and *C*, are dependent upon the relative displacement of the support, a questionable item is introduced. This is especially true because the direction of the reaction at the base of the sheet pile is opposite to *A* and *B*. The apparatus for the first series of tests does not permit the sheeting to take the natural shape, in the lower region, with a resultant effect on the reaction. Because of the pin-connected base, all the bending strain curves (Figs. 12, 15, 17, and 24) should be carried to a zero value at that level.

It is the influence of the *C*-reaction condition, probably more so than that of the lower boundary, as the author states, which has affected the values obtained in the lower regions. In the presentation of so much factual data, compressed by necessity into the permitted length of a Symposium paper, it might have been wise to omit the hypothetical pressure distribution curves in Figs. 19 and 23. Although they are both designated by a question mark, there is no experimental result in the author's tests to prove the shape or location of the curves.

The results of these tests must be used to their full value for the design of sheet piles. However, it should be pointed out that an investigation of a sheet pile bulkhead as a structural unit is also necessary. The writer had occasion recently to show that the lateral movement of an anchored steel sheet pile bulkhead was easily explained by considering the entire structure as a unit. Although the sheet piling by itself had sufficient strength and did not seriously deflect, the total pressure on the structures, consisting of sheet piles and anchorages, was not balanced by a sufficiently large passive soil resistance.

One of the important conclusions of this paper is the proof that the Danish assumption of pressure distribution on sheet piling is not reliable. Another conclusion of great value is the fact that a relatively thin layer of sand against the sheeting does not reduce the liquid pressure of clay fills. This finding should affect the usual reliance on a bagged layer of gravel against a wall as a guarantee that the wall need not be designed to resist liquid pressure, when plastic fills are used. It may be surprising, however, to note that a sand fill equal to one half of the bulkhead height, only partly nullified the fluid clay backfill pressure. This result is equivalent to the findings from other research projects that surcharges beyond the slip or fracture surface do affect the lateral pressure of the fills.

The final report of the later test series should furnish sufficient data for the complete solution of the sheet pile problem. It would be easier to analyze the results of the second and third series, if the first tests were made in more homogeneous soils. The embedment of the sheeting in clay with a backfill of sand introduces possible effects on the deflections beyond those that would be found if the entire soil body was either sand or clay. The explanation of the inconsistent results shown in Fig. 28 for stages 4*a* and 6 may be found in this factor of the test program. Both of these stages of the test series are at sudden changes of the sand backfill, each of which must seriously affect the conditions of the sheeting embedded in the underlying clay. Further tests will undoubtedly be carried on with a smaller ratio of embedded and projecting sheeting lengths. The results having a D-value of 1.18 H cannot be considered as an accurate picture of actual designs used. In many soil types such a length of embedment would make the sheeting entirely stable without top anchorage.

The fact that these tests were instrumental in discovering the economy resulting from a sand-dike backing of sheet piles must not be slighted. It is an important contribution to structural design, and the author should be given proper credit for this advance in engineering knowledge.

E. De Beer,[59] Esq.—The very extensive tests reported by Professor Tschebotarioff have engaged the writer's interest since his visit to the Soil Physics Laboratory at Princeton University (Princeton, N. J.) in 1947. In the intervening time, substantial results have been obtained, one of the most conclusive being the influence of a sand dike on the reduction of the pressure exerted by a backfill of liquid clay.

Some interesting phenomena were observed: (1) The reduction of the pressures on a bulkhead when a more rigid subsoil is near by; and (2) a decrease of the pressure on a bulkhead during the consolidation of the backfill while the relative movement of the wall is inward toward the fill. This second phenomenon has been verified in a report by E. C. W. A. Geuze mentioned by Professor Tschebotarioff and also in observations made during the performance of cell tests by the writer.

The subject of the report is limited essentially to a description of the results, and the author does not offer a mathematical interpretation. Nevertheless, the results are found to be sufficiently conclusive to serve as a basis for rules for the computation of pressures on anchored bulkheads. Since the results have not been given a physical and mathematical interpretation, and since they have not been confirmed by a larger number of tests, it seems advisable to be cautious in applying them in practice. It is in this spirit that the writer places great importance, on the one hand, to the need for continuing and extending the testing program and, on the other hand, to the need for a more refined analysis of the quantitative results.

In Section 5, paragraph *b*, of his paper, Professor Tschebotarioff makes the statement that there was no agreement between the measured pressure and the values calculated by the Rankine-Résal theories, although they are based on shearing resistances obtained from laboratory tests. On the other hand, he states that the measured pressures can be expressed by Eq. 7. That formula can also be written as follows:

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[59] Director of the Institut Géotechnique de l'État, Ghent, Belgium.

$$p_h = \gamma_w h + \gamma_e h (1 - \tan \phi_q) \dots\dots (54)$$

Assigning numerical values to ϕ_q at intervals of 5° an interesting comparison between computed values of $1 - \tan \phi_q$ and $\tan^2 (45° - 0.5 \phi_q)$ is afforded by Table 4. In this table the values of $1 - \tan \phi_q$ for values of ϕ_q higher than 40° have been placed in parentheses, because they become smaller than the values of $\tan^2 (45° - 0.5 \phi_q)$ and thus no longer have any real physical significance. Measured as the slope of Curves II and III in Fig. 11, ϕ_q is about 24°, which by interpolation in Table 4 yields

$$1 - \tan \phi_q = 0.56 \dots\dots (55)$$

The pressure on the bulkhead as measured certainly approaches most nearly the case of the pressure at rest. The function $1 - \tan \phi_q$ should thus vary more or less in relation to the coefficient of the pressure at rest. Table 4 shows that the smaller the value of the angle ϕ becomes, the smaller the difference between the functions $1 - \tan \phi$ and $\tan^2 (45° - 0.5 \phi)$ will be. Consequently, the tests seem to indicate that the consolidation pressure on a moving wall, behind which the soil has the opportunity to consolidate, approaches the active or minimum pressure, as the angle of friction of the soil becomes smaller. This is the general tendency as the soil becomes more clayey. On the other hand, for pronounced sandy soils, the consolidation pressure against a wall that is moving in response to consolidation can be noticeably larger than the minimum or active pressure.

This different behavior of clayey and sandy soils has been known for a long time from the results of the numerous cell tests performed in Holland and in Belgium. The findings of these tests are now confirmed by the large-scale tests described by Professor Tschebotarioff, and they help one to understand partly the problems concerning the difference between the measured pressures and the values calculated from Eq. 5*b*.

TABLE 4.—COMPARISON OF $1 - \text{TAN}\ \phi$ WITH $\text{TAN}^2\ (45° - 0.5\ \phi)$

Quantity	Values of ϕ									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
$1 - \tan \phi$	1	0.913	0.824	0.732	0.636	0.534	0.423	0.300	(0.16)	(0)
$\text{Tan}^2 (45° - 0.5 \phi)$	1	0.840	0.704	0.589	0.490	0.406	0.333	0.271	0.217	0.172

For that matter, Fig. 19 indicates clearly that, when the supports A and B are released after consolidation, the decrease in pressure is larger in the case of the sand-clay mixture than in the case of the pure clay.

In Fig. 19 it would be helpful to know how Professor Tschebotarioff explains values of K_{s+w} less than 0.5. The lateral pressure is given by

$$p_h = \gamma K_{s+w} h \dots\dots (56)$$

With $\gamma = 2 \frac{t}{m^3}$, at depths where $K_{s+w} < 0.5$, the total pressure should be even less than the hydrostatic pressure of the water. In the case of complete consolidation, this seems to indicate some leakage at the considered depth.

The results of the second series of tests (see Part III) are also in agreement with the preceding statements in this discussion. After consolidation of the fluid red clay, $K_s = 0.50$ The value of ϕ derived from $K_s = \tan^2 (45° - 0.5\,\phi)$ is a little smaller than 20°, that derived from $K_s = 1 - \tan \phi$, a little larger than 25°. Both values occur in the range in which the value of the angle of internal friction of a clay may be expected. In that range the values of $1 - \tan \phi$ and $\tan^2 (45° - 0.5\,\phi)$ do not differ greatly, and thus the measured pressure cannot be very different from the minimum or active pressure.

This explains why (see Part III, end of Section B) a release of the ring supports equal to 0.3 in., or to 2.5% of the sample diameter, did not continue to decrease the lateral pressure exerted against the upper rings.

A last question concerns the results of the third series of tests (Part III, Section C) in which a K_s-value of 0.22 is reported. From this value Professor Tschebotarioff claims a substantiation of the earlier statement that the tentative recommendations shown in Fig. 25(*a*) are of a conservative nature. It is not the writer's intention to discuss the recommendations revealed in Fig. 25(*a*), but he cannot agree that the results in Fig. 28 substantiate their conservative nature. Indeed, the third series of tests were performed on a sand backfill with a height of only 39 in. (1 m). Therefore, the maximum effective pressure σ_e does not exceed 1 ton per sq m; yet even sands have some interlocking cohesion c (generally about 0.2 ton per sq m). With $\sigma_e = 1$ ton per sq m the corresponding shearing resistance τ is

$$\tau = c + \sigma_e \tan \phi = 0.2 + 1 \tan 30° = 0.778 \text{ ton per sq m} \quad \text{(57)}$$

The dry density of the sand (Part III, Section C) being 93 lb per cu ft (1,540 kg per cu m), the angle ϕ was adopted as 30°. Consequently,

$$\frac{\tau}{\sigma_e} = 0.778 = \tan 37°50' \quad \text{(58)}$$

and

$$K_s \leq \tan^2 \left(45° - \frac{37°50'}{2} \right) = 0.24 \quad \text{(59)}$$

The value given by Eq. 59 agrees very well with the measured value of 0.22; but the low value of K_s is only obtained because of the low value of σ_e, and thus because of the large relative influence of c. For instance, in a sand backfill with a height of 10 m, one should get

$$\sigma_e = 10 \text{ tons per sq m} \quad \text{(60)}$$

$$\tau = c + \sigma_e \tan \phi = 0.2 + 5.78 = 5.98 \text{ tons per sq m} \quad \text{(61)}$$

$$\frac{\tau}{\sigma_e} = \frac{5.98}{10} = 0.598 = \tan 30°50' \quad \text{(62)}$$

and

$$K_s < \tan^2 \left(45° - \frac{30°50'}{2} \right) = 0.321 \quad \text{(63)}$$

The value given by Eq. 63 is much greater than that given by Eq. 59 and by the tests, indicating that the numerical values found in the third series of tests on small sand backfills may not be extrapolated to the higher backfills encountered in practice.

The results obtained with the tests performed at Princeton are exceedingly valuable. It is to be hoped that the unique equipment available there will also be used in the future to continue the program of testing, and that the profession will receive regular reports of the test results. If an extension of the testing program were possible, it would be worthwhile to include measurements of the pressure changes on the anchored bulkhead when the surface of the consolidated backfill is alternately loaded and unloaded.

D. P. KRYNINE,[60] M. ASCE.—All papers of the Symposium are interesting and merit attention. New ideas are advanced and considerable material for further thinking and research is furnished. The sponsorship of the United States Navy Department is to be highly commended. Skill, energy, and persistence in work by individual authors, particularly by Professor Tschebotarioff, must be given due credit.

Large-Scale Model Earth Pressure Tests on Flexible Bulkheads.—The lateral pressure on a perfectly immovable, indeformable, and frictionless wall obviously would be the same as the horizontal pressure within a natural earth mass made of the same material as the backfill. The lateral pressure diagram in the case of a rigid wall with a backfill possessing some shearing strength may change its triangular shape as the result of translation (yield) of the wall. In the case of a flexible wall, such as a bulkhead, such a change is due to: (*a*) Translation of the wall because of the stretching of the tie rod and forward motion of the anchor and the foot of the sheet piling, and (*b*) deflection of the sheet piling. In the case of a wall that is assumed to be flexible, but in reality is not flexible enough, and in addition is restricted in its forward movement, the lateral pressure diagram would be approximately of a triangular shape. Fig. 18(*b*) indicates that this possibly was the case of the model used in the tests at Princeton University (Princeton, N. J.). This diagram is of great value as a general guide for the designer, however. In fact, a bulkhead will yield at first, being pushed forward by the pressure of the practically liquid, hydraulically placed backfill. There is no redistribution of pressures at this initial stage since liquids do not possess shearing strength great enough to permit shearing stresses of appreciable magnitude to be developed in the backfill. This is the most dangerous time in the service of a bulkhead; and, therefore, the structure should be designed for a triangular lateral pressure distribution.

Required Value of Yield.—The proper way to state the value of the yield of the wall that may produce changes in the lateral pressure distribution is to express it in absolute values (for instance, in inches), rather than in fractions of the span of the wall as usually done. The larger the tendency to the relative displacement of one horizontal soil layer with respect to the other, the larger will be the reaction opposing this movement. A large wall may be flexible, whereas its model may be rigid. In particular, the maximum deflections of the testing plate shown in Fig. 17 are between 0.4 in. and 0.5 in.; and, since the pressure distribution was practically triangular (see Fig. 18(*b*)), it should be concluded that the absolute value of these deflections was insufficient to produce a lateral pressure redistribution. In the prototype, however, the same deflections, duly increased as shown hereafter, would possibly be sufficient to produce lateral pressure redistribution.

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[60] Visiting Prof. of Eng., Univ. of California, Berkeley and Los Angeles, Calif.

Similitude Problem.—The model used in the tests was about one fifth of the prototype size. Increasing the model five times to the size of the prototype, the thickness of the wall would be 5 in. The web thickness of common, full-size, sheet pile sections[61] is only from $\frac{3}{8}$ in. to $\frac{1}{2}$ in. The section modulus of the pile type M107 is 3.0 in.^{3} per ft and that of the heaviest type (M110) is 15.3 in.^{3} per ft, whereas a plate 5 in. thick would have a section modulus of 50 in.^{3} per ft. When computing deflections in engineering textbooks and handbooks the physical properties of the load are not considered at all, the load being shown schematically by arrows. In such a case a fivefold increase of the model would cause a fivefold increase of unit lateral pressures. The moments of inertia (per foot of width) would increase 125 times and the deflections would increase twenty-five times. This would create yields of the prototype that may reach $0.4 \times 25 = 10$ in. However, because of the interaction of elastic forces in the bulkhead and in the material of the fill, such an estimate is exaggerated. Assuming more conservatively that the yield of the wall in the prototype would be between five times and twenty-five times larger than that in the model, there may be yield values of 5 in, or 6 in. that are already sufficient to mobilize shearing stresses of appreciable magnitude in the backfill and thus possible modify the pattern of lateral pressure distribution shown in Fig. 18(*b*).

Decrease of Lateral Pressure Due to Consolidation.—A sheet pile, deformed by hydrostatic pressure of the hydraulic fill, tends to return to its original shape and size and does so as soon as consolidation begins. As the consolidation process continues, vertical and horizontal shears are created in the backfills. The value of these horizontal shears is increased by the elastic action of the material of the bulkhead since the latter decreases its curvature and presses on the backfill. In this sense the lateral pressure as shown in Fig. 17 should be identified as passive pressure rather than as active pressure (stages 4 and 10). The passive pressure below the tie rod acts from the sheet piling toward the backfill, and the displacements are in the same direction. Possibly this will clarify Professor Tschebotarioff's doubt as to the validity of conventional stress-strain equations in the case of consolidation (Fig. 17 and supporting text).

Tension Zone at the Top of a Cohesive Earth Mass.—The tension problem is of importance in the case of unsupported slopes, or vertical slopes supported by yielding rigid retaining walls, rather than in the case of bulkheads. For the case of rigid walls, the writer shares Professor Tschebotarioff's opinion as to the nonvalidity of the formula:

$$z_0 = 2\,\frac{c}{\gamma} \qquad \text{(64)}$$

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[61] "Carnegie Pocket Companion," Carnegie-Illinois Steel Corp., Pittsburgh, Pa., 1938.

for determining the depth of the tension zone at the horizontal top surface of a cohesive earth mass (Fig. 18(*a*)). Eq. 64 may be valid for a stretched semi-infinite mass (if only such a mass can be stretched because of the lack of space required for displacements), but it has no significance in the case of a finite vertical slope, unsupported or supported by a yielding retaining wall. In fact, Eq. 64 may be developed considering the Mohr circle for a point with zero tension located at a depth z_0 below the top of the slope.[12] In the case of a stretched semi-infinite mass, the principal stresses at the given point are practically the vertical pressure γz_0 and the zero tension. In the case of a finite mass, however, the vertical pressures at the points located at a horizontal plane z_0 units deep are not constant, but vary from point to point. The presence of a vertical slope affects the stress distribution in the mass materially only on a short distance from the slope, this distance being perhaps equal to about the height of the slope. Consider conditions of equilibrium of the prism ABCD, Fig. 67, keeping in mind the condition $\Sigma M = 0$. The moment of the couple formed by the total pressure at rest F above plane AD (depth z_0) and the corresponding reaction F equals $\frac{1}{3} F z_0$. To balance this couple another couple I–II, Fig. 67, is necessary. Estimating very roughly the arm of this couple at $z_0/3$, and remembering that

$$F = \tfrac{1}{2} K \gamma z_0^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(65)$$

(in which K is the coefficient of pressure at rest) the value of each of the forces I and II (Fig. 67) would be F. Distributing the overloading force F uniformly over an area of about $z_0/3$, the average additional pressure at a point of plane AD next to the slope is

$$\frac{0.5\, K \gamma z_0^2}{z_0/3} = \frac{3}{2} K \gamma z_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(66)$$

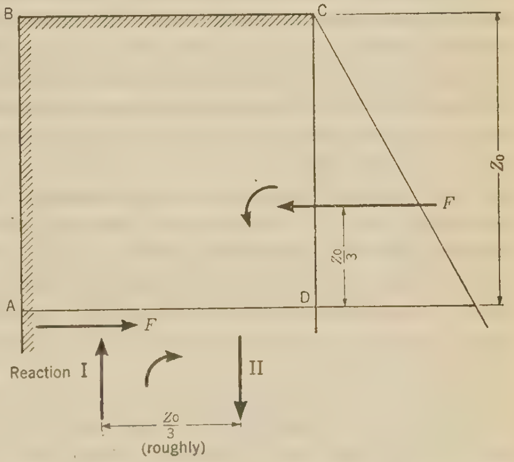

FIG. 67.—CONDITIONED $\Sigma M = 0$ FOR THE END ABCD OF AN UNSUPPORTED EARTH MASS

For $K = 0.6$, the average additional pressure at a point in plane AD next to the slope may reach a value of $0.9\,\gamma z_0$. In reality the overloading and relieving stresses at different points of plane AD are not uniformly distributed, as already stated. More information along these lines has been given by the writer elsewhere.[62]

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[62] "Vertical and Horizontal Shearing Stresses in Earth Masses," *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Holland, 1948, Vol. VII, Paper No. Ie22.

If a retaining wall is perfectly immovable, there is no overloading of the backfill at the surface of contact with the retaining wall. Any translation of the latter (and hence any decrease in the value of the reaction of the wall) approaches a stress pattern similar to that shown in Fig. 67.

Application of Test Results to Quay Wall Design.—Mr. Epstein discusses the possibility of decreasing the active pressure on a bulkhead from a hydraulic backfill by placing a sand dike next to the latter (Fig. 38) or by using vertical sand blankets. Professor Tschebotarioff's interesting tests along these lines have shown that blankets narrower than one tenth of the height of the bulkhead are not efficient. In so far as the dikes are concerned, the writer believes that the difference between the pressure from the hydraulic fill and that from the sand dike is taken up by the earth mass below the bulkhead. In other words, the dike does not decrease the pressure from the hydraulic backfill, but simply redistributes it. To claim otherwise is to contradict the condition of equilibrium $\Sigma H = 0$. It may be argued that pressure can be transmitted to an earth mass or a part of an earth mass, only through some physical body different from the earth mass itself, such as a masonry base or a concrete wall. This is not so, however. As an example, consider the increase in lateral pressure above an unlined tunnel. At a certain distance from the tunnel the natural lateral pressure follows a triangular pattern. This pressure must be balanced at any vertical plane of the mass including the center line of the tunnel; and in this case the condition $\Sigma H = 0$ is satisfied by the increase of the natural earth pressure (that is, the pressure existing prior to the construction) above and possibly below the tunnel.

The shearing stress diagram below and within the dike or a blanket must have a certain shape and a certain maximum value that does not exceed the shearing strength of the material of the dike (or the material at the base of the dike). To be developed, such a curve[62] requires a certain length; and apparently the length of one tenth of the height of the wall is not sufficient for this purpose.

Failure of Quay Wall at Mare Island, California.—Commander Coxe is to be commended for a lucid presentation of the case including the description of the sand drain method of O. J. Porter, M. ASCE. The Mare Island failure was really an object lesson for designers and for the civil engineering profession as a whole. It was proof of the view that, if the submerged backfill consists of a material with a flexible skeleton, the total value of the surcharge is transmitted to the wall by the pore water. It is obvious that, since the lateral pressure on a bulkhead decreases as consolidation progresses, the most dangerous period in the life of a bulkhead is the time of its construction and perhaps a certain brief time afterward.

Gregory P. Tschebotarioff,[63] M. ASCE.—The writer expresses his appreciation to all discussers, but wishes to give an evaluation of such criticisms as have been made.

Professor Roberts offers what he terms a "most serious criticism" of the fact that descriptions of experimental techniques were not given more space in the writer's paper. On the other hand, the original draft of the writer's paper was criticized for the opposite reason on the grounds that the vast majority of the members of the Society are more interested in actual findings than in laboratory techniques. Although he has endeavored to keep a reasonable balance within the space limitations imposed, the writer concurs with the latter point of view. Detailed descriptions of such techniques should be relegated to specialized publications. This was done to the extent possible at the time[36,64] and has been developed since.[65]

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[63] Associate Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

Professor Roberts writes: "* * * the Carlson pressure cells used for measurement of end reactions are stated to have caused trouble by clogging." No such statement is made in the paper. A clear differentiation is made in the paper between the two types of cells—Carlson pore pressure cells (which measured the excess pressures of the water filling the voids of the clay and the porous stones of which got clogged) and the two Carlson cells used for the measurement of vertical reaction pressures of the metal bulkhead. A perfect understanding of this differentiation will serve to clarify many of the questions raised by Professor Roberts.

In commenting on Part I of the paper, Professor Roberts states:

> "The wire type resistance strain gages (apparently) used are subject to many sources of error, but most of these errors are systematic—not random. The scattering of points suggests mechanical difficulties * * * ."

The term "electric resistivity strain gages" was used repeatedly in the text of the paper and the gages were even identified by their trade-mark as "SR-4" gages. Reference was made[64] to an extensive description of the type, mounting, insulation, and other details pertaining to these gages. Thus Professor Roberts is correct when he states that the gages were of the wire type; and he need not have qualified his statement with a parenthetical "apparently." Furthermore, the writer disagrees with the assertion that most errors of such gages are systematic and not random. The reverse is true, as can be seen from Fig. 33 (which shows the erratic deviations from the initial readings of gages mounted on a steel plate which was described in the paper as being "unstressed"). None of the mechanical difficulties suggested by Professor Roberts were thus possible during that test; nor were they possible during any of the tests with model bulkheads. Systematic studies and observations by the writer's laboratory since 1941[64] indicate that even after elimination of instrument errors there occur some erratic variations when readings are taken over long periods of time, because of uncontrollable minute volume changes of the cement by which the SR-4 gages are attached. The erratic nature of these slight variations, when understood, permits their control and elimination by the use of a sufficiently large number of gages. For that reason 105 separately measured gage pairs were used during the third series of tests.

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[64] "Use of Electric Resistivity Strain Gages Over Long Periods of Time," by Gregory P. Tschebotarioff, *Proceedings*, Soc. for Experimental Stress Analysis, Vol. III, 1946, No. 2, pp. 47–52.

[65] "Determination from Bending Strain Measurements of the Distribution of Lateral Earth Pressures Against Model Flexible Bulkheads," by Gregory P. Tschebotarioff, *Géotechnique*, London, England, Vol. 1, No. 2, 1948, pp. 98–111.

Professor Peck admits that the results of the tests performed with backfilled sand dikes "are of interest and of value"; and, on the other hand, he accepts the Princeton testing techniques as giving "highly important" information in that case. Nevertheless, he claims that exactly the same testing techniques give "disappointing" and "misleading" information in the case of backfills composed entirely of sands. The only apparent difference between these cases lies in the fact that sand-dike tests had never been performed before; and they had never been dealt with in textbooks. Consequently, their results were not in conflict with any previous conventional ideas on the matter. Therefore, prior to giving further and conclusive evidence proving the previously unsurpassed accuracy of the entirely new testing methods in the Princeton tests, it appears advisable to glance at the meager factual and experimental basis for these conventional ideas, especially in so far as the so-called "arching" of sands behind flexible bulkheads is concerned. Supplementary reference is made to appropriate passages of the writer's final report[66] on this entire test program.

Discussions related to this topic arose in June, 1948, at the Second International Conference on Soil Mechanics and Foundation Engineering at Rotterdam, Holland (The Rotterdam Conference) concerning papers by the writer and his associates.[67,68,69] These discussions[70,71] revealed the interesting fact that at least one of the authors of the Danish rules[26]—A. E. Bretting—appeared to consider that arching in a vertical direction behind a deflected sheet pile bulkhead must be present since it is known to be effective when the pressure area forms a complete circle around a shaft or a tunnel or when forces act in a horizontal direction behind sheeting of a vertical cut rigidly braced at its top at the start of the excavation. The writer pointed out[71] that the stability conditions of these types of arching are completely different, that they are in no way directly comparable, and that they should not be confused with each other. Some of this confusion may perhaps be the result of improper interpretation of statements such as the one made by Karl Terzaghi, Hon. M. ASCE, who wrote[72] that he was impressed with "the infallible and universal efficacy of arching." The Symposium authors repeatedly emphasized that their findings referred to backfills and to the absence of arching in the vertical direction behind flexible anchored bulkheads—that is, to the type illustrated by Figs. 20 and 22.

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[66] "Final Report to the Bureau of Yards and Docks, Department of the Navy, on Large Scale Earth Pressure Tests with Model Flexible Bulkheads," by Gregory P. Tschebotarioff, Princeton Univ., Princeton, N. J., January 31, 1949.

[67] "Lateral Earth Pressures As a Problem of Deformation or of Rupture," by Gregory P. Tschebotarioff and Philip P. Brown, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Holland, June, 1948, Vol. II, pp. 81–86.

[68] "Reduction of Lateral Cohesive Soil Pressure on Quaywalls by Use of Sand Dykes," by Harris Epstein, *ibid.*, Vol. III, pp. 291–296.

[69] "Effect of Boundary Conditions on Lateral Earth Pressures," by Gregory P. Tschebotarioff and John David Welch, *ibid.*, pp. 308–313.

[70] "General Report on Section V—Earth Pressure, Stability and Displacements of Retaining Constructions," by A. E. Bretting, *ibid.*, Vol. VI, pp. 100–102.

[71] Discussion No. V610, by Gregory P. Tschebotarioff, *ibid.*, pp. 103–105.

[72] "Side Pressure on Retaining Walls," by Karl Terzaghi, *Engineering News-Record*, July 12, 1934, p. 55.

Professor Peck states that the Princeton

> "* * * measurements indicated a moderate degree of arching because the center of pressure of the curves shown in Fig. 23 is at approximately 0.38 of the distance from the dredge line to the top of the bulkhead."

The location of the resultant of lateral earth pressures might have been raised from 0.33 to 0.38 of the height not by any type of arching, but by the transfer of lateral pressures to the rigid bottom of the tank by shearing stresses along it. Furthermore, only horizontal arching could have produced the effect shown in Fig. 23 and then only provided that the anchor support had not moved. This was intentionally not the case in the Princeton tests (Figs. 15, 17, and 27).

Another statement by Professor Terzaghi[4] that should be clarified in this connection is:

> "Some of the Danish bulkheads were exposed for centuries to the pounding of waves, and for decades to heavy traffic, without the distribution of the lateral pressures becoming 'hydrostatic.'"

Originally, this statement led the writer to believe that at least some kind of pressure distribution measurements had been performed on Danish bulkheads. It is now definitely established that nothing of the kind had been done.

A 1940 publication sponsored by the Danish Society of Engineers states[73] that the method known as the Danish rules[26] "* * * although rather complicated, is to be considered only as a rule of thumb." (The exact German word used was: "Faustregel.") The same publication[73] states that the occasion for the development of the Danish rules was "* * * provided by the light quaywall type of construction patented by the Danish firm Christiani and Nielsen." The foregoing views[70] on the matter expressed by Professor Bretting, former chief engineer of that firm, therefore take on added interest. The present chief engineer of Christiani and Nielsen, J. Brinch Hansen, published[74] in 1946 a historical review from which it follows that their first light wharves were designed "* * * by Coulomb theory but with allowable stresses of 3–4 times the stresses applying to ordinary reinforced concrete structures." This procedure is stated to have been adopted in the first place because in some old and stable timber wharves the stresses computed by the Coulomb theory were found to be from 3 to 4 times greater than usually permissible stresses.

The inescapable conclusion is that any other guess which explains the stability of structures built in accordance with the Danish rules[26] has equal validity with the assumption of vertical "arching" indicated by Fig. 22 and by the complicated formulas that support these rules. Foremost among such guesses should be the high factors of safety usually present in the structural members of a bulkhead—a point that has been discussed by Mr. Epstein in the fourth paper of the Symposium and to which Mr. Brinch Hansen has since referred in agreement.[75]

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[73] "*Zur Bemessung des doppelten Spundbauwerkes,*" by I. A. Rimstad, Copenhague, 1940, p. 22.

[74] "Development of the C & N Wharf Type," by J. Brinch Hansen, *Christiani & Nielsen Bulletin No. 56*, Copenhague, 1946.

[75] *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Holland, June, 1948, Vol. VI, pp. 107–108.

There is absolutely no contradiction between the Danish experience with full-sized structures and the Princeton experiments. After the Rotterdam conference the writer was enabled, through the courtesy of Mr. Brinch Hansen, to check the over-all design of three typical wharves and piers built by Christiani and Nielsen and found[66] that the stability was fully explained by the results of the Princeton tests.

The only previous tests of any kind from which attempts could be made to evaluate the effects of vertical arching were performed by J. P. R. N. Stroyer.[21] Messrs. Packshaw and Lake refer to these tests as "practical," presumably having in mind the simplicity of Mr. Stroyer's test setup. This simplicity was carried to such lengths that no attempt was made to measure either the distribution of lateral pressures against a 3-ft by 3-ft flexible steel plate or the distribution of its strains and deflections, and even the measurement of pressures exerted by the plate on its supports was entirely omitted. All evaluations of "arching" therefore had a purely qualitative and indirect character. They were made by comparing the forces actually applied to the overhanging ends of the plate (so as to balance the lateral pressures of the fill) to the forces theoretically necessary to balance the "Coulomb" pressures. The plate was permitted to deflect only after the entire backfill was in place. In this connection Mr. Stroyer stated that his tests were intended to simulate the condition of a "sunk" wall. For that reason the writer pointed out in his paper (Part II, Section C(1)) that during the first series of the Princeton tests:

> "Arching in a vertical direction did develop after completed backfilling and an additional induced deflection between the supports. This corresponded to the small-scale tests reported by J. P. R. N. Stroyer.[21]"

Thus, Messrs. Packshaw and Lake are incorrect when they claim that the Princeton findings are at variance with the Stroyer tests. This error is probably due to the fact that the need for a differentiation between the effects of dredging in front of a "sunk" wall and the effects of backfilling behind a wall (first hinted at by Mr. Stroyer[21]) was disregarded in later publications. Thus both the Danish rules[26] and Professor Terzaghi,[22] when referring to arching in a vertical direction (Figs. 22 and 20), refer only to "backfills" and do not mention the sequence of backfilling in relation to any possible dredging—or of dredging itself, or of any other limiting factors. They most certainly do not mention relieving platforms, although Professor Bretting stated[70] at the Rotterdam Conference that the walls designed according to the Danish rules "are generally combined with a relieving platform."

Professor Terzaghi, however, must have realized the likely limiting effect of the rigidity of the anchor supports, since he included the "yield of the anchorage" in the variables to be studied by model tests of flexible bulkheads, the performance of which model tests he recommended in 1941.[4]

The sponsors of the Princeton research project were mainly interested in conditions under which any necessary dredging is done before the backfilling is started, such as are illustrated by Figs. 37 and 38 in Mr. Epstein's paper. Tests simulating such conditions were therefore given priority in the Princeton test program which, however, excluded tests with a relieving platform. A test simulating a "sunk" wall was to be conducted at the end of the program and was actually performed.

The writer devoted half a page of his paper (see Part II, Section C(1)) to emphasizing the fact that he secured diametrically opposite results concerning "arching" depending on whether the sand backfilling was done in a normal manner by dumping it through water or whether the sand was "in place at both end supports of a yielding structural unit before its final deflection." The specific statement was made with reference to the latter case that "indications were that there was arching under such conditions." Professor Peck's description of the effects of additional dredging in front of a bulkhead therefore represents an elaboration of the writer's stated point of view.

The preceding discussion has shown the flimsy experiments and theories on which conventional conceptions of arching behind bulkheads have been based. Only two specific suggestions were made in discussion. Professor Roberts suggests the "null method," which consists in varying the reaction pressure of a support in such a manner that, supposedly, it does not yield unless the support is released by a controlled amount. Very small but repeated reversals of the direction of support movement are unavoidable in practice, however, with alternating active and passive earth pressures being successively brought into play. Thus, an undesirable variable is introduced into the situation and justified objections to the method are offered under any circumstances. The very principle of the method precludes its application to tests of the type of the most important third series (Part III of the writer's paper), where the bulkhead was actually embedded in the ground. Professor Peck makes a similar suggestion to determine the shears at the dredge line by cutting the piling there for that purpose and by measuring the reactions necessary to hold the piling in its existing position. He admits that even in open air cuts such measurements "are not convenient." This is an understatement when applied to underwater measurements, with which discussion is now concerned. In an open cut the support needed "to hold the piling in its existing position" is provided by the opposite wall of the cut; but in the case of a full-scale anchored bulkhead it would not be possible to provide such a support without erecting an underwater obstruction to docking ships, the foundation of which obstruction, in addition, would alter the normal interplay between the active and passive earth pressures around the bulkhead. The suggested cutting of the flexible piling would also alter such interplay since divers could scarcely be expected to determine the point of contraflexure and to cut the piling there. Unless the point of contraflexure is determined, the conditions of support and of flexure of the sheet piling after the cutting would be completely different from those before the cutting. To determine the point of contraflexure under laboratory conditions by the bending-strain measuring techniques described in the paper and to add to them the cutting method proposed by Professor Peck would require the work of several highly trained men from 4 to 5 months after each test—in building and calibrating new model bulkheads and the gages mounted on them to replace the ones which had been ruined by the cutting. The duration and the cost of the testing program would then run into expenses which no sponsor could be expected to meet, especially since very little would be gained thereby.

The proposals by Professors Roberts and Peck therefore represent an effort to adapt old and inadequate techniques to a new problem, the complexity of which requires radically different and improved procedures for its practial solution. Such procedures have been provided by the new techniques of measurement and of analysis developed by the Princeton project, which have fully met the requirements of the problem at a model scale. They can also be adapted to full-scale observations.[66] The writer always was, and will be, glad to coöperate with anyone interested in such observations.

Sheet piling is designed to withstand certain maximum values of bending moment. The knowledge of the exact shape of the pressure distribution curve is therefore of practical importance to the extent that it permits the determination of anchor pulls and of bending moments. In 1948 Professor Terzaghi stated[76] that:

> "It is doubtful whether a definite rule regarding the pressure distribution on flexible bulkheads can be established at all which would be valid under all conditions to be encountered in practice."

Therefore the direct over-all measurement of values for design, especially of the bending moments, is of the greatest advantage since these values can be used (and in the Princeton tests actually were used[66]) for the direct comparison of test results with design values obtained from various existing theories. The corresponding measurements were performed by the writer and his staff with a precision that has thus far been unsurpassed. In no other previous test had it been possible to perform such accurate measurements by independent methods and then to check them against each other. The writer has already reported in his paper that deflections computed from the electrically measured bending strains agreed within approximately 5% with the deflections measured directly by mechanical methods. A tabulation of such comparisons, as well as an example, are given in the final report.[66] He also provided a diagram (Fig. 12) showing the reasonable agreement between the pressure curve computed from the bending-strain readings during a calibration by water pressure and the actual water pressure curve. The term "reasonable agreement" as used by the writer, of course, is intended to be relative; that is, it should be considered in comparison with the known results of other possible procedures. Direct measurements of pressures by pressure cells easily give variations up to 75% at the same spot.[77] The extrapolated over-all pressures (which are the ones that count in design) are therefore even less precise. The method of estimating the pressure distribution from the location of the resultant of two measured reactions, used prior to 1932 by Professor Terzaghi,[6] is still more inaccurate since the location of the center of pressure can have the same elevation for diametrically opposite curvatures of the pressure distribution curves with differences of from 300% to 400% of the lower values between the actual pressures in certain regions. Therefore Professor Peck's criticism that Fig. 12 shows a maximum difference of 20% between the actual and the computed (but unadjusted) values, and that near the anchor level only, is inconsistent with his use of Professor Terzaghi's outdated and infinitely less accurate technique based solely on the location of the center of pressures, to support critical arguments against the present paper.

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[76] *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam Holland, June, 1948, Vol. VI, p. 111.

[77] "Earth Pressure on Tunnels," by W. S. Housel, *Transactions*, ASCE, Vol. 108, 1943, p. 1053, Fig. 46.

In spite of all the data provided, it soon became evident that the Princeton techniques and findings represented too radical a departure from uncertain but time-honored procedures and concepts to be received without bias unless much more extensive documentation were supplied. Not only Professors Roberts and Peck, but also Messrs. Packshaw and Lake shook their heads about them. So did Professor Bretting[70] and Mr. Brinch Hansen[75] at the Rotterdam conference and, although in a somewhat less definite manner, so did Professor Terzaghi.[76] Professor Bretting expressed regret[70] that "no direct readings of the instruments had been given" by the writer, although this had never been done before by writers of similar papers and could not have been done by the writer because of space limitations. Therefore the writer is greatly indebted to Messrs. R. Glossop and H. Q. Golder of London, editors of *Géotechnique*, for enabling him to announce on the spot[71] the pending publication of a paper that would do just that. This paper appeared[65] in 1948. It gives all relevant bending-strain instrument readings and subsequent lateral earth pressure computations of one typical normal backfilling test stage of test No. 54, which is one of the twenty combined active and passive earth pressure tests of the now completed third series (Part III, Section C, of the writer's paper). To do this required thirteen pages of print[65]—that is, about as much as the entire space provided for the present closure. Reference, therefore, must be made to that paper[65] and to the writer's final report[66] for a description of the numerous checks and controls developed. These checks are based on recognized laws of statics; they are much more effective and, at the same time, less crude than the measurement of reactions recommended by Professor Peck.

The writer's final report[66] gives all relevant bending-strain instrument readings and subsequent lateral earth pressure computations of two more test stages of test No. 41, the general layout of which corresponds to Fig. 70(*b*) presented subsequently. The bending-strain, shear, lateral earth pressure, and deflection curves of these two test stages are reproduced by Fig. 68. Stage V_A reflects the increased bending moments and active lateral earth pressures after the intense vibration of the backfill which was induced by repeatedly lowering on 6-in. centers a concrete spud vibrator from the backfill surface to the dredge line. Stage V_p refers to the decrease of residual passive lateral earth pressures and to the general redistribution of pressures that was induced by the subsequent intense vibration of the sand from the dredge line downward in front of the bulkhead after partial consolidation of the soft clay slope behind it (Fig. 70(*c*)).

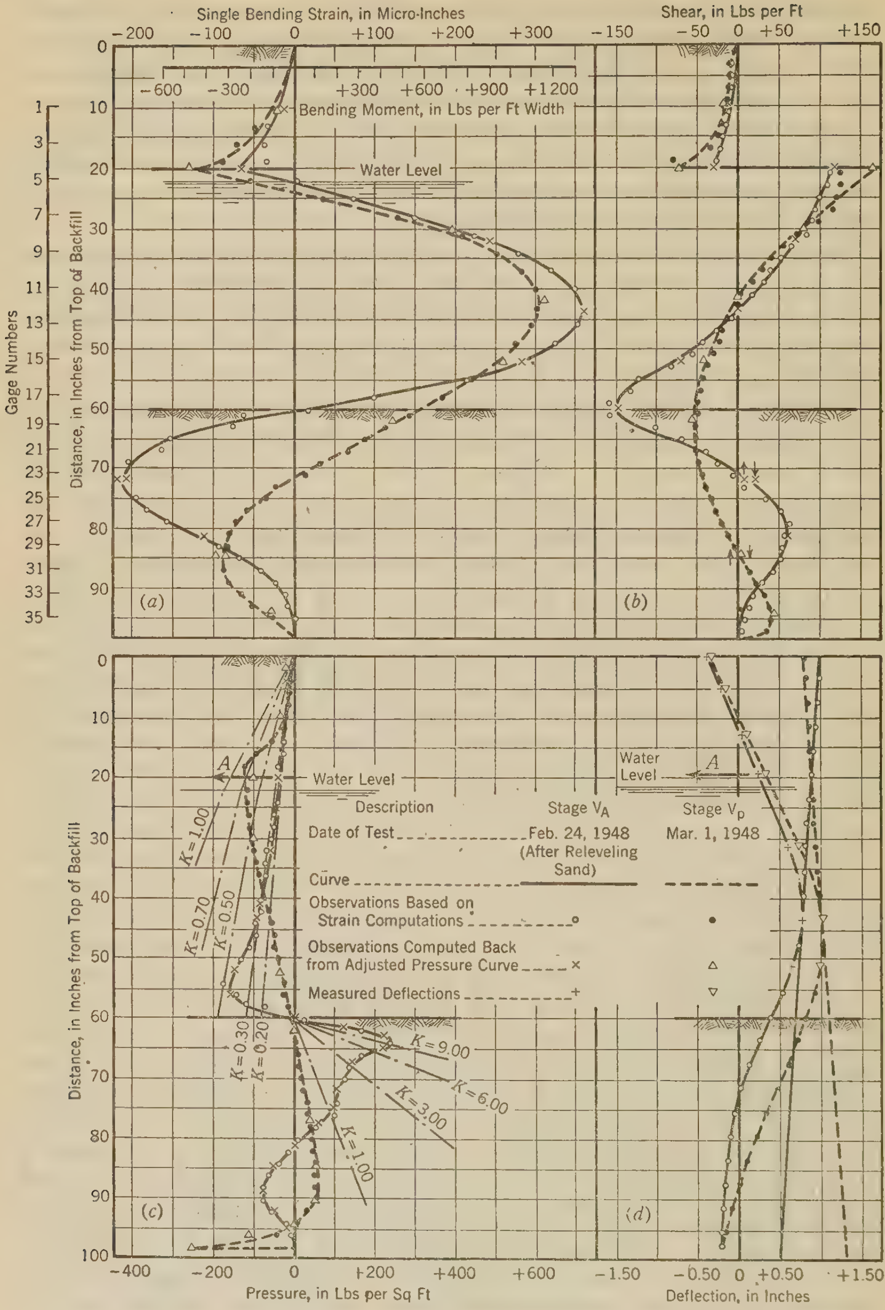


FIG. 68.—TEST 41

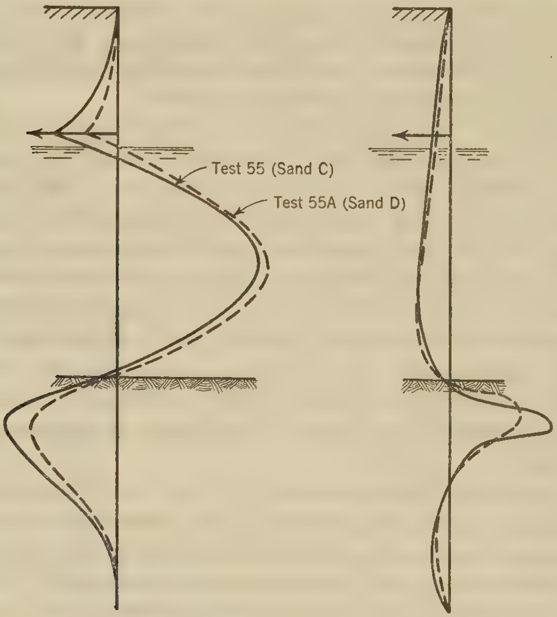
Fig. 68 demonstrates that the procedure followed for the Princeton tests immediately and accurately reflects the changes in the condition of the soil around the bulkhead. The bending moments computed back at eight points from the adjusted pressure curves everywhere agreed closely with the bending moments as actually measured. The agreement was within 7% at all essential points. The writer's final report[66] gives sixteen additional sets of bending moment, shear, lateral pressure, and deflection curves for as many different test stages of the third series when such check backs of the computed pressure curves against the measured bending moments were performed and equally good agreement was obtained. The necessary adjustments were very slight.[65,66] This provides final and conclusive proof of the entirely satisfactory over-all accuracy of the lateral earth pressure computation procedures. Any possible refinements in the determination of actual local deviations from the computed pressure curves are completely inconsequential since they cannot produce any increase in the accuracy of bending-moment determinations which would be of any practical importance for design purposes.

Messrs. Packshaw and Lake raise a pertinent question concerning the possible effect of the grain-size distribution of the sand used on the observed breakdown of arching during backfilling operations. Only an experiment could conclusively settle that point; and an experiment was performed immediately. Test No. 55A was made under conditions identical to those of test No. 55, except that the usual sand was employed in the latter (which is designated as sand C) whereas a different and better graded sand D (the grading of which corresponded very closely to the grading of the sand used by Mr. Stroyer in his experiments) was employed during test No. 55A.[66] Fig. 69 shows that the over-all shape of the bending-moment and lateral earth pressure curves was essentially the same for both tests, with no evidence of arching in either case.

The model similarity factors discussed by Professor Krynine had been fully taken into account during the tests and their subsequent evaluation. The distances between dredge and anchor levels, as well as other dimensions, were changed in different tests until complete model similarity of deflections was obtained.[65,66] The effect of the **Z**-shape of actual sheet piles, as compared to the rectangular shape of the model bulkhead plates (which were $\frac{1}{4}$ in. thick during the third series), was taken into consideration by a corresponding decrease of bending stresses on the model.

Professor Krynine advances the suggestion that the degree of yielding in the wall should be expressed in absolute values "rather than in fractions of the span of the wall as usually done." This idea may be properly questioned since shearing stresses depend on the magnitude of angular deformations and not on linear displacements. The writer knows of no published data that would corroborate Professor Krynine's suggestion, and no observations have been made in the Princeton laboratory that would support it. On the other hand, Mr. De Beer's suggestion that "* * * even sands have some interlocking cohesion c (generally about 0.2 ton per sq m)" has the opposite implication. This view is also contradicted by the results of numerous lateral sand pressure tests at different model scales. Subsequent personal correspondence with Mr. De Beer revealed the fact that his statement was based on indirect reasoning and mainly on the results of triaxial cell tests when a straight line drawn tangent to the Mohr circles did not pass through zero. The writer attributes this observation to the action of the rubber envelope which encloses the soil sample in such tests.

The finding that lateral earth pressures decrease during consolidation of fluid clay backfills, with no outward bulkhead movement being necessary to achieve this reduction, is considered by Professor Peck as "quite normal." Other discussers take a similar attitude. The fact remains, however, that this phenomenon had not previously been so described in modern soil mechanics literature and that the similar, although only qualitative, results obtained by R. O. Comer[58] in 1923 have been generally overlooked. Mr. Feld should be commended for drawing attention to them now.



(a) BENDING MOMENTS (b) LATERAL EARTH PRESSURES

FIG. 69.—COMPARISON OF THE BENDING MOMENT AND OF THE LATERAL EARTH PRESSURE CURVES FOR STAGE IV OF TEST NO. 55 (SAND C) AND OF TEST NO. 55A (SAND D

Messrs. Packshaw and Lake still believe that a movement of more than 1.5% of the height of the wall might reduce the "at rest" pressures of clay backfills to their "active" pressures in accordance with Professor Terzaghi's purely theoretical application of the conventional stress-strain concepts to that problem.[11,12] Professor Peck takes a similar position when he states that after a consolidated equilibrium condition has been reached: "* * * further outward deformations would be necessary to establish a state of failure in any region of the backfill, * * *." The writer, on the other hand, considers on the basis of his tests that the very terms "at rest," or "neutral," or "consolidated equilibrium" imply a limit state of incipient failure where plastic cohesive backfills or remolded natural clays are concerned. This point of view is supported by Mr. De Beer's reference to the results of numerous cell tests performed in Holland and in Belgium. The Department of Public Works of Amsterdam, Holland, indirectly provides further support[78] of the writer's independent findings on this matter[79] by stating: "According to Keverling Buisman[80] however the neutral pressure in cohesive soils approaches the active pressure." The need to recognize the limitations of conventional stress-strain concepts as applied to cohesive soils is therefore definitely indicated.[71]

Mr. De Beer points out an inconsistency in part of Fig. 19. This was caused by a "slip of the pen" which the writer regrets. When extrapolating the curves below the last measured values at the "K_B"-level, their curvatures should be reversed. This correction, however, only serves to emphasize further the great restraining effects of the underlying rigid boundary.

The writer indicated the likelihood of that effect in Fig. 23 by drawing a hypothetical active earth pressure curve of lesser intensity which he suspected might develop behind bulkheads actually embedded in the ground. Mr. Feld believes this should have been omitted. The writer, however, believes that it is desirable to indicate the hypothesis on which further research in progress is based, especially if this warns of possible limitations of previous findings and if its hypothetical nature is clearly indicated—as it was, both in words in the text and by a question mark on the drawing.

The tests of the third series were undertaken not because there were any deficiencies of the previous installation or techniques as such, but because it became evident that an over-all solution of the problem could be obtained only by a new approach which had never been possible before in any detail—that is, by the simultaneous study of active and passive earth pressures and of their interplay. The results confirmed and extended the findings of the first series as well as the suspected effect that had been indicated by a question mark in Fig. 23.

Fig. 70 outlines in sketch form some of the findings of the twenty combined active and passive earth pressure tests of the third series performed in 1948, a few of the main features of which can be summarized as follows:

(1) Vertical arching, because of the flatness of the "arches," is the least stable of the three possible main types. It is similar to horizontal arching in

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[78] "Earth Pressure on Horizontal Circular Conduits" (Dept. of Public Works, Amsterdam), *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Holland, June, 1948, Vol. III, pp. 317–320.

[79] *Ibid.*, Vol. VI, pp. 108–111.

[80] "Grondmechanica," by Keverling Buisman, 2e Druk, Delft, 1943.

that it cannot noticeably develop behind an anchored bulkhead when backfilling is done (Figs. 70(*a*) and 70(*b*)) because no soil is present during most of the backfilling to provide an abutment for the arch at and above the anchor level, and because the elastic elongation of the anchor and the yield of its supports progressively increase as backfilling goes on. The yield induces the breakdown of any arches.

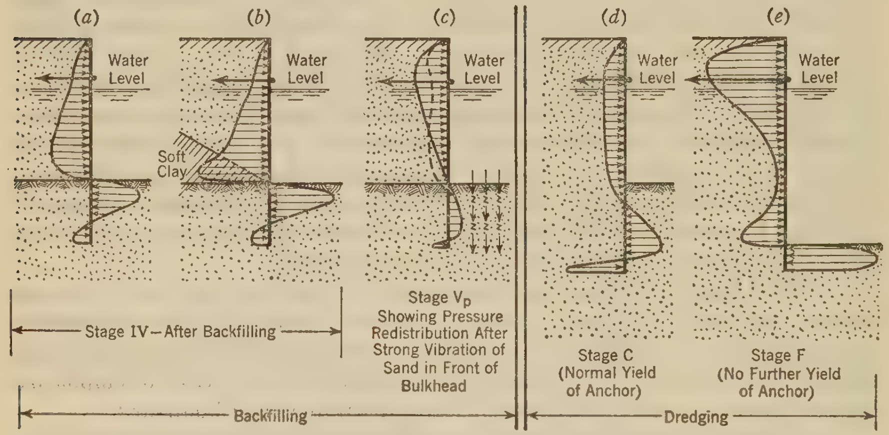
(a)
(b)
(c)
(d)
(e)
Water Level
Water Level
Water Level
Water Level
Water Level
Soft Clay
Stage IV—After Backfilling
Stage V$_p$ Showing Pressure Redistribution After Strong Vibration of Sand in Front of Bulkhead
Stage C (Normal Yield of Anchor)
Stage F (No Further Yield of Anchor)
Backfilling
Dredging

FIG. 70.—FIVE MAIN LATERAL EARTH PRESSURE DISTRIBUTION TYPES DETERMINED DURING THE PRINCETON TESTS WITH MODEL FLEXIBLE BULKHEADS

(2) Definite evidence of vertical arching was obtained at the Princeton laboratory only during the stages of a test that simulated dredging in front of a "sunk" wall, the anchor support of which did not yield any further (Fig. 70(*e*)). Of all the theoretical pressure distribution curves so far advanced which assume arching, only the one proposed by J. Ohde[73,81] closely corresponded to the distribution determined at Princeton under the foregoing conditions. This distribution appears to be caused by a simultaneous action of passive soil resistance above anchor level and of arching both in the horizontal and in the vertical directions. The distribution suggested by the Danish rules (Fig. 22) and by Professor Terzaghi (Fig. 20) does not appear possible under any conceivable conditions.

(3) An absence of lateral displacement at the anchor level is conceivable on actual structures only when the sheet piling is connected to massive relieving platforms. Since the presence of numerous piles under a relieving platform has an unknown but definitely not negligible effect on the redistribution of earth pressures against the sheet piling, it is still an open question whether arching can occur under field conditions even in the case of a "sunk" wall.

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[81] "Zur Theorie des Erddruckes unter besonderer Berueeksichtigung der Erddruckverteilung," by J. Ohde, *Die Bautechnik*, Berlin, Nos. 10/11, 13, 19, 25, 37, 42, 53, and 54, 1938.

(4) The residual passive pressures of sands below the dredge line in front of the bulkhead reached values which in some cases were from three to four times greater than the maximum values that are theoretically possible if the effect of wall friction is neglected. Effective restraint below dredge level was recorded even when the entire embedded length of the bulkhead had moved outward from its original position.

(5) The point of contraflexure of bulkheads embedded in sand coincided (after normal backfilling) very closely with the dredge line. Of all existing bulkhead design methods H. Blum's "equivalent beam" version[82] of the fixed earth support method[22] gave the best agreement with the observations made for backfilling conditions.

(6) Severe vibrations and other causes frequently induced an appreciable downward displacement of the point of contraflexure. However, no correspondingly large increase of bending moments was observed because a considerable simultaneous additional transfer of lateral pressures to the soil beneath the dredge line appeared to take place as a result of the bulkhead deformations which lowered the point of contraflexure. The transfer increased with the flexibility of the bulkhead, on which its displacement at dredge line level directly depended.

In view of these findings, it does not appear advisable to attempt, when making new designs, the actual determination of all possible variations of pressure distribution. Instead the writer proposed,[83] for all designs involving clean sands, a simplified version of Mr. Blum's "equivalent beam" method[82] under full utilization of maximum possible wall friction both for active and passive pressures. Empirical coefficients are suggested to take into account possible variations in the maximum bending moments and in the anchor pulls when uncertainties in the support conditions are known. The proposed design procedure corresponds closely[66] to the bending-moment values directly measured during the third series of tests at Princeton, which are approximately equal to one half of the theoretical bending moments obtained by the free earth support method[22] for the same depth of embedment and which are some 25% higher than the values obtained by the Danish rules.[26,73] Therefore, although the pressure distribution determined at Princeton for normal backfilling is exactly the reverse of the one suggested by the Danish rules, the writer pays tribute to the authors of these rules as good guessers concerning the permissible limit reductions in conventional design values, even though the actual factors of safety appear to be smaller than indicated by these rules.

In addition to the bulkhead design procedure for use with clean sands. the writer's final report[66] (*a*) suggests a design procedure for use when clays are involved; (*b*) brings out a number of secondary but nevertheless essential observations; (*c*) provides extensive documentation of observations made; (*d*) records not yet clarified points; and (*e*) makes specific recommendations for the techniques of full-scale control observations. It is suggested that control measurements be made on all new bulkheads.

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[82] "Einspannungsverhaeltnisse bei Bohlwerken," by H. Blum, Ernst und Sohn, Berlin, 1931.

[83] "Design of Flexible Anchored Sheet Pile Bulkheads," by Gregory P. Tschebotarioff, *Proceedings*, 17th International Navigation Cong., Lisbon, Portugal, 1949.

In closing, the writer wishes to express once more his sincere appreciation to the United States Department of the Navy for the sponsorship of the extensive test program described and to the officers and engineers of the Bureau of Yards and Docks as well as to the staff members of Princeton University who have helped him bring it to a successful completion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2375

STABILITY OF THIN-WALLED COMPRESSION MEMBERS

By H. Dudley Wimer, Jr.,[1] Esq.

With Discussion by Messrs. George Winter, and H. Dudley Wimer, Jr.

Synopsis

Stainless steel has come into the foreground as a material for structures in which lightweight is of primary importance. Its weight economy is inherent in its high strength, permitting the use of thinner sections. Ordinarily, the form in which it is used is that of members fabricated from cold rolled sheet. The thinness of the sections used and the manner of their fabrication tend to make local instability a major factor in their design. This paper describes a method of computing the local stability, or crippling strength, of thin-walled compression members which was developed empirically from test data. Although derived primarily for stainless steel, the method is readily adaptable to other metals. Also, although the tests were conducted on small sections characteristic of aircraft structures, the principles discussed are equally applicable to large-scale members common to civil engineering practice.

Introduction

It is well known that long, slender compression members may fail by buckling, or bending sidewise, at a stress lower than the ultimate fiber compressive stress. This is known as column failure, or primary instability. Hollow, thin-walled members may exhibit a second form of instability known as local instability, or crippling. In this type of failure, the thin walls crumple. The stress at which this occurs depends on the shape and wall thickness of the member.

Although the study of local instability is not new and although there is a large volume of existing dissertation on the subject, the theory is still difficult to apply. Complications are introduced by moduli of elasticity which vary with the stress, lack of clear cut yield-point stresses, and unknown edge restraints and boundary conditions. High-efficiency sheet metal structures,

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Note.—Published in May, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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such as aircraft and all other structures in which guaranteed strength values must be predicted closely, demand many tedious calculations to establish their compressive strength. Even then, the results are so unreliable that they must be substantiated by many physical tests.

This paper describes a method of computing the local stability or crippling strength of thin-walled compression members, which was developed empirically from test data, has been thoroughly proved in practice, and permits great rapidity and ease of calculations. This method was derived for use with 18–8 stainless steel of various hardnesses, having ultimate tensile strengths of from 75 kips per sq in. to 185 kips per sq in. The term "18–8" is a common trade designation[2] for a high-strength chromium nickel alloy. However, it is readily adaptable to other materials having stress-strain curves of the same general shape, and the same approach may be applied to a great variety of structural materials. Test specimens were made with their longitudinal axes parallel to the direction of cold rolling. This is in agreement with usual shop practice and is conservative. The compressive strength is considerably greater across the grain than parallel to the grain.

In comparing stainless steels of different hardnesses, the ultimate tensile strength rather than the yield strength is used as an index. This is done because 18–8 stainless steel has no clearly defined yield point. Tests have indicated that an arbitrarily defined yield strength has little physical significance with respect to local instability. A series of tests, specifically directed toward determining the influence of yield strength on the crippling strength of stainless steel, was run independently of the tests described herein. The specimens (see Fig. 1) were all square tubes with walls 0.050 in. thick. They consisted of three groups, geometrically identical, differing only in the physical properties of the materials from which they were made.

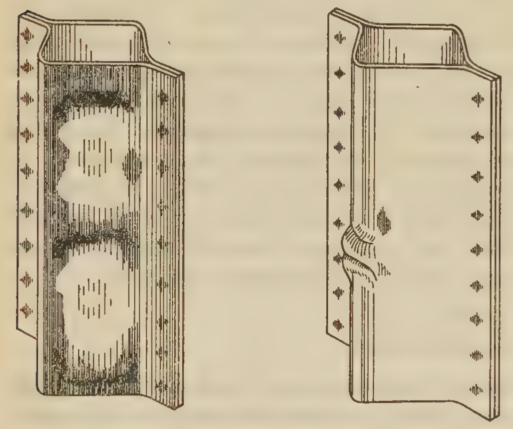

Fig. 1.—Typical Square Tube Specimen

Group I comprised five specimens, $3\frac{1}{16}$ in. long, with sides ranging in width from 0.55 in. to 2.55 in. This group was made from stainless steel having a yield strength of 129 kips per sq in. and an ultimate tensile strength of 156.5 kips per sq in. Group II was dimensionally identical to Group I but made of steel with a yield strength of 104 kips per sq in. and an ultimate tensile strength (UTS) of 154 kips per sq in. Group III was of steel with yield strength s_y of 81.4 kips per sq in. and UTS of 155.9 kips per sq in.

Transactions, ASCE, Vol. 102, 1937, p. 1258, Table 4, item Nos. 5 to 10.

The three groups represented wide differences in yield strength with very nearly the same ultimate tensile strength. Comparable specimens in all groups failed at essentially the same compression load, the greatest single discrepancy being about 10%, based on the average.

Some introductory remarks are necessary to justify the practice, in this paper, of ignoring first buckling as the limit of useful strength. This is a familiar idea to aircraft structural designers, but will not be readily accepted by civil engineers. It is not intended to suggest that structures should be so designed that, under day-to-day service loads, buckling will occur in any of the plates. Also, it is realized that, with structures of the usual proportions found in civil engineering practice, there is probably a rather small margin between the stress at which first buckles appear and that at which permanent damage or failure occurs. However, a knowledge of the ultimate strength becomes very useful under certain special conditions:

(1) When plates of greater width-to-thickness ratios are used (as may be encountered with the use of high-tensile steels), the margin between first buckling and failure becomes greater. This is equally applicable to plates 4 ft wide and 1 in. thick and to sheets 2 in. wide and 0.04 in. thick. Improved data on buckling and behavior after buckling may enable the civil engineer to work closer to such stresses than he normally does at present. There is economic justification for close design in that the huge scale of many civil engineering projects may result in the wastage of tons of material, if ultraconservative methods of design are adopted.

(2) Under certain extraordinary loading conditions, buckling may be permissible as long as strength specifications are met. For example, in railway passenger car design, buckling is not desirable under normal operating loads but is acceptable under loads set up for collision conditions. In civil engineering, analogous conditions may arise from hurricanes, earthquakes, or other unusual sources of loading.

(3) Some structures are designed to fail at a predetermined load in order to safeguard other property. An example is a flashboard on the spillway of a dam, which is designed to collapse before the water reaches a sufficient head to endanger the dam itself. In cases of this nature, the prediction of ultimate strengths within close limits is essential.

NOTATION

The letter symbols used in this paper are defined where they first appear, in the text or by diagram, and are assembled alphabetically in the Appendix.

CRIPPLING OF FLAT AND CURVED ELEMENTS UNDER AXIAL COMPRESSION LOADS

Before presenting the test results, some review of the theory and procedure followed will be helpful. The basic idea has been used for some time by aircraft designers.[3,4] Structural members made from sheet metal, such as those

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[3] "Airplane Structural Analysis and Design," by Ernest E. Sechler and Louis G. Dunn, John Wiley & Sons, Inc., New York, N. Y., 1942.

[4] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1943.

used in aircraft construction, are usually of such a nature that they may be broken down into flat and curved elements, the curved elements being the corners or bends (see Fig. 2).

The curved elements, or bends, may be thought of as segments of a hollow cylinder, whereas the "flats" are analogous to plates with a simple support along two opposite edges. The problem resolves itself, therefore, into two basic problems of elastic stability—(1) the buckling of a rectangular flat plate loaded in compression on two edges and simply restrained along the other two, and (2) the crippling of a hollow cylinder in compression.

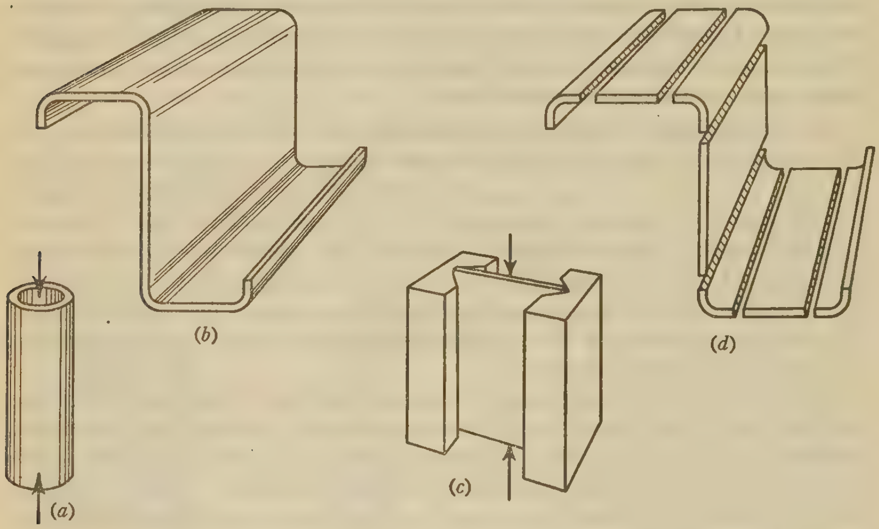

Fig. 2.—Structural Members of Sheet Metal

Consider a short specimen, consisting of several bends with wide flats between, subjected to an increasing compression load. There is a fundamental difference between the behaviors of the curved elements and the "flats." At low loads, the stress is distributed uniformly. As the load increases, the flats tend to buckle into sinsusoidal waves at some critical stress depending on their width, the widest flat buckling first. These buckles are easily visible to the unaided eye (see Fig. 1). As each flat buckles, it behaves like a miniature Euler column, continuing to carry the load at which it buckled, but going out of action as far as any increase of load is concerned. As the load on the specimen continues to increase, the stress distribution becomes nonuniform, remaining constant in the buckled elements and increasing at a more rapid rate in sections that have not buckled. If the load is removed, the specimen will spring back to its original shape, provided that none of the curved elements have buckled and that the yield stress of the material has not been exceeded. If the load is increased until one of the curved elements buckles, it will crumple sharply. The deformation will be permanent rather than elastic and the specimen will collapse with any further increase in load. The curved elements do not exhibit any visible distortion until they fail completely.

Summarizing briefly, at low loads the stress distribution is uniform. At higher loads the flat elements buckle, maintaining their critical loads but shedding any increase so that the corners must carry a disproportionate share. Finally, the ultimate strength is determined by the strength of the curved element with the lowest crippling stress. The unevenness of the stress distribution is illustrated graphically by end impressions left on bearing plates used in the tests, where the corners have dug in deeply—whereas the flats have left only a faint impression.

To predict the load at which a short, thin-walled compression member will fail, it is necessary to determine the load that will be carried by each element, flat or curved. The summation of the elemental loads will be the total for the member. The load carried by each element that buckles before final failure will be its area multiplied by its own buckling or critical stress. The load carried by each element that does not buckle before final failure will be its own area multiplied by the lowest crippling stress of any curved element. Since the lowest crippling stress in a curved element is the index of the highest stress that may be carried by the entire specimen, the first problem is to establish the critical stress of that curved element.

S. Timoshenko has established[5] the fact that the crippling stress is a function of the diameter-to-wall-thickness ratio, being higher for low values of this ratio. However, his test data show wide variance from the theoretical curve because, in cylinders of proportions used in practical design, buckling occurs at so high a stress that it is accompanied by yielding of the material. The problem then becomes one of stability in the plastic range, rather than the elastic range. Instead of attempting to reconcile these complicated phenomena with the theory, it was decided to conduct a test program on hollow, stainless-

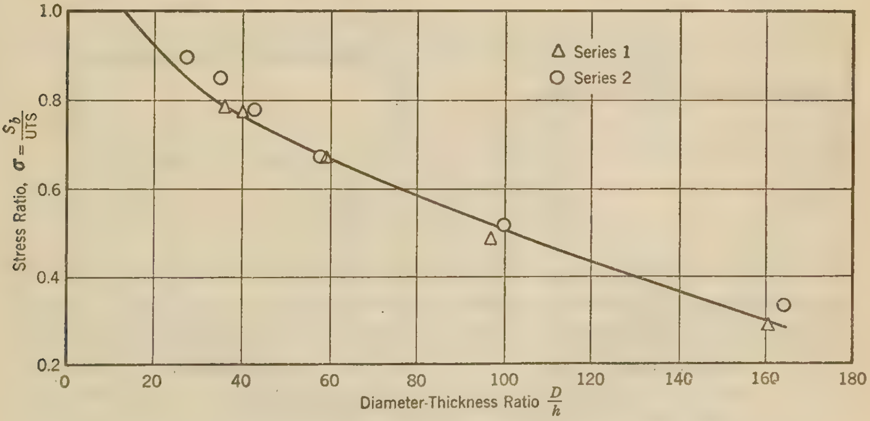

FIG. 3.—CYLINDER CRIPPLING TESTS

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[5] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 936, p. 439.

steel cylinders of various diameters D and thicknesses h, from which the curve of Fig. 3 was determined empirically. The ratios of crippling stress to UTS are taken as ordinates and the ratios of diameter-to-wall thickness are taken as abscissas.

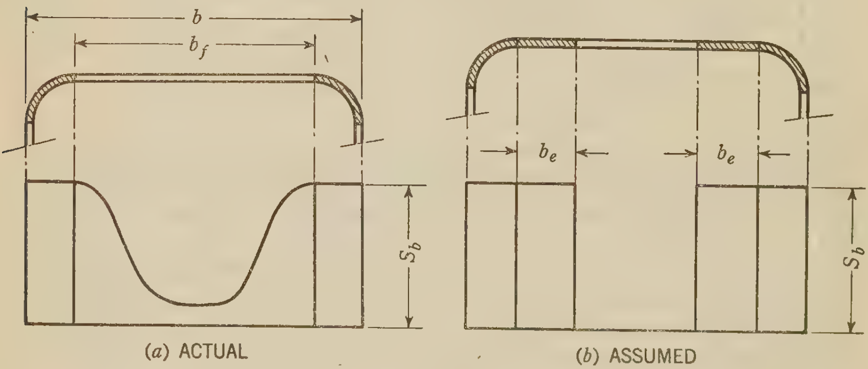
Two series of tests were run. Series 1 consisted of five specimens of constant thickness ($h = 0.017$ in.) but varying from $\frac{5}{8}$ in. to $2\frac{7}{8}$ in. in diameter (D). Series 2 comprised six specimens of 1-in. outside diameter but varying from 0.006 in. to 0.040 in. in thickness. All specimens were $2\frac{1}{4}$ in. long and were made of full hard stainless steel (UTS = 185 kips per sq in.). Welding flanges were trimmed off after welding until they were so narrow that no buckling occurred in them. Each test was conducted in duplicate and the loads were averaged. In no instance did the individual observed values vary from the average by more than 8%. The test results are plotted in Fig. 3.

The purpose of running two series was to detect any influence of the length-diameter ratio. Series 2 (Fig. 3) had a constant ratio of $2\frac{1}{4}$ to 1, whereas in Series 1 there was a range in this ratio from approximately 1 to 1 to 4 to 1. Since all test points fall close to the curve, regardless of the length-diameter ratio, it appears that within the scope of the tests this ratio has small effect.

The critical, or buckling, stress s_b for flat plates, simply supported along two edges, has been derived by Professor Timoshenko,[6] as follows:

$$s_b = \frac{3.62\,E}{(b_f/h)^2} \qquad (1)$$

in which E is the modulus of elasticity; b_f is the width of flat plate; and h is the thickness of plate. Poisson's ratio μ is assumed equal to 0.3.



(a) ACTUAL (b) ASSUMED

FIG. 4.—COMPARISON OF STRESS DISTRIBUTION

Use of Eq. 1 to find the load carried by a plate after buckling is difficult, since the stress distribution in a buckled plate is not uniform. Rather, it varies from the critical stress at the center of the flat to a stress equal to that of the adjacent material at the edges, as illustrated in Fig. 4. This gives rise

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[6] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., p. 329.

to the "effective width" hypothesis,[7] in which a narrow strip along each edge of the plate, working at the same stress as the adjacent round, is assumed to carry all the load in the plate, the stress in the middle segment being zero. The assumption that the central part carries relatively little load is substantiated by many tests,[8,9] which demonstrate that, when the width of a flat plate exceeds certain narrow limits, there is little or no increase in the load it will carry.

Again resorting to empirical means for the development of this method, a series of tests on flat-sided specimens, similar to those shown in Fig. 1, was conducted to determine the effective widths of flat elements. Three series were tested: One group with a constant width of the flat sides equal to $b = \frac{3}{4}$ in. but with thicknesses h varying from 0.006 in. to 0.040 in.; another group of constant wall thickness ($h = 0.017$ in.) but with widths b varying from $\frac{7}{16}$ in. to 2 in.; and a third group whose purpose was to evaluate the outstanding welding flanges. These flanges are necessary for the fabrication of the specimens, but their presence affects the validity of the tests. Hence, the part of the load they carry was determined by testing several specimens with different numbers of flanges, but otherwise identical. In the interpretation of test data, only the net areas of the specimens without flanges and the net loads after subtracting the flange loads were considered.

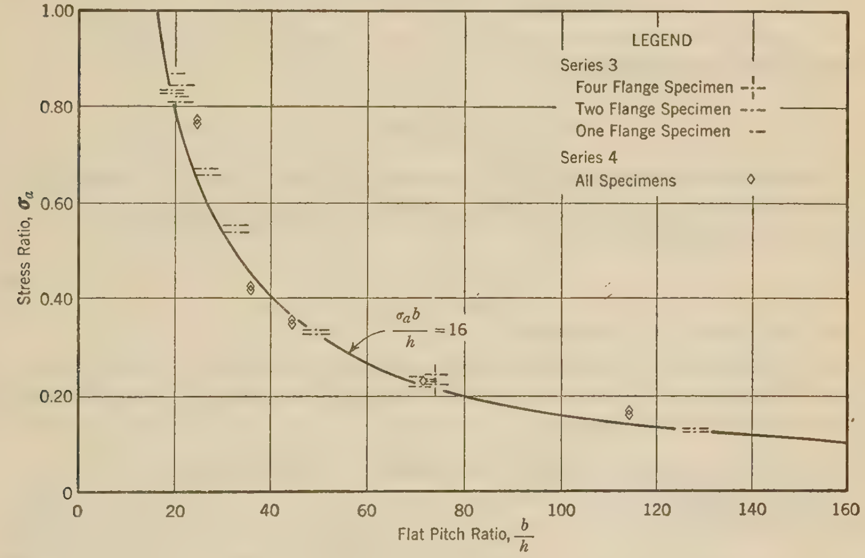

FIG. 5.—TESTS OF FLAT-SIDED SPECIMENS

The test results are plotted in Fig. 5. The ratios σ_a of crippling stress to UTS are taken as ordinates and the flat pitch ratios are taken as abscissas.

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[7] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1943, p. 291.

[8] "Strength of Rectangular Flat Sheet In Edge Compression," by L. Schuman and G. Back, *Technical Report No. 356*, National Advisory Committee for Aeronautics, Washington, D. C., 1930.

[9] "The Strength of Thin Plates in Compression," *Transactions*, A.S.M.E., Vol. 54, 1932, p. 53.

It may be seen that the test points are very well expressed by a hyperbola whose equation is:

$$\frac{s_a}{(\text{UTS})} = \frac{16}{b/h} = \sigma_a \ldots\ldots (2)$$

in which s_a is the average crippling stress, $\frac{P_t}{A}$; P_t is the crippling load; A is the section area; and b is the gross width of flat, including bend radii.

To be of any use Eq. 2 must be expressed in terms of b_e, the effective width of flat adjacent to each bend, and s_c, the lowest crippling stress of any curved element.

Multiplying Eq. 2 by $b\,h$,

$$\frac{s_a\,b\,h}{(\text{UTS})} = \sigma_a\,b\,h = \frac{16\,b\,h}{b/h} = 16\,h^2 \ldots\ldots (3)$$

Since, with very small error, $s_a\,b\,h = \frac{P_t}{4}$:

$$P_t = 64\,h^2\,(\text{UTS}) \ldots\ldots (4)$$

The inside radius of the bends in these specimens was $3\,h$. From Fig. 3 a cylinder with a ratio D/h less than 13 has a crippling stress equal to UTS. The area of four bends is:

$$4\,A_b = 7\,\pi\,h^2 = 22\,h^2 \ldots\ldots (5a)$$

The load carried by the bends is:

$$4\,P_b = 22\,h^2\,(\text{UTS}) \ldots\ldots (5b)$$

The net load on four flats is:

$$4\,P_f = (64\,h^2 - 22\,h^2)\,(\text{UTS}) = 42\,h^2\,(\text{UTS}) \ldots\ldots (5c)$$

Conservatively,

$$P_f = 10\,h^2\,(\text{UTS}) \ldots\ldots (6a)$$

but, by definition of effective width,

$$P_f = s_c\,(2\,b_e\,h) \ldots\ldots (6b)$$

Therefore,

$$\frac{s_c}{(\text{UTS})} = \sigma_c = \frac{10}{2\,b_e/h} \ldots\ldots (7)$$

Eq. 7 is plotted in Fig. 6 from which, once the critical curved-element crippling stress is known, the corresponding effective widths of flat elements may be determined.

A flat narrower than $2\,b_e$ will carry a load equal to its own area multiplied by the stress of the bend having the lowest crippling stress:

$$P = b_f\,h\,s_c \ldots\ldots (8a)$$

A plate 2 b_e in width, or wider, will carry a load—

$$P = 2\, b_e\, s_c\, h \qquad (8b)$$

—regardless of its width.

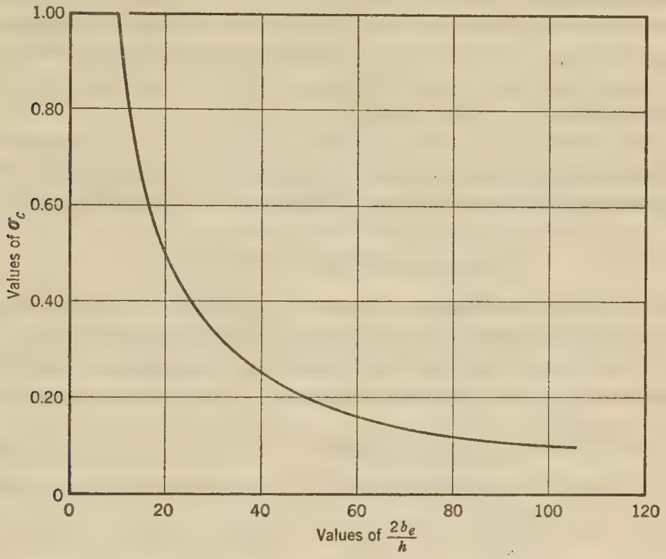

FIG. 6.—FLAT SHEET BUCKLING (EQ. 7)

The general problem of predicting the failing load for the case of local instability of stainless-steel compression members resolves into the following steps:

1. From the curve in Fig. 3 determine the lowest critical stress of any curved element. This will be limited by the curved element with the largest radius.
2. Knowing this stress, find 2 b_e from the curve in Fig. 6. Check all the flats to determine those wider than 2 b_e.
3. Add together the areas of all curved elements plus all flats less than 2 b_e in width, plus 2 b_e for each flat 2 b_e or greater in width. The total is the effective area of the cross section.
4. Multiply the total effective area by the stress found in step 1. The product is the crippling load of the member.
5. If desired, the average crippling stress may be found by dividing the crippling load by the gross area:

$$s_a = \frac{P_t}{A} \qquad (9)$$

A combination of three very fortunate circumstances makes possible the further simplification of this method for the great majority of structural members encountered in practice. The first is that, in usual shop practice, a standard bend radius is set up for each metal thickness. The second is that all these standard radii are small enough so that on the curve in Fig. 3 they fall in the range of D/h in which $\sigma_e = 1.00$. The third is that practically all flats used in practice are wider than $2\,b_e$, which is taken as $10\,h$ for $s_e = \text{(UTS)}$.

Then, for one thickness, all 90° bends will have the same radius, crippling stress, and area; hence, they will have the same crippling load. Similarly, all flats will have the same effective area, which is $10\,h^2$ times the same crippling stress, and, therefore, all flats have the same crippling load. It is possible then to tabulate, for each metal thickness used, the crippling load for a flat and the crippling load for a 90° bend.

Using such a table, all a designer needs to do is add up the number of flats, total the number of 90° bends, multiply each sum by the appropriate load from the table, and add the two products. The result is the crippling load for that member.

EXAMPLE OF USE OF METHOD

Consider an 18–8 stainless-steel square tube, similar to that in Fig. 1, with $h = 0.010$ in. and the sides $b = \frac{3}{4}$ in. The UTS is 185 kips per sq in. Assume the length L to be small, so that column failure is precluded. It is desired to predict the load at which crippling will occur. The inside radius of all bends is 3/64 in.; and $D/h = \dfrac{2}{0.010}\left(\dfrac{3}{64} + 0.010\right) = 11.4$. From Fig. 3, when $D/h = 11.4$, $\sigma_e = 1.00$ so that, in this example, $s_e = 185$ kips per sq in. From Fig. 6, when $\sigma_e = 1.00$, $\dfrac{2\,b_e}{h} = 10$. Then $2\,b_e = 10\,h = 0.10$ in. All the sides are considerably wider than this.

Outstanding flanges are arbitrarily evaluated at half the value they would have if supported on two edges. This is known to be incorrect scientifically, but their proportionate area is small and very little error is introduced.

The total effective area for this specimen comprises two 90° bends, four 45° bends, and four flats and four flanges. By tabulation:

Description	Computation	A
Say, four 90° bends....................	$0.104 \times \pi \times 0.010$	0.00327
Four flat sides at $2\,b_e\,h$ each.............	4×0.001	0.00400
Four flanges at $b_e\,h$ each.................	4×0.0005	0.00200
Total area, in square inches..		0.00927

and $P_t = 0.00927 \times 185{,}000$ lb per sq in. $= 1{,}715$ lb.

The average crippling load of several test specimens answering this description is 1,850 lb. The test specimens were actually 0.0104 in. thick and had an ultimate tensile stress of 195 kips per sq in. Roughly correcting: $P_{\text{test}} = 1{,}850 \times \dfrac{185{,}000}{195{,}000} \times \dfrac{0.010}{0.0104} = 1{,}690$ lb, against 1,715 lb computed.

Care must be taken in applying this method to members subject to other forms of instability. For example, long slender members may fail as columns. Open sections—that is, members whose sides do not form a closed tube—are subject to torsional instability unless well restrained to prevent twisting.

Another manifestation of instability to be guarded against is that illustrated by a square or round tube whose walls are formed of corrugated sheet, the corrugations running longitudinally. In such a member there would be no flat surfaces, and at first glance the over-all strength might appear to be the summation of strength of all the curved sections. The corrugated sheet, however, is subject to a general instability of its own, which may be thought of as a column failure of the corrugations.

EXTENSION OF THE CRIPPLING METHOD TO COMPRESSION OF MEMBERS THAT MAY FAIL AS COLUMNS

The use of the foregoing simplified method of computing the failing load under conditions of local instability is limited to very short compression members. It is obvious that, in extremely long compression struts, failure resulting from primary column instability will occur at a stress so low that there will be no local buckling. In quite a wide range between these two extremes of length, failure may occur as a combination of local and primary instability, each contributing to the other. Local buckling means reduced effective area, reduced moment of inertia, and as a result—reduced radius of gyration. On the other hand, column curvature results in unequal stress distribution, enhancing the buckling on the side where the compression is increased.

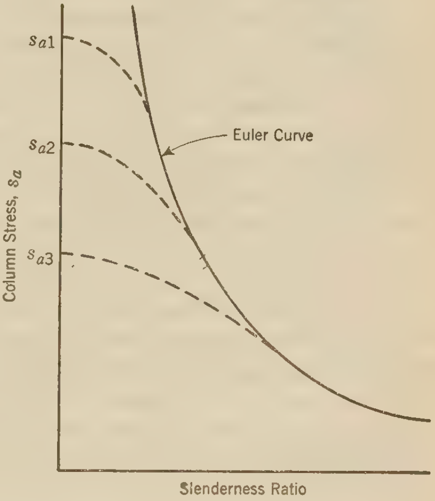

FIG. 7

Because a rigorous treatment of these complex interactions is tedious and complicated, semi-empirical methods will again be adopted. Referring to Fig. 7 in which critical column stress is plotted against slenderness ratio for three columns of different cross sections, having different average crippling stresses, s_{a1}, s_{a2}, and s_{a3}, it is generally conceded that for very long columns all three specimens will conform to the Euler equation.

$$\frac{P}{A} = \frac{\pi^2 E}{(L'/r)^2} \qquad \text{(10a)}$$

It is also evident that the limiting stress for each at very small values of (L'/r) will be its own crippling stress, s_a.

Arbitrarily treating the average crippling stress as the column yield stress and plotting for each case the Johnson parabola—

$$\frac{P}{A} = s^2_y - \frac{s_y (L'/r)^2}{4 \pi^2 E} \qquad \text{(10b)}$$

in which s_y is the column yield stress, assumed equal to s_a—the dotted curves in Fig. 7 are obtained. The Johnson formula is no more justified scientifically than are other short column formulas; that is, it agrees with tests made over a period of years.

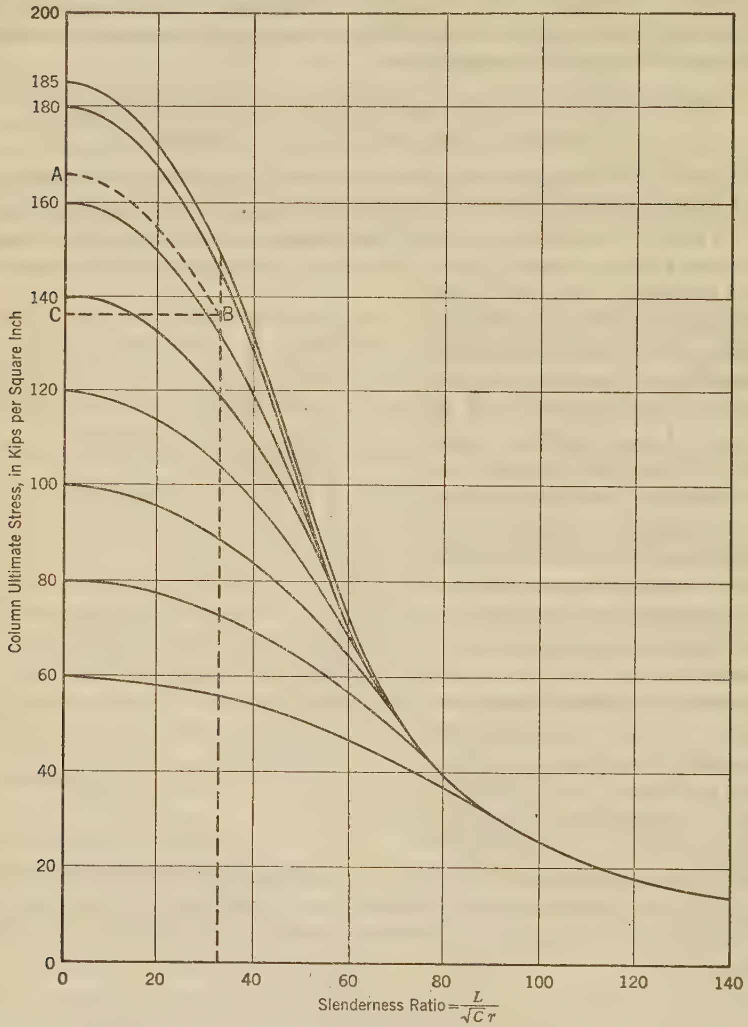

FIG. 8.—JOHNSON-EULER CURVES FOR 18–8 STEEL ($c = 1$; $E = 26{,}000{,}000$ LB PER SQ IN.)

Finally, the chart in Fig. 8 was made by plotting a family of Johnson-Euler curves starting at regular intervals of column yield stress.

The procedure in using the chart is as follows:

(*a*) Determine crippling load by the method previously described.

(*b*) Divide crippling load by gross cross-section area, to obtain average crippling stress (Eq. 9).

(*c*) Beginning with this crippling stress on vertical scale (point A, Fig. 8), follow the curve down to intersection with appropriate slenderness ratio (point B, Fig. 8).

(*d*) Read horizontally across to find ultimate column stress (point C, Fig. 8).

In a large number of tests, in addition, and subsequent to the tests by which the method was derived, predictions have been confirmed within the limits of the variations in thickness, hardness, workmanship, and all other factors involved.

This method has been a tremendous labor saver in the design and stress analysis departments and has been substantiated by experience with a number of structures which have stood up satisfactorily in static tests and in service.

Conclusions

The stress at which thin plates in a structure buckle is not necessarily the limit of useful strength. Compression members having wide, thin plates—particularly those built up from sheet metal—may have an ultimate strength considerably greater than the load at which first buckles appear.

The concept of an "effective area" is used to compensate for the reduced efficiency of the elements that have buckled. Factual data, derived from a test program, are presented for 18–8 stainless steel in the form of curves, from which the effective area and crippling stress may be obtained. The curves are strictly applicable only to stainless steel, but the method of their derivation may be applied to other metals.

Experience in the use of this method leads to the realization that there is much more work yet to be done. To predict the behavior of plates restrained on only one edge, "pile-ups" of several metal thicknesses spot-welded together, and numerous other complex arrangements encountered in practical design, still requires considerable judgment. Care must be taken not to overlook other forms of instability such as general column stability of the entire member, or the stability of loosely connected parts of a member, and torsional instability of open sections.

Within its limitations, the method described has proved to be a rapid and reliable method of predicting the crippling strength of stainless-steel compression members.

Acknowledgment

The writer is indebted to many among the technical personnel of the railway and aircraft divisions of the Budd Company, Philadelphia, Pa., who contributed to the development of this material.

APPENDIX. NOTATION

The following letter symbols, defined where they first appear in the text, or by illustration, conform essentially with the proposed American Standard

Letter Symbols for Structural Analysis prepared by a Committee of the American Standards Association, with Society participation:

A = cross-section area; A_b = cross-section area of a bend;
b = breadth or width of a plate; gross width of flat, including the bend radii:
  b_e = effective width of the flat part adjacent to each bend;
  b_f = the flat part of the breadth;
c = coefficient of fixity;
D = outside diameter of a hollow cylinder;
E = modulus of elasticity;
h = wall thickness of stainless-steel sheet metal;
L = length of specimen; L' = reduced column length, $\frac{L}{\sqrt{c}}$;
P = concentrated load; total load:
  P_b = part of the load P carried by a bend;
  P_f = part of the load P carried by a flat;
  P_t = crippling load; entire member;
r = least radius of gyration;
s = unit stress (ultimate tensile unit stress, UTS):
  s_a = average crippling stress;
  s_b = buckling stress, flat element or plate;
  s_c = critical stress, curved element or cylinder;
  s_y = yield stress;
μ = Poisson's ratio; and
σ = stress ratio:
  $\sigma_a = s_a/\text{UTS}$;
  $\sigma_c = s_c/\text{UTS}$.

DISCUSSION

George Winter,[10] M. ASCE.—The use of an effective design width for thin-walled compression members, as discussed in this paper, has gained increasing acceptance in standard structural design specifications in recent years. The February, 1946, specifications[11] of the American Institute of Steel Construction prescribe the use of an effective width for compression plates whose width exceeds 40 t. The April, 1946, specification[12] of the American Iron and Steel Institute (A.I.S.I.) presents a similar approach in a much more detailed and specific manner, in view of the decisive role of local buckling in the performance of light gage steel members. Both of these specifications apply essentially to mild structural steels, whereas Mr. Wimer's informative tests refer to a hard, high alloy steel.

The effective width stipulations of the second of the previously mentioned specifications[11] are based on the writer's experimental work in this field.[13] It is of practical interest to investigate the degree of agreement or discrepancy between Mr. Wimer's results for hard alloy steels and the writer's results for mild carbon steels.

The writer's findings, in a later, slightly modified form,[14,15] were expressed as follows: Thin, flat compression elements are fully effective ($B_e = b$) if their width to thickness ratio does not exceed

$$\frac{b}{h} = 0.95\sqrt{\frac{E}{s}} \qquad (11)$$

For elements with b/h-ratios exceeding that given by Eq. 11, the effective width B_e can be computed with good accuracy from

$$B_e = 1.9\,h\sqrt{\frac{E}{s}}\left(1 - 0.475\,\frac{h}{b}\sqrt{\frac{E}{s}}\right) \qquad (12)$$

in which s, as in the author's notation, is the edge stress; E is the modulus of elasticity; and B_e is the total effective width (that is, $B_e = 2\,b_e$ in the author's notation).

To apply Eqs. 11 and 12 to 18–8 stainless steel it is necessary to refer to the stress-strain curve of that material. The writer was able to obtain this information only for the very similar 18–7 stainless steel. For 0.02 in. thick, 18.3% cold reduced sheet steel, the properties[16] are practically identical with those given by the author for the 18–8 stainless steel (tensile strength = 186.3 kips per sq in., and E = 26,600,000 lb per sq in.). The slope of the stress-strain curve at high stresses differs materially from this value, resulting in a tangent modulus of about 11,000,000 lb per sq in. at a stress of 110 kips per sq in., and one of about 8,000,000 lb per sq in. at a stress of 120 kips per sq in. Definite data for still higher stresses are not available to the writer; but, from the general shape of the curves of similar steels, it can be assumed that a value of about 7,000,000 lb per sq in. should represent a reasonable approximation up to stresses not too close to the ultimate.

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[10] Prof. and Head, Dept. of Structural Eng., Cornell Univ., Ithaca, N. Y.

[11] "Specifications for the Design, Fabrication, and Erection of Structural Steel for Buildings," A.I.S.C., New York, N. Y., February, 1946.

[12] "Specification for the Design of Light Gage Steel Structural Members," A.I.S.I., New York, N. Y., April, 1946.

[13] "Strength of Thin Steel Compression Flanges," by George Winter, *Transactions*, ASCE, Vol. 112, 1947, p. 527.

[14] "Strength of Thin Steel Compression Flanges," by George Winter, *Reprint No. 32*, Cornell Univ. Eng. Experiment Station, Ithaca, N. Y., October, 1947, p. 53, Appendix.

[15] "Performance of Thin Steel Compression Flanges," by George Winter, *Preliminary Publication*, 3d Cong., International Assn. for Bridge and Structural Eng., Liége, Belgium, 1948.

[16] "Tensile and Compressive Properties of Some Stainless-Steel Sheets," by C. S. Aitchison, Walter Ramberg, L. B. Tuckerman, and Herbert L. Whittemore, *Journal of Research*, National Bureau of Standards, Vol. 28, 1942, p. 499.

On this basis a comparison can be made between Mr. Wimer's results and those of the writer. The author's tests apparently cover a width to thickness range from about 19 to 125. By substitution of these values, and of the appropriate magnitude of E for various stresses, Eq. 12 determines magnitudes of B_e which can be compared directly with Mr. Wimer's results. This comparison is shown in Fig. 9, in which separate curves are drawn for b/h = 19 and b/h = 125, computed, as indicated, from Eq. 12. The curve for b/h = 19 is discontinued at a stress of 66.5 kips per sq in., because, according to Eq. 11, elements with b/h = 19 are fully effective ($B_e = b$) at stresses below that value.

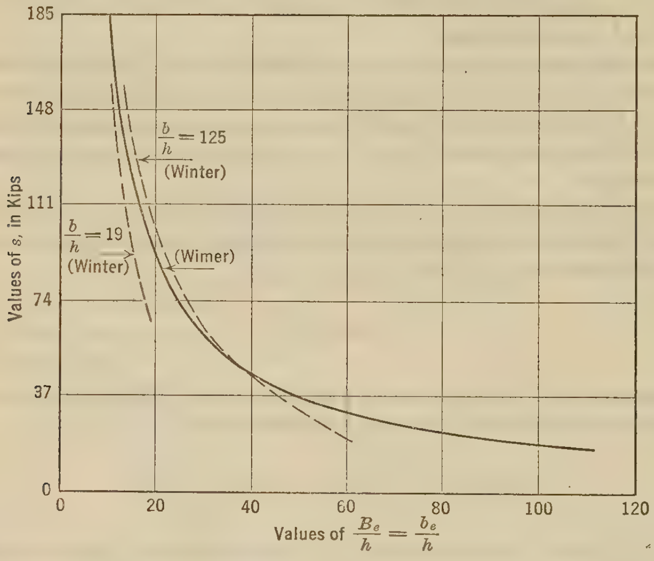

FIG. 9

The findings depicted in Fig. 9 can be stated as follows: In contrast to Mr. Wimer's formula, Eq. 12 indicates that the effective width depends not only on the stress, but also on the flat width ratio b/h, although this influence is small at the high stresses at which most of the author's tests were made. Consequently, the results of these tests are compatible with Eq. 12 if they fall within the narrow range between the two curves for b/h = 19 and b/h = 125—which is indeed the case for high and medium stresses, indicating very good agreement between the two investigations. In the range of the very lowest stresses investigated by the author, his B_e-values seem to be somewhat larger than the writer's. It should be noted, though, that very few of Mr. Wimer's great number of tests refer to this low range, whereas all the writer's results were obtained at low and medium stresses. The writer is inclined, therefore, to regard Eq. 12 as more generally valid than Eq. 7, because: (*a*) Both equations give very similar results in the range of most of Mr. Wimer's tests; (*b*) Eq. 12 is extensively supported by tests[13,14,15] in that range of low stresses which is scarcely covered by the author's experimental data; and (*c*) certain theoretical considerations, also discussed previously,[13,14,15] seem to favor an expression of the type of Eq. 12.

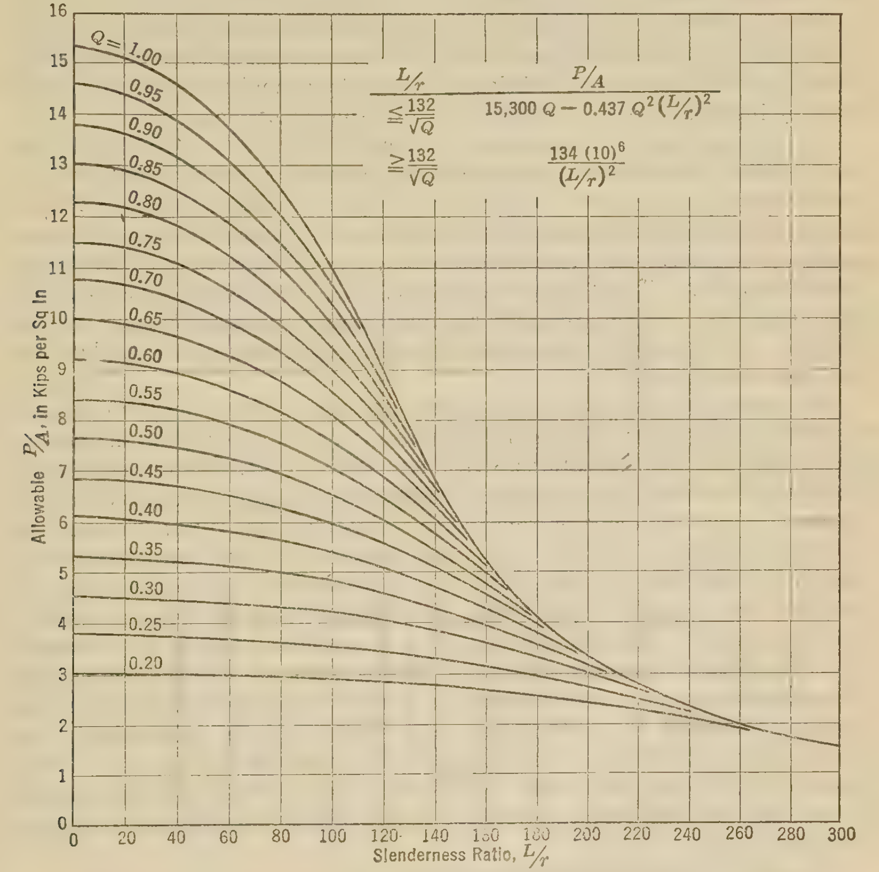

FIG. 10.—COLUMN DESIGN CURVES FOR GRADE C STEEL

It is gratifying indeed to find such excellent agreement between the two investigations over most of the range despite the fact that Mr. Wimer's tests and the writer's tests were carried out by entirely different methods and on fundamentally different materials.

The author's manner of plotting Figs. 5 and 6 could easily be understood to imply that the magnitude of the effective width depends on the ultimate tensile strength (UTS), since these graphs happen to be plotted in terms of $\sigma = s/\text{UTS}$. Actually, the effective width is a function of s/E, rather than of s/UTS, as is evident from Eq. 12 and from similar expressions available in the literature on this subject. The edge stress, to be sure, has the compression strength of the material as its upper limit, but at any stress below that limit the effective width is independent of the strength of the metal. (Unfortunately, no definition of compression strength for materials of the type of hard steels like 18–8 has yet been developed. The properties of these steels in compression and in tension are sufficiently different[16] to make the use of a tensile property in the analysis of compression tests somewhat meaningless.)

Fig. 6 could easily be interpreted as being applicable to steels other than 18–8, simply by substituting the magnitudes of their respective tensile strengths in the determination of the value of σ with which to enter the graph. To be sure, the author has not suggested that such use be made of Fig. 6. To guard against such inadvertent misunderstanding, Fig. 9 is plotted in terms of s rather than of σ. These graphs, therefore, apply strictly to 18–8 steel only; Eqs. 11 and 12, however, are quite general if proper values of E and s are substituted. A convenient design graph derived from these equations has been published elsewhere.[14,15]

The method of modifying column formulas to account for local buckling, as described in the paper, by this time has also found its application in standard structural design specifications. In fact, the A.I.S.I. 1946 specification[11] stipulates allowable column stresses exactly in the manner indicated by the author. The column curves from this specification (Fig. 10) are precisely the same kind as those of Fig. 8. The "form factor" Q for this chart is defined in the A.I.S.I. specification as the ratio of the effective area to the gross area, the former to be computed either for the yield stress, or for the buckling stress of the most unstable unstiffened element of the section, whichever is smaller. Thus, the physical meaning and the use of this factor are wholly identical with the author's method of substituting in a Johnson column formula the average crippling stress for the yield stress. This method, in its application to the specification, was developed jointly by J. H. Cissel, M. ASCE, F. E. Fahy, Assoc. M. ASCE, and the writer. It should be noted that Fig. 10 gives allowable stresses for mild structural steel, whereas Fig. 8 gives failure stresses for 18–8 hard steel.

Summarizing, the writer believes that:

(1) The methods discussed in this paper are not new to structural designers in the field of civil engineering. In fact, they are incorporated to various degrees in several current standard design specifications; and

(2) The paper provides valuable empirical data in the field of high-strength alloy steels. These data, obtained on 18–8 stainless steel and formulated in equations valid for such steel only, are in substantial agreement with the writer's earlier tests on mild steel members, published some years ago in a form which is applicable to almost any metallic material.

H. Dudley Wimer, Jr.,[17] Esq.—The discussion by Professor Winter was most welcome and informative. Several points were emphasized which had not been completely covered in the paper.

As was remarked by Professor Winter, it is very gratifying to note the agreement between his investigation and that described in the paper. Eq. 16 is admittedly a refinement and of more general application to different materials.

However, it should be noted that Professor Winter's tests were performed an open, channel-type sections where torsional instability was at least a contributing factor to failure. Undoubtedly, had closed sections been included, their failing loads would have been higher, and the indicated effective widths greater. Consideration of this factor would tend to bring the two investigations into even closer agreement.

In the opinion of the writer the most unique features of the method presented in the paper are:

(1) The separation of sheet metal compression members into flat and curved elements, and recognition of the fact that their behaviors are fundamentally different; and

(2) The practical simplicity of its use, including the tabulating of values for flats and bends which may then be combined to suit a large variety of sections.

The work has been sponsored for publication as a project of the Committee of the Structural Division, ASCE, on Design in Lightweight Structural Alloys.

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[17] Chf. Analyst, Eng. Development Div., Rheem Mfg. Co., Lester, Pa.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 2376

THE MARINE OPERATING PROBLEMS, PANAMA CANAL, AND THE SOLUTION

By Miles P. DuVal,[1] Esq.

Synopsis

Reconstruction of the Panama Canal to facilitate its operation and to increase its capacity is imperative. The present design is a high level waterway, 85 ft above sea level. An improvement near the Pacific end of the Canal to provide a high level terminal lake analogous to Gatun Lake on the Atlantic side is discussed in this paper.

The need for the change is explained, based on marine operating and safety considerations. The solution proposed is the physical removal of Pedro Miguel Locks, the construction of all Pacific locks in continuous lifts near Miraflores, the elevation of the Miraflores dams, and the creation of a high level anchorage north of Miraflores. Navigational rather than engineering features are stressed.

This paper was originally presented before a meeting of the Panama Section of the Society on May 20, 1943. The opinions and recommendations are purely personal, and have no official connotation.

Present Canal Operating Plan

Any mariner's first transit of the Panama Canal is always a memorable experience. Although he may know only a little of the history of this great waterway he is always deeply impressed by the magnitude of the Canal and the quiet efficiency of its operation.

His ship passes through the Atlantic (Caribbean) sea level section (Fig. 1) and enters the three-lift Gatun Locks. The ship is locked up to the summit level in one continuous operation. After traversing Gatun Lake, it enters the tortuous and rocky artificial gorge known as Gaillard (formerly Culebra) Cut. At the south end of this cut the vessel enters the single-lift Pedro Miguel Locks and is locked down to the intermediate-level Miraflores Lake. It then enters a

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Note.—Published in February, 1947, *Proceedings*. Position and title given are those in effect when the paper or discussion was received for publication.

[1] Capt., U. S. Navy Dept., Washington, D. C.; formerly Capt. of Port, Balboa, Canal Zone, March, 1941, to June, 1944.

second set of locks at Miraflores and is locked down in two steps to the Pacific sea level.

Many shipmasters observe the Canal and the lock arrangements closely. Instinctively they note that only one lock structure was placed on the Atlantic side, and they wonder why two sets of locks are needed on the Pacific side in what they feel is an improper marine arrangement. They would prefer one structure on the Pacific as at Gatun to avoid (1) slowing their vessels in a rocky gorge and (2) undertaking double handling.

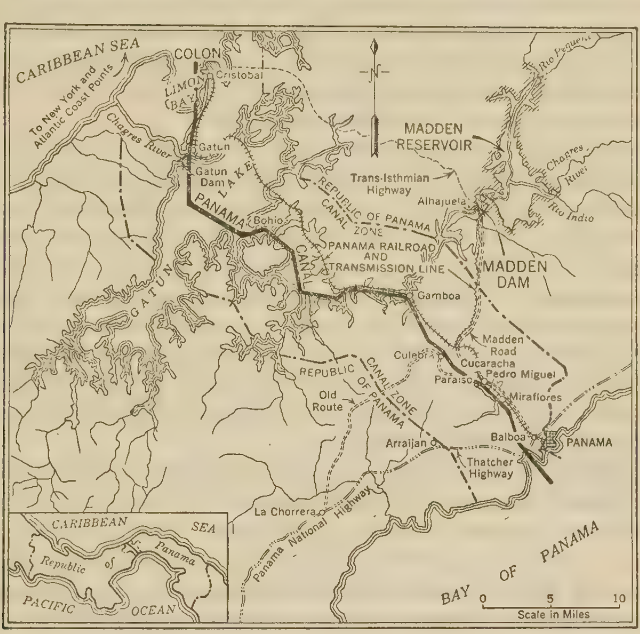

CARIBBEAN SEA
To New York and Atlantic Coast Points
COLON
Cristobal
LIMON BAY
Chagres River
N
Rio Pequeni
MADDEN RESERVOIR
Gatun
Gatun Dam
LAKE
Trans-Isthmian Highway
Chagres River
Alhajuela
PANAMA
Bohio
REPUBLIC OF PANAMA CANAL ZONE
Rio Indio
PANAMA RAILROAD AND TRANSMISSION LINE
CANAL
MADDEN DAM
GATUN
Gamboa
Madden Road
Culebra
Cucaracha
REPUBLIC OF PANAMA CANAL ZONE
Pedro Miguel
Paraiso
Miraflores
Old Route
Arraijan
Balboa
PANAMA
Thatcher Highway
La Chorrera
Panama National Highway
CARIBBEAN SEA
Republic of Panama
PACIFIC OCEAN
BAY OF PANAMA
0 5 10
Scale in Miles

FIG. 1.—GENERAL MAP OF THE PANAMA CANAL AREA

Masters generally do not know the long history of the Canal or the great struggles involved in its building; so they are reminded that this idea of single-lock structures at the Pacific terminus is not new and that it was considered during construction but had to be discarded because it was claimed that suitable foundations were not available and because no other solution seemed possible. This explanation is accepted; and a transit is usually sufficient to impress masters with some of the marine problems affecting operation of the Canal.

The principal marine problems are the traffic bottleneck at Pedro Miguel, lockage surges in Gaillard Cut, fogs, currents, and the suction of banks or lock walls. Only after long service with the Canal and close observation do shipmasters comprehend fully the nature of the operational difficulties and hazards of this waterway.

In addition to the inherent navigational problems of a restricted waterway, the physical layout of the Canal for operation is not symmetrical. On the Atlantic side there is a capacious summit level anchorage in Gatun Lake where vessels bound in either direction can anchor safely until ready to proceed. Such favorable facilities do not exist on the Pacific side. The Pedro Miguel Locks are located squarely at the south end of Gaillard Cut, with no summit level anchorage available as in Gatun Lake. These locks serve as a timing device that restricts the use of the cut to the capacity of the locks, and thus limit the capacity of the Canal.

In spite of these inherent problems, the Canal has operated successfully but it has operated under difficulties. Northbound vessels enter the cut at lockage intervals and no faster. Southbound vessels cannot arrive more rapidly than the locks can be readied to receive them. Hence, the Canal has not been able to develop its maximum obtainable capacity.

This condition can be illustrated by examining a day's traffic in the Canal under the restricted conditions. Vessels start transits on both sides of the Isthmus simultaneously, arriving at Gatun Locks and Miraflores Locks about 7:00 a.m. Northbound vessels are locked up to the summit level continuously until all northbound traffic has passed Pedro Miguel Locks. Then each proceeds in succession at lockage intervals through the cut into Gatun Lake and thus to Gatun where it enters the locks or anchors to await down lockage. Southbound vessels are locked up to Gatun Lake where they anchor at Gatun anchorage or wait under way until they are scheduled to enter the cut. After the northbound traffic clears Gaillard Cut at Gamboa, the southbound vessels spaced at lockage intervals enter the cut.

Thus, Gatun anchorage supplies a stopover station for both northbound and southbound vessels and permits flexible operation of Gatun Locks. At Pedro Miguel there is no comparable anchorage to enter. Vessels have to approach these locks in a relatively narrow and rocky passage; they cannot anchor for they would swing into the bank; they cannot slow too much because of the necessity of maintaining steerage way in a narrow rocky gorge which in the dry season is subject to high winds in the daytime. Thus, vessels must be received as they arrive and they cannot arrive faster than they can be received. This situation creates immense ship handling and traffic control problems that cause delays and at times subject both the Canal and the transiting vessels to danger.

For marine operations the location of the Pedro Miguel Locks at the end of Gaillard Cut constitutes the great traffic bottleneck of the Panama Canal. It was the fundamental error in the design of the Canal.

Canal Capacity

What is the capacity of the Panama Canal? Obviously, if the summit water supply is ample, it is the capacity of the locks. Because of the bottleneck at Pedro Miguel Locks, their capacity measures that of the Canal.

Assuming that one side of the Pedro Miguel Locks is in operation during biennial overhauls and that the lockage interval for single culvert operations is 53½ min, the 24-hour capacity is twenty-seven lockages. This figure has been accepted as the minimum capacity of the Panama Canal.

A more complete picture of canal capacity was presented in a study by the Locks Division of the Panama Canal in 1938 (Table 1). Two methods of operating the Gatun Locks are noted. In the one designated "normal" a vessel enters a lock after the vessel ahead has cleared the distant chamber; in the "follow-up" operation a vessel enters a lock chamber before the preceding vessel has cleared the last chamber. Thus the vessels are always separated by one lock. It should also be mentioned that the filling may be accomplished by the use of either single or double culverts as shown in Table 1, for each lock has two sets of culverts.

TABLE 1.—CAPACITY OF SINGLE LOCKS, PANAMA CANAL; BASED ON STUDY BY LOCKS DIVISION, 1938

Lock	NORMAL INTERVALS (MIN)		TOTAL 24-HR CAPACITY	
	Single culvert	Double culvert	Single culvert	Double culvert
Gatun:				
Follow-up	54	40	26	35
Normal	91	76	16	19
Pedro Miguel	53.5	37.5	27	38
Miraflores	60	48.5	24	29

Thus, the lockage capacity of the Panama Canal is lowest during the overhaul of Miraflores Locks, with the rate of one lockage an hour. This capacity is based on uninterrupted lockages and disregards the effect of certain other important factors to be discussed later. It represents the maximum capacity of the Canal under the most restrictive lock operating conditions—that is, during periods of one side operation at Miraflores when under overhaul.

EFFECT OF FOG

Fog is one of the most frequent and serious interruptions of traffic in the narrow reaches of the Canal. The channel between Pedro Miguel and Bohio is subject to dense fog at frequent intervals, especially in the wet season. South of Pedro Miguel and likewise north of Bohio fog seldom occurs.

Both Gatun and Miraflores Locks can operate on a 24-hour basis under favorable weather conditions because both Gatun Anchorage and Miraflores Lake are relatively fog free. Vessels can lock up from the Atlantic to the summit level and anchor safely at Gatun anchorage without interruption, day or night, and similarly at Miraflores Lake. In fog, vessels cannot enter Gaillard Cut because of the danger of striking the bank; but when Pedro Miguel Locks are clear, northbound vessels may lock up to the north approach wall of the locks to await the clearing of fog in the cut. After the north wall is filled to capacity, all north traffic must stop.

Few persons not directly concerned with the control of traffic in the Panama Canal realize the frequency with which fogs affect marine operations. In 1942 there were two hundred and twenty-three fog reports from marine signal stations in Gaillard Cut during traffic hours, but only 118 fog days. Conditions are worst during the wet season—May to December. Fog forms after 9:00 p.m. but normally clears by 8:30 a.m.

The canal capacity is thus reduced from the rated lock capacity at Pedro Miguel to the lock capacity during favorable weather conditions in the cut.

During periods of long fog duration the capacity is less than the previously assumed minimum of twenty-seven ships a day.

Traffic Restrictions

For certain types of vessels, the Panama Canal has strict regulations requiring one way traffic in Gaillard Cut, known as Clear Cut Rules. Ships laden with explosives, oil tankers, unwieldy vessels, ore ships, large warships, and largest merchant vessels are dispatched only when Gaillard Cut will be clear of vessels to pass. For reasons of safety, traffic in the opposite direction is delayed while they transit the cleared cut.

As a result the capacity of the Canal is still further reduced, particularly when the traffic does not arrive at times convenient for safe and prompt locking. The physical layout of the Pacific locks prevents any compensatory routing of vessels in groups that would overcome these delays.

Accidents in the Panama Canal

In connection with the writer's duties when he was Captain of the Port, Balboa, Canal Zone, the records of all accidents in the history of the Canal that had been formally investigated were examined and indexed. The revelations were impressive. The most serious were found to be lock accidents, groundings, collisions, and accidents resulting from steering gear failure. The study showed that they tended to occur at definite places, and these danger spots were determined.

TABLE 2.—Number of Lock Accidents, Panama Canal, Formally Investigated Between January 13, 1922, and July 13, 1942

Lock		Entering	In chambers	Departing	Total
Name	No. of chambers				
Gatun	3	87	41	24	152
Pedro Miguel	1	79	15	44	138
Miraflores	2	56	28	19	103

Of a total of one thousand and thirty-six accidents formally investigated between January 13, 1922, and July 13, 1942, three hundred and ninety-three were lock accidents (Table 2). The most serious were caused by striking the sharp corners of the lock wing walls or ramming them.

Groundings in Gaillard Cut are feared more than any other type of accident. When a large vessel strikes the rocky bank, the hull may rupture and the vessel may sink and close the Canal. The records are notable especially for the increasing record of sinkings; out of a total of seven sinkings in the 24 years from 1919 to 1942, five occurred in the final 6 years.

Collisions in the relatively narrow waters of the Canal are likely to be serious —especially in Gaillard Cut where the banks are rocky. During this same period there were fifty collisions in the canal channels, of which Gaillard Cut contributed twenty six, causing one out of four sinkings.

Steering gear failure is always on the mind of a master when his ship is in close waters. This has been one of the most prolific causes of groundings and collisions. Of the one thousand one hundred and eighteen accidents during this same period, one hundred and ten were due to this cause. Steering gear failure in Gaillard Cut makes grounding or collision almost inevitable (Table 3). Obviously, the cut is the most serious danger spot in the Canal.

TABLE 3.—ACCIDENTS CAUSED BY STEERING GEAR FAILURE; JANUARY 1, 1919, TO JULY 19, 1942

Year	ATLANTIC SEA LEVEL		GATUN LAKE		BARBACOAS TO GAMBOA		GAILLARD CUT		Mira flores Lake (collisions)	PACIFIC SEA LEVEL		Tota
	Groundings	Collisions	Groundings	Collisions	Groundings	Collisions	Groundings	Collisions		Groundings	Collisions	
1919	..	..	..	..	1	..	..	1	..	..	..	2
1920	..	..	..	..	..	..	..	1	1	..	..	2
1921	..	..	..	..	..	..	1	..	..	..	..	1
1922	..	..	1	..	..	..	1	1	..	..	..	3
1923	..	..	1[a]	..[a]	..	..	4	..	..	1	..	6
1924	1	..	..	1	2	2	3	1	..	1	..	11
1925	1[a]	..[a]	..	..	2	..	5	1	1[a]	1	2	13
1926	1	..	..	..	..	..	3	..	..	1	..	5
1927	1	1	1[a]	..[a]	1	..	4	..	3[a]	2	..	13
1928	2	..	..	..	..	..	4	2	2[a]	..	..	10
1929	..	..	..	1	..	..	1	..	1	2[a]	..	5
1930	..	..	..	..	..	..	2[a]	..	1	..	1	4
1931	..	..	..	..	1	..	2	1	..	1[a]	..[a]	5
1932	..	1	..	..	..	..	1	..	..	..	..	2
1933	..	..	..	..	1	..	1	..	1	..	..	3
1934	..	..	..	..	..	..	4	..	..	..	..	4
1935	..	..	..	..	..	..	..	..	..	1[a]	..[a]	1
1936	1[a]	..[a]	..	..	1	..	2	..	..	..	..	4
1937	1	..	..	..	1	..	1	1	1[a]	..	..	5
1938	..	..	..	..	..	..	..	..	..	..	..	0
1939	..	..	..	..	..	..	2	..	..	1	..	3
1940	..	..	..	..	..	..	1	..	..	2[a]	1	4
1941	..	..	..	..	..	..	1	..	..	..	..	1
1942	..	1	..	..	..	..	2	..	..	..	..	3
Total	8	3	3	2	10	2	45	9	11	13	4	110

[a] Grounding or collision not specified for one case.

The failure of steering gear is not by any means the only cause of serious accidents. Another is the phenomenon known as bank suction, experienced by vessels moving in narrow channels or close to walls. Wall or bank suction often causes grounding as vessels depart from locks or as they pass salients in Gaillard Cut. Usually suction causes a movement of the ship's stern toward the closer bank, center wall, or salient. If the pilot is unable to break the resulting sheer, the vessel may get out of control, strike the wall or bank, or ground on the opposite bank.

In a study by the Panama Canal in 1939, the positions of many groundings were plotted. They tended to occur at bends after the turns were completed, and in some relation to the degree of curvature. The larger turns had the larger number of groundings.

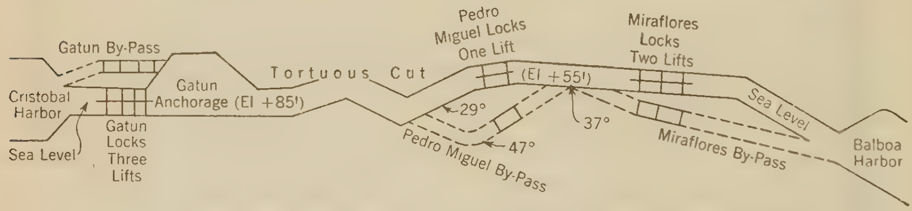
The subject of groundings has not been exhausted. It is a large field for a detailed study that should be undertaken—leading to a scientific determination of necessary changes in alinement of the channels.

Lock accidents have demonstrated the need of modifying the wing walls of the locks so as to eliminate the sharp corners at the lock entrances. The record of lock accidents and groundings shows that the separation of the Pacific locks into two structures has caused more accidents than would have occurred had the canal plan provided a summit level terminal lake on the Pacific analogous to Gatun Lake on the Atlantic.

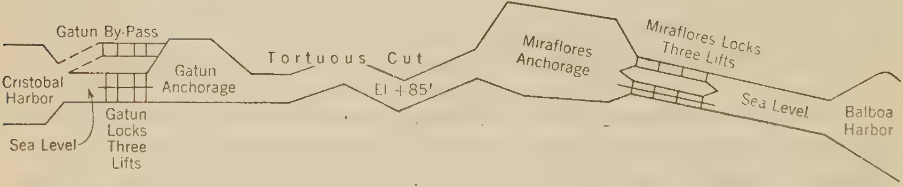
The perspective of a quarter of a century of marine operation shows that the separation of the Pacific locks into two structures, the location of Pedro Miguel Locks at the south end of Gaillard Cut, and the failure to create a commodious summit level terminal anchorage on the Pacific are the great marine operational errors in the planning of the Panama Canal.

THE 1939 THIRD LOCKS PROJECT

Canal authorities have looked forward to increasing the capacity of the Canal for many years and have made several studies of the subject. However, not until August 11, 1939, was construction of the Third Locks Project authorized by Congress, in those months of hectic activity preceding World War II.


Gatun By-Pass
Cristobal Harbor
Sea Level
Gatun Locks Three Lifts
Gatun Anchorage (El +85')
Tortuous Cut
Pedro Miguel Locks One Lift
(El +55')
29°
47°
37°
Pedro Miguel By-Pass
Miraflores Locks Two Lifts
Miraflores By-Pass
Sea Level
Balboa Harbor

(*a*) BEFORE (SHOWING THE BY PASS PLAN OF THE 1939 THIRD LOCKS PROJECT)


Gatun By-Pass
Cristobal Harbor
Sea Level
Gatun Locks Three Lifts
Gatun Anchorage
Tortuous Cut
El +85'
Miraflores Anchorage
Miraflores Locks Three Lifts
Sea Level
Balboa Harbor

(*b*) AFTER (SHOWING THE HIGH LEVEL TERMINAL LAKE PLAN)

FIG. 2.—EFFECT OF ADOPTING THE TERMINAL LAKE PLAN

The purposes were to increase the capacity of the Canal, to permit the transit of large naval vessels, to attain a greater security from bombing attack, and to facilitate conversion from a lock type canal to a "sea level" canal.

The earliest plans for a third set of locks placed the locks alongside the existing structures. The approach of the war and the resulting desire to disperse lock structures because of the danger of bombing caused the adoption of a plan in which the new locks were placed at a distance from the present structures and by-pass channels connected the new locks with the main channels, as shown diagrammatically in Fig. 2(*a*). Except for the segregation of the new locks, the 1939 Third Locks Project represented no fundamental change in the canal plan. It was in principle an acceptance of the present canal arrangement.

Its completion would have perpetuated the bottleneck at Pedro Miguel and would have delayed indefinitely any chance of solving the navigational problems of the present Canal.

Instead of improving and simplifying the Canal, part of the project added certain features dangerous to operation. The proposed new by-pass channel at Pedro Miguel would intersect the present channel near Cucaracha at an angle of 28° 59′. North of the proposed new Pedro Miguel Locks there would be a turn of 46° 17′ in the by-pass channel; and, in Miraflores Lake, still another turn of 37° 30′.

The experience of a quarter of a century has demonstrated that the proposed by-pass channel north of Pedro Miguel would be definitely dangerous, and that it should be abandoned. This channel will not simplify or improve the operation of the Canal; on the contrary, it will complicate the existing situation. The turns of the new channel and the intersection of the new cut with Gaillard Cut would become new foci of accidents; create the most difficult marine operating problems; and make transit of the Canal more hazardous.

Fortunately the suspension of this 1939 Third Locks Project in 1942 occurred at such a stage as to afford an opportunity to re-examine some of its dangerous features.

Sea Level Versus Lock Type Canal

There has been so much discussion of the so-called "sea level" canal as an assumption in the planning of the evolution of the ultimate Panama Canal that an examination of this proposal is essential before focusing attention on the fundamental marine problems that should be solved. The idea is not new. The alluring prospect of the "Strait of Panama" is an ancient historical conception that has had great public appeal. This idea has even been symbolized in the Canal Zone seal which shows a Spanish galleon sailing through Culebra Cut into the waters of the Pacific. Statesmen have made many eloquent speeches setting forth the assumed advantages of the sea level canal over the lock type canal. Yet, in spite of all the rhetoric, a lock type canal was adopted mainly through the efforts of the late John F. Stevens, Hon. M. and Past-President, ASCE. It was completed by the late George W. Goethals, M. ASCE.

In a recent comparative study of the marine features of the sea level and lock type canals, it was assumed that the sea level canal would follow the same general route as the present Canal; would have approximately the same form; would contain a tidal lock at Miraflores; and possibly would have an anchorage or mooring basin north of Miraflores. In effect, such a waterway would not be a sea level canal but a tidal level lock canal.

The conclusions as to the effects on navigation were that the low level lock canal would:

(1) Extend the length of hazardous channel from 7.69 to 31.18 miles;
(2) Increase the number of critical curves (20°+) in the hazardous channel from 2 to 12;
(3) Increase the total curvature in the hazardous channel from 129° to 564°;

(4) Probably extend the channel length subject to fog;
(5) Probably curtail operations during fog periods;
(6) Require the use of ship mooring stations;
(7) Extend the collision, grounding, and steering gear failure area considerably;
(8) Increase the number of transverse streams;
(9) Reduce pilots' vision;
(10) Increase the time required for transit;
(11) Complicate the traffic control problem;
(12) Subject the Canal to the dangers of great floods in the Chagres Valley; and
(13) Reduce the general navigability and operational convenience.

There is only one appreciable marine operational advantage for the "sea level" canal and that is the elimination of the Atlantic locks and the consequent reduction of lock accidents. In comparison, the operational advantages of the lock type canal are overwhelming.

To Solve the Marine Operating Problem

The principal marine operating problems of the present Canal have been described. The way to overcome them is to remove Pedro Miguel Locks from their position at the end of Gaillard Cut; to create a large summit level anchorage in an elevated Miraflores Lake; and to concentrate all Pacific locks near Miraflores in continuous lifts. These changes automatically correct most of the operating problems. This High Level Terminal Lake Plan (Fig. 2(*b*)) is fundamental from the operational standpoint. It will supply the best canal for handling ships.

The land contours of the Miraflores Lake basin are suitable as an impounding perimeter for a terminal lake on the Pacific and will require relatively small dams and dikes. There are several possible lock arrangements. Of these the best marine arrangement is the one in which the present Miraflores Locks are abandoned and all the Pacific locks are placed on a new site that will utilize in someway the excavation for the by-pass locks at Miraflores (Fig. 2). The general layout and relationships of this terminal lake, as of May 9, 1946, are shown in Fig. 3.

The summit level anchorage (Fig. 4) is of greatest marine interest to those charged with the operation of the Panama Canal. It contemplates nine 300-yd berths and thirty 200-yd berths, to be obtained by dredging certain areas. The areas of the several berths, classified as to depth, are as follows:

Depth (ft)	Area (sq miles)
20	0.23
30	0.13
50	0.38
Navigation channel	0.29
Total	1.03

The number of berths in this anchorage should be sufficient for the present traffic even without dredging.

The essential points of the Terminal Lake Plan for the improvement of the Canal are the removal of the bottleneck Pedro Miguel Locks from their position at the end of Gaillard Cut and the creation of a large summit level anchorage on the Pacific side. The lake is the key to the solution.

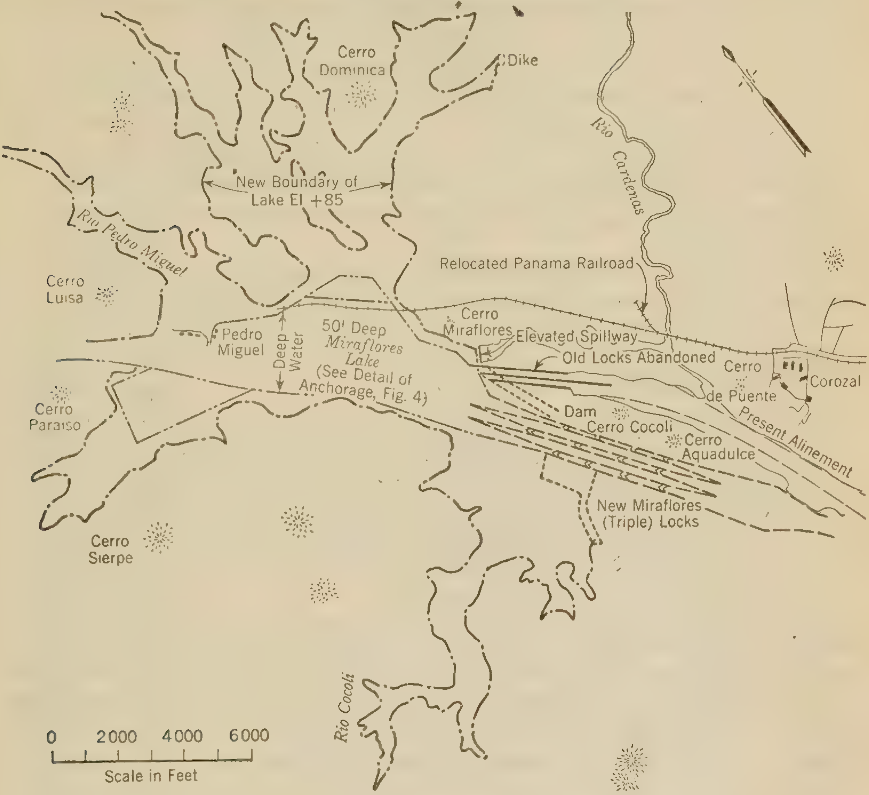

Cerro Dominica
Dike
New Boundary of Lake El +85
Rio Cardenas
Rio Pedro Miguel
Relocated Panama Railroad
Cerro Luisa
Cerro Miraflores
Elevated Spillway
Pedro Miguel
Deep Water
50' Deep Miraflores Lake
(See Detail of Anchorage, Fig. 4)
Old Locks Abandoned
Cerro de Puente
Corozal
Cerro Paraiso
Dam
Cerro Cocoli
Cerro Aquadulce
Present Alinement
New Miraflores (Triple) Locks
Cerro Sierpe
Rio Cocoli
0 2000 4000 6000
Scale in Feet

Fig. 3.—Plan for the Improvement of Panama Canal

Marine Advantages of the Proposed Plan

Anyone who has made many transits through the Pacific locks, viewed the Miraflores Lake from near-by vantage points, or studied the operating sheets, weather reports, and accident records, cannot fail to discern the tremendous advantages of the Terminal Lake Plan. Among its marine advantages are that it:

(1) Improves and simplifies the operation of the Canal;

(2) Increases the capacity of the Canal;

(3) Extends the useful life of the Canal;

(4) Increases the summit level water storage by 50% to 75%, and the watershed by 37.4 sq miles;

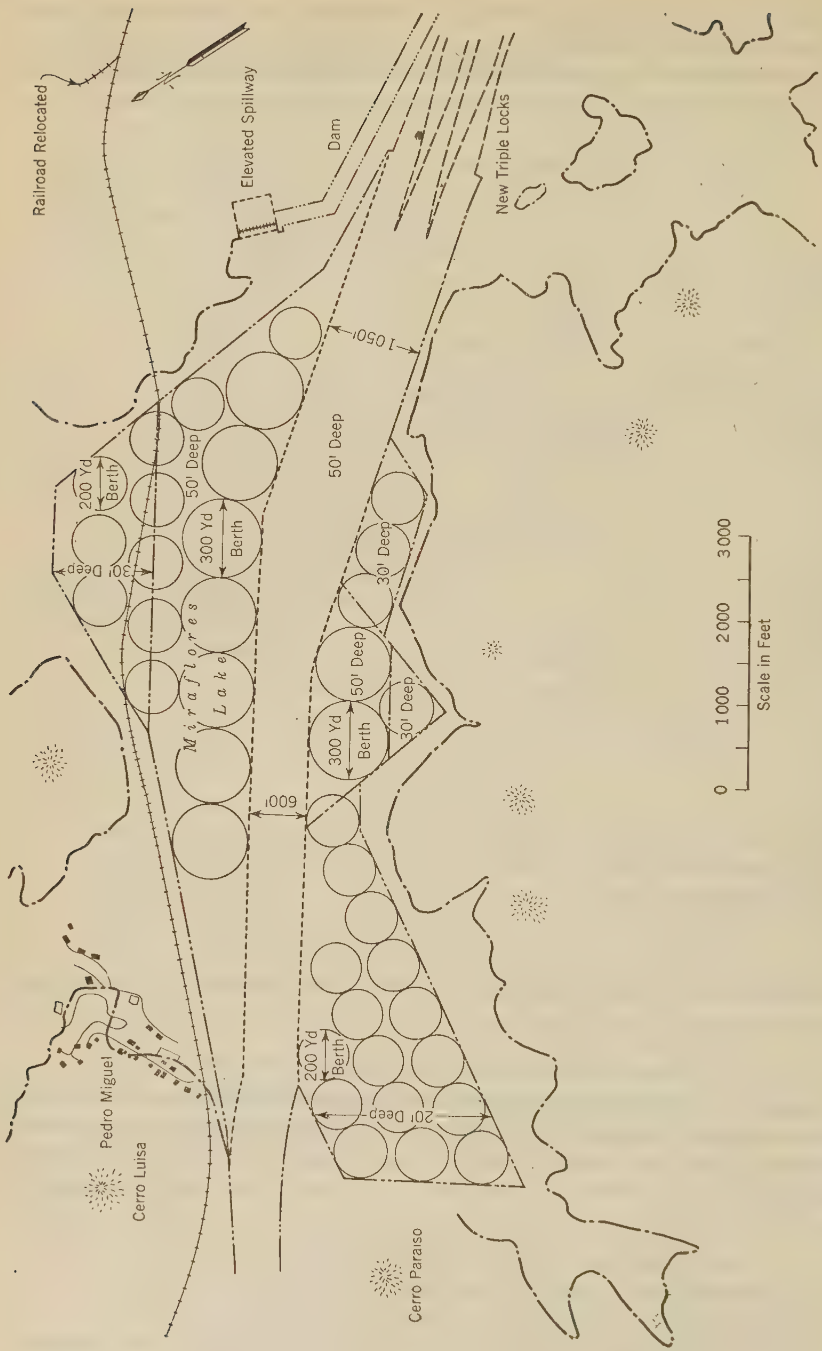

Railroad Relocated
Elevated Spillway
Dam
New Triple Locks
1 050'
50' Deep
200 Yd Berth
50' Deep
300 Yd Berth
30' Deep
Miraflores Lake
600'
30' Deep
50' Deep
300 Yd Berth
30' Deep
200 Yd Berth
20' Deep
Pedro Miguel
Cerro Luisa
Cerro Paraiso
0
1000
2000
3000
Scale in Feet

Fig. 4.—Summit Level Anchorage in Miraflores Lake

(5) Provides a summit level anchorage between Miraflores and the entrance to Gaillard Cut;

(6) Makes the operation of Pacific locks independent of fog;

(7) Simplifies the problem of dispatching transit traffic;

(8) Avoids the three large turns and consequent groundings in the 1939 Third Locks Project channel between Cucaracha and Miraflores which would necessarily be traversed by the largest ships;

(9) Eliminates one lock from the Pacific lock system and the hazards in Miraflores Lake during the approach and departure of vessels to and from Pedro Miguel Locks;

(10) Reduces the number of expected lock accidents at the Pacific locks;

(11) Removes lockage surges in Gaillard Cut as factors controlling depth of channel;

(12) Simplifies the operations of the Pacific locks;

(13) Reduces the channel maintenance operations;

(14) Reduces the time of transit about 1 hour;

(15) Enables a better distribution of Dredging Division equipment in event of slides;

(16) Increases the safety of transit especially for large war vessels;

(17) Enables a better handling of transit traffic in the event of slides;

(18) Eliminates the lock silting problem in the present Pedro Miguel Locks;

(19) Improves ship handling conditions in Gaillard Cut because of the elimination of surges and the increase in channel depth; and

(20) Removes the dangerous traffic bottleneck of the Panama Canal at Pedro Miguel.

Other operational advantages could be added to this list. As far as known there are no operational disadvantages to this plan. The marine advantages of the Terminal Lake Plan are so extensive that this plan will have a tremendous appeal to the United States Navy and to all merchant shipping.

The main engineering problems will be involved in the construction of the necessary locks and dams. A study of the topography shows that the configuration of the land is favorable for the creation of the terminal lake by dikes. Geological reports indicate favorable foundations for locks and dikes. The engineering features of the plan have been examined by engineers and have been given preliminary approval. All of them would have to be subjected to intensive and detailed study. The cost of the Terminal Lake Plan should not differ materially from the revised cost of the 1939 Third Locks Project.

On the other hand, it is not desired to minimize the problems that will be encountered and that will have to be overcome. Among these are:

(1) Inherent difficulties of changing an approved plan now in effect;
(2) Relocation of sections of the railroad, highways, and pipe and cable lines;
(3) Removal of Pedro Miguel Locks;
(4) Elevating the spillway and dams at Miraflores;
(5) Foundation work at Miraflores; and
(6) Maintenance of canal traffic during construction.

Nevertheless, the navigational superiority of the Terminal Lake Plan to the 1939 Third Locks Project on the Pacific end of the Canal is so overwhelming that it should be adopted even at considerable additional cost. It is the plan that will meet the marine operating requirements of the Panama Canal. It will make possible the construction of additional sets of locks at each end of the Canal. It should be the plan for the ultimate canal.

Historical Perspective

One of the first questions that is likely to be raised after this discussion is, "Why were these fundamental ideas not presented before this year?" The answer is that they were, but they were presented differently and by men without marine operational experience.

In recent years Ralph Z. Kirkpatrick, former Chief of Surveys of The Panama Canal, saw the weakness in the present canal arrangement and submitted suggestions. His main purpose seems to have been to combine the Pacific locks into one structure. His plans were not backed with the force of operating experience and were not adopted.

Before Mr. Kirkpatrick there was the late Maj.-Gen. W. L. Sibert, M. ASCE, the builder of Gatun Locks. He wanted to place all Pacific locks between Cerro Cocoli and Cerro Miraflores in one structure as at Gatun. Although he had an excellent grasp of the needs for traffic, his main thesis was economy of construction. His plan was investigated by a board which reported favorably. The report was referred to the President of the United States who decided against adopting the change in the canal plan because the Pacific locks had been started; because it would have meant a delay in completion date; and because any modification would have given the enemies of the Canal an opportunity to seize it as an evidence of weakness in the lock type canal at a time when a political attack could have endangered the completion of the Canal.

Before General Sibert there was Mr. Stevens. In 1906 he proposed the combination of all Pacific locks into one structure near Cerro Aguadulce with a summit level terminal lake formed by a dam between Cerro Aguadulce and Cerro de Puente. He was a transportation man and understood the operational implications of his proposal. Unfortunately, his investigations did not establish the existence of suitable foundations for lock structures. Also during Mr. Stevens' time there was the late William Gerig, M. ASCE, who developed the same idea independently of Mr. Stevens.

Still earlier was the proposal of the French engineer Adolphe Godín de Lépinay at the Paris (France) Congress of 1879. He had worked on the Isthmus and knew the problems that would face canal builders. With a plan of unbelievable simplicity he advocated creating large artificial lakes about 80 ft above sea level at each end of the canal with dams as close to the oceans as permitted by the configuration of the land, and connecting these lakes by locks with the sea level sections of the Canal. The problem then would have been simply one of joining the lakes by digging a channel across the continental divide. This is properly termed the high level terminal lake conception. M. de Lépinay probably was motivated by control of the Chagres River and the reduction of excavation, but he emphasized the navigational advantages of his plan. The conception of this plan has brought an enduring fame to its author.

M. de Lépinay's idea was not adopted until many years later, in 1906, when the adoption of the high level canal with a dam and locks at Gatun was secured mainly through the efforts of Mr. Stevens. Because there was no provision for a terminal lake on the Pacific side, the Canal as completed in 1914 was only a partial realization of the fundamental conception of the Canal as a marine operating unit. The Pacific sector of the Canal when opened for traffic did not conform to the requirements of the full de Lépinay conception.

THE PRESENT ISSUE

The Panama Canal is again in an era of decision. A vast store of operating experience, not available to the early canal builders, is available for guidance.

When the Canal was constructed, engineering considerations were the chief bases for decisions. Now marine operational requirements rather than engineering problems are the factors that should govern decisions. Just as events forced the relocation of Bohio Dam to Gatun, to form Gatun Lake, the time has come to eliminate the Pedro Miguel Locks and Dam and to concentrate all Pacific locks near Miraflores to form a high level Miraflores Lake.

Primarily the purpose of this paper is to present a historic ideal of the Panama Canal improvement in its modern conception, fortified by thirty years of marine operations. Discussion of these points should clarify the whole problem: No claim for engineering or construction sufficiency is made or intended.

The issue is clear. The solution of the marine operating problems of the Panama Canal consists of (1) the physical removal of Pedro Miguel Locks from their position at the end of Gaillard Cut; (2) the creation of a large summit level terminal lake north of Miraflores Locks for use as an expansion chamber for traffic; and (3) the construction of all Pacific locks in single structures at Miraflores. That plan should equip the Canal for ages to come. Those who bring it about will bestow a tremendous service on the naval forces of the United States and on the shipping of the world. They will be the real modernizers of the Panama Canal. They will achieve the rare distinction that will rank them with the builders of the Panama Canal.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2377

SEA LEVEL PLAN FOR PANAMA CANAL

TO PROVIDE MAXIMUM SAFETY AND UNLIMITED CAPACITY

By J. G. Claybourn,[1] M. ASCE

With Discussion by Messrs. Fred Lavis, Raphael G. Kazmann, Hans Kramer, James H. Stratton, and J. G. Claybourn.

Synopsis

National interest and security demand that the Panama Canal be streamlined, in order to secure at the same time maximum safety for operation in war as well as peace and unlimited capacity for shipping. The best method of insuring these improvements is to convert the canal, in approximately its present alinement, to sea level operation.

Supporting such a thesis, this paper proposes a step-down method of deepening and widening the channel, without interrupting traffic and including the possible utilization of mammoth machines for extraordinarily deep dredging. The major problem, to handle floods and runoff on the Atlantic slope, would be solved by building diversion channels and tunnels where needed, supplemented by large longitudinal barrier dams. The flow would be controlled by storage reservoirs and spillways, leading runoff to tidewater independently of the sea level channel.

Incidental problems are discussed, including control of slides, power development, protection in war, and tidal locks. Only enough history is presented to provide a minimum background. Costs are estimated as on the order of $1,121,000,000 to $1,910,000,000 and construction time as from 12 years to 20 years—all depending on the methods adopted.

This paper is condensed from "Streamlining the Panama Canal for Maximum Safety and Unlimited Capacity," which was presented before the Construction and Waterways divisions of the Society on January 16, 1946, in New York, N. Y. The views expressed are personal and have no official status relative to the Panama Canal. A copy of the original paper has been filed in the Engineering Societies Library for further reference and details.

Note.—Published in February, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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INTRODUCTION

Engineers are familiar with the general layout of the Panama Canal—its three locks at Gatun near the northern end leading to a summit level of 85 ft, its lock at Pedro Miguel on the south side of the Continental Divide connecting with an intermediate level, and its two locks at Miraflores reaching to Pacific tidewater. The distance affected by the proposed conversion to sea level, between Gatun and Miraflores, is approximately 35½ miles. The general features of the Canal are shown in the map, Fig. 1.

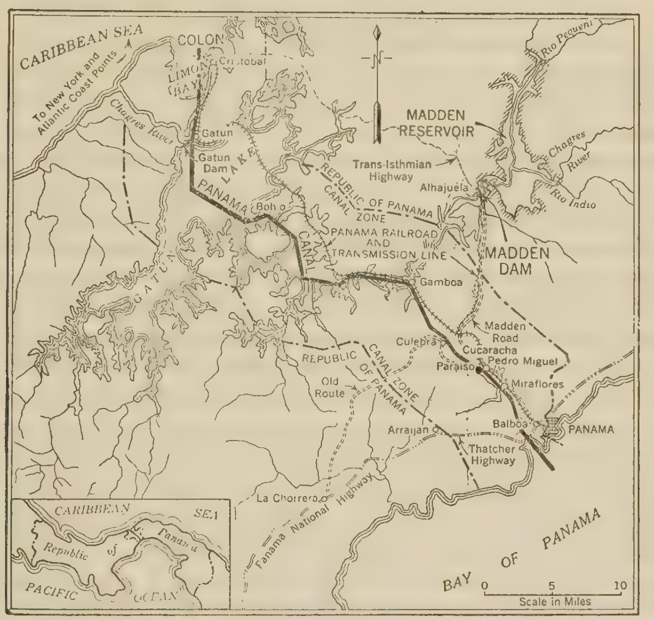
CARIBBEAN SEA
COLON
Cristobal
LIMON BAY
To New York and Atlantic Coast Points
Chagres River
Gatun
Gatun Dam
PANAMA R.R.
MADDEN RESERVOIR
Trans-Isthmian Highway
Alhajuela
Rio Pequeni
Chagres River
Rio Indio
REPUBLIC OF PANAMA
CANAL ZONE
Bohio
PANAMA CANAL
PANAMA RAILROAD AND TRANSMISSION LINE
MADDEN DAM
Gamboa
Madden Road
Culebra
Cucaracha
Pedro Miguel
Paraiso
Miraflores
REPUBLIC OF PANAMA
CANAL ZONE
Old Route
Balboa
PANAMA
Arraijan
Thatcher Highway
La Chorrera
Panama National Highway
BAY OF PANAMA
0 5 10
Scale in Miles
CARIBBEAN SEA
Panama
Republic of
PACIFIC OCEAN

FIG. 1.—CANAL ZONE AND VICINITY

Construction experiences in building the present lock type canal have been tremendously complicated; but they will greatly simplify solution of the vast problems of future development by application of the same close study and coordination. All other construction problems were decidedly secondary to the tremendous task of excavation—piercing the Continental Divide, the greatest obstacle to canal construction. In like manner the amplification of the present Canal, for maximum safety and utility, will be largely a matter of excavation. Questions of economy, time, and defense, coupled with lack of experience and reliable information, precluded earlier realization of the type of canal considered by many as most desirable—a canal without obstructions, or the sea level type.

The controversy of lock versus sea level type (the Battle of the Levels) has been waged virtually from the Canal's inception. In spite of successful operation for more than thirty-one years as a lock canal, it now is imperative to study seriously the manner of securing, not only maximum capacity, but, more important, maximum safety in case of war.

EARLY BEGINNINGS

Many volumes have been written on the historic and economic significance of the fabulous Panama country, dating back to the time of Bastidas and Columbus, through Balboa and the Spanish influence over a period of centuries. In the days of the California gold rush and later in 1852 when Lt. Ulysses S. Grant crossed the Isthmus, the United States became intensely interested in the canal project which persisted to its completion. Meanwhile the French, headed by Ferdinand de Lesseps—the builder of the Suez Canal—started a sea level canal at Panama, changed to a lock type, and failed in both, due to financial considerations.

The memorable trip of the battleship *Oregon* around South America in 1898 focused the attention of the American public on the strategic importance and defense needs of an isthmian waterway. In succession there were two early American commissions to handle the construction—the first with John G. Walker as chairman and John F. Wallace, Past-President ASCE, as chief engineer; and the second with Theodore P. Shonts as chairman and John F. Stevens, Hon. M. and Past-President, ASCE, as chief engineer. The problems were critical, particularly the determination of the type of canal. With this in mind a Board of Consulting Engineers was created on June 24, 1915, by executive order of President Theodore Roosevelt. His instructions in part stated:

> "There are two or three considerations which I trust you will steadily keep before your minds in coming to a conclusion as to the proper type of canal. I hope that ultimately it will prove possible to build a sea-level canal. Such a canal would undoubtedly be best in the end, if feasible, and I feel that one of the chief advantages of the Panama route is that ultimately a sea-level canal will be a possibility. But while paying due heed to the ideal perfectibility of the scheme from an engineer's standpoint, remember the need of having a plan which shall provide for the immediate building of a canal on the safest terms and in the shortest possible time * * *."

The Board of Consulting Engineers, failing to reach a unanimous agreement, presented two reports on June 10, 1906. The majority report, signed by eight members, favored a sea level canal. The minority report, signed by five members, favored a lock canal, with a normal lift of 85 ft; it was concurred in by the Isthmian Canal Commission, with one member dissenting, and also attained the support of Mr. Stevens, the chief engineer. Congress then passed an act requiring a lock canal, which became law by signature of President Roosevelt on January 29, 1907.

At that time the waterway was vulnerable only from land or water attack, hence the security features referred to were principally confined to those inherent in making the structures themselves safe from an operational standpoint. In the matter of time, the capacity of new machinery assured completion of the larger American lock canal in about the same interval as that required for the French company's proposed lock type—or ten years. Because of the prohibitive cost of the sea level waterway as evaluated at that period, the uncertainty of success in the minds of many leaders, and the unknown and unsolved problems which were later encountered but finally mastered, it is generally conceded that the choice of type at the time was amply justified, leaving to later generations any corrective measures deemed necessary.

So great was the progress under the new chairman and chief engineer of the third and final Commission for Construction—Col. George W. Goethals, M. ASCE,—especially in excavation including dredging, that it became possible to put the first ship through on August 3, 1914—with the canal channels in Culebra Cut cleared to full width and navigable depth of 32 ft.

Problem of Slides

Slides were troublesome and costly during dry excavation, but not until after the Canal had been opened to navigation did they attain major importance and become calamitous. Even more important, they were disastrous in neutralizing the Canal's primary purpose—as America's greatest single instrument of defense and offense in wartime.

It was discovered during dry excavation that, whenever the bottom of the cut pushed up, an equal displacement took place in the adjacent banks, caused by the unequal distribution of pressure and in direct proportion to the varying elevations of the adjacent banks above the bottom. Obviously, "* * * if the height of these banks were reduced, the movement would decrease; and if reduced sufficiently, would cease entirely." [2] A contrary view developed—that, by allowing material to move into the cut, the minimum amount would be handled, resulting in a natural slope of the material in addition to reducing cost.

The first assumption was correct. The lack of proper slope of the banks was substantiated as one of the primary causes of slides in the cut, to which was later added the second primary cause for slides—that is, lack of seepage and drainage control. These two causes furnished a well-defined formula for combating, arresting, or preventing slides.

Early it was known that parts of Gaillard Cut were particularly susceptible to landslides, requiring relatively flat slopes, and that the surface must be provided with proper drainage—the latter being considered only a contributory cause. However, because of the failure of the French company and because of immense political and economic pressure brought to bear for early completion of the Canal, a more or less uniform slope throughout the cut section was assumed, and a minimum of drainage was provided. Furthermore, many stretches as originally excavated did stand up, saving temporarily at least the cost of securing a more desirable theoretical slope. The opening of the Canal to traffic was thus expedited, with ever-increasing returns; and the slope problem, of questionable proportions, was left to be solved by the maintenance forces.

In the deep cut sections where slides are definitely to be expected, the procedure of attack is that of working to a predetermined slope in a widening and deepening improvement by removing the high banks immediately bordering the old slope, first sluicing and bulldozing off the overburden followed by mining the open face toward the cut to depths up to 75 ft, with berm widths of approximately 100 ft. Thus, the frontal step is kept one lift below the adjacent one behind, and this procedure is continued until the predetermined slope line is reached. By this method the objectionable superimposed weight is removed, and simultaneously a new and flatter step slope, nearly conforming to the final angle of repose, is obtained and subsequently retained throughout the operation. Furthermore, the new slope carries much of the blasted material directly to the dredges below. In these operations, up to 15 tons of 60% sodium nitrate dynamite have been used on the highest banks of Gaillard Cut, which are most susceptible to slides, without causing any earth movement except that predetermined in the mined area. Therefore, it is reasonable to conclude that bombing of the canal banks would not result in an appreciable slide movement.

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[2] "Annual Report of the Governor," The Panama Canal, Fiscal Year Ending June 30, 1916, U. S. Govt. Printing Office, Washington, D. C., p. 31.

After twenty years of actual experience, a formula for proper slopes in Gaillard Cut has been determined:

From El.+40 to El.+90—slope 1 on 1
From El.+90 to top rock—slope 1 on 3
From top rock to top slope—slope 1 on 5

These slopes are definitely safe for any of the material involved; they are based on nature's great test laboratories—the banks themselves.

The second great development for preventing slides is a systematic plan of seepage and drainage control along both banks of the cut. It consists of diverting all drainage directly into the Canal at suitable places, where rock banks are firm, and, where they are not, of providing permanent works to hold the banks.

Some of the most formidable structural breaks or deformation slides have occurred during the dry season. The presence of impounded water in depressions relatively near the slide areas was apparently given very little consideration by engineers or geologists. Isolated areas still remain to be drained, but the final system for the entire cut area will be postponed, pending definite conclusions as to ultimate development plans in the way of widening and deepening the cut. Possibly some of the slide areas may be extended in the future, and additional bank breaks may be expected because of insufficient slope and lack of proper drainage.

The practicability of conversion of the lock type canal to sea level at Panama has been questioned at times on the score of slides. Results to date definitely indicate that conversion to sea level is entirely practicable in so far as slides are concerned.

Channel and Other Improvements

Immediately after the great slide menace was brought under control, the Canal began to serve its primary purpose as a great instrument of offense and defense in World War I. It increased the effectiveness of American naval and transportation fleets. The tremendous aid to the war effort was apparently taken as a matter of course by the general public.

With the tremendous development of the airplane, every step possible was taken for the Canal's defense by the time of the outbreak of World War II.

The number of berths in this anchorage should be sufficient for the present traffic even without dredging.

The essential points of the Terminal Lake Plan for the improvement of the Canal are the removal of the bottleneck Pedro Miguel Locks from their position at the end of Gaillard Cut and the creation of a large summit level anchorage on the Pacific side. The lake is the key to the solution.

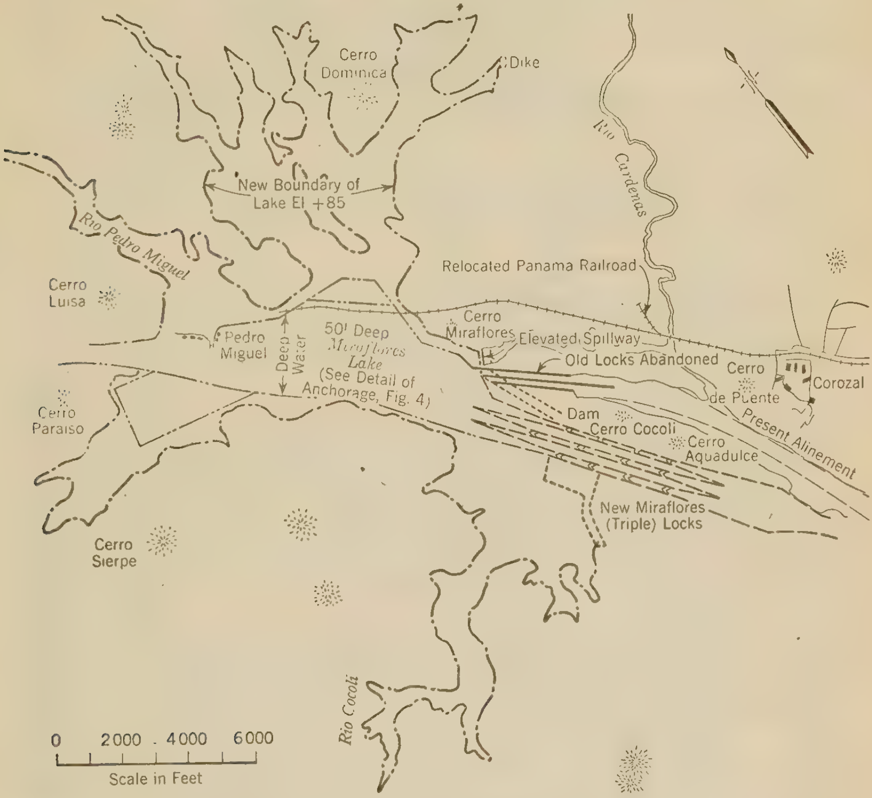
Cerro Dominica
Dike
Rio Cardenas
New Boundary of Lake El +85
Rio Pedro Miguel
Cerro Luisa
Relocated Panama Railroad
Cerro Miraflores
Elevated Spillway
Old Locks Abandoned
Pedro Miguel
Deep Water
50' Deep Miraflores Lake (See Detail of Anchorage, Fig. 4)
Cerro de Puente
Corozal
Cerro Paraiso
Dam
Cerro Cocoli
Cerro Aquadulce
Present Alinement
New Miraflores (Triple) Locks
Cerro Sierpe
Rio Cocoli
0 2000 4000 6000
Scale in Feet

FIG. 3.—PLAN FOR THE IMPROVEMENT OF PANAMA CANAL

MARINE ADVANTAGES OF THE PROPOSED PLAN

Anyone who has made many transits through the Pacific locks, viewed the Miraflores Lake from near-by vantage points, or studied the operating sheets, weather reports, and accident records, cannot fail to discern the tremendous advantages of the Terminal Lake Plan. Among its marine advantages are that it:

(1) Improves and simplifies the operation of the Canal;

(2) Increases the capacity of the Canal;

(3) Extends the useful life of the Canal;

(4) Increases the summit level water storage by 50% to 75%, and the watershed by 37.4 sq miles;

Early estimates for performing necessary excavation were based on a six-year work program—from July, 1940 to June, 1946. Then, because of the likelihood of war, the over-all schedule was accelerated to permit use of the new locks by June, 1945, thus reducing the construction program to a five-year period. However, war developments caused practically all the work on the Third Locks to be terminated in May, 1942, except wet excavation, which was carried on as a backlog of work.

Considering the developments that have been made since Pearl Harbor in the range and size of planes and the destructiveness of their bomb loads, and the further developments that may be expected, the dependability of the lock type canal as a wartime transportation and supply route is very doubtful. It is therefore necessary to consider streamlining the Canal in such a manner that this service at all times shall be available.

Possible Modifications in Third Locks Construction

Following general cessation of Third Locks construction, various changes were suggested in the original plan when and if construction is resumed. The main reasons were: (*a*) If conversion to sea level is considered necessary, and is undertaken in line with the plan of lowering the lake by steps, as contemplated in the report of Governor H. Burgess, M. ASCE, for 1931, the project will have been advanced by the removal of Pedro Miguel locks; (*b*) suitable design can be incorporated in new Third Locks and existing locks at Miraflores to facilitate conversion; and (*c*) certain improvements in operation will have been gained for the existing lock type canal in the interim.

The general scheme conforms to that proposed in 1907 (except for the addition of Third Locks) by the late Maj. W. L. Sibert, M. ASCE, member of the Isthmian Canal Commission, and later advocated by a board of consulting engineers in February, 1909. It was also given consideration by the Interoceanic Canal Board of 1931, the general object being to concentrate all three flights of Pacific locks at Miraflores, similar to the arrangement on the Atlantic side at Gatun; and to create, in the words of the consulting engineers,

> "* * * a Pacific Terminal Lake * * * more than a square mile in area, immediately above the locks, in which ships going in either direction can come to anchor to wait for the lifting of night fogs in Culebra Cut, if east bound, or for any other convenient purpose. This terminal lake would be of great convenience in connection with fleet lockages * * *."

Other advantages claimed for the Terminal Lake scheme are: The elimination of Pedro Miguel locks, the bottleneck at the south end of the cut; the elimination of surges in the cut; the elimination of the construction of third locks at Pedro Miguel and the excavation of a second cut, between Miraflores and Gaillard Cut; and the reduction of traffic time and accident rate caused by the present separation of the locks on the Pacific side. Of the several schemes presented, only one plan seemed reasonably economical and therefore acceptable if conversion to sea level is to be long delayed—namely, that of constructing an additional chamber on the north end of Miraflores Locks, of appropriate height for the summit level, and the construction of all new third locks at Miraflores at the side originally designated for two lifts.

It will be apparent, however, that all the disadvantages enumerated in the lock type canal will be corrected and the additional advantages claimed for the Terminal Lake scheme will be obtained, automatically, upon conversion to sea level. In addition, many advantages of greater, more vital, and more far reaching importance will have been gained by such conversion, for

(1) All high lift lock bottlenecks will have been removed.

(2) Surges in the cut, created by lockages, will have been eliminated.

(3) No additional channels to additional locks will have to be excavated.

(4) The traffic time and accident rate, as affected by these artificial barriers, will have been reduced.

(5) The narrow cut section will have proper side slopes from an increased bottom width. This will allow ships to be maneuvered to the new full 500-ft bottom width instead of an actual 200-ft navigable width, as at present, because of the present vertical bank slopes (10 vertical to 1 horizontal) which prohibit ships from utilizing the full 300-ft width now available.

(6) Electric power requirements for the Canal will have been reduced to navigational aid demands.

(7) Most important, the necessary artificial storage of lockage water, and, concurrently, the annual water shortage fear, for the lock type canal operation, will have been eliminated and with it the greatest danger of prolonged closure of the Canal by enemy action.

Danger of grounding ships in channels of 500-ft bottom width, with bank slopes and increased widths at turns as recommended, cannot be taken seriously when compared to the number of accidents which have occurred under conditions existing in the present lock type canal. Little trouble has been experienced with the transit of average size ships, even in the comparatively narrow channels of Gaillard Cut section, except as may be inherent in the power plant or steering gear of such vessels. The difficulties have been with the large, deeply laden vessels termed "bad handlers" and may be attributed to various causes, such as poor design; underpowered, insufficient rudder surface; uneven trim; and the great displacement of water in relatively confined channels. In the latter case, the hull is too close to the bottom or to the comparatively vertical banks, and thus prevents sufficient accessibility of water to the propeller and rudder—referred to in the "Pilot's Handbook" as bank suction.

From all available data, it appears that fogs were as prevalent before canal construction as at the present time and possibly more so. Furthermore, fogs will probably remain as a navigational problem, should the lock type canal be converted to sea level. The safety of navigation in the cut section, due to fogs, unquestionably would be increased by its widening, permitting any cross wind to aid in fog dissipation. It is possible that even the comparatively slight fog and occasional rain squall that menace navigation may be neutralized by the application of scientific discoveries produced during the war, such as radar or other devices not yet disclosed, which will permit at least directional or one-way traffic during these comparatively short stoppage periods, permitting all-out navigation of the channel under these conditions.

In any event, fogs throughout the cut section are not nearly of such import to the shipping capacity of the Panama Canal as is the one-way traffic required by narrow channels during daylight operations. The widening of the cut section would more than make up for the capacity lost as a result of fog.

PROPOSAL FOR CONVERSION TO SEA LEVEL

The multilock canal at Panama has been in successful operation for thirty-one years. It is agreed that, if a lock type plan is to be retained, it can be improved, if desired, toward the optimum capacity by the addition of locks as required, incorporating the Terminal Lake Plan, but at great expense. There always remains, however, the element of vulnerability of the lock type in time of war and this is increased each time a lock, a dam, a spillway, or other control work is added. Furthermore, great additional cost is involved in widening the cut each time a lock is added, to maintain proper navagational conditions. Otherwise undesirable currents would increase in proportion to the withdrawal of lock water from the summit level; and, moreover, the resulting increased cost of augmenting the necessary water supply is evident in the form of more dams and control works, each in turn increasing the Canal's vulnerability. Even mechanical methods of supplying water for the lock type canal must be considered because the deepening and widening process, for the present summit level and lock structures, is limited, as is the storage of water in reservoir systems at higher elevations.

On the other hand, enlargement for easier navigation or for augmentation of the capacity to the optimum by conversion to sea level operation is simple and relatively inexpensive, because such improvement will be made as traffic demands, with no change in flood or other control works necessary.

It is believed that practically all engineers familiar with the Canal's construction, maintenance, or operation considered the lock type favorably until the destructiveness of aerial warfare was so forcefully demonstrated in World War II. This raised serious doubts concerning the feasibility of protecting such a canal against an aerial attack.

It seems entirely possible that the lock canal could be put out of business without even endangering the personnel conducting the attack. A sea level canal could not be destroyed by such methods. Once the water in the summit level is dissipated, the lock canal could conceivably be out of commission for the duration of a war. As against this, Navy strategists apparently consider it an essential that transit of all vessels between the Atlantic and Pacific oceans shall be rapid, easy, and safe for defensive and offensive operations in any global war.

The feasibility and practicability of converting the Canal to sea level form has received approbation from the Board of Consulting Engineers in 1906, and from Governor Burgess of The Panama Canal and the Interoceanic Canal Board of 1931.

The Board of Consultants, 1906,[3] admonished:

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[3] "Report of the Board of Consulting Engineers for the Panama Canal, 1906," U. S. Govt. Printing Office, Washington, D. C., 1906, p. 39, paragraphs 1 and 2.

> "That it is possible to turn any lock canal which it has considered into a sea level canal without interrupting the traffic upon it * * *"

but such conversion is

> "* * * impracticable from a financial standpoint until the traffic should have so increased as to tax the capacity of the lock canal, or until other good and sufficient reasons exist for such a change."

The good and sufficient reasons have become most apparent. To quote a very applicable statement of the Supervising Engineer, E. E. Abbott:[4]

> "If the hypotheses that a lock canal cannot be made impregnable and transits without delay of many months are essential to the security of the United States, are accepted, it must be assumed that a sea-level canal program must be the ultimate solution * * *."

STEP-DOWN PLANS FOR CONVERSION

The general plan proposed for the conversion from lock to sea level canal follows that described in Governor Burgess' report of August 4, 1931, to the Secretary of War, and the excavation scheme is based on a revision of the plan by S. B. Williamson, M. ASCE, formerly Division Engineer of the Pacific Division, later a member of the Interoceanic Canal Board in 1931. This method (Fig. 2) permits conversion without interference to traffic by excavating one half, or 250 ft, of the channel to an additional 30-ft depth, while traffic is confined to the other half and then by interchanging operations to the opposite side—the latter operation to be followed by the removal of the locks at Pedro Miguel and the upper locks at Gatun. The same procedure is followed in the second lift, after which the top flights of locks at Miraflores and the second flights at Gatun would be removed. Finally, by the same procedure, the last lift would be removed down to sea level. Coincidentally, with the excavation of channels and removal of locks, the diversions, necessary dams or barriers, and control works would be installed progressively at the appropriate periods.

The primary reasons for prior completion of Third Locks as a forerunner to the execution of this plan, in whole or in part, are fourfold: (1) To insure continuous traffic through the Canal during conversion; (2) to permit additional lockages for excavated material and movement of construction plant, when and if required; (3) to provide for lock overhaul during construction; and (4) to allow for normal increase of traffic during conversion. An additional advantage of the step-down scheme is that lifts may be made piecemeal, with intervals between them if required for any reason.

STUDIES FOR FLOOD CONTROL OF THE CHAGRES

Flood control of the Chagres River and its tributaries is considered the most difficult problem to be solved in effecting conversion to sea level as well as in providing a permanent flood control system thereafter. This feature was accomplished for the lock type canal in the creation of Gatun Lake. The French company's sea level plan to accomplish this purpose provided a dam across the Chagres River above Gamboa and a spillway into an East Diversion, which

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[4] "The Hypothetical Sea-Level Project," by E. E. Abbott, memorandum to the Engr. of Maintenance, The Panama Canal, Records Bureau, Balboa Heights, December 2, 1944.

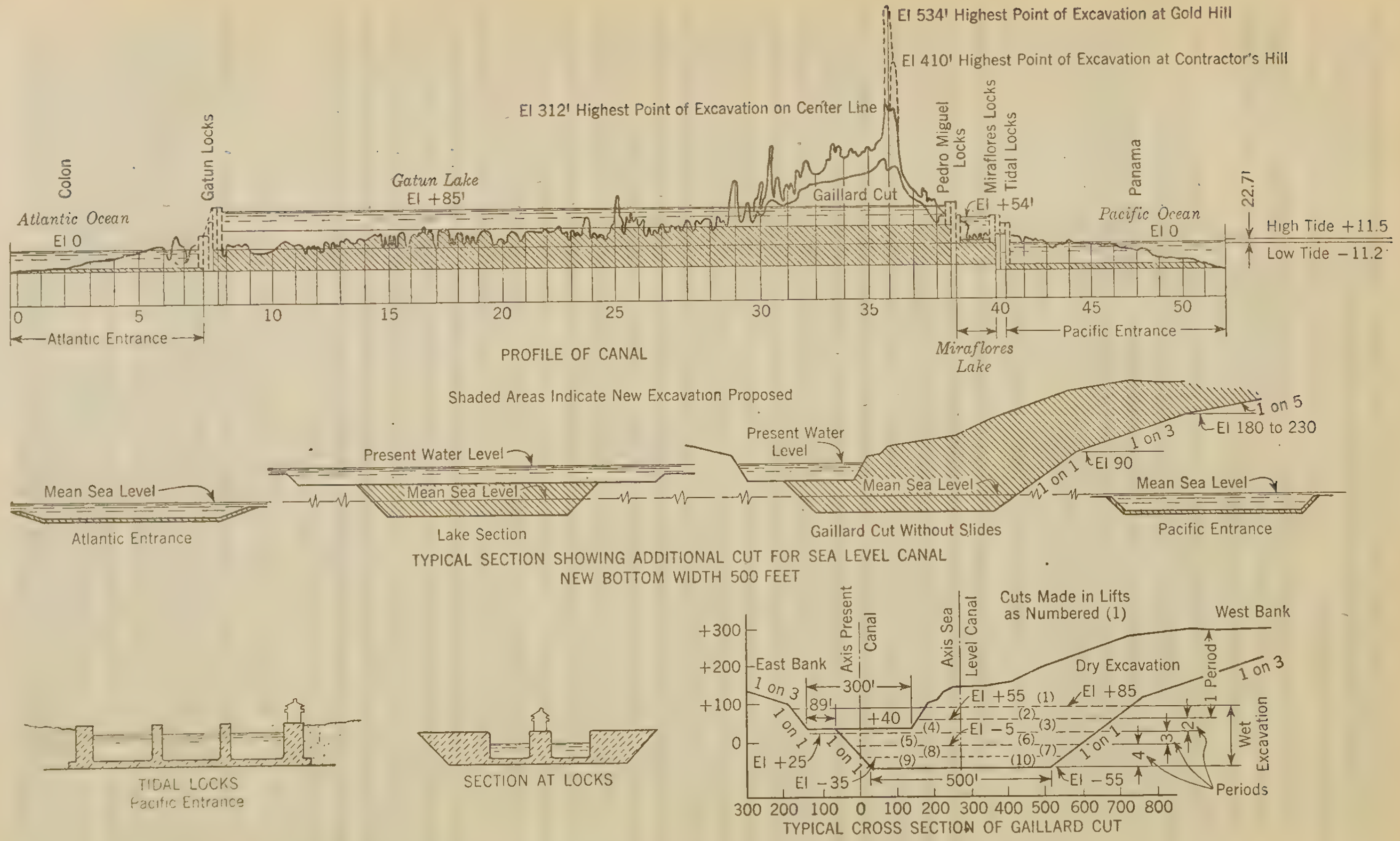
El 534' Highest Point of Excavation at Gold Hill
El 410' Highest Point of Excavation at Contractor's Hill
El 312' Highest Point of Excavation on Center Line
Colon
Gatun Locks
Atlantic Ocean
El 0
Gatun Lake
El +85'
Gaillard Cut
Pedro Miguel Locks
Miraflores Locks
Tidal Locks
El +54'
Panama
Pacific Ocean
El 0
22.7'
High Tide +11.5
Low Tide −11.2
0
5
10
15
20
25
30
35
40
45
50
Atlantic Entrance
Miraflores Lake
Pacific Entrance
PROFILE OF CANAL
Shaded Areas Indicate New Excavation Proposed
Mean Sea Level
Atlantic Entrance
Present Water Level
Mean Sea Level
Lake Section
Present Water Level
Mean Sea Level
1 on 1
El 90
1 on 3
El 180 to 230
1 on 5
Gaillard Cut Without Slides
Mean Sea Level
Pacific Entrance
TYPICAL SECTION SHOWING ADDITIONAL CUT FOR SEA LEVEL CANAL
NEW BOTTOM WIDTH 500 FEET
TIDAL LOCKS
Pacific Entrance
SECTION AT LOCKS
+300
+200
+100
0
East Bank
1 on 3
1 on 1
1 on 1
El +25
El −35
89'
300'
+40
Axis Present Canal
Axis Sea Level Canal
El +55
El −5
(1)
(2)
(3)
(4)
(5)
(6)
(7)
(8)
(9)
(10)
500'
El −55
El +85
1 on 1
Cuts Made in Lifts as Numbered (1)
West Bank
Dry Excavation
1 Period
1 on 3
2
3
4
Wet Excavation
Periods
300 200 100 0 100 200 300 400 500 600 700 800
TYPICAL CROSS SECTION OF GAILLARD CUT

FIG. 2.—CONVERSION TO SEA LEVEL OPERATION, SHOWING PROFILE, SECTIONS, AND ORDER OF PROCEDURE

eventually discharged its waters into Manzanillo Bay at the Atlantic end (Fig. 3). In addition, the canal channel itself was to be used in part for flood control. The tributaries from the south were to be handled by the West Diversion, discharging into the Lower Chagres, and thence into the Atlantic Ocean. These diversions more or less paralleled the canal channels and at some points were joined.

The Board of 1906 planned[5] a solid masonry dam across the Chagres River at Gamboa, with a top elevation of 180 ft mean sea level. The highest flow line of the reservoir thus formed was 170 ft; it included an area of 29.47 sq miles. Through this dam 15,000 cu ft per sec was to be discharged into the canal channel with a minimum depth of 40 ft, and a minimum wetted cross section of 8,000 sq ft. For the size of canal prism planned, this would have resulted in a current, flowing in one direction, of 1¼ miles per hr—a negligible quantity in so far as navigation is concerned, discounting the effects of fresh water mixing with salt water. The board assumed that, with a flood of the proportions of 1879, in which 65,000 cu ft per sec inflow was recorded at Gamboa for a period of 48 hours, there would accumulate in Gamboa Lake, 8,640,000,000 cu ft of water, which is:[6]

> "* * * that portion of the volume of the lake included between the water surfaces at elevations 159 and 170 feet M.S.L. Furthermore, * * * 15,000 cubic feet per second would discharge * * * the 1879 flood in 8.7 days."

Furthermore the board stated that the capacity of the flood control of such a lake:[7]

> "* * * between water surfaces of 108 and 170 feet M.S.L. is sufficient to take the aggregate discharge of three times the maximum average 48-hour flow of the 1879 flood, without any water escaping through the regulating sluices of the dam; or the volume between 128 and 170 feet M.S.L. will hold three times the flow of such a flood, if a uniform discharge of 15,000 c.f.s. be permitted concurrently through the regulating sluices."

This board, therefore, concluded that the capacity of Gamboa Lake would be ample for the exigencies of any flood on the Chagres River.

New Plan for Chagres Control—Eastern Tributaries

Under the new plan herein proposed (Fig. 3) diversions and control works therefor shall be absolutely independent of canal channels and thus shall positively eliminate any ill effects that floodwaters might have on the Canal, or on navigation. This plan also provides a suitable reservoir system to accomplish the same purpose as the present Gatun Lake, in allowing floods to spread over large areas and thus dissipate their force, finally discharging through regulating works on the Atlantic side.

The control of the Upper Chagres and tributaries formerly running into it from the east must be handled in two stages: (1) During conversion and (2)

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[5] "Report of the Board of Consulting Engineers for the Panama Canal, 1906," U. S. Govt. Printing Office, Washington, D. C., 1906, pp. 42–45.

[6] *Ibid.*, p. 44, paragraph 2.

[7] *Ibid.*, p. 44, paragraph 3.

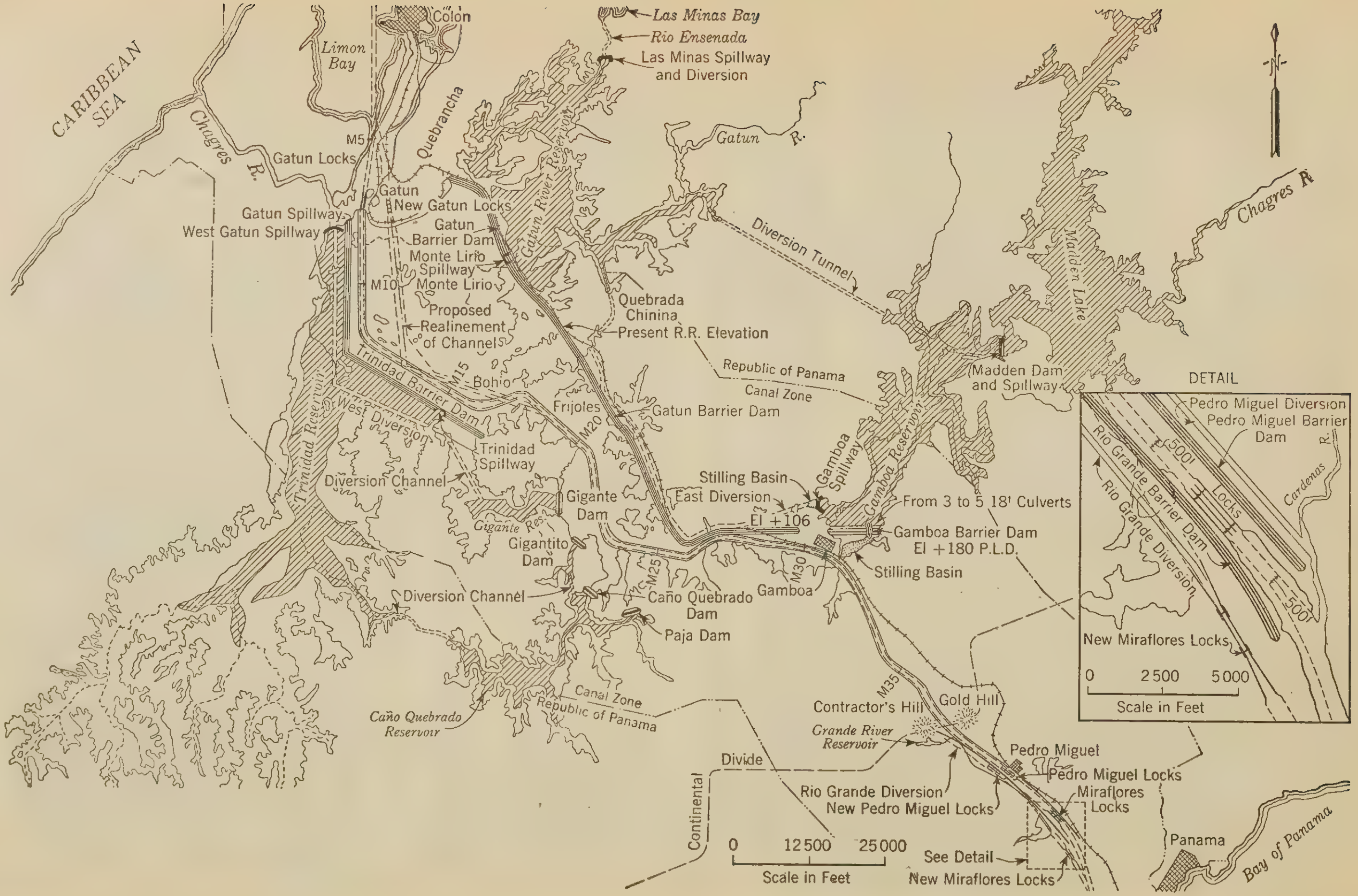
CARIBBEAN SEA
Limon Bay
Colon
Chagres R.
Gatun Locks
M5
Quebrancha
Gatun
New Gatun Locks
Gatun Spillway
West Gatun Spillway
Gatun Barrier Dam
Monte Lirio Spillway
Monte Lirio
M10
Proposed Realinement of Channels
Las Minas Bay
Rio Ensenada
Las Minas Spillway and Diversion
Gatun River Reservoir
Gatun R.
Diversion Tunnel
Quebrada Chinina
Present R.R. Elevation
Republic of Panama
Canal Zone
M15
Bohio
Trinidad Barrier Dam
West Diversion
Trinidad Reservoir
Trinidad Spillway
Diversion Channel
Frijoles
M20
Gatun Barrier Dam
Gigante Dam
Gigante Res.
Gigantito Dam
Diversion Channel
Stilling Basin
East Diversion
El +106
Gamboa Spillway
Gamboa Reservoir
From 3 to 5 18' Culverts
Gamboa Barrier Dam
El +180 P.L.D.
Stilling Basin
M25
M30
Gamboa
Caño Quebrado Dam
Paja Dam
Canal Zone
Republic of Panama
Caño Quebrado Reservoir
Madden Lake
Madden Dam and Spillway
Chagres R.
N
M35
Contractor's Hill
Gold Hill
Grande River Reservoir
Divide
Continental
Rio Grande Diversion
New Pedro Miguel Locks
Pedro Miguel
Pedro Miguel Locks
Miraflores Locks
0 12500 25000
Scale in Feet
See Detail
New Miraflores Locks
Panama
Bay of Panama
DETAIL
Pedro Miguel Diversion
Pedro Miguel Barrier Dam
R.
Cardenas
Locks
500'
500'
Rio Grande Barrier Dam
Rio Grande Diversion
New Miraflores Locks
0 2500 5000
Scale in Feet

Fig. 3.—Flood Control by Diversion, Showing Proposed Facilities to Protect Sea Level Canal

after conversion. The conversion period envisions a barrier dam across the Chagres River at Gamboa, of such proportions that it cannot be destroyed by bombing and raised to an elevation of 180 ft mean sea level, with a maximum flood line of 170 ft. Conduits at low elevation through this barrier, controlled by valves, with a maximum discharge of approximately 30,000 cu ft per sec will serve (*a*) for flood control; (*b*) for water supply during conversion; and (*c*) as safety factors afterward, for flood control if desired.

In this plan a spillway is also provided, with a maximum flow line of 170 ft mean sea level, and a maximum discharge of 50,000 cu ft per sec emptying into an East Diversion extending to Quebrada La Chinina, approximately 13¼ miles northwestward. The channel will have a bottom width of 350 ft, a depth of 20 ft, slopes of 1 to 1, and a bottom elevation 76 ft mean sea level, at the Chagres River end. It will carry 50,000 cu ft per sec at a mean velocity of approximately 6.8 ft per sec on a slope of 0.4 ft per thousand. Detailed studies would be required to determine the optimum cross section and slope. If the channel is designed for part of this floodwater, the remainder may be handled by a diversion tunnel connecting the Upper Chagres and Gatun rivers, or may be permitted to flow into the Canal through the culverts already mentioned.

In addition, a continuous barrier dam from Gamboa to Quebrancha will be provided, of sufficient proportions to withstand bombing, with an elevation of approximately 106 ft mean sea level at the Gamboa Spillway, and continuing 10 ft above the diversion flow line to a point opposite Quebrada Chinina with an approximate elevation of +85 ft, and then to Quebrancha at approximately the same elevation. This design would provide a safety freeboard against floods of approximately 10 ft, corresponding with that for the barrier dam at Gamboa. The material required for these barriers would be provided from excavations of channels and diversions. It may be necessary to shift the Gamboa Reach Channel of the Canal southward slightly, to reduce the cost of excavation of the East Diversion Channel, as well as to provide sufficient width for the Gatun Barrier Dam between the East Diversion and this channel. It will be noted that the outflow of 50,000 cu ft per sec in the East Diversion, combined with other methods enumerated, compares with the maximum volume of 15,000 cu ft per sec contemplated in the 1906 plan for regulating the Gamboa Reservoir.

Discharge and Controls—Eastern Tributaries

During the conversion period it is proposed also to use Madden Dam as a flood control and water supply device in conjunction with the Gamboa Reservoir. The spillway is to be operated at a maximum discharge of 50,000 cu ft per sec. The East Diversion will thus join the high Gamboa Reservoir with a lower Gatun River Reservoir (Fig. 3), to be formed by extending the Gatun Barrier Dam across the Gatun River arm of the existing Gatun Lake.

The Gatun River Reservoir would discharge its floodwaters through Rio Ensenada and thence, into Las Minas Bay through a Las Minas Spillway and a 6,000-ft channel. The dimensions of this spillway and channel would be adequate to take care of the combined floodwaters of the Gamboa and Gatun River reservoirs. The top flow line of the latter would not exceed +75 ft mean sea level during the construction period.

When the sea level canal is opened to traffic, it may be found advisable to have the spillway regulating works, at Gamboa and Las Minas, operate at a lower level than during the conversion period, to provide for greater flood cycles. In any event, they would provide the same outflow to retain the advantages of the reservoir system. Working models would indicate the proper installation so that alterations would not be necessary at the end of the work.

During the conversion period, it is possible to take advantage of a feature proposed by the Hydraulics Section of the Special Engineering Division in 1941. This would provide additional lockage water during the critical or lower stages of conversion, when elevations of the smaller sized Gatun Lake will fluctuate more violently because of reduced area. The installation of a regulating spillway at Monte Lirio (through the Gatun Barrier Dam) would permit storage waters to be conducted from the Gatun River Reservoir into the present Gatun Lake, over a 300-ft overflow spillway, at elevation +45 ft mean sea level, with a discharge of 60,000 cu ft per sec at a maximum water surface elevation of 59 ft. Obviously, the storage in this reservoir may be increased by giving greater range to the spillways at Monte Lirio and Las Minas.

Other features of the plans of the Hydraulics Section provide for a dam at Gamboa, 95 ft above the stream bed or approximately at 135 ft mean sea level, 4,000 ft long, with a control spillway 400 ft long, a crest elevation of 105 ft mean sea level, a discharge capacity of 190,000 cu ft per sec, and outlet conduits having a capacity of 30,000 cu ft per sec with water at crest elevation. This scheme provides a discharge into a stilling basin, and thence directly into the Canal. Its disadvantage is that destruction of the dam would result in a flood of sufficient magnitude to interrupt navigation; however, considering the great increase in cross section of the prism channels, 30,000 cu ft per sec might be discharged into a 500-ft channel without materially influencing navigation, discounting any ill effects produced by discharging fresh water into salt water. As may be noted, this proposal revives the same scheme that was advocated by the Majority Board of 1906.

A complete study of all methods must be made before any one method, or a combination of methods, can be recommended for adoption. Working models of appropriate size will be of the utmost value and importance—to determine a balanced control of flood and feeder problems, to determine the maximum amount of water that can be discharged into the Canal safely, to uncover omissions, or to correct erroneous conclusions. In any event, the independent flood control as described would appear to be the safest from the viewpoint of aerial attack, because any damage to embankments can be repaired quickly and cheaply, and the ill effects of introducing waters of varying quantities and densities directly into the Canal will have been eliminated.

Provision for Electric Power

It is possible to secure some electric power from Madden Dam, from the proposed spillways at Gamboa and Las Minas, and at the outlet of the West Diversion (to be described subsequently). Nevertheless, it is considered decidedly inadvisable to depend on such power after once starting to reduce the elevation of the summit level, for such a system cannot function to greatest efficiency as a source of power, requiring high head and full basin capacity, and at the same time, at a moment's notice, provide for storing floodwaters, requiring a low head or a comparatively empty basin. It is, therefore, proposed to provide for all necessary electric power requirements by installing a number of permanent diesel or steam plants, widely separated, but at convenient points or by using sources entirely independent of the Chagres River watershed, thus obtaining the greatest possible protection by this dispersion.

Obviously, it will be necessary to give the highest priority to the creation of the flood control system for the Upper Chagres watershed (above Gamboa and Gatun Barrier dams) in the over-all construction program so that it will be ready to function immediately the present Summit Lake level is reduced. Such procedure will provide accessible dumps for judiciously utilizing excavated material from the adjacent channels and the diversion, from the start of the job. The essential features of the flood control system for the Lower Chagres watershed must be completed by the time the present Summit Lake area is reduced appreciably below +55 mean sea level datum.

It is proposed that the Madden Dam, after it has served its purpose as a flood control factor during the conversion period, will be made inoperative by the removal of its control works. This structure will then remain only as a highway bridge across the Chagres River.

Questions of Vulnerability and Drainage

Those favoring other basic plans for the Panama Canal will immediately emphasize that the control works of this scheme are also vulnerable. This is true, of course, but they are separate from the Canal itself; and, even if the spillways or regulating works were to be destroyed, no damage to the Canal would result, for the drainage system would still function. Furthermore, as the 1906 Commission reported, the water surface in the reservoirs "* * * would be depressed immediately after any flood, low enough to receive any subsequent sudden flow, which might possibly occur."

Under this procedure all critical control works will have been removed by the time sea level channels are placed in commission and the Chagres River will have been reduced to normal original flow. There will also be the additional safety factors of the large Gamboa and Gatun River reservoirs.

Drainage into the cut section, which is relatively small, would be allowed to enter the Canal as at present. In reducing to sea level, the area thus drained would not be enlarged. Therefore, control works, which must be built in any event, would conduct this water into the Canal without scour and with very little, if any, more serious effect on shipping than at present, which is nil.

Control of Lower Chagres—Western Tributaries

Control of floodwaters from tributaries of the Chagres entering from the west side and below Gamboa will be accomplished by an improved counterpart of the method proposed in the report of the Board of Consulting Engineers of 1906. A system of reservoirs would be created (Fig. 3) and the flow of streams would be reversed by ditching, to discharge through low saddles near their middle or headwaters. For example, the Caño Quebrado and the Gigantito would be reversed and diverted into the headwaters of the Trinidad; and the headwaters of the Trinidad and Gigante, in turn, would be diverted into the basin of the Peña Blanca Marsh, through which the West Diversion will have been excavated, and thence into the Old Chagres river channel through the new West Gatun Spillway and so out to sea.

To the eastward of the West Diversion, between it and the channel proper, the Trinidad Barrier Dam would be constructed more or less paralleling those channels extending south from the present Gatun Dam. Barrier dams at the mouths of the Gigante, Gigantito, Caño Quebrado, and Paja would, in turn, divert these streams as described, thus sealing off all fresh water drainage into the Canal from the old tributaries of the Chagres formerly running into it from the south. Material for dam construction would be obtained from excavating the West Diversion Channel, the adjacent canal channels, and the cut, thus furnishing a disposal area for scow as well as for hydraulic dredge operations. Runoff from the small drainage area on the canal side of this continuous barrier dam would not affect shipping.

Additional lockage water to be provided from the Trinidad Reservoir will, no doubt, be necessary during the excavation of the last lift of the canal prism—a critical period of conversion. For this purpose a temporary Trinidad Spillway would be introduced through the Trinidad Barrier Dam system.

The advantages of the reservoir-diversion-barrier dam scheme are multifold: It prevents interference to traffic by invulnerabilities to bombing and by sealing off freshet water from entering navigable channels; it eliminates the objectionable effects of turbulent currents on shipping as a result of fresh water mixing with salt; it provides an easily accessible spoil area for both dry excavation, by rail, or wet excavation, by hydraulic methods or scow dump; it provides the necessary reservoir capacities and insurance against any floods reaching the Canal; it provides more flexibility for flood control than obtains in the lock system and also safeguards against silting fully as well as the lock canal; on the east side it permits the removal of all bridges on the Panama Railroad, reduces grades, and provides strategic rail and road communication across the Isthmus that is the shortest practicable; it is indestructible; and it is entirely in the Canal Zone, under American control.

Drainage South of the Divide

To take care of the relatively small amount of drainage waters south of the Continental Divide, it may be necessary to re-establish the old diversion on the west bank to accommodate the Rio Grande watershed to the westward of the Canal, discharging into the Pacific. On the other hand, model studies might indicate the feasibility of simply installing control works where the Rio Grande meets the Canal, and also where its largest tributary from the west (the Cocoli River) meets the Canal. Similarly, the tributaries eastward of the Canal could be provided for by a short diversion, discharging below the present Miraflores Locks, or through regulating works where these small branches meet the Canal.

FAVORABLE FEATURES

Because of all these provisions, a flood of the proportions of those of 1879 or October, 1923, could be faced without misgivings. This reservoir system is more effective than Gatun Lake because of a division of the total Chagres River watershed, with a potential discharge through regulating works more than twice as great as is now possible at Gatun Spillway, supplemented by the lock chambers at Gatun and Pedro Miguel. The increased capacity of this flood system, therefore, provides safeguards beyond any reasonable expectancy of flood increase. The maximum safety feature during conversion, obviously, must be sacrificed; but this danger is lessened tremendously the sooner it is accomplished immediately after a war, when the element of war damage is most remote.

The system proposed has greater advantages in guarding the Canal against silting than obtain in the present Gatun Lake.

If conversion is not delayed until the indefinite future, the Third Locks can be integrated into the scheme in a greatly simplified form, resulting in an appreciable decrease in cost. It is probable that no new locks would be built at Pedro Miguel and the new locks at Gatun could begin service at the level of the middle chamber. The lowering of the lake level would progress so rapidly that the upper chambers of the new locks would scarcely be required if constructed.

DEEP DREDGING PLAN

An alternative plan for conversion, recently (1944) proposed by Supervising Engineer, E. E. Abbott, deserves serious consideration. In this "Deep Dredging Plan," all channels in the summit level would be dredged to grade, and flood control works and diversions would be completed, while maintaining the present summit level at an elevation of +85 ft. Obviously, these are great advantages in that problems of hydraulics are largely eliminated, except those inherent in the present lock system and the completed project. Flood control systems identical to those for the step-down plan would be provided; but no doubt savings in execution would be realized. The entire control system would be completed by the time the excavation is finished and the construction of Third Locks would not be necessary. The capacity of the present lock type canal will be sufficient to take care of all traffic until conversion is completed, if the work is started before, say, 1948. This is because only three commercial vessels and a few naval vessels are too large for the present locks.

In this plan enormous dredges of great power, capable of excavating to depths of 135 ft, will be necessary. A plant of such power and capacity is considered definitely practicable. Great savings are indicated in the final costs for this plan, if it proves practicable. However, a combination of the step-down and deep dredging plans would possibly meet conditions more satisfactorily than would either one exclusively.

If the primary lift of the step-down plan is executed first, followed by the last two lifts under the deep dredging plan, practically all benefits of both will have been attained. The following advantages are then indicated: (*a*) Conventional type dredges may be employed by increasing the power, length of ladders, spuds, and dipper handles; (*b*) the proposed flood control system, incorporating the high level reservoirs for the construction period, would make available the necessary lockage water lost in reducing the present summit level by one lift or step; (*c*) construction of one Third Lock at Miraflores and one Third Lock at Gatun, of simple design with one lift, equivalent to the two lifts as of present design, would reduce over-all costs materially and simplify conversion; and (*d*) the solution of other hydraulic and construction problems claimed for the deep dredging plan would still obtain.

TIDAL LOCKS

After many years of debate the necessity of tidal locks in a sea level canal at Panama is still an open question. As a precautionary measure, sufficient funds for such tidal locks have invariably been included in estimates. In most discussions the important point has been overlooked that, if a sea level canal is deemed essential, it should be built whether tidal locks are needed or not. Such locks would affect only the navigation conditions in the Canal. Regardless of the results of the investigations to determine if tidal locks are necessary, the problem is entirely subordinated to the essential decision of converting the existing Canal to a sea level canal.

The installation of tidal locks would be made for the sole purpose of improving nagivation conditions in case tidal currents are sufficiently strong to constitute a hazard to navigation. The majority report of the Board of Consulting Engineers stated:[8] "It is probable that in the absence of tidal locks, the tidal currents during extreme spring oscillations would reach five miles per hour." Recent calculations, based on the formula by Brig.-Gen. G. B. Pillsbury,[9] M. ASCE, indicate that the maximum velocities may be as high as 6 miles per hr (5.2 knots).

Hydraulic model tests will be necessary to determine the intensities of currents in the Canal and the measures that may be undertaken to reduce their effect on navigation. A decision can then be made with respect to the tidal locks. If it becomes established that tidal locks are required, they should be designed so that their total cross-sectional area is at least equal to the cross-sectional area of the canal prism. Then, should one of the locks be placed out of operation in such a manner that uncontrolled tidal flow would take place through it, the other locks can be opened and the velocities at the lock section will not be greater than those in the channel. A smaller cross section at the tidal locks would result in increased velocities and a sudden change in the surface profile.

A tidal lock, which might best be placed at Miraflores (Fig. 3), would be a relatively insignificant structure in comparison to the present Panama Canal locks. It would have to be designed for operation under a maximum head of about 12 ft for control by sector gates that can be operated against flowing water. Should the lock be placed out of operation, transit through the Canal might be limited somewhat, but it would not be suspended as is the case if the present locks were destroyed. Because of their greater simplicity, tidal locks could be repaired with greater ease, in less time, and at much less cost.

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[8] "Report of the Board of Consulting Engineers for the Panama Canal, 1906," U. S. Govt. Printing Office, Washington, D. C., 1906, p. 56, paragraph 2.

[9] "Tidal Hydraulics," by G. B. Pillsbury, *Professional Paper No. 34*, Corps of Engineers, U. S. Govt. Printing Office, Washington, D. C., November, 1939.

Increased maneuverability, if desired, can be provided either during original construction or by progressively widening the channel as regular maintenance after the Canal is in operation. The additional width would not appreciably increase the velocities of the tidal currents and would improve conditions for navigation.

Estimates of Cost and Time

A cursory check of the 1931 estimates for converting the present lock canal to sea level, revised to 1946, indicates that a 500-ft sea level canal channel with all appurtenances, as described, would cost approximately $1,310,000,000. If the conversion is made without constructing Third Locks, this estimate may be reduced by $100,000,000. The estimated cost of the deep dredging plan in which no Third Locks are contemplated is $1,121,000,000.

A change of alinement of canal channels is possible at the north end, in the Gatun Lake section. This would shorten the waterway by 10,000 ft, eliminate a bad turn at Bohio, and shorten the time of transit by 10 min—for an additional expenditure of $25,000,000. The reason for the present alinement, in the French as well as in the American sea level plans, was that, by following the course of the Chagres River, less excavation would be entailed. This condition was also true for the lock canal to a lesser degree.

It is also estimated that an additional $600,000,000 would be sufficient to construct a veritable Strait of Panama, 1,000 ft in width throughout its length, from ocean to ocean. Originally the cost of building a sea level canal was considered prohibitive; nowadays its cost can be considered as only a fraction of what the nation will spend for national defense. At the rate of expenditure in World War II, it would require the equivalent of 5¼ days' cost for a 500-ft sea level channel and about 7⅔ days' cost for a 1,000-ft sea level channel. It is not advocated, however, that the 1,000-ft channel be installed at once, but rather that it be considered as a matter of maintenance when traffic demands.

The cost of conversion will vary with the urgency of the completion date. In the 1931 studies, twenty years was considered a reasonable time requirement. Twelve years will probably be the minimum. The longer period is approximately the span between great wars. Since a depression, or at least a critical adjustment period, inevitably occurs after a war, a job of this magnitude would provide employment for an abundance of available technical personnel, as well as skilled and ordinary labor; and, moreover, as a consequence a great saving in the cost of the project would redound to the government.

No attempt is made to describe the step-by-step procedure in the alteration of locks for the lowering of Gatun Lake. This procedure has been covered, however, in *House Document No. 139*;[10] and modifications have been discussed in a report on *House Joint Resolution No. 143*, prepared in 1941, by the Special Engineering Division.

The cost of operation of both lock and sea level canals would probably be about the same for a number of years, or until all control works governing drainage throughout the Canal had been completed and tested under most severe working conditions and until canal banks had received their initial set. After this there should be a decided reduction in cost of maintenance and operation in favor of a sea level canal, since slides will have been eliminated and since scour from freshly excavated channel banks and runoff from adjacent areas will have been stabilized. Large suction dredges, specially made for the purpose, will be able to cover great channel areas in maintenance, and should show a decided downward trend as to cost, by virtue of large output and relatively close hydraulic dumps, following methods already used at the Pacific and Atlantic entrances for the disposal of hydraulic spoil.

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[10] *House Document No. 139*. 72d Cong., 1st Session, pp. 35–38.

Problems of Maintenance and Operation

In connection with the slide menace, it may be stated that, in the process of widening or deepening the cut section, which must be performed to the westward, the banks will have been reduced to a lower level in many places than at present since the accepted slopes now being applied will cut off the tops of the adjacent hills, intersecting the down slope at the back of these hills. The towering volcanic plugs—Gold and Contractor's hills—through which the Canal now passes at the divide will likewise not constitute a menace, for Contractor's Hill will be removed in the process of widening and Gold Hill will have been cut down to impotency.

Engineers at Panama are no longer working in the dark in excavating through heterogeneous materials or conglomerate strata which make up the geological formations through the cut and other sections of the Canal. For twenty years this deepening and sloping process has been going on in the deepest sections of the cut most susceptible to slides, with positive results.

As an item of disadvantage total curvature (or summation of curves), cannot be taken too seriously, nor held as a valid reason for unsatisfactory navigational conditions in a sea level canal, because additional and adequate widths are provided for maneuverability of ships at the turns. The sea level Suez Canal, whose channels are less than half those proposed at Panama, is ample proof of this statement.

Based on the normal velocity of 7½ knots through Suez, a speed of 10 knots through the proposed uniform 500-ft sea level channels at Panama would be permissible. The time scheduled for the present Canal as a minimum for the passage of ships between the north end of Gatun Locks and the south end of Miraflores Locks—30.4 nautical miles (35 statute miles)—is 5½ hours, or a rate of 5.5 knots (6.38 miles per hr). With a permissible speed of 10 knots through a 500-ft channel, this same distance would be negotiated in 3 hours 5 min or a saving of 2 hours and 25 min.

In Favor of Conversion to Sea Level

Engineers now have the background of experience to build whatever is required at Panama, whether it be a lock type canal of optimum capacity, or a conversion of the present Canal into a relatively invulnerable sea level canal, which, it is believed, will provide unlimited capacity and will be continuously useful in war and in peace. There are no problems in either case that cannot be solved; it is purely a question of ultimate value to the country in time of war.

In so far as national defense is concerned most engineers conversant with the problem agree that it would serve the purpose better to convert the Panama Canal to sea level than to build a second lock type canal at Nicaragua. The costs would be approximately the same. Other advantages of Panama over the Nicaraguan project have been pointed out very clearly by Hans Kramer,[11] M. ASCE.

Conversion of the present Canal is not justified from the standpoint of commercial earning power alone; but if it serves its purpose of defense on one critical occasion its cost will have been justified. The relative invulnerability of the sea level type has been demonstrated at Suez; its serviceability, by the fact that it is passing the largest ships efficiently, at low cost and with practically no hazard. The Suez Canal has been blocked by sunken ships during World War II, but only for days, not months or years. Its effectiveness as a dependable world-wide waterway has been demonstrated, even with channels less than half the minimum recommended for Panama.

The sea level type possesses all the advantages from low cost enlargement to unlimited capacity for traffic, in addition to insurance against prolonged interruption of traffic from natural causes or enemy action. The only advantages of the lock canal over the sea level type, at the time of construction, were the lower first cost and earlier completion date. However, this procedure was justified, for it provided a usable canal, which many thought was impossible at the time; and resulted in timely aid to the United States and her allies during World War I, as a great instrument of defense and offense—the main purpose for which it was constructed. The intensity of the critical period has now abated, but the war value of the Panama Canal is continuous. The Canal was not attacked during the two world wars, but it may not be so fortunate another time.

To emphasize the importance of the Canal as an instrument of defense and offense, it is only necessary to state that immediately when war was declared on December 8, 1941, commercial traffic as such stopped or was curtailed because the necessities of war demanded the transiting of vessels of war—troop ships, or supply ships. It is significant that the number of ships, and also the gross tonnage, passing through the Canal was comparable to that of the greatest commercial traffic during peacetime. The monetary value of this service to the United States government, contributory to the success of the war, is inconceivable and incalculable.

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[11] "The Isthmian Canal Situation," by Hans Kramer, *Transactions*, ASCE, Vol. 94, 1930, p. 406.

DISCUSSION

Fred Lavis,[12] M. ASCE.—There seems to the writer to be no valid reason for considering a so-called sea level canal at Panama at this time. The difference in level between the Atlantic and Pacific oceans would always make one lock necessary. The present canal has not nearly reached its capacity in the thirty or so years it has been open. That capacity could be greatly increased by 24-hour operation. It is unlikely that traffic through the canal will increase more rapidly in the immediate future than it has in the past. The author gives few traffic data and no figures, that would seem to justify the proposed expenditure. A second set of locks would increase the capacity and would obviate the present restrictions of width.

The author confines himself to details of design and construction. No economic justification is offered for the large expenditure proposed. Increased costs of maintenance of the low-level channel and its accessories might offset savings in costs of operation of the present locks. It is doubtful whether a sea level canal would be more beneficial than the present canal. A sea level canal would be as vulnerable to modern high power bombs (not including atomic bombs) as is the present canal. The writer has seen the old channel of the Chagres River in Panama. He will readily acknowledge the ability of engineers, contractors, and modern excavating machinery, but he has a very high regard for the difficulties of the Culebra-Gaillard cut—more so, it would appear, than has the author of this paper.

Raphael G. Kazmann,[13] Assoc. M. ASCE.—There has been a tendency in the past few years to justify a number of projects of dubious utility on the grounds of "national security." As applied to engineering projects this tendency does not favor objectivity and reliance on the merits of a project, when ably presented, to motivate its acceptance.

In times of rapid and fundamental technologic change a "national defense" argument, which is the principal reason for a project, may become entirely invalid in a short space of time, thus nullifying the benefits to be gained by a proposed engineering improvement. The plan for a Panama Canal at sea level seems to be a "textbook" illustration of this—without detracting at all from the technical excellence and soundness of Mr. Claybourn's paper, if his basic assumptions are granted. The most important reason advanced by the advocates of a sea level Panama Canal is contained in the quotation,[4] cited by Mr. Claybourn in the last paragraph (under the heading, "Proposal for Conversion to Sea Level"):

> "If the hypotheses that a lock canal cannot be made impregnable and transits without delay of many months are essential to the security of the United States, are accepted, it must be assumed that a sea-level canal program must be the ultimate solution * * *."

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[12] Cons. Engr., New York, N. Y.

[13] Hydrologic Engr., Ranney Method Water Supplies, Inc., Columbus, Ohio.

It should be noted that this statement was made in December, 1944, six months before the first atomic bomb was detonated in the New Mexico desert. It is not clear to the writer just why anyone should use an atomic bomb on the canal when there are more profitable targets available such as industrial centers, reservoirs and lakes, and the internal transportation network. It is not clear, also, how the United States will identify the source of atomic bombs and then put the canal to work as a transportation medium. If such bombs were delivered by rocket no identifiable remains would be available for examination after the explosion. In its sea level form, how would the canal add anything to the security of the United States during an A-bomb war that the nation does not already possess in the canal's present form?

Even assuming that the canal were a valid military target and thus subject to improvement on purely military grounds, would changing it from a lock canal to a sea level canal improve its security under an atomic attack?

There are two main possibilities—(1) that an A-bomb scores a direct hit on the canal somewhere between the present locks, but near one lock; and (2) that the bomb lands near one of the canal's entrances. Under the first possibility, in either form, the canal would be made inoperative for a long time. The lock canal would stand idle until new locks were built or until the old ones were rehabilitated. The sea level canal would be filled with poisonously radioactive sea water and any ships that were involved in the explosion (including all ships in transit at the time), even if they were not sunk, would be dangerously radioactive for many years. Proof? Some of the test ships at Bikini that remain afloat are still so radioactive that the United States Navy doesn't know how to dispose of them. Between radioactive sea water and poisonous fogs evaporating from the canal surface, it is difficult to imagine either type of canal being useful for several years after the explosion. Under the second possibility, assuming that the locks were not destroyed, the canal could remain open, since any radioactive sea water that fell in the canal could be removed by flushing with water from Gatun Lake. The sea level canal, on the other hand, would be as vulnerable to a near miss as it would be to a direct hit.

The primary purpose of this discussion is not to show that military applications of atomic energy may make the entire "locks versus sea level" discussion meaningless. It is to point out that all engineers, whether or not they are planning projects involving "national security" would do well to add the Smyth report,[14] "One World or None,"[15] and "The Atomic Story"[16] to their shelf of reference volumes, after first studying them. Atomic energy and its possible applications should be considered in detail together with other technical and economic aspects of planned engineering work. Admittedly, this is extremely difficult, both because of the newness of this source of energy and specialized materiel, and of the secrecy connected with the development of atomic energy which seals the lips of those most familiar with its potentialities. On the other hand, it is difficult to understand how future expenditures in the field of civil engineering (for example, hydroelectric power and water supply) can be justified without a mature and detailed re-examination of accepted economic and social assumptions in the light of developments in the atomic field.

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[14] "Atomic Energy for Military Purposes," by H. D. Smyth, Princeton Univ. Press, Princeton, N. J., 1945.

[15] "One World or None," a symposium edited by D. Masters and K. Way, McGraw-Hill Book Co., Inc., New York, N. Y., 1945.

[16] "The Atomic Story," by John W. Campbell, Henry Holt and Co., New York, N. Y., 1947.

HANS KRAMER,[17] M. ASCE.—Although Mr Claybourn's authoritative paper and this discussion do not deal exclusively with current construction operations of immediate technical interest, they do presage construction activities of a nature and magnitude to challenge the capabilities of the construction industry. These potential and not far-distant activities deserve the most sober consideration of the Society from a professional viewpoint, and of the public at large as a matter of national concern.

Since the preparation of Mr. Claybourn's paper, an important bill, directly connected with his subject, has been enacted by the Seventy-Ninth Congress and approved on December 28, 1945, as Public Law No. 280. The significance of this bill warrants quotation of its main provisions:

> "* * * the Governor of the Panama Canal, under the supervision of the Secretary of War, is hereby authorized and directed to make a comprehensive review and study, with approximate estimates of costs, of the means for increasing the capacity and security of the Panama Canal to meet future needs of interoceanic commerce and national defense, including restudy of the construction of additional facilities for the Panama Canal authorized by the Act approved August 11, 1939 (53 Stat. 1409). He shall also make such study without drafting plans or sketches as he may deem desirable to permit him to determine whether a canal or canals at other locations, including consideration of any new means of transporting ships across land, may be more useful to meet the future needs of interoceanic commerce or national defense than can the present canal with improvements. He shall report thereon to the Congress, through the Secretary of War and the President, not later than December 31, 1947."

In orienting this discussion to the over-all review contemplated by Public Law No. 280 it seems appropriate, at the outset, to attempt to dispel at least one popular illusion created by recent news items in connection with the inability of the latest supercarriers to cross the Panama Canal because of insufficient width of the existing locks. Contrary to assertions or intimations, this simple well-known fact does not indicate that someone has been "asleep at the switch," or that there is lack of teamwork between the United States Navy and the Canal Zone authorities. The dimensional limitations of the Panama Canal are, and have been, a matter of common knowledge, but their existence has been no deterrent to the construction and operation of the *Queen Mary* and a few similar superliners. The prewar carriers, the *Saratoga* and *Lexington*, with their 105-ft beams, were always a tight fit in the 110-ft locks, but their transits through the canal were always accomplished successfully. The federal government was well on the way toward enlarging those limitations by a new and larger third set of locks (which were authorized in 1939), when construction was suspended in May, 1942. That suspension, notwithstanding the desirability of having safer and larger facilities for war use, was compelled by serious shortages in shipping, manpower, and critical materials; it was due to the sound judgment of the Navy and Army, and of their commander in chief that these vital resources, then of greater importance to the war effort, were applied to other fields.

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[17] Brig. Gen., U. S. Army (*Retired*) Cons. Engr., San Francisco, Calif.

The Navy's construction in the subsequent and more abundant years of supercarriers too large for the existing locks was programmed with open eyes and in full anticipation of their usefulness in the later phases of the war, despite their inability to transit the Panama Canal. It is obvious, however, that the Navy's adoption of these larger dimensions for its newer vessels dictates canal facilities of corresponding size as soon as it is reasonably practicable to provide them. Whatever form those new facilities may take, the bulk of the $75,000,000 (of the $277,000,000 authorized), which was expended mainly on excavation and appurtenant work prior to suspension of the Third Locks Project, will undoubtedly prove to have been well spent; this work will be directly useful in the ultimate development of the project.

The activities and traffic of the Panama Canal were necessarily censored during World War II, but the uninterrupted functioning of the canal was of inestimable significance in the global war operations of the allied nations. If, and when, the full story is revealed, the United States will be prouder than ever of the tremendous benefits derived from the Panama Canal. Although the traffic capacity of the canal usually has been rated on a per annum basis, peak traffic conditions over shorter periods of weeks (or even days), such as occurred during the redeployment period in mid-1945, will undoubtedly have to be considered in gaging future traffic and facilities.

Thirty years of experience in the operation and maintenance of the present canal are reflected in Mr. Claybourn's paper, together with the advances in engineering technique and in the development of engineering tools, such as geology and soil mechanics over the same period; these advancements have extended engineering knowledge to a stage where engineers are much better able to design and construct works, such as cuts, embankments, and foundations with greater confidence in their ultimate performance. The preceding statement does not imply that all the answers to the engineering problems are simple and readily available. The brief discussion (under the heading, "Deep Dredging Plan") for converting the Panama Canal into a sea level waterway in virtually a single operation with the aid of "* * * enormous dredges of great power, capable of excavating to depths of 135 ft * * *," exemplifies the problems to be solved and the unprecedented methods to be considered. It does imply, however, that the profession is blessed with greater enlightenment, and should, therefore, be cursed by less darkness in the solution of the next series of engineering problems; and it also implies that apprehensions of serious slides in the Panama Canal caused by improper sloping and draining in critical zones need not haunt future efforts.

Opposed to the advances of the past generation in the construction phases of engineering must be ranged the even greater advances in the destructive phases of the physical and military sciences. The destructive potentialities of the atomic bomb stagger the imagination and threaten man's very existence. However, even conventional bombing with long-range aircraft or in the form of remote-controlled missiles has destructive power which dictates a re-examination of the protective installations engineered in 1939. The resultant problem and the concern for the vulnerability of the Panama Canal will challenge all available technical skill and judgment, as never before. The development of a logical, feasible solution will necessarily absorb the attention of the ablest engineers and scientists, and will require the collaboration of the best minds of the Army, the Navy, the atomic bomb group, and other governmental agencies that have a stake in the future of the Isthmian canal.

In the comprehensive restudy to determine the next phase in the development of the Panama Canal, which is envisaged in Public Law No. 280 appropriate reconsideration must be given to the Nicaragua route and to other alternative sites. Although these alternatives have been disposed of in previous studies and decisions, they obviously deserve review, not only because of changes in physical factors affecting construction, but also in the light of the new geopolitical situation. No major field surveys are contemplated or necessary with respect to these alternatives, but an adequate reanalysis of factual data which are essential for purposes of comprehensive review is definitely in order.

In the restudy under Public Law No. 280 some popular fallacies must be dispelled again—for example, the misconception that the Nicaragua route is feasible for a sea level canal. Neither the Nicaragua route, nor any other alternative route has ever been considered practicable for anything but a lock canal; the Nicaragua plan involves a summit lake 110 ft above sea level against the 85-ft summit level of Gatun Lake at Panama. Again, it remains to be proved that the present Panama route was, and is, the only one engineeringly and economically feasible for construction of a sea level canal. Likewise, the oft-quoted adage, "It is unwise to carry all your eggs in one basket," which serves as a popular argument for the construction of another Isthmian canal at a location other than Panama, will have to be re-examined. However, the 1927 analysis of the "eggs-in-one-basket" argument together with the conclusion (stated previously by the writer[11]) that "The Isthmian canal situation is not a question of carrying eggs, but of protecting baskets," is still pertinent and tenable today.

In conclusion the writer wishes to compliment Mr. Claybourn (and to congratulate the Society on its choice of subject and author) on his timely, forceful, and competent presentation of the problems involved in "Streamlining the Panama Canal for Maximum Safety and Unlimited Capacity" (see "Synopsis"). The writer has not attempted here to analyze the author's reasoning, nor to debate his conclusions, because he is confident that Mr. Claybourn's work will prove a valuable contribution and will receive thorough attention in the over-all review of the problem to be undertaken pursuant to Public Law No. 280. It needs to be emphasized, however, that, whereas a sea level canal at Panama has been heretofore a second choice (and rightly so), changed conditions have placed the burden of proof on those who would not advocate conversion to sea level. The restudy of the Isthmian canal problem must necessarily be governed by considerations of national defense which, in turn, are engendered by the specter of war. The successful solution of that problem should constitute another noble achievement for the engineering profession, and the nation, in their dedication to the constructive works of peace.

James H. Stratton,[18] M. ASCE.—The author presents a straightforward, practicable plan for the conversion of the present Panama lock canal to a sea level canal. Mr. Claybourn's proposal for the conversion to sea level is based primarily on the needs of national security, but he points out that the results of the conversion will be to satisfy the growing needs of commerce and, at the same time, provide a waterway both safe and satisfactory from a navigation viewpoint. Excepting for a few features, such as the method of excavation and lowering of the summit lakes of the Canal, some details of flood control, and the method of providing regulation of the Pacific tides, the plan described by Mr. Claybourn does not differ materially from that recommended by the Governor of The Panama Canal, J. C. Mehaffey, in his report submitted to Congress in compliance with Public Law No. 280, Seventy-ninth Congress.[19]

Mr. Claybourn describes both the multiple stage (step down) and single stage (deep dredging) plans for lowering the canal to sea level, and concludes that a combination of these plans would probably be more satisfactory inasmuch as this would permit using conventional-type dredges with minor modifications, instead of requiring special dredges capable of working to depths in excess of 135 ft. More recent extensive investigations of the single stage lowering plan (in which Mr. Claybourn has acted as dredging consultant), under Public Law No. 280, have revealed that single stage lowering of the Canal, requiring the use of special dredges capable of excavating to depths of 145 ft, is entirely practicable. Single stage lowering would result in a savings of approximately $130,000,000.

The plan proposed by the author for the control of floods differs from that recommended in the report under Public Law No. 280, principally in features of design which are the outgrowth of extensive studies of recent date. The control plan for the Chagres River recommended in Governor Mehaffey's report contemplates that the Madden Dam and reservoir will operate in tandem with a dam at Gamboa to effect the control and diversion of floods, whereas Mr. Claybourn proposes that Madden Dam be made inoperative after the completion of construction. Nevertheless, it appears that with the removal of the control works, as he proposes, Madden Dam will serve to retard floods. The plan in the report, it is believed, will give greater flexibility of control than that proposed by the author. Although outlets and a spillway leading from Gamboa Dam are proposed in both plans, they would be used only in emergencies in the plan recommended to Congress. Thus, flows from the Chagres River into the sea level canal itself would be an extreme rarity, rather than one of occasional requirement which might result in the plan offered by Mr. Claybourn.

The author has stated the case for tidal regulation extremely well, pointing out that essential decision is not one of needing such regulation, but one of providing a sea level canal to meet security needs. The tidal regulating plan recommended in the studies under Public Law No. 280 provides for both a tidal lock and a navigable pass, the latter of full channel width. The proposed navigable pass meets the criterion stated by Mr. Claybourn—that, if tidal locks are provided, they should be designed so that their cross-sectional area is at least equal to the cross-sectional area of the canal prism.

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[18] Col., U. S. Army; Supervising Engr., Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

[19] "The Future and the Panama Canal," by James H. Stratton, in "Panama Canal—The Sea-Level Project: A Symposium," *Proceedings*, ASCE, April, 1948, p. 444.

Extensive studies of waterways throughout the world and model investigations at the Unites States Navy David Taylor Model Basin in Carderock, Md., made in conjunction with the investigations under Public Law No. 280, led to the adoption of a somewhat larger channel and an improved alinement compared with that suggested by the author; and this, in part, accounts for the higher cost of the plan for the conversion of the Panama lock canal to a canal at sea level recommended by the governor over that proposed by the author.

J. G. Claybourn,[20] M. ASCE.—Since the presentation of this paper at the Annual Meeting of the Society in New York, N. Y., on January 16, 1946, a

> "* * * comprehensive review and study, with approximate estimates of cost, of the means of increasing the capacity and security of the Panama Canal to meet the future needs of interoceanic commerce and national defense * * *"

has been concluded in accordance with the provisions stipulated under Public Law No. 280.

These investigations and studies were conducted under the immediate direction of Colonel Stratton, supervising engineer of the Panama Canal and under the over-all supervision and review of a Board of Consulting Engineers—all appointed by the Governor of The Panama Canal (J. C. Mehaffey) especially for the purpose. Only one member of this group had ever been associated with the canal organization previously—namely, General Kramer, who was supervising engineer of the Panama Canal when the Third Locks construction was terminated in May, 1942, and a lock type advocate at that time.

The studies made by this group were the most comprehensive ever made of the Panama Canal and these data, together with the conclusions of the Board of Consultants (which were unanimous), were reviewed, approved by the Governor, and included in his report to the President of the United States, Harry S. Truman, through the Secretary of Defense, James V. Forrestal, in October, 1947.

The unanimous conclusions reached in brief were:

(*a*) The Canal as it exists today or any lock type canal constructed to have maximum security feasible cannot be considered as meeting the needs of national defense;

(*b*) Only a sea level canal will satisfy conditions prescribed under Public Law No. 280; and

(*c*) Such a canal can be built most economically at the site of the present waterway.

A thorough review of the Governor's report will furnish unbiased, unequivocal, and incontestable answers to previous controversial issues. These conclusions are based on facts and on calculations that may be reviewed and checked; and then are backed by a wealth of experience and have been confirmed by model tests in all essential particulars.

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[20] Gamboa, Canal Zone; formerly Supt., Dredging Div., The Panama Canal, Gamboa, Canal Zone.

Since the United States first became interested in the Panama Canal, the value of that project as an instrument of defense has always been held of primary importance, transcending commercial requirements or necessities, by the United States Government. The Canal was built primarily as an instrument of defense; it still is the greatest single instrument of defense and it must remain as such, as long as sea power and water transportation are vital to national safety.

The concept of a two-ocean, three-ocean, four-ocean, or five-ocean navy, each with its auxiliary supply, hospital, transport, and other ships to support and supply these several units independently and successfully, is mere twaddle even to the layman, who witnessed the shuttling of war vessels, and their supporting supply ships, through the Canal, to the side most needed, throughout World War II. The axiom of Gen. Nathan B. Forrest—"The side wins that gets thar fustest with the mostest"—was knowingly or unconsciously followed at all times. This was possible because, by good fortune, the Canal was not attacked and was available. Had the Canal suffered an attack, such as at Pearl Harbor (Hawaii), results may well have been more disastrous than the loss of the fleet and, with proper timing, both could have been easily destroyed. By virtue of the Panama Canal, no one but the "Top Brass" eventually knew where or how many fleets the United States had since the number and the place were changed overnight.

It will always be necessary to have a canal available to transfer warships and supply ships from one side of the United States to the other, in the shortest possible time, during any war in which the nation may be engaged.

It is apparent that, if the item of defense is omitted from the requirements of Public Law No. 280, the problem would be simple because, as Colonel Stratton has stated:[19]

> "The present lock canal could be improved at a cost of $129,983,000 to meet the needs of commerce for the remainder of the twentieth century. A lock canal designed to meet the future needs of commerce and constructed to have the maximum security feasible in this type of canal would require new locks and strengthened summit lake impounding dams. It would cost $2,307,686,000 and would still be deficient in resistance to modern weapons."

A sea level canal—an enduring and indestructible item of defense—as estimated in the Governor's report, would cost $2,483,000,000.[19] This cost is but $185,314,000 more than that of the lock type canal "constructed to have the maximum security feasible" which is still vulnerable to modern weapons. Besides, the vital works of a lock type canal—dams, spillways, locks, and their necessary operating machinery—are most vulnerable and easy to sabotage. Conversion will require at least ten years and probably more. The project will never be ready if a start is not made.

The foregoing discussion will answer Mr. Lavis' query as to the "* * * valid reason for considering a so-called sea level canal at Panama at this time."

As to economic justification: From a purely commercial angle the Panama Canal has not yet justified expenditures for its construction except for the boom years preceding the crash of 1929 and for one or two years since, when full interest on the investment and some so-called amortization, apart from the actual cost, was paid. Some intangible benefits would be difficult to evaluate in dollars. There are also good prospects that commercial activities will become profitable in the future, should world conditions stabilize, thus permitting South American trade development. However, in wartime the cost of the Canal has "paid off."

When the United States declared war on December 8, 1941, commercial traffic, as such, stopped or was curtailed in response to the necessities of war. It is significant that the number of ships and the gross tonnage that passed through the Canal during World War II are very comparable to those during the greatest peacetime commercial traffic years. The relative revenues may be computed and would be a persuasive argument, but the actual monetary value of transferring vessels of war, troopships, Army and Navy supply ships, and other vessels to supply materials necessary to the war effort of the United States and its allies is inconceivable and incalculable.

At the rate per day that World War II was costing the United States, the de luxe sea level canal proposed by the Governor in his report of October, 1947, would require the approximate equivalent of ten days of such expenditure. The cost of future wars will probably be greater. It is inconceivable to anyone familiar with the tremendous transportation facilities furnished the war effort by the Panama Canal, that this contribution did not shorten both World Wars I and II materially; and, obviously, it also saved the lives of thousands.

The war value of the Panama Canal is continuous. However, only by conversion to sea level, can the Panama Canal—this key to national and hemispheric defense—be made enduring and indestructible and, in the words of the Hon. Joseph J. Mansfield, "It is one defense item that cannot become obsolete and will be a lasting benefit in war or peace." If the Canal serves its purpose of defense on one critical occasion, its cost will have been justified.

Mr. Lavis states that:

> "The writer has seen the old channel of the Chagres River in Panama. He will readily acknowledge the ability of engineers, contractors, and modern excavating machinery, but he has a very high regard for the difficulties of the Culebra-Gaillard cut—more so, it would appear, than has the author of this paper."

The writer's "regard for the difficulties of the Culebra-Gaillard cut" is based on nearly thirty-eight years of experience on Panama Canal excavation, during and since construction, twenty-six years of which were as head of the Dredging Division. This service involved immediate supervision as well as general supervision of excavation over every section of the Canal from one end to the other; and one of his duties was the removal of slides and their prevention. Under the writer's supervision the Dredging Division accomplished both tasks. There have been no serious slides on the Canal since 1932, even at Culebra, the the area most susceptible to slides. Potential slide areas still exist and slides may still occur, because remedial treatment advocated and used by the writer, has not been applied in many sections, awaiting the outcome of Congressional action on future canal development.

Charges up to 15 tons of 60% sodium nitrate dynamite have been exploded at depths of 75 ft in areas most susceptible to slides without starting, or precipitating, a slide and on slopes that had not attained the calculated angle of repose. It was noted from pictures taken at the focal points of the atom bomb explosion at Hiroshima, Japan, which exploded above ground, that, although all structures in the vicinity were disintegrated, the conditions and outlines of the river channels through the city remained unchanged. It is logical to assume that even an atom bomb, exploding on the surface of an ample predetermined slope (a thirty-year slope shaped in nature's laboratory—the banks themselves), would not cause an appreciable slide; nor would an atom penetrating bomb be sufficient to close a channel of 500 ft or more. The soils mechanics and atom bomb experts apparently are of the same opinion.

Mr. Kazmann comments on the writer's citation of a hypothetical statement by Mr. Abbott[4] as of December, 1944, "* * * six months before the first atomic bomb was detonated in the New Mexico desert." That hypothetical statement is now considered a statement of fact by the Board of Consultants in its study under Public Law No. 280. It has also received the approbation of the Governor in his report to the President as well as that of at least two former commanders-in-chief of the Caribbean Command (Generals George H. Brett and Willis D. Crittenberger) and two former governors (Generals Julian L. Schley and Glen E. Edgerton, M. ASCE) of the Canal Zone who are fully acquainted with conditions on the Canal.

From the foregoing discussion it should be gathered that the Panama Canal is a worthwhile target. It is admitted that if, the United States were destroyed, there would be no use for any canal, lock or sea level. However, present conditions should not be permitted to continue merely on the premise advanced that the "proposed engineering improvement" might be "nullified" by some probable future "fundamental technologic change."

It is the opinion of the Board of Consultants that the sea level canal, proposed by the Governor, can operate with very little danger of closure, except for short periods, under the worst conditions of war now known and will be tremendously more safe from destruction from any future and unknown perils that may arise, than the present lock canal.

At Bikini, radioactive vessels that were not sunk in the lagoon were towed to sea and destroyed. The same procedure would be followed at the Panama Canal. Contaminated water would be flushed out by tide wash; a sunken hulk could be dug around or blown up and removed by derricks.

Locks, dams, and spillways would be the targets of any bombing attack on the lock canal at Panama, conventional or atomic. A hit or a near miss with the atomic bomb would demolish or breach these structures; the lake would be emptied and there would be no canal. Should the vital canal structures remain intact and flushing operations be undertaken from Gatun Lake to clear the summit level from radioactive waters, the lake would probably be drained in the process and there would be no canal, because very little current would be produced, with the sluice gates of the spillway and lock culvert valves wide open, at a point far enough away from an atomic bomb explosion to escape damage to the vital lock canal structures.

With the sea level canal, radioactive waters would be dissipated by tidal flow four times in 24 hours, if the tidal gates were destroyed, and by the same method, but more scientifically, if they were not destroyed.

When fogs occur in the cut section of the Panama Canal, they develop from about 11:00 p.m. to 11:30 p.m., and are usually completely dissipated from 7:00 a.m. to 7:30 a.m. If there is an appreciable wind, fogs will not form. If rain occurs after the fog has formed, the fog is cleared or precipitated by the rain. If the sun hits the fog before the wind currents begin in the morning, the fog is precipitated. If wind currents form early (which is often the case) and from the north, the fog is blown out of the cut. All fog particles passing the divide, between Gold and Contractor's hills are dissipated within approximately 2,500 ft south of that divide. Any ill effects from fog contamination would be about the same for a lock canal or a sea level canal, but would diminish in direct proportion to the lessening of ill effects of the contaminated water and would be in favor of the sea level canal.

The writer was gratified that the plan finally approved by the Governor in October, 1947, and arrived at after intensive study by the supervising engineer and the Board of Consulting Engineers, was basically the same as that covered by the paper.

The same flood control scheme, which operates independently of canal channels, was adopted with minor changes—that is, Madden Dam was retained in operation as an additional flood control item; a tunnel was provided from the Gatun Reservoir through the high ridge to the East Diversion cut around the Gamboa Hills, and a baffle system was introduced in the East Diversion.

It is suggested that a review be made of the writer's suggestion to utilize the present Gamboa Reach for this section of the East Diversion, now that this part of the present Canal would be abandoned, thus saving considerable excavation and cost.

Instead of following the route of the lower lying Chagres River Valley and that of the tributary of the Obispo through the cut section, as in the writer's plan, a new alinement of the Canal from immediately north of Pedro Miguel, almost direct to Gatun was adopted. Reaches, each nearly 5 miles in length, a ruling width of 600 ft, and turns at intersections not exceeding 26° are provided. Wetted surfaces are planned with ample slopes giving usable maneuvering room for the full width of channel for vessels and insuring safety to transiting ships.

Costs submitted for the Governor's plan substantiate the data published by the writer when evaluated on yardage involved. The items of difference are:

Description	Governor's plan	Claybourn plan
Width of channel, in feet	600	500
Depth, in feet (Panama level datum)	−60	−50
Curves, in degrees (maximum)	26	....
Cutoffs	Complete realinement	....

In the writer's plan the total curvature (not stated) would depend on model tests. The plan for cutoffs was to follow the present route except for a cutoff south of Mamai Curve and Gamboa Reach, another cutoff at Bohio, and one from Peña Blanca Reach to the site of the new Gatun Lock.

To summarize:

(*a*) The sea level canal proposed by the Governor in his report of October, 1948, fulfils, in all details, the requirements of Public Law No. 280 and only such a canal would be secure against a determined and resourceful enemy attack with modern weapons.

(*b*) The Canal as it exists today cannot be considered to meet the future needs of commerce or national defense.

(*c*) A sea level canal can be built most economically at the site of the present waterway.

(*d*) The proposed sea level canal does not depend on an artificial lake for its water supply, which, according to General Edgerton, offers the "greatest danger of prolonged closure of the present canal to enemy action," and could still operate even if the tidal locks were destroyed.

(*e*) The sea level canal will provide for all future commercial needs.

(*f*) With tide gates open on the 600-ft canal, the maximum current that will be experienced is 4.5 knots, and ships navigate the 500-ft Cape Cod Canal, in Massachusetts safely with currents as rapid as 6 knots.

(*g*) The invulnerability of a sea level canal has been demonstrated at Suez (Egypt). Its serviceability has been proved by the fact that such a canal passes the largest ships efficiently, at low cost, with practically no hazard and through channels only 270 ft wide, less than half the width recommended at Panama. The Suez Canal was blocked by sunken ships during World War II, but only for days, not for months or years.

(*h*) An average of 2.50 hours of transit time would be saved by the sea level canal as compared to the best lock type that could be built.

(*i*) The sea level canal will provide the United States with an enduring and indestructible item of defense.

(*j*) A canal must always be available to transfer ships of war, supply ships, transports, etc., from one side of the United States to the other in any war in which the nation may be engaged, and only a sea level canal will satisfy these requirements.

(*k*) The sea level canal is entirely feasible and practicable.

(*l*) The only advantages the lock canal had over the sea level type, at the time of construction, were lower first cost and earlier completion date. It served its purpose, however, and justified its construction in World War I. The canal is now obsolete and must be converted to sea level to continue to serve the purpose for which it was constructed.

(*m*) The lock canal is not only vulnerable to air attack, but the operating machinery on which it depends is easily sabotaged.

(*n*) The cost of a sea level canal is but a fraction of what the nation will spend on items of defense. The canal is one item that will not become obsolete, and it will furnish unlimited capacity and maximum safety in peace or war for ships of the United States.

(*o*) It is far better and more economical to build one invulnerable sea level canal than to build another lock canal elsewhere. In the latter case, neither one nor the other could be defended.

(*p*) The war value of the Panama Canal is continuous but can be realized only if converted to sea level.

(*q*) A curtailment of any future war by ten days, by virtue of having a dependable canal, is justification for the conversion to sea level. The United States will have secured unlimited capacity for peace and for all time, in any event, which would not be achieved by the optimum lock type canal.

(*r*) It is the opinion of all high military officials that the Canal is a vital part of the nation's defenses. It will take at least ten years to make the conversion. The sooner the work is begun, the sooner it will be finished.

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THE FUTURE AND THE PANAMA CANAL

By James H. Stratton,[1] M. ASCE

Synopsis

Investigations under Public Law No. 280 (Seventy-ninth Congress, First Session, approved December 28, 1945) disclose that only an Isthmian sea-level canal will meet the future needs of interoceanic commerce and national defense. The broad phases of the studies leading to this conclusion are described in this paper. Various engineering features of the studies of the recommended sea-level canal are described more fully in the other papers of this Symposium.

The Canal Zone offers the most economic site for either a lock or a sea-level Isthmian canal. The present lock canal could be improved at a cost of $129,983,000 to meet the needs of commerce for the remainder of the twentieth century. A lock canal designed to meet the future needs of commerce and constructed to have the maximum security feasible in this type of canal would require new locks and strengthened summit lake impounding dams. It would cost $2,307,686,000 and would still be deficient in resistance to modern weapons.

A sea-level canal at Panama constructed by the conversion of the existing lock canal could not be destroyed by enemy attack or sabotage. Only the atomic bomb could cause significant interruption in service, and then for not more than a few weeks. Navigation would be practicable in the sea-level canal even though tidal currents were not regulated. Nevertheless tidal regulation would be provided for greater safety to shipping. For this purpose a tidal lock and a navigable pass would be provided. The opening of the gates of the navigable pass at selected Pacific tidal stages would permit the routine operation of the canal as an open waterway. In the event of damage to the tidal-regulating structures, the gates of the navigable pass could be removed quickly and the canal could be operated thereafter as an open waterway. Serious slides could be prevented by appropriate flattening of the slopes in cut. The entire excavation to effect the conversion to a sea-level canal would be completed in advance of conversion; the major part would be in the dry. Part of the wet excavation would require the use of special dredges working to a depth of 145 ft. Construction interference with canal traffic would be negligible except for a period of a few days when the summit lakes are lowered. The Panama sea-level canal would be constructed in 10 years and would cost $2,483,000,000.

Introduction

The long history of the search for a passage to the east, and of the early efforts to promote a canal across the Isthmus of the Americas, provide a background of romance and color to the story of actual construction which opened in 1882 with the initiation of work on a sea-level canal at Panama by the first French Canal Company. After years of effort and mounting adversities when funds had run low, the French changed to the less costly high-level lock canal. Because the French believed that a sea-level canal ultimately would be necessary, plans for the locks were drawn to facilitate the transformation. The chapter of the story dealing with the French construction ended with financial failure and the transfer of all rights and interest in the Panama Canal to the United States.

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The opening chapter of the tale of construction of the Panama Canal by the United States is one of controversy over the type of canal to be built. The records disclose that the selection of the lock canal was based on the advantages it offered in the earlier completion at a lesser cost.[2] The issues entering into the selection have a particular interest now, in the face of the challenge presented by the "blockbuster" bomb, the guided missile, and the atomic bomb. When the present canal was planned the critical forms of attack were envisioned as naval gunfire directed against the locks and enemy forces moving overland to capture the canal intact.

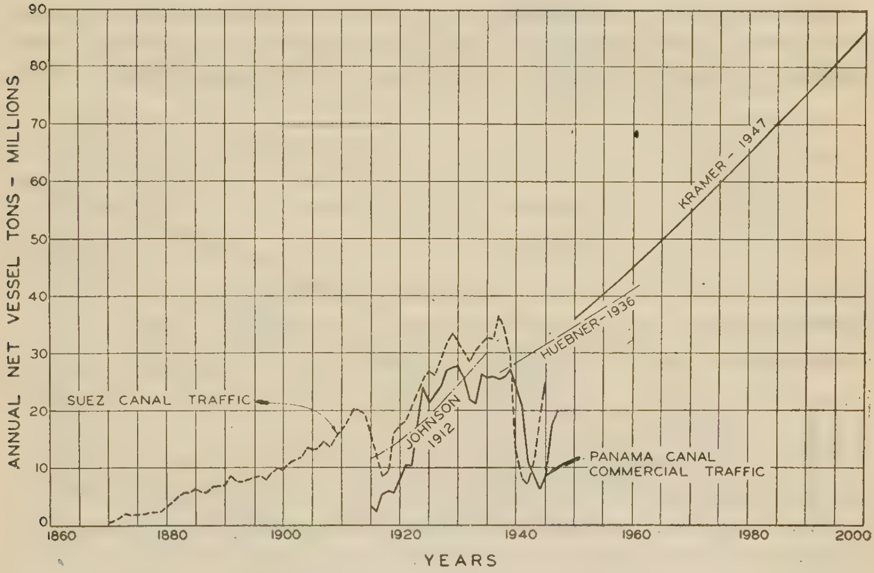

Fig. 1.—Panama Canal Traffic Predictions

The Seventy-ninth Congress expressed the temper of present concern for the security of the canal by passing Public Law No. 280 which was approved by President Harry S. Truman on December 28, 1945, and which provides:

> "Be it enacted by the Senate and the House of Representatives of the United States of America in Congress assembled, That the Governor of the Panama Canal, under the supervision of the Secretary of War, is hereby

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[2] "Report of the Board of Consulting Engineers for the Panama Canal, 1906," U. S. Govt. Printing Office, Washington, D. C., 1906.

authorized and directed to make a comprehensive review and study, with approximate estimates of costs, of the means for increasing the capacity and security of the Panama Canal to meet future needs of interoceanic commerce and national defense, including restudy of the construction of additional facilities for the Panama Canal authorized by the Act approved August 11, 1939 (53 Stat. 1409). He shall also make such study without drafting plans or sketches as he may deem desirable to permit him to determine whether a canal or canals at other locations, including consideration of any new means of transporting ships across land, may be more useful to meet the future needs of interoceanic commerce or national defense than can the present canal with improvements. He shall report thereon to the Congress, through the Secretary of War and the President, not later than December 31, 1947."

Traffic History of the Canal

The volume of traffic through the Panama Canal has steadily increased since it was opened, despite the setbacks of wars and world depressions. With uninterrupted world prosperity a further growth of commercial traffic may be expected generally as predicted by Roland L. Kramer of the University of Pennsylvania at Philadelphia in recent studies which were made for use in the investigations under Public Law No. 280. Fig. 1 records past traffic and depicts the estimated and future commercial traffic, as projected by Professor Kramer.

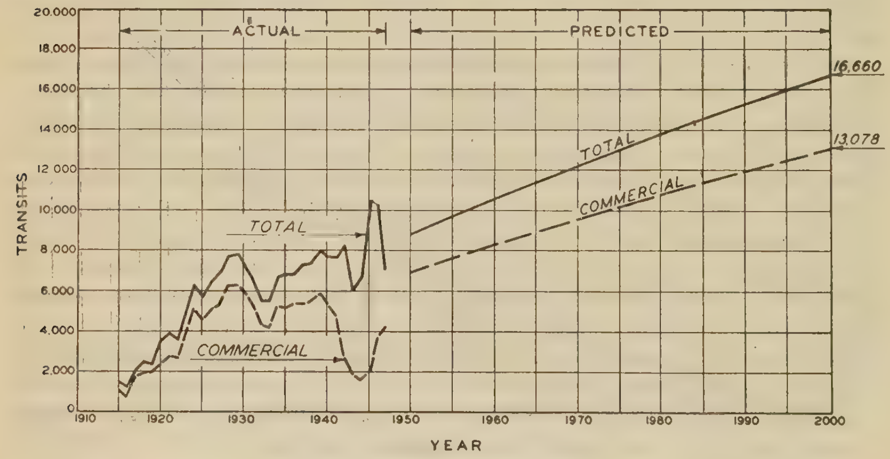

Fig. 2.—Panama Canal Transits

Before the war the "tolls-free" traffic, consisting largely of vessels owned and operated by the United States, accounted for about 15% of the entire traffic. In 1945, the peak war traffic year, this rose to 78%. Predictions of future canal transits are presented in Fig. 2. Since the canal was opened in 1914 it has transited 198,000 ships of which 142,000 were tolls-paying commercial craft. The percentage distribution of transits and tonnage is illustrated in Fig. 3.

War traffic through the canal in the period from 1941 to 1945 totaled nearly 17,000 transits. Had there been no canal during this period, it is estimated that the increased ship-operating costs and the cost of additional shipping and escort craft that would have been required to preserve the schedules made possible by the canal would have exceeded $1,500,000,000.

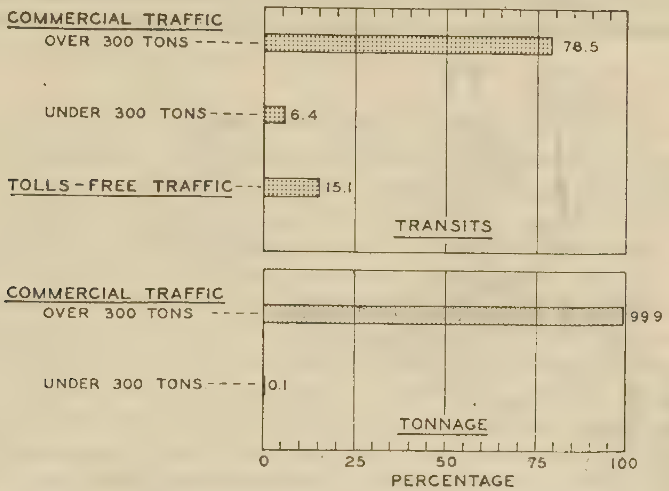

FIG. 3.—PREDICTED CANAL TRAFFIC

From its contribution in peace and war, it is clear that the future of the canal is the future of ships, provided the canal can be made safe against destruction. As Norman Padelford has stated:

> "* * * the Panama Canal is more than a commercial artery. * * * It is a focal point of national defense, a base of operations for the protection of the Hemisphere, an instrument of national influence."
>
> "* * * it seems safe to say that so long as any material part of the commodities of trade are carried in ships, and so long as sea power persists as a determining factor in the relationships of nations, so long certainly will use of the Panama Canal be sought by merchantmen and vessels of war. And so long will the Canal as a waterway remain essential to the United States. * * * No Panama Canal would exist today to pass great ships from ocean to ocean had it not been for vision which saw beyond the limitations of existent realities. The hope for a more ordered future * * * lies in similarly transforming present difficulties through enlightened leadership and continued vision." [3]

THE PANAMA CANAL AS IT IS TODAY

The canal as a waterway has changed little since it was placed in service, except for the addition of the Madden Dam on the Chagres River in 1935 for the benefit of water supply for lockages, flood control, and power generation.

A plan and profile of the existing canal are shown in Figs. 4 and 5. It is estimated that by about 1960 the capacity of the canal will be inadequate to accommodate traffic without inflicting undesirable delays on peak traffic days.

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[3] "The Panama Canal in Peace and War," by Norman J. Padelford, The Macmillan Co., New York, N. Y., 1942.

The delays thereafter will become more serious with the further growth of traffic unless additional capacity is provided.[4,5]

The restricting effect of the small locks (width 110 ft, length 1,000 ft) of the present canal on the design of Navy ships became intolerable with the approach of war and, in 1939, Congress directed the construction of a third set of locks, 140 ft wide and 1,200 ft long, which it was then thought would be adequate for all future needs. The new locks were designed to resist attack by the largest aerial bomb then known to exist. Construction was suspended early in 1942, when it became apparent that the new locks could not be completed before the end of the war because of conflicting demands for men and materials. At that time, excavation for the Gatun and Miraflores Third Locks had been substantially completed, but excavation for the third lock at Pedro Miguel and work on actual lock construction had not commenced. Of the authorized expenditure of $277,000,000, approximately $75,250,000 was spent.

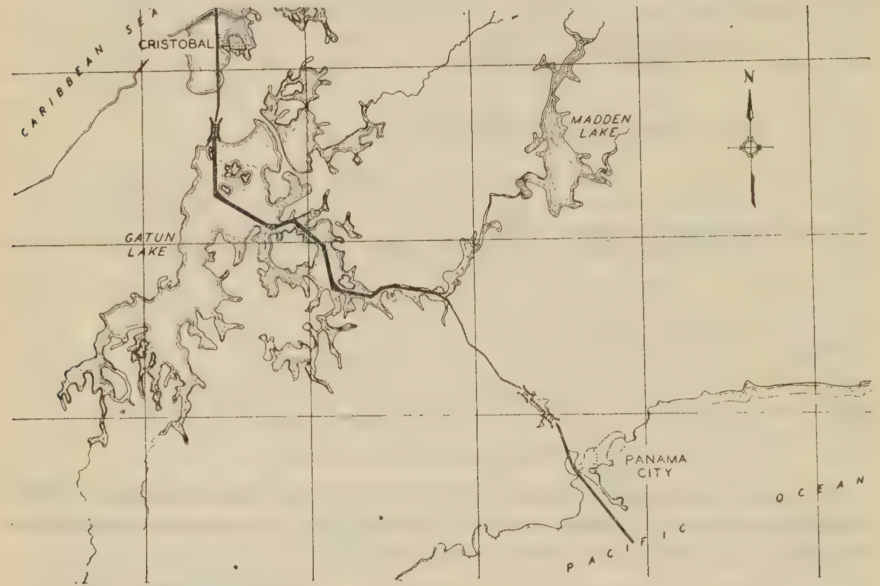

FIG. 4.—GENERAL PLAN, PANAMA LOCK CANAL

CAPACITY LIMITATIONS OF PRESENT CANAL

The present locks are expected to be adequate dimensionally for all commercial shipping for the remainder of the twentieth century except for ships of the "Queen" class which do not, and ordinarily would not, use the sea route through the canal. The limiting effect of lock size on the passage of naval ships is expected to become even more stringent in the future than it is at present.

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[4] "The Isthmian Canal Situation," by Hans Kramer, *Transactions*, ASCE, Vol. 94, 1930, p. 406.

[5] "Additional Lock Facilities for Panama Canal, 1939," *House Report No. 494*, 76th Cong., 1st Session, 1939.

The individual locks of the present canal periodically are taken out of service a lane at a time for overhaul, thus the dependable or firm capacity of the canal is not that established by the twin lanes of locks but is that established by the single lane locks available in the extended periods of overhaul. Occasional night fogs require closing Gaillard Cut to traffic after midnight in the rainy season. The locks are overhauled in the dry season when round-the-clock operation of the canal is feasible, thereby avoiding the reducing effects of both lock overhaul and fog on the canal capacity.

FUTURE TRAFFIC NEEDS

The commercial canal tonnage predictions of Professor Kramer were transformed into expected future ship transits by the engineering staff employed on the studies by taking into account the trends in ship sizes, the expected character of future cargoes, and the expected proportion of transits under full and partial load, and in ballast, as evidenced by past experience. Thus, the transportation of 87,770,000 long tons of commercial cargo estimated by Professor Kramer for the year 2000 would require 13,078 ship transits. In the year 2000, commercial and tolls-free traffic would average 46 transits daily. On peak traffic days 69 transits could be expected. The year 2000 was selected to establish the period during which future traffic needs must be met if any major construction or reconstruction is undertaken.

By locking small ships in tandem and taking into account the sizes of ships estimated for the future, the peak load of 69 ships could be realized with 46 lockages using the present locks. With locks 200 ft wide and 1,500 ft long (which is the size now recommended by the Navy to meet its future needs), it would take 29 lockages to effect the passage of the 69 ships.

HOW SECURE IS THE PRESENT CANAL?

Although no one can say what course World War II would have taken had the Japanese followed up Pearl Harbor (Hawaii) with an attack on the Panama Canal, it is now clear that the locks could have been destroyed and the canal emptied into the sea had an attack been made and the defenses penetrated. The development of larger bombs and new weapons of both conventional and atomic types since Pearl Harbor leaves no doubt as to the vulnerability of the canal to enemy attack and sabotage. If the needs of national defense are to be met, the canal must be made secure against attack and sabotage.

New measures offering effective resistance to the penetration of defenses by rockets and guided missiles have not kept pace with the development of offensive weapons and indeed the prospects, for the future, of the defense are so gloomy it appears that the historic pattern of war may never be restored. The advantage now lies entirely with the aggressor who undertakes to destroy his enemy's ability to wage war by the wholesale destruction of his population and of his instruments of war before he can employ them. Thus, to live as a nation the United States must shield its own vital weapons so that it can take countermeasures to prevent exploitation of any initial advantages the enemy may gain by surprise attack. Prudence requires also that communications,

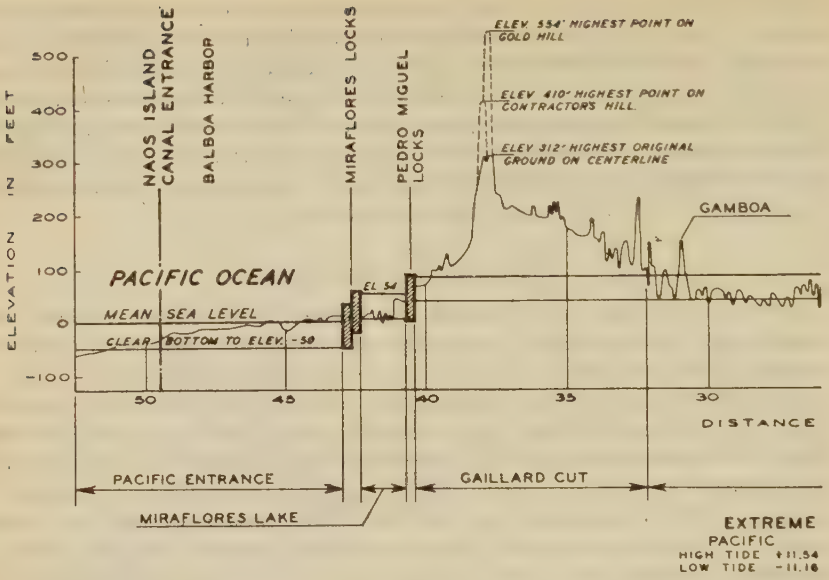


FIG. 5.—PROFILE,

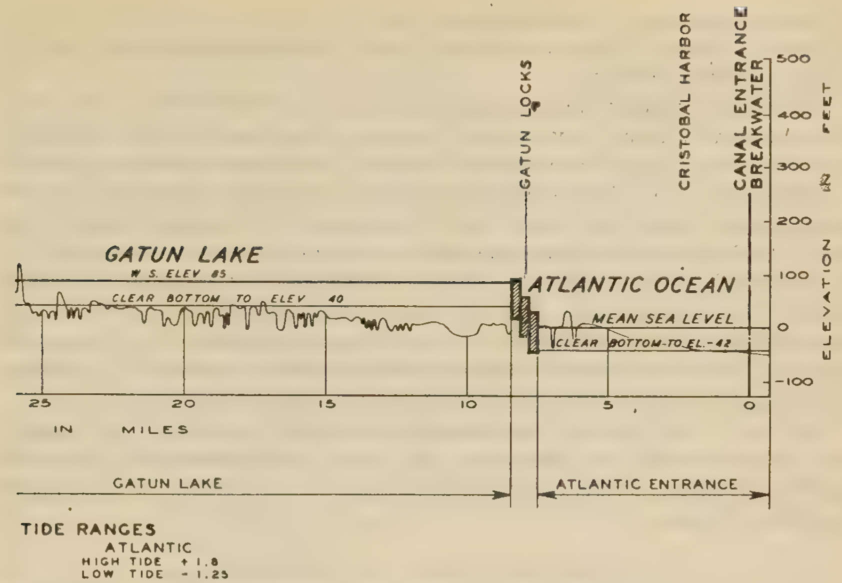
such as the Panama Canal, and essential war industries be planned so that the nation would not be hopelessly crippled if attacked.

The penetration of canal defenses by rockets, guided missles, or robot planes loaded with powerful conventional or atomic explosives launched from the air, from ship or submarine at sea, or from a land base, must be accepted as a possibility. A single sneak attack could destroy the lock gates of the present canal and drain Gatun Lake to the sea.

If the needs of national defense are to be met, steps must be taken to make the canal secure. No matter how resistant to attack the canal is made, however, an alert and modern defense is needed to warn the enemy that an attack will be costly in men and material, to prevent its capture, and to keep the enemy from having a free hand in sinking ships in transit and in making havoc of the auxiliary facilities of the canal.

THIRD LOCKS PROJECT IN REVIEW

A new third set of locks to accommodate large naval vessels, no matter how strongly constructed to resist attack and sabotage, would not add to the overall security of the canal because the existing locks and impounding dams cannot be strengthened sufficiently to give them equivalent protection. If any lock is breached and Miraflores Lake or Gatun Lake is lost, the canal would be closed for months or even years for repairs and for the restoration of the lost lake. In the light of the threat of present weapons, the Third Locks Project would provide for only the peacetime needs of commerce, which, if that were the sole consideration, could be met by other means at a considerably lesser cost.


GATUN LOCKS
CRISTOBAL HARBOR
CANAL ENTRANCE
BREAKWATER
500
400
300
200
100
0
-100
ELEVATION IN FEET
GATUN LAKE
W. S. ELEV 85.
CLEAR BOTTOM TO ELEV 40
ATLANTIC OCEAN
MEAN SEA LEVEL
CLEAR BOTTOM TO EL.-42
25
20
15
10
5
0
IN MILES
GATUN LAKE
ATLANTIC ENTRANCE
TIDE RANGES
ATLANTIC
HIGH TIDE +1.8
LOW TIDE -1.25

Panama Lock Canal

Improvements in the Interests of Commerce Only

The requirements of interoceanic commerce for the remainder of the twentieth century could be met by the elimination of lay-up of the locks for overhaul and by overcoming fog interference with traffic. The first of these is the more restrictive on the capacity of the canal, reducing it from 58 ships to 36 ships per day. Each lock is now overhauled every 4 years, repairs being undertaken every 2 years, alternating between the Atlantic (Gatun Locks) and the Pacific Locks (Miraflores Locks and Pedro Miguel Locks). While one lane of locks is under repair, the adjacent lane of locks is kept open to traffic; thus the availability of only a single operating lane of locks at one end or the other of the canal, during the period of overhaul (about 4 months), establishes the dependable canal capacity. Repairs are made in the dry season when there is no fog, and the canal is then operated 24 hours daily instead of 16 hours per day as is the case in the rainy season when fogs are of relatively frequent occurrence after midnight. Round-the-clock operation and careful scheduling of transits make it possible to hold the reduction in capacity due to overhaul of the locks to less than one half of the normal operating capacity of the canal.

Repairs to the lock gates and their mountings, and to the operating machinery and the filling culvert valves and fittings, are undertaken with the lock chamber dewatered, as are the cleaning and painting of all underwater metal parts. The lay-up and dewatering of the locks for repairs could be eliminated by providing new gate mountings of a special type and new type lock gates having buoyancy chambers to float them out and into position, thus effecting replacements in the matter of a few hours. Repairs to gates would be made in dry dock. Alterations to avoid lay-up of the locks for overhaul and certain channel improvements would raise the dependable capacity of the canal to 65 ships per day.

Fogs of the ground-radiation type which occur in Gaillard Cut in the rainy season (generally from May through December between midnight and daybreak) require closing the canal to traffic during these hours. Various methods of dispersing fog have been employed for the clearing of airfields, but present indications are that they would be too expensive for dispersing fogs in the canal. Developments in electronic aids to navigation offer the prospect of a cheap and fully reliable method of passing ships through a fogbound restricted channel. By the time traffic demands require 24-hour operation of the canal, these devices will be considerably improved and undoubtedly can then be adapted to the canal needs to increase its dependable capacity to 70 ships, which would be adequate for the remainder of the twentieth century.

The cost of the various improvements to extend the life of the present canal in the interests of commerce only would be $129,983,000. Any expenditure in excess of this amount can be justified only by the requirements of national defense.

A New Lock Canal For Security

Although it became apparent early in the studies that an increase in security to meet the needs of national defense could not be attained through the reinforcement of the locks of the present canal, this conclusion of itself did not eliminate the possibility of its attainment by the complete reconstruction of the existing lock canal or by the construction of a new lock canal elsewhere.

A survey of available reports of previous explorations and investigations and the current studies revealed thirty lock canal routes, of which many are merely alternate alinements of well-known routes:[6]

Route	Description	
1	Tehuantepec	
	Nicaragua, via Lake Nicaragua—	
2	Greytown-Fonseca Bay	Via Lake Managua
3	Greytown-Realejo	
4	Greytown-Tamarindo	
5	Greytown-Brito	
6	Greytown-San Juan del Sur	
7	Greytown-Salinas Bay	
	Nicaragua—	
8	Greytown-Salinas Bay	
	Panama—	
9	Chiriqui	
10	Chorrera-Lagarto	Alternate sea-level routes, Canal Zone and vicinity
11	Chorrera-Limon Bay	
12	Chorrera-Gatun	

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[6] "U. S. Army Interoceanic Canal Board Report on Panama Canal and Nicaragua Canal (Sultan Report) 1931," *House Document No. 139*, 72d Cong., 1st Session, 1932.

Route	Description	
13........	Panama parallel	
14........	Panama sea-level conversion	
15........	Panama Canal, route of the present lock canal	
16........	San Blas	
17........	Sasardi-Morti	Caledonia Bay routes
18........	Aglaseniqua-Asnati	
19........	Caledonia-Sucubti	
	Panama and Columbia; Tuyra River Routes—	
20........	Tupisa-Tiati-Acanti	
21........	Arquia-Paya-Tuyra	
22........	Tanela-Pucro-Tuyra	
23........	Atrato-Cacarica-Tuyra	
24........	Atrato-Peranchita-Tuyra	
	Colombia; Atrato River Routes—	
25........	Atrato-Truando	
26........	Atrato-Napipi	
27........	Atrato-Napipi-Doguado	
28........	Atrato-Bojaya	
29........	Atrato-Baudo	
30........	Atrato-San Juan	

Excavation estimates based on existing maps, supplemented in some cases by field and air reconnaissance, resulted in narrowing the choice of routes for a lock canal to the eight listed in Table 1 and shown in Fig. 6.

TABLE 1.—COMPARISON OF LOCK CANAL ROUTES

ROUTE[a]		Length (miles)	Elevation of divide at crossing[b]	Channel excavation[c] (million cu yd)	Elevation of summit lake[b]	Lock lifts per lane[d]	Approximate cost (million dollars)	Action
No.	Place							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Tehuantepec	165	812	3,360	550	10	13,280	Eliminated because of higher cost.[e]
5	Nicaragua..	173	153	1,060	110	2	3,566	Eliminated on the same basis as route 1.[e]
9	Chiriqui....	55	5,000	....[f]	....[f]	....[f]	....[f]	Eliminated because of height of divide.[g]
15	Panama....	51	340	191	92	2	2,308	Retained for final study.
16	San Blas....	40	1,100	1,480	110	2	5,960	Eliminated because of excessive cost.
17	Caledonia...	63	1,100	1,110	110	2	4,751	Eliminated because of excessive cost.
23	Tuyra River	135	470	1,120	110	2	....[h]	Eliminated because of excessive cost.[g]
25	Atrato River	95	932	1,450	177	3	....[h]	Eliminated because of excessive cost.[g]

[a] See Fig. 6. [b] In feet above mean sea level. [c] Channels are 500 ft wide at a depth of 40 ft and 55 ft deep below low water level. [d] Two lanes of locks in each route, the lock chambers being 200 ft wide, 1,500 ft long, and 50 ft deep. [e] After full consideration of shipping benefits from shortening trade routes. [f] Details not developed. [g] Excessive excavation. [h] Not estimated.

None of the seven lock canal routes at locations remote from Panama offers compensating advantages in additional security or in savings through the shortening of trade routes over the Panama lock canal route.

The plan of development used in the comparative lock canal studies was the same in each case and is as described in the next section for a modernized lock canal at Panama having the maximum security feasible in this type of canal.

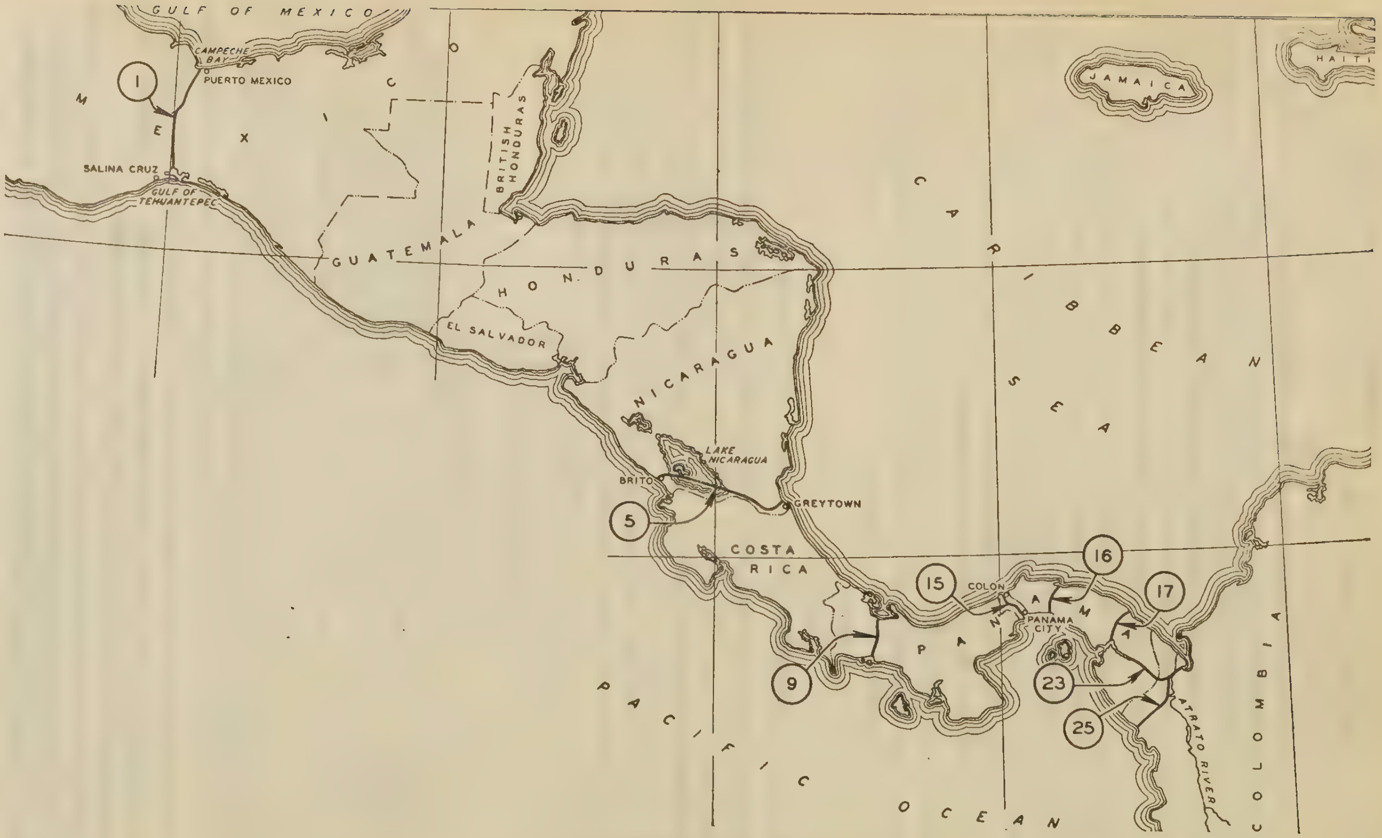
GULF OF MEXICO
CAMPECHE BAY
PUERTO MEXICO
1
MEXICO
SALINA CRUZ
GULF OF TEHUANTEPEC
BRITISH HONDURAS
GUATEMALA
HONDURAS
EL SALVADOR
NICARAGUA
LAKE NICARAGUA
BRITO
GREYTOWN
5
COSTA RICA
CARIBBEAN SEA
JAMAICA
HAITI
9
15
COLON
16
PANAMA CITY
PANAMA
17
23
25
ATRATO RIVER
COLOMBIA
PACIFIC OCEAN

FIG. 6.—TRANSISTHMIAN CANAL ROUTES

A Modernized Panama Lock Canal

A reconstructed lock canal with all Pacific locks grouped at Miraflores to provide full lift to summit level would offer the greatest economy of construction and would create an anchorage area at the head of the Pacific locks that would facilitate the dispatch of vessels through Gaillard Cut. A similar arrangement of the Pacific locks was proposed by Adolphe Godín de Lépinay in 1879 and was advocated by the late W. L. Sibert, M. ASCE, in 1908. Colonel Sibert's suggestion was rejected because of the advanced state of the planning and construction, and doubts as to the foundations for three-lift locks at Miraflores, the site he proposed for the Pacific locks. The Miraflores foundations have since been found satisfactory for locks with full lift to summit level.

Two dispersed locks would be provided at Gatun and Miraflores with two lifts to attain summit level instead of three as at present. Chambers would be 200 ft wide, 1,500 ft long, and 50 ft over the sill. By raising Gatun Lake to El. 92, from its present maximum El. 87, additional storage for lockage water would be provided. This augmented water supply would not meet lockage demands until the year 2000 and supplementary pumping from the sea eventually would be necessary.

The dispersion of the locks and their armoring with concrete and steel to protect the lock machinery and the culverts would provide them with the highest practicable degree of protection. The lock gates do not lend themselves to protective treatment, except against the lightest type of aerial bomb. However, multiple sets of gates, well dispersed, would increase the difficulties of dissipating Gatun Lake. Certain of the gates would be housed in protected recesses when not in use. The closure dams adjoining the locks at Gatun and Miraflores would be of massive earth construction. The Gatun Dam spillway would be channeled in the rock abutment for maximum protection.

The cost of such a fully modernized lock canal at Panama would be $2,307,686,000.

Security considerations restrict public evaluation of the protective designs in relation to the various weapons that could be employed in an attack. It can be stated, however, that the lock canal cannot be made resistant either to atomic bombs or to modern conventional weapons. At best the protection that could be provided a lock canal would only increase the difficulties rendering it useless. The modernized locks could be breached by a determined enemy and the canal could thus be closed to traffic for the period required for reconstruction and for the capture of the tributary runoff to restore the summit lakes, which might require as much as 4 years. The extent of damage and the length of the period of traffic interruption would depend on the nature of the weapon employed and the intensity of attack. Radioactive contamination would make repairs to the locks difficult if not impossible. The lock type of canal, no matter how strongly constructed, would not increase security to meet the needs of national defense.

Sea-Level Canal Possibilities

The various sea-level canal route possibilities were narrowed down, in the same manner as were the lock canal routes, to the eight listed in Table 2, which includes the principal features, quantities of excavation, and the estimates of cost. The Panama route (Fig. 7) is the least costly and has the additional advantages of an operating and administrative establishment and of defenses already in place. These installations would have to be duplicated at any other sea-level route. The plan of development for a sea-level canal at the several routes was similar to that for a sea-level canal at Panama as described in the next section.

TABLE 2.—COMPARISON OF SEA-LEVEL CANAL ROUTES

Route[a] No. (1)	Route[a] Place (2)	Length (miles) (3)	Elevation of divide at crossing[b] (4)	Channel excavation[c] (million cu yd) (5)	Approximate cost[d] (million dollars) (6)	Action
1	Tehuantepec....	165	812	6,130	....[e]	Eliminated because of excessive cost.[f]
5	Nicaragua......	168	760	5,200	....[e]	Eliminated because of excessive cost.[f]
9	Chiriqui.......	55	5,000	68,800	....[e]	Eliminated because of height of divide.[f]
15	Panama........	46	410	1,069	2,483	Retained for final study.
16	San Blas.......	40	1,100	2,080	6,272	Eliminated because of excessive cost.
17	Caledonia......	59	1,100	1,880	5,132	Eliminated because of excessive cost.
23	Tuyra River....	135	470	2,140	....[e]	Eliminated because of excessive cost.[f]
25	Atrato River ...	95	932	1,810	4,594	Eliminated because of excessive cost.

[a] See Fig. 6. [b] In feet above mean sea level. [c] Channels are 600 ft wide at a depth of 40 ft and 60 ft deep below low water level. [d] Exclusive of tidal regulating structures, except at Panama. [e] Not estimated. [f] Excessive excavation.

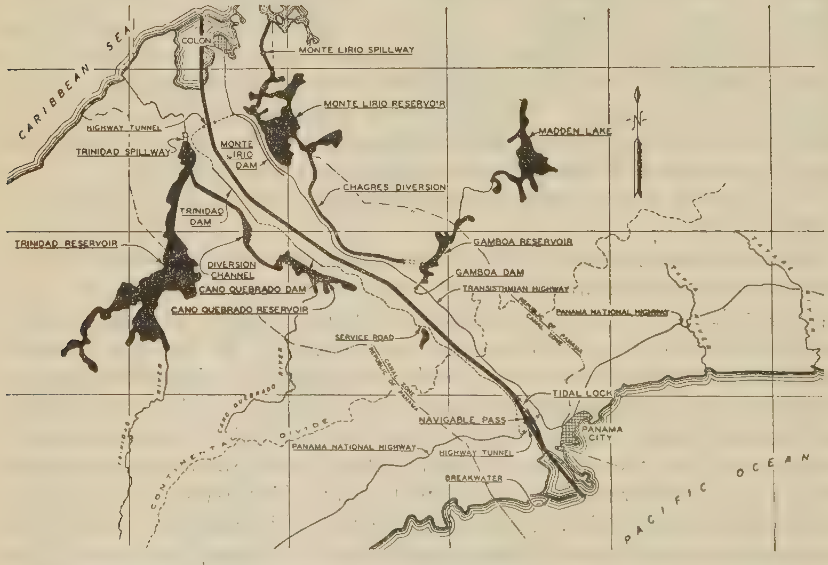
CARIBBEAN SEA
COLON
MONTE LIRIO SPILLWAY
MONTE LIRIO RESERVOIR
MADDEN LAKE
HIGHWAY TUNNEL
TRINIDAD SPILLWAY
MONTE LIRIO DAM
CHAGRES DIVERSION
TRINIDAD DAM
TRINIDAD RESERVOIR
GAMBOA RESERVOIR
DIVERSION CHANNEL
GAMBOA DAM
CANO QUEBRADO DAM
TRANSISTHMIAN HIGHWAY
CANO QUEBRADO RESERVOIR
PANAMA NATIONAL HIGHWAY
SERVICE ROAD
REPUBLIC OF PANAMA
CANAL ZONE
TIDAL LOCK
NAVIGABLE PASS
PANAMA CITY
PANAMA NATIONAL HIGHWAY
HIGHWAY TUNNEL
BREAKWATER
CONTINENTAL DIVIDE
CANO QUEBRADO RIVER
PACIFIC OCEAN
N

FIG. 7.—PLAN OF PANAMA SEA-LEVEL CANAL

Plan of Development of a Sea-Level Canal at Panama

There are numerous possibilities for a sea-level canal in the Canal Zone and in the immediate vicinity;[7] the more favorable of these are shown in Fig. 8. The route that would cost the least and have other outstanding advantages is designated the Panama sea-level conversion route, since it follows generally the alinement of the present lock canal. The distinctive feature of a canal on the conversion route is that its construction would involve lowering the present canal to sea level, whereas this would not be the case if either the Panama parallel or one of the Chorrera canals were constructed.

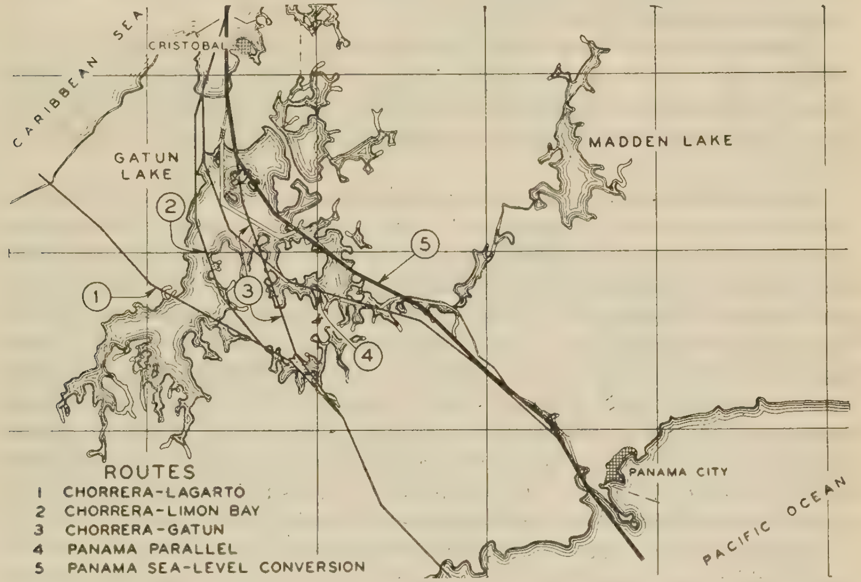

Fig. 8.—Routes in Canal Zone and Vicinity

In the Gatun Lake area, the Panama parallel canal would be separated from the existing lock canal by a barrier dam constructed from spoil material from the excavation for the new canal. From Gamboa south to Balboa Harbor it would follow an alinement separated from the present canal. Thus, the Panama parallel sea-level canal would be completely independent of the lock canal and, as would be the case if the Chorrera-Lagarto canal were constructed, both the existing canal and the new canal could be maintained and operated if this were thought necessary. There are no reasons arising either from capacity or security considerations that would justify the additional $800,000,000 for the Panama parallel sea-level canal. Any one of the Chorrera routes would be even more costly than the Panama parallel route and would be no more justified.

The alinement of the present canal and that of the proposed Panama sea-level conversion canal would be sufficiently separated at several reaches to make it possible to construct a substantial part of the latter in the dry, thus avoiding interference with canal operations. The alinement improvements introduced in the conversion route would shorten the canal by 5.2 miles.

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[7] "Sea Level Plan for Panama Canal," by J. G. Claybourn, *Transactions*, ASCE, Vol. 114, 1949, p. 572.

THE SEA-LEVEL CANAL CHANNEL

The channel of the Gaillard Cut in the present canal is deficient in width for two-way traffic involving large and unwieldy ships, and single directional transiting arrangements for such ships are in effect to avoid their encountering other ships in the cut. The delays and the inconveniences resulting from the special handling of this type of traffic have not thus far been seriously objectionable, but they would be with the further growth of traffic and this fact was borne in mind in designing the sea-level canal.

Currents up to 4.5 knots would be induced in a sea-level canal at Panama without tidal control by the Pacific tides which have a range up to 20 ft. Atlantic tides have a maximum range of 2 ft and would cause currents of 0.5 knot if the Pacific tidal effects were eliminated by control works at the Pacific entrance.

In establishing the design of the sea-level channel, a world-wide survey was made of comparable channels and canals. Several waterways having characteristics similar to the Panama sea-level canal, including the sea-level canals at Suez (Egypt) and Cape Cod (Massachusetts), were visited by members of the staff engaged on the studies. In addition, model investigations were made for The Panama Canal by the United States Navy at the David Taylor Model Basin at Carderock, Md., to determine, for both the lock and sea-level canals, the width, depth, and alinement requirements. For the sea-level channel determinations, various widths and depths of channel using both straight and bend channel sections were tested with currents up to 5 knots. Self-propelled, remote-controlled models of typical ships that transit the canal, operating at a wide range of ship speeds, were employed in testing the channels.

Two-way traffic of most of the ships that would be expected to transit the canal could be accommodated in a channel designed for the safe meeting of a standard "Liberty" ship by the largest naval craft or the largest commercial craft now afloat. The meeting and passing of the largest present-day naval craft and commercial vessels would be unusual and could be avoided by special transiting arrangements similar to those now in effect. Similar arrangements could be made for the transit of the largest ships expected to be built during the twentieth century.

In spite of considerations leading to the recommendation of the Governor of The Panama Canal (J. C. Mehaffey) that tidal currents in the canal be regulated, the channel was designed for safe navigation in currents up to 4.5 knots. This is made necessary by reason of the fact that the structures for the regulation of currents in the canal could be irreparably damaged by bombing and would have to be cleared from the channel. As the result of the studies, it was concluded that all shipping could safely transit the proposed Panama sea-level canal at any condition of current that would obtain up to the maximum of 4.5 knots; only an occasional unwieldy or low powered ship would be held for mean tide to avoid encountering high currents. Tug assistance could be provided such ships to avoid delays and to provide increased safety of transit when thought necessary.

The channel standards adopted as the result of the model studies and other investigations are given in Table 3, as are the controlling standards of the present canal.

TABLE 3.—COMPARISON OF CHANNEL DIMENSIONS

Description	Depth (ft)	Width (at a depth of 40 ft) (ft)	Cross-sectional area (minimum) (ft)	Minimum sight distance (miles)	Maximum deflection angle (degrees)	ANGULARITY	
						Total (degrees)	Per mile of canal length
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Present Panama lock canal.....	42	300	13,860	0.6	67[a]	598	11.7
Modernized Panama lock canal .	55	500	28,400	0.6	67[a]	642	12.5
Panama sea-level canal.........	60	600	36,800	1.5	26	117	2.5

[a] This applies to Gatun Lake; in Gaillard Cut the maximum angle is 30°.

The survey of the world waterways disclosed that channel depths are generally considered inadequate and that better ship controllability would result with more water under the keel. The Carderock tests fully confirmed the latter conclusion. The lesser depth proposed for the projected lock canal results from the lesser ship speed (8 knots) that would be prescribed in the restricted channel of a lock canal. A ship speed of 10 knots was used to establish the depth of the projected sea-level canal. The width of channel is referenced in each case to the 40-ft depth below the low water surface, which approximates the draft of the large vessels of the future. This depth also establishes the point of revolution in setting bank slopes, thus minimizing variations in the surface width of the channel arising from differences in the slopes, which are fixed by the character of the bank materials.

The results of the model tests were interpreted with the assistance of the United States Navy and the pilots of the Panama Canal and of the Cape Cod Canal. The advice and counsel of the Panama Canal pilots on all problems dealing with navigation were invaluable in the conduct of the studies.

SEA-LEVEL CANAL MODEL INVESTIGATIONS

George B. Pillsbury,[8] M. ASCE, has developed a procedure for computing currents in sea-level canals which, when applied to the proposed Panama sea-level canal, yielded results that were closely confirmed by Boris A. Bakhmeteff, Hon. M. ASCE, by independent computations and by model investigations. A model of the proposed sea-level canal at an undistorted scale of 1 : 100 provided methods of accurately determining conditions of flow in the uncontrolled waterway at all ranges of tide, of establishing the design, and of testing the tidal-regulating structures. The maximum velocities in the unregulated canal 60 ft deep and 600 ft wide at 40-ft depth at various Pacific tides as computed and as observed in the sea-level canal model are given in Table 4.

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[8] "Tidal Hydraulics," by George B. Pillsbury, *Professional Paper No. 34*, Corps of Engrs., U. S. Govt. Printing Office, Washington, D. C., 1940.

TABLE 4.—VELOCITIES IN AN UNREGULATED SEA-LEVEL CANAL

Tidal range (ft) (1)	Percentage of time tides are exceeded (2)	Maximum Velocities, Uncontrolled Channel	
		Observed (knots) (3)	Computed (knots) (4)
6.....	99	2.1	2.1
10.....	80	2.7	3.0
13.....	50	3.3	3.5
16.....	20	3.8	4.0
20.....	2	4.4	4.5

TABLE 5.—SCHEDULE OF NAVIGABLE PASS OPENING IN A REGULATED SEA-LEVEL CANAL

Permissible current velocity (knots) (1)	Hours per Day That the Navigable Pass Would Be Open for Tidal Ranges of:				
	6 ft (2)	10 ft (3)	13 ft (4)	16 ft (5)	20 ft (6)
1........	8.5	4.5	3.5	2.9	2.3
2........	20.2	9.7	7.2	6.0	4.9
3........	24.0	24.0	14.9	11.6	9.1
4........	24.0	24.0	24.0	24.0	19.8
4.5........	24.0	24.0	24.0	24.0	24.0

TIDAL REGULATION FOR THE SEA-LEVEL CANAL

The majority (eight out of thirteen) of the Consulting Board appointed by President Theodore Roosevelt in 1905 to consider various plans for a canal at Panama prepared by the Isthmian Canal Commission in voting for a sea-level canal stated with respect to tidal control:

> "The plan [sea-level] proposed by the Board for the Isthmian transit will have twin tidal locks near the Pacific terminus, which if disabled, one or both [by enemy attack or sabotage] would still be usable (after removal of wreckage) for a part of each day (the period of spring tides) in each lunar month, and probably throughout the whole twenty-four hours the remainder of the lunar month (neap tides)."

If tidal regulation were to be omitted, occasional ships would need to be held at the canal entrances for a favorable tide, or tug assistance would need to be provided for their transit. The Carderock tests and experience in other waterways are conclusive in demonstrating that relatively few modern ships would require special transiting arrangements; the majority, having reliable power and rudder control, would be capable of transiting the Panama sea-level canal safely in currents up to 4.5 knots. Nevertheless, because of the serious consequence of collisions with the canal banks (which would be largely of rock), it was decided to provide tidal regulation for added safety to shipping. The loss of tidal regulation as the result of bombing of the regulating structures is a definite possibility, and is accepted as a condition of operation during wartime.

If the control of the Pacific tides were to be provided by tidal locks and a barrier dam and these were to be damaged by the enemy, then the canal would either have to be closed for repairs or the structures cleared from the channel to permit use of the canal as an open waterway. In the latter case, there would be an abrupt transition from slack-water navigation to navigation in currents ranging up to 4.5 knots. Prolonged closure of the canal for repairs would be intolerable, which suggests that the tidal-regulating structures should be planned to permit the rapid clearing of a "channel-way" in case they are damaged. This could be done by excavating an auxiliary channel which would normally be closed by an earth barrier dam that could be rapidly blasted to clear the channel when the locks were rendered unusable by bombing. Obviously, an abrupt change from navigating a canal with currents completely regulated to one with currents of the order listed in Table 4 would be undesirable and this led to an investigation of other methods for providing tidal regulation.

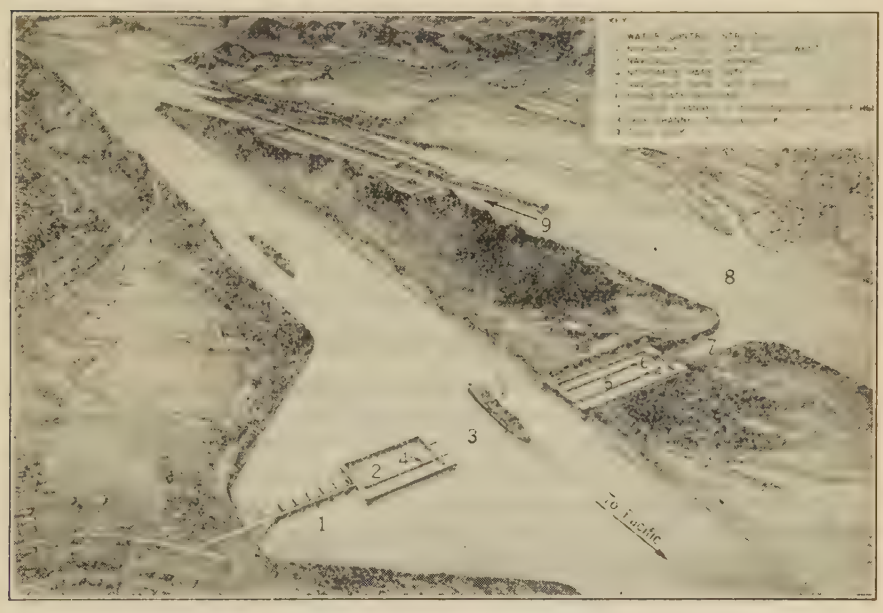

Fig. 9.—Tidal-Regulating Structures Showing the Navigable Pass Open

A sudden transition from slack-water navigation to navigation in a completely unregulated waterway in the event of damage to the tidal lock could be avoided by supplementing the tidal lock with a navigable pass through which ships, as a matter of routine operation, could pass at any selected stage of the tide. The pass would have retractable gates (Fig. 9) which could be opened rapidly to permit navigation without the need for lockage under any set of conditions of current desired, from 0.5 knot (Atlantic tidal currents) to the maximum. The gates of the pass would be of steel construction, and in the event of their damage could be readily removed from the channel. Table 5 shows the hours daily that the pass would be open if the tidal currents were controlled to 1, 2, 3, 4, and 4.5 knots. These data were determined by computations and confirmed by sea-level model tests.

It is probable that, when the sea-level canal is first placed in operation, the navigable pass would be opened only at mean tidal stages to limit channel currents to low values. As operating experience is gained, the periods of opening of the pass gates would be extended to permit higher currents in the canal. Tentatively, currents in the canal would be limited to 2 knots; this value was selected after a survey of operating experience in other waterways and as the result of the United States Navy tests at Carderock. On this schedule for the opening of the pass gates, the canal would operate 32% of the total time as an open waterway.

A gated water-control structure would complement the navigable pass to assist in adjusting the water surface in the canal to extend the period of opening of the pass at each mean stage cycle of Pacific tide (Fig. 9.) The gates of the water-control structure would be open only when the pass is closed to traffic. The tidal lock could accept ships at all stages of the tide and would be the sole avenue of transit when the pass is closed.

The sea-level canal model was an invaluable aid in planning the location, the arrangement, and the hydraulic features of the separate elements of the tidal-regulating works and in developing the operating schedule of the navigable pass.

One important phase of the model study was that of relating the friction factor of the model to the friction factor of the prototype channel. This was accomplished by actual flow tests in Gaillard Cut with the culverts in the Pedro Miguel Locks wide open and discharging 22,000 cu ft per sec.

SLIDES IN THE PANAMA SEA-LEVEL CANAL[9]

A study of the major slides experienced in the period of the canal construction was of great assistance in comprehending the character and probable behavior of the materials to be encountered in any new excavation. These slides, as was stated by the late George W. Goethals, M. ASCE, in 1916, were the result of an attempt to fix uniform slopes throughout the length of the canal, regardless of height of cut and character of materials. General Goethals stated after the completion of the canal:

> "* * * With the geological formation changing so frequently and so suddenly both in the direction of the Cut and up and down there is no possibility of any uniformity in slopes. No uniformity of slopes could be maintained * * *."[10]

For a time after the first major construction slide took place, it was thought that the slopes would eventually stabilize themselves and that the volume of material ultimately to be removed would be less if the slides were allowed to run their course. This proved not to be the result; moreover, the pressing need for opening the canal led to flattening certain of the slopes to prevent further blockage of the channel. Fig. 10 represents for a specific location the design slopes now proposed for the Cucaracha formation, and shows the slope selected in the initial construction and the actual slope attained in this formation after failure.

Consideration was given in the design of slopes to dynamic loading that could be induced by bombing. A study of the resistances of the materials to dynamic (transient) loadings was made at Harvard University in Cambridge, Mass., under an investigational contract. The studies of the resistances of the materials to transient loadings and the tests of their residual strengths after failure will be of particular interest to engineers. The transient-load testing apparatus developed at Harvard University offers promise as a valuable tool in developing fuller understanding of the behavior of soils and rocks under stress.

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[9] "Panama Canal Slides," Rept. of the Committee of the National Academy of Sciences, *Memoirs*, National Academy of Sciences, Vol. XVIII, 1924.

[10] "The Dry Excavation of the Panama Canal," by George W. Goethals, *Transactions*, International Eng. Cong., San Francisco, Calif., 1915.

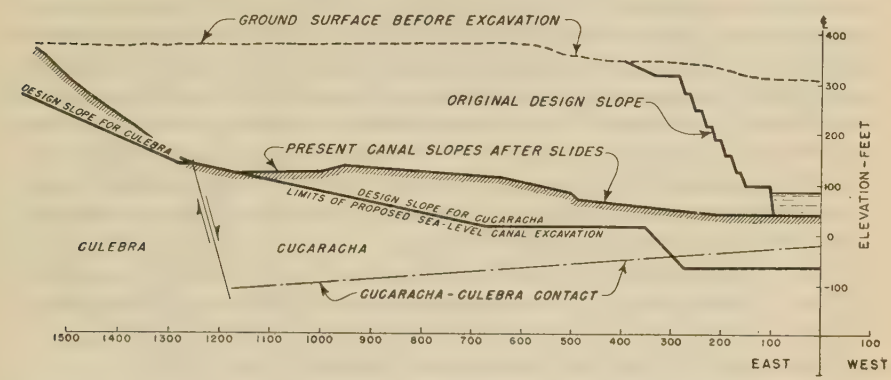
GROUND SURFACE BEFORE EXCAVATION
℄
ORIGINAL DESIGN SLOPE
DESIGN SLOPE FOR CULEBRA
PRESENT CANAL SLOPES AFTER SLIDES
DESIGN SLOPE FOR CUCARACHA
LIMITS OF PROPOSED SEA-LEVEL CANAL EXCAVATION
CULEBRA
CUCARACHA
CUCARACHA - CULEBRA CONTACT
400
300
200
100
0
-100
ELEVATION - FEET
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
100
EAST
WEST

FIG. 10.—ORIGINAL AND PRESENT SLOPE DESIGN CRITERION (OLD CANAL STATION 1785+00)

No allowance was made in the designs for slide failures resulting from dynamic loadings since it is unlikely that slides induced by the atomic bomb would fully block the canal. However, debris from cratering resulting from atomic bombing could block the channel, but widening the channel to overcome this possibility would require such a large amount of excavation that the risk of closure was accepted, since in any case the blocking materials, even though radioactive, could be removed in a few weeks at the outside. None of the conventional weapons could induce slides or throw up craters that would block the channel of either a sea-level or an improved lock canal.

CONVERSION TO SEA LEVEL

Studies of the conversion of the Panama lock canal to a canal at sea level prior to that under Public Law No. 280 contemplated the lowering of Gatun Lake in stages. One study planned for the lowering of Gatun Lake in seven stages. Philippe Bunau-Varilla,[2] the French engineer who negotiated the sale of the canal holdings of the French to the United States, urged the consulting board appointed by President Theodore Roosevelt to provide deep upper sills at each of the locks to facilitate the later conversion of the lock canal by stages to a sea-level canal. His plan was rejected on the grounds that major modification of the proposed twin locks would be inescapable in any case and that a third set of locks would be needed in order that two lanes of locks would be available for traffic at all times during the conversion period.

There is no question that the conversion of the existing locks would involve considerable risk both to shipping and to the integrity of the canal with only one lock lane available and that no plan should be accepted that would have less than two lanes of locks available to shipping throughout the period of conversion. If the lowering of the summit lakes to sea level by stages is undertaken, new special twin conversion locks of minimum construction with lift to El. 53 are preferable to previously considered arrangements involving the progressive alteration of the existing locks for each stage of lowering using a new third set of locks with full lift to El. 85 to insure two lanes being available at all times. The proposed special twin conversion locks would be placed in operation and the existing locks abandoned when the first stage of lowering of Gatun Lake from El. 85 to El. 53 is accomplished—or after the completion of excavation to prepare the channel to carry traffic with the summit lake at El. 53. The next stage of lowering of the canal would be to El. 22, and finally the lowering would reach sea level; in each case excavation to provide necessary depth of channel would precede lake lowering. The twin conversion locks have low upper sills to accommodate traffic at all stages of canal elevation from El. 53 to El. 0. This plan and others considered for the stage lowering of the summit lakes were finally abandoned because of the attractiveness and the cheapness of the plan for the single-stage lowering of the summit lakes.

The conversion of the lock canal to sea level by lowering the summit lakes in a single operation would require dredges capable of excavating to a depth of 145 ft. For this purpose a cutterhead-type suction dredge would be used to excavate the softer materials and a chain-bucket type dredge, to excavate blasted hard materials. The suction dredge would have a booster pump mounted in a well on its ladder to effect the lift of the materials. The chain-bucket dredge would be similar to the conventional type used in mining of gold and tin except that the 2-cu-yd buckets proposed are considerably larger than those ordinarily used to depths of the order proposed. Dredges of the chain-bucket type worked successfully to depths of 128 ft in the mining of gold; however, the buckets generally have not exceeded ½ cu yd. The Board of Consulting Engineers and others advising Governor Mehaffey on the current studies were unanimous in their opinion that deep dredging to effect single-stage conversion would be practical and economical.

In the deep-dredging method of conversion (single-stage lowering of the lakes), there would be a net saving of $130,000,000, principally through the omission of conversion locks. The actual lowering of the water surface to effect the conversion to sea level would be accomplished by progressive demolition of natural rock plugs and the removal of a temporary steel dam left to retain the Gatun and Miraflores lakes. The conversion could be effected with a traffic interruption of only 7 days.

Alinement improvements would permit a large part of the excavation to be done in the dry (750,000,000 cu yd) at a large saving in cost. There would be 318,000,000 cu yd of wet excavation classified as follows:

Cubic yards	Classification
9,400,000	Hard rock
97,700,000	Medium rock
46,600,000	Soft rock
165,000,000	Sands, gravels, and clays

Of this amount, approximately 132,000,000 cu yd would lie between depths of 85 ft and 145 ft below maximum water surface and would require the employment of special dredges.

Flood Control in the Sea-Level Canal

Unless tributary inflows into the sea-level canal are controlled, there would be interruptions in service and occasional hazard to navigation. The most essential control is that of the Chagres River, the major tributary; this would be accomplished by a dam constructed at Gamboa to operate in tandem with the upstream Madden Dam. The regulated outflows of the Chagres River would be excluded from the canal by diverting them to the sea through a short tunnel and a channel as shown in Fig. 11. The flows from all other tributaries on the east side of the canal north of the Chagres River would be intercepted and conducted to the Caribbean (Las Minas Bay) by a system of dams and the channels which convey the regulated flows from the Chagres. This control system, termed the "East Diversion," would have sufficient capacity to

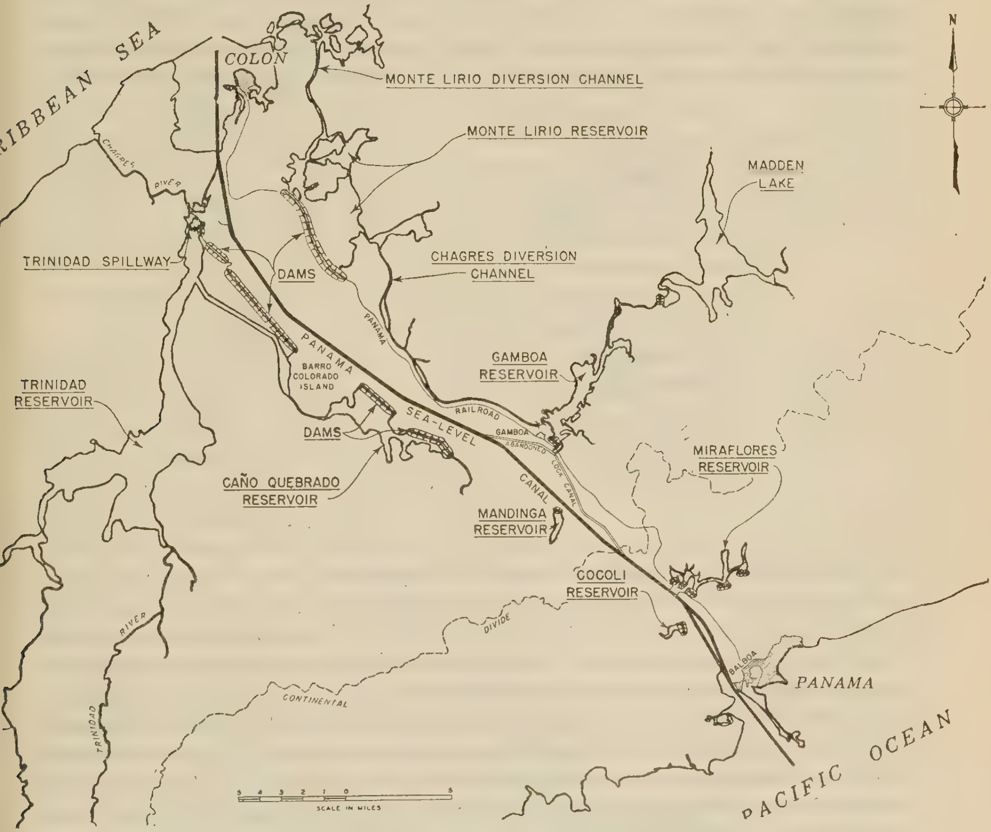
RIBBEAN SEA
COLON
MONTE LIRIO DIVERSION CHANNEL
MONTE LIRIO RESERVOIR
N
MADDEN LAKE
CHAGRES RIVER
TRINIDAD SPILLWAY
DAMS
CHAGRES DIVERSION CHANNEL
PANAMA
PANAMA
BARRO COLORADO ISLAND
GAMBOA RESERVOIR
TRINIDAD RESERVOIR
RAILROAD
SEA-LEVEL
GAMBOA
ABANDONED
LOCK CANAL
DAMS
MIRAFLORES RESERVOIR
CAÑO QUEBRADO RESERVOIR
CANAL
MANDINGA RESERVOIR
COCOLI RESERVOIR
RIVER
DIVIDE
BALBOA
PANAMA
CONTINENTAL
TRINIDAD
5 4 3 2 1 0 5
SCALE IN MILES
PACIFIC OCEAN

Fig. 11.—Sea-Level Canal Flood Control System

divert all floods up to 25% larger than the maximum flood of record. Emergency outlets into the canal are provided for larger floods.

"The West Diversion" system of dams, reservoirs, and channels would exclude the entry of flows into the canal from all the important tributaries west of the canal and north of the Continental Divide by diverting them into the channel of the lower Chagres through an outlet and spillway west of the present Gatun Dam.

South of the Continental Divide there are a number of small tributaries (maximum drainage area 13 sq miles) which could contribute undesirable flood inflows if not regulated. Diversion being impracticable because of the topography and the developments in the area, a system of regulating reservoirs as shown in Fig. 11 would be provided, the corresponding drainage areas being:

Reservoirs (see Fig. 11)	Square miles
East Diversion—	
Madden Lake	393
Gamboa Reservoir	127
Monte Lirio Reservoir	180
West Diversion—	
Caño Quebrado Reservoir	166
Trinidad Reservoir	322
Small Reservoirs—	
Mandinga	10.5
Cocoli	11.5
Miraflores	30
Uncontrolled	118
Total	1,358.0

The proposed combination of reservoirs and diversion systems would control 1,240 sq miles of the total of 1,358 sq miles of area tributary to the canal between Gatun and Balboa. Of the uncontrolled drainage areas, the largest would not exceed 6 sq miles. The flood contribution from the uncontrolled areas would neither inconvenience navigation nor be a hazard to ships in transit.

The flood-control dams would be constructed from excavation spoil except where the length of haul would make local borrow cheaper. The east and west diversion dams would be low structures of earth and rock spoil mounted on broad platforms of fill constructed of excavation spoil placed in the wet, thus insuring very conservative loadings where the foundations are muck and soft clays. The earth and rock flood-control structures would be readily repairable in case of damage by enemy bombing.

SEA-LEVEL CANAL CONSTRUCTION PLAN

It is planned to complete the construction of the sea-level canal in 10 years. The construction plan adopted in the report to Congress provides for the use of large shovels and draglines (25 yd or larger) dumping into scows for the haul of dry excavation, totaling 750,000,000 cu yd, to disposal areas in Gatun Lake. Wet excavation to customary depths would be performed by conventional dredges, but for depths beyond the range of this equipment, special dredges capable of excavating to 145 ft below the water surface would be required. Vehicular tunnels under the canal would be constructed at the Atlantic and Pacific ends to aid construction and for cross-channel access thereafter. Total cost of the sea-level canal is estimated at $2,483,000,000.

CAPACITY OF THE PANAMA SEA-LEVEL CANAL

The capacity requirements for the year 2000 on peak days would be 69 transits. For planning purposes, ship speeds in the sea-level canal were established at 12 knots ground speed traveling with the current and 8 knots against current. A conservative spacing of 1.5 nautical miles between ships was selected after a survey of the practices of other waterways and with the advice of Panama Canal pilots. The daily capacity of the sea-level canal, based on a 16-hour operating schedule for both the tidal lock and the navigable pass, and assuming the latter would be opened to limit currents up to 2 knots, would be 116 transits daily. The average transit time in the sea-level canal would be 4.5 hours. This compares with an average transit time of 8 hours in the present canal.

SECURITY OF THE PANAMA SEA-LEVEL CANAL

The tidal-regulating and flood-control structures of the sea-level canal would not be essential to its safe operation; hence their damage or destruction would result only in temporary interruptions to traffic. An adequate flood-warning system would insure against ships being caught in the channel at the time of incidence of large flood inflows in the event of damage to the flood-control structures. The flood-impounding structures being of earth and rock construction would be highly resistant to bombing but, because they are not absolutely vital to the operation of the sea-level canal, the costly treatment necessary to make them resistant to the atomic bomb would not be warranted. If damaged by bombing, the flood-control structures could be readily repaired and restored to service.

The tidal-regulating works could not be made resistant to modern weapons. Their loss by enemy action would be the loss of convenience to shipping and a lessening of safety in transit, particularly for ships with low power and poor rudder control. The risks in transit of such ships could be avoided by having them await a favorable tide or by providing them with tug assistance. The tidal lock, if heavily damaged, would be a mass of debris that would be difficult to remove. If the tidal lock were contaminated by radioactive particles as the result of atomic bombing, it would probably have to be abandoned. In either case the navigable pass could be used for transiting traffic. If the pass gates were damaged, they could be removed from the channel in a few days to a few weeks, depending on the extent of damage to the channel-way. Thereafter, the canal would operate as an open waterway.

None of the conventional weapons known today could induce slides or blockage of the channel by cratering that would close the sea-level canal to traffic. However, the canal could be blocked for limited periods as the result of an atomic weapon attack. It is estimated that a crater blockage of the canal would require a few weeks, at the most, for the clearing of a traffic lane, even though radioactive contamination would delay the initiation of the removal work. The shielding of the dredges and other excavation equipment to protect crews would be required, and this is considered practicable.

An attack employing conventional or atomic weapons could do great damage to the housing and administrative facilities of the canal and could result in great loss of life but, disastrous as the loss of life would be, the operation of the sea-level canal could go on uninterrupted since ships in an emergency could transit the canal through the navigable pass under the pilotage of their masters.

Acknowledgments

The papers of this Symposium summarize the major studies, except those pertaining to security, made by the staff of The Panama Canal and its consultants and collaborators. Those who contributed to the studies are too numerous to name, but acknowledgment of their contribution is made in the report of the Governor of The Panama Canal under Public Law No. 280. The report consists of text and a folio of eight plates, supplemented by eight annexes and twenty-one appendixes. Appendix 21 of the report contains a complete bibliography for the Isthmian Canal studies. Particularly valuable assistance was provided by the Bureau of Yards and Docks and the Bureau of Ships, Department of the Navy; the Chief of Engineers, Department of the Army; and the Atomic Energy Commission. All major features of the study were reviewed by a Board of Consulting Engineers composed of Boris A. Bakhmeteff and William H. McAlpine, Hon. Members, ASCE; Hans Kramer, John J. Manning, Hibbert M. Hill, and Joel D. Justin, Members, ASCE; and until his retirement in September, 1946, Ben Moreell, Hon. M. ASCE. The writer was in immediate charge of the studies under J. C. Mehaffey, the Governor, The Panama Canal, who was charged by Public Law No. 280 with the responsibility for the studies.

TRAFFIC AND CAPACITY

BY RALPH P. JOHNSON[11] AND SYDNEY O. STEINBORN,[12]
ASSOC. MEMBERS, ASCE

SYNOPSIS

A forecast of future vessel movements through the Panama Canal and a study of the important characteristics of this traffic indicate that 16,660 vessels of all types may be expected to transit a Panama canal in the year 2000. The average daily traffic would be 46 vessels; the peak-day traffic for which the canal should be designed is 69 vessels. By about 1960, the capacity of the present canal, during periods of lock overhaul, would be insufficient to accommodate traffic without undesirable delays on peak days. With modifications principally to eliminate the periodic need for closure of one of the twin locks for overhaul, the capacity of the present canal could be increased to meet all commercial requirements until the end of the twentieth century. Because of the limited size of the lock structures, certain large naval and commercial vessels would be unable to transit the canal.

The construction of new and larger locks or the conversion of the present canal to sea level would create sufficient capacity to handle all traffic until well beyond the year 2000.

INTRODUCTION

The studies under Public Law No. 280, Seventy-ninth Congress, included investigations of the methods of increasing the capacity of the Panama Canal to meet the future needs of interoceanic commerce and national defense. This aspect of the studies required a prediction of the volume and characteristics of canal traffic at some future date which, for planning purposes, was selected as the year 2000. As stated in the first Symposium paper, the prediction was based on a recent evaluation of future Panama canal commercial traffic made by Professor Kramer of the University of Pennsylvania, who was engaged as a traffic consultant by The Panama Canal. The capacities of the present Panama Canal, of two improved Panama lock canals, and of a Panama sea-level canal were then analyzed and their capacities were compared to the estimated design peak-day demand for transits to determine when demand would exceed capacity. These comparisons provided an index of the date at which new facilities would be needed at Panama and of the relative adequacy of the proposed canal improvements.

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Ocean-Going Commercial Traffic

The largest group of vessels transiting the Panama Canal is that which pays tolls. It consists of merchant vessels, foreign naval vessels, privately-owned dredges, yachts, and similar craft. Tolls-paying vessels of more than 300 tons are considered to be ocean-going commercial vessels. In peacetime these make up about 78.5% of all transits, and carry nearly all the commercial cargo passing through the Panama Canal. Tolls-paying vessels of less than 300 tons are engaged principally in local trade and carry an insignificant volume of the commercial cargo.

Future Tonnage of Ocean-Going Commercial Traffic.—Professor Kramer's estimate of future Panama Canal traffic is concerned only with cargo carried in ocean-going commercial vessels. His forecast is based on the following assumptions:

a. No widespread wars or serious political disturbances will take place during the period covered by the forecast.

b. The effects of unforeseeable changes in the sciences, such as development of atomic power and of air transport, are not appraised.

c. Foreign trade will develop as the United Nations organization achieves stature and as trade restrictions are reduced.

d. The United States will maintain a substantial merchant marine in accordance with the mandates of the merchant marine acts of 1920, 1928, and 1936, and of the Ship Sales Act of 1946.

Professor Kramer's forecast, termed an "economic-statistical projection," correlates Panama Canal commercial traffic with the trend in United States national income as based on future United States population and future per capita income. In this forecast, an estimate of future United States ocean-borne imports, exports, and intercoastal trade was made according to the past relationships between national income and these three factors. Next, the past relationships of Panama Canal commercial traffic to these factors were determined. The relationships established from these estimates were used to derive estimates of future intercoastal, United States import, and United States export traffic through the canal. A fourth factor of Panama Canal commercial traffic, "foreign-to-foreign" shipments, was determined on a percentage basis from past records. The summation of the four components of Panama Canal commercial traffic, intercoastal, United States import, United States export, and "foreign-to-foreign" shipments, resulted in the following net-vessel tonnage forecast of predicted growth of ocean-going traffic (see Fig. 1):

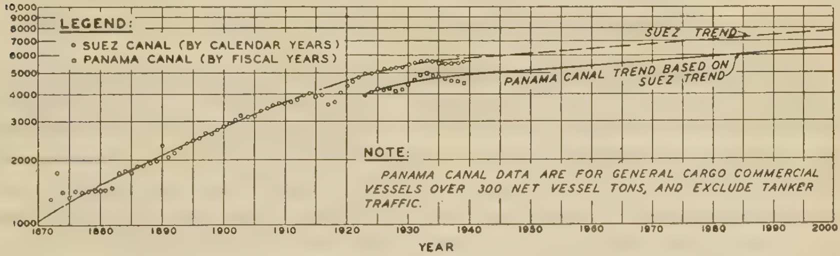
Year	Net vessel tons
1950	36,185,000
1960	44,929,000
1970	54,639,000
1980	65,207,000
1990	75,502,000
2000	86,312,000

This prediction is considered to represent the probable maximum load that will be put on the canal in the future.

For purposes of comparison, earlier predictions of Panama Canal traffic, made by Emory R. Johnson in 1912 and Grover G. Huebner in 1936—both of the University of Pennsylvania, are shown in Fig. 1. As a matter of interest, the traffic history of the Suez canal is also shown.

Conversion of Ocean-Going Commercial Tonnage to Transits.—The number of ocean-going commercial transits expected annually is obtained by dividing Professor Kramer's estimate of yearly tonnage by the estimated average net vessel tonnage per ocean-going commercial transit. The estimate of the average tonnage of ocean-going commercial vessels is based on past experience at Panama and on long-term trends developed at the Suez Canal.

The average net vessel tonnage of oil tankers transiting the Panama Canal has been about 30% higher than that of vessels carrying other types of cargo (general cargo vessels). Also, the relative volume of tanker traffic in the past is very large when compared to its future relative volume as predicted by Professor Kramer. For these reasons general cargo traffic at Panama was analyzed separately from tanker traffic.



NOTE: PANAMA CANAL DATA ARE FOR GENERAL CARGO COMMERCIAL VESSELS OVER 300 NET VESSEL TONS, AND EXCLUDE TANKER TRAFFIC.

FIG. 12.—AVERAGE NET VESSEL TONNAGES OF GENERAL CARGO SHIPS

The average tonnage of ocean-going general cargo vessels at Panama was determined for 17 "normal" years (from 1923 to 1939) during which traffic was relatively independent of influences arising from World War I and World War II. Because the Panama Canal statistics covered so few years, no well-defined trend could be detected in a plot of these data. Accordingly, the plot was compared with the average tonnage of vessels transiting the Suez Canal in the years from 1870 to 1939 (Fig. 12). The trend of Suez data is well defined and may be extrapolated to the year 2000. The average tonnage of vessels transiting at Suez was thus assumed to increase from 5,600 tons in 1939 to 7,800 tons in the year 2000. As the Panama Canal data are roughly parallel to those at Suez, the Panama Canal trend was assumed parallel to that of Suez and the average tonnage of general cargo vessels transiting at Panama was assumed to increase from about 4,500 tons in 1939 to 6,600 tons in the year 2000. Because of the predicted relative insignificance of tanker traffic, the trend in the average tonnage of general cargo vessels is assumed to be applicable to all ocean-going commercial traffic. The number of ocean-going commercial transits for any year up to the year 2000 is then obtained by dividing the estimated tonnage for that year (Fig. 1) by the estimated net vessel tonnage per transit for the corresponding year (Fig. 12). The predicted increase in the number of ocean-going commercial transits is shown in Fig. 3 and in Col. 2, Table 6.

TABLE 6.—PREDICTED GROWTH OF VESSEL TRANSITS THROUGH THE PANAMA CANAL

Year (1)	Ocean-going commercial vessels (2)	Small commercial vessels (3)	Tolls-free vessels (4)	Total transits (5)	Average daily transits (6)
1950	6,959	567	1,339	8,865	25
1960	8,199	669	1,577	10,445	29
1970	9,486	773	1,825	12,084	34
1980	10,796	880	2,077	13,753	38
1990	11,947	974	2,298	15,219	42
2000	13,078	1,066	2,516	16,660	46

In the absence of any reason for change, it is expected that in peacetime ocean-going commercial vessels will comprise the same percentage of canal traffic in the future that they have in the past—namely, 78.5%.

TRAFFIC OF SMALL COMMERCIAL VESSELS

Tolls-paying vessels of less than 300 tons—small commercial vessels—are considered separately from ocean-going commercial vessels. Although these vessels make up about 6.4% of all transits, they carry only 0.1% of the commercial cargo passing through the canal. However, they are a tax on the capacity of the canal in that they are vessels that must be handled and therefore they cannot be disregarded. It is expected that the traffic of these vessels will increase at a rate corresponding to that of ocean-going commercial traffic and thus will continue to comprise 6.4% of total Panama Canal transits in peacetime. The predicted growth in the traffic of small commercial vessels is shown in Col. 3, Table 6.

TOLLS-FREE TRAFFIC

International treaties and federal legislation permit several classes of vessels to transit the canal without charge. They consist of United States Navy combat and auxiliary vessels, United States Army vessels, cargo-carrying or other vessels operated in the service of the United States Government, warships of the Republic of Colombia, Panamanian Government vessels, vessels transiting solely for the purpose of repair at Canal zone shops, and Panama Canal operational and maintenance equipment. In peacetime, these tolls-free vessels account for 15.1% of all transits. There is no indication that the relative volume of this traffic is changing and therefore the 15.1% factor is used to determine the future peacetime volume of all tolls-free traffic (Col. 4, Table 6).

FUTURE TRAFFIC TOTALS

Total Panama Canal traffic in the future will consist of all commercial traffic and all tolls-free traffic whose predicted values are shown in Fig. 2 and in

Cols. 5 and 6, Table 6. These totals are based on percentage factors derived for peacetime years. The volume of military combat and cargo traffic in future wars is indeterminate, but experience during World War II indicates that commercial traffic volume would drop and tend to offset the increase in military traffic. Because there is no assurance that the drop in commercial traffic will completely offset the rise in military traffic, a canal should have some excess capacity to meet the unknown demands that may be put on it during wars.

TRAFFIC CHARACTERISTICS AFFECTING CAPACITY

Variations in Daily Traffic.—The traffic characteristic which controls the capacity requirement of the Panama Canal is the daily variation in the demand for transits. Annual and seasonal variations in traffic volume are so much less than the daily variation that they are of no significance in establishing the capacity requirement of the Panama Canal. Daily variations in traffic volume have been extreme; immediately before World War II, however, the number of transits on the peak day of the year had decreased and appears to be leveling off at 160% of the number of transits on the average day.

The frequencies with which peak traffic days are expected to occur are shown in Table 7. It is evident that very few delays to shipping would occur if the capacity requirement were established so as to provide for traffic peaks 50% in excess of the daily average. With this daily capacity, traffic would be held over to the next day on an average of only 9 days a year, and, in the year 2000, delays would be of such an order that about 20 vessels would be held up a total of about 400 ship-hours with 24-hour operation of the canal in effect. This service is compatible with the high standards expected of the Panama Canal. Therefore, days in which the traffic is 50% in excess of the average are used to establish the capacity requirement of the Panama Canal and are called "design peak days." In the year 2000, this requirement is the capacity to transit 69 vessels of all types in 24 hours.

TABLE 7.—PREDICTED FREQUENCY OF PEAK DAYS, PANAMA CANAL

Peak-Day Conditions		Days per year in which peak-day traffic would be exceeded
Percentage of average daily traffic	Transits in the year 2000	
130	60	45
140	64	24
150	69	9
160	73	0

Direction of Traffic.—In the past, 52% of the yearly ocean-going commercial transits passed through the canal from the Atlantic to the Pacific and 48% transited from the Pacific to the Atlantic. From this, it is assumed that, in effect, future traffic will be evenly divided with respect to direction. The assumption applies only to annual traffic; on individual days traffic may be considerably unbalanced and traffic in any one direction may be nearly three times as great as traffic in the opposite direction. Under certain conditions, the expected maximum unbalance of traffic on peak days would reduce the maximum capacity of a lock canal slightly, but would have no significant effect on the capacity of a sea-level canal. The effect of unbalanced traffic on the capacities of lock canals is described subsequently in this paper.

Vessel Length.—The number of vessels which can be placed in a given lock chamber at one time is controlled by their lengths. This has an important effect on the capacities of the lock structures in a lock canal or in any tidal locks used in a sea-level canal. Using the records of the Panama Canal (from 1921 to 1940), traffic has been grouped into the vessel-length categories shown in Table 8.

TABLE 8.—COMPOSITION OF ALL TRAFFIC THROUGH THE PANAMA CANAL FROM 1921 TO 1940 BY LENGTH OF VESSEL

Length (ft)	Percentage	Transits
> 500	6.2	8,004
400 to 499	47.3	60,870
300 to 399	20.0	25,806
200 to 299	7.3	9,395
< 200	19.2	24,535
Total	100.0	128,610

The records show that the composition of traffic by vessel lengths has varied little from year to year. However, studies made of the relations of net vessel tonnage, beam, length, and tonnage per foot of length, indicated that the predicted increase in average net vessel tonnage would be accompanied by a slight increase in vessel length. Furthermore, it was indicated that a 10-ft increase in the average length of all vessels would be sufficiently conservative for estimating the capacities of lock structures in the year 2000. It was assumed that this increase would be uniform from the present to the year 2000. Table 9 shows the percentages of vessels that may be expected in each length category for the years 1960 and 2000.

TABLE 9.—PREDICTED COMPOSITION OF ALL TRAFFIC THROUGH THE PANAMA CANAL BY LENGTH OF VESSEL

Vessel length (ft)	PERCENTAGE OF TRANSITS		Transits, year 2000
	1960	2000	
> 500	8	11	1,832
400 to 499	46	45	7,498
300 to 399	20	19	3,165
200 to 299	7	7	1,166
100 to 199	8	8	1,333
< 100	11	10	1,666
Total	100	100	16,660

Experience at Panama indicates that the composition of traffic by length of vessel on peak days is very similar to the average annual composition. This fact is of particular significance in establishing the capacity requirements of lock structures which should be able to accommodate shipping on design peak days.

DIMENSIONAL LIMITATIONS OF PANAMA LOCK AND SEA-LEVEL CANALS

The maximum size of vessel that could transit a lock canal or that could pass through the tidal lock of a sea-level canal would be limited by the dimensions of the lock chambers. At present only two commercial vessels, the *Queen Mary* and the *Queen Elizabeth*, are too large to be received in the existing 110-ft by 1,000-ft lock chambers. These vessels are used in the North

Atlantic service and would not be expected to operate through the canal. Until the approach of World War II, the lock chamber dimensions of the Panama Canal were accepted as a limitation in the design of Navy vessels. However, several warships constructed or modified during the war exceed the dimensions of the present locks and are therefore unable to utilize the canal. The proposed lock chamber dimensions of 200 ft by 1,500 ft for either a high-level lock of a lock canal or the tidal lock of a sea-level canal would permit the transit of the largest commercial vessels expected up to the year 2000 and would allow a considerable increase in the size of naval vessels without prohibiting their transit. The 750-ft-wide navigable pass of the sea-level canal would permit the transit of any type of vessel of any size likely to be built in the twentieth century.

Capacity of the Present Panama Canal

The capacity of a lock canal is established by the capacity of its locks as reduced by any conditions which affect the availability of the locks for the transit of vessels or which prevent vessels from entering the locks. Examples of such conditions are the occurrence of fog, restrictions fixed by channel dimensions, or lay-up of the locks for overhaul.

The length of the lock chamber, as related to lengths of future vessels, determines the average number of vessels per lockage. Using the length composition shown in Table 9, it was found that, under ideal conditions, an average of 1.8 vessels could be transited simultaneously in the existing locks (110 ft wide by 1,000 ft long). For conservatism, the estimated number of the vessels per lockage was reduced to 1.5.

The time interval between successive lockages in the same direction has been determined from time and motion studies of operations on the existing locks, and amounts to 45 min with double-culvert filling and emptying of the lock chambers. Thus, operating 24 hours daily under ideal conditions, the present locks could accommodate a total of 96 vessels.

An unbalanced flow of traffic in either direction would require some changes in the direction of lockages. The time interval between successive multiple-lift lockages in opposite directions in the same lane is greater than that for successive lockages in the same direction because, in the first instance, the incoming vessel must wait until the outgoing vessel has cleared the lock structure before entering. The longer interval results in a loss of capacity; in the present canal, however, the physical limitations of Gaillard Cut which necessitate one-way traffic at night would give ample opportunity to balance traffic and capacity losses from this cause would not occur.

The frequency with which fogs occur in the Panama Canal is such that the dependable daily capacity of either a lock or a sea-level canal must be based on the assumption that fog is a daily occurrence. Fogs in Gaillard Cut are most frequent during the rainy season and occur generally between midnight and 8 a.m., with an average duration of 4 hours. At present, it is hazardous for a ship to enter Gaillard Cut whenever there is a possibility of fog. This possibility exists during the wet season and, thus, for 8 months a year Gaillard Cut is closed 8 hours a day. The effect of this closure is a loss in capacity of 32 vessels daily.

Traffic through Gaillard Cut is limited to one direction in the case of unwieldy vessels or of vessels with more than an 80-ft beam, for vessels carrying explosive cargoes, and for all vessels at night. This restriction results in a loss of capacity of 6 vessels daily.

The locks are overhauled biennially at Panama, alternating between the Pacific and Atlantic locks. Thus each lock structure is overhauled once every 4 years. The necessary repairs and replacements are completed in about 4 months, and during that time only one lock lane is available. For most of the overhaul period, only a single culvert is available for filling and emptying the lock chambers, and, therefore, the time interval between successive lockages in the same direction is lengthened to 56 min. The daily lock capacity during overhaul is then 39 vessels.

Since only one lock lane is available during overhaul, the direction of lockages must be changed to permit transits in both directions. Two such changes are assumed to occur daily and this results in a daily loss in capacity of 3 vessels during periods of overhaul.

Overhaul is performed in the dry season when fogs are rare and therefore there are no losses in capacity because of fogs during overhaul periods. Since traffic during the overhaul period is essentially single lane traffic, there are no losses in capacity at such times resulting from traffic restrictions in Gaillard Cut.

Dependable Daily Capacity of Present Canal.—The estimated capacity of the present canal, based on the factors previously described, is developed in Table 10. Table 10 shows that the daily capacity of the present canal is a minimum during periods of lock overhaul. Each lock overhaul lasts 4 months, and it is neither reasonable nor practical to delay vessels over such an extended period. Therefore, the dependable daily capacity of the existing canal is taken at its minimum daily capacity during overhaul—or 36 vessels.

TABLE 10.—DAILY CAPACITY OF THE PRESENT PANAMA CANAL (NUMBER OF VESSELS)

Line	Description	Outside of overhaul periods[a]		During overhaul periods[b]	
1	Theoretical capacity		96		39
	Capacity Losses Resulting from—				
2	Changes in direction of transit	0		3	
3	Possibility of fog in Gaillard Cut	32		0	
4	Restricted channel in Gaillard Cut	6	−38	0	−3
5	Net capacity		58		36

[a] Operation outside of overhaul periods; maximum theoretical capacity of two lock lanes, with double-culvert filling and 1.5 vessels per lockage. [b] Operation during overhaul periods; four consecutive months every two years; maximum theoretical capacity of one lock lane, with single-culvert filling and 1.5 vessels per lockage.

CAPACITY OF THE PRESENT CANAL WITH MINOR IMPROVEMENTS

Table 10 indicates that the greatest improvement for increasing the capacity of the present canal would be to eliminate the necessity of taking one lock lane out of service during overhaul periods. Such an improvement is entirely feasible and could be accomplished by modification of the lock gates, gate settings, and culverts. Lock filling and emptying would then be done with a single culvert for each lock lane while the valves in the third culvert were being serviced. This operation would take place in the fog-free dry season to permit 24-hour operation of the locks. Under these conditions, the net capacity of the canal would be the capacity of both lock lanes, each in single-culvert operation, less losses resulting from channel restrictions imposed by Gaillard Cut—or 70 vessels daily. This is in excess of the 58-vessel daily capacity of the present canal outside of overhaul periods, and the latter capacity would become the controlling capacity of the canal.

To increase the dependable daily capacity of the canal to more than 58 vessels, additional improvements are necessary. The most economical of such improvements is the provision of tie-up stations in Gaillard Cut. These would permit transits to continue until the actual occurrence of fog and it is estimated that they would increase the dependable daily capacity of the canal to 65 vessels.

The foregoing modifications constitute the minimum plan of improvement of the existing canal and would provide adequate capacity to handle traffic until close to the end of the twentieth century.

If a further increase in capacity is desired, it can be obtained by providing the navigation aids that would permit one-way navigation in fog. With navigation in fog the daily capacity of the canal outside of overhaul periods would be 96 vessels less losses caused by channel restrictions imposed by Gaillard Cut—or 86 vessels. However, the daily capacity of the canal during the modified overhaul period would be 70 vessels. Since this is the smaller capacity, it is controlling, and represents the increase in capacity achieved by use of navigation aids. This capacity slightly exceeds the expected design peak-day demand for transits in the year 2000.

Capacity of a Completely Modernized Panama Lock Canal

Complete modernization of the present Panama lock canal would provide for the construction of two new locks (200 ft by 1,500 ft) at each end of the canal, abandonment of the existing locks, widening of Gaillard Cut permitting two-way navigation at night, raising of Miraflores Lake to the level of Gatun Lake to obtain a Pacific summit-level anchorage, and the construction of tie-up stations throughout Gaillard Cut. Using the length composition of vessels for the year 2000 shown in Table 9, an average of 2.9 vessels, under ideal conditions, could be locked through the proposed locks in tandem. For conservatism, however, it was assumed that the locks could accommodate an average of 2.4 vessels per lockage. With the time interval between successive lockages in the same direction estimated to be 57 min, the theoretical maximum capacity of the locks would be 120 vessels daily. Losses caused by an assumed single change in direction to provide for unbalanced traffic would reduce this to 118 vessels.

Without tie-up stations and summit-level anchorages, the loss in capacity in the 8-hour period during which Gaillard Cut would be closed because of the occurrence or possibility of fog would amount to 40 vessels daily.

The provision of a Pacific summit-level anchorage to match that at Gatun would permit vessels to lock-up to the summit level when Gaillard Cut was closed because of fog. With this operating procedure, the loss in capacity due to fogs would be reduced from 40 vessels daily to 25 vessels daily.

With tie-up stations, vessels could proceed through Gaillard Cut until the actual occurrence of fog. The loss in capacity due to fogs would then be further reduced and would amount to 15 vessels daily. The dependable daily capacity of a completely modernized lock canal would therefore be 103 vessels (118 minus 15)—well in excess of the requirement for the year 2000.

The time required to transit, if two new and larger locks were constructed at each end of the canal, would be 7.25 hours, as compared with 8 hours in the present canal under normal operations and 8.5 hours during overhaul periods.

Capacity of the Panama Sea-Level Canal

The capacity of the Panama sea-level canal is established by the conditions under which its tidal-regulating structures are assumed to operate, by the assumed vessel speed and spacing, and by the duration of fogs along the channel.

The tidal-regulating structures of a sea-level canal would consist fundamentally of a tidal lock and a navigable pass. Shipping would utilize the tidal lock when the tidal head between the canal and the Pacific Ocean was large, and the navigable pass when tides were at or near their mean.

The amount of tidal regulation necessary to limit the channel currents to a selected maximum value depends upon the range of the Pacific tides. (This subject is treated subsequently in the fourth Symposium paper.) The capacity of the tidal-regulating structures increases with increased use of the navigable pass—that is, inversely with the tidal range. The dependable capacity of the sea-level canal using tidal regulation has been taken as its capacity on days when 20-ft tides would prevail. Since the navigable pass would be open for longer periods 98% of the time, the assumption is extremely conservative.

The tidal-regulating structures would be designed to control the currents to any value between the maximum velocity resulting from the Atlantic tides alone (about 0.5 knot) and the maximum velocity of about 4.5 knots produced in an uncontrolled waterway by the combination of both Pacific and Atlantic tides. A maximum allowable current of 2 knots has been selected in these studies as the basis upon which to establish the dependable capacity of the sea-level canal.

Vessel Speed and Spacing.—The capacity of a channel depends on vessel speeds and vessel spacings; in these studies an average water speed of 10 knots has been assumed. The factors that bear on the safe distance between vessels traveling in the same direction are their speed, size, power, loading, controllability, operating dependability, channel dimensions, alinement, visibility, weather, and currents. In consideration of these factors, the spacings shown in Table 11 for vessels more than 300 ft long have been selected for the purpose of estimating capacity. Vessels less than 300 ft long can be interspersed with larger vessels spaced in accordance with Table 11 without impairing navigational safety. If vessels were to travel at higher speeds, greater spacing would be required for safety and the net effect would be a reduction in capacity. (Spacings in Table 11 are for vessels longer than 300 ft, traveling at an average water speed of 10 knots.)

Capacity of Tidal Lock.—The daily capacity of the tidal lock depends on the hours of operation, the time interval between lockages, and the number of vessels that would be assembled in the lock chamber at one time. The tidal locks would be the same size as the locks of a modernized lock canal and 2.4 vessels could be handled during each lockage. For 24-hour operation and an average time interval between lockages of 40 min, the daily capacity of the tidal lock would then be 86 vessels.

TABLE 11.—VESSEL SPACING

Channel current (knots)	Spacing (nautical miles)
0	1.0
2	1.5
4 to 5	2.0

Capacity of Navigable Pass.—When open, the navigable pass would give full freedom to the flow of traffic and its capacity would be the same as that for the sea-level channel. The daily capacity of the pass thus depends on the length of time it would be open for navigation and on the adopted vessel spacing (Table 11). If the tidal-regulating structures were operated to limit channel currents to 2 knots, the pass would be open about 4.9 hours during each day having 20-ft tides. The daily capacity of the navigable pass would then be 88 vessels. An increase in the maximum allowable current would permit the pass to remain open for longer periods, and the capacity of the pass would thereby be increased. The daily capacity of the navigable pass for various tides and allowable current is shown in Table 12.

TABLE 12.—DAILY CAPACITY OF NAVIGABLE PASS

Maximum allowable current (knots)	DAILY CAPACITY IN VESSELS FOR A TIDAL RANGE OF:		
	20 ft	16 ft	13 ft
1	53	66	80
2	88	110	130
3	140	180	240
4	260	320	320

TABLE 13.—CAPACITY OF SEA-LEVEL CANAL WITH TIDAL REGULATION (16-HOUR OPERATING DAY)

Maximum allowable current (knots)	NUMBER OF VESSELS PASSING THROUGH:			Notes
	Lock	Pass	Lock and pass	
1	56	35	91	[a] For currents of 4 knots or more, the combined capacity of pass and lock exceeds channel capacity, so the latter controls.
2	57	59	116	
3	59	93	152	
4	61	174	214[a]	

Capacity of Sea-Level Canal with Tidal Regulation.—The 24-hour daily capacity of the sea-level canal with tidal regulation as assumed in these studies would be the combined capacities of the pass and lock—or 174 vessels of all sizes (86 vessels through the tidal lock and 88 vessels through the navigable pass).

Because there are no anchorage areas above the tidal lock or navigable pass, they would not be used when the channel is fogbound, and the capacity of the canal would be reduced accordingly. Observations made prior to the formation of Gatun Lake indicate that fogs in a sea-level canal would have an average duration of 6.6 hours and would reduce the operating day from 24 hours to 17.4 hours and the daily capacity of a controlled sea-level canal from 174 vessels to 126 vessels. In a 16-hour operating day, with operations suspended during the hours in which fog usually occurs, the daily capacity would be as shown in Table 13. It can be noted that, even with the most conservative control of tidal currents, the capacity of a sea-level canal is in excess of the 69-vessel daily capacity required for a Panama Canal in the year 2000.

The average transit time in a controlled sea-level canal would be either 4 hours or 4.75 hours, depending on whether the pass or the lock was used.

Capacity of Sea-Level Canal Without Tidal Regulation.—If operating experience should eventually indicate that tidal regulation is unnecessary or if, during wartime, the regulating structures were not utilized or were rendered inoperative, the canal would have no structures on which the transits of vessels would be dependent. Its capacity would therefore be the capacity of the channel, which is dependent solely on vessel speed and spacing and on the length of the operating day as established by the occurrence of fog.

Using an average water speed of 10 knots and a vessel spacing of 2 miles established for the 4.5-knot current resulting from a 20-ft tide (Table 11), the daily capacity of the canal would be 240 vessels longer than 300 ft. Vessels longer than 300 ft are expected to comprise only 75% of all vessels transiting the canal (Table 9), and therefore 80 additional vessels shorter than 300 ft could be interspersed with the larger vessels; the daily capacity would then be 320 vessels.

Fog of average duration would reduce the operating day from 24 hours to 17.4 hours and the daily capacity from 320 vessels to 232 vessels. If the canal, because of fog or for other reasons, is operated 16 hours a day, the daily capacity would be 214 vessels—far in excess of the requirements for the year 2000.

At an average water speed of 10 knots, the time required to transit the sea-level canal without tidal regulation would vary from 3.3 hours to 5.0 hours.

Summary

The present Panama Canal does not have sufficient capacity to prevent undesirable delays to traffic on design peak days beginning about 1960. With modification of the existing locks to eliminate extensive outages during overhaul periods and with tie-up stations provided in Gaillard Cut to reduce the delays created by fogs, the present lock canal would be adequate until close to the end of the century. In addition to these improvements, if navigation aids were provided which would permit navigation in fogs, the adequacy of the present canal would be further extended. This minimum plan of improvement, however, would not provide for the transit of vessels larger than those accommodated in the present locks.

The Panama sea-level canal or a completely modernized Panama lock canal with new and larger locks would have capacities greatly in excess of requirements for the year 2000. The modernized lock canal and the tidal lock of a sea-level canal would be able to transit any commercial vessel expected to be operating during the twentieth century. They would also permit the transit of much larger naval vessels than now exist. The navigable pass of the sea-level canal would permit the transit of any type of vessel of any size likely to be built prior to the year 2000.

The capacity of a lock canal is fixed by the physical dimensions and characteristics of its locks. In comparison, a sea-level canal has no such limitations, provides faster transit than does a lock canal, and has a larger reserve of capacity which can accept the short-time traffic peaks that may occur during wars.

FLOOD CONTROL

By F. S. Brown,[18] Assoc. M. ASCE

Synopsis

The development of a Panama sea-level canal would require that shipping be protected by adequate control of floods on the major rivers and streams tributary to the canal. If large uncontrolled flows entered the canal, the maximum velocities would be appreciably increased over those normally caused by tides, and dangerous crosscurrents would be generated at each point where large flood flows entered the canal. This paper defines the requirements for flood control in the proposed sea-level canal, describes the hydrological characteristics of the drainage basin, formulates the preliminary basis for hydraulic design of the flood-control projects, and develops the plan of control that has been adopted as an essential feature of the sea-level canal, wherein 87% of the tributary area would be diverted directly to the Atlantic Ocean and an additional 4% would be controlled by retarding reservoirs.

Requirements for Flood Control

Floods have rarely interfered with navigation in the present lock canal because all major tributaries of the watershed, except the upper Chagres River, enter Gatun Lake at remote distances from the navigation lane. Gatun Lake is impounded by a large earth dam 8 miles above the mouth of the Chagres River and has an area of 164 sq miles. The upper Chagres River enters the canal at Gamboa where the lake is narrow. Prior to the construction of Madden Dam in 1934, floods entering the canal at Gamboa twice required the temporary suspension of traffic. The control afforded by Madden Lake has largely ended the likelihood of futher major flood disturbances in the present canal.

Gatun Lake would be drained upon final conversion to a sea-level canal, and the rivers of the watershed, if uncontrolled, would enter the sea-level canal directly. The total area of the watershed (Fig. 13) would be 1,358 sq miles, 94% of which would lie on the Atlantic side of the Continental Divide. The largest tributary would be the Chagres River above Gamboa, with a drainage area of 520 sq miles, of which 393 sq miles are controlled by Madden Dam. Other important tributaries would be the Gatun River, draining 150 sq miles on the east side of the canal, and the Caño Quebrado and Trinidad rivers on the west side, with drainage areas of 120 sq miles and 313 sq miles, respectively. The watershed on the Pacific side of the Continental Divide is only 79 sq miles.

Combined Flood and Tide Flow in a Sea-Level Canal.—Tidal ranges of 20 ft in the Pacific and 2 ft in the Atlantic, which closely approximate maximum tide conditions, would produce a velocity of 4.5 knots near the Atlantic end of

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[18] Chf. of Gen. Eng. Branch, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

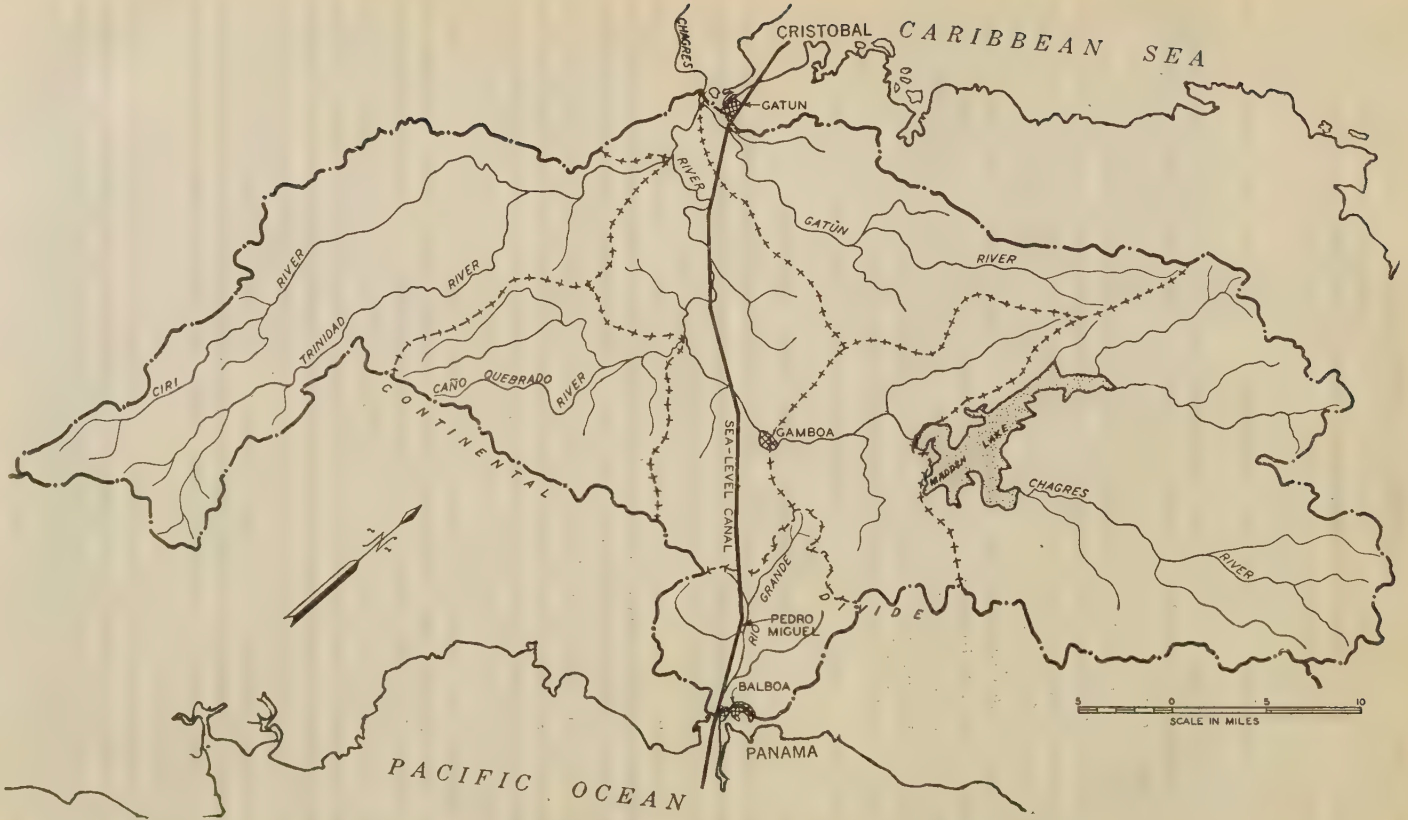
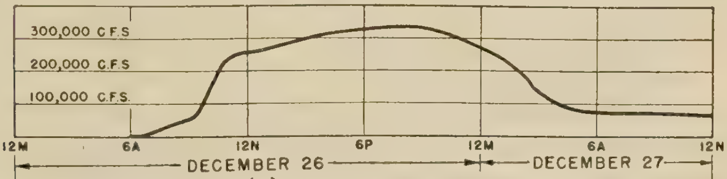
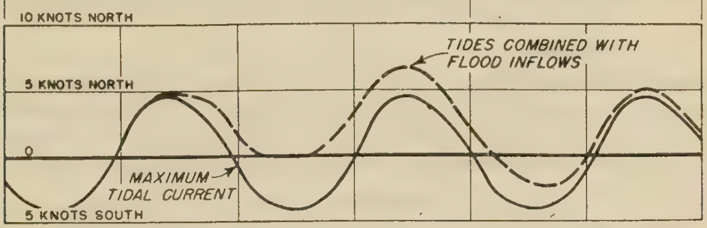

CARIBBEAN SEA
CRISTOBAL
CHAGRES
GATUN
RIVER
GATÚN
RIVER
RIVER
RIVER
CIRI
TRINIDAD
CAÑO
QUEBRADO
RIVER
CONTINENTAL
DIVIDE
GAMBOA
SEA-LEVEL CANAL
MADDEN LAKE
CHAGRES
RIVER
RIO GRANDE
PEDRO MIGUEL
BALBOA
PANAMA
PACIFIC OCEAN
N
5 0 5 10
SCALE IN MILES

Fig. 13.—Watershed, Sea-Level Canal

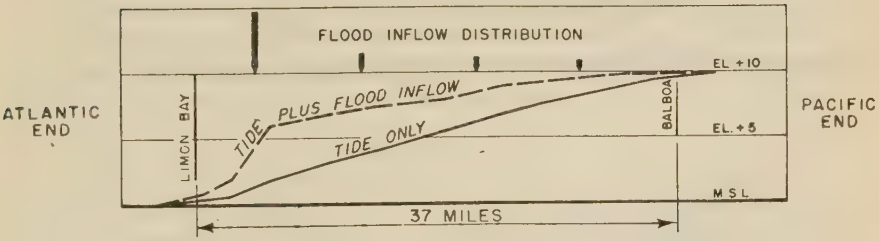
an uncontrolled canal. Ships of adequate power and good controllability could safely transit the canal in currents of 4.5 knots, but any appreciable increase in velocity would be undesirable. Tidal-regulating structures would be installed at the Pacific end of the sea-level canal for the reduction of tidal currents to limits probably as low as 2 knots in the initial period of canal operation. Flood control would be needed regardless of the extent of tidal regulation, although disturbances to shipping from floods would be more pronounced in an unregulated canal, a condition that may be imposed during wartime if the tidal-regulating structures were destroyed.



(a) FLOOD INFLOW



(b) VELOCITIES



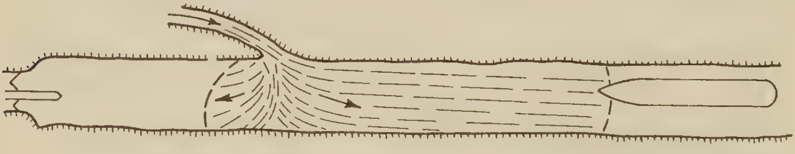
(c) WATER SURFACE PROFILES

FIG. 14.—VELOCITIES OBSERVED IN MODEL TESTS TO DETERMINE EFFECTS OF 1909 FLOOD

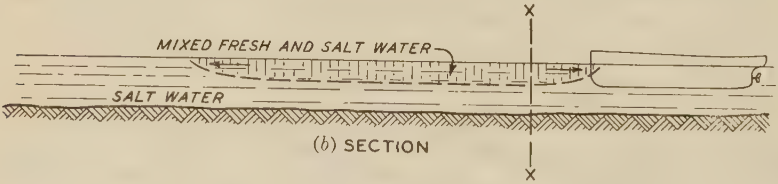
The effect of large floods on velocities in an uncontrolled canal has been demonstrated in the hydraulic model of the sea-level canal by superimposing the flood of December 26–30, 1909, upon the conditions producing maximum tidal flow. This flood, largest of record on the Chagres River at Alhajuela, was extended with suitable adjustments to cover the entire watershed. The net rates of flood inflow into the canal were determined for two degrees of control: Case I, with only Madden Lake in operation (393 sq miles controlled), and case II, with a comprehensive flood-control system in operation (1,240 sq miles controlled). Fig. 14 summarizes the test results for case I and includes a comparison with tide flow only. Flows were introduced in the canal at four locations corresponding to the main tributaries, those entering the canal at Gamboa being modified to reflect the control afforded by Madden Dam. The tests were run without tidal regulation. The profiles of Fig. 14 demonstrate that the gradient of flow is materially flattened upstream from the point of largest inflow. This flattening tends to retard the entrance of tidal flow into the canal and, as a consequence, the combination of tide and flood flow would be less than the arithmetic sum of the separate flows. Maximum combined velocities observed during these tests (open canal; 20-ft tide) were as follows:

Condition	Velocities (knots)
Case I, flood control	7.0
Case II, flood control	5.0
No flood inflow	4.5

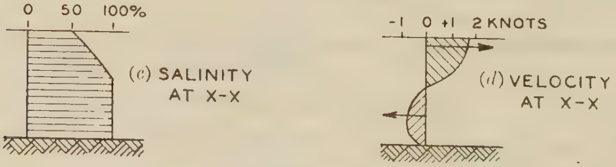
The high velocity of combined flow given by case I and the virtual restoration of the combined velocity in case II to the value for natural tidal flow are indicative of the requirement and effect of a large measure of flood control.



(a) PLAN



(b) SECTION



(c) SALINITY AT X-X

(d) VELOCITY AT X-X

FIG. 15.—INFLOW TESTS, EXISTING PANAMA CANAL

Density Currents in the Sea-Level Canal.—The foregoing tests were performed using fresh water for both flood and tidal flow and hence do not fully represent the conditions that would arise in nature. From observations in tidal estuaries, it is known that fresh-water and salt-water flows tend to retain separate identity, the lighter fresh water, of course, remaining on top of the salt water. Recent full-scale measurements have been made of fresh water from a tributary stream flowing into the salt water of the sea-level approach channel to the Miraflores Locks. The results of these field tests are summarized in Fig. 15.

Fig. 15(a) indicates the pattern of currents near the surface, and the movement of the fresh-water front as it spreads out over the salt water, The successive locations of this front were observed with electrical salinity meters.

The water near the surface of the canal was not wholly fresh water but a mixture of fresh and salt, starting with about 50% salinity at the top and increasing to 100% salinity at about half the channel depth. The water moved in one direction near the surface and in the opposite direction near the bottom. For a tributary inflow of 24,000 cu ft per sec, the velocities in the canal averaged 1.8 knots in the upper zone—or approximately three times the velocities that would have occurred had the discharge over the full cross section been uniform.

Summary of Requirements.—It is difficult to define specific criteria for flood control, since the necessity for control is measured by the safety and efficiency of navigation in the canal, the standards for which are largely qualitative. As a result of the tests and studies performed on flood currents in the canal, however, little doubt exists that the plan of control should be comprehensive, approaching virtual elimination of flood disturbances.

The Drainage Basin

Topography.—The principal subdivisions of the Panama Canal drainage area are illustrated in Fig. 13. The watershed is characterized by conical hills of relatively low relief in the vicinity of the canal, changing to irregular knife-like ridges in the upper regions. Peak elevations of from 500 ft to 1,000 ft above sea level near the canal increase to 2,000 ft in the Gatun River basin, 2,500 ft in the upper watershed of the Chagres River, and 3,500 ft in the headwaters of the Trinidad River. The general steepness of the watersheds produces a rapid concentration of runoff.

Surface Cover.—In the thickly-forested areas of the upper regions, dense foilage prevents sunlight from reaching the ground, and consequently the ground cover is sparse and offers little deterrent to rapid runoff. In regions of lower elevation, the forest cover is less dense, the undergrowth is thick, and open areas are covered with a rank growth of grass, causing rainfall losses and the retardation of runoff to be more pronounced.

Climate.—The Canal Zone region has a distinct seasonal variation of rainfall, a uniform air temperature, and a high relative humidity. The dry season generally extends from January through April, and the wet season spans the remaining 8-month period from May through December. The mean monthly air temperature in the Canal Zone varies only from 2° F to to 3° F for the entire year, and the range between extreme temperature is less than 40° F. The highest recorded temperature is 98° F and the lowest is 59° F, both occurring at Madden Dam.

Rainfall.—The mean monthly rainfall at the Atlantic and the Pacific ends of the canal is shown in Fig. 16. Approximately 92% of the mean annual rainfall occurs during the wet season, and 16% of the annual rainfall occurs in November. All major floods in the Canal Zone region have occurred in the last 3 months of the year.

Fig. 17 is an isohyetal map of the mean annual rainfall on the canal drainage area. Note the gradation of rainfall across the Isthmus, ranging from depths

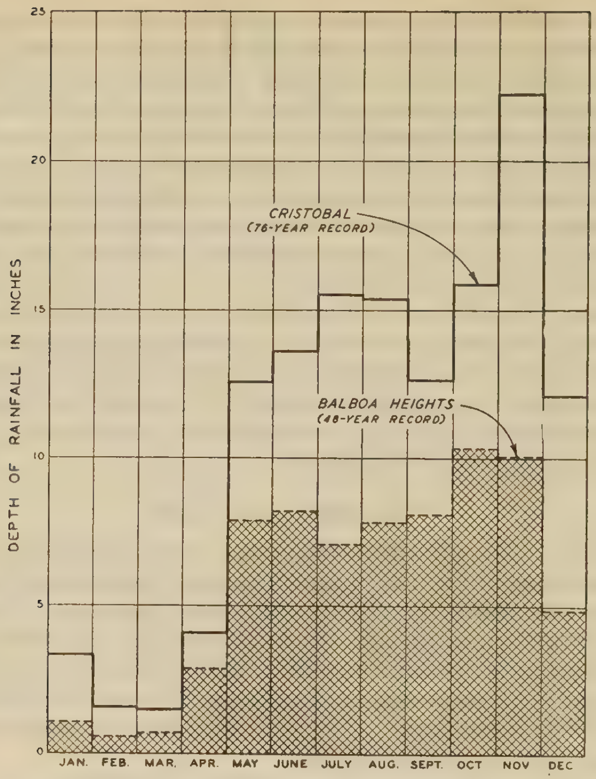


Fig. 16.—Mean Monthly Rainfall

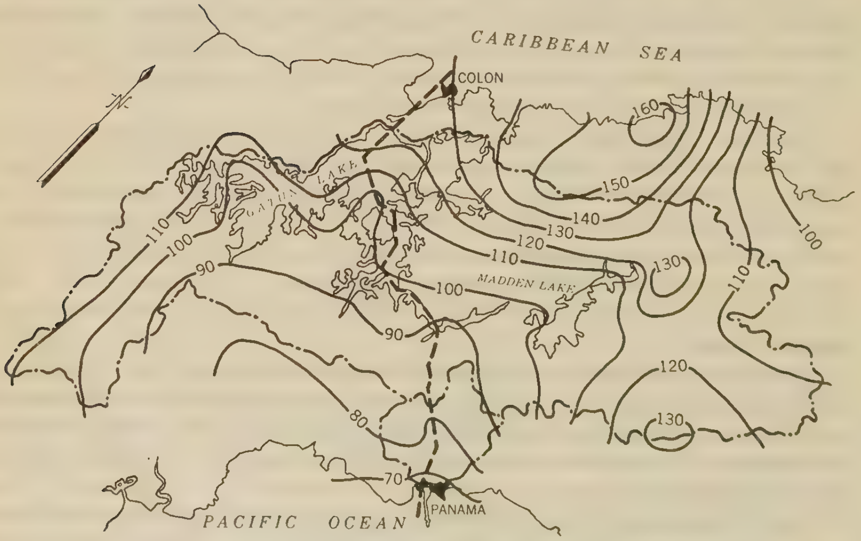

CARIBBEAN SEA
COLON
N
GATUN LAKE
MADDEN LAKE
PACIFIC OCEAN
PANAMA
160
150
140
130
120
110
100
110
100
90
100
90
80
70
130
110
100
120
130

Fig. 17.—Isohyetal Map of Mean Annual Rainfall

of 160 in. on the Atlantic coast to 70 in. on the Pacific coast. This gradation is also characteristic of monthly rainfall and occurs during many of the major flood-producing storms.

A large part of the rainfall results from convective activity in which the areal distribution is limited, the duration is a few hours, and the intensity is relatively high. Large flood-producing storms result from frontal activity and generally extend over the entire watershed of the canal. Rainfall caused by frontal activity usually has a lower intensity but may continue intermittently for a week or longer in storm periods having a succession of frontal passages. Maximum point rainfall in the Canal Zone region and at several locations in the United States is shown in Table 14.

TABLE 14.—MAXIMUM POINT RAINFALL; CANAL ZONE REGION AND UNITED STATES

(Rainfall Depth in Inches)

Duration	Canal Zone and vicinity	First Order Weather Bureau Stations				
		Los Angeles, Calif.	New Orleans, La.	Key West, Fla.	St. Louis, Mo.	Washington, D. C.
5 min.	0.90	0.44	0.77	0.65	0.60	0.80
10 min.	1.76	0.66	1.20	1.03	1.04	1.21
1 hour	5.68	1.51	3.66	4.30	3.47	3.42
24 hours	13.62	7.36	14.01	13.54	8.78	7.31
Annual	237.3	40.3	85.7	58.5	68.8	61.3

Rainfall and Discharge Records.—Heterogeneous rainfall and stream-flow records are available to the hydrologic investigator, some dating back to the 1860's. The primary objective of past observations has been the collection of data for definition of the dependable supply of water for lockages rather than for the development of flood-control projects. Although the number and distribution of rainfall stations in operation in recent years, particularly since 1941, have been sufficient to approximate a pattern of the mean monthly and mean annual rainfalls on the watershed, the records are insufficient for accurate study of storm and flood relations in any tributary area. The best records of discharge are those of the upper Chagres River where observations were started at Alhajuela in 1899 and have been continued to the present time, although the construction of Madden Dam in 1943, immediately upstream, prevented further recording of natural flood discharges at this station. Five rainfall stations have been in operation above Alhajuela since the early 1930's and three more were added in 1941. The general inadequacy of the records has made it necessary to develop data for the hydraulic design of the various projects largely from the Chagres River records.

Comparative Distribution of Rainfall.—Table 15 shows the comparative distribution of rainfall on the major drainage areas of the watershed. The comparison was made for four periods—the annual; the combined flood season months of October, November, and December; the month of November alone; and the average of twenty-two storms. For easy comparison, the values for the Chagres River above Madden Dam have been equated to 100% in each column. This study was undertaken to develop adjustment factors for transfer of storm and flood data from the Chagres River above Madden Dam to other areas. The values for the mean November rainfall were selected because most large floods have occurred during that month and because the November ratios would produce a larger design flood on the other areas.

TABLE 15.—AREAL DISTRIBUTION OF RAINFALL
(Rainfall Depth in Inches)

Drainage area	MEAN ANNUAL RAINFALL		MEAN FLOOD SEASON RAINFALL (OCTOBER–NOVEMBER–DECEMBER)		MEAN NOVEMBER RAINFALL		AVERAGE RAINFALL OF 22 STORMS (1941–1946)	
	Average depth	%	Average depth	%	Average depth	%	Average depth	%
Chagres River above Madden Dam......	116.0	100	45.0	100	15.9	100	4.2	100
Chagres River, Madden Dam to Gamboa..	100.9	87	36.7	82	14.8	93	3.4	81
Gatun River..........................	131.0	113	51.5	114	22.4	141	4.9	117
Caño Quebrado River..................	90.4	78	34.0	76	14.5	91	3.2	76
Trinidad River.......................	103.7	89	41.4	92	14.6	92	3.1	74
Entire Watershed.....................	109.0	94	42.3	94	16.0	101	3.8	90

Major Storms and Floods on the Upper Charges River.—Since the beginning of authentic records in 1899, the greatest peak discharge of the Chagres River at Alhajuela was 140,000 cu ft per sec (356 cu ft per sec per sq mile), recorded during the storm of December 26-30, 1909. The second largest peak discharge was 129,000 cu ft per sec (328 cu ft per sec per sq mile), recorded at Alhajuela on December 3, 1906.

The largest volume of runoff in a single flood period was produced during the storm of November 12–23, 1935, and averaged 36.8 in. above Alhajuela. Summary data of the floods described are given in Table 16.

TABLE 16.—MAJOR FLOODS OF CHAGRES RIVER AT ALHAJUELA
(393 Sq Miles)

Date of flood	RAINFALL		PEAK DISCHARGE		Meyer[a] rating	24-HOUR VOLUME	
	Period (days)	Average depth	Cu ft per sec	Cu ft per sec per sq mile		Day–sec–ft	In.
December 26–30, 1909..	5	14.8[b]	140,000	356	71	94,600	9.0
December 3, 1906.....	1	5.2[b]	129,000	328	65	77,800	7.4
November 12–23, 1935.	12	36.8	....[c]	....[c]	....[c]	52,200	5.0

[a] Expressed in percentages, in which 100% equals 10,000 times the square root of the drainage area. [b] Rainfall observed at Alhajuela. [c] Construction of Madden Dam in 1934 prevented further recording of peak discharge.

Flood Frequency.—The occurrence of Chagres River floods since the beginning of records in 1899 is plotted with respect to the magnitude of peak discharges and 24-hour discharges in Fig. 18. Computations of occurrences have been made by arranging the floods in decreasing order of magnitude and dividing the rank of the flood into the years of record. This assigns a frequency to the maximum value equal to the period of record.

Rainfall-Runoff Relations.—In examining past records, it was noted that the recorded runoff was frequently much more, and occasionally less, than the corresponding recorded rainfall, presumably the result of an insufficient number of stations. As a consequence, rainfall losses were not directly determinable from observed data, and large adjustments of observed rainfall were frequently required for a satisfactory reproduction of recorded flood hydrographs in unit hydrograph verification studies.

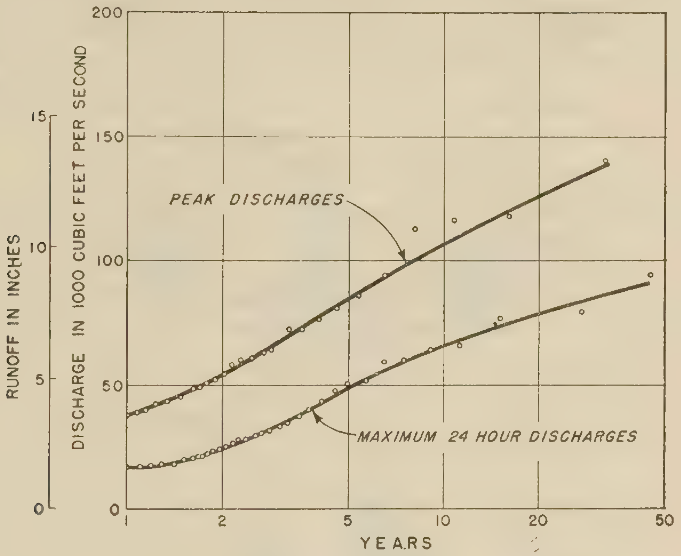


Fig. 18.—Flood Occurrences, Chagres River at Alhajuela

Project Design Flood

Hydrographs of two extended flood periods yielding the largest volumes of runoff on the Chagres River at Alhajuela are shown in Fig. 19. These two floods were studied for the purpose of developing a project design flood that could be used for selecting the storage capacities of reservoirs and the discharge capacities of outlets and diversion channels. The total period of the plotting is 50 days. The 1909 period has three distinct peaks, the last corresponding to the maximum of record cited in Table 16. The 1935 period has two distinct peaks, the first peak corresponding to the 12-day flood of Table 16. The accumulative runoff for the two flood periods is of the same order of magnitude, the highest being 75 in. Because of the relatively short period of record and the importance of positive elimination of flood interference in the canal, the volume of each of these floods was increased 25%. Routing of these floods through the Madden Lake reservoir demonstrated that the maximum storage requirements would result from the augmented 1935 flood. This flood therefore was adopted as the project design flood for the existing Madden Lake and as the general basis for design of projects on other large areas. The flood from the Madden Lake area to other areas was transposed by applying two ratios: (1) The direct ratio of the drainage areas; and (2) the ratio of the mean November rainfall of Table 15.

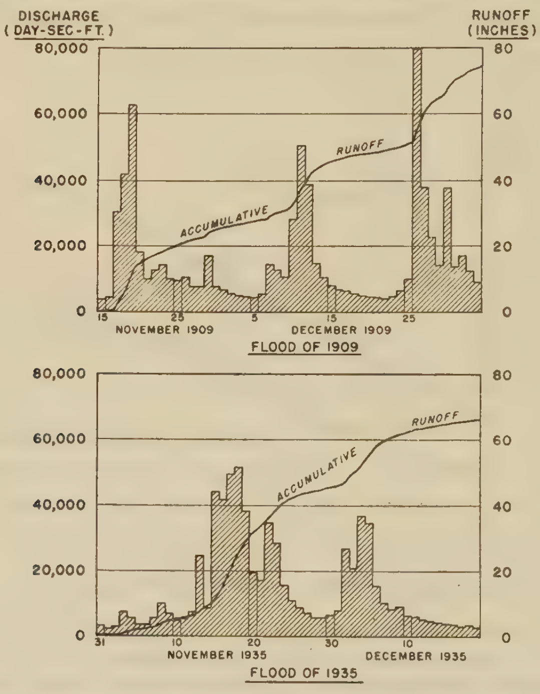


FIG. 19.—FLOOD HYDROGRAPHS, CHAGRES RIVER AT ALHAJUELA

Project Design Flood for Small Drainage Areas.—The proposed flood-control system includes the regulation of runoff on several small tributary areas of from 10 sq miles to 30 sq miles. To assure dependable control on such areas, the project design flood was selected to be 75% of the spillway design flood—a flood of shorter duration but higher discharge than the transposed flood of November, 1935.

SPILLWAY DESIGN FLOOD

The spillway design flood determines the size of spillways and the maximum water level in the reservoir and is derived from the estimated maximum possible storm for the watershed. This storm was established for the Madden Dam areas by the United States Weather Bureau in 1942 following an investigation of all the measurable meteorologic characteristics of the past storms that have produced heavy rainfall over or near the Canal Zone region. Depth-duration data of the storm for the Madden Dam area, extrapolated to apply to areas ranging from 10 sq miles to 600 sq miles, are shown in Fig. 20.

Development of the spillway design flood on the various watersheds required (1) selection of applicable rainfall data from Fig. 20, (2) adjustment of selected rainfall using the ratio of mean November rainfall between the various large areas and the Madden Lake drainage area to permit transposition of data, and (3) conversion of adjusted rainfall into flood runoff using inflow unit hydrographs. On the small drainage areas of 30 sq miles or less, the rainfall values of Fig. 20 were used without adjustment since these values could result from an intense convective storm of short duration which would be of like magnitude in any region.

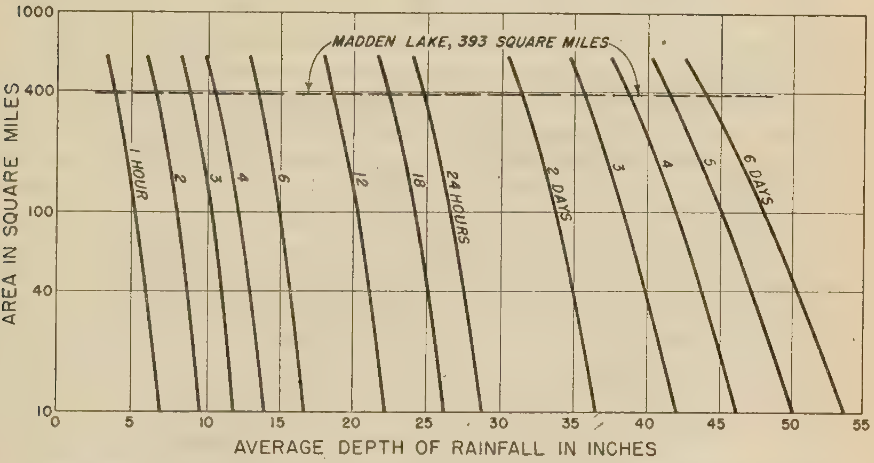


FIG. 20.—DURATION-DEPTH-AREA CURVES OF MAXIMUM POSSIBLE STORM, MADDEN LAKE

Inflow Unit Hydrographs for Reservoir Watersheds.—Unit hydrographs for converting storm rainfall into spillway design flood inflow for each flood-control project were constructed synthetically by the Snyder method.[14] General guidance for appropriate values of the empirical constants C_t and C_p, which reflect the lag time and peaking characteristics of the watershed, respectively, was obtained by constructing unit hydrographs for four gaged areas above Alhajuela and verifying them by the reconstruction of twelve floods. The constants derived from the study are given in Table 17, and a typical example

TABLE 17.—SNYDER'S CONSTANTS FOR ADOPTED UNIT HYDROGRAPHS, UPPER CHAGRES RIVER BASIN

Station	Stream	Drainage area (sq miles)	Unit time of rain (hours)	C_t	$640 C_p$	Peak discharge, q_p (cu ft per sec per sq mile)
Alhajuela	Chagres	393	2	1.00	400	55
Chico	Chagres	160	1	0.40	500	192
Candelaria	Pequeni	52	1	0.45	600	290
Peluca	Boqueron	35	1	0.50	400	200
For general use	....	10–200	1	0.50	500	....

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[14] "Synthetic Unit Graphs," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, Vol. 19, Pt. I, 1938, p. 447.

of verification is shown in Fig. 21. The final inflow unit hydrograph constructed for Madden Lake, using constants summarized in Table 17, was verified by the reconstruction of a number of floods which have occurred on the watershed since the construction of the dam in 1934. An initial loss of

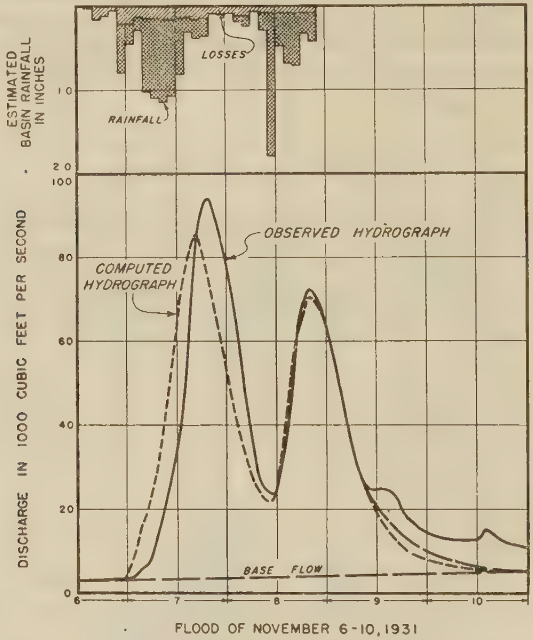

Fig. 21.—Unit Hydrograph Verification, Chagres River at Alhajuela, 1909 Flood

0.5 in. and infiltration at the rate of 0.05 in. per hr were deducted from rainfall in these studies, and a base flow of 10 cu ft per sec per sq mile was added to the computed inflow to complete the spillway design flood hydrograph.

The Flood-Control Plan

Early French and American Plans.—The importance of controlling floods on the tributaries to a Panama sea-level canal was appreciated by both the French and American proponents of a sea-level canal. The French envisioned a dam at Gamboa for regulation of inflow into the canal from the Chagres River and diversion of the Gatun and Trinidad rivers in separate channels leading to the Caribbean Sea. In 1906, the majority group of the American Board of Consulting Engineers recommended the construction of a sea-level canal and proposed the following flood-control plan, similar to that of the early French planners:

> "To control the Chagres River, a dam * * * is proposed at Gamboa * * * forming a reservoir called Gamboa Lake, of which the maximum flow line is to be at elevation 170 * * *.

> "This dam is to be fitted with controlling sluices by which a maximum discharge of 15,000 cubic feet per second is to be admitted to the canal prism * * *. Of the tributaries entering the Chagres below Gamboa, the most important are diverted entirely from the canal and conducted by separate channels to the sea * * *.
>
> * * *
>
> "The Caño [Quebrado] and the Gigante [now included as part of the Caño Quebrado], * * * are to be cut off by dams. The Trinidad will occupy the old channel of the Chagres River and the Chagres diversion. The Gatun will be cut off from the canal by the partly finished Gatun diversion, * * *."

Development of Present Plan.—The physical possibilities for developing flood control for a Panama sea-level canal are abundant and variable. Control could be established by a system of reservoirs located in the lower central part of the main drainage basins, similar to the location of Madden Lake on the Chagres River Valley above Gamboa. Inflow into the canal from such a system would be composed of regulated flood releases from the reservoirs and unregulated runoff from areas below the reservoirs. A study of stream-flow records on the Upper Chagres River indicated that this system of control would permit a peak discharge of 45,000 cu ft per sec to enter the sea-level canal at Gamboa on the average of once in 5 years. This rate of inflow is considered to be from two to three times greater than the maximum that could be tolerated without seriously disturbing navigation. It was concluded that an effective plan of control by reservoirs would require location of dams close to the sea-level canal to prevent runoff from areas of even relatively small size from entering the canal directly.

Diversion of runoff from tributary areas, particularly for the larger streams, would obviously afford the most satisfactory method of preventing floods from interfering with navigation. The large drainage areas lie on the north or Atlantic side of the Continental Divide and, if uncontrolled, would discharge into the canal at distributed locations from Gamboa to Gatun (Fig. 13).

The two main tributaries on the west side, the Caño Quebrado and the Trinidad rivers, could be diverted by excavating a relatively shallow channel in the broad valley of the old Chagres basin, generally paralleling the sea-level canal and connecting with the former channel of the Chagres River below Gatun Dam. Another west side diversion plan would employ material available from sea-level canal excavation for the formation of a series of low dams between west side islands in Gatun Lake to create a diversion reservoir of moderate depth in which a permanent pool could be maintained for the general benefit of better sanitation and access in the region. The material from canal excavation would be available for embankment construction at little or no extra cost because large amounts would be barged to the lake for economical disposal regardless of flood-control requirements.

Control of the two principal tributaries on the east side, the Chagres and Gatun rivers, could be effected generally by either of two plans: (1) By diverting both tributaries, or (2) by diverting Gatun River to the sea and controlling the Chagres River by construction of a regulating reservoir at Gamboa as proposed in the French and American plans. Complete diversion would in-

TABLE 18.—PERTINENT DATA

Line	Reservoir	River	Drainage area (sq miles)	Elevations, Pacific Level Datum (Ft)					Peak outflow[a] (cu ft per sec)
				Normal water surface	Spillway crest	Top of dam	Design Flood		
							Project	Spillway	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Madden (existing)	Chagres	393	210	250[b]	273	239.8	262.9	30,000
2	Gamboa	Chagres	127	95	140	174	129.0	158.8	38,200
3	Monte Lirio	Gatun	180	55	55	92	61.5	82.2	72,000
4	Trinidad	Trinidad and Caño Quebrado	488	55	55	82	58.4	71.3	58,000
5	Madinga	Mandinga	10.5	150	208	235	207.2	223.7	2,970
6	Cocoli	Cocoli	11.5	70	107	135	106.6	122.4	3,130
7	Miraflores	Five small rivers	30.0	70	90	115	89.8	102.3	5,890

[a] Peak outflow, project design flood, in cubic feet per second.

clude a reservoir at Gamboa, a 13.7-mile diversion channel from Gamboa northward to the Gatun River Valley, and a second diversion channel 4.7 miles long from the north rim of the Gatun Valley to the sea. This plan is similar to the one proposed by John G. Claybourn,[7] M. ASCE. Under plan (2), the 13.7-mile diversion channel would not be constructed and the capacity of the reservoir on the Chagres River at Gamboa would be increased to reduce further the reservoir outflow, which in this case would enter the sea-level canal. In either system, Madden Lake would be operated largely for flood control of the upper Chagres River. The plan of complete diversion is considered decidely superior because it would avoid a prolonged inflow of regulated releases at Gamboa during the wettest months. It would be difficult to reduce these releases sufficiently to prevent possible disturbances from crosscurrents or eddies formed by the interflowing of masses of salt water and fresh water.

The Adopted Flood-Control Plan.—The adopted plan of flood control embodies (*a*) complete diversion of all runoff from the major tributaries, and (*b*) control by regulating reservoirs on several small streams entering the canal south of Gamboa. The plan is comprehensive, since less than 9% of the entire watershed would remain uncontrolled. The largest uncontrolled area would not exceed 4.5 sq miles. The location of the flood-control projects and a summary of the areas controlled are shown in Fig. 11. The diversion of major streams lying on the Atlantic side of the Continental Divide and draining 87% of the watershed is accomplished by two distinct projects designated the East Diversion and the West Diversion. General descriptions of the projects and their operation are outlined in the paragraphs following, and statistical data pertaining to the reservoirs and dams are given in Table 18.

EAST DIVERSION

Madden Lake.—The storage in Madden Lake[15] now used principally for power generation and lock water supply would be reallocated, approximately

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[15] "More Water for the Panama Canal," by E. S. Randolph, *Civil Engineering*, May, 1932, pp. 283-288.

—Flood-Control Reservoirs

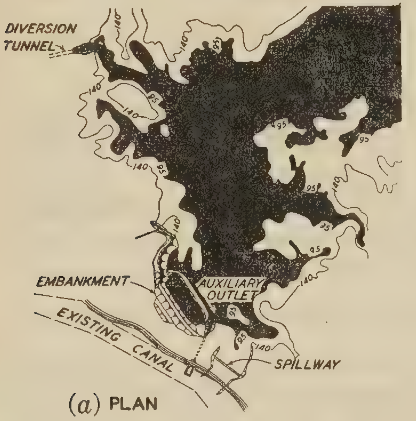
Available Storage Below Spillway Crest		Embankment Dimensions (Ft)			Uncontrolled length of spillway (ft)	Gated Outlets		Ungated Outlets		Line
Acre-ft (10)	In. (11)	Crest length (12)	Crest width (13)	Crest height (14)	(15)	No. (16)	Size (ft) (17)	No. (18)	Size (ft) (19)	
382,000[b]	18.2[b]	5,700	22	183	(4–100)[c]	6	5.67 by 10	2	7-ft needle valves	1
357,000	52.7	3,900	100	129	1,000	1	Diameter, 38 ft	1	Diameter, 45 ft	2
..	..	23,000	100	82	600	5	20 by 20	..	......	3
..	..	47,000	50	72	500	3	20 by 20	..	......	4
6,000	10.7	1,200	50	117	100	1	Diameter, 10 ft	1	Diameter, 9 ft	5
6,820	11.1	1,000	50	95	100	1	Diameter, 11 ft	1	Diameter, 10 ft	6
18,500	11.6	6,500	50	85	200	2	Diameter, 11 ft	2	Diameter, 10 ft	7

[b] Top of 18-ft crest gates. [c] Four, 100-ft crest gates.

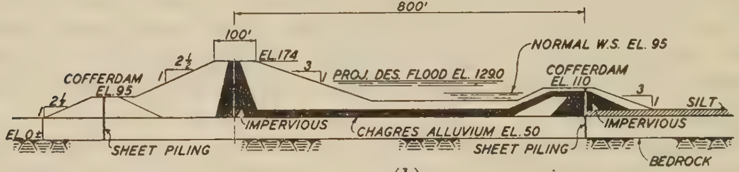
60% being assigned to flood control and 40% to power. The flood-control storage would be equivalent to a runoff depth of 18.1 in. on the 393 sq miles of tributary watershed. Regulated discharges up to a maximum of 30,000 cu ft per sec would be released from Madden Dam. The project design flood would utilize two thirds of the assigned flood-control storage, the remainder constituting a reserve for such contingencies as silting and possible changes in operating procedure.

Gamboa Reservoir.—This reservoir would receive discharges from Madden Lake and runoff from the 127 sq miles of drainage area below Madden Dam. The normal level of the reservoir would be El. 95, 10 ft above the present Gamboa arm of Gatun Lake. The project design flood would raise the water level in Gamboa reservoir to El. 129, leaving approximately one third of the storage below El. 140.0, the crest of the spillway, available as a reserve for operating contingencies. Outflow from this reservoir would be diverted northward via the Chagres River diversion channel to the Monte Lirio reservoir, third and last reservoir in the East Diversion. Any discharge that might occur over the Gamboa spillway would enter the abandoned channel of the existing canal and would flow into the sea-level canal. The top of the Gamboa Dam, an earth and rock embankment (Fig. 22) would be 125 ft above the river bed and the crest length would be 1,800 ft. A 38-ft-diameter tunnel would be constructed through the south abutment for diversion of the Chagres River during construction of the dam and would be retained for emergency release of flood discharges into the sea-level canal under remote contingencies such as closure of the Chagres diversion channel by slides.

Chagres Diversion Channel.—Diverted flows would leave the Gamboa reservoir through a 45-ft-diameter tunnel, 2,700 ft long, and thence through an open channel, 13.7 miles long, varying in width from 100 ft in the sections of deeper cut to 500 ft in the low areas (Fig. 23(*b*)). As shown in Fig. 23(*c*), two weirs would be constructed in the diversion channel—the upper, with crest at El. 95, would be located 6 miles below the tunnel outlet and would control the normal level of the Gamboa reservoir; and the lower, with crest at El. 70, would be located 6 miles farther downstream. These weirs would keep the bottom of the channel submerged in the dry season and thus prevent inception of natural growth. Discharge from the Gamboa reservoir would be controlled by the natural capacity of the diversion channel and upper weir at low reservoir stages and by the capacity of the tunnel at higher stages. Valleys of existing streams crossed by the diversion channel would require nominal diking on the sea-level canal side.

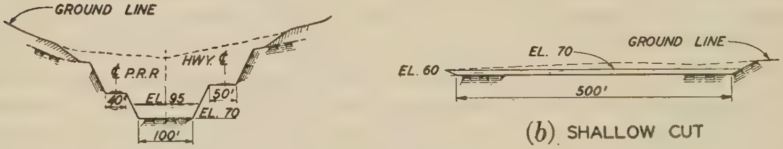

DIVERSION TUNNEL
140
95
EMBANKMENT
AUXILIARY OUTLET
EXISTING CANAL
SPILLWAY

(*a*) PLAN


800'
100'
EL.174
2½
1
3
1
NORMAL W.S. EL. 95
PROJ. DES. FLOOD EL. 129.0
COFFERDAM EL. 95
COFFERDAM EL. 110
SILT
EL. 0±
SHEET PILING
IMPERVIOUS
CHAGRES ALLUVIUM EL. 50
SHEET PILING
IMPERVIOUS
BEDROCK

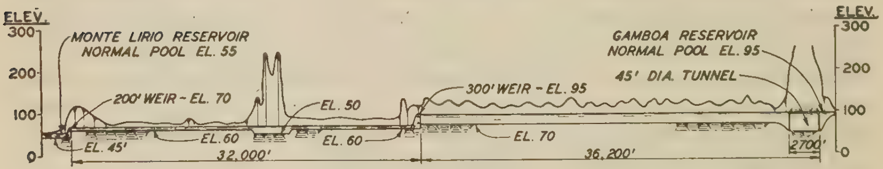
(*b*) SECTION

FIG. 22.—PLAN AND SECTION, GAMBOA DAM


GROUND LINE
℄ P.R.R.
HWY. ℄
40'
EL. 95
50'
EL. 70
100'
GROUND LINE
EL. 70
EL. 60
500'

(*a*) DEEP CUT

(*b*) SHALLOW CUT


ELEV.
300
200
100
0
MONTE LIRIO RESERVOIR NORMAL POOL EL. 55
200' WEIR - EL. 70
EL. 45'
EL. 60
EL. 50
EL. 60
32,000'
300' WEIR - EL. 95
EL. 70
36,200'
GAMBOA RESERVOIR NORMAL POOL EL. 95
45' DIA. TUNNEL
2700'

(*c*) PROFILE

FIG. 23.—SECTIONS AND PROFILES, CHAGRES DIVERSION CHANNEL

Monte Lirio Reservoir.—Monte Lirio reservoir would occupy the lower valley of the Gatun River now inundated by the eastern arm of Gatun Lake and would receive flows diverted from the Gamboa reservoir, runoff from streams intercepted by the Chagres diversion channel, and flow from the Gatun River. The normal level of the reservoir would be maintained at El. 55, 30 ft below the present level of Gatun Lake, a level which is suitable for domestic water supply, area sanitation, and hydroelectric power development. The Monte Lirio Dam (Fig. 24(*a*)) would be formed by strengthening the existing Panama Railroad embankment which extends between several islands across the eastern part of Gatun Lake for a total distance of 4.4 miles.

All flows received in the Monte Lirio reservoir would be diverted to Las Minas Bay on the Caribbean coast through a channel 4.7 miles long (Fig. 24(*b*)). The diversion channel would have a width increasing from 200 ft at the reservoir to 1,000 feet near Las Minas Bay. A 600-ft spillway weir with crest at El. 55 and an outlet structure containing five gates would be constructed in the channel 1 mile from the reservoir.

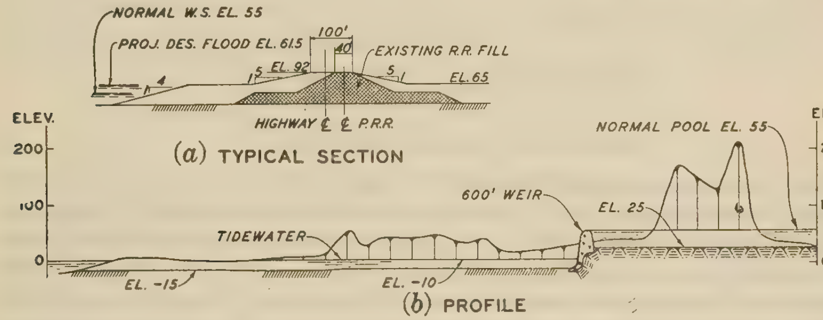


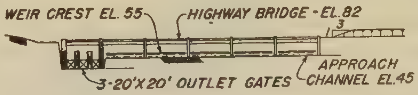
Fig. 24.—Monte Lirio Dam and Diversion Channel

The outlet gates would be operated to maintain the water surface at El. 55 in the Monte Lirio reservoir and, during the passsing of a flood, would be fully open when the reservoir level rose above El. 55. The project design flood would raise the water surface of Monte Lirio reservoir to El. 61.5.

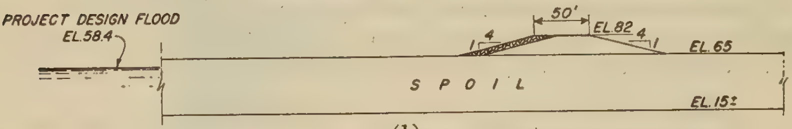
West Diversion

Two interconnected reservoirs, the Caño Quebrado reservoir and the larger Trinidad reservoir, with normal water surfaces at El. 55, would comprise the West Diversion (Fig. 11). Discharge from the Caño Quebrado reservoir would flow through an uncontrolled channel circling the south and west sides of Barro Colorado Island, and would join the Trinidad reservoir near the existing Gatun Dam. A 500-ft spillway with crest at El. 55 and an outlet structure containing three gates constructed in the west abutment of the existing Gatun Dam (Fig. 25(*a*)) would serve both reservoirs and would be regulated in the same manner as the Monte Lirio spillway and outlet structure. Discharges would flow to the sea through the old channel of the Chagres River. The project design flood would raise the water surface in the reservoirs to El. 58.4.

The dams for the West Diversion (Fig. 25(*b*)) would consist of low embankments placed on broad spoil areas of sea-level canal excavation. The surface of the spoil would be at El. 65 and the crests of the embankments would be at El. 82. The dams would extend from Gatun Dam to a point 4 miles north of Gamboa and would connect numerous islands in the present lake.



(*a*) UPSTREAM ELEVATION OF SPILLWAY



(*b*) TYPICAL SECTION

FIG. 25.—TRINIDAD AND CAÑO QUEBRADO DAMS

CONTROL OF SMALL TRIBUTARIES

Three retarding reservoirs on small tributary areas between Gamboa and Balboa Harbor (Fig. 11) are included in the flood-control plan. These are designated as the Mandinga, the Cocoli, and the Miraflores reservoirs, and control areas of 10.5 sq miles, 11.5 sq miles, and 30.0 sq miles, respectively. The last consists of five interconnected small basins with two outlet channels. Each reservoir would have a spillway and both uncontrolled and gated outlets. The maximum discharge into the canal at any one point would approximate 3,000 cu ft per sec.

FOUNDATIONS AND CONSTRUCTION MATERIALS

Preliminary explorations have revealed that sound rock is available at suitable depths in all foundation areas for spillway and outlet structures and in areas of tunnel excavation. Likewise, no major problems are anticipated in excavation of diversion channels. Most of the 13.7-mile diversion channel from Gamboa to Monte Lirio lies in regions of hard rocks of volcanic origin and medium-hard siltstones and limestones. The channel from the Monte Lirio reservoir to the sea would be located largely in sound siltstone and sandstone of the Gatun formation. The river section of the Gamboa Dam would be founded on alluvium, 50 ft thick, composed largely of sand and gravel. Measures for watertightness and safe relief of seepage would be incorporated in the design of the dam, as illustrated in Fig. 22(*b*).

Parts of the embankments for the Monte Lirio, Trinidad, and Caño Quebrado dams would be founded on a relatively soft alluvial deposit of organic clays, silts, and sands that is termed Atlantic muck. Fortunately, the great abundance of spoil material from the sea-level canal excavation would permit Caño Quebrado and Trinidad dams to be founded entirely upon a broad terrace of sound excavation spoil extending not less than 1,500 ft on both sides of the center line—with a top at El. 65, only 17 ft below the crest of the dams.

The Monte Lirio Dam, which would be an enlargement of the railroad causeway completed in 1912 across the eastern arm of Gatun Lake, would be similarly reinforced with broad berms for distribution of load.

Suitable and extensive deposits of gravel are available near the Gamboa dam site, and impervious, random, and rock-fill materials for the Gamboa Dam would be obtained from spillway excavation. Fill for embankments in the lake area would come mainly from canal excavation transported by barge.

Construction

Flood-control construction would be scheduled to utilize the material from canal excavation as it becomes available and to avoid unnecessary peaks in personnel. None of the structures would be required to function during the canal construction period, but Madden Lake would be assigned largely to flood control during the construction of the Gamboa Dam, the only project requiring major cofferdam construction and river diversion. The earth-fill cofferdams for the Gamboa Dam would form a part of the permanent structure.

Operation of Flood–Control Projects During Final Lowering of Lakes

All flood-control projects would be placed in operation prior to the rapid lowering of Gatun and Miraflores lakes upon conversion of the canal to sea level. More than 68% of the normal volume of Gatun Lake would be cut off by flood-control dams and thus the time of emptying the central region of the lake would be greatly accelerated. The total interruption to traffic at this time would be less than 7 days, including the time required for removing the lake-retaining barriers in the channel for the sea-level canal.

Hydrolectric Power

Hydroelectric power is generated in the Canal Zone at Gatun and Madden dams, which have an installed capacity of 46,000 kw. The Gatun station would be abandoned on conversion of the canal to sea level and, as noted previously, the storage assigned to power in Madden Lake would be reduced to one third of the total storage, sufficient to produce 13,000 kw at a 40% load factor. Opportunities would be available at the Monte Lirio and Trinidad spillway structures for small hydroelectric installations. The plant at Monte Lirio, receiving regulated flow from Madden Dam and unregulated runoff from the remaining watershed of 307 sq miles, would have a firm capacity of 13,500 kw at a 25% load factor. The Monte Lirio reservoir would not be regulated for power because drawdown of the normal water surface would affect the quality of water for domestic supply adversely. The Trinidad and Caño Quebrado reservoirs would be regulated between El. 55 and El. 45, permitting the generation of 7,500 kw of primary power at a load factor of 50%.

Safety of Structures During Wartime

The dams in the lake area would actually be low levees ranging from 17 ft to 27 ft in height constructed on broad spoil terraces at El. 65. These terraced areas would be virtually unbreachable by bombing of any type and a breach in the levees would be of no consequence since the occurrence of a flood even as large as the project design flood would not raise the water level in either the East Diversion or the West Diversion above El. 65. If the spoil terraces were ever breached, the outlet conduits in the Trinidad and Monte Lirio spillway structures could be left open for continued diversion of flow to the sea with only minor repair of the breach. A breach in Madden Dam would not result in overtopping the Gamboa Dam even if both reservoirs were filled to spillway levels at the time—an extremely remote possibility. The Gamboa Dam would be difficult to breach by any conceivable conventional bomb, as it would be 350 ft wide at the level of the project design flood and 650 ft wide at normal water surface.

SUMMARY

The control of floods on the tributaries of a Panama sea-level canal, in accordance with the adopted system, would completely eliminate hazards to shipping from flood inflows. The flood-control system would have sufficient capacity to control flows well in excess of the largest flood that has occurred in 47 years of record. Runoff from 87% of the area tributary to the sea-level canal would be diverted directly to the Atlantic Ocean and 4% would be controlled by retarding reservoirs. It would be difficult to breach the flood-control dams adjacent to the canal in wartime. If breached, only minor remedial work would be needed to restore the system to effective operation.

TIDAL CURRENTS

BY J. S. MEYERS,[16] AND E. A. SCHULTZ,[17] ASSOC. MEMBERS, ASCE

SYNOPSIS

A current of about 4.5 knots, at the Atlantic end of the Panama Canal, is estimated both by computations and by hydraulic model tests as the maximum that would be caused in an open sea-level canal at Panama by a tidal range of 20 ft in the Pacific Ocean.

Channel roughness would affect the velocity of the reversing flow in this tidal canal in the same way as in any one-way channel since, at the time of maximum current, the situation is nearly that of ordinary steady flow between the extreme tide levels. Results of measurement of roughness in the existing canal and a summary of roughness data for other large channels are presented. A Manning *n* of 0.024 was selected for the investigations.

The hydraulic model of the sea-level canal, at 1:100 undistorted scale, is described briefly. Close agreement throughout the tidal cycle was obtained between the velocities measured in the model and those computed by the method developed by General Pillsbury.

An example of tidal flow as controlled by tidal-regulating structures is presented and discussed briefly.

EARLY ESTIMATES OF TIDAL CURRENTS AT PANAMA

The currents that would be produced by the tides in a sea-level canal at Panama have been the subject of speculation and concern since the earliest canal proposals. The first careful study known to have been made was that reported on May 31, 1887, by a committee appointed by the French Academy of Sciences[18,19] at the request of Count Ferdinand de Lesseps, chief engineer of the French Isthmian Canal Company, who was undertaking to build a sea-level canal at Panama as he had already done so successfully at Suez. The French analysis was based on the use of the Chézy formula for steady flow, applied to a number of short reaches, with heads adjusted to take account of wave celerity. The committee reached the conclusions that the Atlantic tidal range was so small in comparison with the Pacific range that it could be disregarded. The estimated maximum tidal current for a canal 45 miles long was 2.5 knots.[20]

The Board of Consulting Engineers, Isthmian Canal Commission, reported in 1906: "It is probable that in the absence of a tidal lock the tidal currents during extreme oscillations would reach five miles per hour."[21] This is the equivalent of 4.3 knots. The channel to which this estimate applied was about 40 miles long and 40 ft deep, with bottom width at 150 ft in earth and 200 ft in rock. The method used to obtain this velocity was not stated.

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[16] Chf., Hydr. Section, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

[17] Engr.-in-Chg., Hydr. Models, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

[18] "The Cape Cod Canal," by William Barclay Parsons, *Transactions*, ASCE, Vol. LXXXII, 1918, p. 138.

[19] *Comptes Rendus*, French Academy of Sciences, Vol. 104, May 31, 1887, p. 1484.

[20] *Transactions*, ASCE, Vol. LVI, 1906, p. 211.

[21] "Report of the Board of Consulting Engineers for the Panama Canal, 1906." U. S. Govt. Printing Office, Washington, D. C., 1906, p. 56.

An estimate of 2.6 knots for the mean cross-sectional velocity in a sea-level canal was made by the United States Coast and Geodetic Survey (in a letter to the Panama Canal, dated February 3, 1924), for a channel 40 miles long, 1,000 ft wide, and 50 ft deep. The Eytelwein formula was used to compute the steady-flow velocity that would be produced by an 11-ft difference in head. It was stated that the velocities computed by this formula agreed reasonably well with observed values in the Cape Cod Canal in Massachusetts, which is about 7 miles long, but that progressive wave motions might be created in the greater length of a Panama sea-level canal which would tend to increase the computed value.

TIDES AT PANAMA

The tides at the Atlantic and Pacific termini of the Panama Canal are very different both in magnitude and general character. The Pacific tide at Balboa is remarkably regular, with two highs and two lows of almost equal magnitude occurring in every lunar day of 24 hours and 50 min. Extreme tides have reached levels 10.8 ft above and 11.9 ft below mean sea level. The maximum range between consecutive tides, however, has only occasionally exceeded 20 ft, as indicated by the following percentages of Balboa tides that reach different ranges:

Percentage of total tides	Tidal range (ft)
2	20
20	16
50	13
80	10
99	6

The Atlantic tide at Cristobal is irregular and much smaller than the Pacific tide, with no simple cyclic pattern. Extreme tides have reached 1.8 ft above and 1.25 ft below mean sea level. The mean tidal range is 0.9 ft and the minimum practically zero. Atlantic high tides precede Pacific high tides from zero hours to 6 hours, averaging about 3 hours. Mean sea level on the Pacific side of the Isthmus averages 0.77 ft higher than on the Atlantic side, but the Pacific mean level on individual days has ranged from 2.04 ft above to 0.68 ft below the Atlantic mean level.

Most of the analytical and model studies described in this paper were based on assumed maximum semidiurnal tides of 20-ft range at Balboa and of 2-ft range at Cristobal, with the Cristobal tide preceding the Balboa tide by 3 lunar hours, as shown in Fig. 26. (One lunar day is the time required for one revolution of the moon around the earth. It corresponds to 24 hours 50 min of ordinary solar time.) Selection of the essential features of a sea-level canal generally required consideration of the conditions produced by these maximum tides, but investigations were also made of the effects of average and low tides for some comparisons.

Because the Pacific mean sea level is 0.77 ft higher than the mean level on the Atlantic side of the Isthmus, a greater head and a steeper slope in a sea-level canal would normally be expected when the Pacific tide was at its highest point than when it was lowest. The maximum head in 20 years of record, however, was 11.6 ft from the Atlantic level down to a low Pacific tide level. The maximum head in the opposite direction, from a high Pacific level down to the Atlantic level was 11.3 ft. Since the tidal heads in the two directions are so nearly the same for maximum conditions, the convenient assumption of a common Atlantic and Pacific mean tide level was made for study purposes.

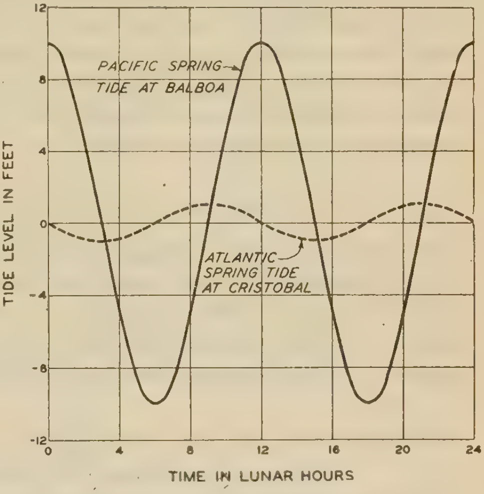


Fig. 26.—Maximum Tides Used for Velocity Studies

Analyses of Tidal Currents for 1947 Investigation

Estimates of velocity for the proposed 60-ft by 600-ft sea-evel canal have been made, using data and methods more complete than those available to earlier investigators. Most of the computations were made by the method developed by General Pillsbury for analysis of tidal flow.[8] By this method the channel is subdivided into a number of reaches, and flow in each reach is computed from an equation of motion in which the frictional resistance is expressed by the Manning, the Kutter, the Bazin, or any desired flow formula. Corrections are made for changes in storage, slopes, and other factors caused by the changing tides. The precision of the results depends on the number of successive adjustments. This method indicates that a maximum current velocity of 4.5 knots would be caused at the Atlantic end of the canal by the tides of Fig. 26 acting on an open sea-level canal with the Manning roughness assumed as 0.024. This velocity, and all others given in this paper, whether from model tests or computations, are mean velocities over the entire channel cross section.

A computation procedure suggested by Professor Bakhmeteff was also used for comparison. The basic assumption was that of steady flow between the maximum differences of water level shown in Fig. 26, with corrections for changes in acceleration, velocity head, and channel storages that would prevail in a tidal cycle. The estimated maximum steady-flow velocity was 4.6 knots, again using a Manning roughness of 0.024.

Tests on the 1:100 scale hydraulic model of the proposed sea-level canal channel, which is described subsequently in this paper, indicated a maximum current of 4.4 knots, for the same 20-ft and 2-ft tides. The roughness of the model channel corresponded to that expressed by a 0.024 Manning coefficient for the full-size channel.

During the preliminary discussions of these analyses and their implications, Professor Bakhmeteff stated that velocities for tidal flow cannot exceed those for steady flow. For the situation at Panama, he reasoned that the steady-flow velocities would be only slightly greater than the tidal velocities, since "preliminary calculations show that at periods of extreme flow about 90 percent of the actuating head is absorbed by friction resistance." The results of the tests on the sea-level model closely support that statement, indicating that channel roughness is a controlling factor in any estimate of tidal velocities in a canal as long as that at Panama.

Hydraulic Roughness of Large Channels

Observations made on open channels of moderate and small size have provided most of the data upon which the current understanding of flow is based. The flow formulas in common use were developed primarily from measurements made on small test channels by Bazin and other early experimenters. In some cases, they were developed with a view to their use for closed pipes as well as open channels. Although the desirability of having the formula fit data for large as well as for small channels was recognized, notably by Ganguillet and Kutter, the information then available on large channels was meager and in some cases unreliable. Because of the importance of the roughness value in this Panama Canal study, special effort was made to obtain and compare information on large channels only, and to weigh the applicability of the different formulas to those conditions.

TABLE 19.—Values of Manning's Roughness Coefficient for Large Channels at Most Efficient Depths

Line	Watercourse	Coefficient n	Notes on channel
1	Emory River	0.029	Earth and ledge rock.[a]
2	Tennessee River	0.026	Natural earth channel of lower Tennessee River.[b]
3	Colorado River	0.028	In newly excavated rock.[c]
4	Mississippi River	0.024 to 0.031	In alluvial valley.[d]
5	Atchafalaya River	0.024	In alluvial valley.[e]
6	Yazoo River	0.026 to 0.031	In alluvial valley.[f]
	Columbia River—		
7	McNary dam site near Umatilla, Ore.	0.030	Stable bottom of sand, gravel, boulders, and solid rock.
8	Central part of State of Washington	0.036	Little bank vegetation.
9	Cape Cod Canal	0.031 to 0.036	Subject to tidal currents.
10	Houston Ship Canal	0.025	Coefficient measured roughly.[g]
11	Panama Canal (Gaillard Cut)	0.026	In rock.[h]

[a] In backwater from Watts Bar reservoir on the Tennessee River below Knoxville, Tenn. [b] In backwater from Kentucky Dam near Gilbertsville, Ky. [c] Downstream from powerhouse at Parker Dam near Parker, Ariz. [d] This range of values was applicable to 1928–1930 measurements, omitting widely divergent values. [e] This range of values was applicable to 1929, omitting widely divergent values. [f] After clearing and snagging. Climate is conducive to vegetation. [g] This coefficient, although roughly measured, is considered applicable to design. [h] Observations in 1946, after channel had been submerged for 33 years.

Roughness Data on Large Channels from Other Sources.—In response to requests made in 1946 by The Panama Canal to a number of agencies in the United States, information on hydraulic roughness of large channels was received from the Bureau of Reclamation, the Tennessee Valley Authority, and several offices of the Corps of Engineers. The data were for long reaches of natural and artificial channels with a considerable variety of slopes, discharges, depths, and bed material. The roughness coefficients have been averaged, and a summary for channels having hydraulic radii exceeding 20 ft and flowing nearly bankfull is given in Table 19.

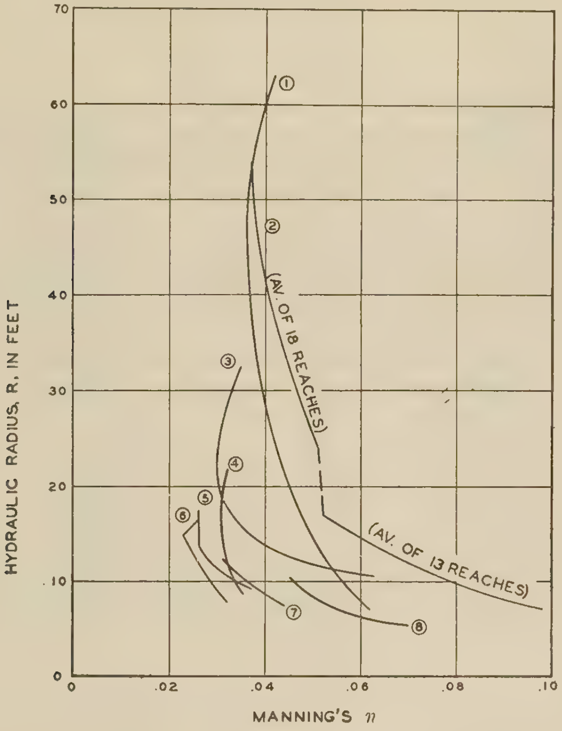
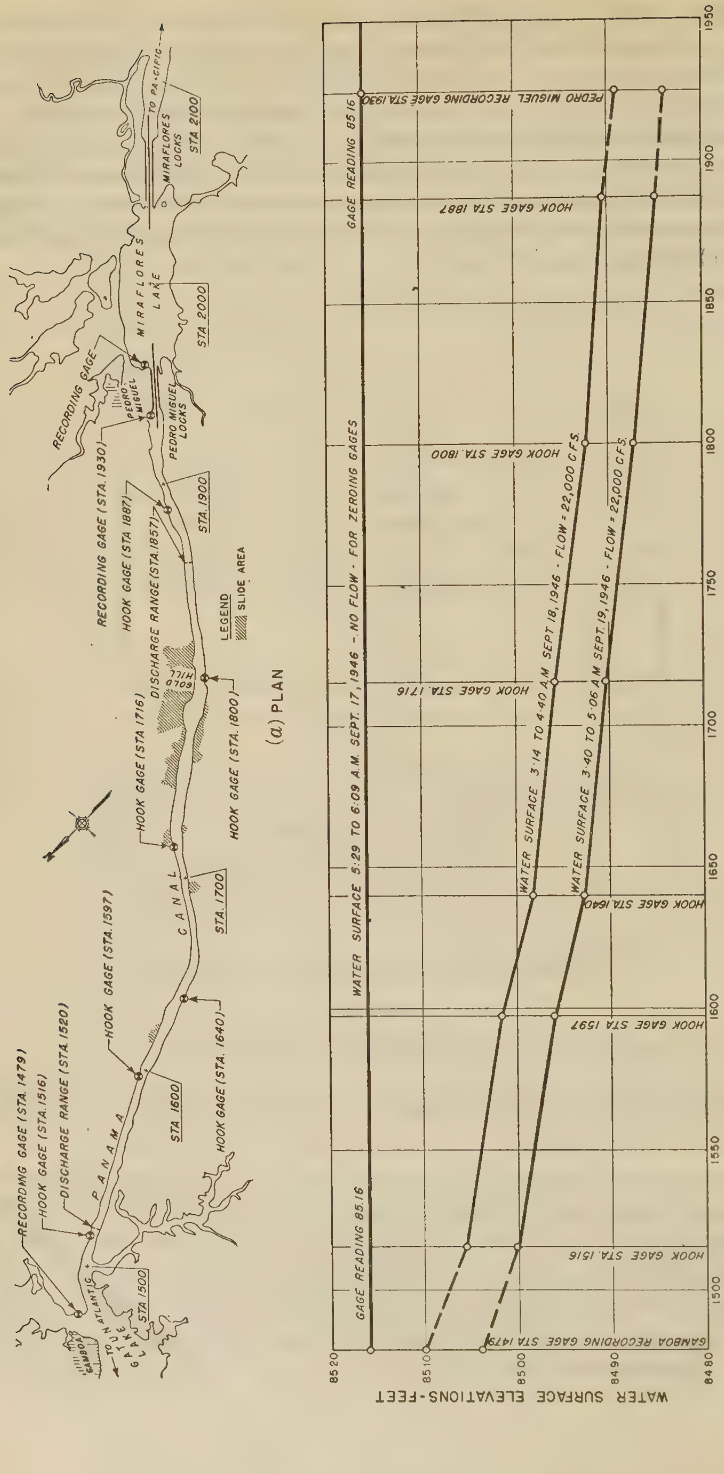


Fig. 27.—Variation of Manning's n, with Hydraulic Radius

The two most important conclusions reached from a study of these data were:

a. The Manning coefficient for a river channel is least when the stage is at or somewhat above normal bankfull stage, and tends to increase for both higher and lower stages. This fact is indicated both by the Yazoo River in Mississippi, flowing in alluvium in an area where vegetation is rank, and by the Columbia River in Washington, with a rocky bed and practically no vegetation on its banks. The effect is shown in Fig. 27, the several curves being identified by numbers as follows:



(a) PLAN

(b) WATER SURFACE PROFILES

FIG. 28.—Roughness Measurements in Gaillard Cut

Curve	Identity
	Columbia River—
1	Foster Creek project above Wenatchee, Wash.
2	Priest Rapids project near Ellensburg, Wash.
3	McNary project above Umatilla, Ore.
	Yazoo River—
4	At Yazoo City, Miss.
5	At Greenwood, Miss.
	Tallahatchie River—
6	At Swan Lake, Miss.
7	At Lambert, Miss.
	Big Black River—
8	At West, Miss.

b. The bankfull roughness coefficients do not vary greatly for rivers and canals in different kinds of material and in widely separated locations. From the summary in Table 19 for channels having hydraulic radii exceeding 20 ft, all but one show roughness coefficient values in the range from 0.024 to 0.031.

Roughness Measurements in Gaillard Cut.—Measurements were made of the roughness of the Gaillard Cut channel of the existing canal, to obtain data under conditions as nearly comparable as possible to those in a new canal channel. A flow of water through the cut was established by opening all the culvert valves at Pedro Miguel Locks after the last ship transit in the evening, and continued until necessary to shut down before the first transit in the morning.

TABLE 20.—ROUGHNESS COEFFICIENTS, GAILLARD CUT

Reach (stations)	DIFFERENCE IN WATER-SURFACE ELEVATION		Average energy gradient slope, S	Mean section area (sq ft)	Mean velocity (ft per sec)	Mean hydraulic radius, R (ft)	Chézy C	Manning n (a)	Kutter n	Modified Kutter n	Bazin m	Kármán-Prandtl k
	Sept. 18	Sept. 19										
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1516–1597	0.040	0.041	0.0000060	14,900	1.48	36.5	101	0.0268	0.052	0.025	3.4	0.35
1597–1640	0.033	0.032	0.0000052	15,800	1.39	36.4	100	0.0271	0.053	0.024	3.5	0.38
1640–1716	0.025	0.025	0.0000055	16,500	1.34	37.4	101	0.0270	0.053	0.024	3.4	0.37
1716–1800	0.033	0.031	0.0000023	21,500	1.02	38.0	108	0.0252	0.054	0.022	2.8	0.22
1800–1887	0.021	0.024	0.0000046	18,000	1.22	36.0	103	0.0261	0.052	0.023	3.1	0.29
1516–1640	0.073	0.073	0.0000057	15,200	1.44	36.4	101	0.0269	0.053	0.024	3.4	0.36
1640–1887	0.079	0.080	0.0000041	18,600	1.18	37.2	103	0.0263	0.053	0.023	3.2	0.31
1516–1887	0.152	0.153	0.0000046	17,200	1.28	37.0	102	0.0265	0.053	0.023	3.3	0.33

[a] From water-surface profile computations. The remaining coefficients have been determined from n, R, and S as tabulated. The slope term, $0.00281/S$, of the Kutter equation is omitted in the modified Kutter equation.

Six temporary gages were established for these tests and their zero elevations were determined by simultaneous readings of all gages during a preliminary run with quiet water surface and no flow through the cut. An average discharge of 22,000 cu ft per sec was measured by current meter at each end of the test reach for each of the two tests.

Areas and hydraulic radii for the channel cross sections were obtained from a detailed survey made in 1945. Fig. 28 shows the locations of gages and the observed water-surface profiles, and Table 20 lists the coefficient values found for different flow formulas. The depth of the channel was quite uniformly 41 ft and the minimum bottom width was 300 ft, but the width and cross-sectional area varied considerably in reaches where the channel passes through bends and old slide areas. The nature of the bed and bank material varies from very hard rock to soft rock and clay. Fig. 29 shows the nature of the channel just before water was admitted, superimposed lines indicating the ultimate water level.

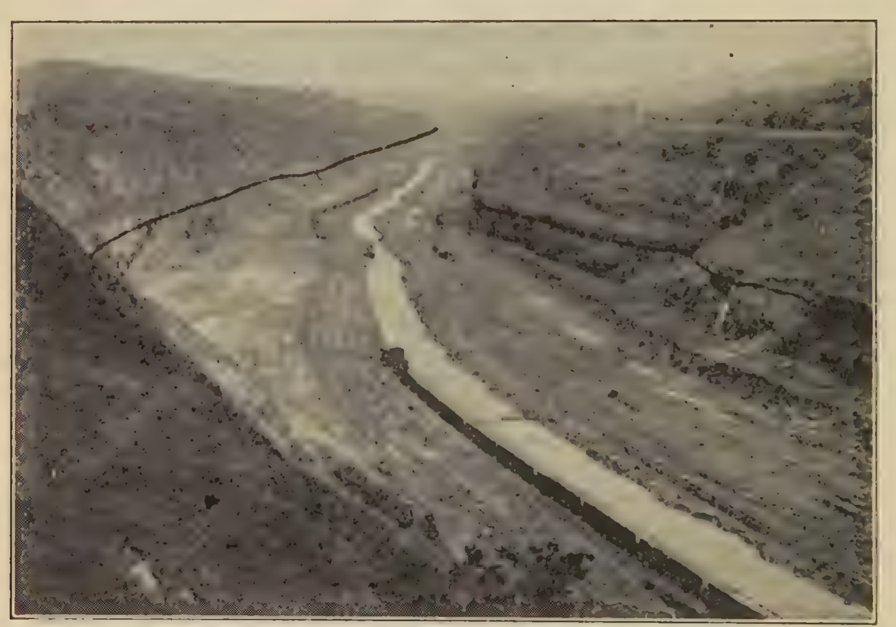


FIG. 29.—CULEBRA CUT (LATER NAMED GAILLARD CUT) AT EMPIRE, AFTER THE COMPLETION OF EXCAVATION

The measurements were made with great care, working at night to avoid interference from traffic and wind, and allowing several hours on each of the two test dates for water levels to become well stabilized. Roughness coefficients for each of the five reaches into which the 7-mile test section was subdivided are very consistent, as shown by Table 20. Observed coefficient values of 0.0265 for the Manning formula, 3.3 for the Bazin formula, and 0.33 for the Kármán-Prandtl formula fall within the expected range. The unmodified Kutter formula, however, gave the high coefficient value of 0.053 because of the very flat slope of 2 in. in 7 miles. An *n*-value of 0.023 was indicated for the Kutter formula with the slope term omitted.

Selected Value for Roughness Coefficient.—The sea-level canal channel would have a mean hydraulic radius of about 55 ft, and would be cut through a variety of materials ranging from muck to basalt, but at least 87% of the length would fall within the classifications of hard, medium, and soft rock. Therefore, a roughness coefficient representative of a rock surface like that of the existing Gaillard Cut should be used.

The Manning formula was adopted for use in the studies for the improvements of the Panama Canal. The range of Manning roughness coefficients from all sources is generally between 0.024 and 0.031, which include the Gaillard Cut observations of 0.026. A coefficient of 0.024 was selected for the computation of velocities in a sea-level canal, where the use of the highest velocity that could reasonably be expected would result in a more conservative design.

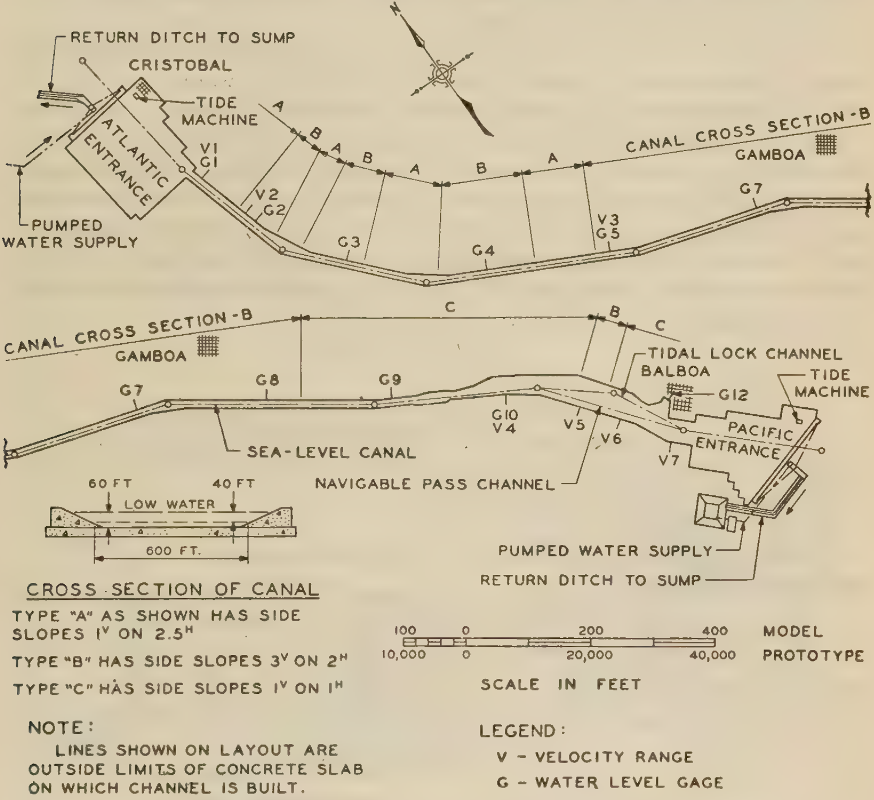

RETURN DITCH TO SUMP
CRISTOBAL
TIDE MACHINE
ATLANTIC ENTRANCE
V1
G1
A
B
A
B
A
B
A
CANAL CROSS SECTION -B
GAMBOA
G7
V2
G2
G3
G4
V3
G5
PUMPED WATER SUPPLY
CANAL CROSS SECTION -B
GAMBOA
C
B
C
TIDAL LOCK CHANNEL
BALBOA
TIDE MACHINE
G12
G7
G8
G9
G10
V4
V5
V6
V7
PACIFIC ENTRANCE
SEA-LEVEL CANAL
NAVIGABLE PASS CHANNEL
60 FT
40 FT
LOW WATER
600 FT.
CROSS SECTION OF CANAL
TYPE "A" AS SHOWN HAS SIDE SLOPES 1V ON 2.5H
TYPE "B" HAS SIDE SLOPES 3V ON 2H
TYPE "C" HAS SIDE SLOPES 1V ON 1H
PUMPED WATER SUPPLY
RETURN DITCH TO SUMP
100
0
200
400
MODEL
10,000
0
20,000
40,000
PROTOTYPE
SCALE IN FEET
NOTE:
LINES SHOWN ON LAYOUT ARE OUTSIDE LIMITS OF CONCRETE SLAB ON WHICH CHANNEL IS BUILT.
LEGEND:
V - VELOCITY RANGE
G - WATER LEVEL GAGE

Fig. 30—Layout of 1:100 Model of Panama Sea-Level Canal

Model of Sea-Level Canal

Fig. 30 shows the layout for the hydraulic model channel that was built and tested in the Canal Zone for the study of maximum tidal currents and of the general hydraulic problems that would arise in the design and operation of a sea-level canal. The entire length of the proposed 60-ft by 600-ft navigation channel was reproduced, and also the Atlantic and Pacific entrances out to deep water, to insure accurate simulation of flow into and out of the canal. At the scale of 1:100, the model was nearly half a mile long, simulating a total prototype length of about 45 miles and including some 35 miles of restricted channel. Varying side slopes were used, to agree with the design slopes for the different geological materials through which the channel would be cut. The model was built outdoors, with a cover of corrugated sheet metal laid across the top of the channel side walls and roofs over the entrances to prevent disturbance of the water levels by rain and wind.

Model Scale.—The scale of 1:100 was selected because it permitted satisfactory measurement of velocity and of differences in water level and because it was expected to reproduce the flow conditions correctly. Experience with other models had indicated that an ordinary concrete surface in a model of this scale would simulate the roughness of the full-size channel more accurately than would a larger or a smaller model. This expectation was fulfilled when the resistance coefficient for the model channel, measured under conditions of steady flow with differential head equal to that for the maximum tides, was found to correspond to the selected Manning coefficient of 0.024 for the full-size canal, and no adjustments were needed.

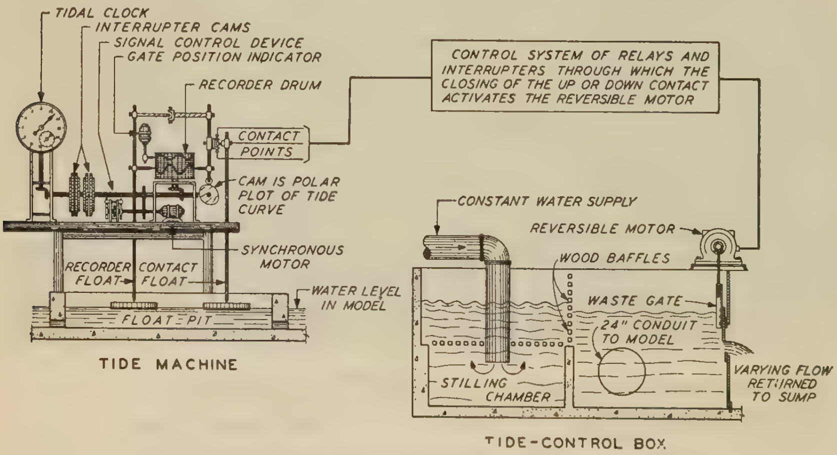

TIDAL CLOCK
INTERRUPTER CAMS
SIGNAL CONTROL DEVICE
GATE POSITION INDICATOR
RECORDER DRUM
CONTACT POINTS
CAM IS POLAR PLOT OF TIDE CURVE
SYNCHRONOUS MOTOR
RECORDER FLOAT
CONTACT FLOAT
FLOAT PIT
WATER LEVEL IN MODEL
TIDE MACHINE
CONTROL SYSTEM OF RELAYS AND INTERRUPTERS THROUGH WHICH THE CLOSING OF THE UP OR DOWN CONTACT ACTIVATES THE REVERSIBLE MOTOR
CONSTANT WATER SUPPLY
REVERSIBLE MOTOR
WOOD BAFFLES
WASTE GATE
24" CONDUIT TO MODEL
STILLING CHAMBER
VARYING FLOW RETURNED TO SUMP
TIDE-CONTROL BOX

Fig. 31—Schematic Layout of Tide-Reproducing Apparatus, Model of Sea-Level Canal

The model was built without distortion to insure, as far as possible, that dynamic similitude was obtained, because the absence of an existing prototype prevented making the extensive tests that are essential for the verification of a distorted model. Since gravity is the predominant force in this case, the time, discharge, and velocity factors were used strictly in accordance with the Froude law, which considers gravity as the only factor affecting the motion of the water. Flow at the Atlantic end of the model for maximum tidal conditions would meet the Reynolds' criterion for turbulence about 96% of the time (based on a Reynolds number of 2,000).[22] This situation covers all the critical conditions where accuracy is needed. Regardless of scale, laminar flow cannot be avoided entirely in this model since the velocity must be zero at each reversal of the tide.

Tide Machines.—Tide-control mechanisms were installed at each end of the model to reproduce the tidal variations of the prototype to proper time and height scales. The machines were made by the United States Waterways Experiment Station at Vicksburg, Miss., following the design which had proved satisfactory in other models, and installed at Panama under the supervision of Experiment Station personnel. Figs. 31 and 32 show the tide-reproducing apparatus and the water-supply system at the Pacific entrance. The solid arrows in Fig. 32 indicate continuous flow in one direction, whereas dashed arrows indicate reversing flow that changes in direction with every change in tide. The basic element of each tide machine is a replaceable cam cut to represent the desired tidal cycle. Mechanical and electronic devices operate a waste gate in the water-supply system which controls the rate of model inflow or outflow so that the actual water surface closely follows the indications given by the cam.

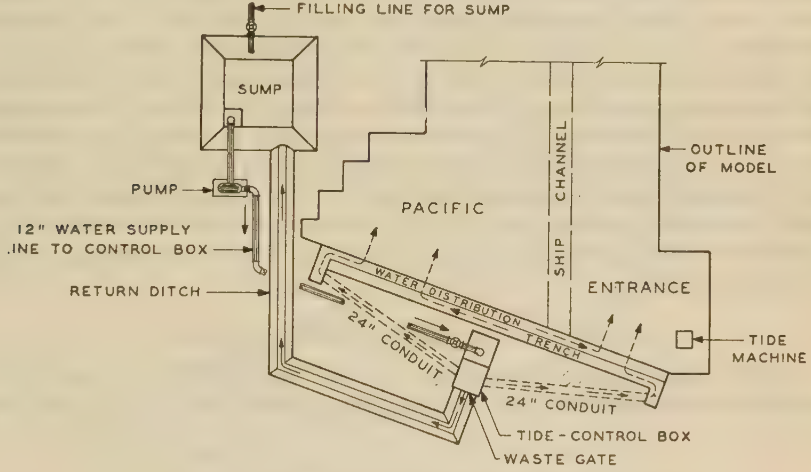


FIG. 32.—WATER-SUPPLY SYSTEM AT PACIFIC END, MODEL OF SEA-LEVEL CANAL

Principle of Tidal-Model Operation.—The principle of operation for this model was to control the water levels at the two ends to follow the desired tides, and to make observations of the water flowing back and forth in the channel under the influence of those tides. Any desired combination of tides could be used, and any desired modifications could be made in the operation of tidal-regulating structures. To observe the effects of any modification, it was always necessary to operate the model through one or more complete tidal cycles, and to coordinate a series of measurements at frequent intervals at a number of locations. . This procedure differs essentially from that used on the usual river model, where tests are normally made with steady flows, and leisurely adjustments and observations can be made until the experimenter is certain that a test run is satisfactory. To repeat any measurement on a tidal model, however, a full tidal cycle must first be run—which, for a 1:100 scale, requires 1.25 hours—and a crew of observers must then make the desired measurements in as short a time as possible, before conditions change appreciably.

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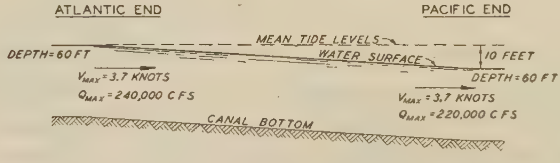
[22] "Hydraulic Models," *Manual of Engineering Practice No. 25*, ASCE, 1942, p. 36.

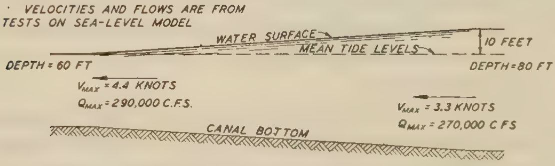
Model Measurements.—Fig. 30 shows the locations at which observations were made in the model. Water levels at the extreme ends were measured by recording gages which form part of each tide machine, and by automatic recorders and manually operated hook gages along the canal channel. Velocities were measured with pigmy current meters of the United States Geological Survey type, which were recalibrated at frequent intervals. For velocities lower than about 0.2 ft per sec (corresponding to full-scale velocity of 2 ft per sec), the meters were supplemented by measurements with floats consisting of corks supporting metal vanes suspended at a six-tenths depth. Observations were made simultaneously by observers at the different stations, at intervals varying from 5 min to 30 min of prototype time—the frequency of the observation depending on its importance and rapidity of change. Electric bells and signal lights were actuated by the tide machines at intervals corresponding to 30 min of prototype time—about 3 min of actual time—to assist in correlating the observations.

CURRENTS IN AN OPEN SEA-LEVEL CANAL

As previously stated, the maximum current that would be created at the Atlantic end of an open sea-level canal by the combination of 20-ft Pacific and 2-ft Atlantic tides has been indicated as 4.4 knots by the sea-level model, as 4.5 knots by the Pillsbury method of computation, and as 4.6 knots by steady-flow computations.


ATLANTIC END
PACIFIC END
MEAN TIDE LEVELS
DEPTH = 60 FT
WATER SURFACE
10 FEET
DEPTH = 60 FT
V_{MAX} = 3.7 KNOTS
Q_{MAX} = 240,000 C F S
V_{MAX} = 3.7 KNOTS
Q_{MAX} = 220,000 C F S
CANAL BOTTOM

(*a*) FLOW TOWARD PACIFIC WITH LOW PACIFIC TIDE


NOTE:
VELOCITIES AND FLOWS ARE FROM TESTS ON SEA-LEVEL MODEL
WATER SURFACE
MEAN TIDE LEVELS
10 FEET
DEPTH = 60 FT
DEPTH = 80 FT
V_{MAX} = 4.4 KNOTS
Q_{MAX} = 290,000 C.F.S.
V_{MAX} = 3.3 KNOTS
Q_{MAX} = 270,000 C F S
CANAL BOTTOM

(*b*) FLOW TOWARD ATLANTIC WITH HIGH PACIFIC TIDE

FIG. 33.—LOCATION OF MAXIMUM VELOCITIES FOR TIDAL FLOW

Location of Maximum Velocity.—Fig. 33, showing velocities and discharges taken from a test on the sea-level model, indicates how the maximum velocity occurs at the opposite end of the canal from the strong tide that creates it. The upper diagram indicates conditions for maximum flow toward the Pacific, when depths and velocities are nearly constant throughout the canal. With maximum flow toward the Atlantic, however, as indicated in Fig. 33(*b*), the depths and velocities vary considerably, the discharge is greater, and the highest velocity occurs at the point of smallest cross-sectional area, which is at the Atlantic end of the channel.

Fig. 34 shows water-surface profiles as observed in the model for successive hours during the tidal cycle. The changing slopes indicate the direction and relative velocity of the flow. When the Pacific tide was high (hour 12, Fig. 34), water throughout the length of the canal would flow toward the Atlantic.

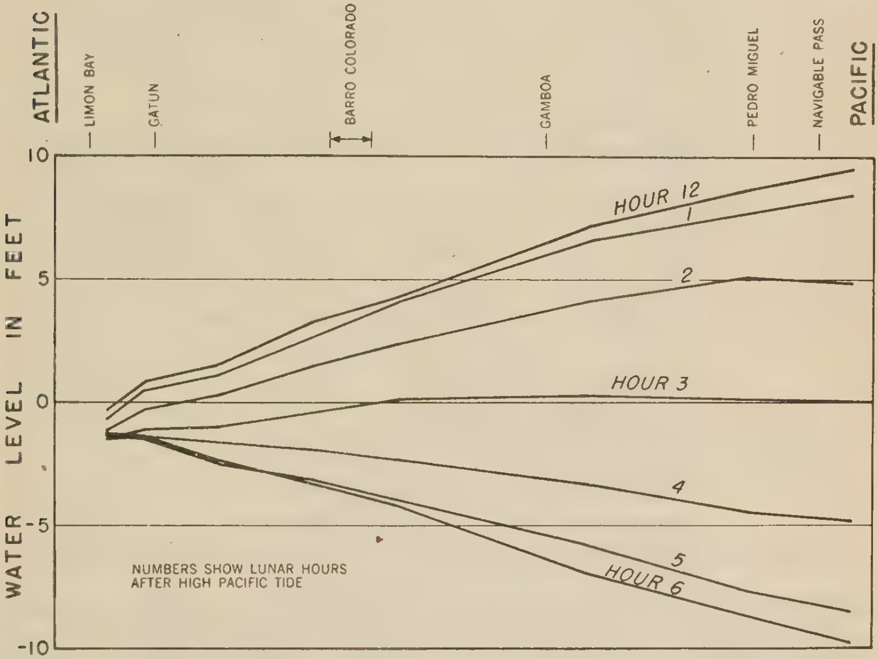


FIG. 34.—WATER-SURFACE PROFILES FOR AN UNREGULATED 20-FT PACIFIC TIDE, MODEL OF AN OPEN SEA-LEVEL CANAL

As the Pacific tide dropped, the flow would slacken and then change direction. For a short period during the change, water would flow out of each end of the canal, and then the flow throughout the canal would gradually increase to maximum strength toward the Pacific (hour 6, Fig. 34). A similar sequence of flows in the opposite direction would occur during the next half-cycle of 6 hours while the tide returned to its starting point. The current in the canal would always vary gradually and continuously with no abrupt differences, and would always change direction every 6 hours. A sinusoidal curve would be a good representation of the velocity at any point.

Comparison of Model Velocities and Computed Velocities.—Fig. 35 shows the observed velocities for unregulated tidal flow at the measuring stations nearest the ends of the model, and also the velocities as computed by the Pillsbury method when carried to different degrees of refinement. The agreement is very close throughout the tidal cycle as well as at the maximum points. The "second adjustment" computations which show the best agreement were carried out only for the combination of 20-ft and 2-ft tides. For most of the analyses the simpler "primary current" computation was used, which agrees very closely for the maximum velocities and is approximate only for the less important smaller velocities. In applying his computation method to an example of tidal flow in a Panama sea-level canal, General Pillsbury (in a letter to The Panama Canal, dated February 1, 1946) has concluded:

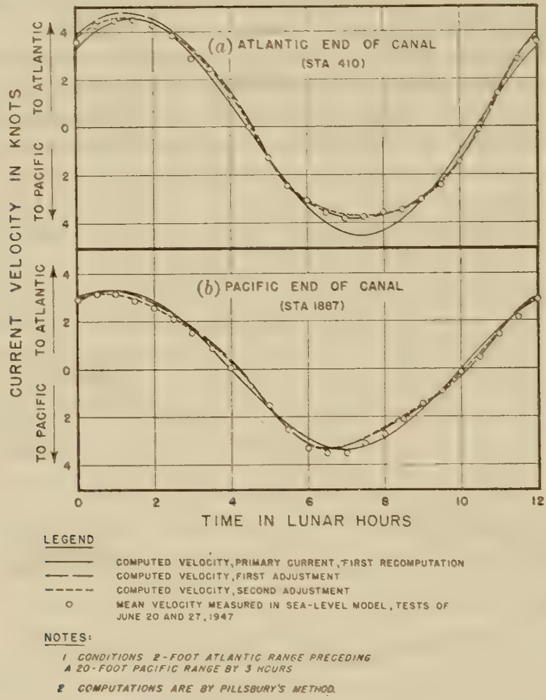


Fig. 35.—Computed and Observed Current Velocities, Unregulated Sea-Level Canal

> (1) " * * * that the corrections produced from the somewhat extensive computations required for the second adjustment are quite small and are in fact well within the uncertainties inherent in the selection of the proper coefficient of roughness, and in the dimensions of the canal as actually dredged, and as eroded by the strong currents";

and that

> (2) " * * * primary currents afford a reliable indication of the magnitude of the currents to be expected in an open sea-level canal across the Isthmus."

Table 21 shows a summary of computed (the Pillsbury primary current) and model velocities for the open sea-level canal. All velocities are the maxima reached during the tidal cycle. The model values listed are the average of velocities measured at seven points across the width of the channel. Velocities at the center of the canal (measured at six-tenths depth) are about 5% greater than these averages.

TABLE 21.—COMPARISON OF COMPUTED AND MODEL VELOCITIES FOR MAXIMUM TIDAL CURRENTS IN UNREGULATED SEA-LEVEL CANAL

TIDAL RANGE (FT)		Computed velocity at Atlantic end (knots)	OBSERVED VELOCITY IN MODEL (KNOTS)	
Pacific	Atlantic		Atlantic end	Pacific end
20	2	4.5	4.4	3.7
16	1	4.0	3.8	3.0
13	1	3.5	3.3	2.5
10	1	3.0	2.7	2.0
6	1	2.1	2.1	1.5

Fig. 36 shows, for any time during a tidal cycle and for any position along the channel, the velocity that would be found at that point. The "contours," which were drawn between values observed for several cycles in the sea-level model, indicate the regimen of velocity for the combination of 20-ft Pacific tides and 2-ft Atlantic tides. Sets of diagrams of this kind, for the full range of tides, have been useful in navigation studies, since lines can be drawn upon them to represent ship transits at different speeds and directions.

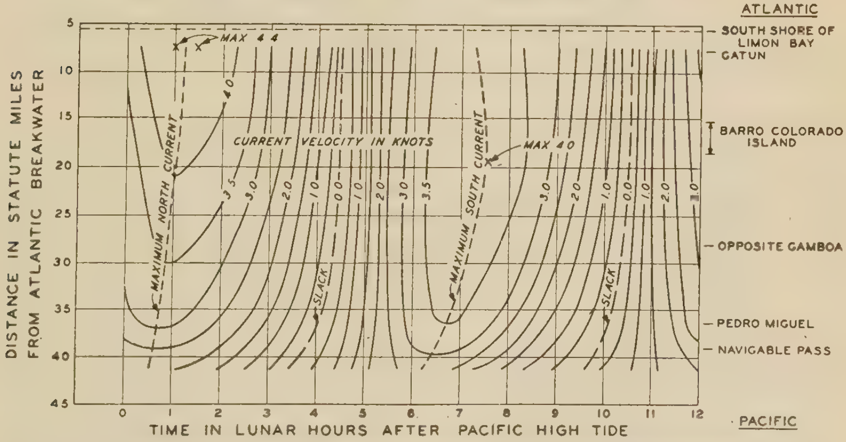


FIG. 36.—TIME-POSITION-VELOCITY DIAGRAM; UNREGULATED FLOW, 20-FT PACIFIC TIDE

Net Flow Toward the Atlantic.—As indicated in Fig. 33, there would be a greater flow toward the Atlantic during half of each tidal cycle than toward the Pacific during the other half. The water in the channel would move alternately toward the Atlantic and then toward the Pacific with the changing tides, with a net advance of about 5 miles per day toward the Atlantic indicated by the model for the 20-ft and 2-ft tides. For the same tidal ranges, but with the Pacific mean level higher than the Atlantic mean level, the total movement would increase. The most extreme situation of this kind which was found was that of December 19, 1937, when the 19.44-ft Pacific tide range averaged 1.74 ft higher than the Atlantic mean level, and the computed net movement of water (a second adjustment computation made by General Pillsbury) would have been about 20 miles per day.

Comparison of Steady-Flow and Tidal-Flow Velocities.—Table 22 compares maximum velocities for steady flow and tidal flow for different tidal ranges, as observed in the sea-level model. With minor exceptions (which are attributable to experimental error), steady-flow velocities are slightly higher. Computed velocities for steady flow agree closely with model results. The data support the opinion that tidal velocities should not exceed velocities for steady flow between the corresponding extreme water levels. Velocity computations on the assumption of steady flows are not difficult to make, but this procedure indicates only the maximum velocity in either direction. The Pillsbury method, on the other hand, provides successive values for velocities and water levels at all times during the tidal cycle. For this reason it was generally used as the basic computation method for analytical studies of velocities in the sea-level canal.

TABLE 22.—TIDAL–FLOW VELOCITIES AND STEADY–FLOW VELOCITIES IN SEA–LEVEL MODEL

WATER LEVELS				End at which velocities are measured	MAXIMUM VELOCITY (KNOTS) FOR:			
For Tides		For Steady Flow			Flow to Atlantic		Flow to Pacific	
Feet (1)	End (2)	Feet (3)	End (4)	(5)	Steady (6)	Tidal (7)	Steady (8)	Tidal (9)
20	Pacific	±10	Pacific	Atlantic	4.6	4.4	3.5	3.7
2	Atlantic	0	Atlantic	Pacific	3.7	3.3	3.8	3.7
16	Pacific	± 8	Pacific	Atlantic	3.7	3.8	3.3	3.4
1	Atlantic	0	Atlantic	Pacific	3.3	2.8	3.4	3.0
13	Pacific	± 6.5	Pacific	Atlantic	3.3	3.3	3.0	3.1
1	Atlantic	0	Atlantic	Pacific	2.9	2.4	3.0	2.5
10	Pacific	± 5	Pacific	Atlantic	3.0	2.7	2.6	2.5
1	Atlantic	0	Atlantic	Pacific	2.4	2.0	2.6	2.0
6	Pacific	± 3	Pacific	Atlantic	2.4	2.1	2.0	1.8
1	Atlantic	0	Atlantic	Pacific	1.7	1.5	2.0	1.4

Minor Velocity Reduction by Channel Enlargement.—As has been indicated in Fig. 33, the maximum tidal current in a sea-level canal with constant width and constant depth below a line connecting the low water levels would occur at the Atlantic end where the cross-sectional area is least. This velocity could be decreased to some extent by deepening or widening the canal at the Atlantic end. Any such enlargement, however, would increase the total flow capacity of the channel, and the velocities would be increased throughout the tidal cycle in the part of the channel not enlarged.

The greatest reduction in velocity could be obtained by a gradually tapered enlargement for about two thirds of the length of the canal, starting near Gamboa and extending to the Atlantic end, where the cross section would need to be enlarged by 16%. The maximum current for a 20-ft Pacific tide could be reduced from 4.5 knots (as computed by the Pillsbury method) to 4.0 knots which would then prevail for most of the length of the canal—the current at the Pacific end being increased while that at the Atlantic end was decreased.

CURRENTS IN A REGULATED SEA-LEVEL CANAL

Reduction of Current by Tidal-Regulating Structures.—Tidal currents in a sea-level canal could be regulated effectively by the structures shown in Fig. 9. The maximum current could then be regulated to any desired limiting value between 0.5 knots and 4.5 knots. The value of 0.5 knot would be the current produced by the action of a 2-ft Atlantic tide in a canal closed at the Pacific end, and the value of 4.5 knots would be that produced by the combination of 20-ft Pacific and 2-ft Atlantic tides in the open channel. Fig. 37 shows the proposed layout for a tidal lock and navigable pass. The pass would normally be closed when the tide was at a high level or a low level to exclude the entry of tidal flow into the canal which would cause currents in excess of the selected maximum value. Ships would use the tidal lock when the navigable pass was closed.

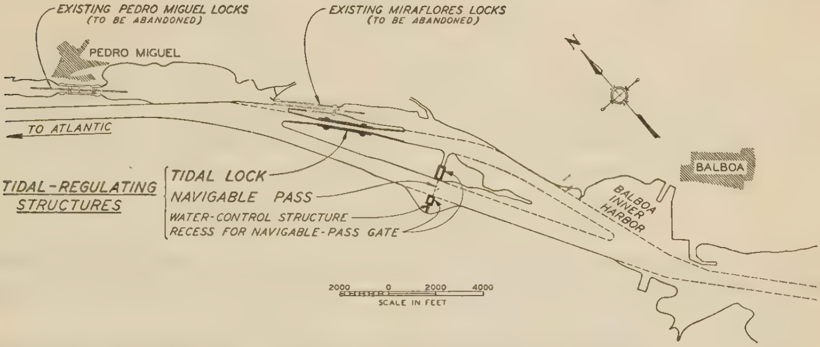

EXISTING PEDRO MIGUEL LOCKS
(TO BE ABANDONED)
EXISTING MIRAFLORES LOCKS
(TO BE ABANDONED)
PEDRO MIGUEL
TO ATLANTIC
TIDAL-REGULATING STRUCTURES
TIDAL LOCK
NAVIGABLE PASS
WATER-CONTROL STRUCTURE
RECESS FOR NAVIGABLE-PASS GATE
BALBOA
BALBOA INNER HARBOR
2000 0 2000 4000
SCALE IN FEET

FIG. 37.—LAYOUT OF TIDAL-REGULATING STRUCTURES

Fig. 38 shows schematically the operation of the regulating structures during a tidal cycle. The action of such structures may be considered as replacing the natural Pacific tide (range A-D, Fig. 38) with a smaller regulated tide (range B-C) on the canal side of the structures which would produce currents not exceeding the selected limits. For a 2-knot limit the regulated tide would be about 6 ft. For a higher limiting current, the regulated range would be greater and the navigable pass could be kept open longer. The water level in the canal would return approximately to Atlantic tide level if no water were released through the tidal-regulating structures after point C, Fig. 38. Reopening of the navigable pass would then be delayed beyond point E until the natural tide level rose to meet the canal level. The function of the water-control structure is to permit the release of a limited flow of water from the canal to the Pacific during this period and to bring the canal level to the desired elevation at point E. The navigable pass could then be opened promptly on schedule, and would stay open for a longer total time. A similar sequence of events, but in the opposite direction, would occur during the remaining half of each tidal cycle.

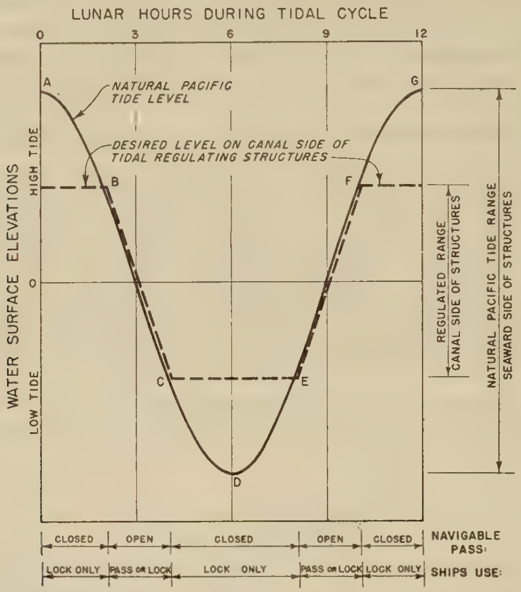

LUNAR HOURS DURING TIDAL CYCLE
0
3
6
9
12
A
NATURAL PACIFIC TIDE LEVEL
G
DESIRED LEVEL ON CANAL SIDE OF TIDAL REGULATING STRUCTURES
B
F
C
E
D
WATER SURFACE ELEVATIONS
HIGH TIDE
0
LOW TIDE
REGULATED RANGE
CANAL SIDE OF STRUCTURES
NATURAL PACIFIC TIDE RANGE
SEAWARD SIDE OF STRUCTURES
CLOSED
OPEN
CLOSED
OPEN
CLOSED
NAVIGABLE PASS:
LOCK ONLY
PASS OR LOCK
LOCK ONLY
PASS OR LOCK
LOCK ONLY
SHIPS USE:

Fig. 38.—Operation of Tidal-Regulating Structures

On the basis of the idealized conditions indicated in Fig. 38, the permissible ranges of water levels on the canal side of the tidal-regulating structures were computed for different velocity limits in the canal. If the controlled water level can range the vertical distance between points E and F, then the period when the pass could remain open is the horizontal distance along the time scale between points E and F, which will vary with the rate of rise of the natural

TABLE 23.—Daily Availability of Navigable Pass

Permissible current (knots)	Hours per Day Navigable Pass Could Remain Open for Pacific Tidal Ranges of:				
	6 ft	10 ft	13 ft	16 ft	20 ft
1	8.5	4.5	3.5	2.9	2.3
2	20.2	9.7	7.2[a]	6.0	4.9[c]
3	24	24	14.9[b]	11.6	9.1[d]
4	24	24	24	24	19.8
4.5	24	24	24	24	24

[a] In model test, pass was kept open 8 hours per day. [b] In model test, pass was kept open 16 hours per day.
[c] In model test, pass was kept open 3.8 hours per day. [d] In model test, pass was kept open 8 hours per day.

tide. This analytical indication of the relation between controlled velocity, natural tide range, and time of opening is summarized in Table 23, which shows hours per day when the navigable pass would be available for traffic.

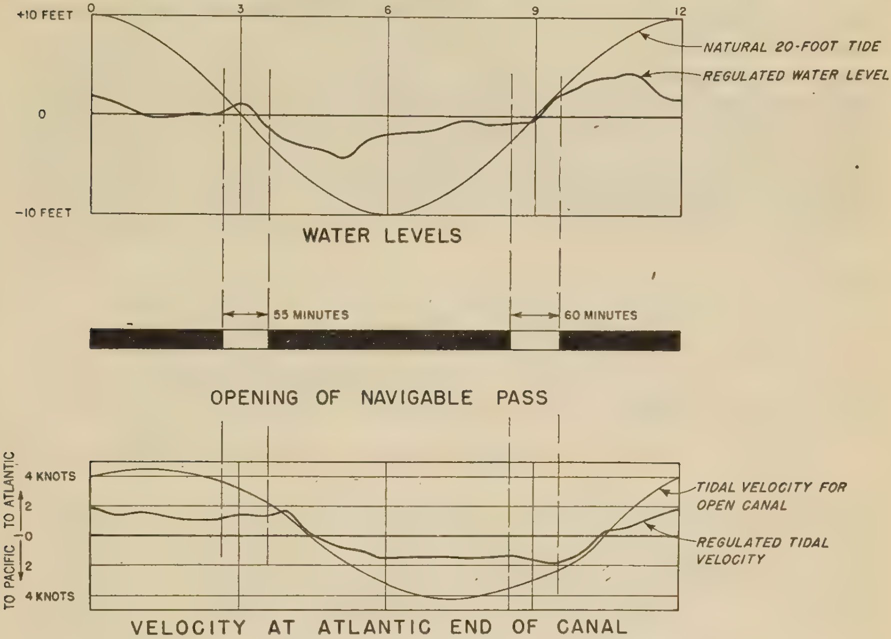


FIG. 39.—MODEL TEST; REGULATION TO 2 KNOTS, 20-FT PACIFIC TIDE

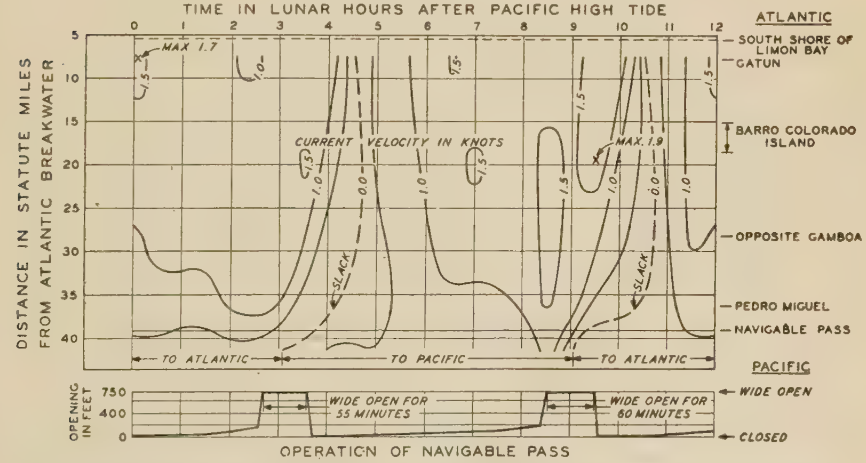


FIG. 40.—TIME-POSITION-VELOCITY DIAGRAM; 2-KNOT REGULATED CURRENT; 20-FT PACIFIC TIDE

Regulated Flow in Sea-Level Canal Model.—Fig. 39 shows one cycle of model test observations for current regulated to a maximum of 2 knots with a 20-ft

Pacific tide, for which the navigable pass remained open for one 55-min period and for one 60-min period during each 12 hours. In this test the navigable pass was used for flow regulation instead of the water-control structure. The regulated velocities are shown also in the time-position-velocity diagram of Fig. 40. Comparison with the similar diagram (Fig. 36) for unregulated tidal velocities shows much the same general velocity pattern, but of course with reduced maximum values. The model tests made thus far (October, 1947), which have also covered other tidal ranges and velocity limits as indicated in Table 23, have demonstrated the possibility of controlling currents and keeping the pass open for periods approximating the computed values. Additional testing and analysis of model results will be necessary, however, to provide data for gate schedules for routine daily operation of the canal.

SHIP PERFORMANCE IN RESTRICTED CHANNELS

By C. A. Lee[23] and C. E. Bowers,[24] Juniors, ASCE

Synopsis

The purpose of the restricted channel model tests at the David Taylor Model Basin was to obtain information which would be of assistance in the selection of the cross-sectional dimensions and in the design of bends for specified conditions of canal operation. The tests were sponsored by the Panama Canal under the authority of Public Law No. 280, Seventy-ninth Congress. The model studies included:

1. An investigation of the effect of varying the cross-sectional dimensions of the channel for both one-way and two-way traffic;
2. The comparative handling characteristics of several different types of ships under various conditions;
3. The effect of current in the channel on the handling characteristics of the ships; and
4. A comparison of several types of bends.

The first tests were conducted with a single ship in a straight channel and in still water. The experience gained from these tests is fundamental to a complete understanding of the more special problems which follow the description and the results of the straight channel, still-water, one-way traffic conditions.

TABLE 24.—General Information Concerning Models Tested in Restricted Channels

Model No.	Scale ratio, λ	Dimensions of Model (Ft)			Dimensions of Full-Scale Ship (Ft)			Propellers		No. of rudders
		Length[a]	Beam	Draft	Length[a]	Beam	Draft	No.	Rotation	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
3769......	45	20.00	2.51	0.717	900	113	32.25	4	Outward	2
3859......	35	20.58	2.855	0.914	720.6	100	32.13	2	Outward	1
3992......	86	10.00	1.256	0.4025	860	108	34.625	4	Outward	2
3748–4....	45	10.01	1.33	0.6326	455	59.84	24.48	1	Right hand	1
3673–A....	44.5	20.00	2.585	0.815	890	115	36.27	4	Outward	2
4018......	45	16.00	2.221	0.714	720.6	100	32.13	2	Outward	1

[a] Water-line length of loaded vessel.

It was thought that the two primary problems in the selection of canal characteristics were (*a*) the controllability of full-scale ships operating at selected speeds in the canal, and (*b*) the change of level of ships in the canal.

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The controllability of full-scale ships from model studies appeared to be the more difficult to evaluate and as a result the test program emphasized this part of the studies.

The models representing larger ships were selected for test on the basis of difficulty of handling in restricted channels. The model of a "Liberty" ship was selected for test as representing an average ship transiting the canal. A particular object of study was to discover the effects on the Liberty ship when meeting and passing the larger vessels. Table 24 gives general background information on the various ship models used during the investigation.

NOTATION

Referring to Fig. 41, let: b_y be the distance from the center line of the channel to the gravity center of the ship (point G); b_z be the distance between the gravity center of the ship and the prism line; V_c be the water speed; V_s be the ship speed (with respect to water); δ be the rudder angle; λ be the scale ratio; and ψ be the angle of yaw. In the restricted channel investigations, Froude's law is used as a basis for adjusting velocity, rate of change of rudder, and revolutions per minute on the propellers, to produce conditions similar to full-scale operations. Froude's law sets forth a nondimensional parameter which is used for model studies in which gravitational influences predominate. Another nondimensional parameter used to express the effect of viscosity on the flow about a vessel is the Reynolds number, **R**.

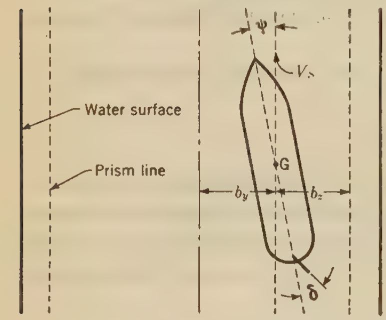

FIG. 41

The model vessels and the models of the restricted channels were constructed geometrically similar for the various test conditions—that is, all linear dimensions were reduced directly in accordance with the selected scale ratios. Under these circumstances the flow characteristics (velocity and pressure) would be similar if the fluid properties were such that the Froude and Reynolds numbers, respectively, are equal for model and full-scale conditions in all cases. The Froude number, which allows for the influence of gravitational forces, may be defined as:

$$\mathbf{F} = \frac{V^2/a}{\gamma/\rho} \qquad (1)$$

The Reynolds number, which allows for the effect of viscosity, is

$$\mathbf{R} = \frac{V\,a}{\mu/\rho} \qquad (2)$$

in which V is the velocity, in feet per second; a is a convenient linear dimension, in feet, defining a fixed boundary condition; γ is the specific weight, in pounds per cubic foot; ρ is the density in slugs per cubic foot; and μ is the dynamic viscosity, in pound-seconds per square foot.

The effects of viscosity will be large enough to read by scale if the Reynolds number is so small that a large part of the boundary layer is laminar rather than turbulent. The actual size and scale ratios of the models used in this investigation insure that the Reynolds numbers are large enough to preclude the scale effects produced by viscosity. Consequently, it was only considered necessary to employ the same Froude number for the model as for the full-scale condition. Thus, the following relationships for time rates and velocity result:

(1) Time rates for the model (that is, rate of rudder movement, revolutions per minute of the propellers, rate of calling orders from "pilot" to "quartermaster," and any other time rates) should equal the time rates for the various elements on the full-scale ship multiplied by the square root of the linear ratio.

(2) Velocity for the model (that is, speed of the model ship and velocity of the water in the channel during moving water studies) should equal the velocity of the full-scale vessel divided by the square root of the linear ratio.

SHIP-HANDLING CONSIDERATIONS IN RESTRICTED CHANNELS

Navigation in restricted channels or canals is very difficult, not only because of the limited space available but also because of various hydrodynamic phenomena that introduce additional hazards. One of these phenomena is popularly referred to as "bank suction.". Bank suction occurs when a vessel is closer to one side of a restricted channel than it is to the other side or when the vessel passes projections in the channel. Its effect is to cause the vessel to sheer or deviate from its original course. It could be described as an interaction between the ship and the channel boundaries. As a result of this effect, an asymmetrical flow distribution develops on the two sides of the vessel, creating unbalanced forces which tend to force the vessel off its original course. If the vessel is under way in a restricted channel, on a course parallel to but to one side of the center line of the channel, the water surface between the bow and the near bank will build up above the level of the normal water surface with the result that the bow is forced away from the near bank. As the water flows aft along both sides of the vessel to fill the void left by the stern, the level of the water surface drops below the normal surface level. The level of the water surface between the vessel and the near bank drops lower than the level on the other side, with the result that the stern of the vessel is forced toward the near bank. The net result of the difference in water level on the two sides of the vessel is to cause the ship to sheer away from the near bank. In some instances the sheer that results cannot be overcome by the rudder and the vessel strikes one of the banks. It is necessary to use a rudder setting which tends to turn the vessel toward the near bank to counteract this effect. Thus, if a large vessel were near the right bank, it would be necessary to use "right rudder" to counteract this effect. If it were desired to return to the center of the channel, the rudder could be eased off enough to allow the vessel to return slowly to the center. If, by using selected rudder angles, the heading of the vessel can be maintained parallel to the bank while the vessel is off center in the channel, the resultant of all side forces acting on the vessel will be a force toward the near bank. If this heading is maintained for a time, the vessel will move bodily into the near bank. However, if the vessel is given a slight angle of yaw away from the near bank and a rudder angle just sufficient to counteract the yawing moment, both the side force and moment will be neutralized and the vessel will maintain a path parallel to the bank.

Another hydrodynamic phenomenon that may be serious during the transiting of vessels through restricted channels is the change of level of the vessel. When the vessel is under way in shallow water or in a restricted channel, the water surface in the vicinity of the vessel drops below the level of the normal water surface because of the increase in the velocity of the water as it flows around the vessel, and the vessel drops with it. If the initial draft of the vessel is quite large with respect to the depth of water in the channel, the ship may touch bottom if under way at relatively high speeds, whereas it would have ample clearance if it were stationary or traveling at a low speed. The magnitude of this change of level is a function of the ship speed, the dimensions and lines of the ship, and the channel dimensions.

Studies of One-Way Traffic

Relative to the straight-channel, one-way traffic, still-water studies of ship performance in restricted channels, primary emphasis was placed on an investigation of (*a*) the relative controllability of specified ships in channels of various cross-sectional dimensions, and (*b*) the change of level of ships in restricted channels. Another factor that may be of general interest, but which was omitted from this investigation, is the resistance of ships in restricted channels. In the past, many investigators have concerned themselves with the resistance of ships in shallow water, but few have treated the problem of ship resistance in restricted channels. Information on ship resistance in shallow water is available in several places.[25,26,27,28,29,30,31,32] Most available information on ship resistance in restricted channels appears in papers by G. S. Baker,[33] Francis Roubiliac,[34] F. E. Nelson,[35] and T. Izubuchis and S. N. Z. Kidkai,[36] although additional data on barges have been obtained.

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[25] "Model Studies of Ship Motions in Canals," by Charles E. Bowers, David Taylor Model Basin, Washington, D. C., November, 1946.

[26] "Barge Canals—Dimensions," by J. M. Rankine, *Encyclopædia Brittanica*, 14th Ed., 1929, Vol. 4, pp. 720–727.

[27] "The Relation of Depth of Water to Speed and Power of Ships," *Engineering News*, Vol. 53, 1905, pp. 275–276.

[28] "Der Schiffswiderstand im bergrenzten Fahrwasser und sein Einfluss auf die Grössenverhältnisse der Schiffahrtskanäle" (On Ship's Resistance in Limited Waters and Its Effect on the Relationships Between the Sizes and the Channel), by M. Graevell, *Der Civil Ingenieur*, 1887, pp. 87–110.

[29] "The Influence of Depth of Water on the Resistance of Ships," by Charles P. Paulding, *Marine Engineering*, May, 1903, pp. 239–243.

[30] "The Resistance of Some Merchant Ship Types in Shallow Water," by Herbert C. Sadler, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 19, 1911, pp. 83–86.

[31] "Modell-Schleppversuche für Lastkähne im Kanalprofil" (Model Towing Tests for Barges in the Channel Profile), by Karl Schaffran, *Schiffbau*, Vol. 16, No. 13, 1914/1915, pp. 321–326.

[32] "A General Discussion of Resistance and Power Consumption of Ships in Different Depths of Water," by David W. Taylor, *Engineering News*, Vol. 53, 1905, pp. 276–279.

[33] "Steering of Ships in Shallow Water and Canals," by G. S. Baker, *Transactions*, Institution of Naval Architects, Vol. 66, 1924, pp. 319–340.

[34] "Speed of Canals," by Francis Roubiliac, *Minutes of Proceedings*, Inst. C. E., Vol. 76, 1884, pp. 160–265.

[35] "Handling Vessels in Restricted Waters," by F. E. Nelson, *Proceedings*, U. S. Naval Inst., June, 1928, pp. 446–456.

[36] "Experimental Investigations on Influence of Depth of Water Upon Resistance of Ships," by T. Izubuchis and S. N. Z. Kidkai, *Paper N4 Autümwerting*, Soc. of Naval Architects—Japan, 1937 (in Japanese).

An investigation of the controllability of ships in restricted channels is necessarily quite complex. It involves the effect of interaction between the vessels and the channel boundaries (bank suction), the steering characteristics of the vessel, and the effect of the restricted channel on the steering characteristics of the vessel. If an attempt is made to maneuver the model in a manner similar to the maneuvers of a full-scale vessel in a restricted channel, the "human element" or skill of the pilot becomes important. In an attempt to evaluate these factors, two general types of tests were set up. In each of these types it was planned to test several ship models in channels with various cross-sectional dimensions.

One type of test, the so-called observational tests, consisted of observing and photographing the models while they were under way and completely unrestrained in a restricted channel. The other type, the force-measurement tests, consisted of the measurement of side forces which developed when the model was held at various transverse positions in a stream of moving water. The side force and yawing moment were measured for various rudder angles and angles of yaw. In addition, the rudder angle required to overcome the turning moment caused by interaction was determined.

Observational Tests.—The observational tests were conducted in a Taylor Model Basin facility known as "the shallow water basin." This basin consists of a concrete-lined channel, 52 ft wide, 10 ft deep, and approximately 300 ft long. Fig. 42 is a photograph of the basin. The central part of the restricted channel is made of steel sections with adjustable sides, so that the angle of slope can be set at 18.25°, 30°, 45°, or 90° to the horizontal. The width of the channel can be varied by moving one or both sides along the basin floor. The depth of water is varied by changing the water level in the basin. The wooden slat structures outside the channel, and at the near end, are arranged to break up waves and surges set up by motion of the model. The ship model, shown at the far end of the channel in Fig. 42, is operated by distant control from the special platform under the towing carriage. The water depth can be set at any value up to 10 ft. A towing carriage, which spans the basin, can be run in either direction at speeds up to about 8 knots. The functions of the towing carriage are (1) to tow models that are being tested for resistance or other performance characteristics; (2) to provide a movable observational and photographic platform, and power supply, for studies of self-propelled models; and (3) to accelerate self-propelled models to the desired speed in a short time. The two steel walls were placed on the floor of this basin to form a smaller channel. The over-all length of this channel was 180 ft, which is equivalent to approximately 1.5 miles of full-scale channel. Its width could be varied from 0 to 23 ft. The side slope of the walls could be varied from an angle with the horizontal of from 18.25° to 90°. During the observational phase of the tests, the walls were set at an angle of 45°. Fig. 43 is a photograph of a model running through a restricted channel 60 ft deep and 300 ft wide with side walls set at 45°. The "pilot" issues orders to the helmsman who steers the self-propelled model by the remote-control rudder gear located on the lower platform of the towing carriage. The flexible cable at the stern carries the electrical leads and is held in position by the movable boom. The reflector mounted

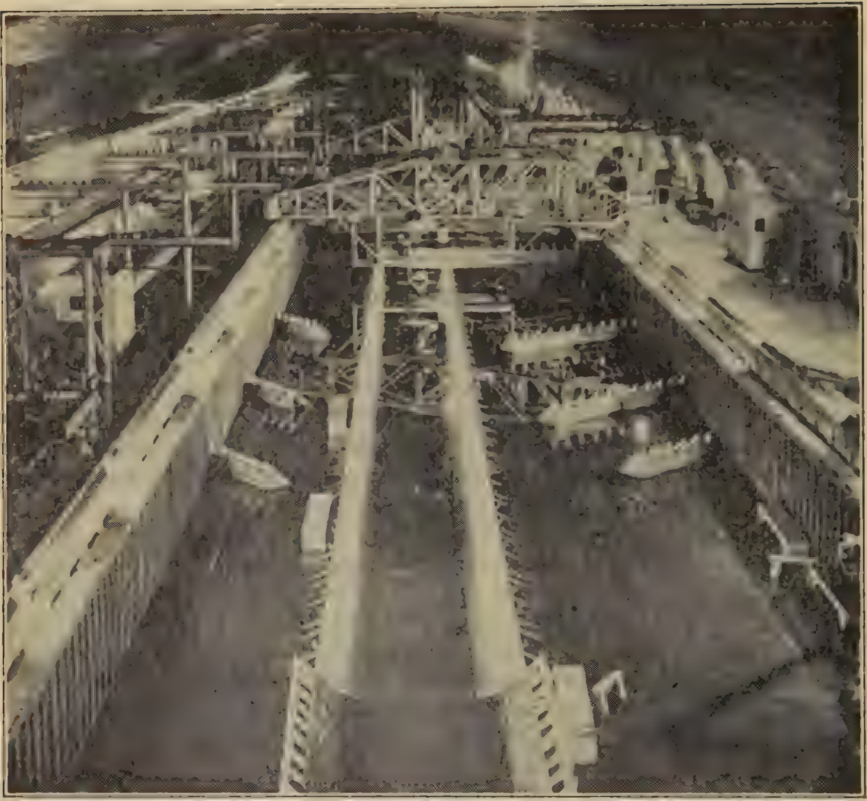


Fig. 42.—General View of Basin Building, Showing the Restricted Channel Setup in the Shallow Water Basin

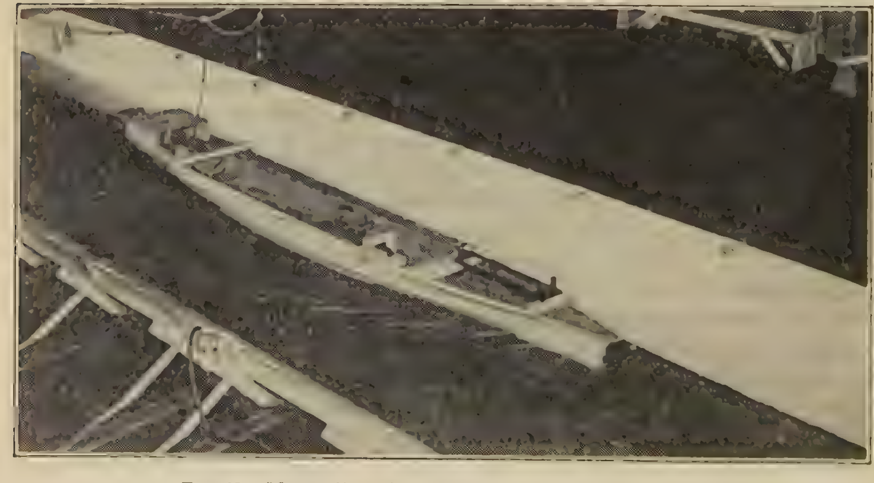


Fig. 43.—Model 3769 Under Way in a Straight Channel

near the bow casts a beam of light on a horizontal scale at the far end of the channel, thus giving the pilot a good indication of changes in the ship's heading.

The tests in this shallow water basin were conducted using Taylor Model Basin models 3769, 3748-4, and 4018; and they represented channels of 300 ft, 500 ft, and 700 ft wide at the bottom with a 1:1 side-wall slope. Each width was tested with depths of water of 45 ft, 60 ft, and 80 ft. The scale ratio for the model studies was 1:45. On the basis of Froude's law, the model speed during the tests was equal to the full-scale speed divided by the square root of 45. For example, the model speed corresponding to 10 knots full scale was 10 divided by 6.71, or 1.49 knots.

Two types of observational tests were conducted. One type (referred to as the "rudder release tests") was for the purpose of obtaining a comparison of the bank suction or interaction for various off-center positions of the ship in the channels. The nature of the tests was such that the magnitude of the yawing moment, caused by interaction between the ship and the channel boundaries, was obtained in terms of the rudder angle required to counteract it.

Some question may be raised with regard to the advisability of expressing the yawing moment in terms of rudder angle rather than in a more orthodox form such as foot-pounds. One advantage of this method is that it expresses the moment in a term which is familiar to most people who are acquainted with the handling of ships. On the average ship the maximum rudder angle that can be used is approximately 35°. Thus, if the rudder angle required to counteract the bank suction or interaction, for some specified condition, is from 25° to 30°, it is obvious that the moment is quite large with respect to the maximum counteracting moment that can be developed by the rudder. A disadvantage of this method, in some instances, is that the turning moment developed by the rudder may not be directly proportional to the rudder angle at large rudder angles.

In conducting the first type of observational tests, the model was attached to the towing carriage by two pins which held the longitudinal axis of the model parallel to the center line of the model canal channel. The towing carriage was then accelerated to the desired speed and at the same time the speed of the propellers of the model was increased to that which would propel the model at the desired speed. The model was then released from all contacts with the towing carriage, with the exception of a light flexible cable that supplied power to the propeller motors. The path and heading of the model were observed from the towing carriage. A light source on the bow of the model cast a 2-in. beam of light along an extension of the model center line to a horizontal scale at the end of the channel. By observing the movement of the beam, the observer could note, instantly, the changes in the heading of the model. It should be noted that the purpose of the towing carriage was to accelerate the model in as short a distance as possible, thus providing the maximum length of the channel available for observation of the unrestrained model. The model was attached to the towing carriage on a line parallel but to one side of the center line. If the rudder had previously been set at zero, it would be noted that the model sheered away from the near wall on release from the carriage. As soon as this sheer was observed, the model and carriage were stopped and the model was again attached to the carriage at the same off-center position. Before releasing it a second time the rudder would then be set at an angle which would normally turn the model toward the near bank. If the model again sheered away from the near bank, it would be returned to the carriage and the rudder angle would be increased. This procedure was repeated until a rudder angle was selected which would just counteract the yawing moment caused by the interaction. After this rudder angle had been selected, the procedure was repeated at several higher speeds. The model was then moved to a point farther off center and the complete procedure was repeated. In this manner, data were obtained for the rudder angles required to counteract interaction at various off-center positions, various ship speeds, and for channels with various widths and depths.

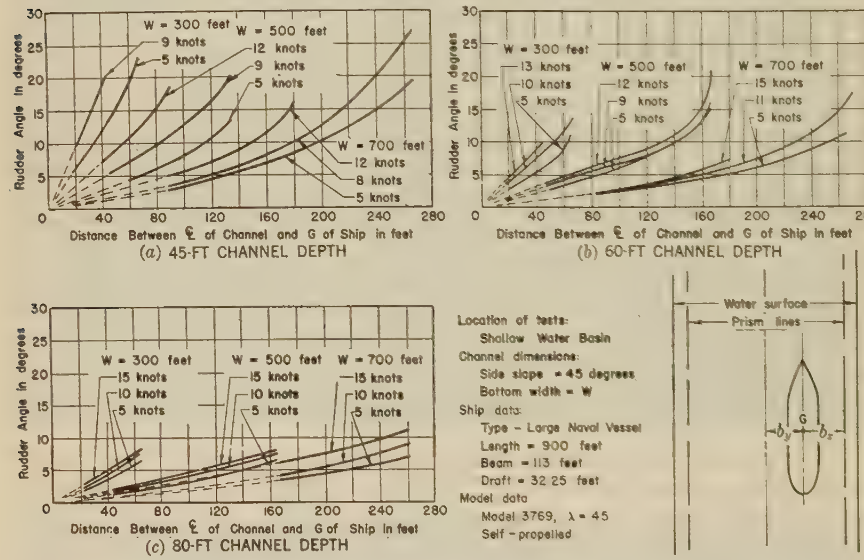
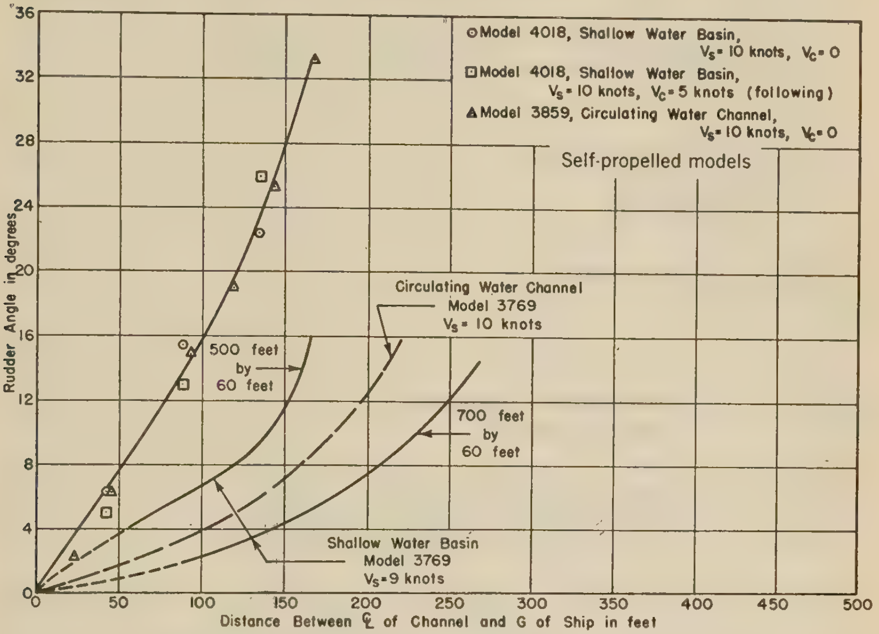


Fig. 44.—Rudder Angle for Equilibrium as a Function of the Distance Between the Center Line of the Channel and the Center of Gravity (Point G) of the Ship

Fig. 44 is a plot of the rudder angle required to counteract interaction or bank suction against distance between the center line of the channel and the center of gravity of the ship. Specifically, these rudder angles are required to counteract the yawing moment that exists when self-propelled model 3769 is released parallel to, and at various distances from, the center line of the channel. Data for several channel widths and ship speeds have been plotted on each graph for purposes of comparison. For a given speed the distance that the ship can navigate off center in the channel without exceeding a specified rudder angle may be selected for each channel width. It may be noted from these data that the rudder angle required to counteract the effects of interaction is especially dependent on the ship speed at a 45-ft depth. Furthermore, the slope of these curves, or the rate at which the rudder angle increases, is much steeper at the shallower depths and narrower widths. In most instances the rudder angle for equilibrium was measured for at least three off-center positions of the ship. These positions were varied for each width of channel so as to cover the proper range of rudder angles. For selected channel widths and ship speeds, the curves are indicative of the effect of channel depth on the yawing moment caused by interaction. It may be noted that, for the 300-ft width, the yawing moment caused by interaction at a channel depth of 60 ft is less than half as great, in terms of rudder angles, as it is for the 45-ft depth. An increase in depth from 60 ft to 80 ft causes a further decrease in the required rudder angle but the additional change is much smaller. For channel widths of 500 ft and 700 ft, there is a similar decrease in the required rudder angle with increasing depth.



	MODEL 3769	MODEL 3859	MODEL 4018	MODEL 3769
Location of tests:	Circulating Water Channel	Circulating Water Channel	Shallow Water Basin	Shallow Water Basin
Channel dimensions:				
Width =	600 feet	500 feet	500 feet	500, 700 feet
Depth =	60 feet	60 feet	60 feet	60 feet
Side Slope =	Vertical	Vertical	45 degrees	45 degrees

FIG. 45.—RUDDER ANGLE FOR EQUILIBRIUM AS A FUNCTION OF THE DISTANCE BETWEEN THE CENTER LINE OF THE CHANNEL AND THE CENTER OF GRAVITY (POINT G) OF THE SHIP

Subsequently, additional tests were conducted on the model of the naval vessel as well as on models of a large tanker and a Liberty ship. Some of these tests were conducted in moving water for the purpose of checking the effect of currents in the channel on the interaction between the ship and the channel boundaries. Others were conducted in the circulating water channel for the purpose of comparing results between that facility and the shallow water basin. The results of some of these tests are presented in Figs. 45 and 46.

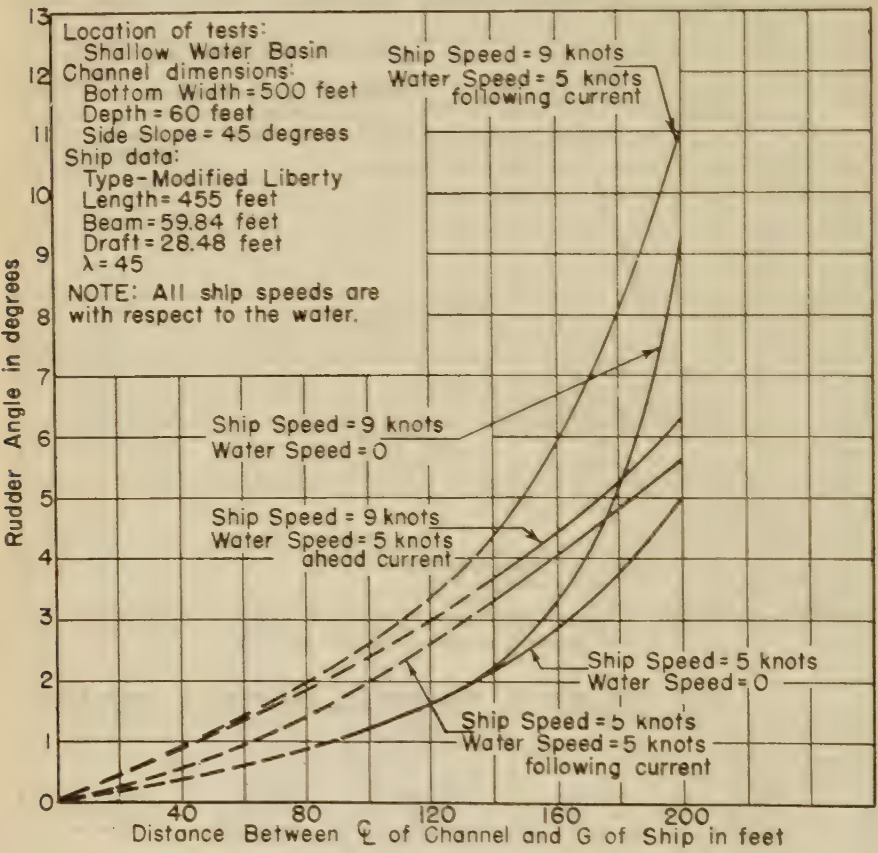


FIG. 46.—RUDDER ANGLES FOR EQUILIBRIUM AS A FUNCTION OF THE DISTANCE BETWEEN THE CENTER LINE OF THE CHANNEL AND THE CENTER OF GRAVITY (POINT G) OF THE SHIP

With reference to Fig. 45, the data for model 3859 (tanker) in the circulating water channel were obtained by holding the model stationary at various transverse positions in a moving stream of water. The test methods are described in the section entitled "Force-Measurement Tests." Similar data were obtained in the shallow water basin, by the methods just described, for both still water and currents. The data indicate that the two facilities produce nearly identical results. Also, the interaction between the ship and the channel boundaries is apparently the same for both still and moving water at the same ship speed with respect to the water.

In Fig. 46, data are shown for the rudder angles required to counteract interaction for a Liberty ship in both still and moving water. (These data are based on self-propelled tests of model 3748-4. The rudder angles are required to counteract the yawing moment that develops when the ship is nearer one wall then the other.) For the same ship speed with respect to the water, the curves indicate that the rudder angles are dependent on the current. However, the maximum difference between the curves for practical operating conditions is less than 3°, which is not considered significant in view of the limited amount of data obtained on this model.

A second type of observational test was based on maneuvering a self-propelled model by remote control. Throughout the tests an effort was made to duplicate full-scale operating conditions as closely as possible. In these tests, the model, when accelerated to the desired speed, was released from all contacts with the towing carriage, with the exception of a light flexible cable which supplied power to the propeller and rudder motors. By closing a double-throw switch on the towing carriage, the rudder on the model could be moved to any desired setting. This setting could be altered as frequently as desired throughout the run. A large indicator was mounted on the stern of the model which showed the instantaneous rudder angle at all times. In addition, the rudder angle was indicated by a "selsyn" system on the model control panel of the towing carriage. By observing the movement of the beam of light from the bow, the observer or pilot could note, instantly, any changes in the heading of the model. During the run, the pilot attempted to maintain the model on a course parallel to or on the center line of the channel. On orders from the pilot, an assistant manipulated the controls which actuated the rudder. Soon after the model was released from the carriage, it was photographed at intervals of about 1 sec by an overhead camera. The camera provided a record of the path of the model, its speed, and the rudder angles used.

During this phase of the tests, an attempt was made to maintain a course parallel to, and at a specified distance from, the center line, as well as directly on the center line of the channel. This procedure differs somewhat from actual full-scale operating conditions in that the pilots usually attempt to stay on the center line of a canal except when meeting another ship. However, it was thought that the off-center runs might provide additional information. Maneuvering runs were taken at about three transverse positions in channels with bottom widths of 300 ft and 500 ft and with depths of 45 ft, 60 ft, and 80 ft. An attempt was also made to conduct separate runs for various speeds ranging from 5 knots to 15 knots, but in numerous instances it was not possible to reach the top speed of 15 knots because of excessive change of level of the model or because of other hazardous operating conditions.

During this phase of the tests, it was noted that for a specified width the depth of water was very important with regard to ease of handling. At the 60-ft and 80-ft depths, it was much easier to control the model and to maintain a course parallel to the bank at greater distances off center than was possible at the 45-ft depths. This condition was especially noticeable at channel widths of 300 ft and 500 ft.

During these tests (which were made to find the maximum off-center distances at which specified ship models could be satisfactorily maneuvered), certain phenomena were allowed to develop and the resultant action of the vessel was observed. It was noted that, in any off-center position in the channel, if the bow was allowed to swing a few degrees away from the near bank there was a sudden drop in the water surface between the stern of the vessel and the near bank. As a result of these conditions, a moment was produced which tended to increase the rate of swing of the bow. If the condition was not corrected immediately, the vessel would develop such a sheer that application of rudder would not bring the vessel into a condition of equilibrium. As a result, the vessel would sheer across the channel and the bow would strike the far bank or the stern would be forced into the near bank. If the heading of the model was maintained exactly parallel to the near bank during an off-center run, by using the correct rudder angle, the model would drift laterally toward the near bank. As it moved closer to the wall, the yawing moment became larger and it was necessary to increase the rudder angle to maintain the heading of the model parallel to the bank. Eventually the model would ground unless the yawing moment became so large that a sheer developed. Therefore, the angular position of the vessel in the channel, especially in an off-center position, is important. Considerable judgment is required of the pilot in balancing the forces acting on the bow and stern by application of rudder and slight changes in heading. It should be understood that application of rudder alone will not always bring the vessel into equilibrium for an off-center position in a restricted channel but that a proper amount of heading must also be maintained.

When the total of all the rudder angles used for a given run were averaged arithmetically and plotted against the average distance between the center line of the channel and the center of gravity of the ship during the run, the resultant curves closely approximated the rudder angles for equilibrium which were obtained in the first part of the observational tests. The primary value of these data is that they substantiate the equilibrium rudder angles which were derived in the first part of the test, as an indication of the yawing moment caused by interaction.

The maneuvering data are still being analyzed and it is hoped that additional quantitative information on controllability may be obtained from them. The preceding results are based on only one model and are intended primarily to indicate some of the general characteristics of the interaction or bank suction between a ship and the channel boundaries. It would undoubtedly be desirable to obtain information on the effect of varying the cross-sectional dimensions of the ship, holding the length constant, as well as information on a variety of ship types and designs. However, a program of this type would be too extensive for the present investigations.

Force-Measurement Tests.—The purpose of the force-measurement tests was to supplement the observational tests in order to evaluate the various factors affecting the controllability of ships in restricted channels. As the term implies, the tests consisted of measuring the forces resulting from interaction between the ship and the channel boundaries. The tests were conducted in a

Taylor Model Basin facility known as the circulating water channel. This facility is similar in principle to a wind tunnel. It consists of a test section 22 ft wide, 9 ft deep, and 60 ft long, a return channel beneath which the water passes from the downstream end of the test section back to the entrance of the test section, pumps in the downstream vertical leg to force the water around this circuit, and related equipment. The water depth in the channel can be varied from 0 to 9 ft. Movable walls may be placed in the test section to vary the width of the channel. The water speed in the channel may be varied as desired up to 10 knots.

The basic difference between these tests and the observational tests is that the model can be held stationary while the water flows past, whereas in the observational tests the water is stationary and the model is moved. The advantage of this type of facility lies in the fact that readings can be taken continuously for as long a period as is desired, whereas, in the towing basin, the length of time available for the observation of a given run is limited by the length of the channel. It should be noted that the force-measurement tests are static tests and involve a different method of analysis than do the observational tests, which are of the dynamic type. In the force-measurement tests, the model was restrained while the forces that tended to make it move were measured. It is recognized that this is an artificial condition, with respect to a normal operating condition for the full-scale ship, but the data obtained provide the background for an analysis of the dynamic tests.

The models were tested at various off-center positions in channels with various cross-sectional dimensions and at the equivalent full-scale speeds of from 4.5 knots to 10 knots. During the tests the model was attached to the dynamometer in the desired position by three arms which extend downward from the dynamometer. Two of the arms were 2.5 ft forward and 2.5 ft aft, respectively, of the center of gravity of the model. These two arms measured the side force acting on the model and restrained any movement laterally. The third arm, referred to as the drag arm, was located forward of the other two and measured any fore and aft force acting on the model. The dynamometer was designed so that the model can be attached at any desired transverse position in the channel. When it is desired to give the model an angle of yaw, the entire dynamometer is turned. Thus, all forces acting on the model are measured parallel or perpendicular to the center line of the model regardless of its angle of yaw.

As previously discussed, when a ship is off center in a restricted channel, the side forces acting on it tend to make it sheer away from the near wall. In the observational tests, the rudder angle required to counteract the yawing moment caused by these forces was determined by trial and error which involved releasing the model at a specified distance from the center line and trying various rudder angles until equilibrium was obtained. It was also pointed out that, in addition to the rudder angle, it was necessary to give a slight angle of yaw away from the near wall to produce a true condition of equilibrium. In the force-measurement tests, equilibrium rudder angles were determined by measuring forces on the model at various angles of yaw. This result was accomplished by varying the rudder angle and the angle of yaw until both the yawing moment and the side force became zero. The model was self-propelled and the speed of the propellers was varied until the drag of the model was equalized. Fig. 47 is a graph of the rudder angle required for equilibrium plotted against the distance between the center line of the channel and the center of gravity of the ship. (If the ship is near the right bank, it is necessary to use "right rudder" and to yaw the ship to port to counteract the moment and side force that develop. The reverse would be true if the ship were near the left bank.) The data are for a large tanker in channels with widths of 268 ft, 500 ft, and 770 ft. The side walls of the channel were vertical.

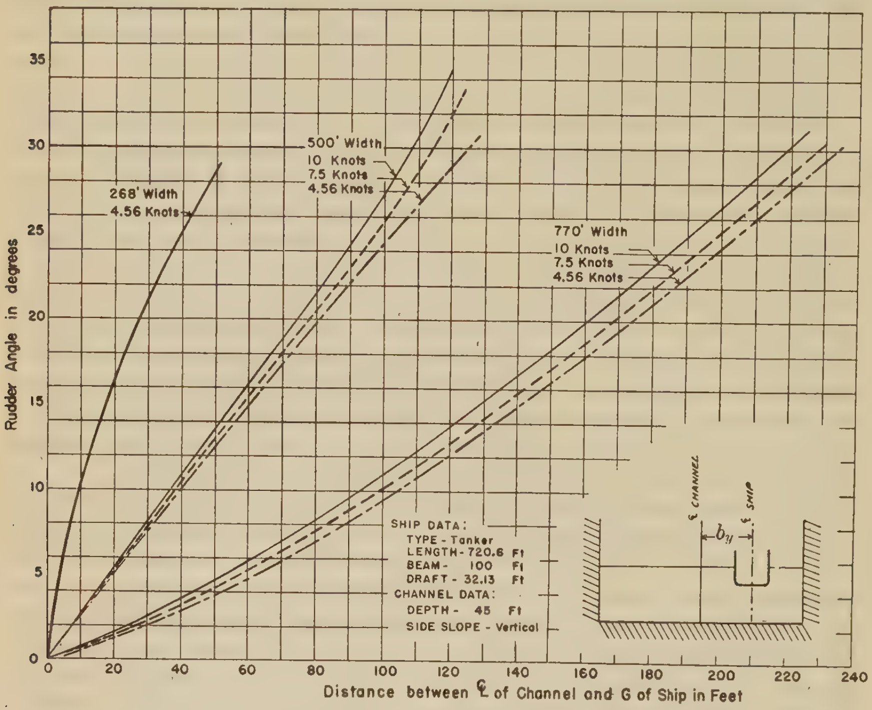


Fig. 47.—Rudder Angles Necessary to Produce a Condition of Equilibrium for Various Distances Between the Center Line of a Restricted Channel and the Center of Gravity of a Ship

In addition to determining the rudder angle and the angle of yaw required for equilibrium, measurements were taken of the side force and yawing moment that developed when the model was held parallel to the wall and at various transverse positions in the circulating water channel. These tests were conducted for the purpose of determining the magnitude of the interaction or bank suction in terms of force and moment as opposed to the preceding data which indicate the magnitude of these effects in terms of the rudder angle and the angle of yaw required to counteract them. The yawing moment and the side force were plotted with respect to the distance between the center line of the channel and the center of gravity of the ship. Fig. 48 is a comparison of the side force and yawing moment acting on the tanker (length 720.6 ft and beam 100 ft) at various transverse positions in channels with three different widths (268 ft, 500 ft, and 770 ft) for one depth (45 ft) and for one ship speed (4.56 knots). The scale ratio was 3.5. The data in Fig. 48 are based on self-propelled tests of model 3859. They are plotted for the condition with the ship to starboard of channel center line. The rudder angle was set at 0° when the yawing moment and the lateral force were measured. The rudder angle required to counteract the yawing moment of zero yaw has also been plotted for comparison. During these tests the rudder angle and the angle of yaw were set at 0°. The rudder angles required to counteract the yawing moment were measured, following the measurement of side force and yawing moment. Previous tests had indicated that there was a slight difference between the true rudder angle for equillibrium and the rudder angle to produce zero moment at zero yaw—thus necessitating separate measurements for the equilibrium condition. The rudder angle for zero moment at zero yaw has been plotted in Fig. 48 for comparison with the moment curves.

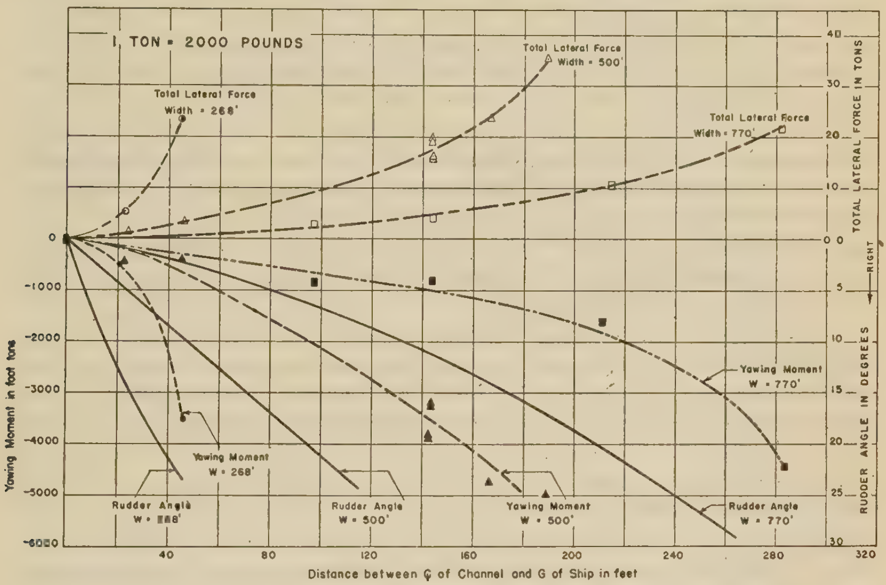


FIG. 48.—YAWING MOMENT AND LATERAL FORCE ACTING ON A SHIP THAT IS PARALLEL TO, AND AT VARIOUS DISTANCES FROM, THE CENTER LINE OF A RESTRICTED CHANNEL

Summary of Observation and Force-Measurement Tests.—According to the original test program for the investigation of ship performance in restricted channels, models of two different ships were to be tested under identical conditions during the straight-channel, one-way traffic studies. The two models selected for the tests were a large naval vessel and a large twin-screw, single-rudder tanker. In accordance with this program, both models would have been subjected to both observational and force-measurement tests. At the request of The Panama Canal, the observational studies on the tanker were deferred until a later date to expedite other phases of the studies.

In both the observational and force-measurement tests, the rudder angle required to counteract the yawing moment caused by the interaction between the ship and the channel boundaries has been determined. Rudder angle is believed to be the most practical term for use in the study of the controllability of ships in restricted channels. The magnitude of the required rudder angle is a function of the channel dimensions, the size, ship lines, rudder and propeller characteristics, ship speed, and the position of the ship in the channel. Thus, the rudder required by two different ships for a specified off-center position will be affected by the steering characteristics of the ships. The study of rudder angles required for many different ships would be valuable for purposes of ship design and for the operation of ships in restricted channels. However, for the immediate purpose of indicating the necessary size and proportions of a canal channel, it is believed that study of the models of the large naval vessel and of the large tanker should provide sufficient information. The smaller and easier-handling ships should not present a problem in channels which are designed to handle the large ships. The naval vessel was originally selected for study in these tests because of its great size, although its steering characteristics are excellent. The tanker was selected as being representative of a type of large twin-screw, single-rudder ships whose steering characteristics in restricted waters are quite poor.

During the present studies, the models were tested in channels with a considerable variation in width and depth. The data have been plotted to facilitate a comparison of the effect of width and depth on the magnitude of the effects caused by interaction between the ship and the channel boundaries. It is thought that the rudder angles required to counteract these effects provide a comparison of the actual difficulty the pilot might have in controlling the ship under the various conditions. In general, it is believed that a condition requiring the use of a relatively high rudder angle would be dangerous for the ship. Available data on the relationship of yawing moment to rudder angle are quite limited.

Change-of-Level Test.—When a vessel is under way in still water, it is found that the water ahead of the vessel moves forward, outward, and downward. At a comparatively short distance aft of the bow the forward motion ceases, but the water still moves outward and downward to make way for the body of the vessel. Near this point the water starts to flow aft. This negative flow continues to within a short distance of the stern where the water closing in and upward behind a vessel has a forward motion. Wherever changes in velocity or direction of flow occur, there is usually a change in the elevation of the water surface. In shallow water or canals, the region of disturbed water about the ship is confined to a much smaller area than in water of unlimited width and depth. Thus, in a restricted channel, the reverse flow past the vessel has a greater velocity for the same ship speed. The net result is a larger change in the elevation of the water surface, with respect to the normal water surface, in the vicinity of the ship. In the past, considerable study has been devoted to the variation in the resistance of ships in shallow water and restricted channels, but very little work has been done on the change of level of ships under these conditions. Studies of ship resistance in shallow water and restricted channels are described extensively elsewhere.[25,27,28,30,32,36,37,38,39] In 1904, Henry N. Babcock[40] described a series of measurements that was taken on change of level of ships that were under way in shallow channels. Measurements were taken from shore with the use of a surveyor's level. He reported a change of level of 4 ft for one of the ships tested. Some additional information on the change of level of model or full-scale ships under way in shallow water or canals has been presented elsewhere by Mr. Bowers.[25]

Fig. 49 is a plot of some actual measurements, during two independent tests, taken on the elevation of the water surface of a model. The lowering of the water surface in the vicinity of the ship causes a corresponding lowering of the ship, with the result that it may ground in an area where the normal water depth is in excess of the draft of the ship. In addition to an increase in the change of level of a ship in restricted waters, as compared to deep water, it has been found that there is also a considerable increase in ship resistance in restricted waters. For shallow water of unlimited width, the increase in resistance depends on the depth of water. In a restricted channel the resistance is also a function of the channel width.

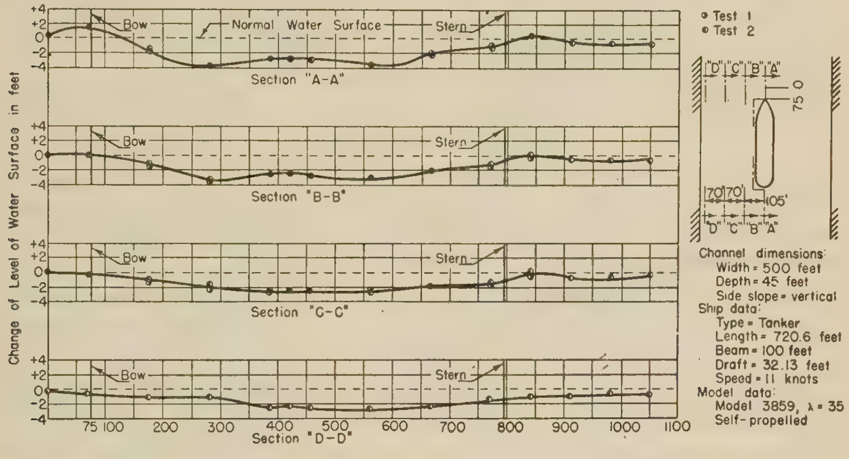


FIG. 49.—WATER-SURFACE PROFILE AT VARIOUS LONGITUDINAL SECTIONS FOR A SHIP ON THE CENTER LINE OF A RESTRICTED CHANNEL

As a part of the Taylor Model Basin investigation of ship performance in restricted channels, the change of level of model 3769 was measured for a range of ship speeds from 5 knots to 15 knots, full scale, in channels with the cross-

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[37] "Tidal Currents and Their Effect on Navigation," by J. A. Conwell, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone, 1941.

[38] "Tests on the Wall Interference and Depth Effect in the Royal Aeronautical Experimental Seaplane Tank and Scale Effect Tests on Hulls of Three Sizes," by L. P. Coombes, W. G. A. Perring, V. W. Battle, and L. Johnston, *Technical Report*, Aeronautical Research Committee, Vol. 2, 1934–1935.

[39] "The Effect of Size of Towing Tank on Model Resistance," by John P. Comstock and C. H. Hancock, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 50, 1942, pp. 149–197.

[40] "Some Model Experiments on Suction of Vessels," by David W. Taylor, *ibid.*, Vol. 17, 1909, pp. 1–21.

sectional dimensions previously given. It was not possible to reach a speed equivalent to 15 knots in some instances due to excessive change of level which caused the model to touch the bottom of the channel. In these tests the model was attached to the towing carriage by connections that permitted a small amount of fore and aft movement and also allowed the model to trim freely. Indicators, which were mounted on the carriage, were attached to the model near the bow and stern by a cable arrangement so that vertical movements of the bow and stern could be read on large dials. The carriage was accelerated to the desired test speed and at the same time the propeller revolution was increased until the model was fully self-propelled. Readings were then taken of the level of the bow and stern with respect to the level when the model was stationary.

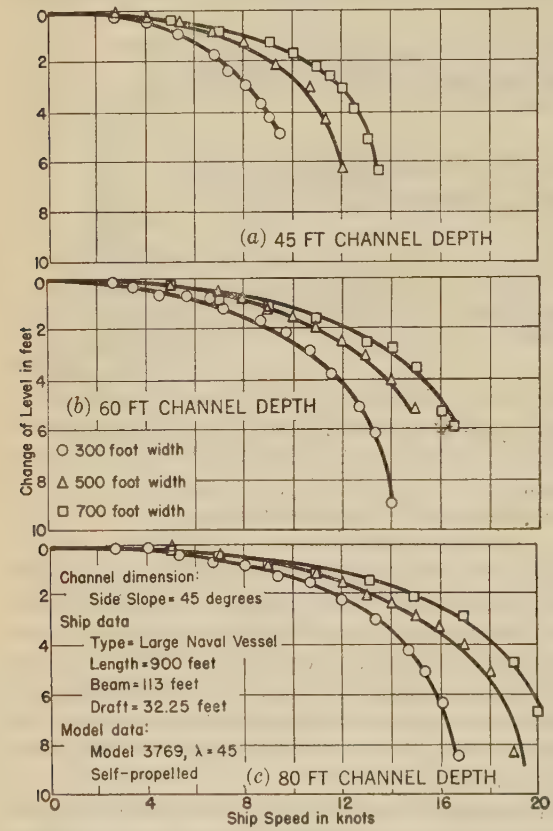


Fig. 50.—Effect of Channel Width on the Change of Level of a Ship on the Center Line of a Restricted Channel

From tests it was noted that the bow and stern change level at approximately the same rate up to the critical speed, where the bow curve reverses direction, and the slope of the stern curve becomes quite large. The critical speed of a vessel in a restricted channel is roughly defined as the speed at which the relative velocity between the ship and the reverse flow past the beam of the ship is equal to the speed of the wave of translation for that particular depth of water. Near the critical speed, the water-surface level around the ship changes quite rapidly with any slight change in relative speed. The models were not tested at speeds above the critical because of possible damage to the model and because there is little possibility of operating the full-scale ship under these conditions, even at the greater depths where there is no danger of grounding. For a specified channel, the change of level of the ship varies approximately as the square of the velocity.

In Fig. 50 the data have been plotted to illustrate the variation in change of level with variations in the width and depth of the channel. (The curves shown are for the stern only. The bow curves are similar.) A large change of level would probably be indicative of excess bank wash and unstable handling conditions due to interaction or bank suction.

The effect of the transverse position of the ship in the channel on its change of level was also investigated. Measurements were taken for various off-center positions of the model for channel widths equivalent to 300 ft and 500 ft and for channel depths equivalent to 45 ft, 60 ft, and 80 ft. The differences recorded for these tests were not of sufficient magnitude to present and would not be determining factors in the selection of channel dimensions.

To determine the effect of the propellers on the change of level of the ship, the model without propellers was towed on several occasions and the results compared with those obtained from the self-propelled tests. In the towed tests the shape of the curves was quite similar to that in the self-propelled tests. The bow and stern curves had the same relationship as in the self-propelled tests, and the critical speed was approximately the same. However, for most of the speed range up to the critical, the towed tests indicated from 8% to 15% less change of level than did the self-propelled tests.

Discussion of Change-of-Level Tests.—The change of level of a ship under way in restricted waters is of importance in the present investigation because (*a*) at relatively high ship speeds, the ship may ground due to excessive change of level; (*b*) a large change of level is indicative of the formation of large waves which may result in serious bank wash; and (*c*) at subcritical speeds a large change of level is indicative of a high ship resistance as compared to the resistance at the same speeds in deep water. The change of level of a ship under way in a restricted channel is a function of the dimensions and lines of the ship, the ship speed, the cross-sectional dimensions of the channel, and the position of the ship in the channel.

In general, an increase in ship speed produces an increase in the change of level at all speeds up to the critical. For a specified channel, the change of level of a ship increases approximately as the square of the speed for subcritical speeds. The exact relationship between the change of level of the ship and the ship speed is apparently dependent on the cross-sectional area of the channel. At speeeds above the critical, an increase in ship speed may result in a reduction in the change of level of the ship. During the present investigation, the range of ship speeds used in the tests did not exceed the critical speed.

STUDIES OF TWO-WAY TRAFFIC

The purpose of the two-way traffic studies was to obtain information that would be of assistance in the selection of the cross-sectional dimensions of a channel adequate for the meeting of two ships of specified types. As part of this study it was desired to obtain information on the interaction between the two ships and between the ships and the channel boundaries.

Prior to this phase of the studies it was decided by The Panama Canal that two-way traffic studies be based primarily on the meeting of a large naval vessel and a Liberty ship. It was further requested that the tests be conducted for: (1)

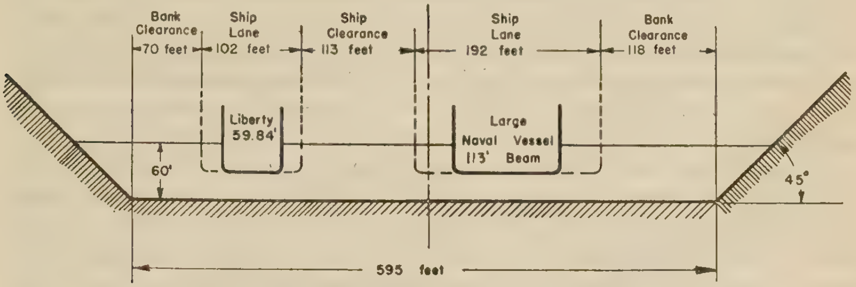
Still water and currents of 3 knots and 5 knots, (2) ship speeds up to 10 knots with respect to the water, and (3) various widths and depths of channel.

When two ships are meeting each other in a restricted channel, each ship interferes with the lines of flow about the other vessel with the result that asymmetrical pressures develop on the two sides of each vessel, tending to divert the ship from its original path. Throughout the maneuver, the turning moment caused by these pressures changes direction several times. For example, as two vessels approach each other, the pilots maneuver them out of the center of the channel. When the bows of the two vessels are almost directly opposite each other there is a tendency for the water surface to build up between the bows forcing them apart, or causing the vessels to yaw away from each other. As the vessels draw abreast of each other, the bow of each vessel tends to move toward the low water surface in the vicinity of the stern of the other, with the result that they yaw toward each other. When the sterns are almost directly opposite each other, there is a tendency for the sterns to move toward each other, thus reversing the direction of yaw. Superimposed on these effects (which result from interaction between the two vessels) is the effect caused by interaction between each vessel and the channel boundaries. In general, this latter effect tends to cause the vessel to yaw away from the near bank. Thus, the maneuvering of the vessels is affected not only by the size, speed, and paths of the vessels but also by the cross-sectional dimensions of the channel.

At the conclusion of most of the straight-channel, one-way traffic studies, certain general criteria were established by The Panama Canal. The decision was made to establish 60 ft, tentatively, as a reasonable depth to be used in further studies of width as related to two-way traffic and bend studies. Drawing upon the experience of well-qualified Panama Canal pilots and Cape Cod Canal pilots, it was decided to establish, tentatively, a reasonable average rudder angle for the maneuvering of a vessel off center in a restricted channel. This average rudder angle would then be used to determine a safe width of canal. Because the vessels vary in size, form characteristics, and rudder power, in addition to the fact that every pilot will maneuver differently, it seemed advisable to set the average rudder angle for equilibrium as low as possible to provide for ample reserve rudder in case of emergency. Pilots considered that a 5° rudder angle was reasonable.

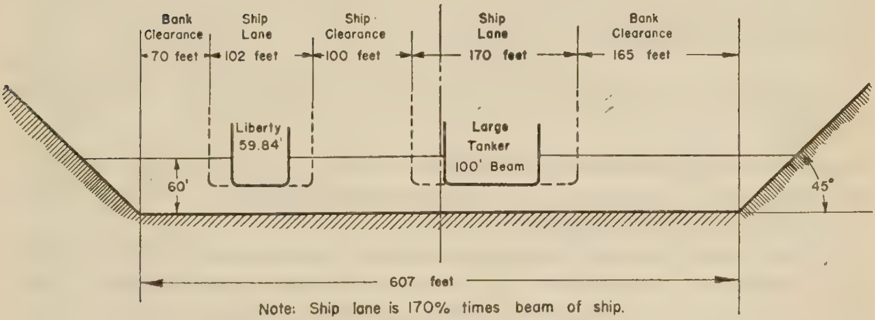
For example, in selecting a width of channel, the following procedure could be used: Fig. 51 shows two cross sections of a channel as determined from model studies, with outlines of the midship sections of (*a*) the large naval vessel and Liberty ship and (*b*) the large tanker and Liberty ship. The dotted outlines represent the average ship lane of the vessels, as determined from observational studies previously mentioned. The widths of the ship lane are approximately 170% of the beam of the vessel for a channel 500 ft wide and 60 ft deep, at a ship speed of 10 knots. The distances of the ships from the prism lines were computed from Figs. 44 to 47 for a rudder angle of 5°, and for a ship speed of from 9 knots to 10 knots. The distance between the vessels was selected as the beam of the larger vessel. This fact was later confirmed with model studies. The curves of Fig. 46 are based on rather limited data and it was thought desirable to select the maximum curve of rudder angle required, which was for the 9-knot ship speed with a following current of 5 knots.

The sum of the various distances from Fig. 51 gives a channel width of 595 ft for the large naval vessel and the Liberty ship, and a width of 607 ft for the large tanker and the Liberty ship, thus presenting one method of analyzing the data. Other selected rudder angles and channel depths will give widths in accordance with the selected values. It should be emphasized that such assumptions will only vary the distance between the ship and the near bank. The widths of the ship lanes and the distance between vessels offer reasonable values for this specific problem. Other combinations of vessels would vary these values. The data in Fig. 51 do not constitute a design intended for the Panama Canal. Other considerations may be necessary in the selection of proper width and depth. The sixth paper in this Symposium will develop this phase of the problem as applied to the Panama Canal.


Bank Clearance 70 feet
Ship Lane 102 feet
Ship Clearance 113 feet
Ship Lane 192 feet
Bank Clearance 118 feet
Liberty 59.84'
Large Naval Vessel 113' Beam
60'
45°
595 feet

Note: Ship lane is 170% times beam of ship

(*a*) PASSING CONDITIONS FOR LARGE NAVAL AND LIBERTY SHIP


Bank Clearance 70 feet
Ship Lane 102 feet
Ship Clearance 100 feet
Ship Lane 170 feet
Bank Clearance 165 feet
Liberty 59.84'
Large Tanker 100' Beam
60'
45°
607 feet

Note: Ship lane is 170% times beam of ship.

(*b*) PASSING CONDITIONS FOR LARGE TANKER AND LIBERTY SHIP

FIG. 51.—PROPOSED WIDTHS FOR TWO-WAY TRAFFIC

Model studies were then conducted with the Liberty ship meeting the large naval vessel in a restricted channel with still water and with ahead and following currents of 3 knots and 5 knots. During the tests one model was towed by an endless cable in one direction while the other model was maneuvered by remote control in the opposite direction. Observations of the handling characteristics and the effect of interaction between the two vessels were limited to the maneuvered model.

With the exception of a limited number of tests at a channel width of 700 ft, all two-way traffic studies were conducted in a channel with a bottom width equivalent to 500 ft and a depth equivalent to 60 ft. It was intended to conduct further tests with a width equivalent to 600 ft but these were deferred to pursue urgent bend studies. The major part of the studies was conducted with both ships traveling at the same speed with respect to the water.

All conclusions were based on visual observations of the models by the Panama Canal pilots (who operated the models) and by the Taylor Model Basin staff. During these tests it was attempted to simulate full-scale conditions as closely as possible. However, it was not possible to satisfy all conditions. The walls of the model channel were quite smooth as compared with the probable irregularities of full-scale channels. Also, the "pilot" had to think and act much faster during the model tests than would be possible on the full-scale ship, because of the time relationship between the model and the full-scale ship. However, the pilots who observed and participated in the tests believed that full-scale conditions were simulated very well and that, where differences did exist, the model was more difficult to control than the full-scale ship.

Subject to the preceding discussion, the following general observations were made:

a. It appeared that interaction between the two vessels did not cause any serious difficulty in handling, in a channel 500 ft wide and 60 ft deep, at the ship and current speeds employed.

b. The current speed did not appear to affect the magnitude of the interaction between the two vessels, for the same ship speeds with respect to the water. However, it would probably have an effect on the handling characteristics of the vessels in the vicinity of bends or large irregularities in the walls of the channel.

c. The Liberty ship steered best when it had a speed with respect to the water of 7.5 knots. A minimum speed at which good control could be maintained was approximately 5 knots.

d. The transverse position of the naval vessel had very little effect on the maneuvering of the Liberty ship as long as the clear distance between the two vessels while passing was at least 100 ft. For more general conditions it was thought that the distance should be at least 100 ft or a distance equal to the beam of the larger vessel—whichever was the larger.

Bend Studies

The primary purpose of the bend studies was to investigate the comparative difficulty encountered in maneuvering a specified ship around variously designed bends at a 1:86 scale ratio. It was also desired to investigate the effects of variations in ship speed and current on the handling characteristics of a specified ship. Later in the investigation it was decided to conduct tests at a 1:45 scale ratio to study the maneuvering characteristics of various vessels and scale effects. Models 3769, 4018, and 3748-4 were used for the 1:45 scale-ratio studies and model 3992 was used for the 1:86 scale-ratio tests. The characteristics of the models are given in Fig. 41.

For the 1:86 scale-ratio tests, model 3992 representing a large naval vessel was selected and constructed and the movable walls were adjusted to the desired width and proper curvature for each test. A section of straight channel was constructed ahead of the bend to permit accelerating the model before it reached the bend. The model was attached to the towing carriage while it was being accelerated, after which it was released from the carriage and maneuvered around the bend by remote control. Veteran Panama Canal pilots issued the helm and engine orders during each run. The control panel, parallel light beam, and other features were similar to those used in the two-way traffic studies and to part of the one-way traffic studies.

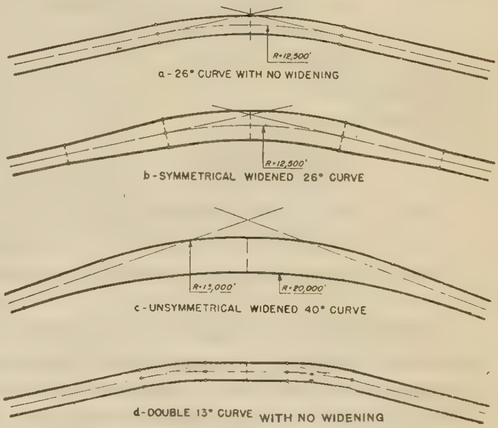

R=12,500'
a - 26° CURVE WITH NO WIDENING
R=12,500'
b - SYMMETRICAL WIDENED 26° CURVE
R=15,000'
R=20,000'
c - UNSYMMETRICAL WIDENED 40° CURVE
d - DOUBLE 13° CURVE WITH NO WIDENING

Fig. 52.—Curves Tested in Model Studies

Tests were conducted on four types of bends at a 1:86 scale ratio, as shown in Fig. 52:

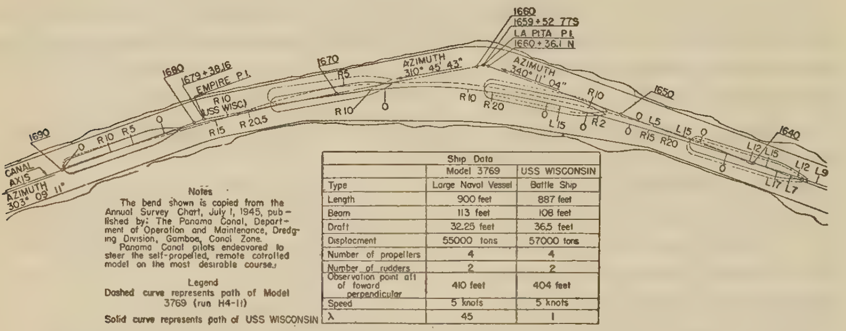
(*a*) A 26° parallel bend;
(*b*) A 26° parallel widened bend;
(*c*) A 40° widened bend; and
(*d*) A double 13° bend;

—and on two types at a 1:45 scale ratio:

(*e*) A model of La Pita bend in the Panama Canal, as generally shown in Fig. 53; and
(*f*) A 26° parallel bend, as shown in Fig. 52.

In connection with test (*e*), Fig. 53 shows a northbound transit of the *U. S. S. Wisconsin* through La Pita bend and the transit of a model of a large naval vessel through a 1:45 scale model of the same bend. The curves show the path of a point that is 0.455% of the ship length from the forward perpendicular and is on the center line of the vessel.

The tests were conducted at ship speeds of 5 knots, 7.5 knots, and 10 knots through the water, with both ahead and following currents of 0 knot, 3 knots, and 5 knots. When the bend studies were initiated, it was planned to investigate arbitrary bend angles of 20° and 40° and to use a straight-channel section at the entrance and exit of the bend with a width of 600 ft and a depth of 60 ft. However, at the conclusion of the first 40° bend test, available information indicated that a channel width of 560 ft (600 ft wide at the 40-ft depth), a depth of 60 ft, and a maximum bend angle of 26° would be of primary interest.



Notes

The bend shown is copied from the Annual Survey Chart, July 1, 1945, published by: The Panama Canal, Department of Operation and Maintenance, Dredging Division, Gamboa, Canal Zone.

Panama Canal pilots endeavored to steer the self-propelled, remote cotrolled model on the most desirable course.

Legend

Dashed curve represents path of Model 3769 (run H4-11)

Solid curve represents path of USS WISCONSIN

Ship Data

	Model 3769	USS WISCONSIN
Type	Large Naval Vessel	Battle Ship
Length	900 feet	887 feet
Beam	113 feet	108 feet
Draft	32.25 feet	36.5 feet
Displacment	55000 tons	57000 tons
Number of propellers	4	4
Number of rudders	2	2
Observation point aft of foward perpendicular	410 feet	404 feet
Speed	5 knots	5 knots
λ	45	1

FIG. 53.—COMPARATIVE SHIP PERFORMANCE IN MODEL AND FULL-SCALE LA PITA BENDS

At first it was felt desirable to attempt to maneuver the vessel both at the center line and at the quarter point on either side of the center line for all conditions of ship speeds and currents—in an attempt to indicate the desirability of meeting and passing a Liberty ship and a large vessel in the curved section. Although a substantial number of such tests were conducted for all the bend designs, the results indicated that such a maneuver might be dangerous if one vessel were large; but little difficulty was foreseen for Liberty ships meeting and passing in the bend. Therefore, the condition of a large vessel meeting and passing another vessel in the bend was eliminated as one of the requirements for the bend design at the present time. Further study may very well give a solution to this problem.

The comparisons of the 1:86 scale-ratio bends were then made for center-line transits only. Table 25 is a summary of the average maximum deviation from the center line of the channel for the various bends tested. ("Deviation" may be defined as the maximum departure of the forward or after perpendicular from the intended path of the center of gravity of the vessel. Width of path is the sum of the maximum port and starboard deviations plus the beam of the vessel for a particular test run.) Referring to Table 25, Panama Canal pilots endeavored to steer the self-propelled, remote-control models along the channel center line of the shallow water basin. Both models represented large naval vessels, with general dimensions as follows:

Dimension	Model 3769	Model 3992
Length, in feet	900	860
Beam, in feet	113	108
Draft, in feet	32.25	34.62

Model 3769 was tested in a 26° parallel bend (λ = 45); and model 3992 was tested in the other four bends (λ = 86). All measured data were taken from streak photographs of the ship's path. The average width of path in Table 25 is based on the total number of runs made under the same test conditions. The average maximum deviation of the ship's path to port or to starboard of the channel center line is based on the total number of runs made under the same test conditions. Included in Table 25 are the average widths of path for the same conditions. The width of path included the beam of the vessel. All runs that were considered as satisfactory test runs were included in the average. Some runs were excluded from the analysis because of failure of the test equipment. The average values of rudder angles or average maximum rudder angles were not used as criteria because of the variations in the techniques used by the pilots in handling the vessel. Some pilots tended to use rather large rudder angles for short periods whereas others tended to use relatively small angles for longer periods of time. Also, various pilots favored one side of the channel more than the other, which will give an even larger variation in rudder angles due to bank interaction.

The first reaction of an observer is that a widened bend would provide more room for maneuvering than would a uniform width of channel in the bend. Actually, the widened bend studies indicated that maneuvering was more difficult, the actions of the vessel were more erratic, and the widths of path were generally greater, as indicated in Table 25. It is considered that this result is principally due to the continual variations in the forces resulting from bank interaction as a vessel transits through a gradually widening section. In the normal transiting of a vessel down a straight channel, it is very difficult to keep it positioned exactly on the center line. In transiting a bend, this task is even more difficult, especially with a large vessel, because it forms a tangent or secant to the curvature of the bend. In either of the latter two cases the vessel will be positioned slightly off center and some rudder angle will be required to prevent a sheering action from developing. Some force or rudder action is usually required to turn the vessel around the bend, although in some cases the bank interaction is such as to turn the vessel around the bend without the use of rudder action. However, if the interaction is too great, the vessel may swing too fast and develop a sheer in the same direction causing the vessel to strike the inner bank. For example, if a vessel enters a right-hand bend slightly off center to the left, the bank interaction is such that the bow will tend to swing around the bend; should the vessel swing too far, a sheer would develop that would require the left rudder to keep the vessel under control. However, if the vessel enters the right-hand bend off center to the right, a larger right rudder would be required to keep the vessel under control and also to turn it around the bend. In a parallel bend the forces created by bank interaction have nearly the same values as similar forces in a straight channel. Therefore, the pilot can judge the required rudder angle for equilibrium more readily than in a widened bend where the values of the forces are changed or are continually changing during the transit. Because the vessel is swinging slightly as it traverses the bend and is not "headed" on a straight course, this maneuver can best be made by constant attention to the swing of the vessel and by the "feel" of the condition.

TABLE 25.—COMPARISON

Line No.	Bend			Still Water; $V_c = 0$				Ahead Current; $V_c = 3$			
	Angle	Description	Scale ratio	Total No. of center-line runs	Width of path[a] (ft)	Maximum Average Deviation (Ft)		Total No. of center-line runs	Width of path[a]	Maximum Average Deviation (Ft)	
						Port	Starboard[b]			Port	Starboard[b]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	(a) VELOCITY OF SHIP IN RELATION TO CURRENT; $V_s = 5$ KNOTS										
1	40°	Widened	86	2	360	59	193	2	763	468	187
2	26°	Parallel; widened	86	6	475	192	175	2	639	316	215
3	13°	Double	86	7	304	141	55	3	295	99	88
4	26°	Parallel	86	7	290	126	56	3	238	115	14
5	26°	Parallel	45	4	277	106	58	3	245	116	16
	(b) VELOCITY OF SHIP IN RELATION TO CURRENT; $V_s = 7.5$ KNOTS										
6	40°	Widened	86	6	466	166	194	5	643	363	173
7	26°	Parallel; widened	86	7	450	215	127	7	411	160	143
8	13°	Double	86	16	363	158	97	3	340	138	95
9	26°	Parallel	86	8	369	172	91	9	263	128	26
10	26°	Parallel	45	4	305	125	67	4	245	115	17
	(c) VELOCITY OF SHIP IN RELATION TO CURRENT; $V_s = 10$ KNOTS										
11	40°	Widened	86	2	723	337	278	1	401	244	49
12	26°	Parallel; widened	86	4	378	174	96	6	402	182	112
13	13°	Double	86	6	389	177	104	2	290	129	55
14	26°	Parallel	86	7	309	126	75	5	342	157	76
15	26°	Parallel	45	4	209	100	−4	4	231	124	−6

[a] Average width of path for all center-line runs. [b] A minus sign in this column

The scale ratio used made it impossible to provide room for the pilot in the model and thus let him "feel" the action of the vessel as it deviated from its heading. A pilot on a full-scale vessel normally "feels" or anticipates the motion of the vessel well in advance of an observer not on the vessel. In addition, the scale ratios were so small that slight motions were difficult to detect from the carriage platform in time to make the proper corrections.

By considering the foregoing factors for the analysis of the 1:86 scale-ratio studies and by comparing the performance of the model vessel in the various bends, not attempting to extrapolate the values to full-scale operation, observers felt that the 26° parallel bend provided the best solution for all conditions of testing. This decision was reached by comparing similar runs of the various bends tested. For this comparison only one ship model was used. No appreciable differences were encountered in the 1:86 scale, 26° parallel bend for still-water or moving-water conditions.

One factor that should be mentioned in the case of moving water is that of variations in time of transit through a bend. No appreciable differences were noted in the model's action for the still-water and moving-water studies, pro-

OF SHIP PERFORMANCE

AHEAD CURRENT; $V_c = 5$				FOLLOWING CURRENT; $V_c = 3$				FOLLOWING CURRENT; $V_c = 5$				Line No.
Total No. of center-line runs	Width of path[a]	Maximum Average Deviation (Ft)		Total No. of center-line runs	Width of path[a]	Maximum Average Deviation (Ft)		Total No. of center-line runs	Width of path[a]	Maximum Average Deviation (Ft)		
		Port	Starboard[b]			Port	Starboard[b]			Port	Starboard[b]	
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	
(*a*) VELOCITY OF SHIP IN RELATION TO CURRENT; $V_s = 5$ KNOTS												
....	....	....	....	4	512	193	213	2	474	170	195	1
....	....	....	....	0	....	....	....	0	....	....	....	2
....	....	....	....	0	....	....	....	0	....	....	....	3
....	....	....	....	4	229	99	22	9	269	103	57	4
....	....	....	....	4	217	93	11	6	244	118	13	5
(*b*) VELOCITY OF SHIP IN RELATION TO CURRENT; $V_s = 7.5$ KNOTS												
7	649	337	204	2	622	214	300	3	342	94	141	6
4	430	185	138	0	....	....	....	0	....	....	....	7
6	406	175	123	0	....	....	....	0	....	....	....	8
7	297	147	41	4	235	80	46	5	255	107	40	9
5	257	92	52	4	183	91	−21	6	190	100	−23	10
(*c*) VELOCITY OF SHIP IN RELATION TO CURRENT; $V_s = 10$ KNOTS												
3	568	244	217	4	486	282	98	3	380	139	133	11
4	417	223	86	0	....	....	....	0	....	....	....	12
3	392	159	123	0	....	....	....	0	....	....	....	13
7	378	159	111	4	253	102	42	5	240	108	23	14
6	231	85	33	4	177	80	−16	6	197	101	−17	15

indicates that the center line of the ship was always to the port of the channel center line.

vided that the speed of the vessel with respect to the water remained constant. There were, of course, differences in time of transit. Appreciably fewer total rudder orders were required in transiting a bend with a following current than with an "ahead" current. However, the rudder orders necessary per unit of time were approximately equal for all conditions of testing in either still water or moving water, provided that the speed of the ship with respect to the water remained constant. Actually the vessel remains in the bend for a shorter period of time for the following current condition than for the ahead current condition. This fact usually gives the pilot the impression, from observations of the speed of the vessel over the ground rather than from observations of its speed through the water, that less time is available for adjusting his course during the transit of the bend—which evidently creates somewhat of a mental hazard for the pilots navigating through a bend in a following current. Experience proves that this condition is no more difficult than conditions of still water or ahead current.

At the conclusion of the 1:86 scale-ratio tests, it was decided to conduct two more bend studies at the larger scale ratio of 1:45:

a. The 26° parallel bend, shown in Fig. 52; and
b. The model of the La Pita bend, Panama Canal, as shown in Fig. 53.

The 1:45 scale ratio of the 26° parallel bend was tested:

a. To determine by comparison with the 1:86 scale ratio test if there were scale effect; and
b. To compare transits of models of other ships—namely, the Liberty ship and a large tanker.

The widths of path and deviations from the center line for the various test conditions are tabulated with the 1:86 scale-ratio tests in Table 25. Similar records were taken, by photographic methods, of all record runs. The results of this study indicate that some scale effect was present in the 1:86 scale-ratio studies. However, the actions of the vessel were quite similar. The scale effect was chiefly present in the additional time permitted the pilot for giving orders. Except for the poor handling quality of the large tanker, as determined in the straight-channel studies, no difficulty was experienced in transiting the 1:45 scale-ratio parallel bend with any of the models under any of the conditions as long as the transits were made on the center line. The Liberty ship model handled very well.

The model of the La Pita bend was tested at a 1:45 scale ratio in a further effort to determine scale effect by comparing model tests with a full-scale transit, as shown in Fig. 53. The model transit of the bend compares very favorably with the full-scale transit. However, studies of the Suez and Cape Cod canals indicated that bends of uniform width throughout were more desirable. In the case of the Suez Canal, a parallel bend had been widened to provide room for larger vessels. Ships which previously had had no difficulty began to experience some erratic action. Therefore, the bend was further widened in such a manner that it again became parallel.

The model studies have been limited to a small number of ships and bend designs. The comparison of widths of path for the 1:86 scale-ratio studies indicated that the 26° parallel bend would give the most satisfactory operating characteristics of the model for all conditions. This comparison was further substantiated by visual observations of the test runs, the number and magnitude of rudder orders, and the greater uniformity of results from pilot to pilot. The 1:45 scale-ratio studies of the 26° parallel bend showed improvement in the operation characteristics of the model over the 1:86 scale-ratio tests. This improvement could be attributed only to scale effect. The width of path in the bend compared quite favorably with the width of path for the straight channel when "crabbing" or "angular position" of the vessel in the channel bend is considered.

The 26° parallel bend was determined to be the most desirable bend of those studied and the test showed that currents did not produce any hazardous operating conditions. The model could be navigated as readily in moving water as in still water.

SUMMARY

The variety of sizes, shapes, rudders, and propulsion characteristics of the many ships that will transit any channel is so great that only representative types from different general groupings could be used for the experiments. For these studies and their relation to the Panama Canal the vessels were selected because of (1) their large size, (2) their poor handling characteristics in restricted waters, or (3) their being representative of a large percentage of the present ships transiting the canal. It is felt that the models of the ships studied "bracket" these conditions and that the results obtained provide a guide for selecting the channel dimensions. Before a final selection of the channel dimensions is made, there are other factors, which could not be inserted into a laboratory investigation, to be studied and considered. Ships of the same class may not have similar steering qualities. The efficiency of operation of a vessel is measured to some extent by the age and the number of days the ship has been under way since overhaul. Seldom do two captains or pilots come to perfect agreement on the proper methods to use in maneuvering a vessel. Wind, rain, fog, and mechanical failures are other items that have to be considered. To a limited extent these factors can be offset by artificial aids to navigation such as range markers, buoys, and beacon lights, but there are always special conditions through which a ship must be navigated.

This paper has presented the information that is available from tests conducted at the David Taylor Model Basin. A study of the maneuverability and controllability of specified ships in specified restricted waters is a multifold problem. The results of the experiments and analysis thus far made give a more complete understanding of a problem which has harassed ship operators, engineers, and pilots for more than a century. A complete solution of the general problem has not yet been obtained; however, limits and trends have been established that may serve a useful purpose in determining (*a*) the speed at which vessels may operate in restricted waters, (*b*) the practical depth and width of a restricted channel, and (*c*) the actions that produce extraneous moments on the vessels. Generally the forces and moments acting on the various ship models were explained and the rudder angle required for controllability and balancing of those moments was used as a measure of the moment.

ACKNOWLEDGMENTS

Acknowledgment is due to the director and many members of the David Taylor Model Basin staff for their advice, cooperation, and active participation in the studies, and to the Panama Canal Zone officials and staff members for their advice, cooperation, and forthright criticism. F. D. Bradley and C. R. Olson conducted the maneuvering studies; B. Rosenberg, the change-in-level tests; and F. W. Puryear, the force-measurement studies. R. S. Garthune, assisted by Miss V. R. Gilchrist and C. Larson, compiled and analyzed most of the data. W. F. Brownell, Jr., Jun. ASCE, D. Cafiero, and W. V. Coyle guided the construction and design of the model setup. The Panama Canal pilots who aided with, and gave honest and valuable criticism of, the tests deserve special acknowledgment.

DESIGN OF CHANNEL

By J. E. Reeves[41] and E. H. Bourquard,[42] Assoc. Members, ASCE

Synopsis

Methods of design and analysis are presented in this paper for use in determining the minimum cross section of the channel and the alinement and treatment of curves in a sea-level canal at Panama, to provide safe and efficient transit of all vessels to the year 2000. The channel design is based on the operation of the canal as an open waterway with a maximum current of 4.5 knots. The design methods described in this paper utilize the marine operating experience of existing waterways and the results of ship model tests conducted at the David Taylor Model Basin, some of which have been presented in the preceding paper in this Symposium. A minimum channel section 600 ft wide, measured at a depth of 40 ft, and 60 ft deep is proposed.

Introduction

The existing Panama Canal has provided commercial and military shipping with a safe and efficient means of transiting the American Isthmus, since its opening in 1914. Almost 200,000 ships have passed through the canal and the total accident cost has been only $1,310,000—equivalent to $7.76 per transit. Any modification of the existing canal, or any new canal to provide the necessary national security, must also provide commerce with a waterway at least as safe and efficient as the existing canal. In a sea-level canal, the adequacy of the channel cross section and alinement are the primary considerations in accomplishing this objective.

This paper describes the methods of designing and selecting the sea-level canal channel. The channel of the sea-level canal referred to herein is the restricted part of the waterway as distinct from open reaches of harbors where the topography does not limit the width of the channel. Although this paper discusses primarily the sea-level canal, there is no essential difference in the method used in the design of either a sea-level or a lock canal channel except that tidal currents in the sea-level canal introduce a factor that must be weighed and compensated for in the design. A comparison of the final selection of the lock canal channel and the sea-level canal channel is included.

The importance of avoiding the construction of a channel larger than necessary is obvious when it is considered that the excavation for the sea-level canal would total more than 1,000,000,000 cu yd. On the other hand, prudent planning is required to meet all reasonable demands of navigation in the future to avoid expensive corrective work.

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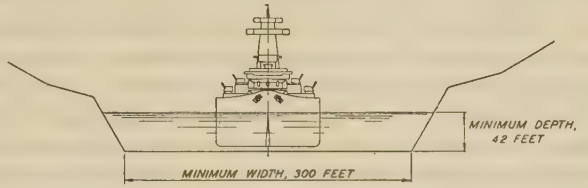
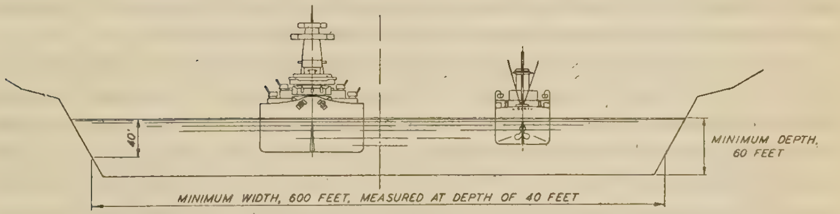
NAVIGATION FEATURES OF EXISTING AND SEA-LEVEL CHANNELS

The channels of the existing canal are of three types: Those through the lake where the width and depth are unrestrictive to vessels; those through harbor or approach areas where both width and depth are restricted, but where channel banks are well below water level and interaction between ship and bank is insignificant; and those where both depth and width are fully restricted by the channel bottom and side banks. In the existing canal, the 8-mile reach north of Pedro Miguel Locks known as Gaillard Cut and a few miles of lock approach channels make up the full extent of restricted channel section. In this restricted section about 70% of all bank-striking and grounding accidents of the existing canal occur. The minimum section of the restricted channel has a bottom width of 300 ft and a minimum depth of about 42 ft. Fig. 54 shows this section with a battleship hull of the *Iowa* class in it. The sea-level canal, in comparison, would have approximately 30 miles of restricted channel section of minimum width extending from Limon Bay on the Atlantic side to the inner Balboa Harbor, Pacific side.



PRESENT PANAMA CANAL



PANAMA SEA-LEVEL CANAL

FIG. 54.—CHANNEL SECTIONS

The existing canal is free of currents except for periodic tidal currents ranging up to a maximum of 1.5 knots in the sea approach channels. Currents in the sea-level canal would be limited by tidal regulation for the safety and convenience of shipping in the ordinary use of the canal to a maximum velocity tentatively established at 2 knots. If the tidal structures were damaged or wrecked by enemy action, the maximum current that would occur for short periods at extreme ranges of tide would be 4.5 knots. The importance of shipping in wartime requires that the channel be designed to provide safe and efficient transit when operating as an open waterway in which currents up to 4.5 knots would occur.

A survey of restricted waterways and canals in which currents are experienced discloses that currents do not significantly increase the problem of ship handling if (1) the ship is adequately powered and ruddered so that it can maintain sufficient headway for good controllability, and (2) the channel section and alinement are satisfactory for navigation under slack-water conditions. However, mechanical failures and personal errors, even if they occur rarely, are essential considerations in analyzing the safety of navigation. The loss of control in a following current would probably result in accidents of greater severity and frequency, because of the shorter time available for regaining control and the greater speed of the ship on striking the bank. To reduce the likelihood of accidents under such circumstances, additional width of channel for maneuvering is desirable.

Climatic conditions, particularly fog, play an important part in operating and scheduling transit in the existing canal; however, they do not have a major bearing on fixing the channel dimensions, alinement, or treatment of curves.

Fogs are of frequent occurrence in Gaillard Cut during the late hours of the night in the rainy season. Operations in the existing canal are scheduled so that ships do not enter the cut during fog periods. Although no exact estimate is practicable of the probable extent, frequency, and duration of fogs in a sea-level canal, some increase in each may be expected. However, no operation in fogs is planned for either an improved lock or a sea-level canal at Panama because the daytime capacity would be sufficient to meet all conceivable demands of traffic to the year 2000. Electronic navigation aids have been perfected which make navigation in fog feasible, and these will be adaptable for use in the Panama sea-level canal if and when required.

Where strong winds are present, vessels having large superstructures, or lightly-loaded vessels, may have difficulties navigating at low speeds in restricted channels. This situation holds at present for large aircraft carriers at the lock approaches and at the curves in Gaillard Cut of the Panama Canal. The difficulties are primarily attributable to the fact that the carrier must necessarily approach the lock or curve at a low speed. The effect of wind is less important in a sea-level canal as it would not be necessary for ships to transit curves at low speeds and lockages would be eliminated or reduced drastically.

Operating Criteria for the Sea-Level Canal

The general regulations under which the canal would be operated form an important part of the criteria for channel design. The essential criteria are those having to do with pilots, the meeting and passing of ships, ships' speeds, and interval between ships.

The design is based on the assumption that all transiting ships will be under the direction of experienced canal pilots. This condition exists in the present canal.

Single-direction traffic with alternating movements of traffic by direction would be feasible in a sea-level canal. This method of operation is used in the narrow and tortuous 8-mile Gaillard Cut for the transiting of large or poor-handling ships and for the transiting of all traffic in the cut after dark. Such a precaution unquestionably has been a major factor in the safety of transit through Gaillard Cut, and shipping has not been unduly delayed. The sea-level canal, by contrast, would have about 30 miles of restricted channel. Since the average time of transit through the sea-level canal would be 4.5 hours as compared to 8 hours in the present canal, it would be possible to operate the new canal by limiting traffic to one direction and causing some additional waiting without loss of efficiency over present canal operation. However, it would be highly desirable to retain the benefit of the shortened transit time in the new waterway, and the imposition of the restriction of one-direction traffic on the efficient use of the canal in wartime is clearly inadvisable. Therefore the canal should be designed for two-way traffic.

A survey by the United States Navy revealed that increases in the size of ships may be expected during the remainder of the twentieth century, and that, for a lock canal, locks 200 ft wide and 1,500 ft long would be required. Whether ships approaching these dimensions will become a reality and will require transit through the canal is quite uncertain. In any case such exceptionally large ships will be few in number, so that the channel needs to be adequate only for the single-direction passage of these ships.

Any future Panama Canal must accommodate the largest ships of the Navy now afloat without restriction or delay, particularly in times of emergency. The transit of large naval vessels would not be required in both directions at the same time, and the canal would not be required to accommodate naval ships of the largest class traveling simultaneously in opposite directions. Likewise, the meeting of the largest of the existing commercial vessels, such as the *Queen Mary*, and large naval vessels need not be considered. The incidence of their meeting would be infrequent and resultant delays to avoid such meetings would be minor.

An index to the size of ships that would transit the canal with such frequency as to make special transiting arrangements inadvisable has been developed by frequency studies of transits of various size ships based on Panama Canal records. Table 26 is based on predicted traffic in the year 2000 and on the assumption that the canal will be operating 16 hours daily. It shows that two ships more than 500 ft long would meet about once a day. It would be impractical to design a canal for the meeting and the passing of two such ships, as the passing could be avoided with little, if any, delay by proper scheduling of transits. Vessels longer than 500 ft would meet vessels from 400 ft to 500 ft

TABLE 26.—THEORETICAL FREQUENCY OF MEETING OF SHIPS IN THE RESTRICTED CHANNELS OF A PANAMA SEA-LEVEL CANAL, IN THE YEAR 2000

Length of vessels (ft) (1)	NUMBER OF TIMES PER DAY THAT THE VESSELS LISTED IN COL. 1 WOULD MEET AND PASS A VESSEL HAVING A LENGTH (FT) OF: 500 or more (2)	400 or more (3)	300 or more (4)
More than 500	1	11	15
400 to 500	10	31	48

long about ten times daily. Since special transit schedules to avoid those passings would cause individual delays of as much as 6 hours to 21 vessels on average days and to 31 vessels on peak-traffic days, the channel should be adequate for such passings. In using this criterion, "design ships" typical of each length group were selected. For the group, 500 ft and longer, a large loaded ore ship and a large naval vessel were selected. A loaded Liberty ship was selected to represent an average ship in the 400-ft to 500-ft class. Therefore, the channel of the sea-level canal is designed to permit a large loaded ore ship or a large naval vessel to pass a loaded Liberty ship with safety and ease. The controlling dimensions of these ships are shown in Table 27. The passing positions of a large naval vessel and a Liberty ship in the proposed sea-level canal channel are shown in Fig. 54.

TABLE 27.—CHARACTERISTICS OF DESIGN SHIPS

Description	DIMENSIONS (FT)				Displacement tonnage	Shaft horsepower	Horsepower per ton	Design speed (knots)	Controllability in proportion to size
	Overall length	Waterline length	Beam	Loaded draft					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Large naval vessel...	986	925	113	36	55,000	213,000	3.87	33+	Very good
Large ore ships......	583	574	78	35	32,450	13,000	0.40	17	Good
Liberty ship........	442	428	57	28	14,250	2,500	0.18	11	Good

Ships longer than 250 ft transiting the present Panama Canal are limited to speeds of 6 knots in the 300-ft channels, 10 knots in the 500-ft channels, 12 knots in the 800-ft channels, and 15 knots in the 1,000-ft channels, except in the case of the lock approaches or the channel through Balboa inner harbor where speeds are further reduced. Overtaking and passing is permitted only when one of the vessels is small or in the channels of Gatun Lake which are from 800 ft to 1,000 ft wide.

For the sea-level canal, a limiting speed of 10 knots was adopted for daily operation with the canal handling two-direction traffic. Naval task forces and convoys would be permitted a higher speed when not meeting opposing traffic. The minimum interval between ships would be about 1.5 miles, which is much greater than is normally required in canals. However, even at this interval, the capacity of the canal, with 16-hour daily operation, would be greatly in excess of the predicted peak-day traffic of the year 2000, so there would be no reason to reduce the interval.

Five criteria were used in the design of the sea-level canal channel, involving (*a*) pilots, (*b*) two-direction traffic, (*c*) single-direction traffic, (*d*) transit speed, and (*e*) ship interval. Specifically it was assumed that:

(*a*) All ships will carry a canal pilot.

(*b*) Naval ships up to the size of the largest existing naval vessel, and commercial vessels up to the size of a large loaded ore ship, will be permitted to meet and pass commercial vessels up to the size of a loaded Liberty ship.

(*c*) The design will be adequate for the clear channel transit of the largest ship that may be constructed to the year 2000.

(*d*) The normal speed of ships will be 10 knots; the passing speed, an average of 6 knots for both vessels.

(*e*) The average distance between ships will be 1.5 miles.

DESIGN METHODS AND INVESTIGATIONS

Previous Methods of Design.—Available information showed that, ordinarily, one of three methods has been used in the past to design navigable channels:

(1) One method is to provide a cross-sectional area of channel equal to a certain multiple of the immersed hull area of the largest ship expected to transit. The channel depth is obtained by allowing about 3 ft of clearance under the hull of this ship, plus an allowance for deposition of silt between dredgings. The channel width is determined from the previously selected channel area and depth.

(2) A second common method involves comparing the channels of existing waterways and, on the basis of operating records, selecting channel widths and depths for the proposed waterway. This method has also been used in selecting the treatment of curves.

(3) The third method employs empirical formulas developed from actual experience or from studies of the maneuvering characteristics of ships.

An analysis of navigation in various major world waterways showed that many factors not taken into account by these methods had an effect on channel requirements. For this reason, none of the foregoing methods was considered satisfactory for the design of a new canal at Panama.

Investigations for Present Studies.—The magnitude and importance of the present Isthmian Canal Studies led to an investigation of all aspects of channel design including many not previously considered. An outline of the investigations follows:

a. Examination of the literature on past practices in the design of restricted waterways;

b. Review of records of ship handling in the Panama Canal and consultation with pilots and others of the marine operating staff, The Panama Canal;

c. Compilation of physical data affecting channel design on the major waterways of the world (those having the greatest similarity to a sea-level canal at Panama being the present Panama Canal, the Suez Canal, the Cape Cod Canal, and the Houston (Tex.) Ship Channel);

d. Consultations with the pilots and operating personnel of the Suez Canal, the Cape Cod Canal, and the Houston Ship Channel on navigation in their respective canals and on the channel requirements for a sea-level canal at Panama, information also being obtained from ship operators regarding the adequacy and limitations of the major world waterways being studied;

e. Operation of ship models in restricted channels of different sizes under various conditions of current, by the Navy at the David Taylor Model Basin;

f. Construction and operation of a model of a Panama sea-level canal at a scale of 1:100, to determine the magnitude of uncontrolled tidal currents and also the range of control that could be established if tidal regulation were adopted; and

g. Advice and recommendations from the Navy on features of the canal, particularly those affecting the requirements for ships operated by the Navy.

The results of these examinations and inquiries are not described in this paper except where they have an important bearing on specific problems of design.

DESIGN OF CHANNEL DEPTH

Both experience and ship model tests have established the fact that depth of section is of prime importance to good navigation, and that, within reasonable limits, the required width of channel is a function of the depth. Consequently, the depth of channel is established prior to the determination of the channel width.

The channel depth must be sufficient to allow transiting of the largest naval and commercial vessels expected to be constructed to the year 2000 under conditions of good controllability and without excessive squat or sinkage of the vessel when traveling at design speeds. Insufficient depth under a ship hull retards the flow of water to the propeller and to the rudder and reduces the effectiveness of both. The Navy has advised that a 50-ft depth over the sills of any new locks should be adequate for the largest naval or commercial vessels expected to the year 2000. As ships pass through the locks at very slow speeds, it is necessary that the channel have a minimum depth greater than 50 ft.

The depth of the channel at any location along the canal is measured below the water-surface profile connecting the low water elevation at each entrance to the canal.

TABLE 28.—RELATIONSHIP BETWEEN CHANNEL DEPTH AND SHIP DRAFT, FOR EXISTING WATERWAYS

Waterway	Depth channel (ft)	Transit speed (knots)	LARGEST VESSELS USING CANAL		LARGEST VESSEL HAVING GOOD CONTROLLABILITY	
			Draft (ft)	$\frac{\text{Depth}}{\text{Draft}}$	Draft (ft)	$\frac{\text{Depth}}{\text{Draft}}$
Gaillard Cut (Panama Canal)	42	6	36	1.17	32	1.31
Suez Canal	43	7.5	34	1.26	28	1.54
Cape Cod Canal	40[a]	6–12	32	1.25	28	1.43

[a] Project depth is 32 ft, but scour has increased this depth to an estimated average of 40 ft.

A study was made of the relationship in certain major existing waterways between the channel depth and the draft of transiting vessels. The depth of the channels of three major waterways and the draft and the transit speeds of the largest ships normally using the waterways are given in Table 28. The draft shown for the "Largest Vessels Using Canal" is either that of the deepest draft vessel to transit the waterway or that of the maximum draft vessel that the canal authorities will permit to transit.

This study, which considered speed and channel width, indicated that for good vessel controllability the channel depth of the Panama sea-level canal should be about 1.5 times the draft of ships comprising the normal daily traffic, which in the year 2000 would probably include vessels having drafts up to about 40 ft. Consequently, a channel depth of approximately 60 ft is indicated. Study of the behavior of ships having a lesser ratio of depth to draft in existing waterways indicates that vessels of maximum size for the year 2000, with draft possibly approaching 50 ft, could safely transit a channel of 60-ft depth at the design speed of 10 knots.

The ship model tests discussed in the fifth paper of the Symposium clearly illustrated the importance of depth on the controllability of a vessel. The rudder angle required to control and hold a vessel parallel to the banks during off center-line maneuvers is the best indicator of relative controllability. Fig. 55 shows a curve, based on data obtained from the model tests, of the rudder angles required to maintain a large naval vessel (length 900 ft, beam 113 ft, and draft 32.25 ft) on a course 115 ft off the prism line of a 500-ft wide channel (bottom width 500 ft with 45° bank slopes and depths ranging from 45 ft to 80 ft). A ship might be in such a position in a passing maneuver following momentary failure of equipment or wrong execution of a rudder command. At a 60-ft depth, a rudder angle of only 9° is required to control the vessel. In a 45-ft depth of channel, the angle required is 20° and the curve is rising rapidly, indicating that an unsafe condition is being approached where full available effort of the rudder would be required to control the vessel. The gain in reduction of the rudder angle as depth increased beyond 60 ft is slight and would not justify the increased construction costs.

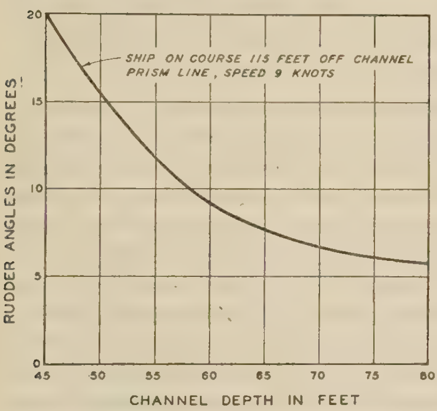


FIG. 55.—EFFECT OF CHANNEL DEPTH ON SHIP CONTROLLABILITY

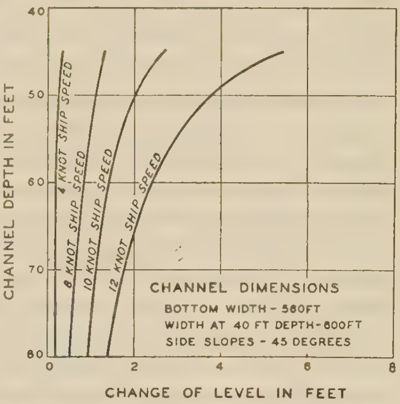


FIG. 56.—CHANNEL DEPTH VERSUS CHANGE OF LEVEL OF A SHIP

In selecting channel depth, sufficient clearance must be provided between the ship and the channel bottom to avoid excessive sinkage or change of level of a vessel when transiting a restricted channel at the maximum allowable speed. Fig. 56 illustrates the effect of channel depth and ship speed on the change in level of the same large naval vessel as that in Fig. 55 in a channel with the same width as the proposed sea-level canal channel. The curves shown are based on data obtained from ship model tests, verified in part by measurements made on actual ships transiting the Panama Canal. It was found that, with a channel depth of 60 ft, sinkage would not be a factor so long as the channel has a width of 300 ft or more. The curves shown are for the stern only, the bow curves being similar. The ship traveled on the center line of the channel.

Based on the experience of existing waterways and experimental work at the Taylor Model Basin, with due consideration of costs, a minimum depth of 60 ft for a sea-level canal is considered adequate. For a lock canal having a shorter length of restricted channel, a lower transit speed, and no currents, a 55-ft depth is considered adequate.

Design of Channel Width

Two methods were used in designing the width of channel, one (method *A*) being based largely on the adjustments of similar existing waterways, and the other (method *B*) being based primarily on observations of the behavior of ships in existing waterways and in ship model experiments.

Method A.—The existing waterways used in the application of method A should have characteristics somewhat similar to the waterway for which the channel width is to be determined. A study is then made to ascertain the sizes of the largest ship or ships of average maneuverability that can easily navigate each of the existing waterways under normal conditions with the degree of safety and efficiency of transit desired in the proposed waterway. The selection of these ships may be on the basis of either single-direction or two-direction traffic. In those waterways where single-direction traffic is used as the basis of analysis, the ship selected is called the "representative ship." In those where two-direction traffic is the basis of analysis, the larger of the two passing ships is called "representative ship No. 1," and the smaller, "representative ship No. 2." The ships that are adopted as the maximum size for which the proposed canal should be adequate to provide two-direction traffic are designated "design ships" and the notations "No. 1" and "No. 2" apply to the larger and smaller ones, respectively. The channel section of each waterway is then adjusted to be adequate for the "design ships" transiting the adjusted waterway under the conditions that would exist in a sea-level canal. This procedure requires four steps, as follows:

1. The channel dimensions of each existing waterway are multiplied by the ratio of the dimensions of design ship No. 1 to those of representative ship No. 1. On the premise that the required channel section is directly proportional to ship size, the resulting channels are then adequate for design ship No. 1.

2. The adjusted channel sections of the existing waterways are now converted to a common selected depth and to the same side slopes. This is done by holding the cross-sectional areas of the channel sections constant and by adjusting the channel width. The adjustment is based on the premise that the navigational characteristics of a channel within reasonable limits are a function of its cross-sectional area.

3. The third step is an adjustment for design ship No. 2. Those channels which in the previous steps were based on single-direction traffic are increased in width by an amount equal to three times the beam of design ship No. 2. Channels based on two-direction traffic are increased (or decreased, if the first step resulted in representative ship No. 2 being larger than design ship No. 2) by an amount equal to five times the difference between the beam of design ship No. 2 and the beam of representative ship No. 2 after the latter has been multiplied by the ratio used in step 1. The ratios of three and five are empirical and were developed from an analysis of the relationship of channel width to ships' beams.

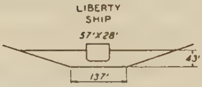
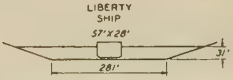
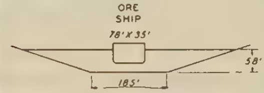
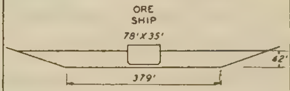
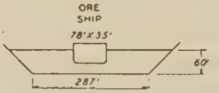
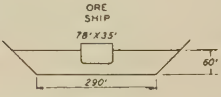
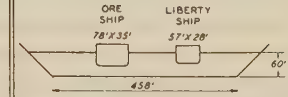
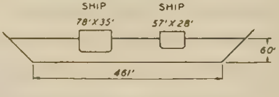
4. The navigating and physical characteristics of each of the existing waterways are compared with those of the proposed sea-level canal and a final adjustment is made in the channel widths obtained in step 3. The characteristics compared are traffic speed, currents, alinement, bank material, and length of restricted channel.

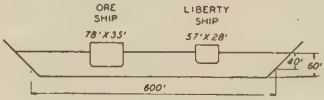
Application of Method *A*.—Channel studies using method *A* included the Cape Cod Canal, the Suez Canal, and the Houston Ship Channel, and Gaillard Cut of the Panama Canal. The procedure followed in adjusting the Gaillard Cut channel of the present Panama Canal is described in detail.

The study of navigation in Gaillard Cut showed that for single-direction traffic the representative ship is a T-2 tanker, and for two-direction traffic representative ship No. 1 is a Liberty ship and representative ship No. 2 is a small ocean-going cargo vessel. The *S. S. Pereira*, a vessel of good controllability with a beam of 32.7 ft and a draft of 15.1 ft, was selected as typical of small ocean-going cargo vessels. Both cases are shown in Fig. 57 which illustrates the step-by-step procedure used in determining channel widths for Panama lock and sea-level canals. The two-direction traffic condition which is shown as case 2 in Fig. 57 will be developed for a Panama sea-level canal in the following paragraphs. In this application the large loaded ore ship (Table 27) is used as design ship No. 1 and the Liberty ship as design ship No .2. One of the requirements of this method is that design ship No. 1 and representative ship No. 1 be of the same controllability classification, thus the large naval vessel (Table 27) could not be used. The ratio of the beam of design ship No. 1 (78 ft) to the beam of representative ship No. 1 (57 ft) is 1.37 and the ratio of of drafts (35 to 28) is 1.25. The average of the two ratios is 1.31 which, for purposes of conservatism, is increased to 1.35. The actual minimum channel dimensions of Gaillard Cut and the dimensions after being multiplied by 1.35 in the first step are shown in Fig. 57.

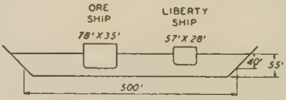
Step 2 is a conversion of the Gaillard Cut adjusted section to the selected 60-ft depth and to 1-on-1 side slopes, which are approximately average bank slopes. The bottom width is decreased to 363 ft as shown in Fig. 57.

Step 3 adjusts for design ship No. 2. When the adjustment in step 1 was made, not only was the size of the channel increased, but representative ship No. 2 was also increased in size from a beam of 32.7 ft and a draft of 15.1 ft, to a beam of 44 ft and a draft of 20.5 ft, respectively. The difference between the beam of design ship No. 2 and that of representative ship No. 2, after ad-

STEP NO	DESCRIPTION OF STEPS	SUEZ CANAL (LAND CUT CHANNEL)	HOUSTON SHIP CHANNEL (250 FOOT PROJECT WIDTH CHANNEL)
	WATERWAY CHARACTERISTICS LENGTH RESTRICTED CHANNEL TYPE BANK MATERIAL CURRENTS ENCOUNTERED BY SHIPS TRANSIT SPEED WITH RESPECT TO LAND LARGEST CURVE — ANGLE — RADIUS ANGULARITY PER MILE OF RESTRICTED CHANNEL MAXIMUM SHIP LIMITATIONS	 75 MILES SAND UP TO 3 KNOTS 75 KNOTS 63 DEGREES 9966 FEET 3.3 DEGREES CURVES	 12.5 MILES EARTH NO APPRECIABLE 4-8 KNOTS 109 DEGREES 3000 FEET 61.7 DEGREES CHANNEL DEPTH
	REPRESENTATIVE SHIPS IN THE SELECTED WATERWAYS	 LIBERTY SHIP 57'X28' 43' 137'	 LIBERTY SHIP 57'X28' 31' 281'
1	ENLARGEMENT OF WATERWAY FOR DESIGN SHIP NO 1	 ORE SHIP 78'X35' 58' 185'	 ORE SHIP 78'X35' 42' 379'
2	ADJUSTMENT TO 60 FOOT DEPTH AND 1/1 SIDE SLOPES	 ORE SHIP 78'X35' 60' 287'	 ORE SHIP 78'X35' 60' 290'
3	ADJUSTMENT FOR DESIGN SHIP NO 2	 ORE SHIP 78'X35' LIBERTY SHIP 57'X28' 60' 458'	 ORE SHIP 78'X35' LIBERTY SHIP 57'X28' 60' 461'
4	WATERWAY CHARACTERISTICS AND INDICATED ADJUSTMENTS	SEA-LEVEL CANAL / LOCK CANAL	SEA-LEVEL CANAL / LOCK CANAL
	TRAFFIC SPEED	INCREASE / NO CHANGE	(INCREASE) / DECREASE
	CURRENT	INCREASE / (DECREASE)	INCREASE / NO CHANGE
	ALIGNMENT	(DECREASE) / (INCREASE)	DECREASE / DECREASE
	BANK MATERIAL	INCREASE / INCREASE	INCREASE / INCREASE
	LENGTH RESTRICTED CHANNEL	DECREASE / DECREASE	INCREASE / NO CHANGE
	GENERAL ADJUSTMENT	SHOULD BE APPRECIABLE MORE / SHOULD BE SLIGHTLY LESS	SHOULD BE APPRECIABLE MORE / SHOULD BE LESS


ORE SHIP
78'X35'
LIBERTY SHIP
57'X28'
40'
60'
600'

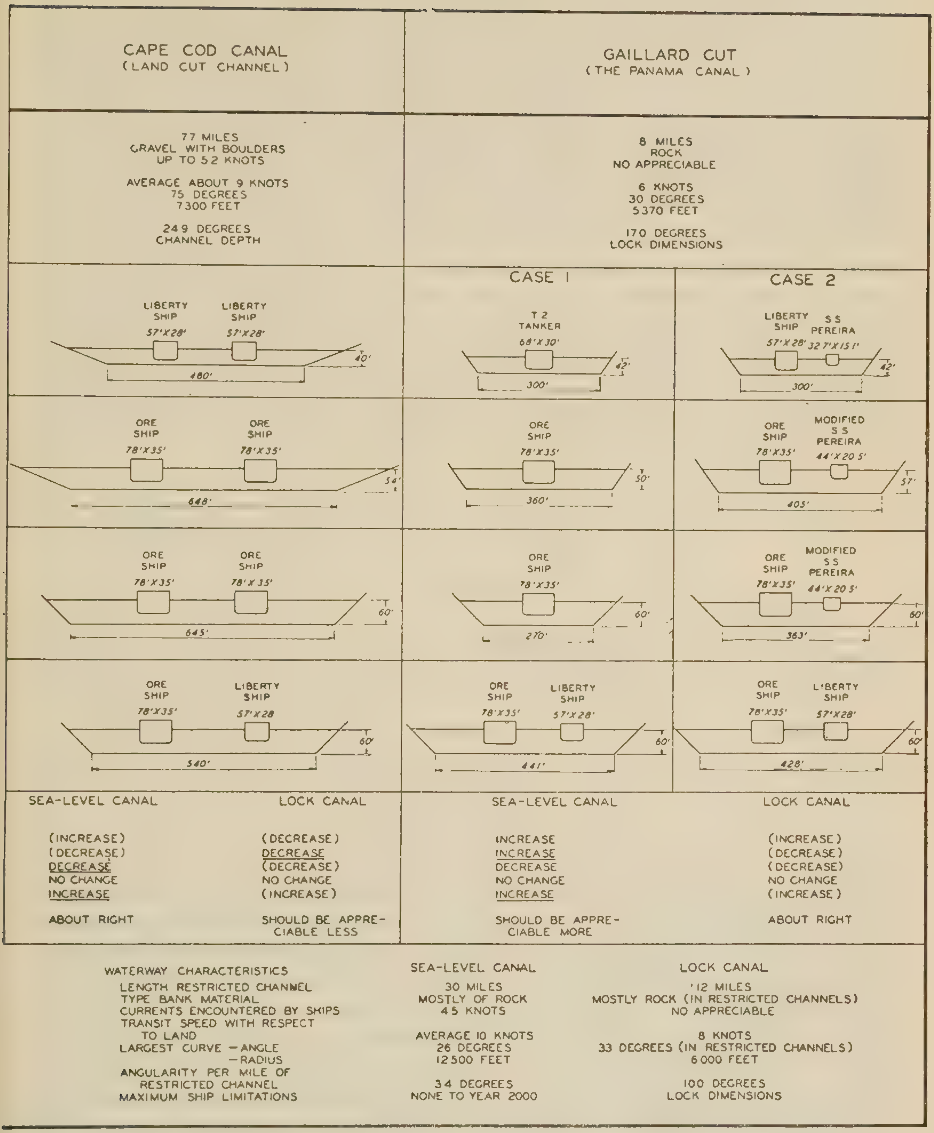
PANAMA SEA-LEVEL CANAL


ORE SHIP
78'X35'
LIBERTY SHIP
57'X28'
40'
55'
500'

IMPROVED PANAMA LOCK CANAL

SELECTED CHANNEL SECTIONS FOR PROPOSED PANAMA CANALS

FIG. 57.—DIAGRAM OF PROCEDURE FOR



CAPE COD CANAL (LAND CUT CHANNEL)	GAILLARD CUT (THE PANAMA CANAL)	
77 MILES GRAVEL WITH BOULDERS UP TO 5 2 KNOTS AVERAGE ABOUT 9 KNOTS 75 DEGREES 7300 FEET 24 9 DEGREES CHANNEL DEPTH	8 MILES ROCK NO APPRECIABLE 6 KNOTS 30 DEGREES 5370 FEET 170 DEGREES LOCK DIMENSIONS	
	CASE 1	CASE 2
LIBERTY SHIP 57'X28' LIBERTY SHIP 57'X28' 40' 480'	T 2 TANKER 68'X30' 42' 300'	LIBERTY SHIP 57'X28' S S PEREIRA 32 7'X15 1' 42' 300'
ORE SHIP 78'X35' ORE SHIP 78'X35' 54' 648'	ORE SHIP 78'X35' 50' 360'	ORE SHIP 78'X35' MODIFIED S S PEREIRA 44'X20 5' 57' 405'
ORE SHIP 78'X35' ORE SHIP 78'X35' 60' 645'	ORE SHIP 78'X35' 60' 270'	ORE SHIP 78'X35' MODIFIED S S PEREIRA 44'X20 5' 60' 363'
ORE SHIP 78'X35' LIBERTY SHIP 57'X28 60' 540'	ORE SHIP 78'X35' LIBERTY SHIP 57'X28' 60' 441'	ORE SHIP 78'X35' LIBERTY SHIP 57'X28' 60' 428'

SEA-LEVEL CANAL	LOCK CANAL	SEA-LEVEL CANAL	LOCK CANAL
(INCREASE)	(DECREASE)	INCREASE	(INCREASE)
(DECREASE)	DECREASE	INCREASE	(DECREASE)
DECREASE	(DECREASE)	DECREASE	(DECREASE)
NO CHANGE	NO CHANGE	NO CHANGE	NO CHANGE
INCREASE	(INCREASE)	INCREASE	(INCREASE)
ABOUT RIGHT	SHOULD BE APPRECIABLE LESS	SHOULD BE APPRECIABLE MORE	ABOUT RIGHT

WATERWAY CHARACTERISTICS	SEA-LEVEL CANAL	LOCK CANAL
LENGTH RESTRICTED CHANNEL	30 MILES	12 MILES
TYPE BANK MATERIAL	MOSTLY OF ROCK	MOSTLY ROCK (IN RESTRICTED CHANNELS)
CURRENTS ENCOUNTERED BY SHIPS	4 5 KNOTS	NO APPRECIABLE
TRANSIT SPEED WITH RESPECT TO LAND	AVERAGE 10 KNOTS	8 KNOTS
LARGEST CURVE —ANGLE	26 DEGREES	33 DEGREES (IN RESTRICTED CHANNELS)
—RADIUS	12 500 FEET	6 000 FEET
ANGULARITY PER MILE OF RESTRICTED CHANNEL	3 4 DEGREES	100 DEGREES
MAXIMUM SHIP LIMITATIONS	NONE TO YEAR 2000	LOCK DIMENSIONS

Selection of Channel Sections

justment by step 1, is 57 − 44 = 13 ft. To adjust for design ship No. 2, the channel width is then increased by 5 × 13 = 65 ft. Thus, the bottom width of the Gaillard Cut channel section is changed from 363 ft to 428 ft.

Step 4 takes into account the difference in the navigating and physical characteristics of the waterways. Traffic speed in Gaillard Cut is 6 knots, but 10 knots is to be allowed in the Panama sea-level canal; therefore, an increase in the width of the Gaillard Cut section, as adjusted by step 3, is indicated. There are no appreciable currents in Gaillard Cut, but currents up to 4.5 knots would obtain in the proposed canal; therefore, an appreciable increase in width is indicated. The alinement of the proposed canal would be an improvement over the existing alinement through Gaillard Cut, so a decrease in width is indicated. Bank material would be the same for both waterways, so no change in width is indicated because of this factor. Gaillard Cut is about 8 miles long, and the proposed canal would have about 30 miles of restricted channel of minimum width; therefore, an appreciable increase in channel width is indicated. These indicated adjustments, which apply to both case 1 and case 2, Fig. 57, for the Gaillard Cut, and the indicated adjustments as developed for the Cape Cod Canal, the Houston Ship Channel, and the Suez Canal, are shown in Fig. 57 for both lock and sea-level canals at Panama. The characteristics are listed in the order of their importance to channel width. The general indicated adjustment for each of the four waterways and the bottom widths of each after adjustment by the first three steps are shown in Fig. 57, slight adjustments being indicated in parentheses and appreciable ones being underlined. The bottom width for the Panama sea-level canal as indicated by this method is 560 ft. However, because of variable slope conditions that would prevail throughout the canal, it is considered desirable to reference the channel width to a depth corresponding to the draft of the larger vessels, which is approximately 40 ft. On the basis of 1-on-1 side slopes used in this method, the indicated desirable width at the 40-ft depth is 600 ft. The indicated width at the 40-ft depth for an improved Panama lock canal is 500 ft.

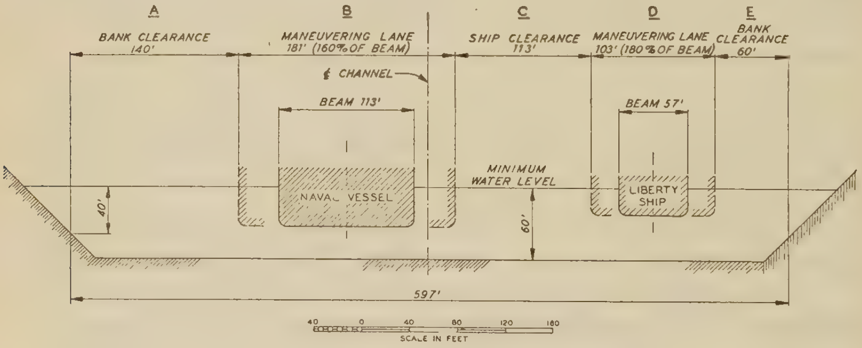


FIG. 58.—ELEMENTS OF CHANNEL DESIGN; SEA-LEVEL CANAL

Method B.—In this method the channel is divided into five parts (Fig. 58) and each part is analyzed separately. The combined widths of the parts then determine the width of the channel. The parts are titled "bank clearance"

(A and E), "maneuvering lanes" (B and D), and "ship clearance" (C). In this method of analysis, use is made of experience at Panama, pilot opinion, and the results of the ship model tests as presented in the fifth paper of this Symposium. The passing of the large naval vessel and the Liberty ship is used in this application since test data are not available for the ore ship.

The bank clearances A and E are considered as the space from the rail of the ship to the bank at a depth corresponding to the draft of the vessel in question. This space must provide sufficient water area between ship and bank so that the ship will retain good controllability when transiting at this distance off the bank. This distance is selected from Fig. 59, which gives the rudder angles required to hold a ship on a course parallel to the bank for various offsets from the bank of a channel 60 ft deep and 600 ft wide at the 40-ft depth. The angularity of the rudder that had to be carried to hold or to maintain the vessel on its course under a particular set of conditions is considered the best measure of controllability. The maximum angle to which a ship's rudder can be set is normally about 35°. The difference between this angle and that required to accomplish a particular maneuver is a measure of the reserve controllability available in the event of an emergency. Thus, a large rudder angle indicates a low safety factor, and a small angle, a high safety factor. The selection of the maximum rudder angle considered safe for approaching the bank or for passing an opposing ship must, therefore, be made primarily on the basis of navigational safety. Also, the selection must take into consideration the discrepancies in model tests as compared to actual navigation, and the factors affecting the channel dimensions which are not fully considered in operating the model. The bank clearance selected is based on a rudder angle of 5°, considered to be a very conservative value but selected to provide an excess of safety to cover conditions that exist in an actual waterway that cannot be reproduced in the model. From Fig. 59, the bank clearance for the large naval vessel is 140 ft, and for the Liberty type ship it is 60 ft—both clearances being measured at the 40-ft depth. The curves in Fig. 59 are based on a 9-knot speed rather than on a 6-knot speed—to take account of head currents.

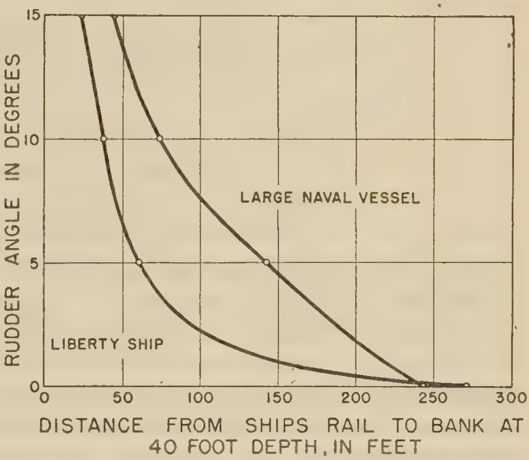


Fig. 59.—Bank Clearance Versus Rudder Angle

The maneuvering lanes are indicated as B and D, Figs. 58 and 60. These lanes represent the part of the channel in which each ship may maneuver without encroaching on the safe bank clearance or without approaching the other ship so closely that dangerous interference between ships will occur. The ship model tests in themselves did not give sufficient data for fixing the widths of the maneuvering lanes because time and funds did not permit the running of a sufficient number of tests on the wide range of ship sizes and types that would be necessary to establish the possible variations of a ship's position when meeting. Based on pilots' opinions and observation of ship courses, the width of the maneuvering lane has been established as ranging from 160% to 200% of the beam of the ship, depending on the characteristics of the ship. The limited observations taken on transits of actual ships through certain reaches of the Panama Canal indicate that these allowances are entirely adequate. Plots of ship courses in the model tests confirm the allowances. Neither experience nor tests indicate that currents have an appreciable effect on the required maneuvering lane if sufficient depth of channel is provided to assure proper response to the rudder. Ships having "very good," "good," and "poor" controllability are considered to require maneuvering lanes of 160%, 180%, and 200% of their beams, respectively. The large naval ship with a beam of 113 ft, therefore, would require a lane width of 181 ft; and the Liberty ship, one of 103 ft.

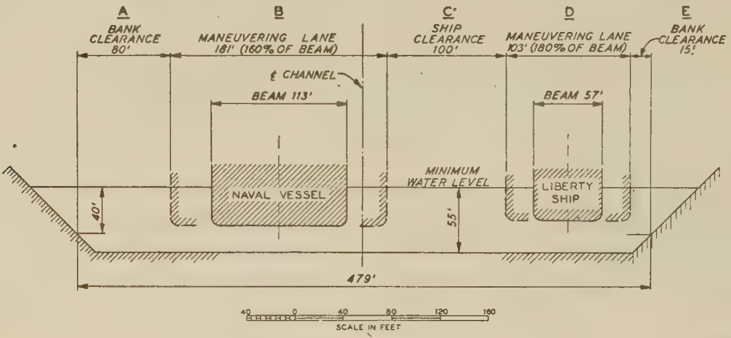


Fig. 60.—Elements of Channel Design; Lock Canal

The ship clearance indicated as lane C in Fig. 58 is the minimum space between two ships in passing that will assure good controllability for both ships during and following the passing maneuver. As both ships approach the inner limits of their maneuvering lanes, the effects of the channel banks decrease, and the interaction between the two ships becomes controlling. With ships at the inner limit of the maneuvering lane, bank action is insignificant. The ship model tests indicate that, if rudder angles of from 10° to 15° are not to be exceeded when two such ships pass in a channel with a current up to 4.5 knots, the distance between ships must be at least 100 ft or equal to the beam of the larger ship, whichever value is greater. Pilot opinion is consistent with this criterion in establishing ship clearance. A larger rudder angle for passing than that used for bank clearance is justified because of the much shorter time that the interaction between ships exists as compared to the time bank action may affect the ship. The minimum clearance required for the passing of the large naval vessel and the Liberty ship is 113 ft.

With a computation based on this method of analysis, the required width of the sea-level canal, measured at a depth of 40 ft, would be 597 ft. The values for each part of the channel are shown in Fig. 58. A similar analysis for the minimum section of a lock canal resulted in the values shown in Fig. 60.

The Panama Canal pilots assisting in the ship model tests were of the opinion that the effect of the proximity of channel banks and channel bottom, the currents, and the interaction between passing ships were clearly and accurately indicated in the ship model tests. A limited series of prototype tests at Panama has generally confirmed qualitative results of the model tests. However, there are certain discrepancies between model and prototype which must be considered in adapting the test data to channel design. The factors favoring the test results included the following:

(*a*) Uniformity of channel section;
(*b*) Weather conditions; and
(*c*) Freedom from mechanical (rudder and engine) breakdowns.

Those factors not favoring the test results included the following:

(*a*) Time scale requiring pilots to act seven to ten times as fast;

(*b*) Stability of ship models about a vertical axis not being comparable with prototype ships (particularly evident in the test run using the 1:86 scale ratio); and

(*c*) The fact that the pilot is not actually aboard the vessel and thus loses the viewpoint and the "feel" of an actual ship.

Pilots were in agreement that navigation under prototype conditions would show some over-all improvement over the action observed in the model. The model tests, although limited in their capabilities to reproduce and take into consideration all factors affecting the safety and security of transiting ships in an actual waterway, have performed an invaluable service in enabling the pilots, technicians, engineers, and consultants to reach a common understanding of the operation of ships in restricted channels.

Final Selection of Channel Dimensions

Pilots and marine operating officials at the Cape Cod Canal, the Houston Ship Channel, and the Suez Canal were polled informally by Panama Canal representatives on their views with respect to the required channel width and depth for two-way traffic in the Panama sea-level canal. The Cape Cod personnel considered that a channel from 500 ft to 600 ft wide, and of adequate depth, would afford easy navigation for the largest commercial and military ships. At the Houston Ship Channel it was considered that for a channel with rocky banks the channel should have a minimum width of 600 ft and a minimum depth of 60 ft. The Suez Canal personnel were of the opinion that a channel from 500 ft to 600 ft wide and from 50 ft to 60 ft deep would be ample for two-direction traffic of the largest commercial vessels operating in currents up to 4.5 knots.

The Marine Division of The Panama Canal and its pilots have been closely associated with all navigation studies made as a part of "Isthmian Canal

Studies—1947," and they are in general agreement that a channel 60 ft deep and 600 ft wide at the 40-ft depth would be satisfactory for the sea-level canal and that 55-ft by 500-ft dimensions would be adequate for the restricted channels of the lock canal.

As a result of the design studies and opinions of marine operating personnel, it was concluded that the Panama sea-level canal should have a minimum channel depth of 60 ft and a minimum channel width of 600 ft at the 40-ft depth, and that corresponding dimensions for an improved Panama lock canal should be 55 ft and 500 ft.

Curves

Navigation around curves is generally more difficult than in a straight channel, particularly if currents exist. The alinement as tentatively accepted for a sea-level canal has a maximum deflection angle of 26°. This value corresponds to maximum deflection angles of 30° for the Gaillard Cut of the present Panama Canal, 63° for the Suez Canal, 75° for the Cape Cod Canal, and 109° for the Houston Ship Channel.

Experience in Existing Waterways.—Analysis of the different types of curves on existing waterways, although illustrating the weaknesses of much of the treatment, has not resulted in the development of a satisfactory method of treatment for a sea-level canal. There are so many differences in operating technique and physical conditions of the different waterways that comparison of curve treatment to establish the best method is not possible. The marine opinion at Suez, where the original curves were widened and modified, is that unwidened long-radius curves are the most desirable. However, this opinion is based on single-direction traffic on curves with no passing even of ships tied up to the bank.

The Cape Cod Canal is a good example of a channel having curves of adequate radius (7,000 ft or more) and channel width (480 ft) to permit two-direction traffic and practically unrestricted operation by all ships that may transit the waterway. There are no straight sections in the land-cut channel of the Cape Cod Canal of sufficient length to establish any relationship between the difficulties of navigating on a curve as compared to a straight channel.

At the existing Panama Canal a true curve is not used. In most cases the outside prism lines are extended to intersection. The inside prism lines are stopped short of the point of intersection and connected by a chord. This treatment, although reasonably satisfactory for a channel without currents, would be unsatisfactory with currents because of poor flow and eddy conditions.

Ship Model Tests of Curves.—Ship model tests of various types of curve treatment were performed at the Taylor Model Basin. Time and funds did not permit the continuation of these tests to the point where finally conclusive results could be established. Most of the tests were conducted at a scale of 1:86 rather than at the scale of 1:45 used in straight-channel tests. Outlines of the various curve treatments tested are shown in Fig. 52. The 1:86 scale tests indicated that the curve with no widening (Fig. 52(*a*)) was superior to all the others for single-direction traffic and that two-direction traffic would not be acceptable for any of the curves. The results of the 1:86 scale tests were very erratic and definitely not consistent with actual experience at either Cape Cod or Suez. It was believed that the instabilities of the 1:86 scale model and of the 9:1 time ratio were the principal causes of the unsatisfactory results. Therefore, tests were made using a scale ratio of 1:45, of a single unwidened curve, 600 ft wide at a 40-ft depth and 60 ft deep, with 1-on-1 side slopes, a 12,500-ft radius, and a 26° intersection angle. These tests clearly demonstrated that complete dependence could not be placed on the results of the 1:86 model tests. The tests to a 1:45 scale demonstrated that single-direction navigation around a prototype of the curve tested is entirely practicable and safe with controllability characteristics similar to those for operation in straight channels. Time did not permit two-way navigation tests in the 1:45 scale curve. The limited number of off-center runs, although showing some improvement in conditions over the 1:86 scale tests, did not indicate that two-direction traffic of "design ships" would be a safe operation.

Whether a practicable method of widening can be developed to permit passing of the large ore carrier or naval vessel and a loaded Liberty ship (design criteria for straight channel) has not been established. It is considered that model tests to scales of 1:20 or of 1:30 should be used in any further tests to determine the best methods of curve treatment.

Should it develop that satisfactory curve treatment for two-direction traffic is unattainable, operating restrictions would have to be placed on the passing of the larger ships on curves. These restrictions would not noticeably affect the transit time or increase delays to shipping but would necessitate more detailed dispatching.

ALINEMENT

The alinement of the present canal contains twenty-three angles with a total angularity of 598°. The largest of the angles is 67°, and four of the angles are larger than 50°. Use of this alinement and a center-line radius of 12,500 ft on the curves would result in almost 50% of the restricted channel of a sea-level canal being in curves, and was considered unsatisfactory for a sea-level canal having a restricted channel for the major part of its length in which currents up to 4.5 knots might exist.

TABLE 29.—COMPARISON OF SELECTED ALINEMENT FOR SEA-LEVEL CANAL WITH PRESENT CANAL ALINEMENT

Canal	No. of Angles		Angularity (Degrees)			Length (Miles)		Restricted Channel	
	<20°	>20°	Maximum angle	Total	Per mile	Minimum length of straight reach	Total length of canal	Length (miles)	In curves (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Sea level......	6	2	26	117	2.54	2.94	46.02	30	14
Present lock...	11	12	67	598	11.68	0.07	51.20	12	33

An investigation was made of possible alinements for a sea-level canal at Panama ranging from the present canal alinement to an absolutely straight alinement from shore line to shore line. The proposed alinement is shown in Fig. 7. The maximum angle in this alinement is 26° and the total angularity is 117°. It was found that any reasonable increase in the size of the maximum angle or in the total angularity would not decrease the excavation cost. A comparison of the characteristics of this alinement with that of the present canal is shown in Table 29.

Summary of Sea-Level Channel Design

A summary of the tentative channel design of a sea-level channel resulting from these studies and investigation is as follows:

Physical Dimensions—	
Total length of restricted section of minimum channel width, in miles	30
Minimum Channel Section—	
Depth, in feet	60
Width, in feet, at 40-ft depth	600
Side slopes (steepest)	3 on 2
Number of angles in restricted section	6
Maximum angle, in degrees	26
Radius of curves, in feet	12,500
Widening at curves (dependent upon further study)	...
Minimum distance between points of intersection, in miles	4.21
Minimum sight distance, in miles	1.52
Qualifying Operation Criteria—	
Pilots required for all ships	...
Maximum transit speed (water speed), in knots	10
Normal passing speed of large ships, in knots	6
Two-direction traffic with maximum limitation of largest existing naval vessels or a large commercial cargo vessel passing an average size commercial cargo vessel	...
Passing on curves limited to average size ships (subject to additional curve study)	...
Maximum length of ships, in feet, that can pass in same direction	300

The design is liberal in physical dimensions and should provide a safe and efficient waterway for all shipping, both commercial and military, that may be expected to use it to the year 2000.

The adequacy of the proposed sea-level canal channel is affirmed by Panama Canal pilots as presented in a review of the Governor's Report to Congress by a pilot committee consisting of four senior pilots whose total experience at the Panama Canal totals 87 years:

> "From a piloting point of view, we feel that this canal [sea level] will have a greater safety factor than the other three plans [lock canal] outlined and will not contain any difficulties or hazards of any importance. The most important safety factors inherent in the proposed sea-level canal are

> the dimensions, alignment, and tidal-control structures, as shown and described in the draft [of the report]. We are of the opinion that in the event the tidal-control structure should be badly damaged or destroyed, it would be feasible to operate the canal as an uncontrolled waterway."

Further Tests and Investigations.—The channel dimensions for the Panama sea-level canal, as developed in this paper, are considered to be satisfactory. However, in the event that construction is authorized, the magnitude of the project and the necessity for more information on curve treatment would make it essential that additional studies be made. These studies would consist of further investigations of navigation in existing waterways and continuance of the ship model tests. Particular emphasis would be placed on the development of data to confirm the width of maneuvering lane and passing clearance and the best method of treating curves. Model studies would be conducted at a larger scale than used previously to reduce the scale effects apparent in former studies.

EXCAVATION SLOPES

BY WILSON V. BINGER,[43] ASSOC. M. ASCE AND THOMAS F. THOMPSON,[44] AFFILIATE, ASCE

SYNOPSIS

The geologic and soil mechanic studies conducted to establish stable slopes for the formations that would be excavated for a sea-level canal at Panama are described in this paper. The authors discuss the early and current geological investigations, the geological features of the area, the difficulties experienced in the slides that occurred during construction of the present canal, and the development of slope standards used in the preliminary design of a sea-level canal.

The paper concludes that adequate exploration and testing to develop the strength properties and distribution of the materials to be excavated, combined with present understanding of the mechanics of slide development, allow design of slopes that would be secure from major slides.

INTRODUCTION

The construction of a sea-level canal across the Isthmus of Panama would require the excavation of more than 1,000,000,000 cu yd of material, or about three times the volume of material removed in the construction of the existing canal. Almost 30% of the new excavation would be concentrated in a 4-mile section through the Continental Divide north of Pedro Miguel, where cuts deeper than 600 ft would be required.

Experience with the great slides that developed during the excavation of the present canal indicates the design of stable slopes for a sea-level canal to be a problem of unprecedented magnitude in engineering geology and soil mechanics. The renowned slides of the Panama Canal occurred in the Continenal Divide area, the greatest troublemakers being limited to the 1-mile length of deepest cut where the weak Cucaracha formation in the East Culebra, West Culebra, and Cucaracha slides (Fig. 61) increased by about 20% the amount of excavation required for building the canal. These slides are object lessons for the design of slopes.

There can be little question of the importance of designing the canal banks for stability. If permitted to occur, slides might block the canal during or following construction. Interruption of traffic by slides at any time would penalize commerce, and in wartime such delays would be intolerable. Furthermore, if slides occur, the total amount of material to be removed would be greater than the excavation required if the canal banks were excavated initially to stable slopes.

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[44] Chf., Geology Section, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

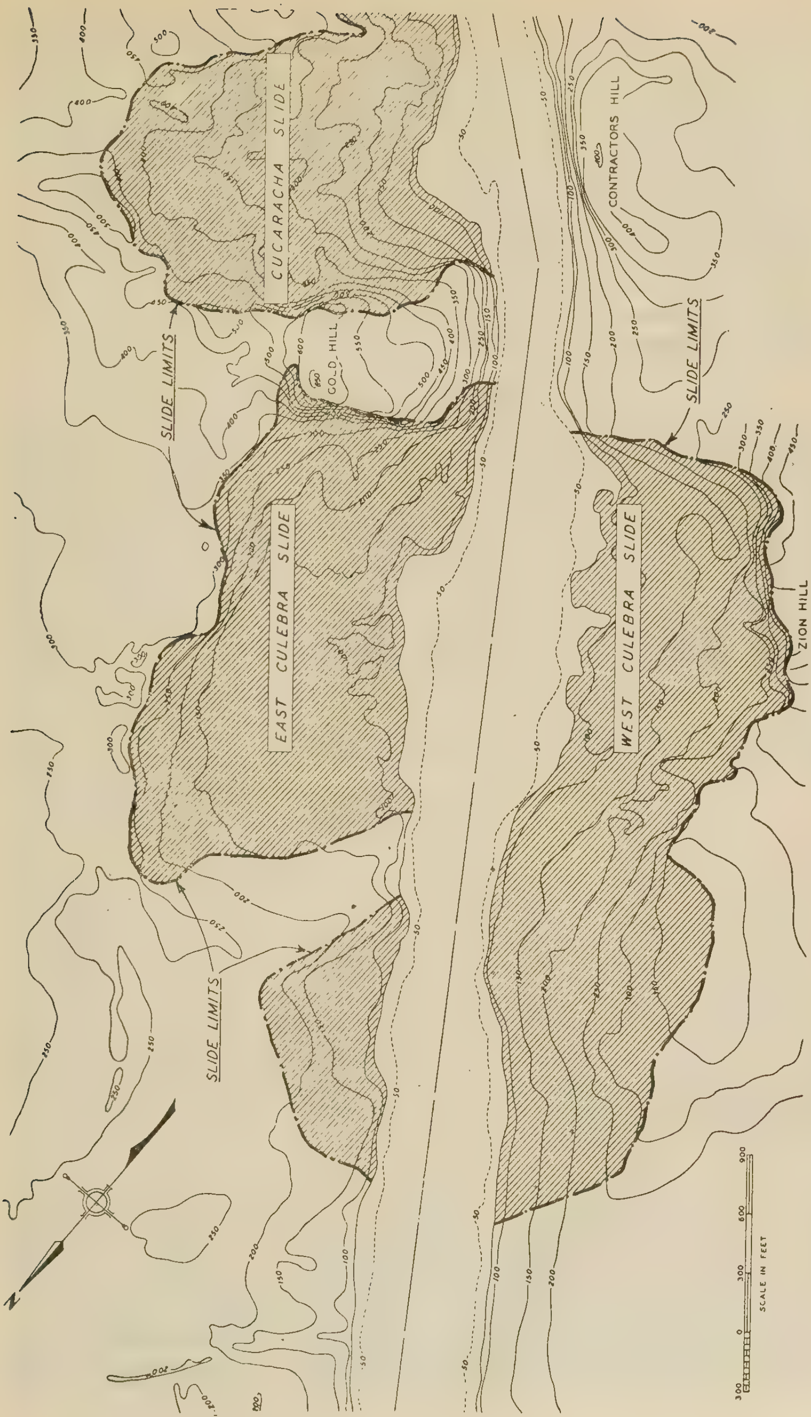

CUCARACHA SLIDE
GOLD HILL
CONTRACTORS HILL
SLIDE LIMITS
EAST CULEBRA SLIDE
WEST CULEBRA SLIDE
ZION HILL
SLIDE LIMITS
SLIDE LIMITS
N
300 0 300 600 900
SCALE IN FEET

FIG. 61.—LOCATION OF MAJOR SLIDE AREAS, PANAMA CANAL

The current studies for Public Law No. 280 have been greatly facilitated by the extensive slope studies made from 1939 to 1942 during the Third Locks Project, particularly those of the areas containing the materials most critical with respect to sliding. Despite the extent of the past and current studies, additional geologic exploration would be required prior to excavation to locate the contacts between rock formations more accurately, as well as to define the location and character of the various weak and strong strata within each formation. More extensive testing and more detailed analysis would also be employed in the design of final excavation slopes. It is believed, however, that the studies of slopes reported herein have been conservative enough that the estimates of excavation cost will more than cover the adjustments in slope design that may result from more extensive studies.

History of Geologic Exploration

The geology of the Canal Zone has been studied intermittently for many years, but such study prior to the arrival of the French was not conducted for engineering purposes and, moreover, was neither systematic nor thorough. By present standards, the geologic investigations by the French during the periods of the first and second canal companies were very inadequate. Explorations were conducted in some detail to determine foundation conditions at dam, lock, and harbor-facility sites by both the early French and early American investigators prior to commencing construction, but only a minimum of information for establishing proper slopes was assembled for the regions of heavy cut through the Continental Divide area where disastrous slides later developed.

TABLE 30.—Subsurface Explorations, Panama Canal, 1881 to 1947

Line	Agency	Years	Total number of holes	Holes in Gaillard Cut	
				No.	Average depth (ft)
(1)	(2)	(3)	(4)	(5)	(6)
1	Old French Company	1881–1894	605	63	(42 holes) 120
2	New French Company	1894–1903	60	27 pits 2 borings	No records available
3	Isthmian Canal Commission	1904–1914	5,073	81	160
4	Third Locks Project[a]	1938–1943	1,968	242	151
5	Studies under Public Law No. 280[a]	1946–1947	230[b]	87[c]	335

[a] The Panama Canal. [b] Includes twenty-three holes on adjacent Panama routes. [c] For purposes of estimating sea-level canal slopes, these holes are of more value than all previous borings combined. They are the only borings completed for that specific purpose—that is, spotted at critical locations, carried down to sufficient depth (El. −100 or deeper), with cores carefully logged by a geologist, and with samples of weakest materials tested in the laboratory.

The principles of slope design to forestall the "deep deformation," shear-type slides were not known at the time of the early studies. It was not until near the end of the American construction period, after serious sliding was well advanced, that the studies of the late D. F. MacDonald, then resident geologist, led to an appreciation of causes and prevention of this type of failure.

Studies for the design of the Third Locks Project between 1939 and 1942 resulted in a thorough understanding of the geologic nature of the relatively small areas in which excavation was to be made for the locks and approach channels. In addition to determining the foundation adequacy of the rock at the sites selected for locks and appurtenant structures, investigations were sufficiently detailed to permit the establishment of slopes that would be secure against major slide development. In this work great benefit was derived from the services of Mr. MacDonald, who had returned to the Canal Zone as consulting geologist.

Table 30 presents the quantities of subsurface exploration performed during each of the several periods of major design or construction at Panama.

Field and Laboratory Investigations

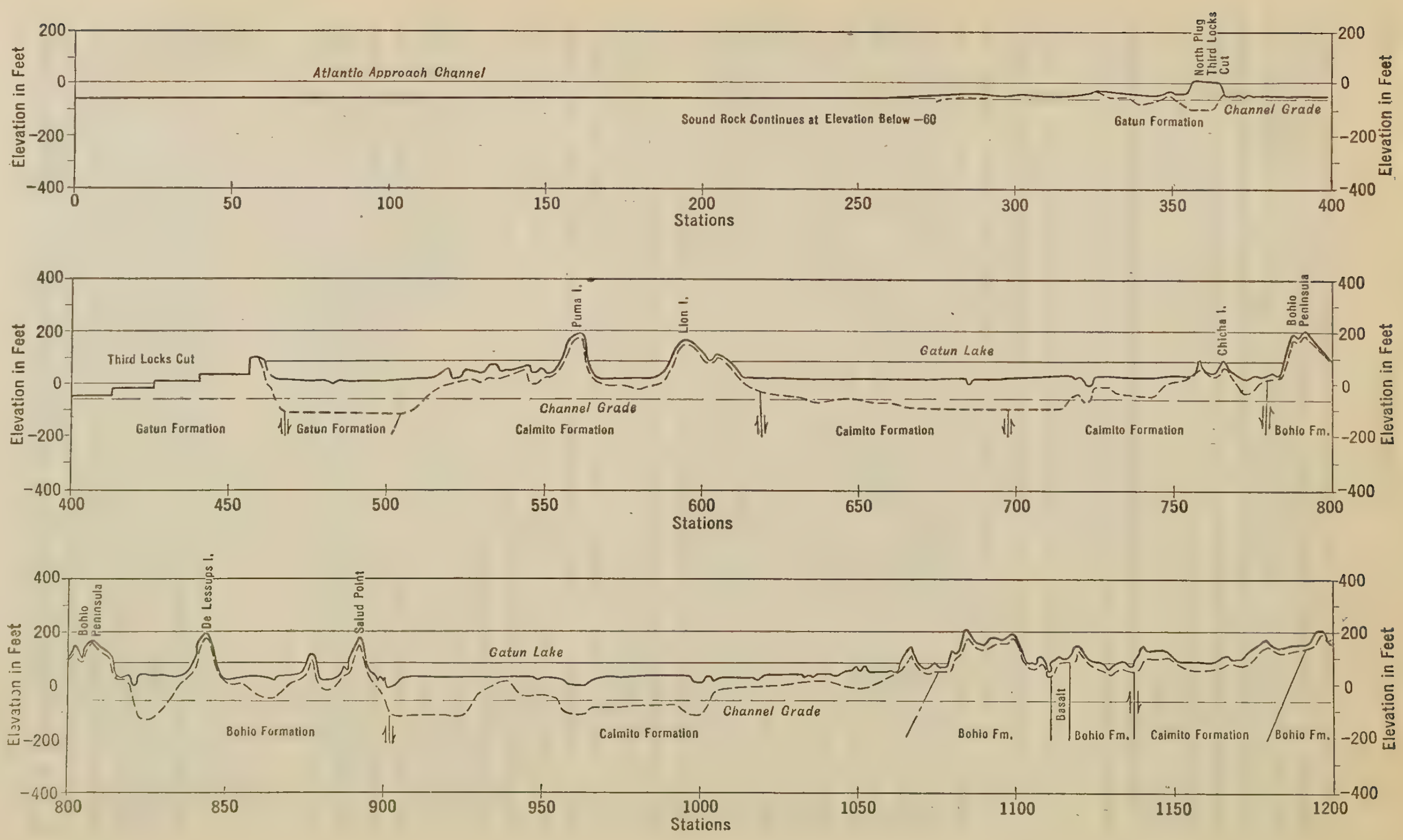
The investigations of the geology of the region traversed by the alinement of the Panama sea-level canal included: (1) Mapping on 1:20,000 scale base maps of all outcrops to be found along the line of the present canal, roads, and natural exposures such as stream channels that have removed the deep soil overburden; and (2) core borings at selected locations to develop further the character, distribution, and structural relationships of the underlying rock formations. Both methods of investigation benefited from stereoscopic examination of aerial photographs, which permitted characteristic topographic expressions to be used in tracing faults and contacts and in establishing the areal extent of certain formations. A large proportion of the borings was located near the Continental Divide where the sliding of oversteep slopes in the original canal excavation attested to the need for careful study in setting of slopes required by the greater depths of cut for the new canal.

The deepest boring made during the present investigations, near the Continental Divide, was 825 ft deep. The average depth of all holes drilled was 236 ft, and the total footage drilled for the 230 holes completed up to July 1, 1947, was 54,376 ft. Drill accessories employed were of "NX" size and yielded $2\frac{1}{8}$-in. diameter cores. Tungsten-carbide hand-set bits were found to be most efficient for the soft and medium rock types, whereas diamond bits proved best for the hardest igneous rocks.

Core-boring operations were inspected by a qualified geologist assigned to each drill rig. Samples of rock were obtained with standard core barrels and samples of the softer types of plastic overburden with 3-in.-diameter brass tubes. Samples were subjected to standard laboratory tests to determine their engiеering properties, of which the strength characteristics were the most important and were determined principally by unconfined and triaxial compression tests.

Geology

Geologic History and Physiography.—The datable geologic history of the Canal Zone commences in late Eocene time, some 50,000,000 years ago, when the local Isthmian region was depressed to the extent that a seaway extended diagonally across it and covered at least part of the present Canal Zone. During the succeeding ages, the land surface was repeatedly raised and lowered by diastrophism, resulting in changes in base level that are reflected in the character and distribution of bedded formations deposited on the lands or in the seas of each period. Recurrent volcanic activity ejected vast quantities of


Elevation in Feet
200
0
-200
-400
Atlantic Approach Channel
Sound Rock Continues at Elevation Below -60
Gatun Formation
North Plug
Third Locks
Cut
Channel Grade
0
50
100
150
200
250
300
350
400
Stations
400
Third Locks Cut
Gatun Formation
Gatun Formation
Channel Grade
Calmito Formation
Puma I.
Lion I.
Gatun Lake
Calmito Formation
Calmito Formation
Chicha I.
Bohio Peninsula
Bohio Fm.
450
500
550
600
650
700
750
800
Bohio Peninsula
De Lessups I.
Salud Point
Gatun Lake
Bohio Formation
Calmito Formation
Channel Grade
Bohio Fm.
Basalt
Bohio Fm.
Calmito Formation
Bohio Fm.
850
900
950
1000
1050
1100
1150
1200

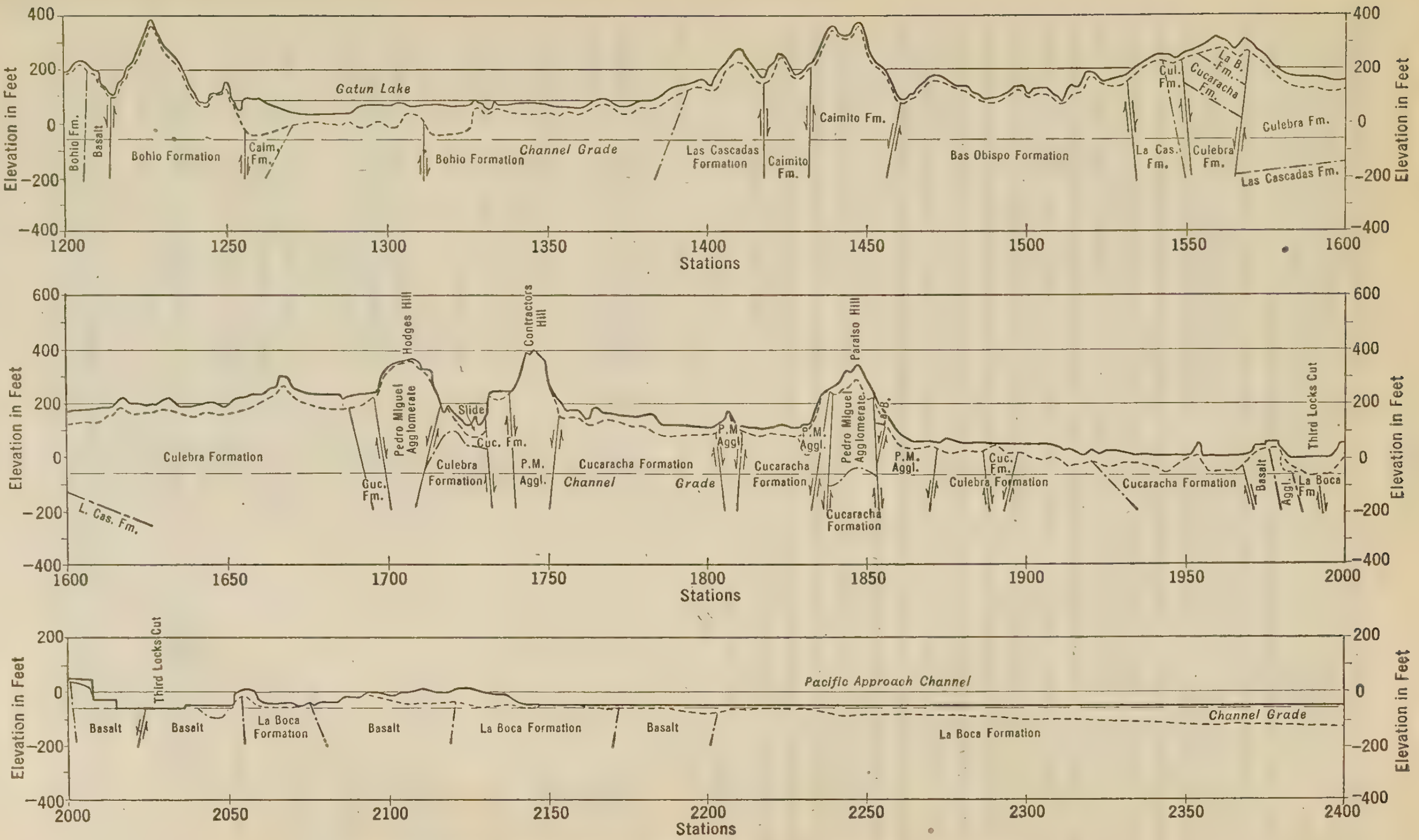

Elevation in Feet
400
200
0
−200
−400
Gatun Lake
Channel Grade
Bohio Fm.
Basalt
Bohio Formation
Calm. Fm.
Bohio Formation
Las Cascadas Formation
Caimito Fm.
Caimito Fm.
Bas Obispo Formation
Cul. Fm.
Cucaracha Fm.
La B. Fm.
La Cas. Fm.
Culebra Fm.
Culebra Fm.
Las Cascadas Fm.
1200
1250
1300
1350
1400
1450
1500
1550
1600
Stations
600
Hodges Hill
Contractors Hill
Paraiso Hill
Third Locks Cut
Culebra Formation
L. Cas. Fm.
Cuc. Fm.
Pedro Miguel Agglomerate
Slide
Cuc. Fm.
Culebra Formation
P.M. Aggl.
Cucaracha Formation
Channel
Grade
P.M. Aggl.
Cucaracha Formation
P.M. Aggl.
Pedro Miguel Agglomerate
L.B.
Cucaracha Formation
P.M. Aggl.
Cuc. Fm.
Culebra Formation
Cucaracha Formation
Basalt
Aggl.
La Boca Fm.
1600
1650
1700
1750
1800
1850
1900
1950
2000
Stations
Third Locks Cut
Pacific Approach Channel
Channel Grade
Basalt
Basalt
La Boca Formation
Basalt
La Boca Formation
Basalt
La Boca Formation
2000
2050
2100
2150
2200
2250
2300
2350
2400
Stations

FIG. 62.—GEOLOGIC SECTION ALONG CENTER LINE, PANAMA SEA-LEVEL CONVERSION ROUTE

materials over then-existing lands and seas or intruded igneous rocks into the pre-existing materials. This period is reflected today by widespread distribution of pyroclastic, flow, and intrusive rock types intimately associated with those resulting from normal sedimentation processes.

The topography of the Canal Zone is the result of stream erosion in a humid climate acting on a land surface, composed of rocks with very different resistances to erosion, which periodically has been elevated and depressed with respect to sea level. The region is one of unique topographic diversity, characterized by conical, irregularly spaced hills and by unsystematic drainages that give a chaotic appearance to the terrain. Sizes and shapes developed by the land forms are controlled principally by their relative resistances to erosion. Structural features, such as faults and folds, play a secondary and relatively minor role in configuration of the landscape. Drainage patterns are well developed and sharply defined despite their comparatively recent geologic age. Areas underlain by soft rocks are marked by broad valleys within which the larger streams have beveled the strata and deposited on them a blanket of alluvial material. Along both coasts, extensive swamps are present in the lower reaches of the main streams.

Rock Formations.—Only the formations that would be cut through by construction of a new canal are described in this paper. Their distribution along the proposed sea-level canal route is indicated in the center line geological profiles, in Fig. 62, to which the following condensed descriptions apply:

Formation	Explanation
Overburden	Poorly consolidated mucks, clays, silts, gravels, etc.
Gatun	Fine-grained argillaceous and calcareous sandstones with interbedded tuffs and conglomerates
Caimito	Medium-hard, medium to fine-grained, coarsely bedded, limy sandstones and tuff
Bohio	Volcanic pebbles, cobbles, and boulders in a tuffaceous sand matrix
Basalt	Flows and intrusives
Cucaracha	Largely soft clay shales with minor intercalations of sandstones and conglomerates
Culebra	Medium-hard sandstones and soft carbonaceous and sandy shales
La Boca	Largely soft silty and sandy shales with sandstone, tuff, and agglomerate interbeds
Las Cascadas	Medium-hard agglomeratic tuffs and tuff-breccias
Bas Obispo agglomerate	Largely angular fragments of andesite and basalt in hard sandy matrix of same composition
Pedro Miguel agglomerate	Hard, fine to coarse textured agglomerates and tuffs

In succeeding paragraphs each formation is taken in order of geologic age, beginning with the oldest and progressing to the youngest materials.

Bas Obispo Formation.—The Bas Obispo formation is a massively jointed agglomerate formed from angular rock fragments and ash that was blown from an old volcanic vent and cemented into a strong dark-gray rock of concrete-like appearance on freshly exposed surfaces. The Bas Obispo is hard and tenacious except where broken or sheared and softened by faulting. Only small slides have developed in the section exposed along the existing canal, and these are related to fault-broken zones. Where undisturbed by faulting, the formation represents one of the strongest rock types involved in new canal construction. About 2.5% of the total excavation for a sea-level canal at Panama would be through this strong formation.

Las Cascadas Formation.—The Las Cascadas formation originated in much the same manner as the Bas Obispo and directly overlies it, but its subsequent history has been dissimilar and it is much more heterogeneous. The Las Cascadas is mostly composed of scattered hard angular fragments of basalt and andesite that are embedded unsystematically in a fine, soft to medium-hard, altered tuff. Some sections are composed entirely of well-bedded tuffs without the hard inclusions. The fines are variably clayey and locally somewhat bentonitic. The formation contains interbedded and random sheets and dikes of fine-grained, hard, igneous rock, mostly either basalt or andesite. During and after the original construction of the canal, numerous small bank failures developed in this formation where the banks had been weakened by faulting. One of the weaker rock units within the Canal Zone series, it would comprise only 1.5% of the excavation for a sea-level canal.

Bohio Formation.—The Bohio formation is composed of heavy beds of conglomerate and sandstone with infrequent shale or tuff layers. The conglomerates are most prevalent and consist of subangular to well-rounded pebbles, cobbles, and boulders in a basaltic sand matrix. The Bohio, in general, is well-cemented, moderately hard, and has widely spaced joints. This formation would comprise roughly 12.5% of the total excavation for a sea-level canal at Panama.

Culebra Formation.—The Culebra formation unconformably overlies the Las Cascadas formation and is in turn overlain by the Cucaracha formation. The Culebra can be divided into an upper calcareous and a lower argillaceous member. Its component beds are mainly soft to medium-hard sandstones, shales, limestones, tuffs, and thin conglomerates. Carbonaceous shales and a few impure lignite beds are found frequently in the upper and middle parts, and it is highly fossiliferous throughout. This is one of the weaker formations found in the Canal Zone. About 15% of the sea-level canal excavation would be rock of this formation.

Cucaracha Formation.—Strata of this formation were involved largely in the historic huge Gaillard Cut slides that developed during and continued for some time after the original digging of the canal. The maximum known thickness of the formation is 625 ft. Its composition is dominated by weak, poorly bedded, variably bentonitic, slickensided, soapy-textured clay shales (altered impure tuffs) interbedded with soft to medium-hard, fine, tuffaceous siltstones; medium to coarse, cross-bedded sandstones; pebble conglomerates; thinly bedded, often lenticular, soft clayey lignitic beds; and one hard bed of agglomeratic, indurated tuff. The formation largely represents an accumulation of fine volcanic detritus that has been reworked by stream action and subjected to a partial chemical decomposition of its component ash particles with resulting creation of hydrous clay minerals of the montmorillonite-beidellite group. It is the weakest rock formation encountered along the Panama sea-level canal alinement and would represent 14.5% of the total volume of excavation for sea-level construction.

La Boca Formation.—The La Boca formation consists of medium-hard, silty or sandy, variably calcareous shales, tuffs, sandstones, and limy concretion beds, with scattered heavy agglomerate layers. It is similar in most essential respects to the Culebra formation but represents only 2% of the material to be excavated for sea-level canal construction.

Pedro Miguel Agglomerate.—This formation occurs near Pedro Miguel, where it overlies and is in intimate association with the Cucaracha formation. Thick beds of agglomerate also are found interspersed within the lower part of of the La Boca formation. The dominant constituents range from small angular fragments to huge blocks of basalt enclosed in a strong, dense tuff matrix, locally cemented by secondary calcite. Beds of hard, black, indurated tuff are found through its section. It is massively jointed and frequently shows crude bedding. It is one of the stronger types of material to be found locally. Only 4% of the materials that would be removed for a sea-level canal are from this formation.

Caimito Formation.—The Caimito formation is widely distributed in the Gatun Lake area, where it overlies the Bohio formation. It has been divided into three recognizable units. Its basal phase is a tuffaceous sandstone conglomerate of spotty, localized outcrop distribution containing abundant igneous pebbles, cobbles, and boulders. The middle phase, slightly fossiliferous tuffs, limestones, and sandstones, is also localized in extent. The upper phase is a widely distributed series of tuffs, tuff-breccias, and sandstones, with occasional sandy or fairly pure limestone beds. It is moderately strong throughout. Of the total sea-level canal excavation, 13.5% would be through its members.

Gatun Formation.—This formation is composed essentially of variably calcareous or argillaceous fine-grained sandstones, tuffs, and a few conglomerate beds that were deposited in a shallow sea in middle Miocene time. Stratification and jointing are massive and tight. As encountered at the site for the Gatun Third Locks, it was easily excavated with a minimum of blasting, and the deep cuts made for this project in 1941 and 1942 show no sliding after 5 years. Rock of this formation would comprise 2% of the material to be removed in sea-level canal construction at Panama.

Atlantic and Pacific Mucks.—These deposits are the combined accumulation of stream-deposited and ocean-deposited fines that were laid down as a result of a general land submergence in late Pleistocene time that drowned the then-existing stream-carved topography. Soft clays, silts, sands, organic swamp deposits (mostly decayed leaves and wood), and layers of sea shells are found intimately intermixed and to highly variable depths. Near Gatun Dam, muck-buried channels locally extend to below El. − 200. The mucks have very high natural water contents and are of low shear strength. They can be removed economically by hydraulic dredging methods.

Igneous, Intrusive, and Flow Rocks.—All formations of the Canal Zone older than the Gatun (Middle Miocene) have been intruded by volcanic dikes, sills, or plugs, and have interlayered flow rocks. These are mostly basalt, but andesites, rhyolites, and dacites are occasionally present. Rocks of this group in their unweathered occurrences are hard crystalline types and are characterized by a high shear resistance, but their other properties are extremely variable. They are found in greatest development in the central and Pacific regions. About 8.5% of the total volume of sea-level excavation would consist of igneous rocks.

Soils and Weathered Rock.—Depth and character of the soil and weathered rock cover are much diversified and are usually related to the type of underlying rock and topography. In many instances, weathering is known to extend to depths of 50 ft or more, but the average depth is in the order of from 20 ft to 30 ft. Clay soils predominate throughout but are usually rather stiff and only moderately plastic. This material and the mucks previously described represent together 24.5% of the total volume of materials requiring removal for new canal construction.

Geologic Structure.—Geologic features cut by the center line of the proposed new canal alinement are shown in profile in Fig. 62. Particular geologic study has been devoted to a number of basalt-capped or agglomerate-capped hills in the section between the Continental Divide and Pedro Miguel. The structural relationships of these hill-forming, hard basalts and agglomerates to adjacent or underlying softer materials of relatively low shear resistance, as typified by the weak Cucaracha beds, are of great importance. Certain hills have been determined to be deep-seated, stable masses of agglomerate or basalt that present no unusual problem in the design of excavation slopes for a canal, but others involve basalt or agglomerate, as either a cap or an overhang, resting on the Cucaracha formation. Experience in the construction of the existing canal has demonstrated that severe slides may result in cases where a deep cut is opened in a competent mass underlain by weak material if the contact between the two materials is above the bottom of the cut or within limited distances below. The potential slide hazard of this condition can be eliminated only by special slope-design treatment involving a removal of at least a part, if not all, of the overlying hard rock mass above the line defining the stable slope of the weaker formation.

PANAMA CANAL SLIDES

Types of Slides.—The slides of the Panama Canal were divided by Mr. MacDonald into four types. His description of these follows:

> "(1) Deep-deformation Slides. These began where the canal channel was cut deep into weak, somewhat plastic rocks. The weight of the high steep banks caused the rocks to deform very slowly and eventually to shear more than a score of yards below the bottom of the excavation. (2) Structural Slides. These took place where sheared zones, joints, bedding planes, or

> other structural weaknesses dipped fairly steeply toward the excavation. (3) Mudflow Slides. These formed where mud or moist clay slipped off or flowed down an inclined surface. (4) Combination Slides. A combination of any of the above types."

The deep-deformation slides were by far the largest and most troublesome. They practically all originated in the Cucaracha formation and, in the area of principal occurrence, are represented by the two slides of greatest magnitude, the East and West Culebra. Slides of this kind began to develop when the cut reached a depth of about 100 ft and increased in size and frequency as the excavation was deepened. The next largest slide was the Cucaracha slide, of the mudflow type. The structural and combination slides were not nearly as large or as troublesome as those of the deep-deformation type. The brief slide history which follows is confined to the East and West Culebra slides and the Cucaracha slide, because they were not only the largest, but also the only slides that actually blocked the channel. The locations of these three slides are shown in Fig. 61.

Slide History.—Shortly after the start of excavation by the first French canal company in 1884, the Cucaracha slide became active. It appeared at first to involve only the surface materials—soil and highly weathered rock from 20 ft to 30 ft thick—and took the form of mudflows down the steep slopes of the cut. This slide was intermittently active between 1885 and 1889, while the first French company was in operation. From 1889, when this company failed, until 1905, the slide was small or inactive, there being practically no excavation in the Cucaracha area by the second French company.

The first French company attempted to reduce the water in the slide material by surface drainage and tunneling. Surface drainage was reasonably successful in diverting the water, but neither method was successful in preventing the reoccurrence of slides.

American excavation for the Panama Canal began in 1905. The Board of Consulting Engineers established canal slopes approximating 3 vertical on 2 horizontal for all cuts in rock through the region of the Continental Divide.

The Cucaracha slide resumed activity in January, 1907, and in October of that year a large mass flowed into the bottom of the cut and caused extensive damage. The slide involved some 500,000 cu yd of material and for 2 weeks moved at the rate of 14 ft per day.

Another slide developed in January, 1907, on the east bank of the canal opposite the village of Culebra, and in October a crack developed 50 ft or more back of the top of the west bank near Culebra. This cracking was followed by bulging of the bottom of the cut. These movements marked the beginning of the East and West Culebra slides. The activity of the slides increased in 1911, and large quantities of material moved into the cut during the years from 1911 to 1913. During this period, excavation operations were delayed as railroad tracks and shovels were overturned. The disruptive effect of these slides is illustrated in Fig. 63.

In 1911 a project was undertaken to stabilize the banks by terracing to flatten the excavation slopes. This work was stopped in December, 1913, when it appeared that the Culebra slides were finally stable.

The Cucaracha slide started anew in January, 1913, and by February it was estimated that from 2,000,000 cu yd to 3,000,000 cu yd of clay and rock were in motion. By October, 1913, the channel was completely obstructed for 90 ft near Gold Hill, and partly blocked for 100 ft south. For the remainder of 1913 and during the first half of 1914, the slide continued to move while failed material was removed from the toe. By August 15, 1914, a channel 150 ft wide and 35 ft deep had been excavated through the Cucaracha slide, and the canal was opened to commerce.

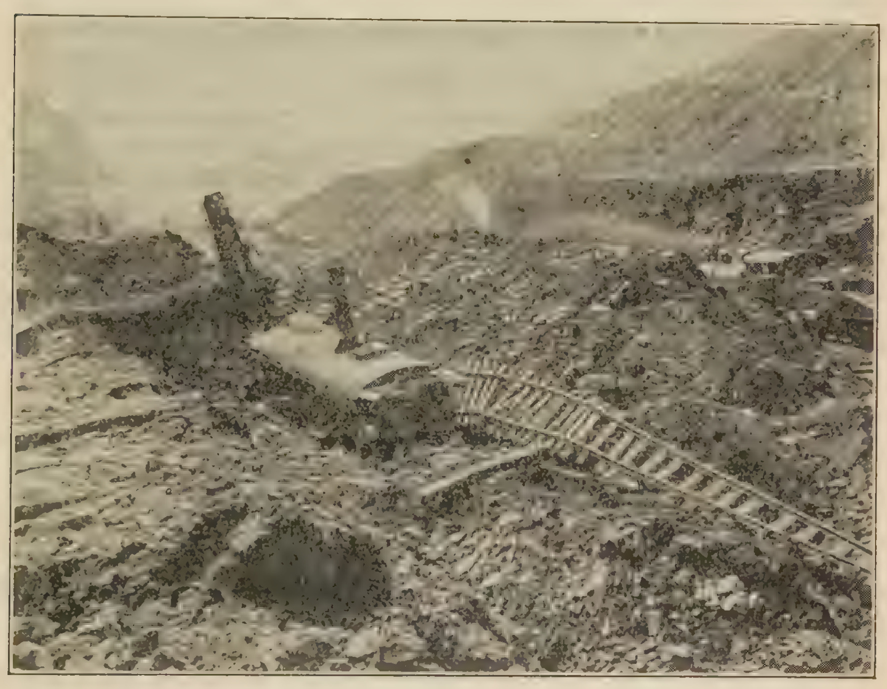


Fig. 63.—East Culebra Slide Showing Upheaved Material Between Stations 1746 and 1758, Facing South, February 6, 1913

Two months later, on October 14–15, a section of the east bank of the cut, north of Gold Hill, extending over 2,000 ft along the face of the cut and about 1,000 ft back from the center line of the channel, settled almost vertically, and about 725,000 cu yd of rock and earth were squeezed out or heaved up into the channel prism. The channel had 45 ft of water when the movement started. An hour later the bottom of the channel had been forced upward in some places to within 9 in. of the water surface. The dredges were able to clear the obstructions and keep the channel open until August 7, 1915, when slides closed the canal for 4 days. The canal was again closed to shipping for 6 days beginning September 4, 1915.

On September 15, 1915, a section of the face of Zion Hill (directly opposite the East Culebra slide) broke away and settled down. Both the East Culebra and West Culebra slides then began to move rapidly toward the channel. An island first appeared near the middle of the canal and later formed a complete barrier, 250 ft wide at the water surface, rising 65 ft above the water level. The canal remained closed to traffic for 7 months until April 15, 1916.

The Cucaracha slide again became active on August 30, 1916, and closed the canal to traffic for 8 days. The East Culebra slide started moving again in January, 1917, and closed the canal to traffic for 2 days. No further closures of the canal were experienced until March 20, 1920, when the Cucaracha slide again broke loose to close the canal for 4 days. In 1931 the canal was closed to traffic for 2 days while a small slide in the East Culebra area was removed. This was the last time that the canal was closed because of slides.

Design of Slopes

Cucaracha Formation.—All the worst slides of the original canal construction involved the weak, slickensided clay shale of the Cucaracha formation. Since the alinement for a new sea-level canal is through an area where the Cucaracha formation is present, the design of slopes in this material is of the greatest concern.

The Cucaracha slope-design problem was studied intensively during the Third Locks investigations, since an important part of the excavation for the new Pedro Miguel locks would have been in this material. The studies included laboratory and field tests of the undisturbed clay shales, analyses of the slides along the existing canal, and the analysis of an existing stable slope in the Cucaracha formation.

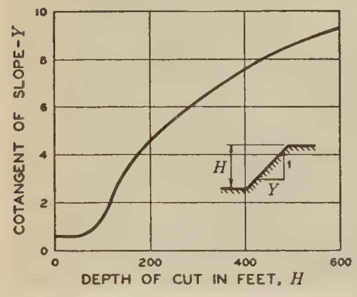


Fig. 64.—Excavation Slope Curve, Cucaracha Formation

Difficulties were encountered in obtaining suitable laboratory samples from the initial core-drilling operations, as the cores recovered from the weaker horizons were often badly broken. The subsequent use of a heavy mud slurry in place of drill water reduced caving and increased the percentage of recovery of undisturbed samples through the more critical and weak horizons.

Many of the samples that were recovered broke apart on the slickensided surfaces when being prepared for testing. As a result of these difficulties, the weakest materials could not be tested, and the average strengths determined were regarded as high.

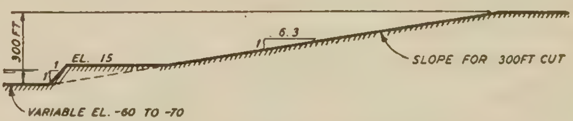


Fig. 65.—Typical Slope Based on Slope Curve in Fig. 64

Since the strength of the Cucaracha formation could not be determined reliably by laboratory tests, the slope design standards for Cucaracha (Figs. 64 and 65) were based on the strength of the material derived from analytical studies of a stable bank cut in Cucaracha, combined with the results of a simple laboratory test to measure the friction that might be expected along the slickensides in the material.

The tests to determine the friction were made in a standard direct-shear machine on specimens cut from solid cores of sound clay shale, polished to simulate slickensides, and brought into contact under water. The minimum angle of friction determined by three series of such tests on three different samples, under various conditions of normal load, was 10° which compares with a value of 13.5° obtained for the angle of internal friction from laboratory compression tests.

The stable bank referred to previously is a cut about 200 ft deep in the Cucaracha formation on the west bank of the canal, just south of Zion Hill, which has an average slope of 1 vertical on 2.7 horizontal. This bank has never been disturbed by slides but is composed of material similar to the Cucaracha material that would be encountered in the excavation of a sea-level canal. Computations were made to establish the cohesion needed for stability of the bank along the most critical sliding arc, assuming a factor of safety of 1.0 and a friction angle of 10°, as found from the laboratory tests. In this way a value for the cohesion of the formation as a whole was obtained which provides a reasonably sound basis for analysis of other cuts in similar material, since the effects of weak and strong bedded units, slickensides, joints, fractures, and gouge areas are automatically accounted for.

In the analyses, the ground-water table was considered to have been at the level of a bed of material known to provide good drainage, and therefore the effect of sudden drawdown, applied as a simple approximation of the forces of steady seepage, was limited to the mass below this bed. The analyses for a factor of safety of unity resulted in a value of 16 lb per sq in. (2,300 lb per sq ft) for the cohesion of the Cucaracha formation. This value was checked by an analysis of the first known shear slide that involved the Cucaracha formation—that of October, 1907—when Gaillard Cut was about 100 ft deep and presumably was being excavated to a slope of 3 vertical on 2 horizontal. The value has also been checked by analytical studies of the East Culebra and West Culebra slides.

The adopted strength values, cohesion equal to 16 lb per sq in., friction angle equal to 10°, and a factor of safety of 1.3, were used in developing the slope curve shown in Fig. 64. Sudden drawdown over the entire height of cut was assumed for the Cucaracha slopes. This assumption simplified the analyses and yet was somewhat conservative with regard to steady seepage out of the bank, a condition which might prevail after excavation. The part of this curve for cuts deeper than 200 ft was developed from data published by D. W. Taylor.[45] The part of the curve for cuts less than 200 ft deep is empirical and is based on recommendations made by Mr. MacDonald on the basis of his broad experience with the Cucaracha formation. Slopes for cuts less than 200 ft deep can justifiably be flatter than theoretically necessary, because the weakening effect of from 20 ft to 40 ft of weathered rock and residual clay overburden may be of some importance in shallower cuts.

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[45] "Stability of Earth Slopes," by D. W. Taylor, *Journal*, Boston Soc. of Civ. Engrs., July, 1937, p. 197.

Atlantic Muck.—The extreme instability of the Atlantic muck was first observed in the early 1900's during the excavation for the north approach walls of the existing Gatun Locks. The muck was excavated to El. −55 on slopes of 1 vertical on 5 horizontal; but, before the dredging was completed, the material had slid in some places making the slopes as flat as 1 vertical on 13 horizontal. After the excavation was dewatered, when the pile foundation for the approach wall was about half completed, the east bank gave way and covered the greater part of the foundation with mud to a depth of from 6 ft to 18 ft. The final slope was 1 vertical on 20 horizontal.

In the design of the Third Locks, a similar slope problem was encountered in the excavation of an extensive muck area for the construction of the approach walls of the new Gatun Locks. Unconfined and triaxial compression tests, supplemented by direct-shear tests, were made on representative undisturbed samples of the material. The first tests on the muck were run quickly; the entire time of the test, after consolidation of the sample to overburden pressure had been completed, was only from 10 min to 20 min. It was later found that lower results were obtained if the tests were run at a slower loading rate. A critical loading rate of 1.6 lb per sq in. per 15 min was found to give the lowest strengths and was adopted for all subsequent tests. The duration of tests for that loading rate varied from 60 min to 300 min. The results of these tests showed conclusively that, independently of the time of loading, the strength of the muck depends on the degree of consolidation of the material at the time of shearing. Thus, within any muck deposit, the strength of the muck is greater at increasing depths, since the consolidating pressures increase with depth. Fig. 66 shows data from the final series of tests, made at the critical loading rate, which indicate the strength characteristics of the muck. (Each circle in Fig. 66 represents one triaxial compression test on a sample consolidated under pressures equal to or greater than the overburden loads and then loaded at the rate of 1.6 lb per sq in. per 15 min to failure.) In the Third Locks design studies for slopes in Atlantic muck to be cut by dredging and left permanently submerged, it was found that stability was influenced more by the height of the bank above permanent water level than by the depth of cut below water level—because the saturated weight of the muck above water level is about 90 lb per cu ft, whereas the weight of the submerged material is

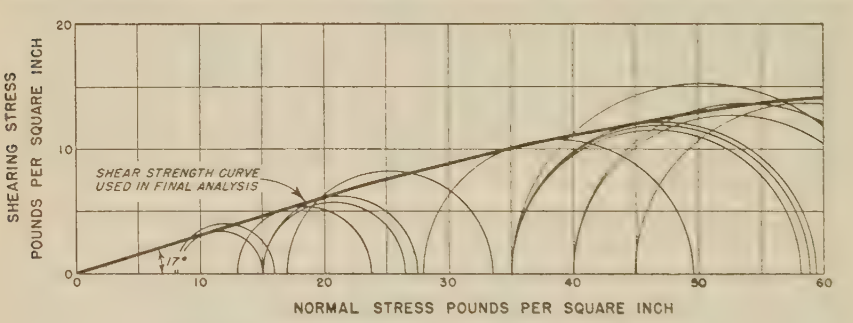


Fig. 66.—Shear Strength Curve for Atlantic Muck

only about 28 lb per cu ft. The proper slope for a permanently submerged cut in muck, therefore, is dictated largely by the height of the top of the cut above water. The slope adopted for use in the Gatun Locks area, where the top of muck was less than El. 10 and the total depth of cut was greater than 30 ft, was 1 vertical on 2.5 horizontal. Such slopes were actually cut during the period from 1941 to 1945 and have remained stable since that time.

A large part of the sea-level canal between the Atlantic entrance and Barro Colorado Island would require cuts in Atlantic muck which would be made by dredges operating on the present level of Gatun Lake. After lowering Gatun Lake to sea level, the top of the muck banks would be exposed above water level, in some areas as high as El. 35. Undisturbed samples of the muck underlying Gatun Lake were tested and found to have strength characteristics approximating those shown in Fig. 66.

One cross section for sea-level canal excavation through muck in the Gatun Lake area, Fig. 67, was designed to meet all conditions. This slope was analyzed by the Swedish circular arc method, using the shear strength curve of Fig. 66, and the minimum factor of safety obtained was 1.3. The slope of 1 vertical on 2.5 horizontal below water level results from the desire to maintain as uniform a channel cross section as possible, in keeping with the cross section through the rock-cut areas, and from the successful experience with this particular slope in the Gatun north approach channel. The uniform slope of 1 vertical on 10 horizontal above water level would simplify the dredging and is amply safe for top-of-slope elevations as high as El. 35.

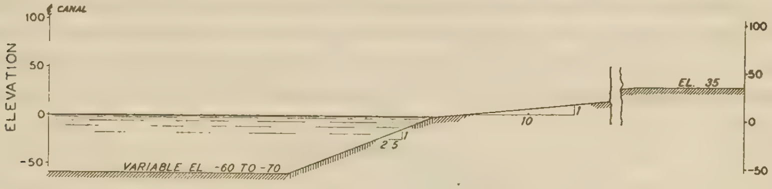


Fig. 67.—Excavation Slopes in Muck

Other Rocks.—Slope-design standards for materials other than the Cucaracha and Atlantic muck formations were developed empirically. These can be classified into three groups: (1) Soft rocks—the Culebra, La Boca, and Las Cascadas formations; (2) medium rocks—the Gatun, Bohio, and Caimito formations; and (3) hard rocks—basalt, the Pedro Miguel agglomerate, and Bas Obispo formations. Slides in these materials, where encountered in the existing canal, were uncommon and usually resulted from structural causes.

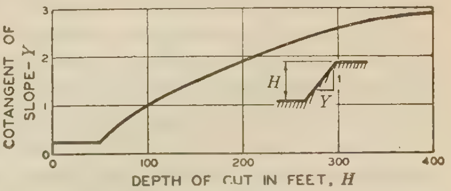


Fig. 68.—Excavation Slope Curve, Culebra Formation

The slope curve for the first and weakest of the three groups is shown in Fig. 68. This curve is based on recommendations made by Mr. MacDonald and on experience with previous excavation in the Culebra formation. A typical slope based on the curve in Fig. 68 is shown in Fig. 69.

Fig. 70 shows the standards for excavation slopes in the other two groups of materials, medium and hard rocks. The two standards differ only in the vertical interval between berms. The channel slope is 3 vertical on 2 horizontal up to El. 15. Above a 45-ft berm at this elevation the slopes are 12 vertical on 1 horizontal with 25-ft berms every 50 ft in elevation for the medium rocks, and every 100 ft for the hard rocks.

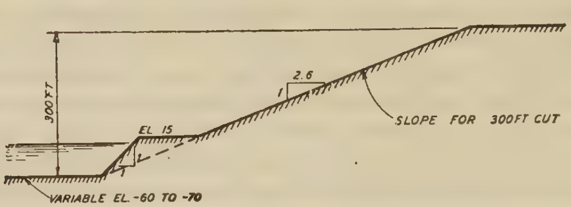
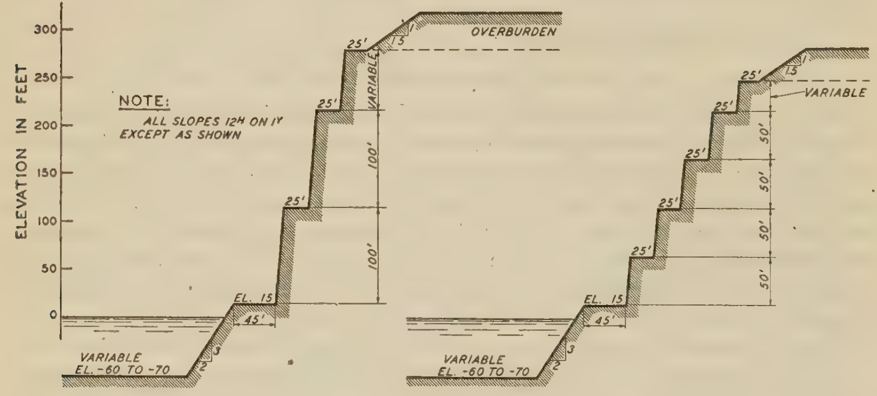

300 FT
2.6
1
EL 15
SLOPE FOR 300FT CUT
VARIABLE EL -60 TO -70

Fig. 69.—Typical Slope Based on Slope Curve in Fig. 68

Considering stability only, it is possible that, in any of the medium or hard rocks, vertical cuts as high as 500 ft would stand without danger of major failures. However, rockfalls are always a possibility and would be particularly dangerous during construction. A 45-ft berm at El. 15 is provided to prevent falling rock from reaching the channel. The 25-ft berms above El. 15 would provide a reasonable degree of safety during construction.


ELEVATION IN FEET
300
250
200
150
100
50
0
NOTE:
ALL SLOPES 12H ON 1V
EXCEPT AS SHOWN
OVERBURDEN
25'
15
1
VARIABLE
100'
100'
EL. 15
45'
3
2
VARIABLE
EL. -60 TO -70
50'
50'
50'
50'

(*a*) BASALT, AGGLOMERATE, AND BAS OBISPO (HARD ROCKS)

(*b*) GATUN, BOHIO, AND CAIMITO (MEDIUM ROCKS)

Fig. 70.—Excavation Slopes for Medium and Hard Rocks

Channel Slopes.—The channel cross section should be as nearly uniform as possible to provide the best navigation conditions. Rock slopes for a sea-level canal designed only for stability would vary from 12 vertical on 1 horizontal for hard rocks to as flat as 1 vertical on 9.4 horizontal for the Cucaracha formation. To reduce this extreme variation for the benefit of navigation, the slopes below El. 15 were modified as follows: In hard rocks, the channel slopes would be 3 vertical on 2 horizontal; in Cucaracha and other soft rocks, a channel slope of 1 vertical on 1 horizontal would be used as indicated in Fig. 65 and 69.

Except for this modification, the over-all slope in these soft materials would be designed by the slope curves.

DYNAMIC LOADS

Large Explosions.—The effects of large bomb explosions in the vicinity of excavations were not taken into account in the development of the slope-design standards described previously. Investigations were made, however, of the effect that large explosions in air and on or under the ground might have upon the stability of slopes. Research was conducted by Harvard University to investigate the effects of explosive forces on the strengths of soils and rocks. This research is reported in the eighth Symposuim paper.

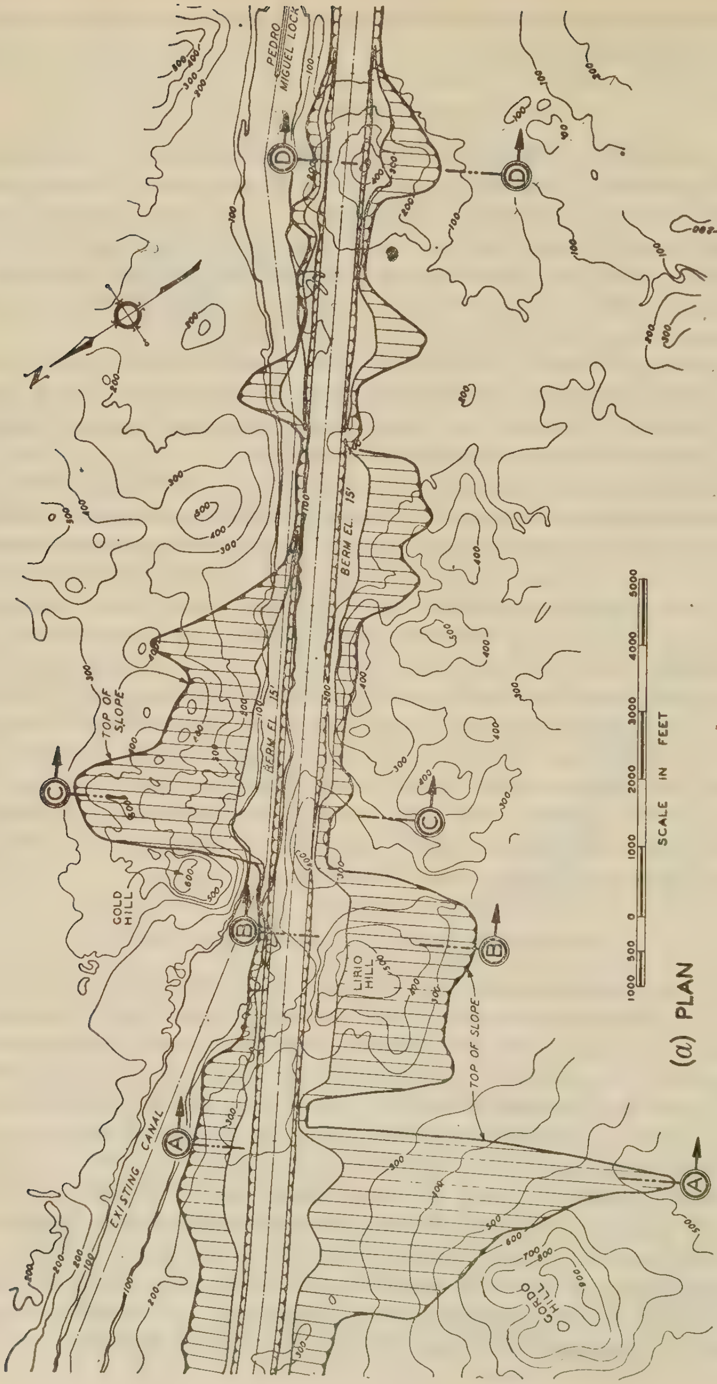
The studies indicate that sliding failures caused by dynamic loads on a statically safe slope would probably not result in closure of the canal. From present evidence, flattening the slopes beyond the requirements for static stability, therefore, is not believed to be necessary.

Earthquake Hazards.—According to information furnished by L. Don Leet, Professor of Geology at Harvard University, landslides do not occur at distances greater than 50 miles from the epicenter of the largest earthquakes known to have occurred, or at more than 20 miles from one as large as that in Japan in 1923. The computed accelerations which produced slides range from around 0.5 g to 0.7 g. The earthquake history of the Isthmian region around the Canal Zone indicates that no serious danger exists of an earthquake large enough and near enough to place the Canal Zone within such a landslide radius. The closest major active zone on record has been off the Los Santos peninsula, more than 100 miles from Balboa. Absolute prediction, either positive or negative, of the time and place of occurrence of earthquakes cannot be made. It is concluded from the study, however, that earthquake-induced landslides in cuts for the Panama Canal would be improbable; therefore, no allowance for earthquake forces has been made in the design of canal slopes.

APPLICATION OF SLOPE-DESIGN STANDARDS

The design standards described previously for excavation slopes under conditions of static loading are sufficiently detailed for quantity estimating and construction planning purposes. The application of these standards to the 4-mile section of a sea-level canal through the Continental Divide north of Pedro Miguel is illustrated in Fig. 71. Approximately 30% of the total required excavation is concentrated in this area. This section of the canal would also include the deepest cuts.

In the development of the standards illustrated by the slopes shown in Fig. 71, minimum strength values have been used for each of the geologic formations encountered. Before actual construction of a sea-level canal, more detailed geologic data would be available, and the slopes would be modified to take advantage of stronger phases of some formations. It is believed that any modification would result in a reduction of excavation quantities.


PEDRO MIGUEL LOCK
EXISTING CANAL
BERM EL. 15'
BERM EL. 15'
TOP OF SLOPE
TOP OF SLOPE
GOLD HILL
LIRIO HILL
GORDO HILL
A
A
B
B
C
C
D
D
N
1000 500 0 1000 2000 3000 4000 5000
SCALE IN FEET
100
200
300
400
500
600
700
800
900

(a) PLAN

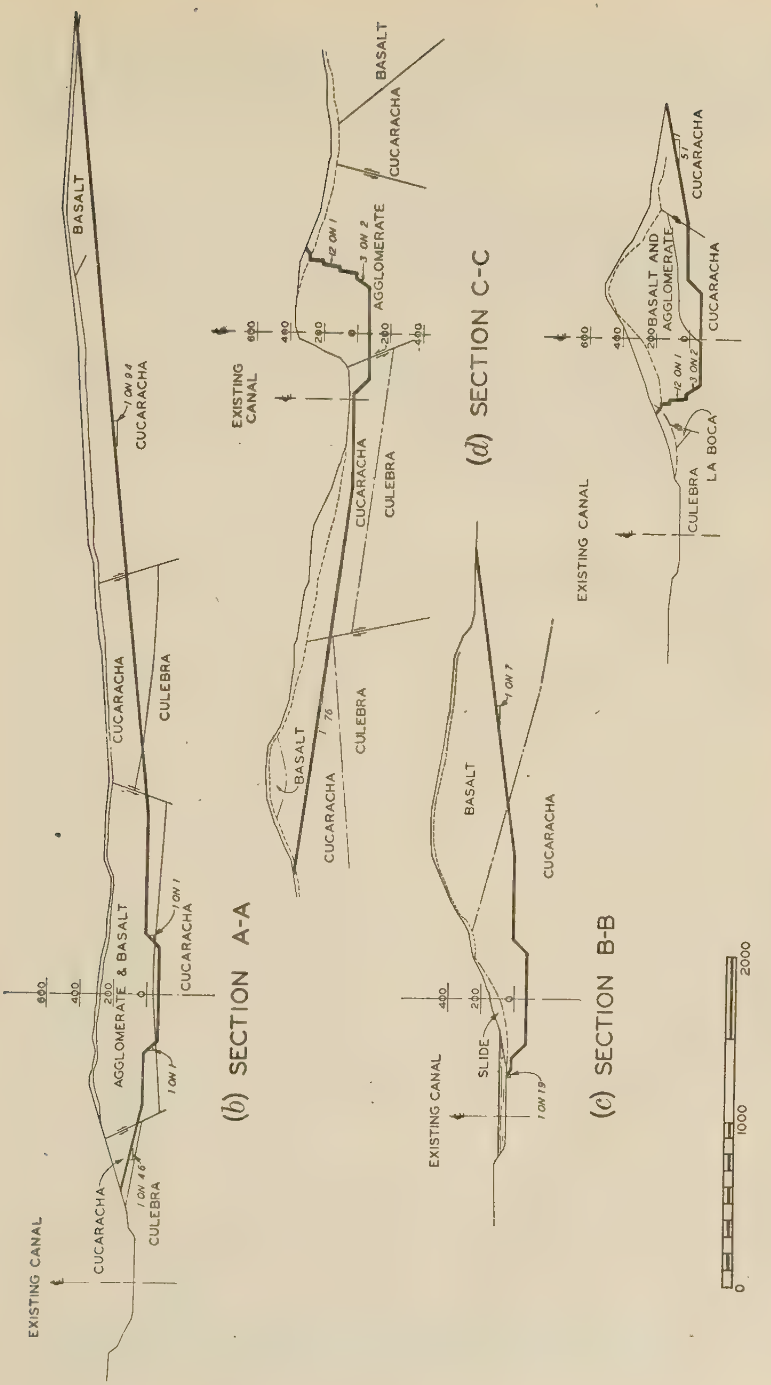

EXISTING CANAL
CUCARACHA
CULEBRA
1 ON 46
AGGLOMERATE & BASALT
1 ON 1
600
400
200
0
CUCARACHA
1 ON 1
CUCARACHA
CULEBRA
CUCARACHA
1 ON 94
BASALT
(b) SECTION A-A
EXISTING CANAL
600
400
200
0
200
400
BASALT
CUCARACHA
1 76
CULEBRA
CUCARACHA
CULEBRA
12 ON 1
3 ON 2
AGGLOMERATE
CUCARACHA
BASALT
(d) SECTION C-C
EXISTING CANAL
SLIDE
1 ON 19
400
200
0
BASALT
CUCARACHA
1 ON 7
(c) SECTION B-B
EXISTING CANAL
CULEBRA
LA BOCA
12 ON 1
3 ON 2
600
400
200
0
BASALT AND AGGLOMERATE
CUCARACHA
CUCARACHA
51
(e) SECTION D-D
0
1000
2000

FIG. 71.—EXCAVATION LIMITS IN DEEP CUT SECTION

CONCLUSIONS

Although large slides occurred during and following construction of the present Panama Canal, major slides would be prevented in the proposed Panama sea-level canal by excavating the channel banks initially to the proper slopes.

The analytical methods and the testing employed for the design of safe slopes conform with established methods that had not been developed when the present canal was constructed. Studies of the slides that occurred during canal construction were of value in the design of stable slopes for the Cucaracha formation.

The design standards presented in this paper are considered to produce slopes that would be stable under static loading. Any slides that might be initiated by dynamic forces from earthquakes or large explosions would not be expected to cause closure of the canal.

STRENGTH OF SOILS UNDER DYNAMIC LOADS

By A. Casagrande,[46] M. ASCE, and W. L. Shannon,[47]
Assoc. M. ASCE

Synopsis

This paper describes apparatus developed and results of tests performed to investigate the strength characteristics of soils and soft rocks under dynamic loads.

Introduction

In connection with studies of the stability of slopes under the effects of bombing, laboratory investigations on the strength of soils and soft rocks under dynamic loading are being conducted at Harvard University for The Panama Canal.

This investigation is expected also to benefit other engineering problems in which soil is subjected to dynamic loading, such as the effects of earthquakes on dams and their foundations, or the effects of transient loading by fast moving traffic on airfield and highway pavements and the underlying materials.

Conventional strength tests on soils are either unconfined compression, or triaxial compression, or direct-shear tests. Any of these tests can be performed with either a controlled stress or a controlled strain loading apparatus. Loading of the specimen in such tests is performed over a period of at least several minutes. Such tests will be referred to herein as static strength tests, to distinguish them from the dynamic tests described in this paper.

It has been recognized that the strength of soil increases as the rate of loading increases. For example, in connection with the design of the third locks for The Panama Canal, a series of triaxial compression tests was conducted to determine the strength of undisturbed, soft organic clay by producing failure within a range of from 1.7 min to more than 7 hours. These tests indicated that the strength at the fastest rate of loading was about 40% greater than that at the slowest rate. D. W. Taylor,[48] Assoc. M. ASCE, investigated the strength of a clay that was remolded at the liquid limit and then consolidated under 4.22 kg per sq cm. Failure was produced within the range of from 4 min to 8 days. In these tests the strength of specimens that were loaded to failure quickly was found to be about 25% greater than the strength of specimens that were loaded slowly.

Investigations have been performed on metals to determine their strength at various rates of strain. One comprehensive series of tension tests was performed by M. J. Manjoine[49] on a mild steel within the range of from 1×10^{-6}

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[46] Prof. of Soil Mechanics and Foundation Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass.

[47] Research Associate in Soil Mechanics, Graduate School of Eng., Harvard Univ., Cambridge, Mass.

[48] "Progress Report on Triaxial Shear Research and Pressure Distribution Studies on Soils," U. S. Waterways Experiment Station, Vicksburg, Miss., April, 1947, p. 95.

[49] "Influence of Rate of Strain and Temperature on Yield Stresses of Mild Steel," by M. J. Manjoine, *Journal of Applied Mechanics*, December, 1944, p. A-211.

strain per sec to 1 × 10^{+3} strain per sec, which corresponds to a range of time necessary to reach the ultimate strength of approximately 2.3 days to 0.0002 sec. The ultimate strength of specimens tested in the shortest time was found to be about 50% greater than that of specimens tested in the slowest time.

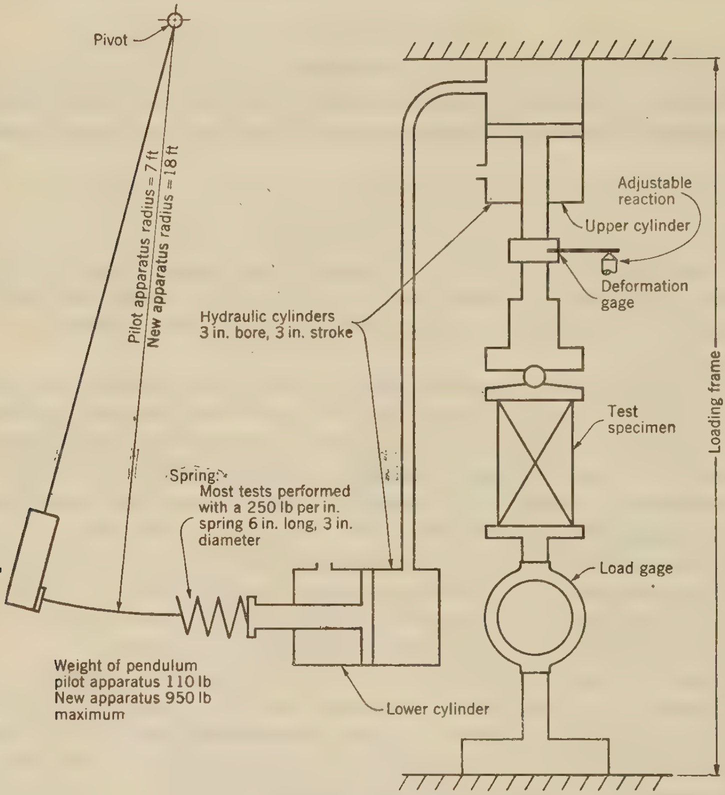

Pivot
Pilot apparatus radius = 7 ft
New apparatus radius = 18 ft
Hydraulic cylinders
3 in. bore, 3 in. stroke
Adjustable reaction
Upper cylinder
Deformation gage
Loading frame
Test specimen
Spring:
Most tests performed with a 250 lb per in. spring 6 in. long, 3 in. diameter
Load gage
Weight of pendulum
pilot apparatus 110 lb
New apparatus 950 lb maximum
Lower cylinder

FIG. 72.—PENDULUM LOADING APPARATUS

For fast transient tests on soils and soft rocks, it was necessary to develop apparatus for applying dynamic loads and for measuring and recording the loads applied and the resulting deformations of test specimens.

APPARATUS FOR APPLYING TRANSIENT LOADS

After a comprehensive review of dynamic testing apparatus developed for various purposes, the writers realized that none of these apparatus would be suited for this investigation, and they were obliged to develop new apparatus for applying transient loads. The type of loading desired was a transient load in which the test specimen is subjected to a rapid loading and unloading, simulating the effect of the first stress wave created by an explosion. As a criterion for the speed of load application, it was found convenient to define the time of loading as the difference in time between the beginning of test and the time at which the maximum compressive stress is reached. The value for the fastest time of loading for use in this investigation was determined in consultation with H. M. Westergaard, M. ASCE, and L. Don Leet, special consultants to The Panama Canal. The value thus decided on was 1/100 sec. The time for the slowest loading was determined by the desire to overlap with the fastest loading time used in static strength tests.

LEGEND

1. Loading frame (see Fig. 74)
2. Lower cylinder
3. Spring
4. Pendulum, weighted to 700 lb
5. Strain indicators
6. Power supply
7. Oscillator
8. Oscillograph

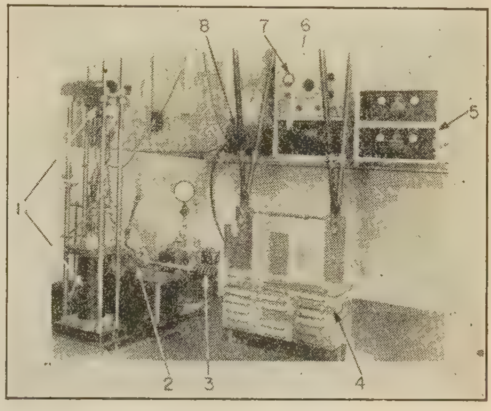


Fig. 73.—General View of Pendulum Loading Apparatus, with Recording Instruments in the Background

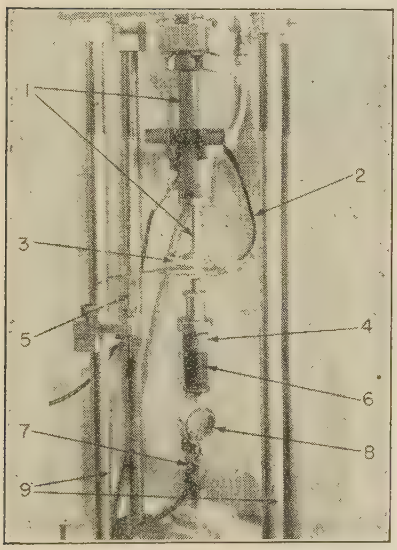


Fig. 74.—Close-Up of Loading Frame of the Pendulum Apparatus, Showing the Arrangement for the Unconfined Transient Compression Test

LEGEND

1. Upper hydraulic cylinder and piston
2. Shielded cable to deformation gage
3. Deformation gage
4. Tilting cap
5. Reaction for deformation gage
6. Unconfined compression specimen
7. Shielded cable to load gage
8. Load gage
9. Tie rods

Three different types of apparatus for applying transient loads in triaxial compression and unconfined compression tests were developed simultaneously, since it was not certain what method of load application would be best suited to this investigation. It finally was found that all three types were needed because they supplemented one another in the range of time of loading for which each type was best suited.

Pendulum Loading Apparatus.—Figs. 72 and 73 show a diagram and a photograph of the pendulum loading apparatus which utilizes the energy of a pendulum that is released from a selected height and strikes a spring connected to the piston rod of a 3-in. bore cylinder. This lower cylinder, in turn, is connected hydraulically to an upper cylinder of the same bore, which is mounted within a loading frame. Fig. 74 is a detailed view of the loading frame of the pendulum apparatus.

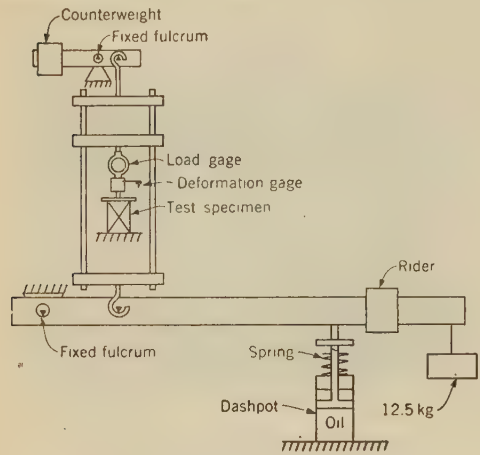

Counterweight
Fixed fulcrum
Load gage
Deformation gage
Test specimen
Rider
Fixed fulcrum
Spring
Dashpot
Oil
12.5 kg

Fig. 75.—Falling Beam Loading Apparatus

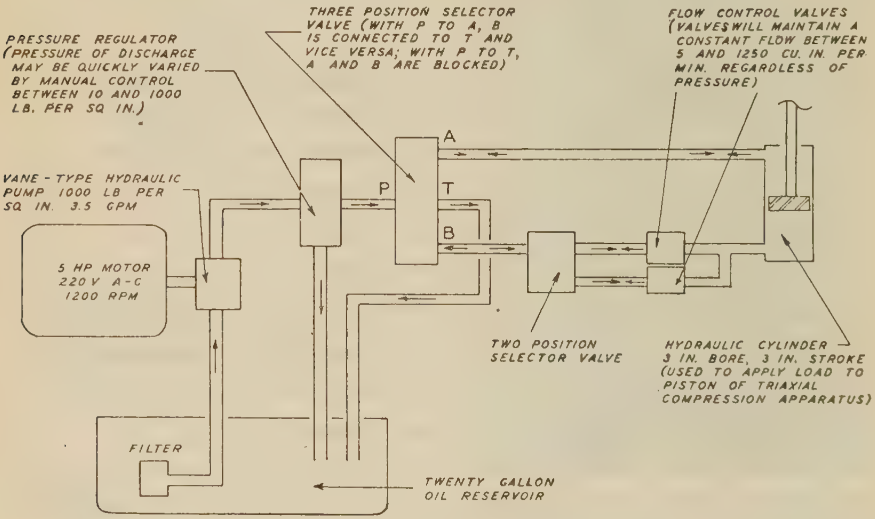
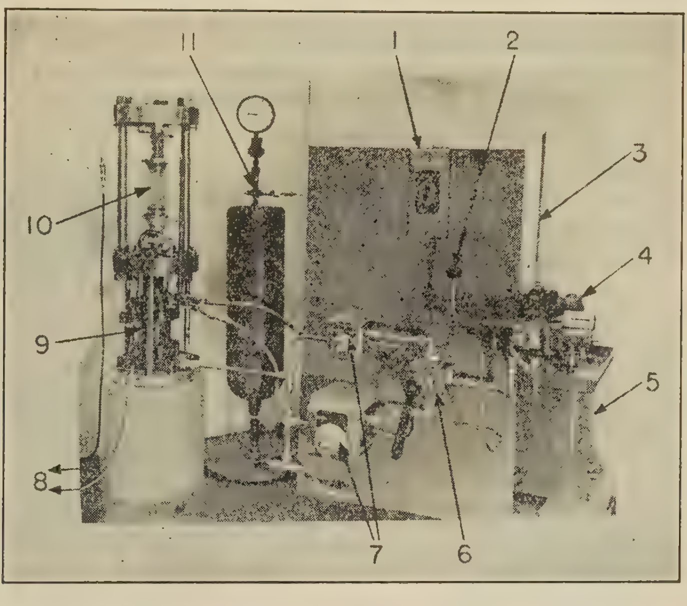

THREE POSITION SELECTOR VALVE (WITH P TO A, B IS CONNECTED TO T AND VICE VERSA; WITH P TO T, A AND B ARE BLOCKED)
FLOW CONTROL VALVES (VALVES WILL MAINTAIN A CONSTANT FLOW BETWEEN 5 AND 1250 CU. IN. PER MIN. REGARDLESS OF PRESSURE)
PRESSURE REGULATOR (PRESSURE OF DISCHARGE MAY BE QUICKLY VARIED BY MANUAL CONTROL BETWEEN 10 AND 1000 LB. PER SQ IN.)
A
P
T
B
VANE - TYPE HYDRAULIC PUMP 1000 LB PER SQ IN. 3.5 GPM
5 HP MOTOR 220 V A-C 1200 RPM
TWO POSITION SELECTOR VALVE
HYDRAULIC CYLINDER 3 IN. BORE, 3 IN. STROKE (USED TO APPLY LOAD TO PISTON OF TRIAXIAL COMPRESSION APPARATUS)
FILTER
TWENTY GALLON OIL RESERVOIR

Fig. 76.—Diagram of Hydraulic Loading Apparatus

The time of loading for which this apparatus was found best suited ranges between 0.01 sec and 0.05 sec.

The Falling Beam Loading Apparatus.—The loading apparatus in Fig. 75 utilizes the unconfined compression test apparatus of the "universal soil loading machine"[50] for the application of a transient load. The apparatus consists essentially of a beam with a weight and rider, a dashpot to control the velocity of the fall of the beam, and a yoke for transmitting the load from the beam to the specimen. A small beam mounted above the yoke counter-balances the weight of the beam. This apparatus was found to be suited for a relatively long time of loading, ranging from 0.5 sec to about 300 sec.



LEGEND

Point	Description
1	Starter switch for 5-hp electric motor which is behind panel
2	Three-position selector valve
3	Handle of pressure regulator
4	Vane-type hydraulic pump operated by electric motor
5	Oil reservoir
6	Two-position selector valve
7	Flow control valves
8	Cables from load and deformation gages (inside pressure chamber) leading to recording instruments
9	Hydraulic cylinder
10	Test specimen inside transparent pressure chamber
11	Air pressure reservoir

FIG. 77.—PHOTOGRAPH OF HYDRAULIC LOADING APPARATUS

The Hydraulic Loading Apparatus.—The loading apparatus in Figs. 76 and 77 consists of a constant volume vane-type hydraulic pump connected to a hydraulic cylinder through valves by which either the pressure in the cylinder or the volume of liquid delivered to the cylinder can be controlled.

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[50] "Recent Developments in Soil Testing Apparatus," by P. C. Rutledge, in "Contributions to Soil Mechanics 1925–1940," Boston Soc. of Civ. Engrs., Boston, Mass., 1940, pp. 243–250.

The peak load that can be produced with this apparatus is much greater than can be obtained by either the falling beam or the pendulum types of loading apparatus.

This apparatus is used for testing soft rocks from the Canal Zone with a time of loading between 0.05 sec and any desired slow loading.

APPARATUS FOR APPLYING STATIC LOADS

Two types of loading apparatus were used for determining the static compressive strength.

The Fairbanks Scale Loading Apparatus.—This type consists of a conventional 500-lb Fairbanks platform scale equipped with a loading yoke and a mechanical jack.[51] It is used extensively in performing soil tests with stress control.

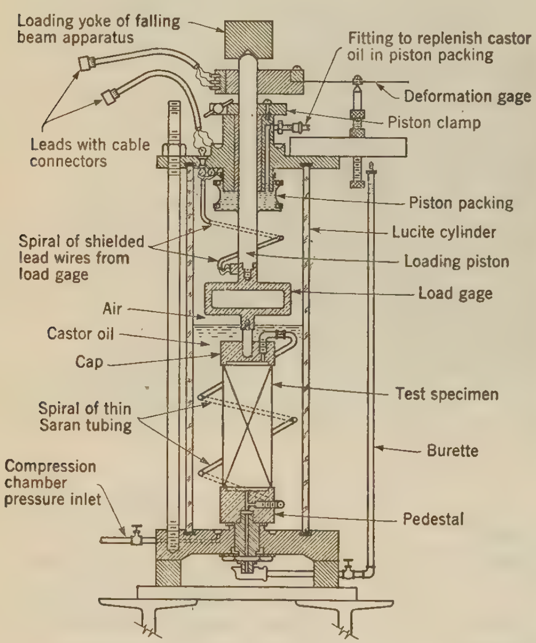
Loading yoke of falling beam apparatus
Fitting to replenish castor oil in piston packing
Deformation gage
Piston clamp
Leads with cable connectors
Piston packing
Lucite cylinder
Spiral of shielded lead wires from load gage
Loading piston
Load gage
Air
Castor oil
Cap
Test specimen
Spiral of thin Saran tubing
Burette
Compression chamber pressure inlet
Pedestal

FIG. 78.—DIAGRAM OF TRIAXIAL TRANSIENT COMPRESSION APPARATUS

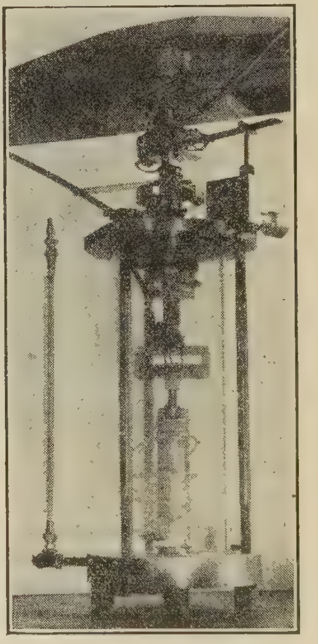

FIG. 79.—PHOTOGRAPH OF TRIAXIAL COMPRESSION APPARATUS

The Hydrostatic Loading Apparatus.—This type consists essentially of the two hydraulic cylinders of the pendulum apparatus and a motor-driven drum and lever with which the travel of both hydraulic pistons can be controlled at a constant, slow rate.

TRIAXIAL AND UNCONFINED COMPRESSION APPARATUS

All triaxial compression apparatus used in this investigation for transient load tests have been adapted from triaxial apparatus previously constructed at

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[51] "The Soil Mechanics Laboratory at Harvard University," by P. C. Rutledge *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., June, 1936, Vol. II, pp. 85–97, particularly Figs. 14 and 15.

Harvard University for static testing.[52] One of these is a vacuum type and three are compression types. The vacuum type of apparatus is limited to tests on dry, noncohesive soils under a minor principal stress less than one atmosphere. The compression-type apparatus illustrated in Figs. 78 and 79 is suitable for tests on either cohesive or noncohesive soils under a minor principal stress up to about 6 kg per sq cm.

Figs. 72, 73, and 74 show the pendulum apparatus assembled for unconfined compression tests; and Fig. 75 shows the falling beam apparatus similarly assembled.

APPARATUS FOR MEASURING AND RECORDING TRANSIENT LOADS AND DEFORMATIONS

A comprehensive study was made of instruments available for measuring and recording transient compressive forces and deformations. In addition, several months of intensive work were required to develop suitable load and deformation gages. To measure and record rapidly changing loads and deformations, metal-electric (SR-4) strain gages with companion strain indicators and oscillographs were found most suitable, in part because such equipment was readily available. A brief description of this instrumentation follows:

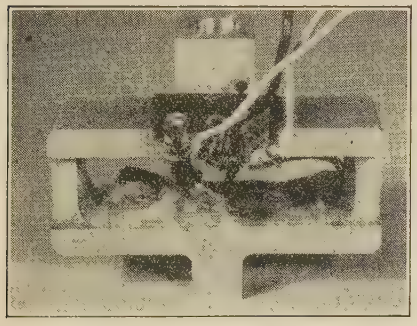

FIG. 80.—LOAD GAGE

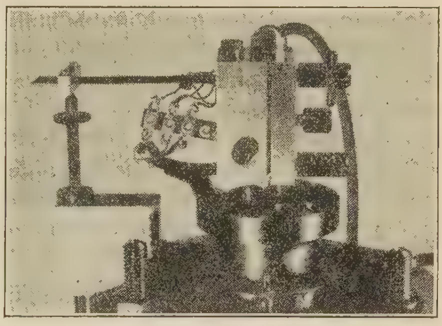

FIG. 81.—DEFORMATION GAGE

Load Gage.—For measuring load, a load gage of rectangular or cylindrical shape is used, with four metal-electric (SR-4) strain gages mounted on the inside face. (The irregular mass on the interior surfaces of the load gage in Fig. 80 is wax placed there for the protection of the SR-4 gages.) Load gages are also shown in Figs. 72, 74, 75, 78, and 79.

Deformation Gage.—For measuring deformation, a thin flexible steel spring cantilever is used (Fig. 81) with metal-electric (SR-4) strain gages mounted on the cantilever, the base of which is clamped to the loading piston. The tip of the cantilever reacts against an adjustable screw which is mounted on the head plate of the triaxial compression apparatus. Deformation gages are also shown in Figs. 72, 74, 75, 78, and 79.

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[52] "Progress Report on Triaxial Shear Research and Pressure Distribution Studies on Soils," U. S. Waterways Experiment Station, Vicksburg, Miss., April, 1947, pp. 130–154.

Recording Apparatus.—Equipment for amplifying and recording the signal produced by the SR-4 strain gages consists of two strain indicators,[53] an oscillator, a power supply, and oscillographs. The strain indicators contain two arms of a Wheatstone bridge and a carrier type amplifier. The other two arms of the Wheatstone bridge are the SR-4 strain gages on the load or deformation gages. This equipment can be seen in the background of Fig. 73.

MATERIALS TESTED AND TECHNIQUE OF TESTING

Static and transient compression tests have been performed to date on the following materials:

(*1*) *Manchester* (*N. H.*) *Sand.*—This is a clean, medium sand obtained by screening from a glacial-fluvial deposit and contains only the fraction between 0.42 mm and 0.21 mm. It consists principally of subangular quartz grains, and has a void ratio in the densest state of about 0.61 and in the loosest state of about 0.88.

(*2*) *Cambridge* (*Mass.*) *Clay.*—This is a medium soft, inorganic clay, with occasional thin silt partings, brittle in the undisturbed state and soft and sticky when remolded. Its natural water content ranges from 30% to 50%. For layers of this clay having a natural water content of from 40% to 50%, the liquid limit was found to range from 44 to 59 and the plastic limit from 21 to 27. For layers of this clay having a natural water content of from 30% to 40%, the liquid limit was found to range from 37 to 44 and the plastic limit from 20 to 23.

(*3*) *Boston* (*Mass.*) *Clay.*—This is similar, geologically and in its appearance, to the Cambridge clay. The samples tested had a natural water content between 32% and 36%, an average liquid limit of 42 and a plastic limit of 20.

(*4*) *Stockton* (*Calif.*) *Clay.*—This is a tough, brown clay, locally called adobe. The sample tested was obtained from the compacted fill on which the Stockton pavement traffic test[54] was conducted, and was about 90% saturated, with a natural water content of about 25%, a liquid limit of from 60 to 64, and a plastic limit of from 20 to 23.

(*5*) *Atlantic Muck* (*Canal Zone*).—This is an organic clay, having a natural water content ranging between 50% and 135%, a liquid limit of from 55 to 95, and a plastic limit of from 30 to 55. The samples tested were obtained in 3-in. diameter, thin-walled tubes from a boring into a natural deposit located adjacent to the Panama Canal.

(*6*) *Cucaracha Shale* (*Canal Zone*).—This is a slickensided clay-shale. The samples tested were obtained from core borings in the Cucaracha formation located in the vicinity of the Gaillard Cut, Panama Canal.

Details of Testing.—The Cambridge and Boston clays were tested both in unconfined compression and in triaxial compression, whereas the Stockton clay was tested in triaxial compression only. Cucaracha specimens were tested in triaxial compression and Atlantic muck in unconfined compression. Manchester sand was tested in the dense state, dry, in a vacuum-type triaxial compression apparatus.

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[53] "A Carrier Type Strain Indicator," by George W. Cook, *Report No. 565*, David Taylor Model Basin, U. S. Navy, Washington, D. C., November, 1946.

[54] "Flexible Pavement Test Section for 300,000-Lb. Airplanes, Stockton, California," by Ralph A. Freeman and O. J. Porter, *Proceedings*, Highway Research Board, National Research Council, Vol. 25, 1945, pp. 23–44.

Most unconfined compression test specimens of Cambridge clay were 6.3 cm square and about 16 cm high; all triaxial compression test specimens of clay were 3.56 cm in diameter and about 9 cm high. Unconfined compression test specimens of Atlantic muck were 2.5 cm square and about 6 cm high. Triaxial compression test specimens of Cucaracha shale were 5 cm in diameter and 12 cm high, and specimens of Manchester sand were 7.1 cm in diameter and about 18 cm high.

Two types of triaxial compression tests were performed, designated "quick" and "consolidated-quick." A quick triaxial compression test is one in which there is no preliminary consolidation and no drainage of pore water during the test. A consolidated-quick triaxial compression test is one in which the specimen is allowed to consolidate under a hydrostatic pressure, but, during the subsequent quick axial loading, there is practically no drainage of pore water. A detailed discussion of these types of tests appears elsewhere.[55]

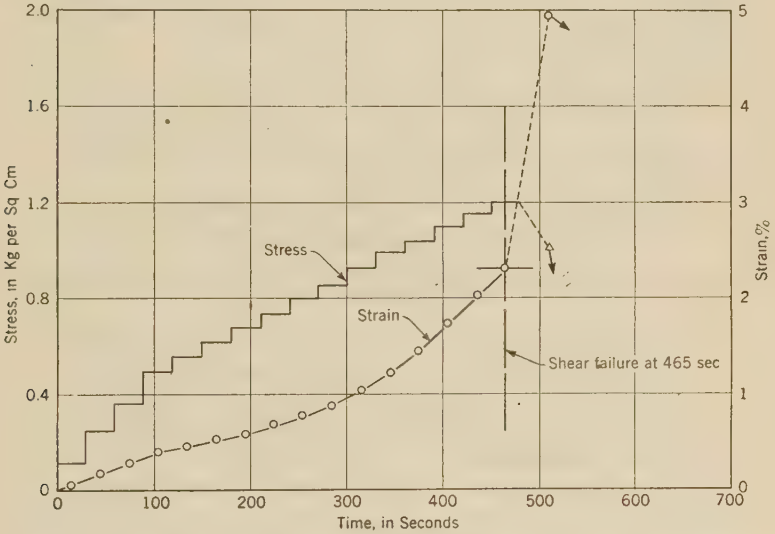

FIG. 82.—TIME CURVES FOR STRESS AND STRAIN; UNCONFINED STATIC COMPRESSION TEST ON CAMBRIDGE CLAY

The triaxial compression test specimens were surrounded by two rubber membranes, each averaging approximately 0.05 mm thick. Specimens of clay and Cucaracha shale for consolidated-quick triaxial compression tests were fully consolidated under a lateral pressure of either 3 kg per cm² or 6 kg per cm² before axial compression was started. Manchester sand specimens were tested under a lateral pressure of 0.3 kg per cm² and 0.9 kg per cm².

Two procedures for performing transient compression tests were used, which are described below:

Controlled Impulse Method.—The specimen is subjected to a controlled impulse. The peak load exerted on the test specimen and its deformation depend on the stress-deformation and strength characteristics of the specimen. Typical stress-time and strain-time diagrams for an unconfined transient compression test on clay, using the controlled impulse method, are shown in Fig. 82.

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[55] "Progress Report on Triaxial Shear Research and Pressure Distribution Studies on Soils," U. S. Waterways Experiment Station, Vicksburg, Miss., April. 1947.

Controlled Strain Method.—The specimen is subjected to a rate of strain which is maintained approximately constant from the beginning of the test to the desired maximum strain. The peak load exerted on the specimen and the time of loading depend on the stress-deformation and strength characteristics of the specimen.

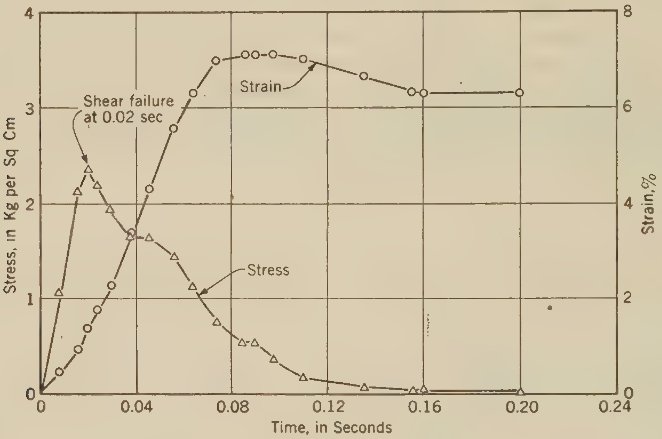


Fig. 83.—Time Curves for Stress and Strain; Unconfined Transient Compression Test on Cambridge Clay

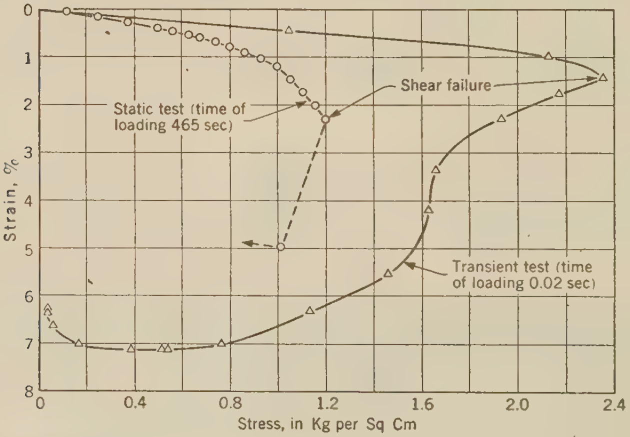


Fig. 84.—Stress-Strain Curves; Unconfined Compression Tests on Cambridge Clay

Results of Tests

Representative results of a transient and a static unconfined compression test on Cambridge clay are shown in Figs. 82, 83, and 84. Figs. 82 and 83 are time curves for stress and strain and Fig. 84 shows the stress-strain curves.

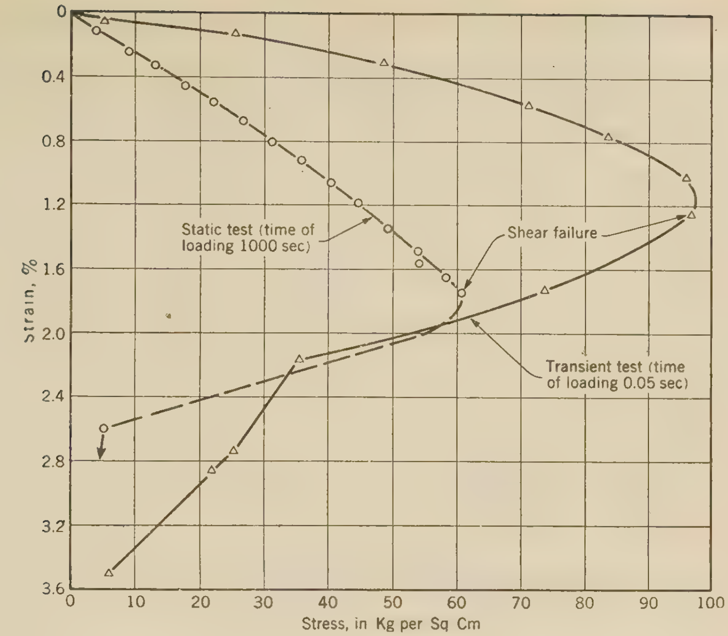


FIG. 85.—STRESS-STRAIN CURVES FOR TRANSIENT AND STATIC TRIAXIAL COMPRESSION TESTS (CONSOLIDATED-QUICK) ON CUCARACHA SHALE (σ_3 = 6 KG PER SQ CM)

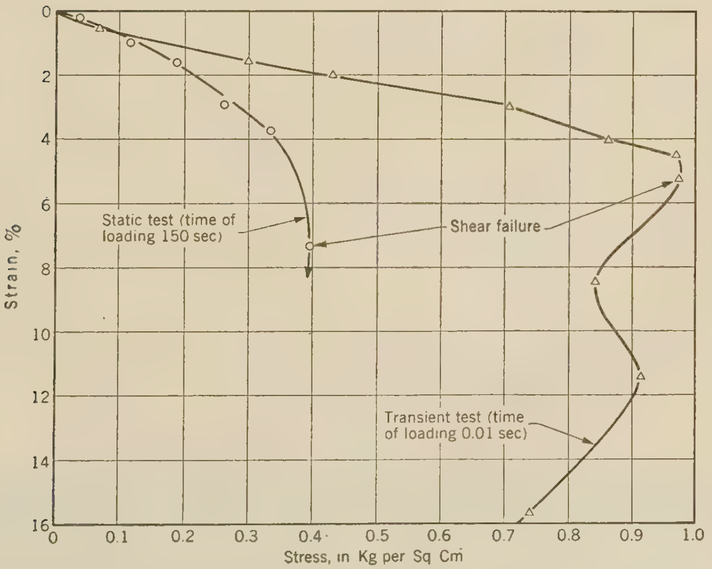


FIG. 86.—STRESS-STRAIN CURVES FOR TRANSIENT AND STATIC UNCONFINED COMPRESSION TESTS ON ATLANTIC MUCK

(The plotted points shown in Fig. 83 were taken from a continuous oscillograph record.)

All unconfined compression specimens of Cambridge clay failed with one or more clearly visible shear planes. To measure the slope of the shear planes, seven transient tests, with a time of loading of about 0.02 sec, and four static tests on Cambridge clay were so conducted that loading was stopped as soon after the shear planes appeared as was technically possible. The strain at the end of these tests ranged between 2% and 3% and it is believed that the distortion of the shear planes in these tests was negligible. The slopes of the shear planes were carefully measured to determine whether there was a significant difference in these angles between static and fast transient loading. For the

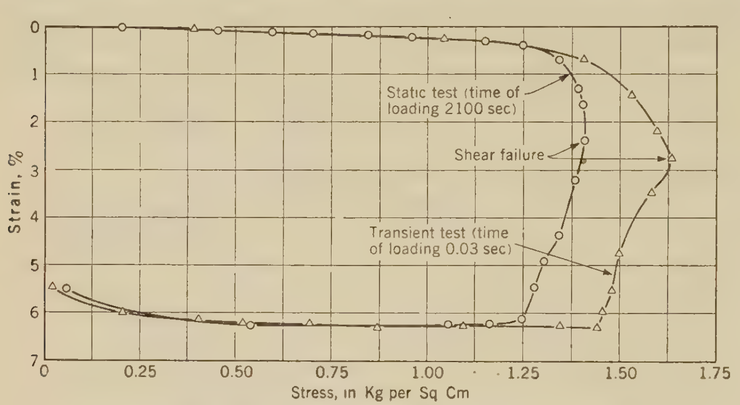


Fig. 87.—Stress-Strain Curves for Transient and Static Triaxial Compression Tests on Manchester Sand (σ_3 = 0.3 Kg Per Sq Cm)

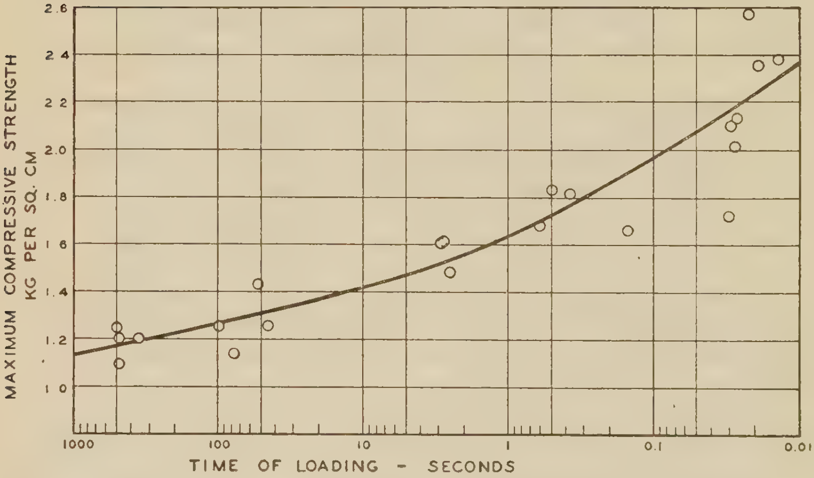


Fig. 88.—Unconfined Compression Tests on Cambridge Clay

static tests, the slopes ranged between 53° and 68°, with an average of 61°; and for the transient tests, between 47° and 65°, with an average of 58°.

Stress-strain curves for a transient and a static triaxial compression test on Cucaracha shale are shown in Fig. 85. This material failed along one or more clearly defined shear planes sloping from 35° to 65°. Usually the position and slope of the shear planes were influenced by jointing and the presence of weaker phases of the Cucaracha shale. Both specimens tested represent a strong phase of Cucaracha shale. The points shown for the transient test were taken from a continuous oscillograph record.

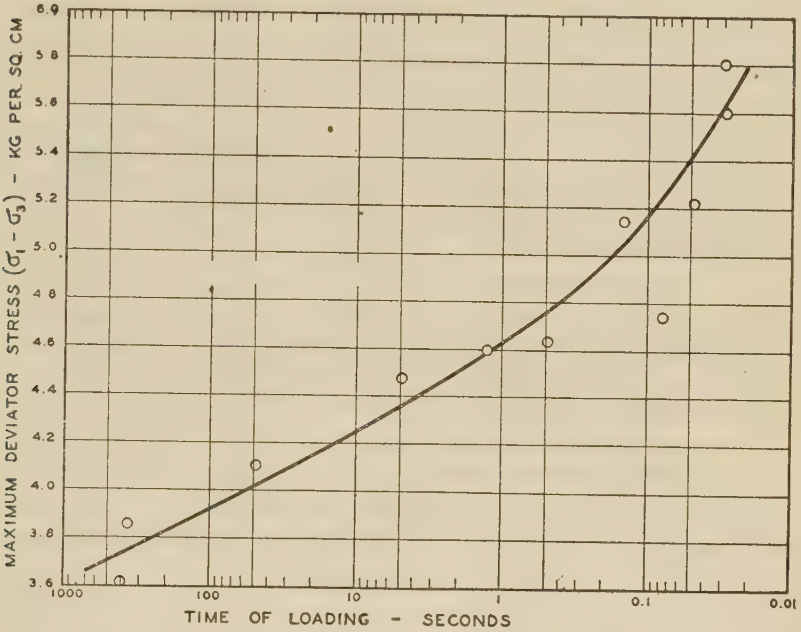


FIG. 89.—TRIAXIAL COMPRESSION TESTS ON CAMBRIDGE CLAY
(σ_3 = 6 KG PER SQ CM)

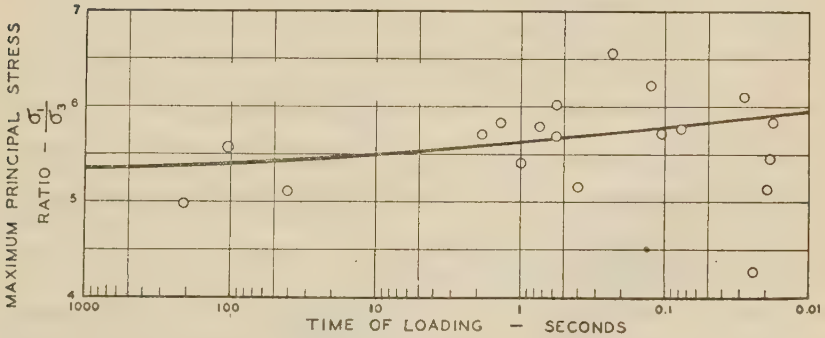


FIG. 90.—TRIAXIAL COMPRESSION TESTS ON SAND
(VOID RATIO = e_p = 0.620; VACUUM PRESSURE $-P_V$ = 0.30 KG PER SQ CM)

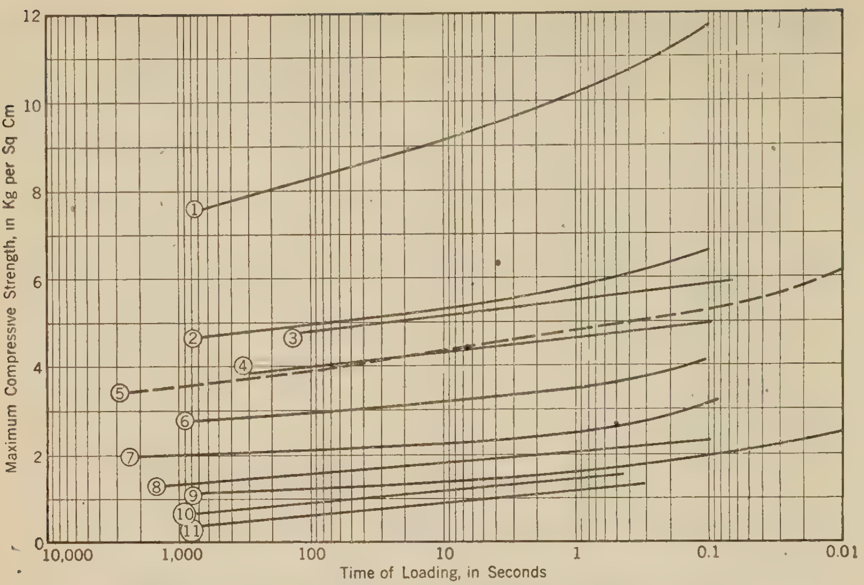


FIG. 91.—COMPRESSIVE STRENGTH VERSUS TIME OF LOADING FOR ALL TEST SERIES ON CLAY

TABLE 31.—LEGEND FOR FIG. 91

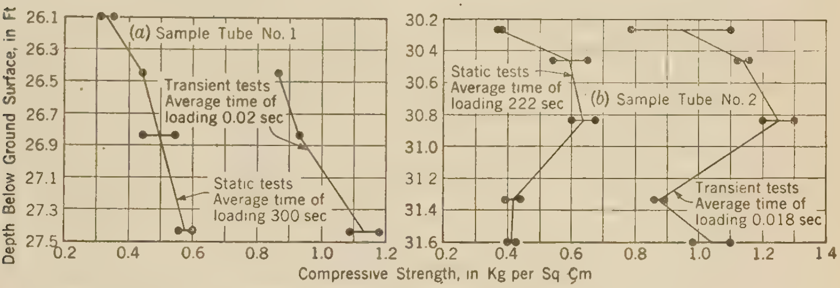
Curve No.	Sample No.	Source of clay[a]	Type of test	Water content at time of test[b] (%)
1	HP-6-C	Cambridge clay which was slowly dried to slightly higher than its shrinkage limit	Unconfined compression	27
2	HP-6-C	Cambridge clay which was slowly dried from its natural water content of about 47% to about 33%	Unconfined compression	33
3	HP-6-B	Cambridge clay	Consolidated-quick triaxial compression $\sigma_3 = 6$ kg per cm^2	24 to 32
4	HP-6-A	Cambridge clay	Consolidated-quick triaxial compression $\sigma_3 = 6$ kg per cm^2	32 to 35
5	HP-6-D	Cambridge clay	Consolidated-quick triaxial compression $\sigma_3 = 6$ kg per cm^2	40 to 44
6	HP-6-C	Cambridge clay which was slowly dried from its natural water content of about 47% to about 37%	Unconfined compression	37
7	HP-7-A	Stockton clay	Quick triaxial compression $\sigma_3 = 3$ kg per cm^2	24 to 27
8	HP-5-A	Boston clay	Quick triaxial compression $\sigma_3 = 6$ kg per cm^2	32 to 35
9	HP-2	Cambridge clay	Unconfined compression	34 to 39
10	HP-6-B	Cambridge clay	Unconfined compression	35 to 40
11	HP-6-A	Cambridge clay	Unconfined compression	40 to 42

[a] All specimens were tested undisturbed. [b] For curves Nos. 1 to 6, the specimens were reduced in water content from their original by consolidation or drying. For curves Nos. 7 to 11, the specimens were tested at their original water content.

Stress-strain curves for a transient and a static unconfined compression test on Atlantic muck are shown in Fig. 86. This material usually failed along one or more clearly defined shear planes sloping about 50°. Some specimens failed by bulging with vertical tension cracks.

Stress-strain curves for a transient and a static vacuum-type triaxial compression test on Manchester sand are shown in Fig. 87. Both of these tests were performed in a vacuum-type triaxial compression apparatus on dense sand ($e = 0.61$). The points shown for the transient test were taken from the continuous oscillograph record. Failure of all specimens was by bulging without noticeable shear planes.

The relation between compressive strength and time of loading for a given material is determined from a series of static and transient compression tests. Such relationships are shown by Figs. 88 and 89 for Cambridge clay, and by Fig. 90 for Manchester sand.



Sample tube No.	Average Time of Loading (Sec)		Average Modulus of Deformation (Kg per Sq Cm)	
	Static tests	Transient tests	Static tests	Transient tests
1	300	0.02	8.1	16.0
2	222	0.019	8.7	19.4

FIG. 92.—UNCONFINED COMPRESSION TESTS ON ATLANTIC MUCK

Fig. 91 is a summary sheet showing the relationships between compressive strength and time of loading as obtained from all the tests on clays performed to the time this paper was prepared. An explanatory legend for Fig. 92 is given in Table 31.

For the weakest clays the transient strength for the fastest time of loading was about twice the 10-min static strength. The ratio between these two values appears to decrease with increasing strength of the material. For the strongest clays included in Fig. 92, the fast transient tests showed a strength that was only about 50% greater than the 10-min static strength. Sufficient data are not available to plot the corresponding relationships for Cucaracha shale and Atlantic muck.

The stress-strain curves for Cucaracha shale in Fig. 85 indicate that the strength in a transient test, with a time of loading of 0.05 sec, is about 60% greater than that for a 10-min static test.

Results of unconfined compression tests on Atlantic muck are summarized in Table 32. Fig. 91 shows profiles of results for two sample tubes. These tests indicate that the transient strength for a time of loading of about 0.02 sec is about twice the 5-min static strength.

TABLE 32.—RESULTS OF UNCONFINED COMPRESSION TESTS ON ATLANTIC MUCK

Tube No.	Depth (ft) below ground surface	Water content[a] (%)	Static compressive strength[b] (kg per cm²)	Transient compressive strength[c] (kg per cm²)
1	24.1 to 28.1	109	0.31 0.35	....[d]
		78	0.44	0.87
		77	0.54 0.44	0.94
		85	0.55 0.60	1.18 1.09
2	28.1 to 32.1	67	0.38 0.37	0.69 1.00
		63	0.54 0.65	1.12 1.16
		59	0.60 0.67	1.30 1.20
		60	0.44 0.39	0.89 0.86
		52	0.40 0.43	0.98 1.10
4	35.4 to 39.2	120	0.33 0.33	....[d]
		118	0.26 0.20	....[d]
		121	0.37 0.31	0.64 0.91
		123	0.40 0.43	1.00 0.95
		122	0.47 0.53	....[d]
6	43.1 to 46.8	114	0.18 0.18	....[d]
		134	0.42 0.45	0.89 0.55
		135	0.43 0.34	0.68 0.88

[a] Water contents are average values for the static and transient tests for which the strength values are shown in the same horizontal column. [b] Time of loading ranging between 3 min and 9 min. [c] Time of loading approximately 0.02 sec. [d] Recording instruments failed to function satisfactorily.

Modulus of Deformation.—The original purpose of the investigation did not include the determination of the stress-deformation characteristics. Therefore, precise measurements of the deformation at small loads in transient tests have not yet been made. However, all tests have been analyzed as well as possible for the purpose of arriving also at some information regarding the effect of time of loading on the initial slope of the stress-deformation curve as represented by the modulus of deformation. For the purpose of this investigation, the modulus of deformation is defined as the slope of a line drawn from the origin through the point on the stress-deformation curve corresponding to a stress of one half of the strength.

The order of magnitude of the modulus of deformation, as obtained from the tests performed on clay and Cucaracha shale samples, is summarized in Table 33.

TABLE 33.—MODULUS OF DEFORMATION OF CLAYS AND CUCARACHA SHALE

Material	Type of test	AVERAGE VALUES OF MODULUS OF DEFORMATION (KG PER CM²)	
		Static tests	Fast transient tests
Cambridge clay	Unconfined compression	200	400
	Consolidated-quick triaxial ($\sigma_3 = 6$ kg per cm²)	650	1,300
Boston clay	Quick triaxial	250	500
Stockton clay	Quick triaxial	250	500
Atlantic muck	Unconfined compression	9	18
Cucaracha shale	Consolidated-quick triaxial ($\sigma_3 = 6$ kg per cm²)	5,000	12,000

From Table 33, it is concluded tentatively that, for these cohesive materials, the modulus of defromation for fast transient loading is about twice the value for 10-min static loading.

As can be noted in Figs. 82 to 86, the slope of the stress-strain curves for transient tests appears to be almost constant to 50% of the failure load. Hence, the order of magnitude of the modulus of deformation for transient tests is independent of the stress, provided a reference stress of not more than one half of the compressive strength is selected. For this reason the ratio of the moduli for transient and static tests would remain about the same, even if the reference stress for a transient test is assumed equal to one half of the static strength.

The modulus of deformation of dry Manchester sand had the same order of magnitude in transient tests as in static tests, as shown by the nearly identical slopes of the stress-strain curves, Fig. 87.

CONCLUSIONS

The findings presented in this paper can be summarized as follows:

(1) The strength of the clays and the Cucaracha shale loaded to failure in about 0.02 sec was found to be between 1.5 and 2.0 times greater than their 10-min static strengths.

(2) The strength of sand increases only slightly with decreasing time of loading. The strength for the fastest time of loading (0.02 sec) was about 10% greater than that for 10-min static tests.

(3) The modulus of deformation of clay and Cucaracha shale for fast transient tests (time of loading about 0.02 sec) was found to be approximately twice that for 10-min static tests.

(4) The modulus of deformation of sand was found to be independent of the time of loading.

ACKNOWLEDGMENT

The writers wish to express their appreciation to Colonel Stratton for initiating this investigation and for his encouragement. The following members of the research staff of the Harvard Soil Mechanics Laboratory, who were engaged on this project, contributed to its success by valuable suggestions: L. S. Chen, J. V. Grasso, H. B. Sutherland, Edward Wenk, Jr, Jun. ASCE, and S. D. Wilson. The David Taylor Model Basin, United States Navy, assisted materially on questions of instrumentation and by the loan of strain indicators and recording instruments.

In connection with the use of the SR-4 strain gages, advice was obtained from A. C. Ruge, M. ASCE. The hydraulic loading apparatus was largely designed by Lessells and Associates, Consulting Engineers.

CONSTRUCTION PLANNING AND METHODS

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Synopsis

The principal construction features of the proposed Panama sea-level canal are described in this paper, including the various plans for the conversion of existing lock canal to sea level, and the construction methods on which the cost estimates are based.

Conversion of the existing canal to sea level in a single stage by deep dredging is preferable to conversion by stage dredging because it would be cheaper, would require a shorter construction period, and would interfere less with canal traffic.

For purposes of construction planning and estimates, the bulk of the dry excavation would be performed by large shovels or draglines of at least 25-cu-yd capacity, loading directly into dump scows which would haul the spoil to disposal areas in Gatun Lake. The wet excavation would be performed by conventional dredges supplemented by special hydraulic and bucket-ladder dredges capable of dredging to a depth of 145 ft below water surface. These methods of dry excavation and wet excavation are the most feasible and economical, and can be adapted most readily to the over-all project.

Introduction

Public Law No. 280 which authorized the Isthmian Canal Studies—1947 directed "a comprehensive review and study, with approximate estimates of costs, of means of increasing the capacity and security of the Panama Canal * * * [and] study [of] * * * a canal or canals at other locations * * *."

Compliance with Public Law No. 280 required initially the following studies:

1. Selection of general construction techniques and estimating methods to obtain approximate and comparable estimates of cost for the many canal routes to select the routes offering the best possibilities for development; and
2. Determination of the most practicable construction procedures and equipment, and preparation of detailed and dependable comparative estimates of costs of the best canal plans, to assist in the final selection of route and type of canal.

These procedures led to the selection of the Panama sea-level canal as the most economical means of increasing capacity and security to meet the future needs on interoceanic commerce and national defense. After the selection of the sea-level canal at Panama, more detailed construction studies and estimates were made. The main features of these studies were:

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(1) Investigation of new and special equipment for converting the Panama Canal to sea level; and

(2) Investigation of the major construction problems involved in executing the recommended plan of converting the present Panama Canal to sea level in a single stage by the "deep-dredging" method.

For the initial construction studies under Public Law No. 280, thirty possible routes or alinements (Fig. 6) were examined, of which eight were found on map inspection and, in some cases, on field reconnaissance, to offer the best possibilities for development. Cost estimates for the more favorable of the sea-level and lock canals on eight routes were prepared by using excavation quantities obtained from center-line map profiles and, in each case, approximate classifications of materials. Detailed investigations were made of the problems involved in the construction of both a lock canal and a sea-level canal in the Canal Zone and the immediate vicinity, and estimates of costs based on standard construction procedures were prepared for use in comparative analyses. Unit costs for excavation were those developed for the Panama route. Costs of structures, such as locks, were developed for each route on the basis of height or lift, using the cost of locks at Panama as the estimating standard. As shown by Tables 1 and 2, the Panama route is the most economical for both a sea-level canal and a lock canal.

After selection of the Panama sea-level canal, detailed studies and cost estimates were made of the most efficient and economic construction methods. The investigations and studies used as the basis for the estimates of cost for the construction of the Panama sea-level canal are described in this paper.

Features of the Main Channel

The Panama sea-level canal would have a channel 600 ft wide at a 40-ft depth, with a total depth below mean sea level varying uniformly from 60 ft at the Atlantic entrance to 70 ft at the Pacific end. The principal features of the canal are shown in Fig. 7. The total length of the canal from deep water in the Atlantic to deep water in the Pacific would be approximately 46 miles. The alinement would follow, generally, that of the present canal. Sharp curves would be eliminated and tangents lengthened to meet navigation requirements. The sea-level canal alinement would be contiguous to the present canal in the section of deep cut through the Continental Divide and would permit the removal of a large part of the required rock excavation more economically by dry methods. The alinement would utilize the excavation accomplished for the Third Locks structures and the approach channels at Miraflores and Gatun. The total required excavation is approximately 1,070,000,000 cu yd, of which 750,000,000 cu yd could be excavated in the dry. Dredging would be required for the removal of the remaining 320,000,000 cu yd. For comparison, the channel excavation by the United States to complete the existing canal totaled approximately 120,000,000 cu yd of wet and 125,000,000 cu yd of dry. The total usable channel excavation, including that done by the French, was approximately 275,000,000 cu yd.

The maximum depth of cut to be excavated would be 650 ft; however, the length of canal at this depth would be less than ½ mile. Approximately 3½ miles of canal would be in cut deeper than 300 ft; 7 miles, in cut between 200 ft and 300 ft; 9 miles, in cut between 100 ft and 200 ft; and the remaining 26 miles, in cut less than 100 ft deep. Of this 26 miles, 10 miles would be in a muck formation, a soft overburden material in which the maximum depth of cut would be less than 80 ft.

For convenience in estimating and for facilitating orderly completion of the excavation, the channel excavation has been divided into ten parts. The material to be excavated has been classified into the following four types according to the relative difficulties of excavation: Common, soft rock, medium rock, and hard rock, as shown in Table 34. The linear distribution of the material is shown in Fig. 93.

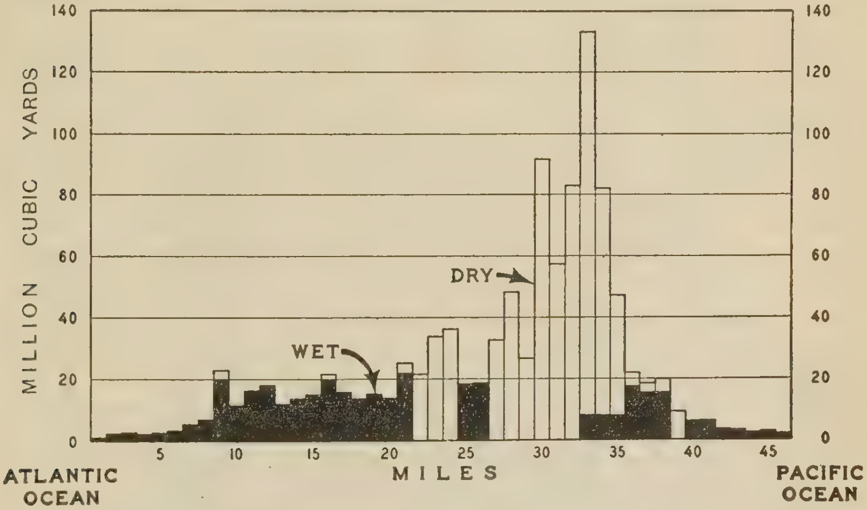


Fig. 93.—Distribution of Excavation Quantities

Structural Features

Tidal-Regulating Structures.—These structures consist of a tidal lock, a navigable pass, and a gated water-control structure, and involve 52,000,000 cu yd of excavation and 1,800,000 cu yd of concrete. The construction of these structures would involve only conventional methods.

Flood-Control Structures.—The flood-control structures consist principally of earth dikes and dams, constructed mainly with excavation spoil. Construction of the flood-control structures except as a method of spoil utilization is conventional.

Housing and Facilities for Construction Workers.—The Canal Zone is a government-administered area in which, with rare exceptions, there is no private industry, business, or ownership of land or buildings. Consequently,

TABLE 34.—EXCAVATION CLASSIFICATION; CANAL ZONE ROCK UNITS

Formation	Description	Unit weight (lb per cu ft)	Quantity (Thousands of Cu Yd)		Dynamite (lb per cu yd), dry excavation	Dynamite (lb per cu yd), wet excavation
			Dry	Wet		
(a) COMMON EXCAVATION						
Overburden and muck	Muck, silts, sands, clays, gravels, etc.	Variable	108,439	165,044	0	0
(b) SOFT ROCK						
Cucaracha	Largely dense greenish-gray clay shales highly slickensided within certain horizons. Black carbonaceous shales, sandstones, and conglomerates in subordinate proportions	135–140	125,584	27,406	0.30 (40%)	0.65 (40%)
Culebra	Medium-hard sandstones, soft sandy and carbonaceous shales	140 ±	146,647	8,216	0.30 (40%)	0.65 (40%)
La Boca	Dense silty or sandy dark-gray shales with intercalated sandstone beds sporadically present	140 ±	17,993	3,572	0.30 (40%)	0.65 (40%)
Gatun	Fine-grained argillaceous and calcareous sandstones with interbedded dense tuffs and conglomerates	120–125	14,330	7,409	0.30 (40%)	0.65 (40%)
(c) MEDIUM ROCK						
Caimito	Coarsely bedded medium-grained and fine-grained medium-hard limy sandstones and tuffs	130 ±	94,497	47,236	0.50 (40%)	1.00 (60%)
Las Cascadas	Agglomeratic tuff and tuff-breccia consisting of angular fragments of hard dark-gray andesite in a clayey dark-gray to light-green altered tuff matrix	140 ±	13,778	0	0.50 (40%)	1.00 (60%)
Bohio conglomerate	Subangular to rounded pebbles, cobbles, and boulders up to 2 ft in diameter in a dark-gray or brown generally coarse friable tuffaceous sand matrix	145 ±	81,722	50,463	0.50 (40%)	1.00 (60%)
(d) HARD ROCK						
Pedro Miguel and Bas Obispo agglomerate	Hard light-gray to dark-gray fine-textured to coarse-textured agglomerates and tuffs	155 ±	64,645	1,884	0.75 (60%)	1.50 (60%)
Basalt	Hard columnar-jointed basalt flows and intrusives	160–170	82,288	7,516	0.75 (60%)	1.50 (60%)
Total	....	....	749,923	318,746	....	....

housing, water supply, sewage disposal, power, communications, hospital and health facilities, schools, fire and police protection, and other facilities would have to be provided for the expanded population of the Canal Zone during canal construction. Housing and other facilities would be largely temporary in character but certain permanent construction would be performed to replace existing housing and other facilities which date back to the early 1900's and which are, or soon will be, obsolete.

PROGRAM

Various programs of construction were evaluated, and it was determined that a 10-year construction program to convert the existing canal to sea level would be the most favorable. A shorter program would result in less economical use of special plant for dry and wet excavation and would require more housing. A longer program would probably result in somewhat lesser costs, but would delay the fulfilment of the requirements of national defense.

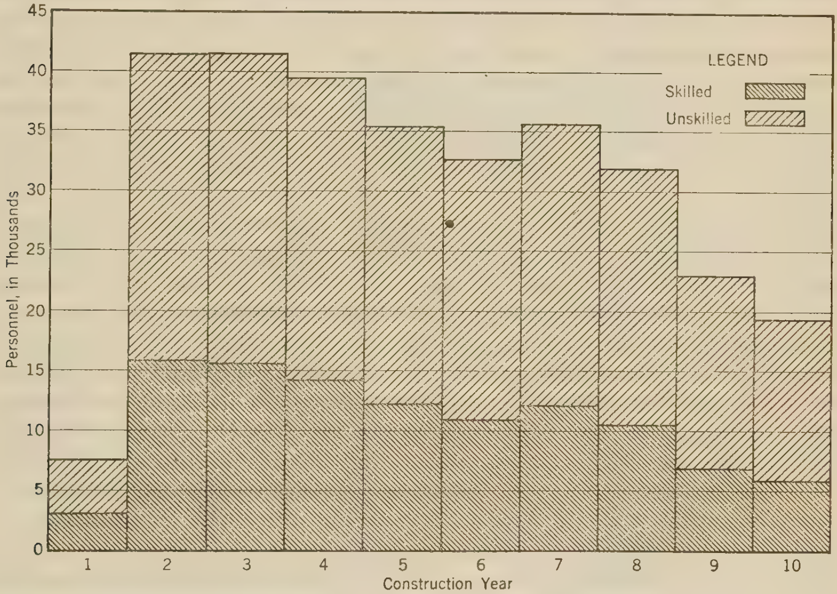


FIG. 94.—TOTAL PERSONNEL REQUIREMENTS, PANAMA SEA-LEVEL CANAL

PERSONNEL

Skilled and technical personnel would be obtained from the United States. The unskilled employees would be largely indigenous to the Caribbean area. Contracts with skilled and unskilled labor would include payment of transportation from the place of recruitment and return. Transportation of government workers' families and household furnishings would be provided. The labor requirements are shown in Fig. 94.

MATERIALS

Satisfactory concrete aggregates may be obtained from alluvial deposits of sand and gravel in the Chagres River or from basaltic rocks from the channel excavation. Local sources for cement, steel, and lumber are not adequate, and these items would be obtained from the United States. The large volumes of excavation throughout the entire length of canal and water transportation make available adequate and economical embankment materials for earth dam construction.

CONVERSION TO SEA-LEVEL CANAL BY STAGE DREDGING

The method of converting the present canal to sea level has a direct effect on the method of excavation; and, in turn, the various possibilities of performing the excavation have a direct bearing on the selection of the method of conversion.

In all previous studies of plans for effecting the conversion of the present canal, it was contemplated that Gatun Lake would be lowered by stages. In one study, seven stages of lowering were planned; in others, three stages were selected. Stage lowering would require progressive alteration of the existing locks so that traffic could be accepted at all stages of the lake. This program would involve certain risks to shipping and to the canal if auxiliary conversion locks were not built, since only one lane of locks would be open to traffic at a time while the other was being modified to accept traffic at the next lower stage.

In the current studies it was found that stage conversion could best be effected by lowering the lake in three stages—from El. 85 to El. 54 (the elevation of Miraflores Lake), from El. 54 to El. 22, and from El. 22 to sea level. For this plan of conversion, equipment would be required capable of dredging to depths of 72 ft, a not uncommon depth of excavation. It was concluded that the risk in depending on one lane of locks during conversion was not warranted and that continuous operation of two lanes of locks would be essential. Consideration was first given to the use of a third lane of locks, as planned in earlier studies, constructed to full summit height (El. 85) to overcome this serious handicap to the stage conversion plan. However, it was determined later that the three-stage lowering plan made it practicable and economical to construct the special two-lane, single-lift conversion locks, to El. 54, at Miraflores and at Gatun. The conversion locks would have a lower lift and would be no more costly than would the alternate plan involving a third single-lane, high-lift lock at each end of the existing lock sites and the conversion of the existing locks. The conversion locks would have upper sills at a low elevation to permit the transit of ships at each successive stage of lowering below El. 54. In this plan the existing locks would be abandoned when Gatun Lake was lowered to El. 54, and the temporary conversion locks at Gatun and Miraflores would then be placed in operation.

The disadvantages of the stage conversion plan are:

a. Conversion locks would be required and would add considerably to the cost of the project.

b. All dry excavation, approximately 750,000,000 cu yd of the total of more than 1,000,000,000 cu yd, would have to be excavated prior to the initial lowering of Gatun Lake (thus requiring a minimum construction program of 15 years to permit sufficient time for the two remaining stages of dredging).

c. Traffic in the canal would be interrupted during the drawdown for each stage.

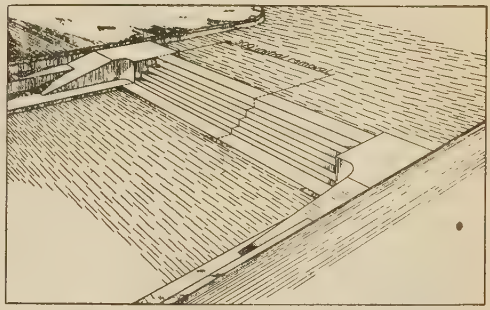
d. Part of the material dredged during the second stage and all the material dredged during the final stage would require locking to sea for disposal, because the drawdown would prevent using Gatun Lake as a spoil area, and use of the locks for this purpose would interfere with canal traffic. (Approximately 55,000,000 cu yd would have to be transported to sea.)

e. The lowering of Gatun Lake for the final stages would deplete the storage area to such an extent that pumping of lockage water would be required.

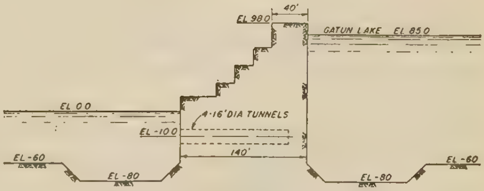
SINGLE-STAGE CONVERSION BY DEEP DREDGING

A proposal by E. E. Abbott for conversion by deep dredging led to an investigation of the possibilities of adapting or developing dredging equipment so that all wet excavation could be performed in advance of the lowering of the summit lake. This investigation indicated that such a method would be entirely feasible. Drilling, blasting, and excavating to depths of 145 ft below the surface of Gatun Lake would be required to provide the design depth in a sea-level canal.

In this plan, the present operating levels of Gatun Lake (El. 85) and Miraflores Lake (El. 54) would be maintained during the entire period of excavation of the canal. The level of Gatun Lake would be dropped to El. 81 a short time prior to final lowering to permit placing the flood-control system of the canal in operation. The canal would then be closed to traffic for about 7 days, during which the lakes would be lowered to sea level and the channel would be cleared of the barriers used to retain Gatun and Miraflores lakes during excavation.



PERSPECTIVE



CROSS-SECTION

FIG. 95.—GATUN CONVERSION PLUG

The barriers or channel plugs used to retain the lakes would consist of the following: (1) A natural rock barrier across the sea-level channel on the north shore of Gatun Lake (Fig. 95); (2) a natural rock barrier in the sea-level channel separating Gatun and Miraflores lakes near the existing Pedro Miguel Locks (Fig. 96); and (3) a temporary steel dam in the north approach to the tidal lock (Fig. 97). The natural rock plugs at Gatun and Pedro Miguel would be removed progressively by blasting and dredging during the 7 days that Gatun Lake was being lowered. The temporary steel dam in the approach to the tidal lock would be removed in sections by derricks when the water in the canal reached sea level.

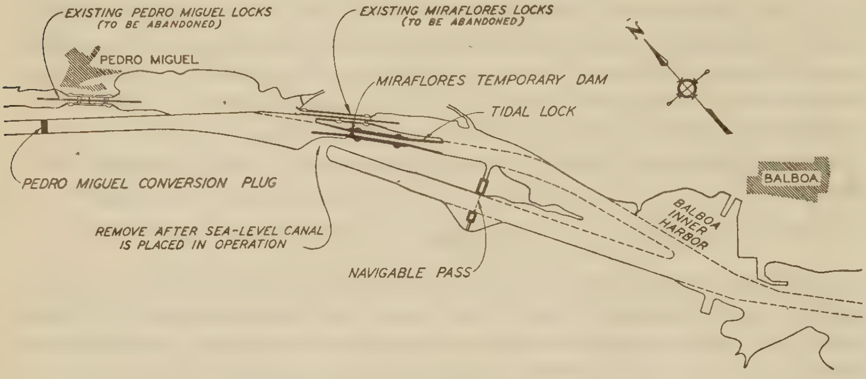

EXISTING PEDRO MIGUEL LOCKS
(TO BE ABANDONED)
PEDRO MIGUEL
EXISTING MIRAFLORES LOCKS
(TO BE ABANDONED)
N
MIRAFLORES TEMPORARY DAM
TIDAL LOCK
BALBOA
BALBOA INNER HARBOR
PEDRO MIGUEL CONVERSION PLUG
REMOVE AFTER SEA-LEVEL CANAL
IS PLACED IN OPERATION
NAVIGABLE PASS

FIG. 96.—LOCATION PLAN, PEDRO MIGUEL PLUG AND MIRAFLORES TEMPORARY DAM

Gatun Conversion Plug.—The Gatun conversion plug would be of sandstone (Fig. 95). The bedding of this formation is massive, and the infrequent near-vertical joint planes are tight. The lake approach to the Gatun plug would be excavated by deep-dredging methods, but the immediate face of the plug would be carefully formed by line drilling and light blasting. The Atlantic approach would be excavated by dry methods. Four discharge outlets, 16 ft in diameter, would be tunneled in the plug from the downstream side of the plug to within about 20 ft of the face. The entire mass of the rock plug would be prepared for blasting and final removal by drilling vertical holes in a 5-ft pattern down to El. −50. Initially, the lake would be lowered through the flood-control spillways and the lock culverts. Later the tunnel outlets in the rock plug would be opened by blasting, using drill holes from the top of the plug for loading and firing, and finally the entire remaining mass of the plug would be blasted.

Pedro Miguel Conversion Plug.—The Pedro Miguel conversion plug would be generally similar to that at Gatun. It would be located near the upper approach to the existing Pedro Miguel Locks (Fig. 96). It would be placed as close to Miraflores Lake as sound rock permits, to reduce the volume of channel excavation required to be transported through the Miraflores Locks for disposal at sea. The agglomerate of which this plug is composed is a relatively impervious rock massively jointed. Any leakage through material of this type would occur along joints or fracture planes, and the condition would become evident during the progress of excavation. Grouting and sealing of the upstream side would be undertaken if found necessary. During the lowering of the lake, the plug would be removed progressively in a manner similar to that used at Gatun.

Miraflores Temporary Dam.—The upstream arm of the construction cofferdam, enclosing the tidal lock, was considered for use as the plug for Miraflores Lake. The removal of the cofferdam and the excavation of the channel under the cofferdam that could not be done until drawdown of the lake had been completed would not be accomplished as rapidly as the removal of the Gatun and Miraflores plugs, and traffic would be suspended for a longer period. Therefore, a temporary steel dam is planned in the north approach to the tidal lock, illustrated in Fig. 97, which would be capable of holding Miraflores Lake at its present elevation. It would be constructed in the completed lock approach above the upper gates prior to the removal of the construction cofferdam. Short sections of earth embankments extending to high ground, with temporary concrete-retaining abutments on the lock walls, would be required to complete the closure. The dam would be constructed in a manner permitting rapid disconnection and removal of individual sections in 1 day by two derricks mounted on the lock walls.

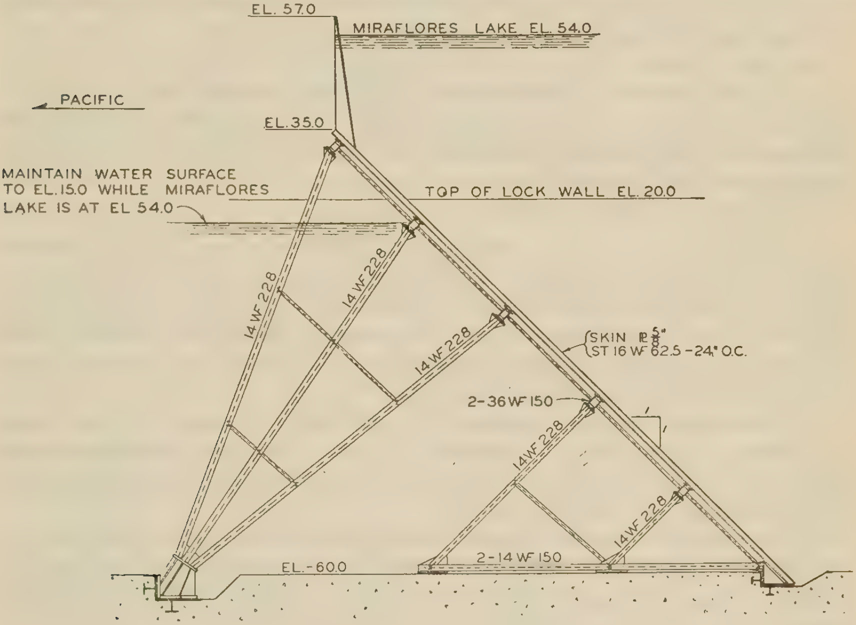

EL. 57.0
MIRAFLORES LAKE EL. 54.0
PACIFIC
EL. 35.0
MAINTAIN WATER SURFACE TO EL. 15.0 WHILE MIRAFLORES LAKE IS AT EL 54.0
TOP OF LOCK WALL EL. 20.0
14 WF 228
14 WF 228
14 WF 228
SKIN PL 5/8"
ST 16 WF 62.5 - 24" O.C.
2-36 WF 150
14 WF 228
14 WF 228
2-14 WF 150
EL. -60.0

Fig. 97.—Cross Section, Temporary Steel Dam in Tidal Lock

Advantages of Deep-Dredging Plan.—The principal advantage of the deep-dredging method of excavation over the stage-dredging plan is in the saving of lock conversion costs. Also, the deep-dredging plan permits full use of the Gatun Lake area for deposit of all excavation spoil, with corresponding saving in excavation costs. Using Gatun Lake for the hauling of excavated materials would make it possible to construct the Caño Quebrado, Trinidad, and Monte Lirio flood-control dams with greater facility and at less cost. This plan would also result in less interference with canal traffic and would save about 5 years in construction time.

The net saving of the deep-dredging plan over the stage-dredging plan is estimated at $130,000,000.

DRY EXCAVATION

Conversion of the present Panama lock canal to a sea-level canal would require the excavation in the dry of 750,000,000 cu yd of material, practically all being between miles 21 and 24 and miles 27 and 36, Fig. 93. Considering the character and the volume of material to be removed, the nature of the terrain in this area, and the accessibility of spoil areas, it is evident that various types of equipment could be employed in the excavation and removal of material.

Operation Phases.—The most suitable method of excavation is that which has the widest general application and which results in the least cost for removal and disposal of the excavated material. It can be selected only by an examination of the principal phases of the excavation operations, namely: (*a*) Preparation of the material for excavation; (*b*) excavation; and (*c*) haul of excavated material to disposal areas.

(*a*) Preparation of the Material for Excavation.—For common material no preparation is required, and for soft rock preparation may or may not be required, depending on the selection of excavation equipment. For medium and hard rocks, preparation requires systematic drilling and blasting, the extent of which is determined by the character of the material and the type and and size of excavation equipment selected.

(*b*) Excavation.—A wide choice of equipment of various types and sizes is available. The utility of the different types of equipment depends on the character of material, the nature of its occurrence, and other factors discussed more fully elsewhere in this paper.

(*c*) Haul of Excavated Material to Disposal Areas.—This factor, because of the large quantity of materials involved, is of major, if not controlling, importance in the selection of the method of dry excavation. Accordingly, the methods of haul are discussed first.

Haul.—Haul refers to the transportation of the material from the place where it is excavated to the point of disposal—by truck, railroad, belt conveyer, scraper, scow, or other methods. The selection of the method is determined by the character of the excavation and its location in relation to the disposal area, the gradient between the two points, the location for suitable haul roads or conveyer lines—and in the case of scow haul by the availability of water transportation between the point of excavation and the disposal area.

Because the alinement of the proposed Panama sea-level canal takes advantage of the low terrain, the available dry spoil areas in the reach from Gatun Lake to Miraflores Lake are higher than the excavation, and are at an average distance of more than 5,000 ft from the center line of the canal. The bottom of the cut through this section is at approximately El. −70 and the average elevation of the top of the spoil areas would be between 400 ft and 500 ft. The distance and necessary gradient to overcome the difference in elevation would necessitate approximately 2 miles of truck haul on an upgrade. Truck haul, therefore, would not be an economical operation for most of the material.

However, a small part of the excavation (that above El. 270) could be hauled economically by trucks because the difference between that elevation and the spoil areas would be sufficiently small to permit removal with an average haul of about 1 mile. In some locations, truck haul could be used economically below El. 270. The exact elevation would be dependent on the location and elevation of the dry spoil areas as well as on the haul distance to the wet spoil area.

The difference in elevations between the bottom of the cut and the disposal areas would require approximately 5 miles of railroad on a 2% grade for the entire distance. Removal by this method, therefore, was not considered feasible or economical.

The large amount of rainfall in this area, combined with the adhesive or sticky quality of the shale which forms a large part of the excavation, and the necessity for breaking the material into small sizes do not favor the use of belt conveyers. In some sections of the canal excavation, this method might be used successfully but it does not appear feasible or economical for the operation as a whole.

The lack of suitable spoil areas within reasonable overland haul distances led to the consideration of a method of disposal in areas in Gatun Lake remote from existing and future channels. Actually, this use of Gatun Lake is not new since the lake has been used for wet disposal of excavated materials since it was first created. The principal new problem which this method of disposal presents, therefore, is that of effecting the transfer of dry excavated materials to suitable haulage equipment and their transportation to the dumping areas. These areas could be made accessible to railroad haul, averaging 15 miles, but trestles would have to be constructed in Gatun Lake. Furthermore, it was found that the material could be hauled more economically in scows. The Gatun Lake spoil areas are readily accessible to bottom dump scows and have capacity for more than 3,000,000,000 cu yd of spoil material below El. 65. Accordingly, a method of dry excavation was developed so that the material could be handled directly into scows to take advantage of the low cost water transportation. The haul distance from point of excavation to point of disposal would vary from 1 mile to 23 miles and would average about 15 miles. The average cost of disposal by truck haul would be approximately twice as much as by scow haul.

Practically all disposal of dry excavation could be handled by the scow haul method by using a system of auxiliary barge channels, which would lie within the limits of the excavation area and would roughly parallel the center line of the new channel for the entire length of the dry excavation (Figs. 98 and 99). The initial auxiliary barge channels would enter the sea-level canal channel from the lake through natural inlets, or in some instances by way of an inlet or a channel excavated to connect with the present canal. In constructing the barge channels, a pilot cut 180 ft wide to El. 90 would be made first along the length of the cut, followed immediately by a channel 110 ft wide excavated to El. 65, giving the channel a depth of 20 ft with the lake surface at El. 85. Material adjacent to both sides of the barge channel would be excavated to El. 90 casting directly, or recasting, into bottom dump scows in the barge

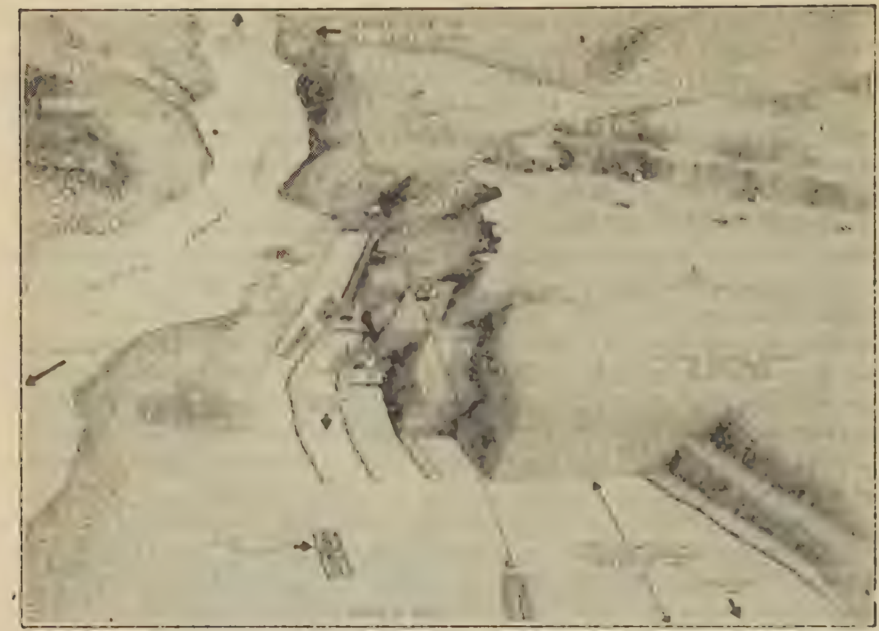


Fig. 98.—Excavation Above El. 90, Approaching Contractors Hill from the North

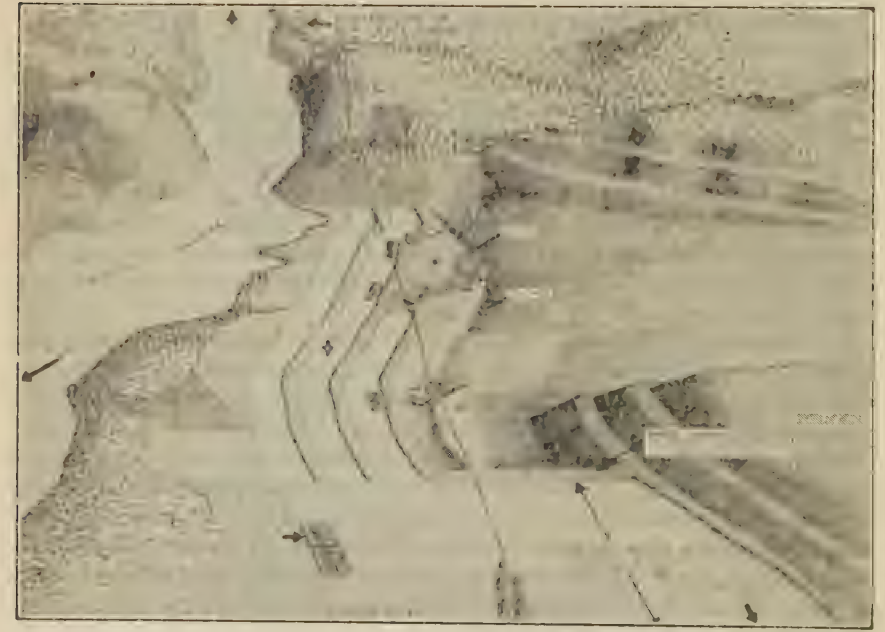


Fig. 99.—Excavation Below El. 90, Approaching Contractors Hill from the North

channel. When a width of 180 ft had been excavated on either side of this channel, another barge channel would be excavated parallel to it, leaving a 70-ft berm between the two channels as shown in Fig. 98. This process would be continued until the excavation to El. 90 had been extended laterally to intersect the design side slopes at this elevation. All the material above El. 90 could be loaded or cast directly into the scows except that for the excavation of the very flat slopes, which would require recasting or trucking of the material to the scows.

For the excavation of material below El. 90 to project depth (Fig. 99), the barge channels would be blocked and dewatered progressively from one side of the channel to the other, each being excavated in turn to project depth. After drilling and blasting, the material in each dewatered channel would be removed by a dragline operating from the 70-ft berm separating the dewatered channel from the adjoining open or wet channel. Material below El. 90 would be excavated in two steps. In the first step, when the material was excavated to El. 0, the draglines would pick up the materials and load directly into the scows. In the second step, below El. 0, the materials would be cast on a bench and then picked up by draglines and loaded into scows.

The scow haul method of disposal facilitates the construction of the broad flood-control dams in Gatun Lake. This method would materially reduce the construction costs of these dams because they could be built to El. 65 with the 2,000-cu-yd bottom dump scows with very little extra haul distance. The dams could then be raised by using smaller barges, which would permit dumping to El. 75. The only important cost for the construction of the dams would be the special handling of a small part of the material from El. 75 to the required height at El. 82.

The scow haul method of disposal of all material excavated below El. 270 was adopted for construction planning purposes because it was the most economical method. The dry spoil areas are favorably located for the disposal of material excavated above El. 270 and the use of 20-cu-yd trucks was adopted for planning purposes. Additional economies may be realized by further study of truck haul for material below El. 270 in some locations.

Excavating Equipment.—The size and type of the hauling unit and the type of material to be excavated are the principal factors in selecting the size of shovel to be used. The capacity of the hauling unit should be equal to one or more times the load handled at one time by the excavator. For the material hauled by truck, studies have indicated that the 5-cu-yd shovel would be the most economical type of equipment for leading 20-cu-yd trucks.

Large excavating equipment, shovels and draglines with capacities greater than 25 cu yd, has proved economical in strip-mining operations where there is no problem of transportation. However, no records are available, involving the use of these large shovels or draglines, where the excavated material had to be transported. The scow haul method, previously described, using auxiliary channels, would lend itself readily to the use of this large equipment. This equipment has a wide operating radius, which permits it to make a cut about 180 ft wide. The large swinging radius of this equipment would facilitate the direct loading of scows reaching the equipment through barge channels.

This equipment could work high faces of 90 ft and the size of the dippers would require less drilling and blasting of the hard material. Also, the rugged construction of the large scows would withstand the loading shock of the material deposited by the large dippers. Furthermore, the large equipment would require less operating personnel for the same volume of material excavated than would smaller excavation equipment.

The lack of mobility of the large equipment would not be a serious disadvantage on this job because the excavation is highly concentrated. Large quantities of material would have to be removed for each position of each piece of equipment, because of the great depths and widths of cut. For the foregoing reasons, the large excavating equipment, having buckets of 25-cu-yd capacity or greater, was adopted for construction planning purposes for the excavation of practically all the material below El. 270.

Drilling and Blasting.—The methods of excavating and of hauling each have a definite bearing on the type and quantity of drilling and blasting required. The character of the material to be blasted determines the selection of the type of drilling equipment. The spacing of holes and the amount of blasting, expressed in terms of pounds of powder per cubic yard, are determined by the size of the hauling, loading, and drilling equipment.

The classification of the material to be drilled and blasted is shown in Table 34. From studies of drilling performed by contractors and The Panama Canal in this area, it was determined that the rotary drill would be the most suitable and economical for soft and medium rocks; and the percussion or churn drill for hard rock.

Excavation with small shovels would be on rather low faces, about 30 ft high, and would require fairly close spacing of drill holes varying from 8 ft by 8 ft for the hard rock, drilled by percussion-type wagon drills, to 18 ft by 18 ft for the soft rock, which would be drilled with rotary drills.

Most of the dry excavation, 700,000,000 cu yd out of 750,000,000 cu yd, would be handled with the large shovels and draglines, 25 cu yd and more, and hauled in large scows. The digging height of the large shovels is about 90 ft, and therefore depths of drilling of 90 ft would be practicable. The size of the dippers and the fact that the scows could accept pieces as large as those handled by the dipper, would permit 30-ft by 30-ft spacing of the drill holes in soft and medium rocks, materially reducing the cost of drilling. For the hard rock, drilling could be done to the full depth of the 90-ft faces, but coyote or tunnel blasting would be employed to effect savings in drilling and in blasting.

The weight of powder required would vary with the material, size of blast, and local conditions. Past experience in the Canal Zone indicates that the powder requirements will vary between ¼ lb per cu yd for the soft rocks to ¾ lb per cu yd for the hard rocks, when the drilling and blasting are done in the dry.

WET EXCAVATION

The deep-dredging scheme for conversion of the Panama lock canal to sea level requires the development of special equipment for excavation to the unprecedented depth of 145 ft below the existing level of Gatun Lake. It also calls for subaqueous drilling and blasting to far greater depths than have previously been required in wet excavation. Since these two considerations are controlling factors in setting up the construction procedure for wet excavation, they will be discussed first, followed by a brief discussion of proposed construction procedure.

Dredge Design Contracts.—To obtain the views and advice of the dredging industry as a whole regarding the feasibility of dredging to depths of 145 ft, a preliminary conference was held in Philadelphia in March, 1946, attended by representatives of thirty dredging contractors, dredge designers and builders, the Corps of Engineers, and The Panama Canal. At this conference, problems involved in construction of a sea-level canal were discussed, and it was the consensus that deep dredging was practicable. Subsequently, contracts were made with dredge manufacturers for design of the following types of dredges:

Type	Company	Maximum dredging depth (ft)
Hydraulic	Panama Contractors	145
Bucket ladder	Yuba Manufacturing Company	145
Dipper	Bucyrus-Erie Company	85

Revised specifications of the bucket-ladder dredge show that, with the ladder at a 45° inclination with the horizontal, the maximum dredging depth would be 148 ft.

Panama Contractors, organized especially for the hydraulic dredge contract, is a combination of the Atlantic Gulf and Pacific Company, Standard Dredging Corporation, and Gahagan Construction Company.

The maximum dredging depth of 148 ft specified for the hydraulic and bucket-ladder dredges is that required for excavation to grade, working from the normal Gatun Lake elevation of 85 ft. The maximum dredging depth of 85 ft for the dipper dredge is presently considered to be the greatest depth to which a dredge of this type can economically excavate. It is proposed, therefore, that dipper dredges will excavate rock to an 85-ft depth, followed by deep-digging bucket-ladder dredges, excavating to grade. Bucket-ladder dredges, capable of dredging to a maximum depth of 90 ft, would also be utilized for initial rock excavation. The hydraulic dredge would be used only for the excavation of common material and soft rock, such as the Gatun sandstone. Development designs and cost estimates for the construction of the three types of dredges have been completed.

Hydraulic Dredge.—In present hydraulic dredge design, the main pump of the dredge is located near the water surface. Atmospheric pressure, in forcing the dredged materials to the pump from depths of 40 ft or 50 ft, is sufficient to supply ample velocity head, overcome friction losses, and support a column of mixture having a density substantially greater than that of water. As deeper dredging is undertaken, the entrance loss and velocity head remain the same, but the friction head and the pressure necessary to support the columns of mixture increase. Various schemes to bring a richer mixture to the dredge pump were considered, such as utilizing nozzles in the suction, depressing the elevation of the main pump, placing a propeller-type pump in the suction, or placing a booster pump on the dredge ladder below the water surface. It was

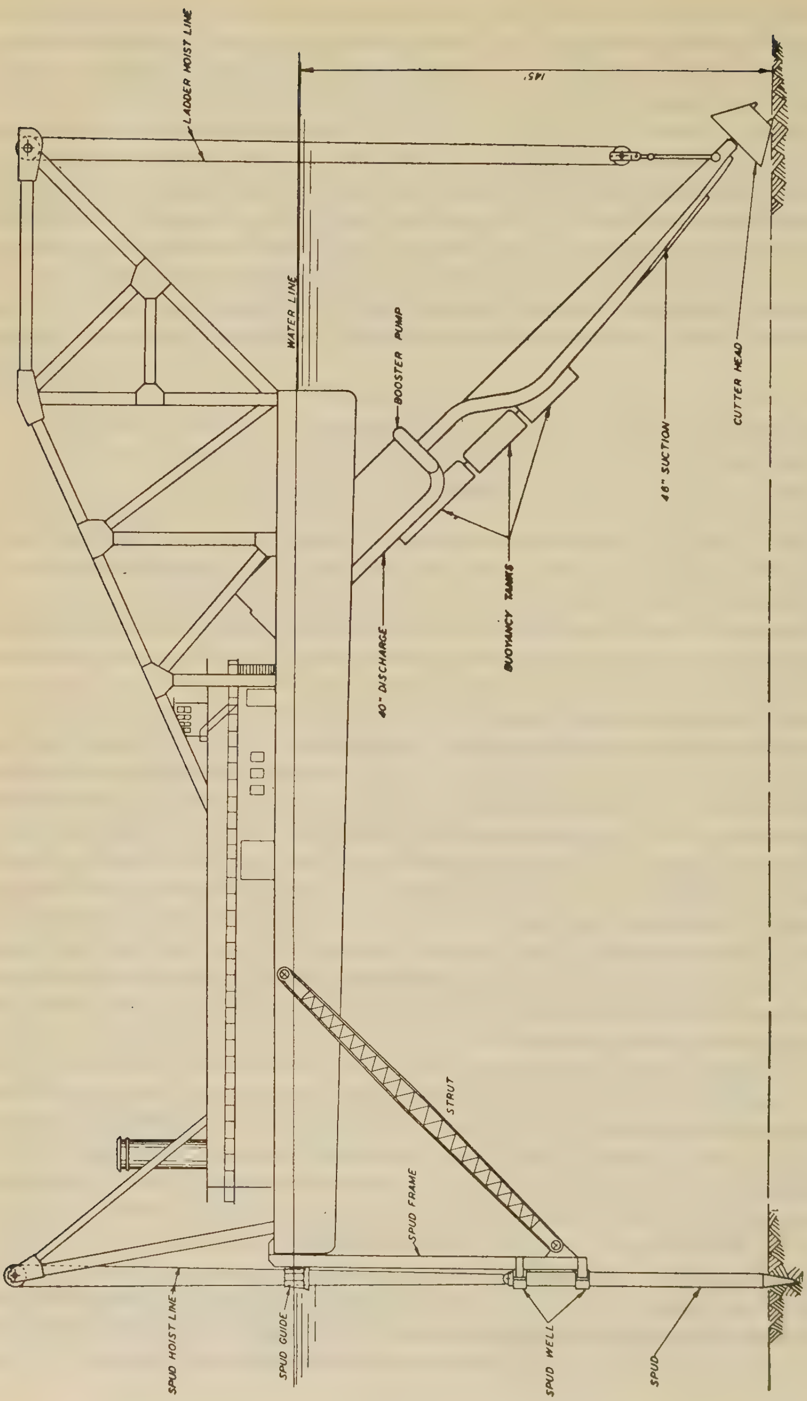

LADDER HOIST LINE
145'
WATER LINE
BOOSTER PUMP
CUTTER HEAD
48" SUCTION
BUOYANCY TANKS
40" DISCHARGE
STRUT
SPUD FRAME
SPUD HOIST LINE
SPUD GUIDE
SPUD WELL
SPUD

Fig. 100.—Proposed 40-In. Hydraulic Dredge

decided that the last scheme was the most practicable of those considered. Fig. 100 shows a general arrangement of the ladder and booster pump.

Two basic mechanical problems that required consideration were: First, the design of a dredge ladder approximately 200 ft long and capable of supporting the booster pump and attendant machinery; and, second, the design of spuds and a spud frame to hold the dredge in place. Both designs were developed sufficiently to indicate that they are entirely practical.

The deep-dredging ladder is provided with buoyancy tanks (Fig. 100) for assistance in hoisting and for better stress distribution. The booster pump is enclosed in a cofferdam to permit removal of obstructions from the booster pump when the ladder is not completely raised.

For holding the dredge in place for deep dredging, a frame extending 90 ft below the water surface is used to hold the spuds. This frame, which is braced to the hull, would be replaced by shorter frames when dredging to more conventional depths. The frame would consist of two main vertical members of sufficient size and section to serve as wells for the spuds which would extend below the wells to a depth of some 160 ft. Fig. 100 shows a schematic layout of this arrangement. General features of the hydraulic dredge are as follows:

Feature	Description
Suction	46-in. diameter
Discharge	40-in. diameter
Hull	265 ft long, 60-ft beam, and 20 ft deep
Power: Main pump	Steam turbine, reduction gears, 8,000 hp normal, and 10,000 hp maximum
Auxiliaries	Electrically driven from turbogenerators
Cutter motor	Electrically driven, 2,000 hp to 2,500 hp
Booster	Electrically driven from shore, 8,000 hp

Bucket-Ladder Dredge.—The bucket-ladder dredge has been used quite extensively in Europe for many types of subaqueous excavation and is generally recognized as a European development. It was the most successful type of dredge used by the French at the Panama Canal, and under the American regime several of the old French bucket-ladder dredges were rebuilt and placed in service, one of these remaining in commission until 1920. Use of the bucket-ladder dredge in the Western Hemisphere, the Dutch East Indies, the Federated Malay States, and Russia has been confined principally to placer mining for gold, platinum, and tin. The bucket-ladder dredge would be used on the Panama Canal project for excavation of blasted rock, work which has been performed successfully by such equipment but which is more unusual than the excavation of gravel and conglomerate.

The bucket-ladder dredge has certain distinct advantages over other types. For example, its operation involves a minimum of lost motion as compared with the dipper dredge; it does not require large quantities of water, as does the hydraulic dredge; it requires relatively little power; and it can dredge to great depths. One bucket-ladder dredge in the California gold fields is excavating to a depth of 124 ft, and 200-ft dredging depths have been predicted for the near future by leaders in the placer dredging industry.

Bucket-ladder dredges of two limiting digging depths are under consideration—one for dredging to a maximum depth of 90 ft and the other for dredging to a maximum depth of 145 ft. The general scheme of operation would be to utilize the 90-ft dredge for excavation to 90-ft depths, followed by deep-digging dredges for excavation between 90-ft and 145-ft depths. For excavation to 90 ft below water level, a dredge hull of shorter length could be utilized than for 145-ft digging; also the length of the digging ladder could be reduced from 239 ft to 163 ft. The original and operating costs of the 90-ft dredge would obviously be less than those of the 145-ft dredge. However, machinery for dredges of both sizes would be identical to permit interchangeability of parts.

The major item of cost involved in rock excavation in the wet is that of drilling and blasting, and, in general, unit costs decrease rapidly as the spacing between drill holes is increased. In the bucket-ladder dredge design, therefore, it was desired to provide the largest practicable buckets to permit maximum spacing of drill holes. Consideration was given to bucket sizes of 6 cu yd, 5 cu yd, 4 cu yd, 3 cu yd, and 2 cu yd. Representatives of the contractors and The Panama Canal agreed that the 2-cu-yd bucket is the maximum size that should be adopted, because of difficulties in casting very large buckets and because of the tremendous weight of the digging ladder and the bucket chain.

The 2-cu-yd bucket has three times the capacity of the largest buckets now in use in placer dredging. A few European dredges have been equipped with buckets having capacities up to 2 cu yd, but the working conditions that they encountered were not severe, the maximum dredging depth being only about 50 ft and the material excavated being sand, silt, shell, and clay. The bucket-ladder dredge *Corozal*, formerly owned by The Panama Canal, was equipped with 2-cu-yd buckets for soft digging and $1\frac{1}{4}$-cu-yd buckets for rock excavation.

Spoil disposal would be accomplished by scows, loaded by conveyers on the starboard and port sides of the dredge. Consideration was also given to a stern conveyer system composed of the necessary conveyer unit for final disposal of spoil ashore. However, detailed study indicated that the latter method of disposal is not desirable because of the very long conveyer lines required and because of difficulties in designing the spud arrangement. By elimination of the stern conveyer system, spud design is simplified. Fig. 101 shows an elevation of the proposed bucket-ladder dredge, including the proposed spud arrangement.

General features of the bucket-ladder dredge design for 145-ft dredging as developed under contract are as follows:

Description	Quantity
Dimensions of Hull, in Feet—	
Length	371
Beam	100
Depth	13
Bucket capacity, in cubic yards	2
Power; number of 1,600-hp, deisel electric engines with identical direct-connected generating equipment	4
Main drive, in horsepower	2,500
Total connected load, in horsepower	5,294
Length of digging ladder, in feet	239

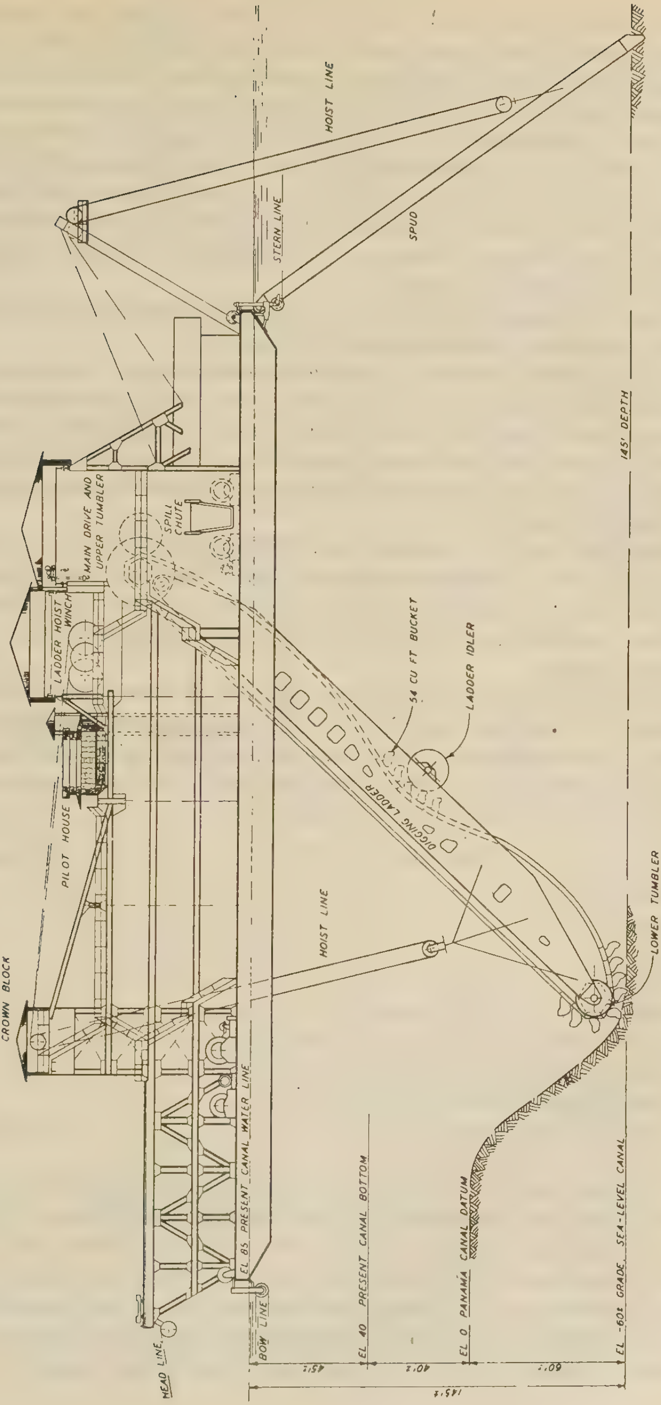

HOIST LINE
STERN LINE
SPUD
145' DEPTH
MAIN DRIVE AND UPPER TUMBLER
SPILL CHUTE
LADDER HOIST WINCH
54 CU FT BUCKET
LADDER IDLER
DIGGING LADDER
PILOT HOUSE
HOIST LINE
LOWER TUMBLER
CROWN BLOCK
EL 85 PRESENT CANAL WATER LINE
EL 40 PRESENT CANAL BOTTOM
EL 0 PANAMÁ CANAL DATUM
EL -60± GRADE SEA-LEVEL CANAL
HEAD LINE
BOW LINE
45'±
40'±
60'±
145'±

FIG. 101.—PROPOSED 54-CU-FT BUCKET-LADDER DREDGE

Dipper Dredge.—The dipper dredge is a proved tool for rock excavation, its principal limiting factor being that of digging depth, since the time required for the digging cycle increases rapidly as depths increase. Although, under the deep-dredging plan, the dipper dredge contemplated could not excavate to final grade from the existing level of Gatun Lake, its maximum dredging depth of 85 ft is considerably in excess of that of any dipper dredges previously developed. Furthermore, large quantities of hard rock are located above the 85-ft depth in Gatun Lake (that is, above El. 0) within the economical digging range of the dipper dredge. After canal construction, the dipper dredge could be used effectively for maintenance.

The dipper dredge design incorporates a balanced hoist and a walking stern spud, which are shown diagrammatically in Fig. 102.

The balanced hoist consists of a movable counterweight connected to the main hoisting machinery by a double three-part hoist. The counterweight is lowered as the dipper is raised, or vice versa, effecting an approximate balance for the dead weight of the dipper and handle, so that practically the entire hoist motor power is available for digging. The movable counterweight is located at the stern.

The stern spud would be mounted as a trailing or walking spud in a well at the stern of the hull. For the purpose of controlling the angular movement and applying power to walk the spud or push the dredge ahead, a powerful hydraulic ram on the main deck would be pin-connected to a heavy frame surrounding the spud.

The dredge would be electrically operated from a 4,000-v, three-phase, 60-cycle alternating current generated aboard by four identical diesel engine generating sets. The combination of any three sets would provide sufficient power for full-speed operation of the dredge, and the fourth would be a stand-by unit General features of the dipper dredge design are:

Description	Quantity
Length of Hull, in Feet—	
Length	200
Beam	85
Depth	14
Dipper capacity, in cubic yards	20 to 30
Power; diesel electric power from 1,000-hp engines (adaptable for shore power)	....
Maximum digging depth, in feet	85
Bail pull, in pounds (stalling)	500,000

Deep Drilling and Blasting for Wet Excavation.—Preparation of rock for excavation under the deep-dredging plan requires subaqueous drilling and blasting to depths far in excess of those ever before encountered in canal excavation. Therefore, it was considered necessary to determine the effects of these depths, as compared to conventional depths, on (1) drilling and blasting operations, including maneuvering of equipment and rate of drilling; (2) requirements of explosives; and (3) dimensions of the blasted rock.

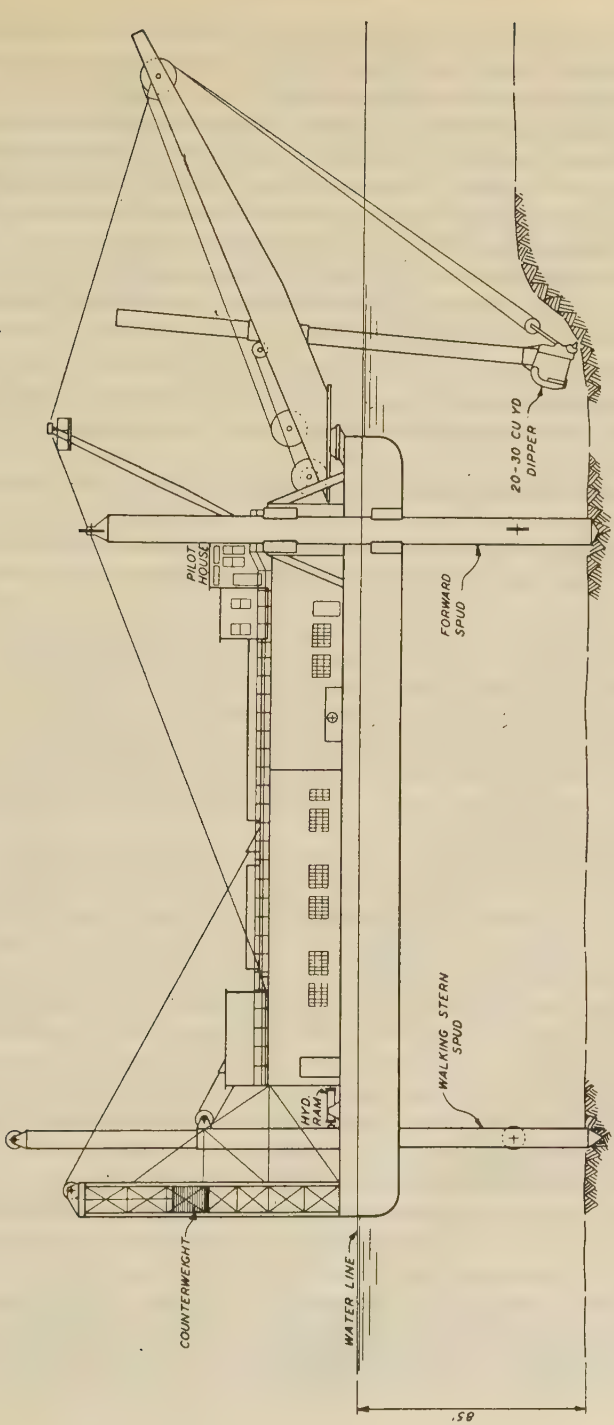

20-30 CU YD DIPPER
PILOT HOUSE
FORWARD SPUD
WALKING STERN SPUD
HYD. RAM
COUNTERWEIGHT
WATER LINE
85'

FIG. 102.—PROPOSED 20-CU-YD TO 30-CU-YD DIPPER DREDGE

To obtain the desired information, a deep-drilling and blasting test was conducted in which rock was drilled, blasted, and excavated in three lifts of approximately 25 ft each from 50 ft to 125 ft below water surface. An accurate record was kept of all drilling, loading, and blasting operations to determine drill performance, the difficulty and time required for maneuvering equipment, setting casing, and loading. Fragmentation of the excavated material was observed to ascertain the effect of hydrostatic head on breakage. The same quantity of explosive per cubic yard was used for each lift.

All drilling was performed by rotary drill, mounted on a barge with necessary winches for maneuvering. Excavation was accomplished by derrick barge equipped with a 3-cu-yd clamshell bucket. From the test, it was concluded that:

(1) Drilling operations with a properly designed drill boat are not appreciably affected by the depths of water that would be encountered in the deep-dredging plan.

(2) The effect of increased depth of water (within the limits of this test) is negligible.

(3) Dimensions of blasted rock obtained in all three lifts did not differ appreciably. The same unit explosive loading and hole spacing were used for each lift.

The rotary drill was selected because it could be utilized in the test without extensive and costly alterations, whereas other types of equipment would have required major reconversion with resulting delay of the test. Furthermore, sufficient data regarding drilling rates at conventional depths with the percussion and rotary types of drills were available to permit reasonably accurate conversion of deep-drilling results with one type of drill to those that might be expected with the other.

In the preparation of estimates for drilling, blasting, and dredging, rock has been divided into three classifications—soft, medium, and hard—on the basis of extensive core borings. Experience on the Panama Canal indicates that, for soft and medium rocks, subaqueous drilling rates are higher and costs are lower, using the rotary type of drill. For hard rock, drilling rates of the rotary and percussion drills are almost equal, or slightly favor the rotary type, but bit costs are higher for the rotary drill. The net result is that in hard rock the drilling cost per foot for the rotary drill may be somewhat higher than that for the percussion type. This latter factor, however, is not important because hard rock comprises only 5% of the total subaqueous drilling and blasting in the project, or 4% of the deep subaqueous drilling and blasting.

Wet Excavation Procedure.—Excavation would be performed in the dry throughout the Gatun Lake section to El. 90 (5 ft above normal lake level) and throughout the Miraflores Lake section to El. 60 (6 ft above normal Miraflores Lake level). Excavation below El. 90 in Gatun Lake and below El. 60 in Miraflores Lake, except for certain sections adaptable to dry excavation behind plugs, would be performed by dredges. Dredging conditions in both the Atlantic and Pacific sea approaches are suitable for the use of existing hydraulic and dipper dredges.

The deep-dredging plan would require that about 60% of the total dredging be performed to depths of 85 ft or less; and about 45% of the total dredging, to depths of 65 ft or less. Therefore, a number of present-day dredges that can excavate to a depth of 65 ft with minor or no alterations and some dredges capable of excavating to 85-ft depths without extensive modification could be used on the project.

Plans contemplate the use of conventional hydraulic dredges for the initial excavation of common material to a depth of 65 ft, or possibly to a depth of 85 ft. Common material from the bottom of the initial excavation to a depth of 145 ft would be removed by the 40-in. hydraulic dredges previously described. The number of passes or lifts that would be required in excavation to final grade would vary with materials and their ability to flow to the dredge suction without objectionable caving of the bank, but usually the excavation would be performed in about 30-ft lifts. Hydraulic dredge spoil would be pumped to the nearest available disposal area, and for most of the material in Gatun Lake only a floating pipe line would be required. Pipe-line lengths would vary from 3,000 ft to 12,000 ft and, when in excess of about 5,000 ft, shore-powered boosters would be introduced into the line. Pipe lines would be laid to disposal areas on both sides of the canal so that there would be no interference with traffic. For example, a dredge working east of the center line would discharge to the east, and vice versa. Under certain conditions, where it might be necessary to pump across the channel, submerged pipe lines would be used.

Following the removal of common material or overburden by hydraulic dredge, rock would be drilled and blasted for subsequent removal by dipper and bucket-ladder dredges. Most, if not all, of the drilling and blasting would be performed by rotary drill boat.

Blastholes would be overdrilled the equivalent of one half of the hole spacing, which would vary from 10-ft centers in hard rock to 14-ft centers in soft rock. It is anticipated that lifts of about 30 ft each would usually be blasted, but this estimate would vary considerably, depending on local conditions and type of excavating machinery utilized. Future drilling and blasting tests may indicate the desirability of drilling and blasting to grade in one operation. Blastholes would be drilled approximately 6 in. in diameter, and for the larger blasts as much as 20,000 lb of dynamite would be fired in one shot.

Following drilling and blasting operations, rock would be excavated by dipper and bucket-ladder dredges, loaded into 2,000-cu-yd dump scows and towed to the dumping grounds by 1,500-hp tugs. Spoil from the Gatun Lake area would be utilized in the construction of the flood-control dams across Gatun Lake, whereas that from Miraflores Lake and from the Atlantic and Pacific approaches would be deposited at sea.

SUMMARY

The methods presented in this paper are feasible and show a decided saving over other methods studied. The estimate showed that approximately 15% could be saved on the cost of the dry excavation alone, whereas the use of the large dry excavation equipment would reduce the required personnel by 9,000 workers, or by approximately 30%. This saving would be reflected in the lowered cost of housing, utilities, services, and mobilization, making an over-all saving of between 10% and 12% for the project. In addition to the cost saving, there would be a saving in the time required for construction. Any reasonable schedule to perform the required excavation by stage dredging would cover a 15-year period, whereas the adopted plan could be scheduled for a 10-year construction period.

DISCUSSION

HANS KRAMER,[60] M. ASCE.—When the late Theodore Roosevelt, then President of the United States, received the Board of Consulting Engineers for the Panama Canal at Oyster Bay, N. Y., on September 11, 1905, he stated the main considerations of that distinguished body as follows:[2]

> "What I am about to say must be considered in the light of suggestion merely, not as direction. I have named you because in my judgment you are especially fit to serve as advisers in planning the greatest engineering work the world has yet seen; and I expect you to advise me, not what you think I want to hear but what you think I ought to hear.
>
> "There are two or three considerations which I trust you will steadily keep before your minds in coming to a conclusion as to the proper type of canal. I hope that ultimately it will prove possible to build a sea-level canal. Such a canal would undoubtedly be best in the end, if feasible, and I feel that one of the chief advantages of the Panama route is that ultimately a sea-level canal will be a possibility. But while paying due heed to the ideal perfectibility of the scheme from an engineer's standpoint, remember the need of having a plan which shall provide for the immediate building of a canal on the safest terms and in the shortest possible time.
>
> "If to build a sea-level canal will but slightly increase the risk, and will take but a little longer than a multilock higher-level canal, then of course it is preferable. But if to adopt the plan of a sea-level canal means to incur great hazard and to insure indefinite delay, then it is not preferable. If the advantages and disadvantages are closely balanced, I expect you to say so. I desire also to know whether, if you recommend a high-level multilock canal, it will be possible after it is completed to turn it into or to substitute for it, in time, a sea-level canal without interrupting the traffic upon it. Two of the prime considerations to be kept steadily in mind are—
>
> "(1) The utmost practicable speed of construction;
> "(2) Practical certainty that the plan proposed will be feasible—that it can be carried out with the minimum risk.
>
> "The quantity of work and the amount of work should be minimized so far as is possible.
>
> "There may be good reason why the delay incident to the adoption of a plan for an ideal canal should be incurred; but if there is not, then I hope to see the canal constructed on a system which will bring to the nearest possible date in the future the time when it is practicable to take the first ship across the Isthmus; that is, which will in the shortest time possible secure a Panama waterway between the oceans of such a character as to guarantee permanent and ample communication for the greatest ships of our Navy and for the largest steamers on either the Atlantic or the Pacific. The delay in transit of the vessels owing to additional locks would be of small consequence when compared with shortening the time for the construction of the canal or diminishing the risks in the construction.
>
> "In short, I desire your best judgment on all the various questions to be considered in choosing among the various plans for a comparatively high-level multilock canal, for a lower-level canal with fewer locks, and for a sea-level canal. Finally, I urge upon you the necessity of as great expedition in coming to a decision as is compatible with thoroughness in considering the conditions."

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[60] Brig.-Gen., U. S. Army (*Retired*), Cons. Engr., San Francisco, Calif.

Substantially that same precept, which now sounds almost prophetic, has guided the present engineering staff and consultants in the comprehensive review of the Isthmian canal situation directed by Congress on December 28, 1945. Now, as then, this waterway between the oceans must assure permanent and ample communication for the greatest ships of the United States Navy and it must accommodate all commercial traffic. The present problem consists of evaluating the military security, or invulnerability, and the traffic capacity of the Panama Canal in the direct light of past experience, both peace and war, and in the dim emanations from a crystal ball. The studies presented in the Symposium cover all aspects of the problem—comprehensively, competently, and objectively.

The writer subscribes unqualifiedly to the conclusions stated by Colonel Stratton. The optimum and the only proper course of action is the conversion of the present canal to a sea-level waterway at a cost averaging about $250,000,000 for 10 years. If it were possible to disregard national security, an expenditure of only $130,000,000 would suffice to meet the needs of commerce for the remainder of the twentieth century; but there is no sound middle ground between these two alternatives.

It is not the writer's purpose to discuss the basic considerations which entered into the engineering studies. Colonel Stratton has stated them so adequately—indeed, so eloquently—that they do not need reiteration. It is desirable, however, to mention briefly a few points of particular interest to the civil engineering fraternity.

One of the most debatable as well as intriguing aspects of the sea-level proposal, but fortunately not one of vital significance in the over-all plan, is the question of the regulation of tidal currents. It is fully recognized that tidal-control structures may be made inoperable by enemy action and therefore, willy-nilly, the canal must be dependable for open channel navigation in time of war. The writer is convinced by present evidence that tidal regulation can be dispensed with—not only in time of war but also for normal peacetime operation. However, with the problem of navigation in tidal currents still under study, he certainly does not question the prudence of making adequate allowance, in preliminary estimates, for the eventuality of tidal-control structures—especially since they represent only about 7% of the total costs.

Undoubtedly the most challenging aspects of the sea-level conversion project, from both the engineer's and layman's viewpoints, are the stupendous excavation job and the accompanying hazard of serious earthslides. The present canal involved a maximum height of cut, vertically, of 514 ft and the removal of something less than 300,000,000 cu yd of material of which about 30,000,000 cu yd was taken out during the French regime. The conversion plan will involve cuts as high as 660 ft, vertically, and will require the removal of something more than 1,000,000,000 cu yd of material, or about four times the original undertaking. Staggering as those simple statistics may seem, modern methods and machinery have been expertly analyzed, during the course of Colonel Stratton's studies, which lead to the firm conclusion that the excavation task can be accomplished efficiently, economically, and without obstructing normal canal traffic. Those who are particularly interested in the excavation problem should read the last Symposium paper, dealing with methods and machinery for both dry and wet excavation.

The seventh paper in the Symposium constitutes a detailed report on excavation slopes and slide prevention. It yields testimony that Colonel Stratton, his staff, and his advisers have tackled the slide problem as competently and thoroughly as any individual or group possibly could. The history of earthslides in the digging of the original canal gives just cause for grave concern in judging the feasibility of much deeper excavation in the very same material. The Canal Zone is a topsy-turvey place where the sun not only rises in the Pacific and sinks into the Atlantic, but where hard bedrock is likely to be only a crust over plastic, slippery substrata; the characteristics of its geology and soils are sometimes diabolically treacherous, inconsistent, and unpredictable. Having given "the devil his due," however, it should be recognized that he is certainly not inscrutable, thanks to the modern technique of soil mechanics and much more complete knowledge of subsurface conditions.

Specifically, three broad conservative principles have been adopted in the general treatment of the slope problem in the critical Cucaracha material. The first of these is the use of generous factors of safety in the static analysis for the design of slopes; secondly, a liberal yardage allowance for the contingency of slides has been made in the basic estimates; and, finally, more detailed drilling and testing to supplement the preliminary data now available are to be performed during the construction stage to determine actual design slopes, station by station—or even closer if indicated. Despite all these precautions it would be foolhardy to assert that there will be no slides in the new excavation; but it is safe to assume that the hazard of slides will be minimized and that the engineers are justified in being confident of a successful job.

Recent political occurrences in Latin America have dramatically highlighted the Isthmian canal situation. Renewed concern for the security of the Panama Canal has brought forth proposals for new canal construction in alternative locations, on the premise that "it is unwise to carry all your eggs in one basket." That adage, however, is fallacious when applied to defensive strategy in respect to the Panama Canal because, as was pointed out in an earlier analysis,[4] guaranteed interoceanic communication is not a matter of carrying eggs but of protecting baskets. The Panama sea-level recommendation of the Governor of the Panama Canal, as presented in Colonel Stratton's paper, has been critically reviewed at the highest levels of United States strategic policy with the irrefutable conclusion that the Panama Canal converted to sea level would be our stoutest and safest "basket."

Philip G. Nichols,[61] Esq.—The authors of the second Symposium paper have produced a clear and comprehensive dissertation on the subject matter. The conclusions reached regarding future tonnage of ocean-going commercial traffic and the conversion of this tonnage into daily peak and average transits produce extremely liberal criteria on which to establish future canal designs. Canal improvement based on these data should provide more than adequate capacity for any foreseeable contingencies, including the eventuation, in any conceivable manner, of either or both of the negative assumptions, *a* and *b*, of Roland L. Kramer (under the heading, "Ocean-Going Commercial Traffic: Future Tonnage of Ocean-Going Commercial Traffic").

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[61] Capt., U. S. Navy; Marine Supt., Dept. of Operation and Maintenance, The Panama Canal, Balboa Heights, Canal Zone.

The effect of the past two world wars was to decrease, markedly, the commercial tolls-paying traffic and to increase the military traffic. The amount of this increase in military traffic is unpredictable and, as the authors state, there is no assurance that in a future war the loss of commercial transits would offset completely the gain in military traffic. However, previous war experience shows that this change in preponderance from commercial to military traffic results in little variation in total transits. During the war years (1942–1946) the annual average was 6,515, whereas the average for the preceding twenty years of peace, was 6,431 transits. The traffic for the first six months of the present fiscal year indicates an annual rate of about 6,000 transits for 1948.

The great danger, from the aspect of canal capacity, has been that this increased military traffic would result in excessive peak days caused by the transit of large task forces. Advances in the sciences (such as those in the field of atomic energy), as they affect air power and new weapons, should reduce the probability of these peak days occurring. Such developments should discourage the heavy concentration of military shipping, at least in confined waters, and tend to force its dispersal. The effect of scientific developments on transportation in general is unpredictable, but the writer's opinion is that it will be to reduce rather than to increase martime freight and passenger traffic.

During the first six months of the fiscal year, 1947–1948, variations in the daily traffic volume continued to be extreme, as they were prior to World War II. The directional inequality of traffic reversed itself and is now preponderantly northbound rather than southbound. This shift may not be significant; undoubtedly, with a return to a normal, peacetime world economy, it will level off or reassume its prewar tendency. This change in directional volume is of importance to the Marine Division of the Panama Canal, since the present canal lends itself to more efficient operation with preponderant northbound transits (rather than with the opposite), because of the existence of a commodious fog-free anchorage in the vicinity of Gatun Locks in the north, and the presence of Gaillard Cut, with its high incidence of night fog, in close proximity to the Pacific terminal. These conditions make it possible to dispatch the heavier northbound traffic through the cut during such greater part of the early daylight period as is necessary. This traffic can be anchored as required in Gatun Lake, and locked down throughout the night. By proper scheduling, the lighter south traffic can be dispatched through the cut during the shorter period remaining, and prior to the hours of high incidence of night fog. Hence, the canal has a somewhat greater one-way capacity northbound and, with two-way traffic, its capacity increases with a northerly preponderance.

The estimate of 1.5 vessels per lockage in the present canal is the least conservative in this paper. On January 13, 1939, a total of 77 warships transited north during a period of 24 hours. This force consisted of 2 large carriers, an aircraft tender, 17 cruisers, 46 destroyers, and 11 submarines. All southbound traffic was discontinued until the next day. These transits were accomplished with 49 lockages, or an average of 1.57 ships per lockage. This later value tends to make the estimate of 1.5 ships per lockage seem safe, but conditions were ideal as to number of ships in each length class. Delays to individual units were acceptable if they expedited the transit of the entire force. On January 10, 1939, a peak day, a total of 35 ships transited the canal; 22 transits were northbound, with 18 lockages or 1.22 ships per lockage, and 12 transits were southbound, with 11 lockages or 1.09 ships per lockage. By accepting delays up to several hours the transits could have been effected with more tandem lockages, increasing the ships per lockage somewhat, but not approaching the estimate of 1.5 unless delays into the next day were accepted.

Messrs. Johnson and Steinborn state that the present canal capacity can be increased to 70 vessels per day by minor improvements, such as the installation of tie-up stations in Gaillard Cut; the modification of lock gates, gate settings, and culverts; and the provision of modern electronic aids to navigation in the cut. This conclusion is based on a number of assumptions with which the writer is in full agreement, but the use of the unit 1.5 ships per lockage for planning purposes appears overly optimistic unless substantial delays are imposed. One might argue that the delays will have to be accepted. If this is granted the situation may tend to rectify itself, because the imposition of delays may result in a loss of traffic. Upon release of the biennial lock overhaul schedule for the winter of 1947–1948, several local steamship agents inquired as to the probability of delays to shipping. They stated that their principals, engaged in shipping between the Antipodes and Europe, had intimated that delays would result in rerouting via the Straits of Magellan or the Suez Canal. This trend of thought was probably considerably influenced by the dollar scarcity existing in all European maritime nations. The presence of such thinking is indicative of the necessity to increase capacity if the canal is to continue to provide adequate service to the shipping industry.

A completely modernized canal with an increase in lock sizes to 200 ft by 1,500 ft, the widening of the cut, and the provision of a Pacific summit-level anchorage would greatly increase the operational efficiency and capacity of the canal. This capacity is sufficiently in excess of the predicted traffic so that the lack of conservation in basic assumptions, if there is any, is more than compensated for. Further discussion of this plan is perhaps beyond the scope of the Symposium. However, the magnitude of the task in man power and material makes it prohibitive if the gain is only in capacity (ship dimensions and traffic volume) and operational facility. Only two commercial vessels, the *Queen Elizabeth* and the *Queen Mary*, cannot be accommodated by the present canal. There is no evidence of plans to duplicate these mammoth liners; in fact, from contacts in the maritime industry, the writer infers that the trend is away from large express passenger liners approaching their dimensions. The Cunard White Star liner *Caronia*, launched in 1947, has an over-all length of 715 ft and a beam of 91 ft. Her gross tonnage is less than that of the new *Mauretania* because the technical developments of recent years have enabled ship designers to produce more earning capacity per ton of ship than was possible in the past. This trend (away from large ships of the Queen class) will receive further impetus from the expansion of overseas air services.

If this reasoning is sound the necessity for increasing the dimensional capacity of the canal must be predicated on combat ship sizes; the problem then becomes one of national defense and a prime requisite is security. Little or no increase in security can be obtained by any high-level lock canal.

The capacities of a Panama sea-level canal are arrived at in the paper by what appear to be conservative assumptions. As in the completely modernized lock canal, the capacities obtained are so far in excess of the predicted volume of traffic or ship dimensions that any errors in the assumptions are covered adequately. The operational difficulties experienced in the present canal, with the exception of fog hazards, are removed. The fog area will undoubtedly be increased, but there is no evidence that prevalence of fog from a daily durational or seasonal aspect will suffer any appreciable change. The operational difficulties imposed by the introduction of tidal current and the extension of the fog area are more than compensated for by the improved width, depth, and alinement of the channel and by the provision of tidal regulation with tie-up stations in the channel above the lock and navigable pass. The continued improvement in ship design, as it produces better maneuverability, and the further development of aids to navigation (particularly in the electronic field) will tend to reduce the hazards of ship operations, in general, and those hazards peculiar to all narrow waterways, in particular. Operation without tidal regulation is entirely feasible, and experience will indicate that this method can be adopted with safety, except in unusual circumstances.

The writer concurs in the final summary of the Johnson-Steinborn paper, as far as it relates to the capacities obtained by converting the present canal to a sea-level canal, or by completely modernizing it to a terminal lake canal. He doubts the wisdom of depending on the minor modifications to provide adequate canal capacity through the remainder of the twentieth century, if Professor Kramer's predictions of traffic growth are accepted. The minor improvements described would not increase the security or the dimensional capacity but should provide sufficient canal capacity to serve commercial shipping for many years—if not through the entire century. Canal improvement in excess of such minor modifications should be based entirely on the predicted effect of present and future scientific developments, on commercial and combat ship dimensions, and on canal security. The question of security seems relatively simple to answer. The only method of reducing the over-all vulnerability of the canal to present or projected weapons is a change to a sea-level canal. The extent of this gain in security is important; perhaps it is so small in comparison to the cost that the possibility of damage should be accepted as a calculated risk. The effect from a dimensional aspect is more involved and less easy to predict. Logical reasoning or intuition (whichever one prefers to call it) induces the writer to believe that the effect of scientific progress will tend to make numbers of ships, both commercial and combatant, more profitable and more valuable than the great size of individual units. The effect of the new sciences on weapons and on shipping, both commercial and military, should be the subject of exhaustive study, because this is the crux of the problem.

GEORGE B. PILLSBURY,[62] M. ASCE.—The basic differential equations for hydraulic flow in open channels have long been established. For the varying flow in tidal channels the term dependent on the acceleration of the velocity with respect to time must be included. On the other hand, the turbulence set up by the slowly changing tidal currents cannot differ appreciably at any moment from that produced by a steady current of the same velocity. The friction term may then be taken as that applicable to steady flow under the same conditions, although it must change its sign with the direction of the current. Since the slope of the water surface, in a given direction, is alternately upward and downward, it is more convenient to consider surface slopes and heads, as well as elevations of the water surface, as positive in an upward direction. The basic equations for the currents in a tidal channel may then be written

$$\frac{\partial y}{\partial x} + \frac{v}{g}\frac{\partial v}{\partial x} + \frac{1}{g}\frac{\partial v}{\partial t} + \frac{v\,[v]}{C^2 R} = 0 \qquad (3a)$$

and

$$\frac{\partial Q}{\partial x} + b_W \frac{\partial y}{\partial t} = 0 \qquad (3b)$$

In Eqs. 3*a* and 3*b*—at any given cross section distant x from an assumed origin, and at the time t—y is the elevation of the water surface above any assumed datum; v is the algebraic value of the mean velocity in the cross section and $[v]$ its arithmetical value; R is the hydraulic radius; C is the Chézy coefficient applicable to the hydraulic radius under any of the formulas derived for steady flow; b_W is the width of the water surface; and Q is the discharge.

In a navigable channel, the velocities cannot be so great that the second term in Eq. 3*a* (which may be called the "velocity-head" term) becomes of major importance.

The form of these equations is such that the currents at any station along a tidal channel must depart somewhat from simple harmonic fluctuations, even when the tides at the entrances are assumed to have such fluctuations. These departures may be eliminated by omitting the velocity-head term, disregarding the changes in the area of the cross sections with the variations in y, and adopting an harmonically varying friction term most nearly conforming to its true fluctuation. Under these assumptions, a mathematical solution of the equations for tidal flow in a long canal of uniform cross section was derived many years ago.[63] The resulting exponential equations are unwieldy, and do not afford a satisfactory method for computing even an approximation to the currents in a canal of varying cross section, such as is proposed at Panama.

Under the same assumptions, Eq. 3*a* affords an algebraic relation between the amplitude and phase of a simple harmonic fluctuation of the surface slope in an elementary reach of a tidal canal and those of the resulting harmonically varying current; and Eq. 3*b*, the means for determining the changes in the current from reach to reach because of the storage and release of water with the rise and fall of the tide, as well as the variation due to changes in the cross section. Through a procedure developed by the writer,[8] which does not warrant repetition here, the harmonically varying, or "primary" currents produced by "primary" tides (with a simple harmonic fluctuation) at the entrances to a canal may be computed from these relations by successive approximations. For these computations, the canal is divided into sufficiently short reaches, such as those used in backwater and similar hydraulic computations. In a canal of the length and dimensions proposed at Panama, three successive computations afford a satisfactory determination of the amplitudes and phases of the "primary" currents in the several reaches and of the tidal fluctuations which establish the surface profiles through the canal.

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[63] "The Cape Cod Canal," by William Barclay Parsons, *Transactions*, ASCE, Vol. LXXXII, December, 1918, p.1.

By a somewhat laborious set of computations, the primary currents and surface profiles may be adjusted at sufficiently short intervals of time (such as the half lunar hour) to develop their distortions because of the factors disregarded in their derivation—including any departures of the selected entrance tides from the simple primary tides, from which the primary currents were determined. For a first adjustment—in order to develop the major distortions—the currents and discharges in the middle reach of the canal may be corrected (at the adopted time intervals) to conform to the correct values of the friction term as it varies with the changing values of R and C with the rising and falling tides, and also to the consequent variations in the acceleration term. The currents in the other reaches are then derived by dividing the resulting discharges in these reaches by the varying area of their cross sections. If the entrance tides selected for the computation depart from a simple harmonic variation, these departures are distributed between the several reaches.

A more involved second adjustment includes the effect of the velocity-head term, the entrance and recovery heads, and the minor changes in the water-surface profiles resulting from the current distortions developed in the first adjustment. The diagrams in Fig. 35 indicate the smallness of the distortions developed by this second adjustment, even when the current velocities are as large as those that would occasionally occur in an uncontrolled sea-level canal at Panama. It may be noted that the model tests afford a highly satisfactory confirmation of the adjusted currents as computed; but without such adjustment the primary currents afford an approximation sufficient for all practical purposes.

Needless to say, the currents derived by these calculations are the mean values at a given cross section at a given time. The midstream currents encountered by vessels navigating the canal would be somewhat larger. Current measurements in the Cape Cod Canal in Massachusetts before it was enlarged to its present dimensions showed midstream currents 25% in excess of the mean; but the model tests are understood to show that, in a canal of the dimensions proposed at Panama, the midstream currents are not much in excess of the mean velocities.

It should be emphasized, perhaps, that the tides at the Pacific entrance, which would largely determine the currents in an uncontrolled sea-level canal at Panama, run through a marked variation from springs to neaps in a period of about a week, and back to springs the following week. The semidaily fluctuations of the spring tides vary from fortnight to fortnight, but occasionally attain a range of about 20 ft, or 10 ft above and below mean sea level at the time. The average spring range is given as 16.4 ft. The neap tides may have a semidaily fluctuation of as little as 3 ft above and below their mean elevation, although sometimes they are as much as 6 ft above and below that level. The vagaries of the small tides at the Atlantic entrance are described by Messrs. Meyers and Schultz, and as stated by them the long-term mean sea level at the Pacific entrance is nearly a foot above that at the Atlantic entrance. In addition, the monthly mean sea level at the Pacific entrance varies by about a foot during the year, coincidently with the changing temperature of the water in the Gulf of Panama. Long-term records do not show marked disturbances of the water levels at either entrance such as those sometimes produced in other regions by violent storms.

Because of the tidal variations, the strength of the currents in an uncontrolled sea-level canal would vary widely each week, and from week to week. The currents would not reach their strength simultaneously throughout the canal, and the currents at the one end of the canal would be increasing quite rapidly at the time of slack water at the other end. It does not seem possible to time the transits of the large traffic through the canal in such manner so as to avoid the maximum currents for the day. Furthermore, shipping will be concerned with the currents that might be encountered at any date of arrival, rather than with those which would be met were their arrivals so timed as to afford a passage at the time of neap tides, or even when the currents would have their average strength. For this reason, the principal emphasis is given to the currents produced by the highest spring tides to be expected ordinarily at the Pacific entrance. These should not be interpreted as a prediction of the currents usually encountered during a transit of an uncontrolled canal.

As shown in the Meyers-Schultz paper, the current strengths would increase toward the Atlantic entrance where the tides are the least. The main reason for this seeming anomaly is the reduction in the cross section of the canal in this direction at the higher tidal stages, as is clearly shown in Fig. 33. Fig. 33 also shows that the maximum inflow and outflow is somewhat the larger at the Atlantic entrance, as the flow through this entrance has a larger part in the filling and emptying of the tidal prism of the canal than the flow through the Pacific entrance. The maximum currents in a tidal canal are generally, but not always, found at whichever entrance the tidal range is the least.

The extended investigations, computations, and tests described by the authors of the fourth Symposium paper have removed any doubt as to the strengths and the timing of the currents to be expected in an uncontrolled canal at Panama of the dimensions now proposed. Although all evidence indicates that such a canal could be navigated by the largest vessels in the strongest currents that might at times be encountered in a transit, its navigation would be safer and easier if the currents were reduced to an insignificant value by the construction of tide locks. A grounding of a vessel against the bank of the canal, because of failure of its steering gear or propelling machinery, might result in the complete loss of the vessel and the temporary blocking of the canal if it occurred at a time when the currents were as strong as they often would be were they uncontrolled. Tide locks, moreover, would eliminate the possibility of a more or less serious maintenance problem because of the deposit in the approaches and at the teminal wharves of material eroded from the bed and banks of the canal. Without entering into a discussion of the relative advantages of a high-level lock canal and a sea-level canal, the writer believes that (if the latter is decided upon) the inclusion of tide locks is advisable, but that the canal would remain usable even if these were destroyed by hostile air attack.

KENNETH S. M. DAVIDSON,[64] ESQ.—An extensive series of model experiments relating to the handling of ships in restricted channels is presented in the fifth Symposium paper by Messrs. Lee and Bowers. This is a subject on which there has been all too little reliable information in the past.

The writer is particularly impressed by the bold choice of method, which makes a "direct frontal attack" on the problem of selecting channel dimensions. The phenomenon of "bank suction" and the practical difficulties to which it may lead have been recognized for many years. There has been, too, a certain amount of scattered information on the force and moment magnitudes introduced by bank suction, and on other technical factors involved. Ultimately, however, the proper selection of channel dimensions depends only partly on these basic aspects of the matter, and rests primarily on the answers to the specific questions: "What is feasible, and what is practicable, in the actual business of steering ships in channels?" Hence, the method of employing free, self-propelled models, with provisions for steering them in the same fashion as full-scale ships, strikes at the heart of the problem.

The adoption of this method, however, does not in itself dispose of all questions. There are two separate aspects of steering: (1) "What pattern of rudder angle changes is required under given circumstances, and is it physically possible for the rudder to control the ship?" and (2) "What responses will the helmsman make—that is, how successful will he be in getting the rudder to the proper angles at the proper times?" Of course, if the circumstances impose requirements beyond the capacity of the rudder for control, steering is obviously impossible and the helmsman's responses are of no consequence. On the other hand, it does not follow that steering is necessarily possible if the requirements are not beyond the capacity of the rudder. Even with an ideal helmsman, it might be impossible to maneuver the ship into the kind of nearly steady motion needed for practical operation, and, further, it might be impossible to do this with a "human" helmsman although possible with an ideal one. Thus, the helmsman's responses may become an exceedingly important, if not a controlling, consideration.

The employment of professional Panama Canal pilots to steer the models during the experiments, therefore, is at least as significant a feature of the method adopted in this instance as the use of free, self-propelled models. There is, however, one last, but by no means least, question. A pilot may have little trouble in handling a full-scale vessel, and plenty of trouble in handling its model, under exactly corresponding circumstances. A 1:45 scale model has only about 1/90,000 of the ship's mass and, perhaps more important, goes through the same (geometric) cycle of events in about one seventh the time.

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One would expect a professional ship's pilot to be bothered by both these ratios, and particularly by the time ratio. A reduction to one seventh of the allowable reaction time could, in principle, make quite impossible a chain of human responses that would otherwise be simple enough. Hence, there must have been, at the start of the present experiments, some misgivings regarding the chances of success, and it is no small tribute to the experimenters that they had the courage to go ahead nevertheless.

They seem to have been well rewarded. The writer talked with one or two of the professional pilots while the experiments were in progress, and can vouch for their feeling of confidence in the method. One can rationalize, of course, that nothing happens very fast in a ship and that, especially at the low speeds imposed by canal operation, rudder movements are seldom needed more frequently than, say, once in 10 sec, which corresponds to about once in 1.5 sec in a 1:45 scale model, so that no great strain is imposed on the pilot. Fighter airplane pilots are generally believed to respond to control needs in approximately 0.2 sec and the ordinary person driving an automobile, in from 0.4 sec to 0.5 sec; but it is easier to accept this kind of reasoning after the event than before. Flying-boat models of only 1:12 scale are being handled in laboratory tests—mostly, however, with specially trained pilots.

Without belaboring the point unduly, the writer nevertheless considers that the attempt to integrate ship pilots' reactions with the more purely technical factors constitutes perhaps the most significant feature of the present experiments, and one that deserves emphasis. These experiments are among the very few instances in which "the feel in the seat of the pilot's pants," a factor that has been taken seriously by airplane designers for years, has been taken directly into account in a ship handling problem. The Panama Canal pilots have expressed satisfaction that their experience and advice were sought and they have evinced interest in having similar model facilities permanently available for pilot training and practice.

The scope of an experimental project of this type can all too easily expand to impossible proportions, as anyone can testify who has tried to lay down a program for one of them. Strict economy, sometimes amounting to almost "brutal" curtailment, is usually unavoidable. This accounts, of course, for the limited number of ship types worked with in the present instance. The writer does not believe, however, that a serious limitation has been imposed thereby on the generality of the experimental indications; the ship types were well selected.

The experiments go far toward providing a "workable" answer to the problem at hand. Even a casual examination of the force and moment data in Figs. 47 and 48 shows, for instance, that "perfect" equilibrium may be difficult to attain, but that an approximation to equilibrium is good enough in practice; that moment balance is probably more important than lateral force balance; and that the effectiveness of the rudder varies with the lateral position of the ship in the channel. Similar conclusions could probably be drawn regarding the effect of speed, and of other factors, and these would form a substantial framework upon which to hang further work directed toward a more "complete solution"—a phrase used by Messrs. Lee and Bowers.

Going a little more into detail, consider the 720-ft tanker traveling at 4.5 knots in the 770-ft channel of 45-ft depth, on a path 60 ft to the right of the center line. Then (Fig. 48) the force toward the bank is 1.0 ton and the counterclockwise moment, 400 ton-ft. The displacement of the ship, for comparison, is approximately 50,000 tons. A rudder angle of 3.3° is sufficient to offset the moment, the water force on the rudder acting away from the bank. Since the rudder is about 350 ft aft of the center of gravity, the rudder force must then be about 400/350, or 1.14 tons. Fig. 47 shows, however, that in free running a rudder angle of 5° is in fact needed, corresponding presumably to a rudder force of 5/3.3 times 1.14, or 1.7 tons. The forces and moments then are not balanced, and so the ship must assume a small yaw angle. The required force is $1.7 - 1.0 = 0.7$ ton, toward the bank; the required moment is $(1.7 \times 350) - 400 = 200$ ton-ft, counterclockwise. This combination would require the ship to be yawed toward the near bank and the force resulting from yaw to be concentrated at a point on the hull aft of the center of gravity and very near the stern. A yaw toward the near bank, however, does not agree with the experimentally determined "equilibrium" conditions and a yaw force concentrated aft of the center of gravity is out of line with independent open water tests of models of similar vessels. One is almost forced to the conclusion that it must be very difficult, if possible at all, to work a ship into a position of complete equilibrium. Apparently, however, small unbalances are not serious.

For instance, if the ship is assumed to be yawed away from the near bank in a manner to produce a balance of moments, the resulting force unbalance can be shown to have very minor consequences. Moment equilibrium would insure a constant ship heading and could be attained with a slight counterclockwise yaw. This attitude would result in a force of perhaps 0.7 ton away from the bank. The force unbalance would then be 1.7 tons of rudder force plus 0.7 ton of yaw force minus 1 ton of bank suction force, or a net of 1.4 tons of force away from the bank. With a ship weight of about 50,000 long tons, the accelerating force proves to be a very small fraction of g—that is, $\frac{1.4}{50{,}000 \times 2{,}240/2{,}000} = 0.25 \times 10^{-4}\, g$. Neglecting the resistance of the ship to lateral motion, this accelerating force, α, in 100 sec, or about the time required for the ship to travel one length, would move the ship laterally a distance of $x = \frac{1}{2}\alpha t^2 = 0.125 \times 10^{-4} \times 100^2 = 0.125$ ft. In ten ship lengths (which is more than a mile and would require some 16 min of travel time) the lateral travel would be only 12.5 ft, an almost insignificant amount.

It appears, then, that unbalance of forces is acceptable as long as it is not excessive; unbalance of moments is more serious, however, since a small change of heading soon results in a change of lateral position. The pilot's ability to apply forces and moments is limited to use of the rudder (with perhaps some help from the propellers in ships having more than one); the saving grace is that the rudder can apply a substantial moment without imposing a dangerously large lateral force.

The data amply bear out the danger of having canal dimensions that are too small. If the limit of 5° of rudder, as set by Messrs. Lee and Bowers, is accepted (and it seems entirely reasonable to do so), Fig. 47 shows that the pilot must keep his ship very close to the center line. If the ship wanders appreciably, the forces and required rudder angles mount rapidly. The bank suction force on a 720-ft tanker 200 ft off the center line of a 770-ft channel (Fig. 48) is 9 tons (about half of the propeller thrust required to drive the ship), and a rudder angle of 24½° is required to provide a "workable" equilibrium (Fig. 47). In the narrower 500-ft channel, the conditions are almost identical when the ship is only 100 ft off the center line. Thus, if a ship is allowed to move very far from the center line, complications mount rapidly—and more rapidly in a narrow channel than in a wide one. Here lies the real danger, it seems.

Taken as a whole, the fifth Symposium paper does not seem to do full justice to the experiments—or at least to the experimental method. The method appeals to the writer as a sound one for the practical purpose of selecting channel dimensions in relation to operating conditions. Beyond that, it seems a logical choice for the deeper purpose of clarifying the issues preparatory to undertaking "a complete solution to the general problem." It could scarcely have been expected to yield a complete solution. Messrs. Lee and Bowers might well have made more of the method in its own right, by emphasizing practical conclusions more strongly, and by indicating the gaps that will need to be filled before a complete solution is obtained.

It may be pertinent to state herein that the general problem of ship control —steering and turning—is currently under study, both analytically and experimentally, by several agencies and organizations interested primarily in ships. The work described by the authors of the fifth Symposium paper deals with a special problem of ship control, and demonstrates the need for continued study of ship handling problems, and of the fundamental factors that affect control.

JOEL D. JUSTIN,[65] M. ASCE.—At the time of construction of the Panama Canal the late Donald F. MacDonald was a relatively young geologist in the employ of the canal. He realized during the construction that the philosophy, still held by some engineers, of "Let her slide and dig it out afterwards until she stops sliding" was unnecessarily expensive. As the result of full-scale laboratory experiments in the great outdoors, he decided that it was much better to determine the slope that would be safe in advance and cut to that slope, thus avoiding considerable unnecessary excavation. The essential correctness of his conclusions has been demonstrated throughout the years.

Mr. MacDonald had the advantage of observing slides which had already taken place in the canal. He did determine in advance one slope in Cucaracha and the engineers cut to that slope. The slope stands today substantially as he left it as a testimonial to his judgment.

As Messrs. Binger and Thompson have shown, the history of the Panama Canal has been deeply concerned with the Cucaracha formation. It is difficult to find a piece of this formation large enough to test, and when a piece is found it shows a strength much greater than the 10° internal friction and the 2,300-lb cohesion per sq ft stated by the authors of the seventh Symposium paper.

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[65] Cons. Engr., Justin & Courtney, Philadelphia, Pa.

However, some of the "weakest materials could not be tested." Much of the Cucaracha formation breaks down to little slickensided, more or less cubical, pieces of perhaps ½ cu in. Each piece feels slippery to the touch.

Confronted with conditions like these, Messrs. Binger and Thompson have done well in adopting the method of determining Cucaracha slopes which they did. Friction was determined by test on small slickensided samples, and cohesion was determined from an actual slope just south of Zion Hill which has stood for many years. In other words, the friction as determined by laboratory test was used, and the necessary cohesion required to keep the slope in place with a factor of safety of 1 was determined. It should be noted herein that the error, if there be one in the adopted method, would be on the safe side for great heights.

In setting slopes for a sea-level canal, the factor of safety decided on from the extensive Binger-Thompson studies was 1.3 for sudden drawdown.

With the accumulated mass of data now available, the establishment of safe slopes for static conditions has become relatively simple compared with the problem of establishing safe slopes when the Cucaracha formation is overlain with the harder denser rocks.

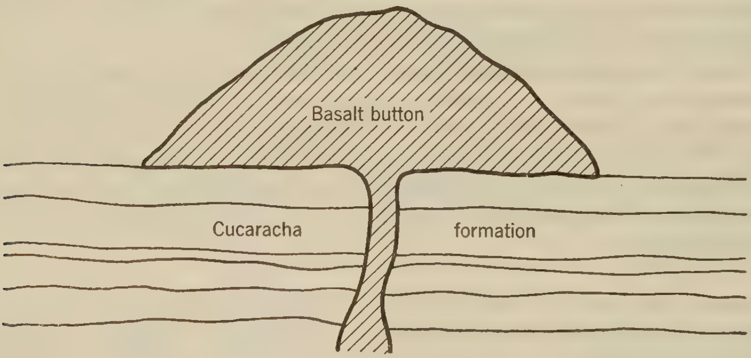

Fig. 103.—Igneous Material Breaks Through Strata of Cucaracha Forming Mushroom-Like Structure (Idealized)

As Mr. MacDonald explained to the writer, many of the hills prevalent in the Canal Zone were like mushrooms. They had been formed by molten volcanic rock forcing its way up through the soft greasy Cucaracha formation roughly forming the stem of a mushroom-like structure. When it had passed through the Cucaracha it blossomed out, forming the top of the mushroom. On cooling these buttons of igneous material became agglomerates or basalt, or other igneous rock (Fig. 103).

The resulting pressure on the Cucaracha, tending to crush it because of the overlying igneous rock, was often excessive. The surrounding softer Cucaracha was often worn away by erosion, leaving the igneous rock buttons sticking up on the landscape as the caps of the steep hills. In many cases the erosion of the Cucaracha left slopes unstable because of the heavy overburden of denser rock which was overhanging and which tended to crush the weak Cucaracha. In particular, the basalt often gave way and broke up in sliding down the steep hill. Hence, slides are nothing new in the Canal Zone, but started long before the white man discovered America (Fig. 104).

Particularly hazardous in canal excavation is the condition in which basalt overlies the Cucaracha formation. The structure of the basalt, which is columnar, is geologic evidence that this igneous rock is unable to span material distances by cantilever action, which means that most of the pressure from the weight of the basalt will be directly downward. Zion Hill is one of the cases in which an overburden of basalt on the Cucaracha formation has led to considerable slides.

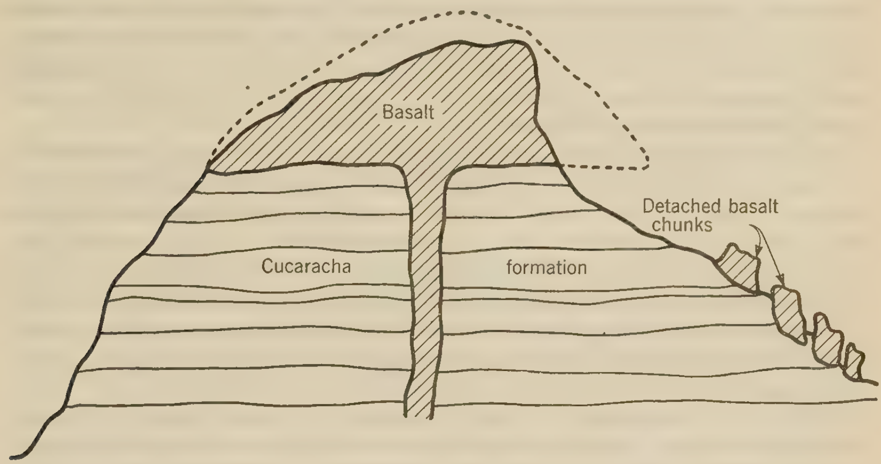

FIG. 104.—EFFECT OF WEATHERING ON MUSHROOM OF BASALT (IDEALIZED)

To make canal slopes stable under such conditions requires considerable investigation and study of each particular case. In some cases it requires flattening the slopes to a Cucaracha slope as 1 on 6.3 for a 300-ft cut.

W. H. McALPINE,[66] HON. M. ASCE.—An excellent general description of the construction features of the proposed sea-level canal is presented in the last Symposium paper. It is mainly devoted to a description of the methods considered for (*a*) conversion to a sea-level canal by multiple-stage and single-stage dredging, (*b*) dry excavation, and (*c*) wet excavation. Such features involving conventional methods of construction as tidal-regulating structures, flood-control structures, and housing facilities are only mentioned.

(*a*) *Conversion to a Sea-Level Canal by Multiple-Stage and Single-Stage Dredging.*—It was found that conversion by the multiple-stage method is best accomplished in three steps. In this plan, the construction of two temporary single-lift locks is proposed in order to provide two lanes of traffic, because without them dependence for uninterrupted traffic would be placed on a single lane. The single-stage conversion, or deep-dredging plan, permits traffic and pool levels to be maintained until the tidal structures have been completed and the canal made ready for draining to sea level in a single operation, requiring only a few days.

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The advantages of the single-stage conversion plan are quite apparent, as shown by the authors of the last Symposium paper, with the exception that it involves excavating in Gatun Lake from a pool elevation of +85 to −60, or a total depth of about 145 ft. Dredging to such depths has very little precedent. However, designs for hydraulic and ladder dredges to excavate to such a depth have been prepared by expert manufacturers. Furthermore, it is understood that one of each type will be constructed for trial prior to their actual needs. This should serve to correct any defects that may develop and also to furnish valuable data to bidders. It is the writer's belief that the deep-dredging plan is feasible and should be adopted.

(b) *Dry Excavation.*—It is noted that of the total excavation of 1,000,000,000 cu yd or more, three fourths, or 750,000,000 cu yd, is dry excavation, which (except for about 50,000,000 yd) is to be handled with large shovels and draglines of 25-cu-yd capacity or more. The 50,000,000 cu yd lying above El. 270 is to be excavated by 5-cu-yd equipment and hauled by truck or other land carriers to spoil areas. The remaining 700,000,000 cu yd of dry material which is to be excavated by large shovels and draglines will be deposited in dump scows of 2,500-cu-yd capacity and towed to Gatun Lake or to the Pacific Ocean for disposal. The average towing distance is stated as 15 miles.

The plan of excavating a system of auxiliary or feeder canals within the excavation areas to permit loading the scows directly from the draglines and shovels is an ingenious one and has considerable appeal. The writer is quite aware of the economy of water transportation over land methods when the loading facilities and the waterways are adequate, but questions whether conditions would exist under the proposed plan sufficiently favorable to justify disposal of such a large proportion of the excavation in scows, particularly that in the upper levels. Conditions for loading the material from the draglines into scows lying alongside the sloping banks are particularly bad and quite different from loading a scow lying alongside a dredge. These scows must be either constantly held in position by a tug, or mooring facilities would have to be provided. Dropping the material directly into the scow from the higher elevation is dangerous if not unfeasible. Lowering the bucket directly over the scow from such a height is time consuming. Then, there is the time lost waiting for scows delayed by fog and other causes tending to disrupt a highly organized schedule of towing operations. Towing approximately 70,000,000 cu yd of material, or 100,000,000 tons per yr, an average distance of 15 miles—coupled with the commercial traffic—would slow up scow movements to some extent.

The writer questions the statement by Messrs. Rose, Dye, Watson, and Crook that the average cost of disposal by truck haul would be approximately twice as much as by scow haul. In this connection, it is understood that most of the excavation for the Third Locks Project on the Atlantic side was removed by truck haul at low cost. The plan of dry excavation should be re-examined with a view to a much greater disposal by land operations than now contemplated by limiting it to that lying above the 270 contour. An examination of the contours indicates that a considerable proportion of the excavation lying below the 270 contour could be disposed of economically by truck, belt conveyor, or other means of land transportation. This should tend to result in a better balanced and more flexible system of disposal.

(*c*) *Wet Excavation.*—The plan outlined for wet excavation, including the deep-dredging scheme, has been studied carefully and appears quite feasible.

In general, the construction, planning, and methods for conversion of the existing Panama lock canal to a sea-level canal, as outlined by the authors of the last Symposium paper, appear basically sound. Further and more intensive studies and conferences with contractors and equipment suppliers will modify or confirm the estimate and methods for disposal of excavated material by scows.

W. E. R. COVELL,[67] M. ASCE.—After 40 short years engineers are again discussing the same problem so bitterly fought over in 1905—the problem of the sea-level canal versus the lock canal at Panama. Once more a basic, and now a controlling, item is safety.

In February, 1904, the Hay-Bunau-Varilla Treaty was ratified and the Isthmian Canal Commission was appointed, consisting of two retired officers of the United States Army and the United States Navy (Maj.-Gen. George W. Davis and Rear-Admiral John G. Walker), William Barclay Parsons, Hon. M. ASCE, and four other civilian engineers. In June, 1904, John F. Wallace, Past-President, ASCE, was appointed Chief Engineer of The Panama Canal, being notified of his selection by a letter from General Parsons.[68] President Theodore Roosevelt appointed a new commission in March, 1905, with Theodore P. Shonts as chairman. In June of the same year, Mr. Wallace resigned and John F. Stevens, Past-President and Hon. M. ASCE, was appointed in his place. The latter stated[69] that the question of the lock or sea-level canal was still undecided, and that the advocates of both were pressing their claims on Congress. About this time, Congress authorized the appointment of a consulting board made up of eminent foreign and American engineers to advise President Roosevelt on this question. Their reports[70] were made public in 1906.

The "majority report," signed by General Davis, General Parsons, William H. Burr, William Henry Hunter, M. ASCE, Ad. Guérard, Eugen Tincauzer, J. W. Welcker, and E. Quellennec, advocated the sea-level route, whereas the "minority report," signed by Alfred Noble, Past-President, ASCE, Henry L. Abbot, Frederick P. Stearns, Past-President, ASCE, and Joseph Ripley and Isham Randolph, Members, ASCE, recommended a lock canal. The latter were all Americans, whereas five of the eight signers of the majority report were Europeans.

The main argument submitted by the majority was one of safety as is acknowledged in the minority report,[71] as follows:

> "The most plausible arguments advanced by advocates of a sea-level canal to justify its greater cost and the greater time required to build it are the alleged danger of carrying away the lock gates at either end of the summit level if a ship moving at speed should strike them, and the possible danger to structures through malice or in time of war."

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[67] Parsons, Brinckerhoff, Hogan and Macdonald, Engrs., New York, N. Y.

[68] "History of the Panama Canal," by Ira E. Bennett, Historical Publishing Co., Washington, D. C., 1915, p. 187.

[69] "Early Days at Panama," by John F. Stevens, *Engineering News-Record*, Vol. 115, 1935, p. 216.

[70] "The Report of the Board of Consulting Engineers for the Panama Canal," *Engineering News*, Vol. 55, 1906, p. 202 and p. 234.

[71] *Ibid.*, p. 238.

The very first paragraph in the majority report[72] covers "safety and protection," part of which states as follows:

> "The modern lock for ocean going vessels is a work which an enemy through stratagem could with no great difficulty put out of use in an hour or even a few minutes. If a small detachment from the enemy's fleet, armed with high explosives landing secretly by night at some nearby shore or inlet, hiding in the neighboring jungle, should surprise the canal guards, or if a few malicious individuals in disguise should succeed in exploding against the lock gate under high water pressure as much explosive as they could carry, they could disable the lock and could probably cause damage of such colossal magnitude as would put the canal out of use for months. This danger is one that very strict watch and guard might prevent in great measure, but it is well nigh impossible to provide effectually and always against such peril."

The last sentence quoted is especially true today, even though the writers of the report in 1905 did not and could not have been expected to foresee either the modern airplane or the atomic bomb!

Another interesting section of the 1906 majority report[72] quotes part of the instruction of President Roosevelt to the Consulting Board dated September 11, 1905, as follows:

> "I desire also to know whether if you recommend a high level multi-lock canal it will be possible after it is completed to turn it into or to substitute for it in time a sea level canal without interrupting the traffic upon it."

The Board answered that query[72] in the following language:

> "1. That it is possible to turn any lock canal which it [the Board] has considered into a sea level canal without interrupting the traffic upon it. 2. That it is practicable from an engineering standpoint to transform any lock canal which it [the Board] has considered, into a sea level canal but that the cost and difficulty of such transformation would be so great as to render such a change impracticable from a financial standpoint until the traffic should have so increased as to tax the capacity of the lock canal, or until other good and sufficient reasons existed for such a change. 3. That if a sea level canal is to be constructed in the future, it should be built directly without first building a lock canal."

The majority report recommended the sea-level canal[73] because of its "indispensible features of a convenient and safe ship canal" and estimated the cost of its construction to be $247,000,000, with a total cost including houses, floating equipment for operation, etc., at $333,000,000.[74] It estimated the time of construction at from 12 to 13 years.

Increased economy and speed won over increased safety, however, and the lock canal was approved and built. Today Colonel Stratton is recommending (and most properly) the expenditure of more that $2,000,000,000 for safety alone! It might be of a little interest, therefore, to make a rough approximation of the financial saving, or loss, which would have resulted if safety had been the original, controlling factor, and if the sea-level canal had been built in 1905.

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[72] "The Report of the Board of Consulting Engineers for the Panama Canal," *Engineering News*, Vol. 55, 1906, p. 205.

[73] *Ibid.*, p. 210.

[74] *Ibid.*, p. 209.

The following calculation is admittedly neither accurate nor thorough, but it is a good and probably a sufficient indication of the correct answer. Any savings that would accrue from a reduced military garrison are omitted as are any savings resulting from shorter passage and waiting time. Similarly, the increased safety is ignored. After all, either by good luck or by good defense, the canal has passed successfully through two major wars, unharmed to date.

The construction of the present canal was begun in 1906 and finished in 1914 at a cost of approximately $350,000,000. Additions made up until June 30, 1947, have increased this amount to $372,000,000. To this sum should be added interest during construction which amounted to $129,000,000, resulting in a total of $501,000,000. Annual interest charges at 3% computed but not charged on the total amount comparable to that given previously, as it varied from year to year, equal $409,000,000. The actual operating cost of the canal as represented by appropriations was $316,000,000. Thus, the original cost of the canal, plus interest and operating expenses—all as of June 30, 1947—equals $1,226,000,000. The income from the canal deposited with the United States Treasury amounted to $587,000,000 for the same period. This has been omitted from the totals on the theory that the sea-level canal proposed in 1905 would have returned at least an equal income. This latter canal would not have been finished as early as the present canal, but the difference probably would have been made up by the attractiveness of quicker transit and less waiting.

To advance the total cost figure of $1,226,000,000 from 1947 to 1960, the assumed date of completion of the new canal, one must add interest amounting to 39% of $372,000,000 and also assumed operating expenses at $10,000,000 a year, or $145,000,000 and $130,000,000, respectively—a grand total of $1,501,000,000 as the cost of the present canal to 1960.

To arrive at a comparable cost of the originally proposed sea-level canal requires several assumptions, all wide open to just criticism. The first, of course, is the question of whether the canal could or would have been built. Considering the well-known impatience of the American public and Congress, it is entirely possible that, with the inevitable huge slides and the cost (which at that time would have been considered astronomical), the construction of the canal might very well have been abandoned in disgust. The lock canal originally estimated at $140,000,000[75] actually cost $350,000,000, or 2.5 times the estimated cost, due to improved cross sections, increased excavation, and other causes. Taking the original estimate of the sea-level canal and increasing it in the same proportion gives $833,000,000. The Special Engineering Division of The Panama Canal estimates the ratio of the 1946 construction costs to the 1906–1914 costs as 2.5 to 1, or 3 to 1. Applying this ratio to the estimated cost ($2,483,000,000) of the proposed canal results in $830,000,000 to $1,000,000,000—a reasonable check. To be conservative, assume the cost to be $1,000,000,000. Add to this sum the interest during construction at $347,000,000 (the same ratio as for the present canal) and the annual interest (also in the same proportion) at $1,489,000,000. The result is a total of $2,952,000,000, to which must be added the cost of operation (assumed at 75% of this cost for the present canal), $334,000,000, which yields a grand total of $3,286,000,000.

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[75] *Engineering News*, Vol. 55, 1906, p. 239.

The 1960 cost of the present canal plus the new canal (including $372,000,000 for interest during construction) amounts to $4,356,000,000, indicating an apparent saving of more than $1,000,000,000 if the sea-level canal had been adopted in 1905. It is evident that had the canal originally been built at sea level, there would have been a very substantial financial saving, in addition to an increase in safety and ease of operation. Time, and the atom bomb, have proved the soundness of the recommendations of General Parsons and his associates.

It would be neither fair nor just, however, to cast aspersions on the minority report of 1905. The recommendations of that group were correct as based on their assumptions. Thus far, the Panama Canal has accomplished its mission superbly in peace and war. As one of the commanders of a theater service of supply in the Far East, the writer can testify to the great service it rendered in World War II—the value of which cannot be stated in dollars alone; but the nation has been "riding its luck." The fact that the canal has come safely through two wars is no guarantee of future security. As the advertisements point out, a new element has been added. The present canal is nearly defenseless against modern weapons of attack and the quicker the proposed canal is completed the better.

William Herbert Hobbs,[76] Esq.—A possible sea-level, ship canal across the Tehuantepec Isthmus in Mexico does not appear to have been very seriously considered in these studies, inasmuch as attention was given to that

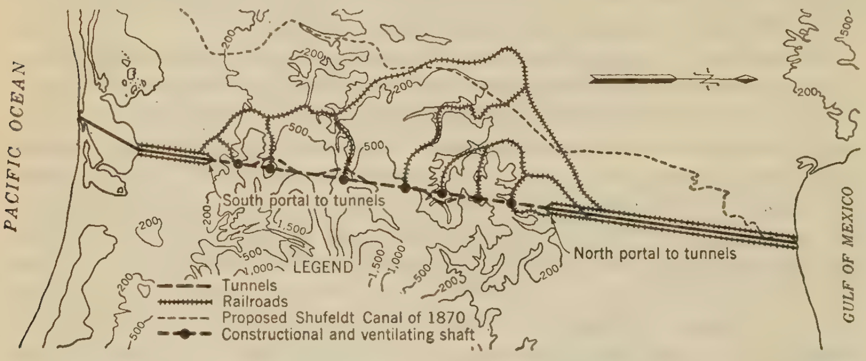

Fig. 105

along the circuitous route proposed for a canal by R. W. Schufeldt, a Captain in the United States Navy, in 1870. This was for a lock canal which would have been 165 miles long (see route in Fig. 105). The Isthmus of Tehuantepec offers the best possible location for an interocean, combined open ditch and tunnel ship canal at sea level, which could be made largely secure from successful attack.

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[76] Prof. Emeritus of Geology, Univ. of Michigan, Ann Arbor, Mich.

In contrast to a canal which would follow the Schufeldt route, the Tehuantepec canal herein proposed would be on a direct line from Puerto Mexico, the Atlantic terminus, almost due south to the Lower Lagoon on the Pacific front (see Fig. 105). Its length is thus reduced from 165 statute miles to 126 miles, and of this distance one half, or 63 miles, is across a featureless plain generally less than 50 ft above the sea. The subsurface rocks of this plain are soft Tertiary sediments which could be easily excavated by power shovel or bulldozer.

It is proposed to pierce the central hardrock mountain, which is the backbone of this isthmus, by a tunnel 63 miles in length. By reason of seven constructional and ventilating shafts, it is, in reality, a continuous succession of eight tunnels between its two portals. The longest of these tunnels would be 14 miles, with five tunnels each less than 8 miles. The tunnel cross section would have a maximum width of 175 ft and the same maximum height, which gives a cross section of 2,222 sq yd (see Fig. 106). This should accommodate the largest proposed battleships and "flattops," and the present trend is toward increased length, rather than beam, in the interest of greater speed of both types and of longer runways on aircraft carriers. The very largest ships may find it necessary to provide a knee joint in their lofty radar masts for partial lowering when making the passage through the canal.

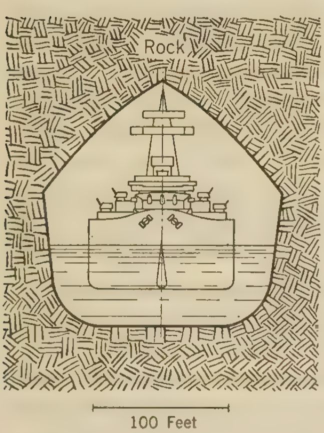
Rock
100 Feet

FIG. 106

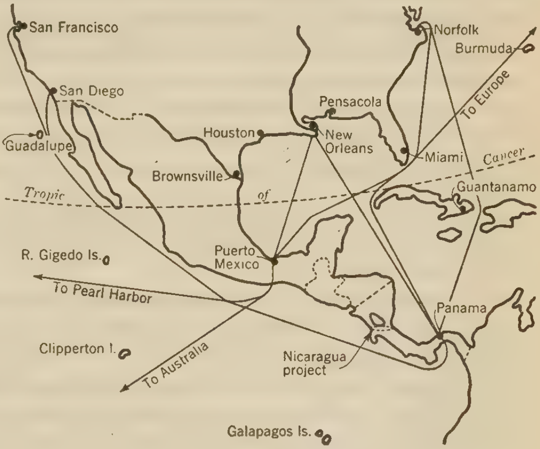
San Francisco
San Diego
Guadalupe
Houston
Brownsville
Pensacola
New Orleans
Norfolk
Burmuda
To Europe
Miami
Cancer
Guantanamo
Tropic
of
R. Gigedo Is.
Puerto Mexico
To Pearl Harbor
Clipperton I.
To Australia
Nicaragua project
Panama
Galapagos Is.

FIG. 107

From the proposed constructional shafts of the tunnel, each only a few hundred feet in depth, conveniently located canyons will permit the building of railways, all with downgrades to the open ditch sections of the canal, so that the excavated rock can be dumped to form high riprap walls on both sides of the open ditch, and to build harbor moles in addition. This should protect the ditch from all but direct bomb hits. For the tunnel itself protection is complete, provided that the portals are of thick heavily reinforced concrete.

Behavior of the German submarine pens so constructed proved them to be almost indestructible, both during World War II and after it as well.

To the east of three of the shafts (the third, fifth, and sixth counting from the north in Fig. 105) the dams erected in the canyons would provide large reservoirs which, if combined with hydroelectric plants, would furnish power for the operation of the hoists and railway, as well as for electrification of the tunnel. These reservoirs would also provide water supply for the workmen's villages high above the unhealthy lowlands.

Three or more tunnel sidings, each 2 miles in length would permit uninterrupted two-way traffic through the tunnel, and would also afford protection for the fleet if the canal were to be under attack from the air in wartime. The seven shafts of the tunnel would permit its construction, simultaneously, in two directions from each shaft, and in one direction from each portal. This should reduce, considerably, the time required to complete it, which is especially important in the event of an early national emergency.

The open sections of the canal should be separated into two waterways, each for one-way traffic. Excavation should thus be greatly reduced and danger of collisions within a single narrow channel be obviated. Two ditches, each 175 ft wide, would serve as much traffic as a single ditch very much wider than the combined breadth of the two. Intercommunicating passages would maintain the normal surface level of the water when disturbed by the bow waves of the moving vessels.

The advantages of this proposed Tehuantepec canal, over that recommended for conversion of the Panama Canal to sea level, are many and convincing. They are as follows:

1. The Panama project would provide only a single interocean canal, as it does at present. The Tehuantepec project provides two, which are more than 1,000 miles apart.
2. During the period of construction (possibly as much as 10 years) the Panama Canal, in the case of the Tehuantepec project, would continue to be in unobstructed use.
3. Throughout its length the Tehuantepec canal would be almost invulnerable to attack, the tunnel section completely so. The converted Panama Canal would be insecure under attack and the studies have indicated that, if blocked, it would require weeks to reopen it. The portals of the proposed tunnel at Tehuantepec, if constructed of heavily reinforced concrete (as were the German submarine pens during World War II), would be almost completely indestructible.
4. In time of war with menace from both ocean fronts, the fleet, when at the canal, could have a fully protected two-ocean base from which to move to either front as might be required. Such protection for a fleet would be quite unique anywhere in the world.
5. The proposed Tehuantepec canal would cut 3,000 miles of sea lanes from the Panama route between the naval base at Norfolk, Va., and the San Diego (Calif.) or Pearl Harbor (Hawaii) bases; and equal reductions would be secured for commercial sea travel between ports on the East Coast and the Gulf of Mexico and those on the West Coast (see Fig. 107).

6. In time of war, the sea approaches to the Tehuantepec canal could be guarded on the Atlantic front by air patrols and by radar operated from airfields at Brownsville, Tex., Houston, Tex., New Orleans, La., Pensacola, Fla., Miami, Fla., Guantánamo, Cuba, and Yucatan, whereas the Panama Canal cannot be thus guarded. The recent enforced withdrawal of air outposts from Panamanian territory, and the 1948 uprisings at Bogotá, Colombia, only 500 miles from the Panama Canal, demonstrated only too clearly the insecurity of the Canal Zone.

7. Off the Pacific front the sea approaches to the Tehuantepec canal could be guarded in part by air patrols and by radar operating from bases in the Revilla Gigedo and the Galápagos island groups and from Clipperton Island (see Fig. 107).

8. A canal on the Isthmus of Tehuantepec is much favored by its known immunity from destructive earthquakes throughout the last three centuries, although earthquakes have been disastrous for most of Mexico and Central America.[77]

9. The northern third of the Tehuantepec Isthmus is a highly productive oil field, so that local fuel oil would be available for vessels transiting the canal. This field, which was opened as late as 1921, yielded 5,842,000 bbl of oil in 1946.

10. The Pacific terminal of the proposed Tehuantepec canal is within the Lower Lagoon (see Fig. 105), and inasmuch as this lagoon has a narrow connection with the sea, no tidal locks will be required on this front, although they are necessary for the Panama project.

11. The operational advantages of the straight course of the proposed Tehuantepec canal over the sinuous one at Panama are considerable. Electrification will be available not only for illumination, but also for the propulsion of vessels; and thus it should obviate the wear on the canal walls by the currents from churning screws.

Without unnecessary delay a precise profile of the line of this proposed canal should be surveyed, and drill borings should be put down within the tunnel section. Until this has been done, no reliable estimate of costs is possible. Purely tentative ones based on an assumed average depth of 120 ft for the open ditch sections (probably a liberal estimate) are as follows: For the 63 miles of tunnel, plus 6 miles for three tunnel sidings, 271,000,000 cu yd; and, for the two open ditch sections, 340,000,000 cu yd—a total of 611,000,000 cu yd. This is roughly two thirds that estimated as necessary to convert the existing canal at Panama to a sea-level canal.

In conclusion, the writer would emphasize as the greatest of all advantages of the Tehuantepec project over the Panama one, its very much greater measure of security. Colonel Stratton, for the committee, has admitted for the latter its vulnerability under bombing attack, but he believes the blocked canal could probably be reopened in a matter of weeks. In a modern war, this period of weeks, during which the canal was blocked to the U. S. Navy, might well spell the difference between ultimate victory and crushing defeat.

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[77] "New Volcanoes and a New Mountain Range," by William Herbert Hobbs, *Science*, Vol. 99, 1944, pp. 289–290, maps.

Hibbert Hill,[78] M. ASCE.—The slides that occurred during construction of the Panama Canal made a deep impression on the popular consciousness. The undertaking to construct the Panama Canal was of world-wide interest. The construction forces were in some degree the representatives of all men in a struggle against gigantic and imperfectly understood natural obstacles—a struggle whose outcome was far from certain. Nature, in the slides, brought up her shock troops. To the construction forces and to the world, the entire contest became epitomized in the struggle with the slides.

Now that a new and deeper excavation is under consideration, the fact will be recalled that the slides were overcome by only a narrow margin and at great added expense, and the question will be asked why can this greater work be undertaken with assurance now. The answer to this question, in brief, is that, in the four decades since the United States undertook to construct the canal, great progress has been made in understanding the cause and prevention of slides.

A Board of Consulting Engineers, drawn from the world's most eminent engineers, aided in the design of the present canal. As stated by Messrs. Binger and Thompson this Board established slopes of 3 vertical on 2 horizontal for all slopes in rock through the region of the Continental Divide. The slide history of the French construction and some few borings in the area of the Continental Divide were available to the Board. The Board and all those concerned with the construction seem to have been satisfied with the information available and with the slopes established. The record indicates that with these slopes some slides were anticipated, but that the volume of slide material to be removed was expected to be less than the additional excavation which initially flatter slopes would have caused. This was substantially proved by the event, except in the case of the great slides in the Cucaracha formation.

The East and West Culebra slides in this formation began to move in October, 1907, and continued to move at an increasing rate. The method of combating them was principally that of digging to remove the slide material from the cut. It was not until 1910 that C. W. Hayes, Chief Geologist of the United States Geological Survey, was sent to study the slides. He made a brief study in which he diagnosed the causes correctly. In January, 1911, D. F. MacDonald was appointed resident geologist, and at that time began a continuous study of the slides and of the soils and geology of the Canal Zone.

This history is not recited in criticism but rather in support of the view that, although the engineers responsible took all measures which they thought justified to prevent and combat the slides, the state of knowledge at the time did not provide a basis for anticipating the slides, nor for corrective measures once the slides were encountered. It is much to the credit of those in charge that, before construction was completed, a fundamental understanding of causes was developed, and a beginning made in the application of that understanding.

The designers of the canal did not appreciate that the 3-on-2 slopes which they adopted were certain to result in slides of great magnitude in the weak rocks of the cut. Furthermore, they were unaware that they were compounding trouble by inducing slides in the Cucaracha formation. With respect to the latter point, Messrs. Binger and Thompson remark briefly that if slides occur the total amount of material to be removed would be greater than the excavation required had the canal banks been excavated initially to stable slopes. This statement is based on the fact that the Cucaracha loses strength upon disturbance; hence, the stable slope of the material disturbed by a slide is considerably flatter than the stable slope of the undisturbed material.

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[78] Hydr. Engr., Northern States Power Co., Minneapolis, Minn.

This characteristic is now well recognized as common to many clay soils, but the possibility of such an occurrence was totally unknown to the builders of the canal. Study of the slides of the present canal has shown that the residual strength of the Cucaracha formation, after sliding, is about 20% of its undisturbed strength. Therefore, the existing slopes in the slide area are now as flat as 1 on 12, whereas the same slopes would have been stable if initially excavated about 1 on 5. This characteristic of the Cucaracha formation provides an impelling reason for stable initial slopes in the cuts of the proposed sea-level canal.

Since completion of the canal the science of soil mechanics has become a well-established branch of engineering. Also, study and exploration have immensely expanded knowledge of the geology of the Canal Zone—of the properties of the soils and rocks, and of the behavior of those rocks in cut slopes. As a result engineers now not only understand the mechanics of slides but are able to compute with reasonable certainty the slopes required to avoid the slides. The gulf between the knowledge of 1905 and the present understanding is dramatized by the difference between the 3-on-2 slopes adapted for the original cut, and the slopes ranging from 1 on 4.6 to 1 on 9.4 shown in Fig. 71.

The stable slopes determined by Messrs. Binger and Thompson depend in part on observations, by Mr. MacDonald and others, of actual slopes in the rocks of the region; in particular, for deep cuts, they depend on analyses by the Swedish circular slide method, a tool of modern soil mechanics. The responsibility which soil mechanics is thus assuming may be measured in the light of the description of past slides, given by Messrs. Binger and Thompson, and in the light of the tremendous volumes of excavation required to obtain the stable slopes shown in Fig. 71. Very important reliance is thus being placed upon the method, and it is proper to inquire whether it is justified.

The writer believes that such reliance is justified. The method has been in wide use since before 1928 and has met with general acceptance. It has been used to design many slopes that have proved stable, and, perhaps more important, it has explained many observed slope failures very well. The writer recently had occasion to apply the method to a past failure, and not only did it explain the failure accurately, but it also located the failure surface in the position actually occupied. The authors of the seventh Symposium paper mention that the value of cohesion for the Cucaracha formation was checked by analytical studies of the East and West Culebra slides. These studies were made by the Swedish circular slide method, and were particularly convincing to the writer in that the computations not only predicted the occurrence of a slide, but also predicted (by the location of the critical arc) the position of the break and the upheaval of the canal bottom as observed.

The eighth Symposium paper, by Messrs. Casagrande and Shannon, describes the beginning of a research which may provide important new conceptions and, in any case, will explore a field—the effect of transient loading—which, in a world subjected to high speed traffic and higher speed explosions, needs exploration. The immediate objective—determination of the effect of explosive impulses on the stability of the Panama Canal slopes—is an important one, and the indications of the published results are reassuring in that respect.

As Messrs. Casagrande and Shannon state, the effect of transient loads on soils is important in many applications. It seems to occur to every questioner to ask, for example, why airfield pavements do not fail at the point of impact of landing planes. Measurement of stresses in the landing gear has demonstrated that such impact loads are often several times the static load, yet failures of pavements at the point of landing are not observed. In accelerated traffic tests by the Corps of Engineers, Department of the Army, pressure cells recorded subgrade pressures approximately 30% less in the case of a wheel moving at from 5 miles per hr to 7 miles per hr than when the same wheel was static. These tests indicated that, for unexplained reasons, the load when moving was spread over an area 20% greater than when static. It is reasonable to suppose that at higher speeds these effects would be intensified. The investigation reported in the eighth Symposium paper should aid considerably in explaining these observations and similar phenomena.

The writer realizes fully that this research is in its initial stages. It is his hope, however, that Messrs. Casagrande and Shannon will be able to clarify the indicated parts of their presentation in their closure.

KENNETH C. REYNOLDS,[79] M. ASCE.—The interesting and valuable paper by Messrs. Meyers and Schultz presents two methods used to predict the velocities to be expected in the proposed 40-mile Panama sea-level canal. The first method utilized computations based on analytical methods developed by George B. Pillsbury,[8] M. ASCE; the second method involved observations made on an undistorted model of the entire canal.

Velocities in the new channel could have been computed had the flow in the canal been steady. Such will not be the case because the water surfaces of the two oceans pass through tidal cycles of different heights and phases (Fig.26), with successive days having different tidal ranges.

General Pillsbury has developed a basic equation for examining the velocity of current in a tidal canal. The rate of flow is increasing or decreasing continuously, and the direction of the flow is reversed periodically. The usual equations for steady flow were expanded to account for the work done in the acceleration and deceleration of the current. The basic equation included the following four terms: The surface head, to describe the slope of the water surface; the velocity head; the acceleration head; and the friction head.

The term for the friction head depended on the Chezy coefficient, C, which at once involved some roughness coefficient such as Manning's n. Both the computations and the model investigations required an evaluation of the roughness coefficient for nature or for the prototype, whereas the model studies also necessitated knowledge of the roughness coefficient of the miniature canal.

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[79] Head, Dept. of Gen. Eng., College of Eng., Univ. of Southern California, Los Angeles, Calif.

The investigators obtained information as to the probable roughness coefficient of the prototype from two sources—from data on large channels in the United States and from field observations of flow in Gaillard Cut for two trial runs. These will be examined in detail subsequently.

The roughness data on large channels were obtained for several waterways in the United States and led to two conclusions. The first was:

> "* * * that the Manning coefficient for a river channel is least when the stage is at or somewhat above normal bankfull stage, and tends to increase for both higher and lower stages."

The velocity of flow in an open channel depends in part on the shear at the boundaries. This must be reflected in the methods used to evaluate Manning's n. The latter varies with the boundary roughness and probably with the viscosity of the water, particularly for small values of the Reynolds number. The foregoing conclusion that, for a natural channel, n has a minimum value at a certain stage or hydraulic radius (Fig. 27) raises the question as to what other factors enter the picture. Does the relative roughness of the channel vary because of changes in the vegetation or the presence of silt? A clarification of this conclusion is desirable. Data from other large natural channels should be studied.

The second conclusion was:

> "* * * that the bankfull roughness coefficients do not vary greatly for rivers and canals in different kinds of materials and in widely separated locations."

In view of the foregoing discussion, this conclusion needs further study. The important point has not been the geographical location of a channel, but the type of boundary roughness; and yet this conclusion is for different kinds of bank material.

The second source of data on the roughness coefficient was the field observations of flow in Gaillard Cut. The outermost stations were 37,100 ft apart, and this distance was divided into reaches of 8,100 ft, 4,300 ft, 7,600 ft, 8,400 ft, and 8,700 ft, respectively. The mean areas varied from 14,900 sq ft to 21,500 sq ft and the hydraulic radii, from 36.0 ft to 38.0 ft during the two tests. The gages at the end stations were recording gages; the six intermediate ones were hook gages. It was essential to "zero" the gages carefully since the observed differences in water levels were to be small for the anticipated flow.

Supposedly, no flow was established through the cut. It appears to the writer that it would be difficult, if not impossible, to quiet the 7-mile-long body of water so that there would be no motion. To eliminate surges, wind action, or any velocity of flow is indeed difficult. Of course, errors in "zeroing" the gages would affect data based on subsequent gage readings.

The test runs were made during two succeeding nights. Current meters were used at each end of the test reach to evaluate the mean velocity and the average discharge of 22,000 cu ft per sec. For the several reaches, the mean velocity was from 1.02 ft per sec to 1.48 ft per sec. It would require about 8 hours for a particle of water to traverse the entire 37,100 ft, and, hence, presumably it would take more than this time for equilibrium conditions to become established.

The mean difference in water-surface elevations lay between 0.021 ft ($\frac{1}{4}$ in.) and 0.041 ft ($\frac{1}{2}$ in.). Small differences in these values would have affected the computed slope of the average energy gradient, which ranged from between 0.0000023 and 0.0000060.

From the foregoing data it was possible to evaluate the roughness coefficients of Manning, Kutter, and others. The average Manning *n* was about 0.026, which in the writer's opinion is only an approximate value. The field difficulties of stilling the large body of water, of setting hook gages at night, and of obtaining uniform flow made it difficult to obtain reliable data.

The second method for predicting velocities was by making observations on a model of the entire 40-mile canal. This model intrigues the writer, as he directed the research program at the Massachusetts Institute of Technology in Cambridge, on the 110-ft-long model of the Cape Cod Canal in Massachusetts for the Boston (Mass.) Office, Corps of Engineers, Department of the Army. Space limitations necessitated a 10:1 distorted scale model, with its attendant uncertainties. Messrs. Meyers and Schultz are to be commended on using an undistorted model, even though it required a model nearly $\frac{1}{2}$-mile long.

The procedure involved creating, artificially, a rise and fall of the water surfaces at the ends of the model to simulate the tides of the two oceans. The resulting motion of the water within the canal section was presumed to be indicative of that to be expected in the prototype. Such a procedure is accepted model practice, provided the requirements for a true model are met fully.

The model scale was 1:100. Based on the authors' experience, an ordinary concrete surface in a model of this scale was used as simulating best the roughness of the prototype. When the model was completed, steady flow was established, and the resistance coefficient for the model "was found to correspond to the selected Manning coefficient of 0.024." The significance of the word "correspond" is not clear. Does it mean that *n* for the model and for the prototype was the same; or were these *n*'s unequal?

Subsequently, it was pointed out that, since gravity was the predominant force, the Froude law was used to interrelate times, velocities, and discharges. This does not preclude the existence of any friction along the model canal. To the writer it would imply that, with the times for the model one tenth those for the prototype, the roughness of the model should be so adjusted that, under gravity flow conditions, the loss of head in the model should be geometrically similar to that in the prototype. Since no prototype data were available, it is assumed that data from other studies of comparable prototype and model roughnesses by Messrs. Meyers and Schultz led to the use of the Froude law.

Using the adopted model scale, the minimum depth of water was 0.60 ft and the minimum bottom width of channel was 5.0 ft. The velocities in such a model would have produced some side-wall effects, although these might not have been serious.

Water levels along the model canal channel were measured by automatic records and manually-operated hook gages. It is difficult to visualize how the recorders gave reliable data. Usually they consist of a float on the water surface whose vertical motion is recorded on a paper mounted on a slowly rotating drum. If the float were in the canal, the flowing water would create uncertain errors introduced by surface tension and by dynamic effects. Should the water surface in the canal be connected to a stilling well and the float be on the water surface in the well, other errors would arise. If the stage in the canal were rising, the water in the well would lag behind that outside. As the rate of rise decreased, the lag would grow less, and at high tide, or soon thereafter, there would be no lag. The correction for a falling stage would be different from that for a rising stage. Were corrections applied to the data from the automatic recorders? Since prototype errors would be one hundred times the model values, great care was needed in collecting model data.

At other stations manually-operated hook gages were used. The writer has always cautioned students not to use hook gages in flowing water, since the conventional hook dips below the water surface so that in flowing past it, water creates a surface disturbance which is likely to affect the gage setting. The magnitude of the disturbance would change with the varying velocity and the reversal of flow. The writer would have recommended the use of point gages. To set such gages accurately and to read them at 30-sec intervals in the model required considerable skill by the observers. The writer believes that the electric water level indicator developed by H. L. Hazen[80] for the Cape Cod Canal model, or some other similar device, would have given more accurate readings and would have given average values of the elevation of the wavy water surface at an observation station.

Gregory P. Tschebotarioff,[81] M. ASCE.—The present discussion is concerned mainly with the paper by Messrs. Casagrande and Shannon. Other papers of the Symposium will be mentioned only in so far as the recommendations contained in them are based on the reported findings of these two authors.

The writer is in general agreement with these findings, but only in a qualitative sense. The manner in which the tests were performed does not permit any quantitative evaluation of the results for the purpose of their direct application to slope-design problems. The writer's qualitative agreement is based on the results of an earlier extensive program (from 1934 to 1946) of dynamic tests performed under his direction at Princeton University (Princeton, N. J.) for the Technical Development Service of the Civil Aeronautics Administration.[82,83,84] This program was intended to provide some insight into the relative importance of the various factors affecting the bearing properties of soils in airport subgrades and base courses when subjected to vibratory and to slow repetitional loading by warming up and by taxying aircraft. Inspite of the different objectives and corresponding differences in the test equipment employed by the two test programs, the data obtained at Princeton University may nevertheless serve for an analysis of, and comparison with, the results reported from Harvard University (Cambridge, Mass.).

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[80] "Electrical Water-Level Control and Recording Equipment for Model of Cape Cod Canal," by H. L. Hazen, *Electrical Engineering*, February, 1937, p. 237.

[81] Associate Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[82] "Effect of Vibrations on the Bearing Properties of Soils," by Gregory P. Tschebotarioff, *Proceedings*, Highway Research Board, National Research Council, Vol. 24, 1944, pp. 404–425.

[83] "Vibratory and Slow Repetitional Loading of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *ibid.*, Vol. 26, 1946, pp. 551–562.

[84] "The Effect of Vibratory and Slow Repetitional Forces on the Bearing Properties of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *Technical Development Report No. 57*, Civil Aeronautics Administration, October, 1947.

The Princeton tests[82,83,84] have shown that sands could be very strongly affected both by vibratory and by slow repetitional loading. For uniform types of sand the deformation produced by the vibratory force could be as much as one hundred and forty times greater than the deformation produced by a single application of an equivalent static load.[85,86] However, in the case of clay soils, other factors remaining equal, the continuous application of a vibratory load produced approximately the same deformation as a single application of an equivalent static load.[87,88,89]

"Continuous load application" means the application of a vibratory force for a period of from 10 to 20 min—that is, more than 10,000 load repetitions. The rate of deformation produced by the continuous application of dynamic load decreases with time;[90,91] the time-deformation curve has a parabolic shape, as illustrated in Fig. 108. Since after a large number of dynamic load repetitions the final deformation Δ is approximately equal to the static deformation produced by a single application of an equivalent static load, it would follow that a single application of a dynamic load should produce only a fraction of that deformation. Therefore, conversely, to produce a deformation leading to failure, by a single loading, a dynamic load greater than a static load would

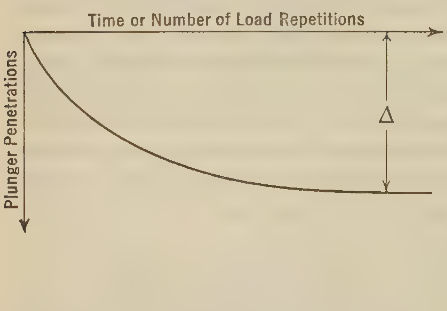

FIG. 108.—RELATIONSHIP BETWEEN TIME AND THE INCREMENT OF SOIL DEFORMATION PRODUCED BY THE CONTINUOUS APPLICATION OF A VIBRATORY LOAD SUPERIMPOSED ON A STATIC LOAD OF CONSTANT INTENSITY (1944 TESTS, PRINCETON UNIVERSITY, PRINCETON, N. J.)

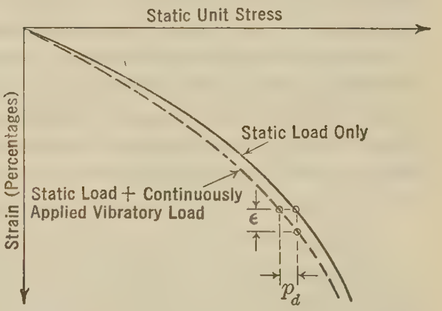

FIG. 109.—EFFECT OF CONTINUOUSLY SUPERIMPOSED VIBRATORY LOAD ON THE STRESS-STRAIN RELATIONSHIP OF CLAY SAMPLES SUBJECTED TO STATIC UNCONFINED COMPRESSIVE STRENGTH TESTS (1946 TESTS, PRINCETON UNIVERSITY, PRINCETON, N. J.)

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[85] "Vibratory and Slow Repititional Loading of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *Proceedings*, Highway Research Board, National Research Council, Vol. 26, 1946, p. 556, Fig. 14.

[86] "The Effect of Vibratory and Slow Repetitional Forces on the Bearing Properties of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *Technical Development Report No. 57*, Civil Aeronautics Administration, October, 1947, p. 561, Fig. 31.

[87] "Effect of Vibrations on the Bearing Properties of Soils," by Gregory P. Tschebotarioff, *Proceedings*, Highway Research Board, National Research Council, Vol. 24, 1944, p. 421, Fig. 28.

[88] "Vibratory and Slow Repetitional Loading of Soils," by Gregory P. Tschebotarioff and George W. McAlpin. *ibid.*, Vol. 26, 1946, p. 554, Figs. 5 and 6.

[89] "The Effect of Vibratory and Slow Repetitional Forces on the Bearing Properties of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *Technical Development Report No. 57*, Civil Aeronautics Administration, October, 1947, pp. 53 and 54, Figs. 38 and 41.

[90] "Effect of Vibrations on the Bearing Properties of Soils," by Gregory P. Tschebotarioff, *Proceedings*, Highway Research Board, National Research Council, Vol. 24, 1944, p. 413, Figs. 8 and 9.

[91] "The Effect of Vibratory and Slow Repetitional Forces on the Bearing Properties of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *Technical Development Report No. 57*, Civil Aeronautics Administration, October, 1947, pp. 36 and 37, Figs. 12 and 13.

be needed. This is exactly what was found during the Harvard tests, and such results are therefore in agreement with the deductions that can be drawn from the earlier tests at Princeton.

The writer, however, has one criticism concerning the manner in which the Harvard tests were performed with the aim of providing design data for the Panama Canal slopes. It would appear from the paper under discussion that, during both the unconfined compressive strength tests and the triaxial shear tests, the dynamic load was not superimposed on a static load which had already induced shearing stresses in the test specimen; instead, all the shearing stresses during the tests were produced entirely by dynamic loads in one series of tests and entirely by static loads in another series of tests. This procedure has undoubtedly given results of considerable theoretical interest which have been reported in the paper. However, no comparable condition can ever exist in nature in connection with the problem of the stability of slopes in cohesive soil. Certain static shearing stresses are always present in an embankment and, depending on how closely these static stresses approach the limit of resistance of the material, an increase of these stresses as the result of dynamic loading may or may not produce failure. The writer is familiar with a case of a clay bank, a 500-ft section of which slid out into a river during the driving of a cluster of piles for a dolphin close to its edge. The relatively small shocks caused by this pile driving obviously produced stress intensities that exceeded the resistance of the bank. In this particular case the bank was composed of varved glacial clay. The rock flour characteristics of the material may have rendered it particularly susceptible to impact stresses superimposed on a near-critical static stress condition. The Harvard installation can provide valuable data by extending its studies to include different combinations of dynamic and static loading applied to various soil types.

However, the Harvard tests, as reported thus far, cannot yield any numerical data on the effect on the shearing strength of soils of dynamic forces superimposed on static forces—that is, under conditions generally existing in nature and as are certain to exist in the slopes of the Panama Canal.

Nevertheless, where clays are concerned the writer does not believe (on the strength of his earlier, 1946, tests) that this condition of superposition of dynamic forces on static forces would cause an appreciably detrimental increase of the effect of dynamic loads. In 1946 several unconfined compressive strength tests were performed on clay cylinders of 3.4-in. diameter and 7-in. height.[92,93] The results of these tests are illustrated diagrammatically by Fig. 109. In one test series the unconfined compressive strength test was performed as a controlled stress test by the gradual increase of only the static load. In the other test series the same type of unconfined compressive strength test was performed while a small vibratory force, equal to approximately 7% of the ultimate load, was applied from the very start of the experiment. It may be seen from Fig. 109 that for the same value of the static unit stress, the strain increment ϵ of the vibrated specimen can be related to an increment of stress p_d

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[92] "Vibratory and Slow Repetitional Loading of Soils," by Gregory P. Tschebotarioff and George W. McAlpin, *Proceedings*, Highway Research Board, National Research Council, Vol. 26, 1946, p. 555, Fig. 10.

[93] "The Effect of Vibratory and Slow Repetitional Forces on the Bearing Properties of Soils," by Gregory P .Tschebotarioff and George W. McAlpin, *Technical Development Report No 57*, Civil Aeronautics Administration, October, 1947, p. 60, Fig. 51.

which was found to have the same order of dimension as the dynamic unit stress produced by the centrifugal force of the vibrator.

Therefore, the writer believes that the factor of safety of 1.3 which has been used for the design of the slopes, as reported by Messrs. Binger and Thompson, may be sufficient to take care of the increment of shearing stresses in the banks such as will be produced by dynamic loading. However, the paper by Messrs. Casagrande and Shannon leaves the impression that dynamic forces may well be neglected entirely in most slope stability problems. Such an impression would seriously hamper further research in this field and might lead to unsafe design proposals. This actually happened at the January, 1948, Annual Meeting of the Society when an oral discusser suggested decreasing the static factor of safety of the proposed Panama Canal slopes to 1.0.

The purpose of this discussion is to warn against any such over-simplifications. It is to be hoped that the Harvard tests will be amplified to determine the effect of impact forces on the strength of soils when such impact forces are superimposed on existing static stresses. A comprehensive further program of research along these lines in coordination with other related studies should permit, in time, the numerical evaluation of dynamic loading effects under natural field conditions and the consideration of these effects in computations of the factor of safety, which otherwise should be considered as still requiring the inclusion of a "factor of ignorance" component.

Charles W. Dohn,[94] Jun. ASCE.—It is significant that this Symposium does not mention the income-producing capabilities of the Panama Canal. The need for a sea-level canal by commercial traffic definitely exists. The decrease in ship operating costs alone nearly equals the differential in cost between a sea-level canal and a lock canal designed to meet the needs of commerce for the remainder of the twentieth century. Significance of the canal as an aid to commerce is often missed and it would be well if studies were made and kept accessible and up to date, as well as publicized, of the amount and value of commerce carried and the cost of alternate routes.

Hence, the Panama sea-level canal can be justified as an economic enterprise. It would be well for domestic economy, however, if construction were postponed for some years. In the intervening years experimental research and actual exploratory, advanced, and perhaps unique methods of construction could be developed and tested. Two steps which should be used to improve financing of the canal are: (1) Increase the tolls, and (2) decrease the canal construction costs. The consulting services of private design and construction engineers should be utilized to a large extent to report on and supervise both of these steps. Canal tolls can be increased on a yearly increment basis extended over a long period with provisions for flexibility. The project can be financed from tolls and the over-all cost reduced by the development of special construction methods. For example, the cubic yards of excavation times the average length of haul can be multiplied by an efficiency factor and reduced to power units and compared with available and potential local sources of hydraulic power. From Table 34, in the last Symposium paper, the total excavation is approximately 1×10^9 cu yd and, assuming an average haul of 10,000 ft, an average unit weight of 120 lb per cu ft, and a factor of 4 to allow for removal, friction and dredge losses, and reserve—this yields: $\frac{1 \times 10^9 \times 10{,}000 \times 4 \times 120}{33{,}000 \times 60} = 2{,}500 \times 10^6$ hp-hr required.

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[94] Engr., Chf. Engr's. Office, Headquarters Dept., New York City Board of Water Supply, New York, N. Y.

There are indications that it might be possible to obtain one half this amount of power over approximately a 10-year period with modifications and additions to existing reservoirs, utilizing almost all the runoff. Runoff from these reservoirs would still be available for lockage use. Additional power sources could be sought within, say, a 200-mile radius of the canal. Development of these sources could be partly of a temporary nature and partly of a permanent form to be sold under agreement to local governments in return for from 10-year to 20-year power rights. Thus, the canal would be excavated by the same methods that nature uses to form valleys.

To operate hydraulic giants and to service hydraulic pool excavation methods use would be made of high-level sources of water, in addition to possible booster pumps run by hydroelectric power and supplementary steam electric power. In hydraulic earth dam construction, small boulders and rock are sluiced out. This need not be done for canal excavation. Hydraulic dredge pipe and sluiceways with renewable linings would be advantageous. The dredge pipe can be siphoned over ridges into low areas and also used to construct dams and dikes as needed. A cutter head and pump such as illustrated in Fig. 100, supplemented by hydraulic giants, might be adapted to hydraulic pool methods and even to excavation on dry land. Much of the rock excavation is indicated to be of a soft nature (see Table 34). It may be possible to break the soft rock by hydraulic methods, inserting high pressure lines into seams, fissures, and drill holes sealed at stategic points, etc. Blasting is not always economical in soft rock because of the dissipation of concussion energy. This is particularly true of rock that has been exposed to weathering at some time in its life cycle. It might be economical also to break large rock fragments somewhat by mechanical crushers and dispose of this product with other hydraulic excavation. It would not be necessary to convey all material to Gatun Lake, which is at a high elevation and somewhat distant from much of the excavation. Such haulage also would impede operation of the lock canal.

Furthermore, using hydraulic methods, a higher continuous rate of removal can be maintained with less load on construction personnel working under tropical climatic conditions. The application of atomic energy to construction projects at present can only be surmised, but, it might be available for later stages of sea-level canal construction.

Donald F. Horton,[95] Assoc. M. ASCE.—All the papers presented in the Panama Canal Symposium are of interest and value not only to civil engineers but to the public generally. The paper by Messrs. Meyers and Schultz was of particular interest to the writer. The major results of the analyses which are used as the bases for determination of tidal currents agree with those derived from similar studies of the Cape Cod Canal in Massachusetts, with which the writer was associated some years ago. This fact is of importance to engineers who in the future may be faced with the problem of determining probable tidal currents in a canal differing in size from the Panama Canal. In tackling this problem, reference can be made profitably not only to this paper but also to the results of analysis of tidal currents in the Cape Cod Canal which have been presented previously.[96]

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[95] Hydr. Engr., Beach Erosion Board, Corps of Engrs., Dept. of the Army, Washington, D. C.

Reference is made in the paper by Messrs. Meyers and Schultz to an estimate of 2.6 knots for a Panama sea-level canal based on the Eytelwein formula with the comment that this formula was stated to agree reasonably well with observed velocities in the Cape Cod Canal. Since this velocity differs radically from the 4.4 knots to 4.6 knots reported by Messrs. Meyers and Schultz as having been determined from the methods described, some readers may infer that these methods would not be applicable to the Cape Cod Canal. However, the writer believes that methods of computation applicable to the proposed Panama sea-level canal are equally applicable to the sea-level Cape Cod Canal and vice versa.

Since the range of applicability of the methods described by Messrs. Meyers and Schultz is a matter of importance to future investigators, some amplification is warranted. The crux of the matter is the conclusion by Messrs. Meyers and Schultz that, at the time of maximum current, the situation is nearly that of ordinary steady flow between the extreme tide levels. Exactly the same conclusion was reached during model studies of the Cape Cod Canal.[97]

The proposed Panama sea-level canal will have a 60-ft by 600-ft cross section and a length of about 35 miles. Maximum heads between the ends of the Panama Canal approximate 10 ft. The Cape Cod Canal has a 32-ft by 540-ft (project) cross section and a length of about 9 miles (not including approach channels). Maximum heads between the ends of the Cape Cod Canal approximate 7 ft. For canals ranging between these characteristics, maximum velocities may be computed on the assumption of steady flow with sufficient accuracy for all practical purposes.

Review of the various methods which have been proposed and used for determination of tidal flow in the Cape Cod Canal indicates the comparative simplicity of computations based on the assumption of steady flow. For example, a rather complicated method of solution by harmonic analysis was developed by William Barclay Parsons, Hon. M. ASCE, and applied by him to the Cape Cod Canal.[63] This method was based on the theory of tides in canals derived by Sir George Biddell Airy, as interpreted by Maurice Lévy.

Complicated tidal-flow formulas may have to be used for canals with characteristics which differ markedly from the range between the Panama and Cape Cod canals. For example, the steady-flow method of computation will not give accurate results for a very long canal of very small cross section. However, whenever applicable, much time and effort can be saved through use of the steady-flow method in the manner outlined in the paper by Messrs. Meyers and Schultz.

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[96] "The Proposed Improvement of the Cape Cod Canal," by E. C. Harwood, *Transactions*, ASCE, Vol. 101, 1936, p. 1440.

[97] "Model Study of Cape Cod Canal and Approaches," by K. C. Reynolds, Rpt. to Dist. Engr., Corps of Engrs., U. S. Army, Boston, Mass., December 21, 1936.

E. MONTFORD FUCIK,[98] Assoc. M. ASCE.—Messrs. Binger and Thompson have very properly emphasized that the major problems in arriving at proper slopes for the sea-level canal concern two materials—the Cucaracha formation, a rock, and the Atlantic muck, a soil. The authors state that the slope curves developed for use in setting slopes for estimating quantities were based on minimum strengths for the materials, and that further study for detailed design might result in steeper slopes, and reduced excavation quantities. The writer heartily agrees with this statement, especially in regard to slopes for cuts in the Cucaracha formation.

Slopes in the Cucaracha Formation.—Fig. 64 shows the curve of slope versus depth of cut which was used for setting slopes in the Cucaracha formation, and Fig. 71 shows typical slopes for a part of the proposed canal in which the Cucaracha formation is encountered. Referring to section A–A in Fig. 71, a slope of 1 on 9.4 is proposed. The writer believes that it is entirely possible that this slope, and others set by the use of the curve shown in Fig. 64, are too conservative, and that a material steepening of the slopes could be effected without any great risk of incurring catastrophic failures of the banks. The authors state that a stable bank of a cut about 200 ft deep in the Cucaracha formation having a slope of 1 on 2.7 was analyzed to determine the over-all strength of the Cucaracha formation. Seepage forces were considered to act on part of the height of the cut, and the over-all strength of the Cucaracha formation in this bank was obtained by considering a safety factor of unity, which assumes that the bank is on the verge of failure. The cohesive strength obtained in this manner was found to be 16 lb per sq in., with an assumed angle of internal friction of 10°.

In arriving at the slope versus depth of cut curve used in setting slopes for the estimate, a condition of sudden drawdown over the entire height of cut was assumed, and a factor of safety of 1.3 was introduced. It can be seen that this resulted in slopes as flat as 1 on 9.4 for the cut illustrated in section A–A in Fig. 71. It is the opinion of the writer that the slope versus depth of cut curve finally arrived at is unduly conservative for design purposes, when the over-all problem of the sea-level canal is considered, and that the magnitude of the excavation quantities involved is such that the use of a lesser over-all factor of safety would be entirely justified in the final design analysis.

The strength value for the Cucaracha formation was derived by a careful study of a stable bank of considerable height, after it had been determined by core borings that the material in the bank was typical of the Cucaracha formation. The condition of sudden drawdown was applied to this bank only below a layer having good drainage. This assumption provides a hidden factor of safety in the final slope curve, which was based on sudden drawdown throughout the full depth of the cut. The factor of safety of the stable bank was assumed to be unity, which is the most conservative assumption possible, because the bank has been standing for 30 years and must have a safety factor of at least unity. It is entirely possible that the actual safety factor is as high as 1.2, because no evidence of incipient failure was found in the bank. A factor of safety of 1.3 was then added to the foregoing unknown safety factor when making the final slope curve in estimating.

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[98] Associate, Harza Eng. Co., Chicago, Ill.

In an excavation project as large as the proposed sea-level canal there is considerable question as to whether it is more economical to design every slope so that it will be absolutely safe against failure, or whether the slopes should be set close to the steepest possible slope, by using a low safety factor. If the latter scheme is employed, some slides might possibly occur, but they probably will involve less additional excavation than if all the slopes were originally flattened so that there would be an ample safety factor against all conditions.

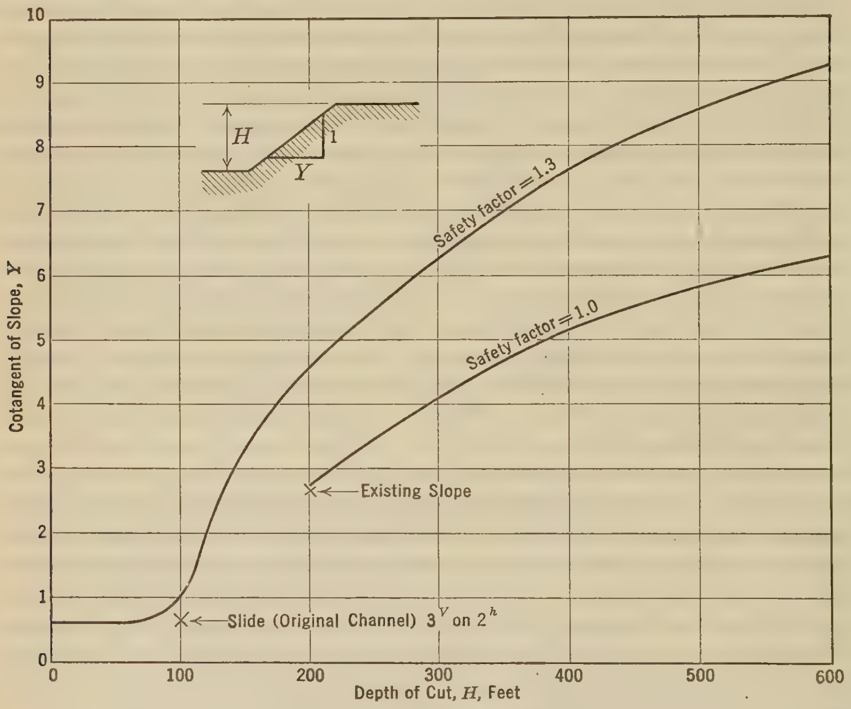

FIG. 110.—EXCAVATION SLOPE CURVE FOR CUCARACHA FORMATION

Using the Cucaracha formation strength values on which the slope curve in Fig. 64 is based, but using a factor of safety of 1.0 instead of 1.3, the writer has worked out the curve shown in Fig. 110. The slope curve for a factor of safety of 1.3 is also plotted, as are the points showing the stable bank from which the strength constants used in these studies were derived, and the slope of 3 on 2 for a 100-ft bank, which failed during the original construction period.

Fig. 71 shows, in plan, the section of the proposed sea-level canal which cuts through the Cucaracha formation. This section is about 17,000 ft long. If an average depth of cut of 300 ft is assumed for this length, and if only one bank is taken to be in the Cucaracha, as seems to be indicated in Fig. 71, the saving in excavation quantities by using the slope curve for a safety factor of 1.0 (Fig. 110), rather than that for a safety factor of 1.3, is about 65,000,000 cu yd.

This estimate indicates the magnitude of quantities involved, and is large enough to merit serious consideration as it represents about 15% of the total volume of the excavation in this section of the canal.

It should be pointed out again that the safety factor of 1.0 does not necessarily represent a condition of incipient failure for the slopes, but merely indicates that these slopes have the same safety factor as an existing slope 200 ft high which has been standing for 30 years without any signs of distress. In all probability, slopes designed with the same relative stability as this existing bank would suffer from only minor slides, if at all.

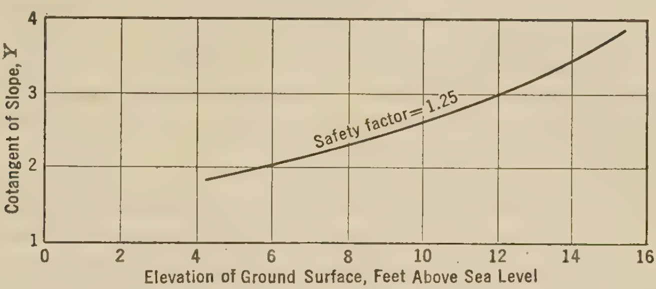

Fig. 111.—Slope Curve for Atlantic Muck; for Cuts Deeper Than 30 Ft Not Unwatered

Slopes in Atlantic Muck.—The Atlantic muck was thoroughly tested during the studies for the Third Locks Project, from 1940 to 1942, and slopes designed from these tests have remained stable after excavation, as stated by the authors. The most interesting and unusual condition in connection with slopes in this material is the great influence which the height of bank above the water level has on the stability for excavations made below sea level and never unwatered. This resulted in the adoption of a uniform slope for all excavation through the Atlantic muck for the proposed sea-level canal. The simplification in dredging a uniform section, rather than a varying section, apparently justifies the removal of some additional yardage.

Fig. 111 shows how much the height of bank above the water affects the slope required for cuts in the Atlantic muck greater than 30 ft and having a safety factor of 1.25. This curve was developed at the Canal Zone during the Third Locks Project.

Conclusions.—The writer's remarks concerning the apparent conservatism of the slope curve for the Cucaracha formation have not been made as a criticism of using Fig. 64 to arrive at estimating quantities for the excavation, because the report of the Governor should present an over-all estimate which is on the high side, rather than on the low side, of the probable cost. However, when, as, and if the sea-level canal becomes an actuality, the writer feels that careful and thorough study should be given to the possibility of using steeper slopes in the Cucaracha formation than are presented in the paper of Messrs. Binger and Thompson.

If the Congress determines to proceed with the construction of a sea-level canal, there will exist an unparalleled opportunity for the advancement of engineering knowledge concerning excavation slopes. The unusual depths of cuts which will be encountered, plus the variety of material to be found—from the softest ocean deposit to extremely hard basalt rock—will yield information which should permit great advancements in the art of excavation slope design.

Charles M. Romanowitz,[99] Assoc. M. ASCE.—Several types of equipment are proposed for the excavation of the sea-level project of the Panama Canal in the paper on "Construction Planning and Methods." The bucket-ladder dredge mentioned therein was the subject of an extensive engineering study carried on by the Yuba Manufacturing Company of San Francisco, Calif.

This company has for many years specialized in bucket-ladder dredge equipment for the mining of placer gold, platinum, and tin ore (cassiterite). A dredge of this type is recommended for the proposed excavation of the rock when suitably broken up by blasting. An experimental blasting project mentioned in the Symposium has proved that rock can be broken up satisfactorily for bucket-ladder dredge excavation.

Two dredges are proposed, each having buckets of 54-cu-ft capacity, one digging to a depth of 90 ft below water level and the other digging to a depth of 145 ft below water level. As designed, these dredges embody the experience of the harbor type, bucket-ladder dredge, and the placer mining type, bucket-ladder dredge, with the units being modernized with proved designs.

A record of the performance of bucket-ladder dredges in the present Panama Canal construction is given by one of the authors of the paper on "Construction Planning and Methods."[100] Seven bucket-ladder dredges, taken over from the French operations, were found to have inadequate power and bucket capacity (17 cu ft). Although their production was not as great as desired, their performance led to ordering a larger and more powerful dredge, the *Corozal*, which started operation on April 11, 1912. Besides having buckets of 54-cu-ft capacity for soft materials and 34-cu-ft capacity for digging rock, this apparatus could dredge to a depth of 50 ft, dumping the spoil either into hoppers on the dredge or into barges alongside. In soft to medium hard materials its maximum monthly yardage was 176,574 cu yd; in 100% rock, 173,267 cu yd. Considerable trouble was caused by breakages, although when digging the *Corozal* gave a good account of itself.

Bucket-ladder dredges are used extensively for placer mining and require a large space on board for treating the materials dug. This has been the main factor in limiting the bucket size of the largest placer mining dredges to 18 cu ft each, although such dredges have not reached a maximum digging depth. In California there are dredges digging the deepest gold placer ground on record —one digging to 112 ft below water level, and the other digging to 124 ft below water level with banks above water up to 50 ft high, making a total depth of 174 ft below ground level. In the same area some of the property is to be equipped with dredges digging to greater depths below water level than those proposed for the Panama Canal sea-level project.

In Malaya, according to official records, there were in June, 1940, eight dredges digging to depths from 110 ft to 130 ft below water level, with five of these digging to the greater depth. It has been stated by responsible mining men in Malaya that dredges digging to depths of at least 150 ft below water level will be considered in the future.

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[99] Director of Sales, Yuba Mfg. Co., San Francisco, Calif.

[100] "Dredging with Ladder Dredges," by Forrest L. Dye, *Memorandum No. 19*, Isthmian Canal Studies, The Panama Canal, Diablo Heights, Canal Zone, July 8, 1946.

The bucket-ladder dredge developed rapidly because values found in placer gravels are low. Every factor that increases the efficiency of the dredge must be given serious consideration. Most of the innovations and improved designs have come directly either from the operators to the designing engineers, or from the engineers who have developed them as a result of having direct contact with the operations. This close association has contributed greatly to the rapid development of the California type dredge.

Credit must also be given to the manganese steel foundries serving the placer mining industry. They have contributed much to the development of the buckets, an important unit on the dredge. Although the largest placer mining dredge has buckets of 18-cu-ft capacity, which is only one third of the capacity of the largest buckets used for harbor work, there is a substantial difference between the two types of buckets. The placer mining bucket is an efficient cutting and digging tool, equally efficient in dumping, and capable if necessary of dislodging large boulders and digging hard bedrock. It has great strength and resistance to wear which makes it a unit of long service. It also has pins of large diameter as compared with those used on harbor dredge buckets, and a bolted lip that is quickly replaceable.

A placer dredge is expected to show an average daily running time of at least 21 hours for a period of several years. There are cases, under extreme conditions, where running time as low as 18 hours daily is considered satisfactory. The yardage dug by placer mining dredges naturally depends on the conditions and the formations encountered. A dredge having buckets of 12-cu-ft capacity, digging in extremely hard formation, has dug approximately 3,500,000 cu yd per yr. Another, with 18-cu-ft buckets digging to 124 ft below water level, has averaged over 4,000,000 cu yd annually for more than 3 years. A dredge of the same bucket capacity but digging only 80 ft below water level will dig under normal conditions from 15% to 20% more. Deeper digging presents operating conditions which reduce the yardage dredged. In the Hammonton area on the Yuba River in California, where the deep digging dredges are operating, approximately 900,000,000 cu yd have been dug. In the Folsom area on the American River in California, the total yardage dug is over 850,000,000 cu yd. The operating costs, including operating labor and supplies, power, repair labor and materials, and replacement parts usually have been 5¢ per cu yd or less for large placer dredges.

The two bucket-ladder dredges designed for use on the Panama Canal sea-level project incorporate a wealth of experience. All units have been designed for easy and quick replacement and for long service. Manufacturers of the various parts—buckets, gearing, electric and power equipment—were consulted. After a prolonged study, buckets of 54-cu-ft capacity were recommended. Buckets larger than this create many difficult problems in other machinery units making up the dredge, and this fact suggested limiting the designs to dredges having bucket capacities of 54 cu ft.

The dredge which digs 90 ft below water level will have units interchangeable with the deeper digging dredge, except for such units as the hull, superstructure, etc. The hull for the dredge digging 90 ft below water level will be approximately 245 ft long, 100 ft wide, and 15 ft deep; for the dredge digging 145 ft below water level, the hull will be approximately 371 ft long, 100 ft wide, and 13 ft deep. The installed horsepower will be 4,900 and 5,294, respectively. For the dredge digging 90 ft, the power plant will consist of three 1,600-hp diesel engines, each direct connected to a 1,250-kva, 2,300-volt generator. Four of the same units will constitute the power plant of the deeper digging dredge.

The digging ladder for the dredge digging 90 ft will be 163.5 ft between tumbler centers. For the deeper digging dredge the ladder will be 239 ft long. The buckets will have a 6-ft pitch and a height of 86.5 in. At the base the buckets will be 60.5 in. wide; at the lips, 74 in. There will be 62 and 88 buckets, respectively, on these dredges—the maximum bucket speed being 15.5 buckets per min.

The disposal system will provide for dumping the material from the buckets into the hopper, diverting them either to port or starboard, into barges. The weight of the dredge which digs 90 ft is estimated at approximately 5,900 tons, and that of the deeper digging dredge, approximately 7,200 tons. These weights do not include live loads.

In determining the estimated yardages and costs, dredges operated in deep and hard digging were considered. Although the maximum bucket speed will be 15.5 buckets per min, a speed of 12 buckets per min was used in the calculations. It is estimated that the dredge digging 90 ft will dig approximately 5,400,000 cu yd per yr and the deeper digging dredge, approximately 4,327,000 cu yd per yr.

Included in the calculations of costs were operating labor, operating supplies, power, repair labor, repair materials, and replacement parts. The salaries used were those in force in the Canal Zone under the schedule of July, 1946, including a sick leave of 20%. A work week of 6 days was considered, and a power cost of 2¢ per kw-hr. On this basis the estimated unit operating cost is slightly more than 20¢ per cu yd for the deep digging dredge, and slightly more than 15¢ per cu yd for the dredge to dig 90 ft.

In conclusion, it is to be noted that harbor dredges have been using buckets as large as 54-cu-ft capacity, and placer mining dredges have worked as deep as 130 ft below water level. Therefore, the dredges described and recommended for the Panama Canal combine the practices of both types of dredging and do not present any insurmountable difficulties.

Raphael G. Kazmann,[101] Assoc. M. ASCE.—An extremely valuable published addition to technical knowledge of some of the possibilities of relocating and redesigning the Panama Canal is presented in this Symposium. However, it is difficult, if not impossible, for an engineer not directly connected with the canal studies to criticize the application of the techniques described therein, simply because criticism implies differing experience on projects of the same order of magnitude in the same, or closely allied, fields. Such projects are few in number. Furthermore, engineers may criticize components of the findings in the Symposium (such as the applicability of soil mechanics, the navigational problems caused by tides, or the design of special dredges) without affecting the over-all correctness of the findings.

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[101] Hydrologic Engr., Ranney Method Water Supplies, Inc., Columbus, Ohio.

The converse is also true: Although the technical excellence of the methods is accepted, the very basis of the project may be so weak that technical excellence is a purely academic matter in determining its feasibility or desirability. The Symposium has clearly shown that engineering techniques now make possible a sea-level Panama Canal. The real question is, is one needed? The Symposium assumes that one is and hardly bothers with the point. It is possible, however, that the point is debatable and that someone who has not been as close to the details of the problem as the authors may be able to suggest a fresh viewpoint.

It is well to emphasize that the security of the canal is only one facet of the problem of national security. Others have already pointed out that in 1942, when the national security was actually in jeopardy, Panama Canal improvements could not complete with other projects for men and materials although more than $75,000,000 of a total of $277,000,000 had already been invested in the third locks "insurance" program. It should be remembered that certain Tennessee Valley Authority dams were still under construction and that all manner of large scale engineering construction projects, including a "fantastic" investment in the production of atomic energy, were being undertaken at the very moment when the canal improvements were being stopped because they were not vital to national security. If the canal were as necessary as the sea-level canal proponents claim, the third set of locks should and would have been completed during World War II. Surely the strategic situation in 1948 is far more favorable than it was in the dark days of 1942. Thus, a sea-level canal is probably needed less at present than a third set of locks was needed in 1942, and apparently at that time it was not needed at all.

It should be remembered that the United States has only so much man power and material at its disposal for construction projects. At present the labor and materials shortage is well known to every engineer. The expenditure of $2,500,000,000 (minimum estimate) over a 10-year period would be enough to accomplish the unified development of, for instance, the entire Missouri Valley Basin, thus increasing the physical resources of the country by 5,000,000 irrigated acres and by about 5,000,000,000 kw-hr per yr of firm power—in addition to preventing the damage due to droughts and floods. Moreover, about one third of the expenditure would be repaid within 50 years through the sale of power and water. The flood-control benefits and navigational improvements would account for the rest of the expenditures.

The question before the public in general, and the engineering profession in particular, when dealing with engineering proposals of this order of magnitude, is: As the United States has only so much man power and materials, is national survival made more secure by doing this or that, or the other, inasmuch as it is manifest that all cannot be done? A project of such magnitude cannot stand solely on its internal merit; it must stand on its utility as compared to other projects of the same magnitude and of equal internal merit, when viewed objectively in the national and international framework.

A sea-level Panama Canal seems to rate far down the list of desirable projects from the standpoint of national security. The unified development of the valley of the Tennessee River made possible the atomic energy program. The development of other basins will make possible a higher standard of living and may form the material basis for similarly great technical advances. The most potent construction equipment of the United States was designed in the course of river basin development, not only on the Tennessee Valley but also on the great flood-control and reclamation projects in the west. Similarly, the great advances in techniques of soil mechanics and hydraulic design resulted from river basin development. The further development of part of the internal transportation network—for example, the joining of the Ohio River and the Great Lakes-Saint Lawrence River systems, and the improvement of the latter—could be fairly compared with the sea-level canal project. Which of these would to a greater degree improve the national ability to win a war?

It should be borne in mind at all times that military technology is based on the nonmilitary material and technological resources of a country. Would the nation be more likely to win a war with a developed Missouri Basin and an improved lock canal as compared to an undeveloped Missouri Basin and a sea-level canal? Similarly, compare a fully harnessed Columbia River, a unified and operating Saint Lawrence and the Ohio River waterway system, a completed Central Valley project, a complete new major road net, urban redevelopment, or completed atomic energy power plants with the utility of a sea-level canal. Which of these would be more important to national existence in the event of a conflict? Which would add more to the striking power of the United States? These are the alternatives that an engineering symposium may fairly be expected to mention, thus putting the proposed technical studies into a proper perspective rather than giving the impression that the issue had already been decided in favor of a sea-level canal and that the question is solely one of technique. Thus, the highest type of professional judgment could be brought to bear on proposed engineering projects, evaluating them as objectively as possible, both as to technical details and as to the possible alternatives of accomplishing the prime objectives—national strength and security. This procedure would assist the public and its duly elected representatives in arriving at a proper decision.

George B. Massey,[102] M. ASCE.—Three of the papers in this Symposium are of especial interest to the writer—the fourth, fifth, and ninth.

Tidal Currents.—In India, the large irrigation canals must have a certain water velocity between very narrow limits so that silt will not be deposited or will not be picked up from the bed of the channel and carried away. To arrive at the necessary hydraulic radius some very unusual cross sections for canals are developed.

The problem in the sea-level canal at Panama is to provide a free, open channel for ships and, as much as possible, to prevent the tidal conditions from causing an undesired current. With that idea in view, consideration should be given to the advantages and disadvantages of submerged natural stone weirs left in place athwart the channel at intervals of 1,000 ft, with top elevation at −42 ft, a top width of 10 ft, and 45° side slopes. It is probable that the reaction of a passing ship to these weirs would scarcely be noticed; and, as a retarding agency to the velocity of the current, they should have a considerable effect.

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[102] Cons. Engr., Chicago, Ill.

If these weirs were left in place when the canal was first used, the result would be that less rock would have to be excavated before the canal was opened. If found ineffective, they could be removed with the canal in use.

Ships would have all the advantage of a 60-ft depth of channel, with the further advantage of a decreased velocity.

Ship Performance in Restricted Channels.—In connection with the many tests in the towing basin, it would be interesting to know if any were made with a central dike having a top elevation of −42 ft, a width of 10 ft, and side slopes of 45°, to determine whether this factor would tend to a favorable modification of the effect of bank suction. A ship could approach or pass over such a dike without danger of grounding.

Construction Planning and Methods.—It is quite possible that the occasion never again may arise requiring dredges to dig blasted rock at depths of 145 ft below the water surface. Certainly it has never arisen heretofore. Unusual conditions require and inspire the conception of unusual equipment.

The dipper dredge described evolved naturally from dipper dredges digging from 50 ft to 60 ft deep. The increase to 85 ft is an over-all increase in size. The 500,000-lb bail pull indicates a horizontal thrust of the dipper teeth of from 250,000 lb to 300,000 lb.

The hydraulic dredge has the unusual feature of a pump located well down the ladder. This presents no mechanical difficulties and the digging reaction in a hydraulic dredge is not severe.

A bucket-ladder dredge for digging to a depth of 145 ft is a monstrosity with a 230-ft-long ladder, consisting of two trusses 20 ft deep and many heavy buckets. Bucket-ladder dredges have operated in Europe for decades, digging soft and medium hard material in harbors and rivers to a depth of 40 ft or more. Also, they have been used most successfully in digging for gold, platinum, and tin, sometimes in compacted gravel, but usually in fairly loose material.

Most placer deposits consist largely of coarse sand, gravel, and small boulders, all of which easily enter the buckets of the bucket-ladder dredge and are carried up to the dumping point without loss of water or mineral through leakage. The high central discharge point adapts itself to feeding screens and tables, with stackers located at the stern. Sometimes these dredges excavate bedrock when it contains fissures apt to hold minerals, but usually the bedrock is shale or slate and only about a foot would be excavated. There is no other type of dredge at all suitable for placer work.

The bucket-ladder dredge purchased by the Panama Canal Authorities about 1912 in Scotland utterly failed to dig blasted rock in the Culebra Cut and was replaced as soon as possible by dipper dredges, which were ideal for the work because of the absence of any space or size restriction on the pieces of rock to be handled. Photographs show these dipper dredges raising to the water surface great pieces of rock too large to pass through the dipper. These were lowered into the water, blasted, and the smaller pieces picked up by the dipper and loaded on barges.

With any system of drilling and blasting there are oversize pieces of rock, just as in the case of a primary crusher, where 15% of the rock will be larger than the crusher opening size. If drilling and blasting are adapted to produce rock that can be pushed into a 2-cu-yd bucket and carried up the ladder, what will happen to the inevitable oversize pieces? They cannot be handled by the 2-cu-yd buckets; they cannot be blasted again at that depth. The only solution would be to dig holes and bury them, or to use a grapple for each individual rock.

To equal the digging thrust of the dipper dredge, the spuds of the bucket-ladder dredge would have to provide a reaction of from 250,000 lb to 300,000 lb. As the width of the 2-cu-yd bucket is probably one third of the width of the dipper on the dipper dredge, the thrust could be reduced, but the efficiency is also reduced in the handling of large rock pieces.

The point of discharge on the proposed bucket-ladder dredge is where the buckets turn over the upper tumbler. This point is 50 ft above the water surface and the rock will have to be loaded into the barges by gravity or pan conveyors. In addition, the buckets must be built with a high factor of safety because one broken bucket will often mean the loss of the whole chain, which causes days of delay. About half of the buckets are subjected to the digging strains at all times.

Two other types of dredge should not be overlooked as possible solutions to the problem—the clamshell and the dragline.

Clamshell Dredge.—A clamshell dredge was used at Pearl Harbor, Hawaii, for excavating blasted rock in a seaway where it was impossible to use a dipper dredge because of the large swell. The closing power of the bucket is dependent on its weight, which is necessarily limited, but this type of equipment can excavate with nothing but wire ropes between the dredge and the sea floor.

Dragline Dredge.—Since the publication of this Symposium, there has been devised a dragline dredge which will dig at 90 ft or 148 ft without change, and which will have a 30-cu-yd bucket 12 ft wide and a digging thrust of 250,000 lb by direct pull of the draglines in an almost horizontal direction. The theoretical capacity is less than that of the bucket-ladder dredge, but the actual capacity would probably not be much less. The 30-cu-yd drag bucket would be raised to 10 ft above the water and dumped into the barge under perfect control.

All rock resulting from suitable blasting would be handled by this dredge; the drilling and the blasting would be 40% of those required by the bucket-ladder dredge. The excavation of blasted rock can be accomplished only by the use of a large bucket with teeth and of a large dragline pull at an angle as nearly horizontal as possible at first, and of not more than 45° after opening up a hole or a pit. The dragline dredge would dig as effectively at the 148-ft depth as would the dipper dredge at the 85-ft depth.

Although it may have been considered, no mention is made of the advisability of excavating with the special dredges to a depth of about 130 ft (El. −42) and of leaving the rest of the excavation to El. −60 to be done by regular dredges after the canal is opened for traffic. The difficulty would probably be the disposal of the spoil with the water surface at sea level and with large spoil areas eliminated, but it would advance the date of opening the sea-level canal to traffic.

William Allan,[103] M. ASCE.—In solving the tidal current problem for a sea-level canal at Panama, Messrs. Meyers and Schultz have made a notable contribution to the science and art of hydraulic computation and experimentation.

The results of extended calculations and of model tests confirm the evaluation of marginal limiting velocities computed by the steady-flow method and suggest the desirability of an explanation of the argument of approach and of the procedures followed in this method. The writer cooperated with Boris A. Bakhmeteff, Hon. M. ASCE, in making the preliminary calculations mentioned in the paper (under the heading, "Analyses of Tidal Currents for 1947 Investigation") and presents the method used as applied to a conventionalized Panama Canal based on the adopted cross section.

Because the Pacific and Atlantic levels, by which flow in a sea-level canal would be actuated, change continuously with time, the flow is variable (unsteady). The governing equations of motion and of continuity, respectively, are

$$S = \frac{1}{g}\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}\frac{v^2}{2\,g} + \frac{Q^2}{A^2\,C^2\,R} \qquad \text{(4)}$$

and

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{(5)}$$

Eq. 4 is the equation of instantaneous dynamic equilibrium, and Eq. 5 is the equation of storage.

A solution using these equations is possible only by adopting a simplifying assumption to make friction vary as the first power of the velocity in selected reaches and periods of time[104,18] or by methods of successive approximations.[8] By these methods a complete solution of a given problem may require many thousand man-hours of computation.[105] The method presented in this discussion, although mentioned in the paper, has not heretofore been published. In the present instance a complete solution is essential, but it is especially desirable to know the limiting maximum tidal currents. These limiting values and other features may be found in from 25 to 30 man-hours by adopting justifiable simplifications which enable one to make a solution based upon varied (steady) flow. This assumes no change of flow characteristics with time, thereby making the local acceleration slope, $\frac{1}{g}\frac{\partial v}{\partial t}$, zero and removing storage from consideration. These conditions are satisfied approximately when the Pacific tide is at about its highest and its lowest level, respectively, at which points in the cycle the change of elevation with time is small compared to the total rise or fall of the tide. The interval of time to be used in observing the comparative smallness of change of elevation is the period required for a change (cause) at the Pacific end to be felt (effect) at the Atlantic end. It is the time required for a translation wave to travel from the Pacific to the Atlantic. This impulse or wave travels at a rate of $(c + v)$ with respect to the bank when flow is from the Pacific end toward the Atlantic end and at the rate of $(c - v)$ when flow is from the Atlantic toward the Pacific, v being the flow velocity and c, the celerity of propagation of a wave in still water. This propagation rate is evaluated by

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[103] Prof. of Civ. Eng. and Dean, School of Technology, The City College of New York, New York, N. Y.

[104] "Leçons sur la Théorie des Marées," by Maurice Lévy, Gauthiers-Villars, Paris, 1898.

[105] "Analogy Between Tides and A. C. Electricity," by Johan van Veen, *The Engineer*, November 28, 1947, p. 498.

$$c = \sqrt{g\frac{A}{b}} \qquad \text{(6)}$$

in which b is the width of the canal at the surface level and A/b is the average depth. The effect of friction on the wave travel, acknowledged to be very small, is omitted.

The conventionalized canal has a channel 220,000 ft long; a bottom width of 560 ft; side slopes of 1 on 1; a depth at the Pacific end of 60 ft at low tide and 80 ft at high tide; a constant depth at the Atlantic end of 60 ft (no tide); a bottom slope downward from the Atlantic to the Pacific of 10 ft in 220,000 ft $(= 0.455 \times 10^{-4})$; and Manning's $n = 0.024$. After completing the basic computations, the ± 1-ft tidal range at the Atlantic end used in the Panama studies was considered, local effects at each end were assessed, and adjustments were made.

In steady flow, when the Pacific stage is below that of the Atlantic, flow is toward the Pacific and in the direction of the bottom (sustaining) slope. Discharges (delivery curves) and surface profiles were computed in this case by use of tables of varied flow functions.[106] When the Pacific stage is above that of the Atlantic, flow is toward the Atlantic and against an adverse bottom slope.[107] For this case no tables were available, so the integral functions were calculated by step-by-step integration and then used to compute the discharges and surface profiles. Steady flow toward the Pacific, if the stage there were held at −10 ft, would be 214,000 cu ft per sec at a uniform depth of 60 ft; and the average velocity of flow would be $v = Q/A = 214{,}000/37{,}200 = 5.75$ ft per sec. At this limit $c = \sqrt{32.2 \times 37{,}200/680} = 42$ ft per sec. Hence, at low Pacific tide, changes there will be transmitted toward the Atlantic at the rate of about $42 - 5 = 37$ ft per sec, and the "time lag" required for a change originating at the Pacific to produce an appropriate effect at the Atlantic will be about $t_w = L/(c - v) = 220{,}000/37 = 5{,}950$ solar sec $= 1.65$ solar hr $= 1.60$ lunar hr.

Steady flow toward the Atlantic, if the Pacific stage were held at + 10 ft, would be 267,000 cu ft per sec and the surface profile would be of the M_2 type.[106] An approximate over-all flow velocity on the basis of a 70-ft average depth would be $v = 267{,}000/44{,}100 = 6.05$ ft per sec and for this same average depth $c = \sqrt{32.2 \times 44{,}100/700} = 45$ ft per sec. Hence, changes in Pacific stage at high levels will be transmitted toward the Atlantic at the rate of about 45

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[106] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

[107] "Varied Flow in Open Channels of Adverse Slope," by Arthur E. Matzke, *Transactions*, ASCE, Vol. 102, 1937, p. 651.

$+ 6 = 51$ ft per sec and will be felt there approximately $t_w = L/(c + v) = 220{,}000/51 = 4{,}320$ solar sec = 1.20 solar hr = 1.16 lunar hr later. Referring to Fig. 26 it will be noted that in 1.60 lunar hr at Pacific low tide the stage falls from − 9.14 ft to − 10 ft and then rises again to − 9.14 ft, a vertical change of 0.86 ft. In one half of this time lag the change down to the lowest level is only 0.22 ft. At Pacific high tide, in 1.16 lunar hr the stage rises from + 9.54 ft to + 10 ft and then falls again to + 9.54 ft, a change in elevation of 0.46 ft. In 0.58 lunar hr at the peak the change is only 0.12 ft.

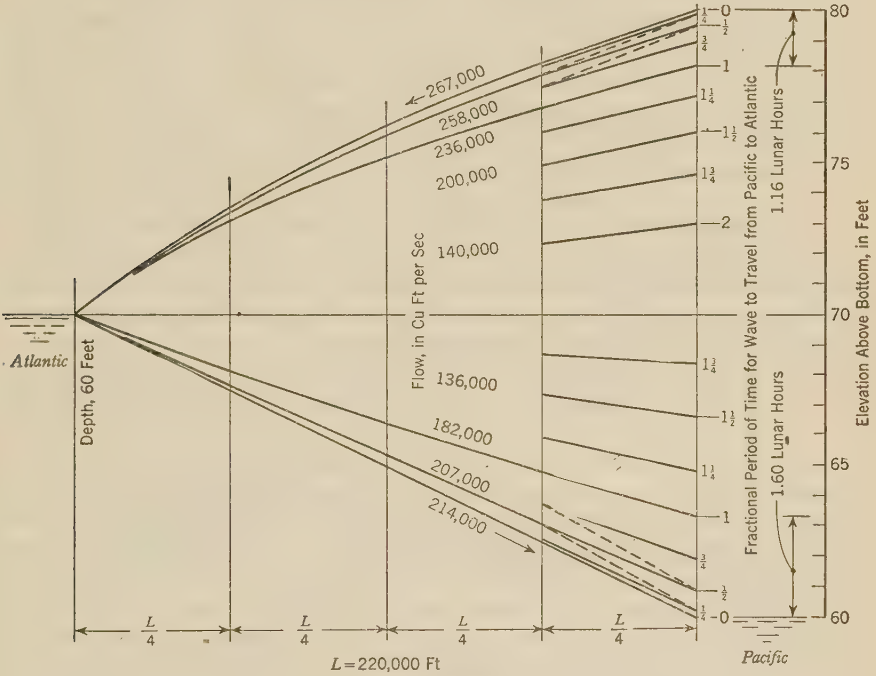

FIG. 112.—STEADY-FLOW CURVES FOR CONVENTIONALIZED CANAL SECTION
(Bottom Slope = 0.455×10^{-4}, Manning's n = 0.024,
Bottom Width = 560 Ft, and Side Slope, 1 on 1)

For comparison, a calculation of steady flow toward the Atlantic from Pacific stage +9.54 ft gives 258,000 cu ft per sec as against 267,000 cu ft per sec from stage +10 ft; toward the Pacific with a −9.14-ft stage a similar calculation gives 207,000 cu ft per sec as against 214,000 cu ft per sec with −10-ft stage. These values and the corresponding surface profiles permit an assessment of the total effect of the local accelerations, which were neglected. At the upper and also at the lower limits of the Pacific tide this value is about 5% of the total fall over the entire length of the canal. In terms of discharge less than 3% error is indicated.

Several steady-flow surface profiles and discharges are shown in Fig. 112. These suggest a method of tracing approximate successive instantaneous surface profiles near the upper and lower limits of the Pacific tide. For example, if the

Pacific level were imagined to stand for a long time at El. 79.54 (marked, "1/2 wave travel time") the corresponding steady-flow surface profile would be formed. If the Pacific level were now free to follow the tide curve it would rise in 0.29 lunar hr (the time for a wave to travel one fourth the length of the canal) to the point marked "1/4" (El. 79.88) and in that time interval the effect of the rise would extend to a point $L/4$ along the canal. The surface on the original "1/2" curve at that $L/4$ point would begin to rise only at that instant. Consequently, the approximate water surface would extend from the "1/4" curve at the Pacific to the "1/2" curve at a point $L/4$ in from the Pacific and the local influx would correspond to this surface slope. In the next interval of 0.29 lunar hr the Pacific level would reach the "0" curve at El. 80, the surface would reach the "1/4" curve at the $L/4$ point, and the effect of the Pacific rise begun 0.58 lunar hr previously would now have extended to the midlength of the canal on the "1/2" curve, and so on. Only after a total interval of 1.16 lunar hr from the start will the effect of the initial movement at the Pacific end of the "1/2" curve be felt at the Atlantic end. The efflux at the Atlantic will correspond to the instantaneous slope of the water surface there and will reflect the effect of the change in Pacific level initiated 1.16 lunar hr previously. This time lag is borne out by the model tests. By this procedure, under the assumption of zero tidal range there, the limiting water-surface slopes at the Atlantic are those cor-

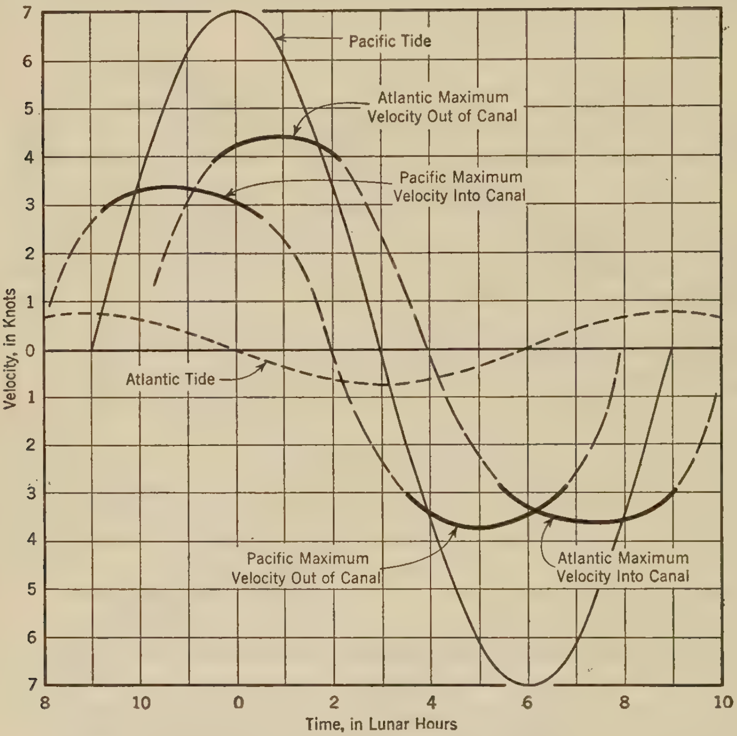

FIG. 113.—CURRENT VELOCITIES AT ATLANTIC AND PACIFIC ENDS OF CONVENTIONALIZED CANAL

responding to the extreme upper and lower steady-flow profiles with their respective flows of 267,000 cu ft per sec out of the canal, and 214,000 cu ft per sec into the canal. Adjustment of these profiles to account for the tidal range of ± 1 ft used in the tests increases the computed flows to 277,000 cu ft per sec and 226,000 cu ft per sec, respectively, with velocities of 4.4 knots out of the canal and 3.6 knots into the canal at the Atlantic end compared to 4.4 knots and 3.7 knots by test on the canal model.

A succession of approximate instantaneous local water-surface slopes at the Pacific end appropriate to rising and falling tides near high water and low water gives corresponding maximum flows of approximately 284,000 cu ft per sec into the canal near high tide, and 243,000 cu ft per sec out of the canal near low tide. These values include a small adjustment to account for the ± 1-ft tide range at the Atlantic and give maximum velocities of 3.4 knots into the canal and 3.7 knots out of the canal at the Pacific end, as compared to 3.3 knots and 3.7 knots, respectively, by model test. The order of some of the approximate instantaneous water surfaces is shown by dashed lines in Fig. 112.

Before adjusting for the Atlantic tide range, a check was made between Pacific influx and Atlantic efflux and the storage between approximate instantaneous water-surface profiles for the period from 0.58 lunar hr before high tide to 0.58 lunar hr after high tide at the Pacific end. The result was excellent.

Velocity values determined by this calculation for stations 0 (Atlantic) and 2,200 (Pacific) of the conventionalized canal are shown in Fig. 113, in which the tide cycle is shown to emphasize the time relationships.

BORIS A. BAKHMETEFF,[108] HON. M. ASCE.—The investigations reported on in the paper by Messrs. Meyers and Schultz set a landmark in the studies of tidal currents in ship canals. In each major engineering project there are certain crucial questions to which clear and dependable answers are required at an early stage in order to judge the feasibility of the whole enterprise. In the instance of the sea-level canal over the Panama Isthmus, in addition to the problem of stable slopes and comprehensive methods of mass excavation, the question was to know the size and the regimen of the currents to be engendered by the tides. The problem is one of unsteady motion, variable in time. The general differential equations are easily written and have been known for years. Thus far, however, satisfactory mathematical solutions in the case of variable flow only could be made either by disregarding the effects of friction (as in the theory of surges and translational waves), or by neglecting inertial effects (as in "hydraulics of floods"). In tidal canals such as the Panama Canal, both inertia and friction exercise comparable influence and a reliable solution requires accounting for the effects of both.

There are no direct mathematical methods that permit a complete and exact integration of the equations under such circumstances. Accordingly, in the past, each time a major ship canal was envisaged outstanding mathematical talent was called upon to offer solutions.[19,109] At best, however, simplified assumptions were necessitated as substitutions for the actual laws of nature.

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[108] Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[109] "The Cape Cod Canal," by William Barclay Parsons, *Transactions*, Vol. LXXXII, 1918, p. 105.

Another method of attack is by gradual arithmetical integration carried out by successive approximation.[8] More recently, recourse has been suggested to the electric analogy.[105]

Notwithstanding the remarkable ingenuity of these methods, there is a practical drawback in that it has not been possible to appraise beforehand the magnitude and even the sense of the possible error. In other words, it was not known whether the values arrived at were greater or less than those to be anticipated in practice. On the other hand, even in the preliminary stages of the canal design it is imperative to know with certitude at least the extreme limits within which tidal currents are bound to occur. The writer vividly recalls the sense of oppressive responsibility when faced with the first stages of the work early in 1946, the final results of which are reported on now with such confident certainty by Mr. Meyers. Indeed, it was for the very limited purpose of obtaining marginal current values, and with no claim for a complete solution of the tidal regimen, that the writer conceived the approach referred to in the paper, which amounts to checking the flow at the extreme tide levels by methods of steady varied motion. It was self-evident, of course, that the outflow in variable motion at the Cristobal end of the canal, where conditions are crucial, would under all circumstances be below the maximum figures computed for steady conditions. With that, however, the possible excess of the calculated values over actual flow values could be appraised only approximately.

A complete and exhaustive answer to all the questions concerning tidal currents could be confidently given solely by experimenting on a suitable model, and the decision to take recourse to model investigations in the preliminary stage of design stands as an eloquent testimony to the vision and courage of the Supervising Engineer under the egis of the Governor of The Panama Canal. The fact is that in order to give effective and indisputable answers, the model had to be undistorted and, moreover, built on a sufficiently large scale. Mr. Meyers and his assistants deserve all possible credit for the design, construction, and operation of the model, which is of unprecedented type and scale.

With the results described in the paper at hand, it may be stated confidently that the sea-level canal project, as submitted by the Governor to Congress, has answered the problems concerning tidal flow in a complete and indisputable fashion. Actually, it is the first time in the history of engineering that such an answer has been given with certitude and finality.

The work reported on by Messrs. Meyers and Schultz carries further significance in that, for the first time, observational data are available to compare and check results of computation by the different approximate methods. Thus, as stated in the paper (under the heading, "Currents in an Open Sea-Level Canal") the method devised by George B. Pillsbury, M. ASCE, after the necessary successive approximations, yields an over-all account of the regimen in close accord with actual facts. The writer was most gratified to find, also, that the marginal figures of maximum flow obtained by his own approach only negligibly exceeded the observed velocities. On the whole, engineering science has moved a step forward in its knowledge of tidal phenomena and in the appraisal of methods to calculate and predict the course of tidal phenomena.

A serious point of weakness, which Mr. Meyers was compelled to face, is the lamentable lack of observational data relating to frictional resistance in large canals. The values in Table 19, assembled with a great deal of effort, clearly exhibit how negligent the profession has been in making the necessary observations on existing canals of large size—one more testimony to the urgency of the task assigned by the Society to the Committee on Hydraulic Data and Facts of its Hydraulics Division.

It is only natural that under the pressure of time the Panama Canal model could be used only for obtaining answers of immediate practical character. As built, however, the model offers unprecedented facilities for systematic study of variable channel flow subject to frictional and inertial effects. Knowledge in this field is very meager and quite important. Naturally, for carrying out such work the model would require more delicate instrumentation than has been available. However, the basic equipment is there and it is confidently hoped that the model will be preserved and put to use to continue its valuable pioneer work.

ROBERT C. SHELDON,[110] Assoc. M. ASCE.—A clear solution of a very complex problem has been presented by the several authors of this Symposium. The writer will discuss some interesting possibilities in connection with the foundations of the proposed Monte Lirio and Trinidad diversion dams and their relation to the proposed sea-level channel. Having had some experience with foundation conditions on the Atlantic side of the Isthmus, while engaged in the construction of a marine railway at Mount Hope during World War II, the writer believes that the material classified as Atlantic muck must be thoroughly studied before being used to support any load, and that these studies must be made in the immediate location of the load.

The spectacular slides in Culebra Cut or in Gaillard Cut so far overshadow the difficulties of construction on the Isthmus that other troubles have been forgotten. In the history of construction on the Isthmus there have been failures or partial failures of construction founded on the Atlantic muck and on the Pacific muck. A number of these failures have occurred on projects that had been adequately investigated by competent engineers.

During the construction of the original Panama Railroad in the period from 1849 to 1860, great difficulties were encountered in crossing a bottomless swamp between Gatun and Tiger Hill. Settlement of the railroad in this region continued for many years, until somewhat stable conditions were reached.

This stable condition remained until the start of construction of the American canal. The new program called for a double track along the line of the original railroad, but, as the fill for this double track was being made, both the new and the old fills settled. Fills, track, rails, as well as dump cars, were all swallowed up in this swamp as if by magic. Piles placed to support temporary track disappeared, to reappear in other parts of the swamp. With all the waste material then available, the double track was never completed; traffic was, however, maintained on the old single track.

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[110] Civ. Engr., Marietta, Ga.

During the canal construction period it was necessary to build the so-called Tiger Hill line from a relocated high level line at Gatun to join the original Panama Railroad at Tiger Hill—requiring fills to a level of approximately +50 ft across this same swamp. The fills were designed for side slopes of 1 on 2; but, as construction of the fill approached finished grade, settlements of 15 ft to 20 ft occurred overnight.

As the existing Panama Railroad embankments to El. +92 were being made, settlements were encountered in the crossings of the Gatun, Buebrancha, Brazos and Baja rivers. At all four crossings the fill was made in successive lifts of 30 ft, 45 ft, and 15 ft. The settlement, which amounted to as much as 30 ft, occurred as the last lift was nearing completion. It was necessary to widen the base to prevent uplift of the swamp at the edges of the fill. Final embankment slopes were 5 on 1 rather than 2 on 1 as designed and as shown in Fig. 24(*a*).

Throughout the records[111] of all these railroad fill settlements the same typical story is told: As the fill was nearing completion everything was in order as the work was left at night—in the morning everything was gone. All these settlements were accompanied by an uplift of the muck outside the fill area.

At the location of the marine railways at Mount Hope a very thorough investigation was made, consisting of closely spaced drill holes and two cofferdam pits in an area 1,200 ft by 300 ft. This investigation disclosed the fact that the top of the sound material underlying the soft, plastic overburden varied abruptly in elevation. Slopes of the sound rock surface running as high as of 1 on 1. In this small area differences in elevation of the sound rock surface were as great as 100 ft, although surface elevations were practically level.

It is the writer's opinion that most of the settlement that has occurred in the fills placed on Atlantic muck are caused by the material sliding on these slopes, rather than by failure in the muck. Also, this Atlantic muck seems to be present at great depths (approximately −200 ft) throughout the area covered by the northern part of Gatun Lake, and the steep slopes of the underlying sound rock found at Mount Hope seem characteristic of this area.

The location of the proposed channel (from Fig. 8) will cross the Gatun River Valley parallel to and about 1,500 ft east of the locations of the original Panama Railroad and the Tiger Hill line, and about 3 miles west of the existing railroad crossing. The location of the west diversion dam is approximately 4,000 ft west of the proposed sea-level channel. The complete section of the proposed construction is shown in Fig. 114.

With the construction program as proposed for the sea-level project, there is little prospect of encountering serious difficulties during the construction of the diversion dams at Monte Lirio and Trinidad. In the dredging of the proposed channel across the Gatun River Valley there is the possibility that considerable excess excavation will be required near the location of the swamp encountered by the original Panama Railroad because of the flow of the soft material there into the dredged channel.

It is proposed to empty Gatun Lake in 7 days and to have the sea-level channel in service at the end of the draining period. As the lake is lowered, the pressure that has been holding the uncovered areas of Atlantic muck in a static condition will be released, permitting this material to flow or to slide under the new loads that will have been placed on it in the construction of the spoil banks (El. +65) and the diversion dams (El. +92).

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[111] From "Annual Reports of the President of the Panama Railroad," U. S. Govt. Printing Office, 1903–1913.

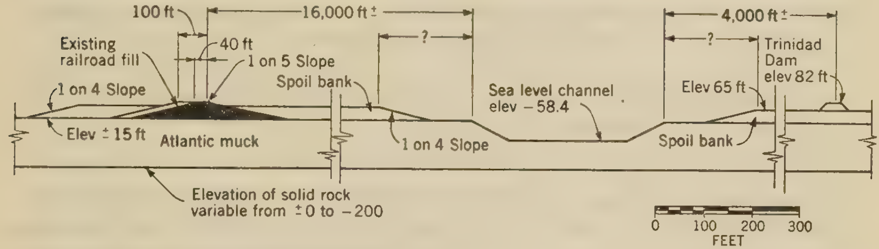

Fig. 114

If these slides occur, they will cause settlement in the spoil areas and uplift of the muck in the areas between the spoil banks and the sea-level channel. If the edges of the spoil bank are near the proposed channel the uplifted muck will flow into the channel, causing a serious block at a critical time.

Although the possibility of a flow of the muck under the Monte Lirio Diversion Dam (with the addition of wide spoil banks to strengthen it), caused by the 55-ft head of the Monte Lirio Lake, is remote, it is a possibility. In view of the importance of this structure to the project and the generous factors of safety planned for other structures proposed, the addition of a sheet pile cutoff wall driven to sound rock at the upstream edge of the existing railroad fill would be warranted.

None of the possibilities mentioned in this discussion constitutes a serious threat to the sea-level project. They can all be avoided by an adequate drilling program aimed at determining the existence of any steep slopes underlying the areas of Atlantic muck at the proposed dams and spoil banks and the proper location of the edges of these spoil banks with respect to any underlying steep slopes. Design of the edge slopes of the spoil bank so that they are very flat (approximately 15 on 1) might be insurance against any settlement.

F. W. Edwards,[112] M. ASCE.—This Symposium is complete, interesting, and particularly impressive. Problems unsolved in the past have been studied in detail. Definite values for maximum tidal currents have been determined, and deep dredging has been established as a practical method for developing a sea-level canal. The report that forms the basis for the Symposium reflects the magnitude of work that can be accomplished in a relatively short time by a well-organized and competent engineering staff. Considering the fact that less than 2 years elapsed between the date the law was passed (December 28, 1945) and the date the report was submitted to Congress, the amount of data collected and the number of detailed studies completed are enormous. During this period the organization was recruited, facilities were constructed, contracts were let, research studies and preliminary designs were prepared, and, finally, the report was written and assembled. The organization deserves credit not only for the magnitude of the work accomplished but also for the quality of technical thought.

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It is interesting under these circumstances to reflect on the relation of these studies to the purpose of the original directive. The following questions appear to cover the essentials of the law passed by Congress—

1. What type of canal would serve best the interests of the United States?
2. Where should such a canal be located?
3. What would be the approximate cost?

The engineering studies reported in the Symposium do not, and could not, provide the basis for the answer to the first question. Whether tidal currents are greater than 4 knots or smaller than 3 knots has little bearing on the answer; nor does it matter how details of flood control are to be handled. Flood control is merely a necessary requirement for a sea-level canal. Reports not available to the general public should furnish some information but the final answer must be based primarily on judgment. The conclusion that a sea-level canal is the only one which might possibly be satisfactory from the standpoint of security was reached on this basis and there is no adequate basis for contradicting this opinion. There is, however, ample reason to believe that a lock canal would not remain usable as a powerful defense facility. It seems, therefore, that the question regarding the type of canal could be answered without such detailed technical studies.

The second and third questions may be considered together as they both involve cost. Relative cost is the primary factor in the second question, whereas estimated actual cost is needed to answer the third question. An examination of Table 1 indicates that the difference in cost between various canal locations is so great that the conclusion that Panama is the most favorable location from a cost standpoint would be reached from less detailed estimates. As evident from the Symposium, only those estimates for locations in the vicinity of the present canal could be considered detailed in any case. Once questions 1 and 2 are answered, question 3 becomes less significant at this time. Would the decision to build, or to reject, the sea-level canal be different if a less detailed estimate indicated the cost to be $3,000,000,000 instead of the reported estimate of $2,483,000,000? It is doubtful, as security appears to be the dominant factor. Congress, which makes the final decision, no doubt will place emphasis on the answers to the first two questions.

The importance of comprehensive studies as detailed as those reported varies with time. When they are made at the beginning of the period of actual design and construction they are invaluable, when made too far ahead of design and construction they are less important. If the decision to proceed with any of the plans outlined is made promptly, the value of the studies will not be lost. However, if the decision is delayed, the details of the studies will become less valuable as time passes. Practically a year has now (November, 1948) elapsed since the report was submitted to Congress and there appears to be little likelihood that a decision will be reached soon.

The importance of the relationship between time of study and time of decision may be illustrated. As late as 1942 the third locks for the Panama Canal were being designed with lock chambers 140 ft wide and 1,200 ft long with a 45-ft depth over the sill. A short 5 years later the Navy (according to the Symposium) estimated that the ultimate width should be 200 ft, the length 1,500 ft, and the depth 50 ft. A report[113] in the popular press indicates that the latest aircraft carrier is 190 ft wide. By the time the sea-level canal or its substitute is authorized probably many of the detailed studies will be obsolete. This situation should not be created in the case of studies reported to cost $4,500,000.[114] Such detailed studies would be more valuable if they were made immediately before final design and after the basic decisions have been made on the broader questions of what to do and when. Engineers responsible for public works programs would perform a more valuable service if they would insist on this approach in planning projects.

In contrast with the foregoing general discussion the writer would like to direct attention to one specific point, involved in the statement by Mr. Brown (under the heading, "Requirements for Flood Control"), that "The control afforded by Madden Lake has largely ended the likelihood of further major flood disturbances in the present canal."

Engineers in the Panama Canal organization believed for a number of years that Gatun spillway was inadequate. This belief was substantiated by a comprehensive hydrology study covering flood control, water supply, and hydro power development, made in connection with the Third Locks Project in 1940–1941.

This earlier report made use of every available record of rainfall, stream flow, and general storm descriptions. Many old French records that had remained in the vaults for years were transcribed and pertinent data were used in the study. The United States Weather Bureau at that time made an independent study and derived the maximum possible storm over the canal drainage basin. Mr. Brown refers to this storm under the heading, "Spillway Design Flood." It is the storm used to develop the spillway design flood in the present studies. Although the results of the United States Weather Bureau study were received too late to be incorporated in the 1940–1941 hydrology report, the flood-producing characteristics of the storm developed by that agency checked reasonably well with those of the design storm used in that report. The 1940–1941 report emphasized the need for augmenting the spillway capacity for Gatun Lake.

The data available for the studies in 1946–1947 were more complete than those for the 1940–1941 studies. This should be true particularly because automatic rainfall stations were installed in 1941 at strategic locations. However, it is doubtful if the recent studies indicate that the Gatun spillway is now adequate. In fact, there is no indication in the paper that the Gatun spillway requirements were studied. This is a problem that should receive further careful study and for which remedial action should be taken.

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[113] "Armed Forces," *Time*, October 11, 1948, p. 28.

[114] "Can We Defend the Panama Canal?" by Sidney Shalett, *Saturday Evening Post*, October 9, 1948, p. 15.

H. R. Cedergren,[115] Assoc. M. ASCE.—The investigation of the effect of transient loads on soil and soft rock (in the eighth Symposium paper) is a valuable contribution in this relatively unexplored field. Unfortunately, much evidence supporting the conclusion of Messrs. Casagrande and Shannon, to the effect that shocks will not endanger otherwise stable slopes in clays and Cucaracha rock, was not included in their paper. An excellent discussion of natural phenomena substantiating this conclusion has been presented by the authors elsewhere.[116] In the Symposium, the authors also point out that loose, cohesionless soils are more susceptible than clays and soft rocks to shear failure under shocks such as might be caused by bomb explosions.

Failures of cohesionless formations under shock are relatively common. Examples of railroad embankments sliding out immediately following the passage of trains are classics in engineering literature. Failures of this type also occurred in the Pacific Northwest during the disastrous 1948 Columbia River flood. A section of a main line railroad embankment on a cohesionless soil foundation slipped into the river taking several cars of a moving freight train with it. At least one major dike failure may have been caused by liquefaction of the foundation that possibly was initiated by the passage of a heavy train over the fill a short time before the failure. Numerous additional examples can be cited as evidence of the potential weakness of cohesionless, saturated soils under shock. The writer knows of no parallel examples for clay soils. Nevertheless, it would seem advisable to collect further factual data regarding large slopes, as recommended by Messrs. Casagrande and Shannon,[116] before ignoring all possible detrimental effects of earthquake or bomb shock on slopes in clays, Cucaracha rock, and other similar materials. A thorough study of records of slides attributed to earthquakes or other shocks would be helpful in this respect. Also, large-scale shock tests of slopes barely stable under static loading conditions should provide valuable information regarding the strength of soils and soft rocks under transient loading.

The authors of the eighth Symposium paper state that the adobe clay subgrade of the Stockton (Calif.) test section (the special test section constructed and tested by the U. S. Corps of Engineers) had a higher modulus of deformation under transient loads than under sustained loads, as typical of clay soils. This observation is consistent with the performance of the test pavements under the heaviest loads ever carried by a single pneumatic tire. For nearly 2 years the writer observed the performance of the specially constructed pavements under both standing and moving wheel loads. Consistently the slowly moving loads (about 5 miles per hr) were less severe on the pavements and caused less deformation in the soil subgrades than standing loads of the same magnitudes. These observations evaluate the combined effect of many factors; however, they are of interest because they reflect the indirect effects of transient and sustained loads on soil masses.

In the seventh Symposium paper Messrs. Binger and Thompson have presented an excellent discussion of the design of cut slopes for the sea-level canal. The design is based on strengths obtained from the analysis of existing Panama Canal slopes supplemented with laboratory tests on representative samples of weaker materials. This approach greatly increases the dependability of new designs. One of the weaknesses of attempting to predict required slopes of excavations from laboratory tests alone is the probable effect of slickensides, faulting, joint planes, and other inconsistencies on the strength of large masses of soil or rock. In the Pacific Northwest, for example, many formations in the hilly regions traversed by highway alinements or proposed relocations of existing highways and railroads around dams contain numerous inconsistencies that are potential sources of trouble from slides. Conditions often vary radically within short distances. In fact, intrusions of weak materials, difficult to detect by field explorations, may cause slides where they are least expected. Because of these conditions, new terrain must be examined closely for indications of old slides, and obviously unstable terrain avoided whenever possible. Careful observation of topography together with well-planned explorations and soil tests can aid in preventing many slides; however, they will not necessarily eliminate all slides. Use of a factor of safety of 1.3 on the Panama project with strengths determined from the study of existing slopes should largely eliminate trouble from slides, as a fairly liberal allowance is made for unknowns. A much larger factor would require considerably more flattening of slopes than proposed. Since the strengths used in the analysis take advantage of extensive experience with the formations in question, the risk of major slides should be slight.

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[115] Senior Engr., U. S. Corps of Engrs., Portland, Ore.

[116] "Research on Stress-Deformation and Strength Characteristics of Soils and Soft Rocks Under Transient Loading," by A. Casagrande and W. L. Shannon, *Publication No. 447*, Graduate School of Eng., Harvard Univ., Cambridge, Mass., June, 1948.

HARRY O. COLE,[117] M. ASCE.—From the nature of the papers in this Symposium, together with another extremely able paper,[118] it is clear that the investigation of the canal under Public Law No. 280 (Seventy-ninth Congress, First Session) was solely directed toward securing the authorization of the predetermined objective of a sea-level project on the premise that the present lock canal "is totally lacking in security to meet the future needs of national defense," and that the only alternative is a sea-level canal.

It appears that the investigation reported in the Symposium was not directed toward securing any major improvement in the operating deficiencies of the present lock canal. At least if there was any such objective, the writer has not found it in the Symposium papers.

The first major contribution toward the solution of operating difficulties in the canal was developed by Miles P. DuVal[119] in a paper published in 1947. The plan presented in that paper—its recording and analysis of accidents, hazards, and difficulties encountered by ships in transiting the canal, over a period of 30 years—was so revealing and informative that it has apparently affected all subsequent engineering thinking at Panama. As all informed engineers with experience in building the Panama Canal know, the problems there are not problems of complexity as much as they are of magnitude.

Viewing with alarm the breakneck speed at which the United States has been spending hundreds of billions of dollars during the 7 years since 1941

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[117] Cons. Engr., Morgantown, W. Va.; formerly, Res. Engr., Pacific Div., The Panama Canal, Canal Zone.

[118] *Congressional Record*, April 22, 1948, p. A2565.

[119] "The Marine Operating Problems, Panama Canal, and the Solution," by Miles P. DuVal, *Transactions*, ASCE, Vol. 114, 1949, p. 558.

(and still doing so) it is clear to all thinking men that unless the nation safeguards its resources, present investments, and future expenditures, it will further seriously jeopardize the national security at a critical time.

It is fair to ask the question, and insist upon a truthful and an unbiased answer: Is an expenditure of $2,500,000,000 justified at Panama or elsewhere on the Isthmus for any canal, lock or sea level, under any circumstances?

The recognized superiority of the Panama Canal as a navigable waterway for ocean vessels over any feasible canal at Nicaragua, Tehuantepec, the Atrato River, or elsewhere, and the high vulnerability of any type of canal to the atomic bomb certainly makes the very idea of expending $2,500,000,000 (and probably much more) for a sea-level project at Panama, or for a canal at any other location, entirely indefensible.

The national resources of the United States should not be squandered in a futile Isthmian project at Panama, or any other place, that would inevitably divert funds and resources from other urgently needed projects and defense programs in the United States.

From evidence presented elsewhere by the writer,[120,121] it is very clear that the great fundamental defect in the original design of the Panama Canal was the separation of the Pacific locks into two groups and the placing of one of them squarely across the south end of the Culebra Cut (later Gaillard Cut) at Pedro Miguel. The writer has given the definite reasons why that was done, and has proved beyond reasonable doubt that, had not that decision been reached and carried out at that time, the United States would have had no canal at Panama during World War I!

The Third Locks Project of 1939, had it been completed (it was abandoned in 1942), would not have been the cure, but it would have greatly aggravated and increased the difficulties at Pedro Miguel because this project was not navigationally sound!

Also, revived with the 1939 Third Locks Project was the idea of "converting" the Panama Canal to sea level, with a plan that had not been studied adequately. The Panama sea-level project reported in the Symposium apparently grew out of that plan and will make it necessary to abandon a great part of the present canal. It is not a true "conversion" but really a great project for a new Panama Canal.

Authorities who are considered well informed have expressed the opinion publicly that any type of canal is definitely vulnerable to the atomic bomb and that the only function of the canal is to transport ships safely and conveniently. It is as self-evident now, as it was when the Panama Canal was constructed, that a canal formed by large lakes affords safer and more convenient navigation than any canal that has a restricted, canyon-walled, channel 30 miles long and whose tidal waters are changing in velocity and direction throughout every day in the year.

Furthermore, the proposed huge "tidal lock control works" with a lock 200 ft by 1,500 ft, contemplate an independent 750-ft wide, "navigable-pass channel" controlled by a 6,000-ton, 6-link, chain-gate system. This project

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[120] *Congressional Record*, May 4, 1948, p. A2872.

[121] *Ibid.*, May 21, 1948, p. A3338.

definitely rules out the possibility of "converting" the lock canal to a true sea-level canal.

According to one of the highest naval authorities, the expenditure of building a so-called sea-level canal, or even huge locks, to expedite the transfer of 2 or 3 "super-width" Navy ships from one ocean to the other, is not justified, since the United States now has a two-ocean Navy, and since these "super-width" ships can readily go around Cape Horn when it becomes necessary for them to be transferred from one ocean to the other.

One other technical phase of this subject is of the most vital importance to the American people. The estimated first cost for "converting" the lock canal to sea level, of $2,483,000,000 alone (which appears to be grossly underestimated), is prohibitive, unless the cost is to be borne by the national defense budget. The annual interest of 3%, not to mention paying off the debt, amounts to about $75,000,000. This would mean an increase in toll charges of about $2 per ton over and above the present toll charge of $1. Even now, shipping interests are clamoring for a reduction in toll charges. If authorized as a defense project with a debt to be paid off in the next 100 years, the American taxpayers would have to do it.

A review of the facts and references as set forth in the foregoing should give a better perspective of the canal investigation that is under discussion.

Apparently the major premise (national defense), on which the recommendation for "converting" the canal to sea level was based, has now definitely been removed.

The sea-level plan recommended in the Symposium proposes to abandon the present high lake type of canal (which has demonstrated its convenient operation since 1914) and substitute a sea-level canal—at an expenditure of untold billions of dollars with no positive proof that in the end it will prove to be as good, or any better, or more economical, or safer for transiting ships, than an improved lake-type canal at only a fraction of the cost and the time to build!

OLE P. ERICKSON,[122] M. ASCE.—In the ninth Symposium paper there were discussed various methods of excavation, such as large shovels, dredges of the dipper and bucket-ladder types with dump scow disposal, and 40-in. hydraulic dredges.

The proposed hydraulic dredges of only 40-in. discharge seem to the writer to be too small. Consideration should be given to much larger dredges, say, with 60-in. discharge pipe lines. It is fully realized that such dredges with a theoretical capacity four times that of the 30-in. dredges now in use, and two and a fourth times the capacity of the proposed 40-in. dredges, are unusual. However, this is a project where large and special equipment with high production capacity can be used advantageously. The friction loss in the 60-in. discharge line would be about half that in the 40-in. discharge line, which would mean a great saving in horsepower. Larger solids could pass through the pump and pipe line, thus reducing delays from this source.

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[122] Cons. Engr., Bureau of Reclamation, Boulder City, Nev.; Pres., Erickson Eng. Co., Cons. Engrs., Tampa, Fla.

There is no particular reason why 60-in., or larger, discharge pipe line dredges, with one or two booster plants, cannot be designed and built to pump the excavated material directly into the dam or levee sections in Gatun Lake. Furthermore, the dams or levees could be pumped to the final height required and could of course be made of any cross sections desired. By allocation of the materials available for fill there should be no need for pipe lines more than 20 miles long. The average pipe line lengths would apparently be 15 miles or less.

With variable cutter speeds and required horsepower on the cutter, a 60-in. hydraulic dredge could excavate all the softer rocks and shales without blasting, except for bringing down high banks. Proper cutter design would screen out all rock or materials that would not pass through the dredge pump. The cutter would have a variable rotative speed up to 1,500 ft or more per min, and the cutter teeth would split the rock by impact rather than with pressure as do the dipper type and bucket-ladder type dredges with their slow bucket and tooth speeds.

In the rainy season hydraulic dredges would operate without delays from rains, which would slow down other methods of operation, because no shore line handling would be required, except a small amount for topping the dam sections. The sticky clay shales, which would be very nasty to handle by other methods, would make ideal hydraulic dredge pumping material.

Two of these 60-in. dredges, it is estimated, would handle approximately 70,000,000 cu yd per yr if pumped into Gatun Lake or into alternate spoil areas where shore line handling would not be required. A comparison of the cost per cubic yard of unclassified material delivered by 60-in. hydraulic dredges into the dam sections and special areas in Gatun Lake with the cost by the other proposed excavating equipment, and loading into dump scows, towing, and disposing in the same spoils areas, would be very instructive.

As regards the various other types of excavating equipment contemplated, the dipper dredge can excavate from 85 ft to 90 ft and has the distinct advantage of being able to handle large boulders or rock sections. Delays may be expected, however, from possible spud and dipper stick failures in the deep dredging. The bucket-ladder type dredge cannot efficiently handle large boulders or hard rock, and it is also low in production although excellent for tin and placer mining. The dredge *Corozal*, built for work in the present Panama Canal, did not show too much promise in this type of dredging. The large shovels and draglines, 25 cu yd or larger, are ideal tools for dry excavation. However, loading into dump scows from canals and towing to Gatun Lake will create some problems. Dump scows of capacities from 3,000 cu yd to 5,000 cu yd should be given consideration for this method of disposal, instead of the 2,000-cu-yd size contemplated.

Clarence S. Jarvis,[123] M. ASCE.—The papers in this Symposium, together with the extensive underlying investigations and critical studies, have established beyond reasonable challenge the vital need for the project as clearly outlined by specialists in line of their official duty. Even the most costly and apparently fantastic of the alternate routes, plans, and suggested procedures has served a good purpose in focusing attention on the more meritorious ones for final studies and revisions.

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[123] Cons. Engr., Salt Lake City, Utah.

The writer is in complete agreement with those who believe that tidal regulation is unnecessary and therefore inadvisable, even though the technical studies were in order. Recalling the situation in 1933, when a tidal lock for the Cape Cod Canal in Massachusetts not only was an approved feature of the enlargement program then under way, but also was on the verge of being let to contract or other definite construction commitment until challenged by the division engineer with technical data assembled by the writer in line of duty, it is pleasing to note the years of satisfactory service without tidal control. As forecast at that time, an enlarged sea-level canal would be effective in reducing the differentials of bay surface elevations as compared with those previously observed near the canal terminals.

If the maximum differentials of about 7 ft fail to produce objectionable velocities in the 9-mile length of the Cape Cod Canal between approach channels, then it appears doubtful that a 10-ft or 11-ft head would produce flow velocities much greater, if as great, at the Panama Canal terminals, nearly four times as far apart, even though the canal cross section will be nearly double in area.

The proposed tidal lock for the Cape Cod Canal was regarded by the writer as a nearly useless obstruction, entailing further hazard to shipping when vessels slowed down and maneuvered for anchorage.

Although recognizing the apparently overwhelming weight of authoritative opinion already in favor of the Panama sea-level conversion route either along or near the present alinement, the writer has as yet been unable to reconcile the following elements and factors that should be considered:

1. Would not the Panama parallel sea-level canal as proposed fully justify the additional estimated cost of $800,000,000, or approximately one half of the total construction and operating costs of the present installation, together with accrued interest at 3% per annum until 1960? Advantages would include assurance of noninterference with canal traffic during construction, a dependable stand-by or alternate route, conservation of the hydroelectric power and flood-control facilities and potentialities, and avoidance of wet excavation totaling nearly 300,000,000 cu yd, nearly half of which would be classified as rock. Finally, some 132,000,000 cu yd of this total excavation on the present canal would be taken from depths ranging from 85 ft to 145 ft below maximum water surface, probably involving unprecedented trouble.

2. Would not the unavoidable interference with normal canal traffic during the period of construction for the Panama sea-level conversion route, together with the estimated 7-day interruption during the final stages of conversion and demolition (or possibly a much longer interruption due to unforeseen conditions), offset a considerable part of the estimated saving in construction cost? Much uncertainty attends the deep blasting and dredging of rock since small-bucket dredging for gold from alluvial bars to depths of 128 ft offers no real precedent.

3. Construction on the Panama parallel sea-level route, or such revised route as further study may dictate, would have all the advantages that come with mass production or mass excavation, with the possibility, if not the probability, of reducing unit costs below those first estimated.

4. The sea-level conversion project, substantially utilizing the present route, but grappling with massive rock at such unprecedented depths and denied the observational and other advantages that attend excavation in the dry, might well overrun the estimated costs as well as the estimated time limits; and, when it reached completion, there would be just one usable Panama route with no alternate or stand-by.

The writer has doubtless overlooked many of the basic data that led to authoritative support of the Panama sea-level conversion project in preference to all others yet suggested. However, can these in the aggregate outweigh the considerations herein listed as needing reconcilation? The writer is reassured that continuous research and review may be maintained, not only pending final decision but also throughout the construction period, because the project is in safe hands, guided by the highest concepts and ideals.

Anson Marston,[124] Past-President and Hon. M. ASCE.—This admirable Symposium on one of the greatest and most important projects in the history of engineering is of special interest to the writer. Every rereading and restudy of the nine papers, keeping in mind his own study (1929 to 1931) of the same problems as a member of the Interoceanic Canal Board, has confirmed the conclusion on first reading—that, starting immediately, the present canal should be converted into a sea-level canal.

The present canal was made a lock canal because of the eagerness of the United States to be able to send its ships across the Isthmus at the earliest possible date. Soon after the Spanish-American War, J. P. Dolliver, then United States Senator from Iowa, speaking to an Iowa audience, stated: "As for the Panama Canal, because the *Oregon* had to go around Cape Horn, you may consider it as dug"—and it was, by the earliest date possible.

The present canal has served its purpose well, in spite of some mistakes in its original planning and construction. The mistakes nearest to being inexcusable were the location of the Pedro Miguel Lock close to the Pacific end of the Gaillard Cut and the overruling of the advice of the late Gen. W. L. Sibert, M. ASCE, later to abandon work already done on it and to construct instead three-step locks at Miraflores. It is debatable whether this mistake was excusable, but the writer thinks that making the locks too small for future ship sizes was excusable; it was impracticable to predict correctly the future sizes of the large ships of the twentieth century. In 1931, locks 125 ft by 1,200 ft in size, instead of 110 ft by 1,000 ft, were adopted on advice of the Interoceanic Canal Board.[6] In 1939, Congress ordered these locks constructed 140 ft by 1,200 ft; and, by 1948, 200-ft by 1,500-ft locks were recommended by the United States Navy.

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[124] Dean Emeritus of Eng., Iowa State College, Ames, Iowa.

The fact is that the sizes of the largest commercial ships and warships needing to pass through the canal within the next 50 years cannot even yet be estimated with certainty. A sea-level canal with a "navigable pass" of the full dimensions of its channel is the only sure solution of this problem.

The writer is convinced that the conclusion stated in the Symposium that only a sea-level canal can insure security to meet the needs of national defense against modern air attack is absolutely correct. Consequently, it is imperative that the sea-level plan be adopted immediately and that construction be begun. The world situation is extremely dangerous. Experience in World War II with the Third Locks Project shows how hopeless it was, and would be again, to wait until another war becomes certain before starting construction on any major Panama Canal security or other improvement project.

In 1931, the late Sydney B. Williamson, M. ASCE, a member of the interoceanic Canal Board,[6] made an estimate of the probable future canal traffic across the Isthmus, on three bases, namely—(1) percentage of world fleet tonnage, past and probable future; (2) percentage of Suez Canal tonnage, past and probable future; and (3) data of average gain in actual Panama traffic, 1915 to 1930. His resulting estimates are given in Table 35.

TABLE 35.—1931 ESTIMATES OF PROBABLE PANAMA CANAL TRAFFIC

Year	Ships	Net Tons	Year	Ships	Net Tons
1940	8,600	40,300,000	1960	13,900	65,400,000
1950	11,100	52,100,000	1970	16,700	80,100,000

Comparison of Table 35 with Figs. 1 and 2 shows that the 1931 estimates for both net tons and transits are larger than those of the Symposium—which, being based on later data, may be presumed to be more accurate. Fig. 1 shows the great effect of World War I. If there is a World War III, what will be its effect? Mr. Williamson and the late Harry Burgess, M. ASCE, then Governor of the Canal Zone, estimated that the maximum capacity of the twin-locks Panama Canal would be reached from about 1960 to 1970; the symposium states 1960.

In 1931, a Nicaragua lock canal was entirely feasible, at a cost estimated at $722,000,000 (which, of course, would be much greater for 1948). Although, its length would have been more than three times that of the Panama Canal, it was estimated that about two thirds of the traffic then crossing the American Isthmus, especially intercoastal United States shipping, would save distance by going through a Nicaraguan rival to the Panama Canal. To avert possible disastrous rivalry, either financial or international, in 1916 the United States concluded a convention with Nicaragua whereby it acquired:

> "* * * in perpetuity * * * forever free from all taxation or other public charge, the exclusive proprietary rights necessary and convenient for the construction, operation, and maintenance of an interoceanic canal by way of the San Juan River and the great lake of Nicaragua, or by way of any other route over Nicaraguan territory * * *"

In return the United States paid Nicaragua $3,000,000 in gold.

Costa Rica, Salvador, and Honduras objected. The United States and Nicaragua held firm, but before 1931 the United States State Department gave "every assurance" that prior to the building of any canal it would enter into negotiations with Costa Rica, Salvador, and Honduras.[6]

In 1931, Governor Burgess, of the Panama Canal Zone, made the first report on a definite, detailed project for doing what had long been talked and written about—conversion of the Panama lock canal into a sea-level canal. For this project he gave full credit to Mr. Williamson, of the Interoceanic Canal Board, who proposed to accomplish the conversion by lowering the canal in three stages: From El. 85 to El. 53; from El. 53 to El. 27; and from El. 27 to El. 0. The final result was to be a canal 42 ft deep and 500 ft wide at the bottom, with triple-tidal locks at Miraflores, each 140 ft by 1,500 ft. The total cost was estimated to be $1,000,000,000. It was admitted that lowering the locks by stages would involve serious hazards, admissible only after constructing third locks.

Although approving this plan then, the writer has always doubted the necessity for the tidal locks, and now is glad to learn that actual computations, using the theory of Gen. George B. Pillsbury, M. ASCE, and measurements from an undistorted generous scale model, combine to prove that the canal will be navigable through the proposed navigable pass even if the tidal locks should be destroyed.

The writer is not a dredging expert, but has studied with some care the plans for special hydraulic dredges working to a depth of 145 ft, kindly sent him in advance, with Colonel Stratton's consent, by Panama Contractors. Single-stage lowering to sea level will do away with at least most of the lock lowering hazards unavoidable with multiple-stage lowering.

The writer is much impressed by the paper by Messrs. Binger and Thompson. Great slides have been a continuing dangerous and expensive characteristic of the construction and maintenance of the present canal. To an engineer who has been interested, and to some extent engaged, in engineering research for 57 years, it is pleasing to note that the Panama Canal has served as a great research laboratory to solve the important problem of safe slopes for a sea-level Panama Canal.

It is pleasing, also, to read in the other Symposium papers the reports of the researches made on the design of the channel to afford safety in ship handling. Enlisting the participation of Panama Canal pilot captains in these researches was especially wise.

In conclusion, the writer earnestly believes that Congress should at once authorize a sea-level Panama Canal and provide funds for starting the work. The present world situation is such that it is dangerous to procrastinate. The expenditure of from $250,000,000 to $300,000,000 annually for 10 years is small in comparison with the billions of dollars being spent to prevent another war, and the hundreds of billions of dollars another war would cost. A sea-level canal, safe to transit the largest war and commercial ships at any time, is vital to the national security. It is unthinkable that the United States should ever again be forced to send them around Cape Horn, like the *Oregon*, in 1898.

RALPH Z. KIRKPATRICK,[125] ESQ.—An effective answer to the basic thesis of this Symposium is a paper published in 1947.[119] To the writer, that paper is an infrared light through the "confusion of ideas" begot by the various plans for the best method of modernizing and enlarging this tropical waterway. The term "confusion of ideas" was used by the late John F. Stevens, Hon. M. and Past-President, ASCE, the chief engineer of the Panama Canal during the earlier part of the construction period. Although he used it as early as 1906, oddly his subject was almost identical to the current one.

Perhaps the sufficiency of the water supply in the driest dry season may be questioned, in what is termed either the "Balanced Lake Plan" or the "Terminal Lake Plan," because it is a fact that the balance which the Madden Lake Dam made possible in flood handling, power making, and water supply for navigation has been slightly upset by the subsequent additional power units operating at the Madden and Gatun hydroelectric plants. Also, the proposed wider locks will use more water.

Fortunately this problem is easily solved: During the reconstruction period (without interference to navigation) simply dredge the bottom of Gaillard Cut from El. +40 to El. +37$\frac{1}{3}$, the ruling bottom elevation of Gatun Locks (that is, the tops of the emergency dam sills); and, since the Terminal Lake cancels the Gaillard Cut lockage surges, there is no further need for the allotment of an extra 1$\frac{1}{2}$ ft of dry-season prism-storage water. Thus, a realistically useful storage water prism of 2.67 + 1.5, or 4.2 ft atop Gatun Lake becomes available for dry season use. In terms of water, this is about 18,600,000 cu ft, or an increase of nearly 75%.

Incidentally, during the fiscal years 1940–1945, the annual power generation in the Canal Zone increased from about 100,000,000 kw-hr to 266,800,000 kw-hr; and, subsequently, the decrease has been only slight. The four turbines at the Gatun plant are now (1948) augmented by two new and larger ones. Under the sea-level plan, that plant must close, and a virtually new electrical system for the Canal Zone must be designed and built, with but minor salvage. All that useful power-making water from Gatun Lake will pass to the sea through the proposed East and West Diversion systems! Compared to the present system or proposed Terminal Lake system the contrast in efficiency and dollar savings will be marked!

The capacity of Gatun Lake to absorb safely, and to make profitable use of the 60 in. to 150 in. of annual rainfall (8 months) in the Chagres River Valley has been tested for one third of a century without a failure. It is one of the triumphs of the pioneers of the Panama Canal that their potential worst enemy has actually been made a best asset. Why gamble? Sooner or later "bugs" are nearly certain to bedevil the operations of a sea-level canal through faults in the proposed extensive and complicated West and East Diversion systems. What of those miles of levees, dams, and spillways proposed on foundation conditions still virtually unknown?

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[125] Rochester, N. Y.; formerly Chf. of Surveys–Chf. Hydrographer, Panama Canal, Canal Zone.

The slides in the 8 miles of Gaillard Cut have a soul-searing history; but those troubles were caused by geological, and maybe other, conditions above the present canal bottom elevation of +40. On the face of it, going down 100 ft further to El. −60 (see Figs. 98 and 99) is daringly unprecedented even to the point of recklessness. Furthermore, what is known of the probable bank and bottom stabilities of that other 20 miles of proposed canal now safely hidden under Gatun Lake? Nothing is known, except geological conjectures!

What foundation tests have ever determined that sufficient bearing values exist for those proposed dams across the Peña Blanca and Caño Quebrado swamps? Those deep quagmires of Atlantic mucks have been forgotten under Gatun Lake for 37 years! Nevertheless, there those important dams are to be found, on the maps of the proposed sea-level canal. For a long time, those swamps were the dreads and troubles of the pioneer French and American engineers—now sanguine hopes make them dam foundations!

Apparently through well-meaning but oversanguine proposals, the future safe and smooth transit of world commerce is being endangered. It must not be!

ERNEST SHANKLAND,[126] ESQ.—In the fifth Symposium paper, Messrs. Lee and Bowers cite the paper by G. S. Baker[33] as one of several sources of information on ship resistance in restricted channels (see heading, "Studies of One-Way Traffic"). Using Mr. Baker's studies as a background, the writer investigated the maneuvering factors in the Thames tideway in England, which is a very restricted waterway considering the trade that it carries. Mr. Baker's laws of safe steering were found to be rather conservative. Observing the starboard hand rule, it is sometimes possible within Mr. Baker's limits to have several vessels navigating the bends at high tide, with a stream fluctuating in force and direction.[127] By experimentation when designing new dredged cuts it was discovered that curves should be held within a 10° limit, to control course alteration, and to retain a channel having parallel sides during the dredging; and, as advocated by Messrs. Lee and Bowers, half-moon concave additions should be avoided at the bends, as they tend to pull the convex side over to them.

JOHN S. MCNOWN,[128] ASSOC. M. ASCE.—Perhaps the most striking of the several significant features of ship behavior in canals described by Messrs. Lee and Bowers is the change in level with speed indicated in Fig. 50. Although the measured changes in level of several feet are surprising on first consideration, they are readily confirmed—quantitatively as well as qualitatively—by the straightforward application of elementary concepts. In order for the familiar form of the Bernoulli equation to apply, the flow pattern must be changed from unsteady to steady by the simple superposition of a uniform flow equal and opposite to the motion of the ship. In this way the ship is brought to rest and the water in the canal is given a steady motion as shown in the two parts of Fig. 115. The resulting pattern of flow then differs only in minor detail from that for flow in an open channel with bottom or side contractions. The reduction in cross-sectional area leads to an increase in velocity which, in tranquil open-channel flow, must be accompanied by a drop in surface level similar to that occurring over a broad-crested weir or in a Parshall flume.

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[126] Assessor and Surveyor, Folkestone, England.

[127] "Modern Harbours: Conservancy and Operations," by E. C. Shankland, James Brown & Son, Glasgow, Scotland, 1926.

[128] Associate Prof., Dept. of Mechanics and Hydraulics, and Research Engr., Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

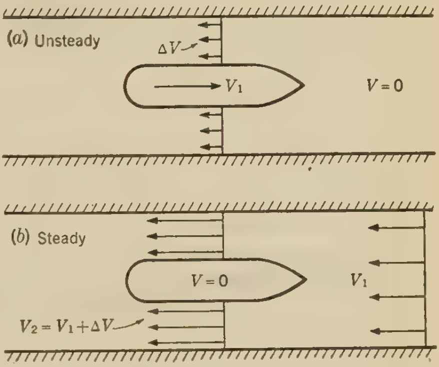

FIG. 115.—APPLICATION OF PRINCIPLE OF RELATIVE MOTION TO SHIP AND CANAL

Because the changes in level seemed large to the writer, representative calculations based on the aforementioned concepts were made. Such remarkably close agreement was found that the computations were completed for nine cases shown in Fig. 50, the assumption being made that the ship is long enough for the flow past it to be essentially uniform throughout the channel cross section along the entire length of the ship. Although the change in water surface is surely less than average near the bow and stern, the effect on the buoyancy is also less in these regions, and no correction is necessary to estimate the effect in question. Although Fig. 49 serves as an approximate check on this assumption, the reversed flow past the ship for the conditions of Fig. 49 is so nearly critical in the reduced section that complete uniformity could scarcely be expected. In one respect the application of the usual one dimensional form of the Bernoulli equation is more accurate for this case than in the usual applications to open-channel flow—the velocity in the approaching flow is actually, instead of approximately, constant over the entire cross section.

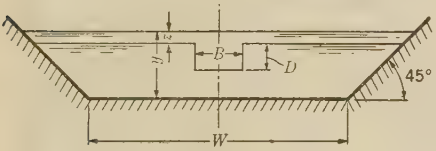

FIG. 116.—DEFINITION SKETCH FOR FLOW CROSS SECTIONS

For steady flow, with the water moving and the ship stationary, the velocity of the undisturbed water V_1 is equal to the ship speed, whereas the velocity in the reduced section V_2 is somewhat greater as shown in Fig. 115. The change in level, which may be designated as z, is simply related to the two velocities,

$$z = \frac{(V_2)^2}{2\,g} - \frac{(V_1)^2}{2\,g} \qquad \text{(7)}$$

Also, from the continuity equation written for the two sections shown in Fig.

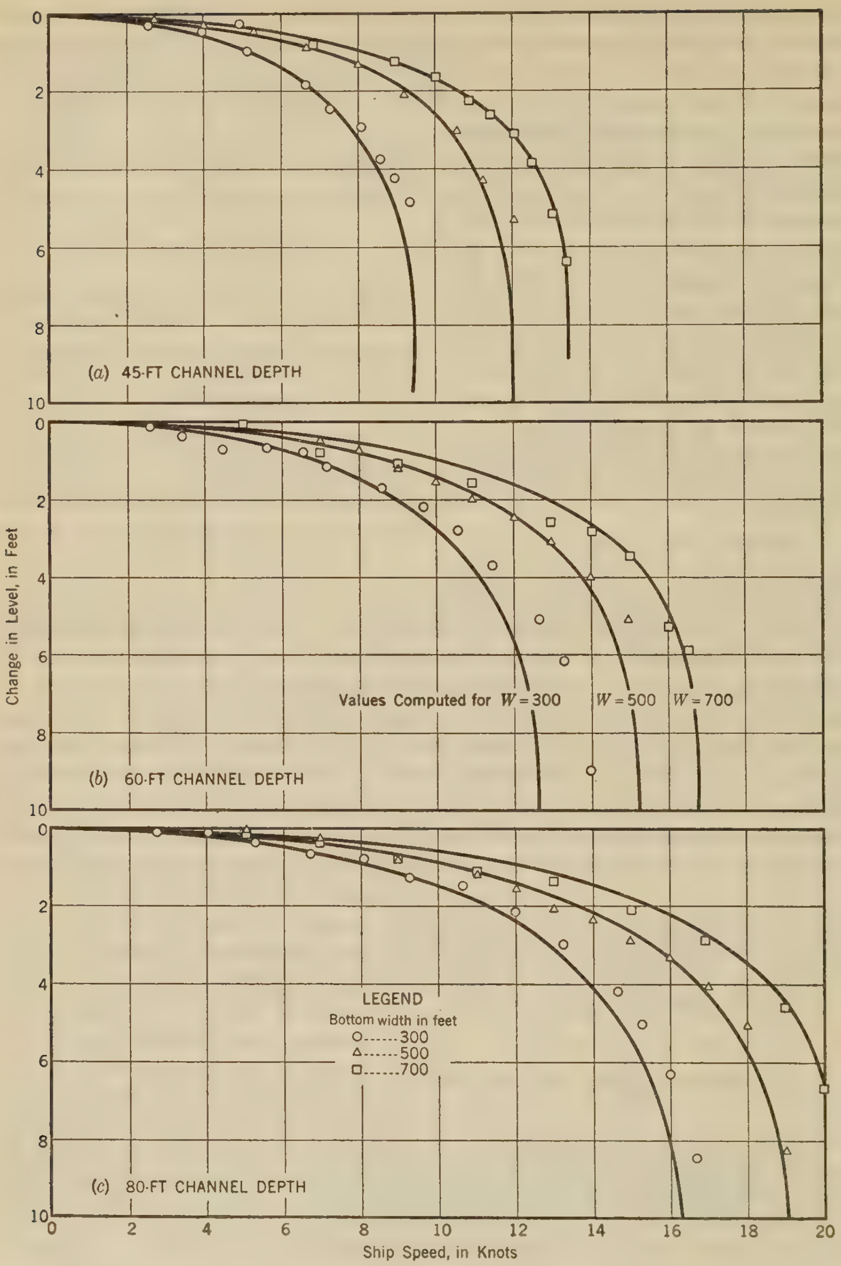

FIG. 117.—COMPARISON OF COMPUTED AND MEASURED CHANGES IN LEVEL AS A FUNCTION OF SHIP SPEED

(Ship and Model Data Are the Same As Those in Fig. 50)

116, a second relationship can be obtained:

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{(W + y - z)(y - z) - B\,D}{(W + y)(y)} \qquad (8)$$

in which B is the width, or beam, of vessel; and D is the depth of vessel displacement. From Eqs. 7 and 8 the variation of V_1 with y can be computed directly for assigned values of the beam (B) and draft (D) of the ship and of the depth and the width of the channel. The area of the submerged cross section was arbitrarily assumed to be 110 × 32 = 3,520 sq ft rather than 113 × 32.25 = 3,650 sq ft, since the shape is not a true rectangle. The computed curves are shown in Fig. 117, together with the experimental points scaled from Fig. 50. The agreement between the results from the analysis and those from the model study is remarkably good.

Both the theoretical curves and the measured values indicate the existence of a maximum velocity, which, as the authors have shown, is that which leads to critical flow in the reduced section. Once this maximum is reached, the change in depth becomes unstable since the slope of the ($V - y$)-curve is infinite. If an even higher speed were attempted, water would gradually pile up ahead of the ship. The critical velocity, therefore, should be used as a design maximum, which should not be closely approached and surely never exceeded.

The simplified analysis utilized herein is restricted to cases in which the channel dimensions and the ship dimensions are comparable in magnitude. Motion of a ship in a large river, or in the ocean, could not be analyzed in this manner since the increases in velocity would be large near the ship, decreasing to zero some distance away laterally, in contrast to the constant velocity assumed in the derivation of Eqs. 7 and 8. For a very large canal cross section the change of surface elevation near the ship is thus a function of the local flow pattern rather than of the canal dimensions. The analysis for this limiting case would be extremely complex even if the induced surface wave pattern were neglected; observations of the magnitude of the comparatively small effect are doubtless available from model tests.

The change in surface level near a ship, and hence the change in ship level as well, are readily explainable and also predictable in magnitude on the basis of a simplified analysis. Although limitations on the applicability of the proposed analyses are evident, within the range of practical canal design, surprisingly good results can be obtained. Logical conclusions that can be drawn from the comparison in Fig. 117 are that further model studies in the range of variables tested are unnecessary, and that many of the runs made could have been omitted once agreement had been established for a few representative conditions. Since the theoretical curves in Fig. 117 are easily prepared, prediction for given ship and channel dimensions of the variation of surface level with speed, together with the critical or maximum ship speed, is reduced to a simple procedure.

H. H. LITTLE,[129] ESQ.—Much was written, in the paper by Colonel Stratton, about the "capacity" of the Panama Canal. What does that term mean? Great stress is laid on "time of transit." To avoid the creation of exaggerated impressions as to their importance, the relation of the terms, "capacity" and "time of transit," to the broad field of strategic planning should be explained by the author, in his closing discussion.

In regard to "capacity," very little in the way of factual analysis has been published on the actual "capacity" of the Panama Canal as it now exists. Messrs. Johnson and Steinborn report (see "Capacity of the Present Panama Canal: Dependable Daily Capacity of Present Canal"): "* * * the dependable daily capacity of the existing canal is taken at its minimum daily capacity during overhaul—or 36 vessels." This would be a minimum of 13,140 ships a year. Colonel Stratton reports (under "Traffic History of the Canal") that there were nearly 17,000 transits of war traffic through the canal from 1941 to 1945. Assuming that this period includes 4 years, the average yearly transit was 4,250 ships, which would indicate that, during World War II, the present canal was operated for war traffic to only about 30% of its minimum capacity.

Under "The Panama Canal As It Is Today," Colonel Stratton states:

> "* * * It is estimated that by about 1960 the capacity of the canal will be inadequate to accommodate traffic without inflicting undesirable delays on peak traffic days."

In other words, the canal is not to be used to "capacity" at other times, but by 1960 it will be used to "capacity" only on "peak traffic days." What does industry do when confronted with similar problems of "peak traffic"? They control them! It spreads the load by proper planning; and, if need be, it increases charges for "peak" business, peak electric rates, peak labor costs, peak charges of any description. Has any attempt been made to control peak traffic at the canal? None is mentioned in the Symposium. There should be many ways of controlling sailings to, or arrivals at, Panama and also of removing bottlenecks in the canal so that traffic will not jam on peak days. Certainly no canal could be built which would prevent occasional peak loads when no attempt is made to control traffic or improve its functioning. The obvious answers to the question are the control of traffic and the proper modification of the present canal, not building canals at other locations across the Isthmus or attempting the construction of a new sea-level Panama canal.

As for "time of transit," it should be realized that plans for military operations are prepared far in advance of the dates of embarkation. Personnel, supplies, and ships are calculated, checked, and rechecked dozens of times. Material is ordered, manufactured, shipped, and stored weeks and even months in advance; D-day is never set until the required resources are on hand and allowance is made for shortages—material which would not arrive because of sinkings, bombings, or failure to load. No general or admiral ever had to "tear his hair out" because some ship or convoy of ships was late in passing through the Panama Canal. Although the expeditious handling of vessels at Panama (or anywhere else) is expected as normal performance, it is doubtful if any saving in time at the canal ever had any bearing on the outcome of an important military undertaking.

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[129] Capt., U. S. Navy (*Retired*), St. Margarets, Annapolis, Md.

Colonel Stratton's repeated use of the term "security" deserves to be properly analyzed or circumscribed in his closing discussion. The writer was startled by the statement in the "Synopsis" that:

> "A sea-level canal at Panama constructed by the conversion of the existing lock canal could not be destroyed by enemy attack or sabotage. Only the atomic bomb could cause significant interruption in service, and then for not more than a few weeks."

A statement such as this can be misleading without essential amplification. As a major premise it represents the greatest weakness in the attempt to justify the sea-level proposal. The statement is true, of course, in the sense that earth and water cannot be destroyed, but it is not true in the sense that the damage caused to a sea-level canal, even by conventional weapons, would be insignificant. A bomb or a thousand bombs dropped in the Mojave Desert in California would have no effect in destroying the desert, but determined bombing attacks or sabotage on the many miles of proposed impounding dams, spillways, tidal locks, and other structures of the Panama Canal (which are shown in Fig. 11 to be necessary for the control of the floodwaters of the Chagres River Valley and for the regulation of tidal currents in a sea-level canal) would create almost incalculable problems and would make the canal unnavigable for long periods. These dangers together with the greater structural damage and channel closures that may be caused by atomic bombs should be just as effective militarily in interrupting ship transit as would be the draining of Gatun Lake. From the evidence presented in the Symposium it seems questionable if the vulnerability of the many flood-control installations and of the other massive structures in the sea-level plan was explored as it should have been. Until such an exploration is made, the question of which type of canal has the "maximum security" against atomic bombing appears to be one for theoretical discussions and not for practical planning, because no amount of static defense measures can give a real "security" to any type of canal.

Conversion of the existing lock canal (see "Synopsis") is another term that needs a more complete analysis. With the exception of the channel sections near the ends of the present canal the plans outlined by Colonel Stratton envisage an entirely new canal with the greater part located at varying distances from the present canal. This feature, because it does not conform to canal treaties, introduces a great international political question that would require complicated negotiations for a new treaty with the Republic of Panama and the payment of a huge indemnity and an increased annual subsidy by the United States government. In view of the rejection of the Defense Base Treaty by Panama in 1948, the negotiation of a new canal treaty no doubt would present even more serious difficulties. This important subject might well be included in the Symposium. Certainly any cost estimate should make some provision for the costs that would be encountered in this connection, in order to justify, fully, the ultimate cost of the sea-level project.

Much might be offered in comment on that part of Public Law No. 280 which Colonel Stratton quotes in his "Introduction." For example, it is fair to question whether passage of this act really "expressed the temper of present concern" of the Congress rather than the ideas of those proponents who were determined on a sea-level revision. Regarding "increasing the capacity" it might be asked whether the sea-level project if built would work better than the present canal—an important phase not covered in the Symposium.

Again, regarding the law's reference to "Canals at Other Locations," it should be remembered that the Nicaragua and other canal routes have always had large followings which had to be given some hope or the bill would not have passed Congress. In other words, it was a foregone conclusion that no scheme other than a sea-level canal at Panama would receive any real consideration under the act.

Colonel Stratton states (under the heading, "The Panama Canal As It Is Today") that:

> "The restricting effect of the small locks (width 110 ft, length 1,000 ft) of the present canal on the design of Navy ships became intolerable with the approach of war * * *."

As far as has been ascertained the Navy has made no strong demands; nor has it indicated that the present conditions were "intolerable." The *U.S.S. Missouri*, the latest type and only first-line battleship in active use, successfully transits the present canal. All the others are in "moth balls," peacefully awaiting "overage" status. These include the older battleships, which were modified by the installation of buoyancy blisters on their sides, thus increasing their beams beyond the capacity of the 110-ft locks.

Naval strategy, however, does look ahead. Should it appear that the wider ships are needed on the opposite coast it would be no great hardship for these vessels to round Cape Horn. They carry sufficient oil for the full trip, coast to coast. Note well that this refers to the largest combatant ships, for there never has been any question about the adequacy of the present canal for the transit of commercial ships.

If the world situation ever requires a precautionary redistribution of United States naval forces, the largest vessels could be sent around South America with the utmost secrecy. There can be no secrecy once vessels are seen in the Canal Zone. As some of the larger aircraft carriers and three older decommissioned battleships are the only war vessels that cannot now transit the canal, the situation would appear far from "intolerable" from the naval standpoint.

It is true that the Navy asked for locks 140 ft wide for the Third Locks Project. It also recommended locks 200 ft wide as a beginning for the recent studies. These naval statements must have been in the form of guarded replies to specific inquiries from canal authorities rather than as strongly worded recommendations for the 200-ft locks. As a leading question, how many times a year would the larger carriers or oversize battleships use the canal if it could accommodate them? The answer is probably not once in 2 years.

Next, how about the navigational safety of a sea-level canal? The related statements in the Stratton paper, based largely on ship model tests, are assumptions and not established facts. In this connection, also, further elaboration is requested to support the incomplete facts presented. In the unregulated sea-level canal, with currents as much as 4.5 knots, an element of great danger that cannot be minimized is immediately introduced. In the regulated sea-level canal the much increased length of restricted channel and the ordinary problems of navigation—night fog, heavy rains, and machinery failures—do not seem to have received the consideration that is due them. To the writer it seems obvious that the lake-type canal with its wide current-free channels is far better adapted to safe navigation than any necessarily restricted sea-level canal at Panama can ever be.

In the "Synopsis" and under "Improvements in the Interests of Commerce Only," Colonel Stratton states that with new navigational aids and other minor changes the present canal could be improved sufficiently to accommodate the transit of 70 ships a day at a cost of $129,983,000. The sea-level canal as projected, estimated as costing $2,483,000,000, he states, would transit 69 ships. This comparison indicates that the Symposium authors are recommending something that will cost twenty times as much as improvements in the present canal and that the completed works will not be as efficient.

In their closing discussions the Symposium authors need to balance the presentation with an adequate treatment of the present operational deficiencies long recognized by all maritime agencies that use the canal. These are such that the proposed minimum improvement program ($129,983,000) would not be a realistic solution; it would be only a repair job. For a fair comparison with the sea-level project, the alternate improvement program should include a cost estimate for the Third Locks Project modified to include a Pacific terminal lake based on more reasonable lock sizes. The terminal lake plan estimate, furthermore, should not be balanced, unfairly, with the costs of protective features against modern weapons when the leading authorities admit there is no effective defense against such weapons. In brief, the relative value of each case should be fairly stated with parallel tables of unit costs so that the best canal can be built at Panama.[130]

The naval aspects of the present canal question have been well summarized by Rear-Admiral D. E. Barbey, Commander of the Caribbean Sea Frontier, when he stated that the United States has a two-ocean navy large enough to permit reinforcements to be sent around Cape Horn, that their is little likelihood of meeting a first-rate naval power in the foreseeable future, and that the Panama Canal has become a primary waterway for merchant ships in time of peace and a secondary route for critical raw materials in time of war. He minimized the probability of any enemy making a major effort to destroy it. "Why," he asked, "should they waste an atomic bomb on the Godforsaken Canal Zone, when they have New York Harbor as a target?"[131]

The military aspects for Admiral Barbey's thesis have been supported by Lt.-Gen. Leslie R. Groves, former head of the Manhattan Atomic Bomb Project, who has also predicted that in a future war the first attack on the

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[130] "Let Us Build the Best Canal at Panama," by H. H. Little, *Journal*, Am. Soc. of Naval Engrs., November, 1946, p. 560.

[131] "Can We Defend the Panama Canal?" by Sidney Shalett, *Saturday Evening Post*, October 9, 1948, p. 162.

United States would be "an atomic attack on New York."[132] General Groves further emphasized that until effective international control of the A-bomb is established the only protection is to keep the A-bomb away by means of a "well-balanced, effective, military strength." From this it follows that the only "security" for any type of canal project across the Isthmus depends on the combined industrial and military might of this nation and not on the inherent merits or physical features of the canal.

When the present confusions about "vulnerability" and "security" engendered by the atomic bomb are cleared away, and when these terms are understood in better perspective, the way should be open for a practical solution of this great problem as determined by the functional needs of the Panama Canal as a navigable waterway.

A. Casagrande,[133] M. ASCE.—The prompt publication of the results of these comprehensive investigations is an invaluable service to the profession. Much of this work will also prove of great value to other engineering projects. In this discussion the writer wishes to elaborate on the question of safe excavation slopes in the Cucaracha formation, which is one of the topics in the seventh Symposium paper, by Messrs. Binger and Thompson. The writer became acquainted with this problem officially in 1940; and, during the subsequent studies under Public Law No. 280 (Seventy-ninth Congress), he concurred in the conclusions as to the slopes in the Cucaracha formation, believing at the time (and believing still) that they are the best that could be reached on the available evidence and knowledge of the behavior of this material. It has been questioned during these investigations whether long-time effects may reduce the shear strength of the Cucaracha as a result of very slow swelling caused by reduction in load. This possibility received recognition in certain proposals by the Special Engineering Division which were briefly as follows:

(*a*) Install telltale devices to observe movements of the Cucaracha and from these evaluate the action taking place and adopt corrective measures where called for.

(*b*) Undertake long-range field and laboratory studies of the long-time effects in the Cucaracha formation when project authorization is given by Congress. Until the sea-level canal would be opened to traffic, there would be a period of 10 years or more for such studies.

In the analysis of this problem it seemed agreed by all concerned, including the consultants, that with the proposed slopes forewarning of such slides as might occur would be given by observable evidences which would permit remedial action (unloading), and that it was unlikely that such slides would involve great quantities of material moving with rapidity into the canal. Even if it is assumed that rapid movement would develop, the wide cross section of the canal prism would admit a large volume of material without causing closure of the canal to traffic.

Since the termination of the active studies by the Special Engineering Division in mid-1947, the writer has become increasingly concerned over the lack of knowledge of the long-time effects as they bear ultimately on the stability of the Cucaracha slopes.

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[132] *Cosmopolitan*, January, 1949, p. 41.

[133] Prof. of Soil Mechanics and Foundation Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass.

Long-Time Effects Influencing Stability of Slopes in Cucaracha.—The Cucaracha clay-shale is so difficult to test in the laboratory that the results of strength tests on this material have so far been of limited value. One was obliged, therefore, to rely on empirical rules derived from the behavior of the slopes and on the results of analytical studies of such slopes. Of special importance in these studies was the analysis of a "model slope" which has not failed, and of another slope which failed in 1907.

In Fig. 118[134] the cross sections through the Culebra slide areas are shown for three different dates, January, 1912, July, 1915, and March, 1947. When comparing these profiles (together with numerous photographs and a detailed description of the slides contained in the report by the National Academy of Sciences[135] and in unpublished reports of the Panama Canal), one is struck by the extraordinary reduction in effective shear strength (that is, the average shearing resistance along the probable failure surface) which has gradually developed over many years. For example, in January, 1912, the East Culebra slope, at the station shown in Fig. 118, had an effective shear strength of 42.8 lb per sq in. and the west slope, one of 29.9 lb per sq in. Still higher values can be computed for earlier construction stages, since substantial sliding had already been taking place in 1911. By July, 1915, the effective shear strength had dropped to 33.9 lb per sq in. and 18.6 lb per sq in. for the east and west sides, respectively. During the period since 1916 there has been no major slide. In 1940 the writer witnessed very slow movement toward the canal along the banks of the west side which would indicate that the computed effective strength of 5.77 lb per sq in. (Fig. 118, March, 1947, west side) would correspond to a factor of safety barely above 1.0.

Even if it is assumed that the early slides had already affected the clay-shale as far as 1,000 ft inside the face of the slopes, it would be remarkable that the effective shear strength of such disturbed material would continue to drop in the course of 35 years quite gradually to about one fifth or one sixth of the value that governed the stability of the slopes in January, 1912. However, the writer believes that in 1912 at least one half of the mass bounded by the underlying Culebra and on the east and the west by faults, as shown in Fig. 118, was not yet disturbed by the slides and that intermittently additional parts of the mass became involved in the movement. If the assumption is made that the strength characteristics of the undisturbed material have not considerably changed with time and that they are equal only to those governing the slopes in January, 1912, then it can be shown that the mass should have come to rest with much steeper slopes than those existing today.

Based on the foregoing considerations the writer is forced to conclude that the strength not only of the disturbed material, but also of the undisturbed material, has been gradually and very substantially decreasing in the course of many years as a result of the reduction in normal stresses and the increase in shear stresses.

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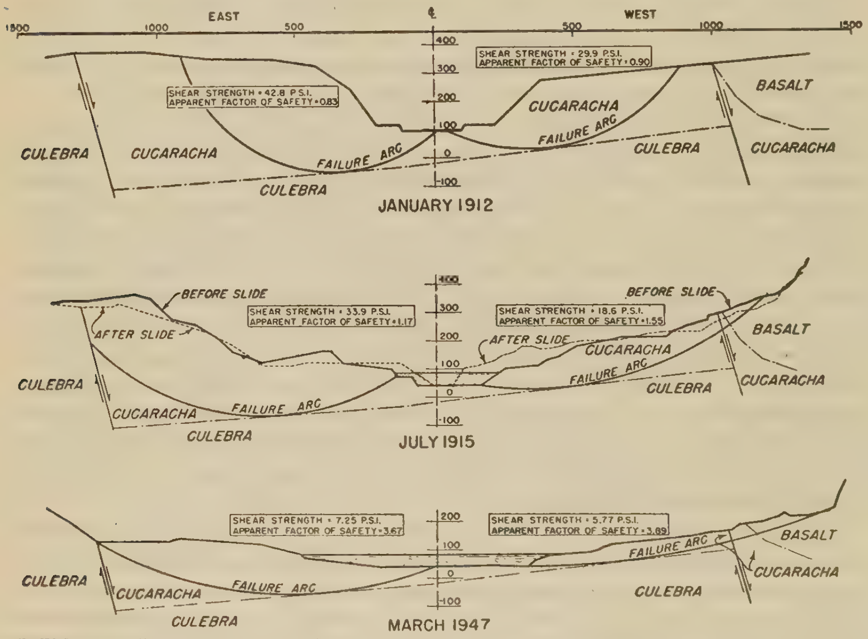
[134] "Report of the Governor of the Panama Canal Under Public Law No. 280, 79th Cong., 1st Session," Annex 4, Fig. 6.

[135] "Report of the Committee of the National Academy of Sciences on Panama Canal Slides," *Memoirs of the National Academy of Sciences*, Vol. XVIII, Washington, D. C., 1924.

Many examples could be cited to show that in clay-shales and stiff clays any substantial reduction in load due to excavation is followed by a long period of swelling. Whereas on this project the engineer's concern is the reduction in strength, the volume increase due to swelling is of importance in connection with several large earth dam projects along the Missouri River where extensive deposits of clay-shales form the foundations for the structures. For example, the Bearpaw shale at the Fort Peck spillway has been causing a continuous heaving of the spillway chute pavement with a maximum of about 2 ft since 1936, and the heaving is still progressing.


EAST
℄
WEST
1500
1000
500
500
1000
1500
400
300
200
100
0
-100
SHEAR STRENGTH = 42.8 P.S.I.
APPARENT FACTOR OF SAFETY = 0.83
SHEAR STRENGTH = 29.9 P.S.I.
APPARENT FACTOR OF SAFETY = 0.90
BASALT
CUCARACHA
FAILURE ARC
CULEBRA
CUCARACHA
CULEBRA
CUCARACHA
FAILURE ARC
CULEBRA
JANUARY 1912
BEFORE SLIDE
SHEAR STRENGTH = 33.9 P.S.I.
APPARENT FACTOR OF SAFETY = 1.17
SHEAR STRENGTH = 18.6 P.S.I.
APPARENT FACTOR OF SAFETY = 1.55
BEFORE SLIDE
AFTER SLIDE
AFTER SLIDE
BASALT
CUCARACHA
FAILURE ARC
CULEBRA
CUCARACHA
CULEBRA
CUCARACHA
FAILURE ARC
CULEBRA
JULY 1915
SHEAR STRENGTH = 7.25 P.S.I.
APPARENT FACTOR OF SAFETY = 3.67
SHEAR STRENGTH = 5.77 P.S.I.
APPARENT FACTOR OF SAFETY = 3.69
FAILURE ARC
BASALT
CUCARACHA
CULEBRA
CUCARACHA
FAILURE ARC
CULEBRA
CULEBRA
MARCH 1947

Notes:

1. Shear strength shown is that required for stability of slope under conditions illustrated.
2. Apparent factor of safety is determined for slope using strength of Cucaracha as selected for new slope designs. Actual factor of safety was less than 1 until movement stopped.
3. P. S. I. denotes "pounds per square inch."

Fig. 118.—Slide Stability Analysis, East and West Culebra, at Old Canal Station 1785 + 00

Whether the slow reduction in strength of the Cucaracha is caused by an ordinary swelling process which follows the theory of consolidation, or whether this reduction is the result of molecular adjustments to the reduced normal and increased shear stresses which gradually weaken the cohesive bonds without significant swelling of the entire mass, is a question that will require much research along the lines already contemplated before it can be answered. Such research should also answer the question whether measurable plastic deformations precede, for a considerable time, any major movement—a likely reaction for such a highly colloidal material. It may well be that by such observations the possibility of additional movements could be determined so far in advance that necessary flattening of a slope could be undertaken in time to prevent a slide. It is conceivable that slopes could be designed to be safe for a period of, say, about 20 years, with the intention of flattening when the need is indicated by analysis of the observed deformations. Such a proposal is unorthodox, but it is what (for all practical purposes) has been done since about 1914 on the slopes shown in Fig. 118, apparently without any special difficulty. If such an approach were followed in the design and construction of a new or improved canal, it would result in an earlier completion of the canal and would also have the advantage that additional flattening of slopes would be carried out only if, when, and where needed. The latter advantage is important because the stability characteristics of the Cucaracha seem to vary widely.

Comments on Model Slope and Old Slide Used as Criteria for Undisturbed Shear Strength.—The presence of a layer of relatively strong and pervious ash flow in the model slope indicates the possibility that the sequence of various layers is not identical with that in the Culebra slopes. Furthermore, even though the borings indicate layers of the soapy phase of the Cucaracha, it is not the mere presence of this material but its relative position to the stronger phases of the Cucaracha which determines the over-all resistance to sliding. It is not difficult to conceive a distribution of strong and weak layers, or of faults and other irregularities, such as to interrupt a continuous shear surface through the soapy phase so effectively that the total resistance of this slope may be much greater than that of the Culebra slopes. If this slope had a real factor of safety very much greater than unity (which is a possibility), there would apparently be no rational basis for using this slope for derivation of those strength characteristics which govern much weaker slopes such as those shown in Fig. 118. However, for the purpose of the recent studies this procedure appeared to be the best that could be followed at that time.

In regard to the 100-ft cut that failed in 1907 with a slope believed to have been 3 vertical on 2 horizontal, the writer understands that the stability analysis of the slide checked the design criteria for a factor of safety of unity (see Fig. 110). If one takes into consideration the probable time effect, one is forced to the conclusion that, to begin with, this slope must have been weaker than the slopes illustrated in Fig. 118, and very much weaker than the model slope which is still standing. In the writer's opinion the tacit assumption that for the purpose of analysis these slopes have similar strength characteristics is not tenable.

The Cucaracha clay-shale is so thoroughly slickensided that the usual differentiation between undisturbed and disturbed material can scarcely apply. Considering the extreme weakness of the finest-grained phases of the Cucaracha, and having in mind the rather rough topography of the Canal Zone, the writer is inclined to believe that some of the Cucaracha which has been affected by slides as a result of the construction of the canal had been exposed to excessive shear deformations at some previous period and, therefore, must be considered as disturbed material. Whatever the causes of the slickensides, they represent disturbance of the material.

Many of the observed slides have moved slowly, showing only a small reduction in the effective shear strength before and after each movement. This condition is particularly significant when compared to the large difference between the maximum and residual (after failure) strength values obtained in all laboratory tests on Cucaracha because it makes it appear probable that the resistance of the banks to sliding had already the character of residual strength when the cuts were made. (This large difference was found both for the tests in the Canal Zone and for those made in the laboratory at Harvard University in Cambridge, Mass. The latter showed that the residual compressive strength in all tests on Cucaracha was about one half of the compressive strength, regardless of the time of loading.)[136] Adding to these observations the important time effects, the writer cannot escape the conclusion that the strength characteristics of the Cucaracha clay-shale are not similar to those that can be expressed by the Coulomb equation with constant c-values and ϕ-values. In addition, the basis on which these values were determined is open to objections.

Conclusion.—Having thus critically reviewed the method of arriving at the slopes in the Cucaracha, the writer reiterates that the proposed design curve represents the best that could be accomplished with the tools and the knowledge available. The writer believes that the Special Engineering Division would not have been justified in spending more money on these investigations even if the necessary additional time had been available. However, as soon as construction of a new or improved canal is authorized by Congress, the additional investigations on the stability of slopes in Cucaracha should be carried out, with special attention to the time effects. As recommended by the Special Engineering Division, it would then be necessary to explore and study thoroughly the materials in each slope, by additional exploratory borings as excavation of the slope progresses, with the purpose in mind that additional flattening of the slope will be undertaken if it is found that in any place the strength falls below expectations.

In addition, the writer recommends extensive observations on the pore-water pressures in the Cucaracha formation, and on the possibility of reducing these pressures by drainage measures (wells and tunnels) for the purpose of increasing the stability of slopes. The effect of time on the shear strength should be investigated also by long-time laboratory tests. Although this is a very difficult task, every effort should be made to bridge the large gap that now exists between the results of laboratory strength tests and the strength characteristics that govern the stability of slopes in the Cucaracha.

JAMES H. STRATTON,[137] M. ASCE.—In this closure to the Symposium, the writer wishes first to acknowledge on behalf of all the authors their gratitude for the generous attention given by the discussers to their efforts to compress within reasonable bounds a description of the extensive investigations and studies undertaken by a fairly large engineering organization over a period of nearly 2 years. Several discussers have evidently misunderstood the main purpose of the Symposium, which was to present to the Society for discussion and criticism the engineering features of the recommended plan. In consonance with this purpose only enough information was included in the Symposium regarding the reasons for adopting the plan to serve as a background for the presentation; and a complete discussion of those reasons, even if it were appropriate in an engineering paper, would have been impossible because of security requirements. The selection of material for presentation in the Symposium papers necessarily involved omissions which, no matter how discriminating, could not fail to create the impression that there were corresponding omissions or oversights in the studies. Unfortunately, in correcting this impression where it is in error, space limitations do not permit more than a passing reference to this fact. For those who are interested, a complete copy of the Governor's (J. C. Mehaffey's) report on The Panama Canal has been placed in the Engineering Societies Library. The record of these discussions should serve as an invaluable guide in the reevaluation of certain phases of the investigations during the design and the construction of the project when authorized by Congress.

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[136] "Research on Stress-Deformation and Strength Characteristics of Soils and Soft Rocks Under Transient Loading," *Bulletin No. 31*, Soil Mechanics Series, Cambridge, Mass., June, 1948.

[137] Col., Corps of Engrs., U. S. Army, Div. Engr., New England Div., Boston, Mass.; formerly Supervising Engr., Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

In most instances where discussers have challenged the conclusions leading to the recommendation for the conversion of the Panama lock canal to a sea-level canal, they have not backed up their discussions by scientific demonstration or by engineering and navigating experience. Just as broad judgments and an effort to penetrate the future are essential to an understanding of the place of the Panama Canal in the national economy and in the planning for national defense, so do scientific predictions of future traffic needs, the evaluation of weapon effects, and careful engineering appraisals of such problems as slides and navigation of the proposed sea-level canal also play a vital part.

For this reason the writer is particularly indebted to General Kramer for quoting the farseeing and prophetic instructions of Theodore Roosevelt, then President of the United States, to the Board of Consulting Engineers convened in 1905 to consider the type of canal to be constructed in Panama.[2] These instructions, and also the report of the Roosevelt Board and the conclusions of many other eminent investigators, not only established the "precept" (in the words of General Kramer) that guided the recently completed Panama Canal Studies; but, by test, proved to be a logical forecast that the decision to convert to sea level was ultimately inevitable when, for reasons of expediency, economy, and speed of construction, the lock-type canal was chosen. The Governor's recommendation to Congress under the provisions of Public Law No. 280, Seventy-ninth Congress, that the present Panama lock canal be converted at once to sea level, in effect implements a decision that was implicit in the acceptance of the lock canal in the first instance. In the light of experience during World War II and of recent weapon developments, the Governor's recommendation is as inescapable as it is timely.

Captain Nichols has had ample opportunity to evaluate the consideration given to the problems of navigating the proposed sea-level canal. In his discussion of the Symposium, he draws conclusions that should be reassuring to those having latent misgivings and should satisfy even the most skeptical that the proposed canal will be entirely adequate and will provide a safe and easy waterway for the navigation of ships.

Acceptance of the tidal computations for the Panama sea-level canal by General Pillsbury and Professor Bakhmeteff strengthens the case for the use of the computation method in evaluating conditions that would obtain in waterways of this kind. It is the view of the writer that for future waterway projects the large expense of model studies may well be avoided where (1) the extreme tidal conditions are reasonably predictable, (2) the channel cross section can be made adequate dimensionally and of reasonable uniformity, and (3) the computed currents are found to be lower than those now encountered in the Cape Cod Canal (Massachusetts)—that is, of the order that may be expected in the proposed Panama sea-level canal.

It is hoped that the extensive investigations in the sea-level canal model suggested by Professor Bakhmeteff can be undertaken. Steps have been taken to protect and preserve the model in good condition for just such purposes.

Professor Davidson's interesting analysis gives clarification to the descriptions in the Symposium; and his acceptance of the model experiments related to the handling of ships in restricted channels adds authority to the testing method. Professor Davidson notes that in this testing the model operator (pilot in this case) is of vital importance to the effective operation of the model. The writer was impressed by his unqualified statement, when he deduces that perfect equilibrium may be difficult to attain, that: "* * * approximation to equilibrium is good enough in practice * * *." Professor Davidson's analysis of the model results by application to an actual sailing problem provides a clearer understanding than can be gained from an examination of the test results expressed in mathematical tabulations and graphs. Professor Davidson regrets that Messrs. Lee and Bowers did not emphasize the practical conclusions more strongly. This omission is the unfortunate result of the limitations in scope of treatment agreed on by the several authors to insure the fullest possible coverage in the several Symposium papers. Messrs. Lee and Bowers in the course of the investigations expressed full confidence in the practical application of the test results as did the Panama Canal pilots who were familiar with the Carderock (Md.) tests.

Messrs. Justin, Fucik, Hill, and Casagrande have contributed interesting sidelights on the Cucaracha formation, which is the most treacherous that will be encountered in the excavation for the sea-level canal. Mr. Fucik's analysis of the safety factor suggests that certain chances might well be taken with the Cucaracha to limit the volume of initial excavation with the expectancy that sloughing and slow moving slides, although contributing to the maintenance problem, may not entail total quantities approaching those that would be excavated initially if a high factor of safety is selected. A rather complete analysis by Professor Casagrande of what may happen subsequent to the completion of the sea-level canal makes it clear that a slow movement of the Cucaracha toward the canal cut may be expected in any event. This was the view of the staff employed on this phase of the investigations and was the consensus of the consultants. None felt, however, that there was a risk to shipping involved, since movements would be very slow and the quantities involved relatively small. Thus the material could be readily excavated as it moved into the canal. Because of the great depth and width of the proposed canal there would not be interference with traffic as a result of such movement of material or in the operation of equipment in its removal.

Mr. Hill indicates that the method of analysis employed in establishing the slopes of the Cucaracha is reasonably reliable. There is a seeming point of difference between Mr. Fucik and Professor Casagrande in that the former takes the view that the proposed slopes may be conservative and the latter the view that they may not actually be conservative enough. Those are really expressions of the same opinion with respect to the future behavior of the Cucaracha from two different viewpoints—presumably Mr. Fucik's is related to that of probable ultimate cost whereas Professor Casagrande's is related to the quantity of Cucaracha material that may ultimately have to be removed. Both seem to agree that there will be some movement of the Cucaracha, and both are fully aware of the need for further laboratory and field investigations, for which plans have been suggested in the Governor's report. Fortunately the time available from the start of construction until the new cuts are opened to traffic will permit extensive tests and field observations of movements in the Cucaracha if any should occur. Whether the safety factor can be reduced is a design consideration; a reduction undoubtedly would not be undertaken unless the investigations in the course of design and construction warrant such action.

The admonition by Mr. Cedergren that "* * * new terrain must be examined closely for indications of old slides, and obviously unstable terrain avoided whenever possible" is one that will surely guide those who undertake the design and construction of the proposed sea-level canal.

It is believed that the estimates of cost provide fully for the difficulties in the dry excavation and in the disposal of materials by scows foreseen by Mr. McAlpine. The excavation and the handling of large volumes of material will both stimulate and support improvements of equipment and of method as construction progresses, but for reasons of conservatism of estimates this possibility has been discounted to provide against unforeseen contingencies.

General Covell presents an interesting analysis of the savings that would have resulted had the Panama sea-level canal been constructed as recommended by the majority of the Board of Consulting Engineers in 1906. General Covell's presentation of the views of both the majority and minority of this board in its report to President Theodore Roosevelt rounds out the discussion by General Kramer and shows that the final acceptance of the lock canal because of ease and speed of construction in no way constituted a condemnation of a sea-level canal in Panama.

Professor Hobbs' design for a sea-level canal at Tehuantepec (Mexico) completely overrides the conclusions as to channel requirements established by the investigations and tests under Public Law No. 280 and presented in the Symposium, and ignores minimum acceptable hydraulic standards for constricted ship channels. A one-way channel 175 ft wide which he proposes cannot be reconciled with a lock, 200 ft in minimum width, established by the United States Department of the Navy. The excavation involved in the canal suggested by Professor Hobbs is given as approximately 400,000,000 cu yd less than the proposed Panama sea-level canal. For a sea-level canal with adequate canal cross sections on the Hobbs alinement, the quantity of excavation would greatly exceed that required for the Panama sea-level canal.

A one-way tunnel, 63 miles long, would be a serious bottleneck to traffic. Towing locomotives would be required (for large ships at least) in the tunnel section and in the proposed single-way open channels. Under the most favorable conditions, almost 2 days would be required for the transit of such a canal; delays waiting to transit the tunnel would heavily penalize shipping. The expense of installation and operation of towing equipment is apparently overlooked by Professor Hobbs, as is the fact that a new canal at any place other than Panama would require duplication of defense facilities and of administrative facilities, housing, shops, harbors, dry docks, etc., at a cost that may be expected to exceed $1,000,000,000.

The defense agencies take the view that a ship tunnel could be closed by demolitions despite the portal construction of the type proposed by Professor Hobbs. Forced ventilation for the disposal of stack gases and lining of the tunnel to prevent the falling of spalls or large blocks of rock on passing ships apparently are considered unnecessary by Professor Hobbs. If a ship tunnel were acceptable, the San Blas route would be much more favorable than the Tehuantepec route recommended by Professor Hobbs.

The reduction in travel time using the Tehuantepec route, where not offset by delays in transit, would apply principally to intercoastal shipping which is about 30% of the total traffic. For many trade routes the port-to-port travel distance via a Tehuantepec canal would actually be increased. The net annual saving in ship operating costs, if capitalized, would not justify the selection of the more costly Tehuantepec route.

In his list of advantages Professor Hobbs concludes that the combination of the existing lock canal and the Tehuantepec sea-level canal would provide the highest degree of security. Mr. Jarvis has somewhat the same view with respect to the combination of the Panama parallel sea-level canal and the present lock canal. Because of its vulnerability the lock canal would contribute nothing to security; and it would not be required to supplement the capacity of a sea-level canal. Therefore, the lock canal should be scrapped, rather than continued on either an operating or a stand-by basis at high expense. It is not apparent to the writer in what sense or degree the Tehuantepec canal would be less vulnerable to an attack than the proposed Panama sea-level canal as contended by Professor Hobbs. Rather, the reverse would seem to be true because of the highly vulnerable tunnel and tunnel portals and the narrow channels and because of its greater target length.

Mr. Dohn proposes that the canal be justified as an economic enterprise and suggests: (1) Increase of tolls and (2) decrease in canal construction costs. An increase in tolls in terms of the cost thus to be amortized would drive shipping away from the canal. His desirable objective of decreasing construction costs will undoubtedly be sought by those who undertake the canal construction. No one intimately familiar with the geologic formations and other local conditions would propose the use of the hydraulic giants for the excavation, or the development of power to be disposed of subsequent to construction.

Mr. Romanowitz voices the conclusion that rock excavation using bucket-ladder dredges would be both practicable and economical. He demonstrates that, since the employment of this type of dredge in the construction of the present canal, there have been major improvements which would reduce, very materially, lay-up time for repairs. He notes that the dredge *Corozal* did a very effective job of rock removal in the Panama Canal construction during its hours of operation and that only plant breakdown adversely affected its over-all usefulness.

Mr. Massey notes that no mention is made of the advisability of excavating to a depth of 130 ft with special dredges operating on Gatun Lake and, after lowering of the lake, of excavating to final project depth (15 ft of material). This procedure was considered but was left for decision in the course of actual construction. The bucket-ladder dredge will be tested fully before commitment is made to perform all deep rock dredging with this type of equipment.

Mr. Erickson suggests the use of a 60-in. hydraulic dredge for excavating the softer rocks and shales without blasting. Unfortunately a major part of the deep excavation in rock will require blasting. Nevertheless, in the design period consideration should be given to the size of the dredges, speed of cutter, etc., as he suggests.

Mr. Kazmann questions the advisability of expending large sums of money on the proposed sea-level canal and makes comparisons with such projects as the Saint Lawrence Seaway, the Missouri Basin development, and the Tennessee Valley Authority to the detriment of the proposed canal in its relation to national existence in the event of conflict. In reaching this conclusion Mr. Kazmann assumes that the canal project on the one hand and the development of national resources on the other are mutually exclusive. The extension of this reasoning would condemn the appropriation of funds for the further development of the atomic bomb and the development of rockets and improved airplanes and, in fact, for anything pertaining to the national defense. In any event a comparison of the canal project with other development projects was beyond the purview of the report, and the determination of related priorities among such projects is a function of Congress.

It is regretted that the Symposium led Mr. Kazmann to the conclusion that the investigations under Public Law No. 280 hardly bothered with the point: "* * * is one [a sea-level canal] needed?" Space limitations and security considerations may have resulted in an inadequate demonstration of the essentiality of the sea-level canal to the national defense and to the well-being of the American people; but the study nevertheless went into this problem exhaustively. From an examination of the Governor's report, the National Defense Establishment including the Joint Chiefs of Staff and the Department of the Navy, which has a special interest in the problem, fully supported and endorsed the need for a Panama sea-level canal. There is ample engineering proof contained in the classified studies for this conclusion. Professor Edwards notes that "* * * the question regarding the type of canal could be answered without such detailed technical studies.". To the writer it also appears that by taking thought even a layman could not fail, from a knowledge of the features of the two types of canals and of present-day weapons gathered from the popular press, to arrive at the same conclusion.

Mr. Sheldon makes some pointed observations on the foundations of structures on Atlantic muck and warns against slides in this material after the lake has been lowered as the result of overloading the canal banks with spoil material. The setbacks from the canal slopes in the Atlantic muck to the edges of the spoil banks established in the studies are tentative; in the final design these and the rate and manner of disposal of spoil material should be carefully reviewed. Mr. Sheldon concludes that none of the possibilities of failure which he describes constitutes a serious threat to the sea-level canal project; nevertheless, these possibilities should be anticipated and avoided in the designs.

The statements by Mr. Cole that—

> "* * * the investigation of the canal under Public Law No. 280 (Seventy-ninth Congress, First Session) was solely directed toward securing the authorization of the predetermined objective of a sea-level project on the premise that the present lock canal 'is totally lacking in security to meet the future needs of national defense,' and that the only alternative is a sea-level canal."

and

> "It is fair to ask the question, and insist upon a truthful and an unbiased answer: Is an expenditure of $2,500,000,000 justified at Panama or elsewhere on the Isthmus for any canal, lock or sea level, under any circumstances?"

and

> "The estimated first cost for 'converting' the lock canal to sea level, of $2,483,000,000 alone (which appears to be grossly underestimated), is prohibitive, unless the cost is to be borne by the national defense budget."

—have the effect of impugning the integrity of the organization employed on the studies under Public Law No. 280—the Governor of the Panama Canal, the Board of Consulting Engineers who fully concurred in the conclusions and recommendations reached in the studies, and the authorities of the National Defense Establishment, including the Navy General Board and the Joint Chiefs of Staff, and also the Atomic Energy Commission in its evaluation of atomic bomb effects. Heretofore the writer has been disinclined to reply to such charges; but, since they have now reached the Society's publications, the writer challenges them and expresses his resentment of these implications. Honest differences of opinion are certain to develop on many features of the canal problem, now as in the past, but they should be expressed without the invidious implications used by Mr. Cole.

Mr. Cole's charge that Congress and the government officials had "the predetermined objective of a sea-level project" is best answered by the record of Congressional hearings and debate on the bill which became Public Law No. 280.

Mr. Cole and Captain Little both note that the investigation reported in the Symposium was not directed toward securing any major improvement in the operating deficiencies of the present lock canal. In Public Law No. 280, Congress explicitly directed a

> "* * * comprehensive review and study of the means of increasing the capacity and security of the Panama Canal to meet the future needs of interoceanic commerce and national defense * * *."

The exclusive pursuit of other secondary objectives would have been in violation of the realistic and comprehensive directive of Congress. It was considered that the factor of the operating deficiencies of the present canal was a concomitant phase of the studies and this was looked into fully. It is unfortunate that the details of the studies could not be presented within the scope of the Symposium; nevertheless, the designs of the lock canals studied incorporated features to correct the deficiencies noted by Mr. Cole in his references. These are fully set forth in the Governor's report.

Lest it be gathered from Mr. Cole's discussion that the present canal is unduly hazardous to shipping, it should be noted that the accident records of all the major waterways throughout the world were carefully examined and it was found that the Panama Canal is as safe, from an operating viewpoint, as any other major waterway. Damage to ships in more than 200,000 transits through the Panama Canal since it was placed in service has required repairs at an average cost of $7.45 per ship transit. Truly this is a notable record and a complete answer to those who have dwelt unduly on the operating deficiencies of the present canal. The facts are well recognized by marine insurance underwriters, for they attach no special risk to the Panama Canal.

Mr. Cole does not identify the authorities who have expressed the opinion that any type of canal is definitely vulnerable to the atomic bomb. To weigh any such statement, the damage must be related to the period of closure that would result from atomic bombing. In the case of a sea-level canal this would be a matter of weeks at most. In the case of a lock canal it would be a matter of years, for if Gatun Lake were lost it would take 18 months to recapture sufficient water to make the transit of the canal possible and to this there must be added the necessary period for repairs and reconstruction to make impoundment possible. It is not understood, as contended by Mr. Cole, how "* * * the major premise (national defense), on which the recommendation for 'converting' the canal to sea level was based, has now definitely been removed"; or how he reconciles this conclusion with that of the National Defense Establishment that only a sea-level canal will serve the future needs of national defense. It would be interesting to have an evaluation of the effects of conventional bombing on both the sea-level and the lock-type canals, by the authorities to whom Mr. Cole alludes.

Mr. Kirkpatrick evidently holds with Mr. Cole that the solution of the operating problems of the present canal will provide for the future needs of commerce and the national defense, thus satisfying the requirements of Public Law No. 280, Seventy-ninth Congress.

Mr. Kirkpatrick's warning that: "* * * going down 100 ft further [below the present canal bottom] * * * is daringly unprecedented even to the point of recklessness" suggests that had the Panama Canal constructors taken counsel of such fears they would have abandoned the undertaking, for they too had nothing to go on but, to use Mr. Kirkpatrick's words, "geological conjectures." Their conjectures, moreover, were on a much sketchier basis than are those of today, for out of their great accomplishment there is an accumulation of knowledge and of experience which, supported by present-day techniques of exploration, test, and stability analyses gives assurance that none of the dire events which apparently haunt Mr. Kirkpatrick will happen.

Inspection by the Panama Canal engineering staff of certain mining operations disclosed that the bucket-ladder-type dredge is capable of lifting boulders and rocks very much larger than the buckets. This is also true of clamshell and dipper dredges, but one could scarcely contend that the excavating of oversize rocks would not interfere with the over-all plant efficiency of any of these excavators. In making his point that the bucket-ladder-type dredge would not be efficient in the removal of large boulders or hard rock, Mr. Erickson indirectly emphasizes the importance of adequate drilling and blasting.

Mr. Jarvis is of the opinion that tidal-regulating works are unnecessary. His studies of the Cape Cod Canal and actual operating experience there give authority to his conclusion. There is considerable merit to the Panama parallel sea-level canal, as pointed out by Mr. Jarvis. However, it was the conclusion of the Board of Consulting Engineers, as well as of the Panama Canal authorities, that the advantages did not outweigh the additional estimated cost of $800,000,000 for a sea-level canal on the Panama parallel route. Moreover, since a single sea-level canal would fulfil the future needs of commerce and national defense there would be no justification for subsidizing, in addition, a lock canal that could be rendered useless even by light bombing.

Much has been written (as Captain Little remarks) about the capacity and the time of transit of the canal; and, contrary to Captain Little's observation, a great deal of factual material bearing on these quantities is published in the Symposium and more in the Governor's report. All this has been done in an effort to arrive at facts, without creation of any impression, exaggerated or minimized, contrary to the truth.

Captain Little is apparently in disagreement with the use of the term "capacity" as defined and employed in the Symposium. The canal, of course, could transit the number of ships computed by Captain Little, if these ships reached the canal at the unvarying rate of 36 a day—that is, if shipowners would tolerate delays of the necessary, and in individual cases uncertain, amounts of time required to realize such an assumption. If one were to take for illustration the capacity of 13,140 ships per yr (designated by Captain Little) (36 ships × 365 days) one would find from Table 7 that the abnormalties of traffic flow that would obtain would result in 495 ship-days of delay when traffic is 130% of the daily average; on 140% peak days, 336 ship-days delay; and, on 150% peak days, 162 ship-days delay. Using ship-operating cost data furnished by the Maritime Commission, the delay penalties inflicted on the shipping industry for such peak days only would exceed $3,000,000 annually. For all delays, taking into account every day on which shipping would be delayed because the number of arrivals for transit would exceed the daily average of 36 ships, the total annual penalty to shipping would be very much higher.

Captain Little evidently does not take exception to the indisputable fact that there is a wide daily fluctuation in traffic arriving for transit of the Panama Canal (equally true of Cape Cod, Suez (Egypt), and all other canals), for he proposes regulation of traffic to avoid peak congestion; yet he dismisses this factor in his computation of the capacity rating of the Panama Canal by assuming that the daily fluctuations can be easily ironed out. This same problem, the cost imposed by peak loads, plagues many of man's activities, as engineers are well aware. The electric and rapid transit industries would be grateful for a practicable solution of it. They have not been able as yet, in Captain Little's triumphant phrase, to "control them [peak loads]!"

It would be interesting to have a canvass of the shipping industry to ascertain its views on Captain Little's proposal of "controlling sailings to, or arrivals at, Panama." The evident procedure for establishing such control is: Speed up loadings or delay ships in loading ports or en route to avoid the peak traffic condition. An assumption as to the workability of the former implies that shippers now generally conduct loadings inefficiently, and thus could at will improve loading rates to lessen time in port to fit traffic call-up schedules for transiting the canal. Does Captain Little truly believe that shippers will willingly accept delays (and the costs that go with delays) to meet man-imposed regulations to overcome canal operating deficiencies that are susceptible of correction by engineering methods?

In taking exception to the significance given to "transit time" in the studies described in the Symposium, Captain Little states: "No general or admiral ever had to 'tear his hair out' because some ship or convoy of ships was late in passing through the Panama Canal." This is the finest compliment that could be paid the existing canal. It was, in part, to insure for the future this same quality of service that Congress directed the studies under Public Law No. 280, Seventy-ninth Congress. The report of the Governor is fully responsive to the Congressional directive and his recommendation meets this objective and also the additional one imposed by Congress, to wit—that of insuring that an enemy cannot seriously interfere with the capacity performance of the canal, no matter what weapon is employed. By using military operations to analyze transit time Captain Little gives only a part of the picture. In peacetime the Navy traffic through the canal is less than 15% of the total. Commercial traffic, which constitutes the remainder, would most certainly welcome reduction in transit time and resulting savings.

The answer to Captain Little's objections to the conclusions reached in the studies with respect to the vulnerability of the proposed sea-level canal is given in the writer's paper and elsewhere in this closing discussion. Specifically it is stated in the writer's paper (under the heading, "Security of the Panama Sea-Level Canal") that:

> "The tidal-regulating and flood-control structures of the sea-level canal would not be essential to its safe operation; hence their damage or destruction would result only in temporary interruptions to traffic. * * * The flood-impounding structures being of earth and rock construction would be highly resistant to bombing but, because they are not absolutely vital to the operation of the sea-level canal, the costly treatment necessary to make them resistant to the atomic bomb would not be warranted. If damaged by bombing, the flood-control structures could be readily repaired and restored to service."

These conclusions were the product of an unbiased study by competent experts, and were formally accepted by the Secretary of the Navy, the Navy General

Board, the Joint Chiefs of Staff, and the Atomic Energy Commission. The details of the studies are classified and cannot be published. It goes without saying that Congress, in its hearings on the Governor's report prior to the enactment of legislation, will have such access as it desires to the classified studies and can satisfy itself that the conclusions are tenable.

The political aspect of the problem of conversion posed by Captain Little will receive the attention of Congress in its hearings. The Governor's report in dealing with the problem raised by Captain Little states:

> "Arrangements would have to be made with the Republic of Panama and compensation made to landowners for the right to inundate an additional area of about 10,000 acres downstream from Madden Dam. The completion of the sea-level canal, however, would permit the return to Panamanian jurisdiction of lands now inundated by Gatun Lake and having an aggregate area greater than the additional area on which flowage rights would be needed."

Except for such land exchange the present treaties with Panama are adequate to cover conversion of the canal within the Canal Zone.

In using the words, "* * * it was a foregone conclusion that no scheme other than a sea-level canal at Panama would receive any real consideration under the act" Captain Little takes the position with Mr. Cole that some preconceived conviction guided those responsible for the studies. The answer given to Mr. Cole is extended to Captain Little.

The color of language employed by the writer in stating that with the approach of war the restricting effect of the small locks became intolerable on the design of Navy ships seems to have bothered Captain Little. The literal fact is that the 140-ft width of the proposed Third Locks, and the 200-ft width of locks used in the present studies, were established by the Navy. Captain Little assumes that the canal authorities established these widths from "guarded replies" by the Navy to specific inquiries. This is pure conjecture. His leading question on the frequency of the use of the canal by large carriers or oversize battleships should be addressed to the Department of the Navy.

Captain Little takes exception to the writer's statement on the navigational safety of the sea-level canal and implies that fogs, heavy rains, and machinery failures were given inadequate consideration. All that can be said is that these matters were studied, it is believed adequately, and that the published report gives the data used and the conclusions reached from study of similar canals and from the advice of the pilots and various naval and maritime authorities.

The capacity of the Panama sea-level canal would not be 69 ships as stated by Captain Little, but under the conditions stated in Tables 12 and 13 would be much greater.

Captain Little suggests that for comparison with the proposed Panama sea-level canal there should be a lock canal with a Pacific terminal lake and without any protective features. Such a plan with locks 200 ft wide was investigated. It would cost $1,600,000,000 approximately. A reduction in lock dimensions to provide "reasonable lock sizes," as suggested by Captain

Little, would decrease the cost of this plan in proportion to the dimensions determined by the Navy to be acceptable.

Rear-Admiral D. E. Barbey's views[131] cited by Captain Little seem to be at variance with the official views of the Department of the Navy. Of the 17,000 vessels which transited the canal in war years (3½ years approximately), 30% were naval fighting craft. This fact would seem to refute the statement that: "* * * the Panama Canal has become * * * a secondary route for critical raw materials in time of war." It is devoutly to be hoped that the enemy will subscribe to Admiral Barbey's view as quoted and will not attack the canal. Congress had in mind that the enemy might act differently and in effect asked the question: "What if he does?" It may be remarked that a canal easy to destroy would be a much more tempting target than a canal on which the same effort could inflict but minor damage and so the first might be regarded as a profitable target whereas the second might not.

In discussing target values Gen. Leslie R. Groves apparently did not state that the Panama Canal would be an unprofitable target as implied by Captain Little. No contention is made in the report that the enemy would find the Panama Canal a more profitable target than New York, N. Y., or any other industrial center. The answer to such a question can only be an indirect one: Is the Panama Canal essential to national defense? The National Defense Establishment has found that it is.

The Governor's report and the Symposium have very properly emphasized the urgent need for a sea-level canal to meet the future needs of national defense. As described in the Symposium the study of future traffic needs revealed that delays to ships awaiting transit will be experienced on "peak traffic days" beginning about 1960. After 1961 the cost of delays at the entrances to the canal will increase in direct relationship to the growth of traffic. In 1970 they will total more than $3,000,000 unless remedial measures to increase canal capacity have been undertaken. After 1970 there will be a sharp rise in the penalties arising from inadequate capacity averaging $20,000,000 until 1980 when the added cost will be more than $70,000,000 annually. The heavy financial penalties resulting from the failure to provide against the deficiencies in canal capacity which will first be felt in 1960 will fall upon shipping interests who must either accept delays or, when delays become intolerable, route their ships around Cape Horn.

The cheapest method of increasing the capacity of the canal to meet the needs of commercial shipping would entail certain modifications to the locks of the present canal at a cost of approximately $130,000,000. These corrective measures, however, would not add to the security of the canal or in any way reduce the risk of loss of the canal by enemy action. Since the penalties to shipping as a result of inadequate capacity will soon become sufficiently oppressive to demand a large expenditure of public funds, prudent planning dictates that the remedial measures provide both for future traffic needs and for the requirements of national defense. The only answer is early conversion to a sea-level canal.

If the lock canal is ever seriously damaged by enemy action, and particularly if the damage inflicted results in the drainage of Gatun Lake, as would be the case with even minor bombing of the lock gates, not even its most earnest proponents could advocate its reconstruction as a lock canal. A successful attack on the canal would mean a denial of the Isthmian crossing to shipping for the 10-year period required for the construction of the sea-level canal, unless traffic could be restored as construction progresses. The loss of the canal prior to 1960 would increase shipping costs as the result of lengthening of trade routes around Cape Horn an average of $400,000,000 per yr. After 1960 the increased cost per year would be $500,000,000 and would rise to $600,000,000 per yr after 1980. Thus, unless, through some expediency, early restoration of traffic in the damaged lock canal proves practicable, the increase in commercial ship-operating costs alone in the construction period would total approximately twice the estimated cost of peacetime conversion to sea level. The disruptive effects of the loss of the present canal on the conduct of war combined with the penalties that would devolve upon shipping in the postwar construction period are sufficiently alarming to warrant its conversion at once.

If the lock canal is ever badly damaged, every effort would be made immediately to restore traffic in the lock canal if this is practicable. The first step in any case would be the restoration of the summit lake (Gatun Lake) which in itself would greatly facilitate the conversion to sea level. Whether it would be possible, again, to place the locks in operating condition after enemy attack would depend on the extent of damage inflicted. Atomic bombing of the locks would require their abandonment.

Even though restoration of traffic through the lock canal is found practicable, it would probably be on a restricted basis and a considerable part of postwar shipping would have to be diverted around Cape Horn. Moreover, if the sea-level conversion is deferred until the lock canal is crippled by enemy action, the cost of the conversion would be materially increased. The increase would result from the cost of the temporary repairs to the lock canal, from the inevitable lengthening of the construction period, and from the added difficulties during construction if the lock canal is not fully and immediately available for utilization in effecting the conversion to a sea-level canal.

Dean Marston's references to the studies by the late Sydney B. Williamson, M. ASCE, and those of the Interoceanic Canal Board lead the writer to acknowledge that much dependence was placed on these studies in the recent investigations under Public Law No. 280. Dean Marston's acceptance of the navigable pass and his conclusion that the canal would be navigable even if the tidal locks should be destroyed are noted with special interest as are his views on the studies made to avoid slides in the proposed sea-level canal. His appreciation of the fact that the present canal has been a full-scale laboratory for the study of safe slopes high lights a consideration on which major reliance has been placed in establishing the slopes for the sea-level canal.

Ralph P. Johnson,[138] M. ASCE, and Sydney O. Steinborn,[139] Assoc. M. ASCE.—Captain Nichols has stated that use of the estimate of 1.5 ships per lockage through the present locks may be overly optimistic for planning purposes. It is believed, however, that a brief elaboration of the derivation of this estimate will establish its essential conservatism.

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During the past 20 years, the average number of ships per lockage has been about 1.2. However, the canal has not had to operate near its capacity and a greater percentage of tandem lockages would have been possible if required to meet traffic demands. Assuming a group of 100 vessels of the length composition expected in the year 2000, the existing lock chambers could accept lockages composed as follows:

Description	Lockages	Ships
All ships longer than 500 ft, singly	11	11
One ship of the 400-ft to 499-ft group with one below 399 ft	44	88
Remaining 400-ft to 499-ft ship, alone	1	1
Total	56	100

(In Table 9, vessels are estimated to increase in length; this has the effect of reducing capacity and the assumption is thus conservative.) In this manner, an average of 1.8 ships could be transited per lockage. The soundness of this determination was established by routing actual traffic on peak days of record through the canal without delaying any ship more than 2 hours. In the further interests of conservatism, however, additional consideration was given to delays that might be caused by attempting to compose lockages according to the foregoing tabulation; for this reason the 1.8 factor was reduced to 1.5 for planning purposes.

Captain Nichols questions the application of the 1.5 factor, which was determined on an annual basis, to daily traffic. This question also arose during the course of traffic and capacity studies and a comparison of peak-day traffic and annual traffic (in average percentages) was made as follows:

Length of ship, in feet	22 peak days in 1933 and 1939	1921–1940
Over 500	6.1	6.2
400 to 499	49.0	47.3
Under 400	44.9	46.5
Total	100.0	100.0

This comparison made it evident that, historically, there was little difference between the length composition of traffic on peak days and the length composition as averaged over many years. Thus, it was concluded that an estimate of 1.5 ships per lockage was conservative for both annual and daily traffic. It is acknowledged, however, that traffic abnormalities will occur and there may very well be days when few tandem lockages can be made. Fortunately, the occurrence of such days will be rare and thus they have little design significance.

Captain Little has stated that "* * * very little in the way of factual analysis has been published on the actual 'capacity' of the Panama Canal * * *." It was the intention of the writers to correct this situation in preparing the section of their paper under the heading, "Capacity of the Present Panama

Canal." In that section were given the results of time and motion studies which established the capacity of the locks, and which described the effect of fogs, one-way night traffic, and "clear-cut" transits. Similar treatment was given to the capacity of the locks during the overhaul period and finally, as a convenience to the reader, the entire matter was summarized in Table 10. The conclusions thus summarized were the results of months of intensive analysis of lock operations, in which the researching engineers consulted frequently with the experienced lock operating personnel of the Panama Canal. Whether or not the capacities shown are "actual capacities" will not be evidenced until the demand for transits requires that operations conform more or less closely to the procedures and timing evolved in the course of this investigation.

It is certain, however, that, in referring to capacity, the writers are not dealing with annual capacity. The capability of the Panama Canal is similar in some ways to the peaking capability of a hydroelectric power system in that it is established by its performance during a critical period. In this particular instance, the critical period occurs during the 4-month biennial lock overhaul. Its annual capacity has little practical significance because it is manifestly impossible to expect shipping interests to wait until after the 4-month overhaul period to be assured that their cargoes will move without being delayed in Canal Zone waters.

It is equally impracticable to speak of controlling shipments during the overhaul period in order that peak traffic days may never materialize. It is impossible to conceive any amount of planning, short of that which might be possible in a world totalitarian state, that would prevent traffic peaks at the Panama Canal. It should not be forgotten, however, that rigid traffic control does exist in Canal Zone waters. Whenever the dispatchers are confronted with a backlog of ships awaiting transit, scheduling is accomplished with considerable skill and efficiency, the limiting features of the transiting program being set by the physical plant of the waterway.

Captain Little's inferred recommendation that traffic be controlled by increased tolls for transits during peak traffic periods is completely unacceptable when applied to a public utility like the Panama Canal. Faced with increased traffic demands the canal must provide additional facilities. The alternative is a reduction in service which is not acceptable to the shipping industry. This fact has been brought out by Captain Nichols in his discussion:

> "Upon release of the biennial lock overhaul schedule for the winter of 1947–1948, several local steamship agents inquired as to the probability of delays to shipping. They stated that their principals, engaged in shipping between the Antipodes and Europe, had intimated that delays would result in rerouting via the Straits of Magellan or the Suez Canal."

The Panama Canal has a fine record of efficient public service; it was built as an asset to national defense and as an aid to world trade. Its defense has now become a major problem and its inadequate capacity will soon begin to complicate world trade. Therefore, it is time to think about modifications that will solve the defense problems and that will permit the canal to continue its long history of accelerating the movement of world cargoes.

F. S. BROWN,[140] M. ASCE.—The writer concurs with Mr. Edwards that the present Gatun spillway does not have the capacity to handle the "spillway design flood." However, the statement questioned by Mr. Edwards that "The control afforded by Madden Lake has largely ended the likelihood of further major flood disturbances in the present canal" was not intended to refer to this deficiency, but rather to the benefit afforded by the Madden Lake control in reducing objectionable crosscurrents in the canal at Gamboa at the mouth of the Upper Chagres River.

J. S. MEYERS[141] AND E. A. SCHULTZ,[142] ASSOC. MEMBERS, ASCE.—The serious interest evinced by the discussers of this paper is gratifying. Because of the diverse nature of the tidal-current problem, the discussers have generally been concerned only with certain features, and it is considered best to reply separately to each one.

General Pillsbury's discussion of the precision and the reliability of velocity computations is a welcome amplification of the condensed explanation made in the basic paper. Professor Bakhmeteff's discussion, written in similar vein but directed toward a more general appraisal of the entire tidal-hydraulic problem, is also a very welcome addition to the briefer summaries in the original paper.

Professor Reynolds comments on the scanty knowledge of channel roughness coefficients for large channels, and questions whether the conclusions which were drawn from available data (conclusions (*a*) and (*b*) under the heading, "Hydraulic Roughness of Large Channels: Roughness Data on Large Channels from Other Sources") really indicate the true situation. The writers agree wholeheartedly that more data need to be measured and studied, which may well lead to different conclusions. The conclusions as expressed, however, are quite well supported by the information that was collected.

Interest was expressed also in the roughness measurements made in Gaillard Cut. The uncertainties inherent in such observations, as indicated by Professor Reynolds, were recognized and overcome as far as possible. It was a source of some surprise to the writers that the measurements were so consistent and successful, and that the anticipated difficulties did not prove to be serious. The measurements of velocities and of cross sections posed no difficult problems, but the determination of the very small differences in water level was approached with great trepidation. Fortunately the weather was ideal for the purpose on all three nights when observations were made, and careful and intelligent observers were secured. There was absolutely no discernible wind from sundown to sunrise; the water had the glassy appearance characteristic of perfect calm; and barometric pressures were steady.

Surges created by operation of the lock valves persisted, with diminishing intensity, throughout the tests. Consistent surge patterns were observed at each of the hook gages; and, after the lapse of a few hours, the difference in level between trough and crest of a surge dropped to approximately 0.010 ft. Average water levels used were those taken over an 86-min period—the time required for a full surge cycle.[143] The roughness coefficients computed from the measured velocities and differences in water level were remarkably consistent, in spite of the very flat slopes and the considerable irregularity in cross-sectional area for successive reaches. The results are firmly believed to be reliable, and not just the product of a series of fortunate agreements.

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The statement (under the heading, "Model of Sea-Level Canal: Model Scale") in the paper that the resistance coefficient for the model "* * * was found to correspond to the selected Manning coefficient of 0.024 * * *" should have been better explained. Actually, the roughness coefficient for the concrete surface of the model was 0.011 when computed for the measured model velocity (Fig. 33(*a*)) of 0.37 knot, in the model channel 0.6 ft deep and about 6 ft wide. When the same observed slope and the prototype values of 3.7 knots, 60-ft depth, etc., were used in the Manning formula, the roughness value for the corresponding full-sized channel worked out as 0.024. Different sets of model observations, for the different test channels and for steady flow in both directions, gave model roughness coefficients varying from 0.011 to 0.012, corresponding to prototype coefficients from 0.024 to 0.026.

The question was raised as to the reliability of water-level measurement in the model by recording gages and hook gages, in contrast to the measurements obtainable by point gages. Actually, point gages mounted directly over the model channel were used in a number of tests as a check on the indications of the other gages. It was found that hook gages, on which long series of successive readings could be made more quickly and easily, also gave more consistent and accurate results. By the use of small stilling cans and connecting pipes of ample size and the shortest possible length, water levels in the hook-gage wells were made to follow those in the main channel without perceptible deviation. To insure reliable water-level data, the model was left filled with water after each day's operation, and all gage zeros were checked each morning from a still pool. This method, although it was never popular with the individual whose turn it was to reach the model at 4:00 a.m. in order to complete the series of zero readings before daybreak, was much quicker as well as more precise than the alternate method of spirit leveling. Machined reference marks on the metal posts supporting each hook gage and recorder, and an auxiliary hook gage in the stilling well of each recorder, made it easy to to keep account of the small variations in zero that did occur, and to make daily corrections when necessary.

The automatic recording gages were convenient for making adjustments and for trial runs; but, even after the original instruments were extensively modified and improved, they were less precise and dependable than the hook gages; so the latter were used for all record runs. Electric water-level indicators of several types, including the type developed for the Cape Cod model,[80] were considered for the Panama model. However, the comparatively gradual changes in elevation, the smooth water levels in this model, and the problems involved in an outdoor installation for widely separated measurement points, as well as the probable delays in design and manufacture of such equipment, soon led to a decision against the electric gages.

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[143] "Surges in Panama Canal Reproduced in Model," by F. W. Edwards and Edward Soucek, *Transactions*, ASCE, Vol. 110, 1945, p. 345.

Mr. Horton refers to the velocity estimate of 2.6 knots based on the Eytelwein formula. This estimate was made in 1924 by the United States Coast and Geodetic Survey at the request of the Panama Canal, and was included in the basic paper more as a point of interest than as a carefully considered prediction. The writers do not know the basis for the statement of the Coast and Geodetic Survey that the 2.6-knot estimate agreed with the observed values at Cape Cod. It should be remembered that the Cape Cod Canal channel was then a much shallower and narrower waterway, and the body of information developed during the investigations for the enlarged Cape Cod, Chesapeake and Delaware, and Panama canals did not then exist.

No computation of Cape Cod Canal velocities was made during the Panama investigation as a check on the validity of the computation methods. The writers agree fully with Mr. Horton, however, in the belief that the computation methods used for the Panama sea-level canal are equally applicable at Cape Cod.

Since the paper was prepared, several letters on the subject of tidal velocities were exchanged between the Panama Canal and Johan van Veen, chief engineer of the Rijkswaterstaat of the Netherlands, whose work has been referred to in the discussions.[105] Using methods developed for use in the many tidal canals in the Rhine lowlands, Mr. van Veen made an independent computation of velocities for a Panama sea-level canal.[144] His predicted velocities agreed closely throughout the tidal cycle with those computed by the Pillsbury method, and provided further reassurance of the reliability of the existing computation methods.

The description by Dean Allan of the steady-flow method for computing the limiting values for tidal velocities is a useful addition to the original paper. The general agreement between the steady-flow results and the Pillsbury results was very reassuring in the early stages of the Panama investigations, before other supporting evidence had been developed.

For the information of those who wish to make detailed comparisons of the computed steady-flow velocities (Fig. 113) with the velocities obtained at Panama (Figs. 35 and 36), it should be stated that the canal channels used in the two instances were not quite the same. Dean Allan used an effective channel length of 220,000 ft with uniform side slopes of 1 on 1, whereas the examples in the basic paper were for the finally recommended length of about 200,000 ft with side slopes varying from 1 on 2.5 to 3 on 2, as indicated in Fig. 30.

The steady-flow method is regarded by the writers as a desirable check on the maximum velocities estimated by other methods, but not as a satisfactory substitute for a more complete computation method. It was found, after computers became familiar with both procedures, that a Pillsbury computation did not take much more time than a steady-flow computation. The expenditure of "many thousand man-hours of computation," as stated by Mr. van Veen and cited by Dean Allan, was never needed at Panama, although such an amount of time might well be required for the highly complicated networks of canals with which Mr. van Veen was concerned in the Netherlands. As nearly as can now be recalled, it is believed that little more than 1,000 man-hours of computation time were expended for all the sets of velocity computations made by the Pillsbury method during the 2-year period of the investigation.

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[144] "Estimates of Tidal Currents," by Johan van Veen, *Isthmian Canal Studies Memorandum No. 299*, Diablo Heights, Canal Zone, February 18, 1948.

Mr. Massey's suggestion of the possibility of reducing average velocity by transverse submerged dikes is of interest. Tests made in 1933 at the United States Waterways Experiment Station[145] in Vicksburg, Miss., indicated that this idea would be helpful for the situation at the outlet of Lake Huron, where it was desired to raise the lake levels and thus produce deeper water in the harbors. The conditions for those tests were different from those that would be found in a sea-level canal at Panama, however, and the results of the earlier tests apparently should not be applied directly to the Panama situation.

Since the proposal for transverse sills would interfere with the passage of larger ships than those now in use, and particularly with emergency transits of crippled ships, it would probably not be acceptable from a navigation standpoint, but the effect of such sills on velocities could readily be determined by tests on the sea-level model. Restricted channel tests on the performance of ship models operating over such sills have already been recommended as a feature of a schedule of further investigations to be undertaken if a sea-level canal project is approved.

Professor McNown's discussion, showing the close correlation between the observed sinkage values for the ship models and the theoretical change of level as computed by the Bernoulli relationship, duplicates an analysis used for checking results during tests at the Taylor Model Basin. An explanation of this analysis was one of the items omitted from the Lee-Bowers paper because of space limitations, but it has been explained in the later Taylor Model Basin report[146] on the restricted channel tests. W. van Lammeren,[147] among others, has also worked on this subject and developed equations similar to Professor McNown's Eqs. 7 and 8. The comparisons between theoretical and observed sinkage on ship models as plotted in the Taylor Model Basin report are virtually identical with Fig. 117.

In 1947 sinkage was measured on four full-sized ships transiting the Panama Canal, for a comparison between full-scale observations and the theoretical values indicated by the Bernoulli relationship. Two measuring stations were used: One in Gaillard Cut where surface width was 370 ft, water depth was 46 ft, and cross-sectional area was about 15,000 sq ft; and one in the upper end of Gatun Lake where surface width was 1,100 ft, depth in the navigation channel was 46 ft, and cross-sectional area was about 35,000 sq ft. Ship speeds were from 6 knots to 7 knots past the first measuring station, and from 9 knots to 10 knots past the second. Measured sinkage for individual transits varied from 0.37 ft to 1.9 ft, whereas computed theoretical sinkage varied from 0.37 ft to 1.3 ft. In general, the computed sinkages were good approximations of the observed sinkages. The practical difficulties of measuring, accurately, the sinkage of a full-sized ship in normal operation, and the irregularities in the canal channel led to a greater spread between observed and computed values than was the case for the laboratory tests where all factors could be kept under better control, and any questionable observations could be repeated, but the validity of the analytical method was well demonstrated.

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[145] "Experiments to Determine the Backwater Effects of Submerged Sills in the St. Clair River," U. S. Waterways Experiment Station, Vicksburg, Miss., April, 1934.

[146] "The Performance of Model Ships in Restricted Channels in Relation to the Design of a Ship Canal," by R. S. Garthune, B. Rosenberg, D. Cafiero, and C. R. Olson, *David W. Taylor Model Basin Report No. 601*, Washington, D. C., August, 1948.

[147] "Weerstand en Voortstuwing van Shepen" (Resistance and Propulsion of Ships), by W. van Lammeren, H. Stam, Holland, 1942.

C. A. Lee[148] and C. E. Bowers,[149] Juniors, ASCE.—The discussions of this paper are greatly appreciated and the writers sincerely hope that their paper, clarified and enlarged on by these discussions, will form a base from which a clearer analysis of these troublesome problems may be presented.

Professor Davidson's objections, perhaps, need further clarification. Certainly there was never any intent on the part of the writers other than to show their appreciation for the counsel of the Panama Canal pilots. It is well understood that the experience of the pilots afforded great assistance to the personnel engaged in maneuvering tests and also aided in the decision to use approximately 5° of rudder angle as a basis for estimating the proper width and depth of the channel. It should also be emphasized that the selected rudder angle of 5° is a practical one and certainly does not account for extreme conditions of operation when the vessel is a considerable distance off center in the channel.

It is not clear to the writers how Professor Davidson proposes to use his analysis of moments, forces, and time to compute the most efficient width and depth of channel for a given ship speed. A minimum time factor would have to be established which would include time of reaction of the pilot, the rate of rudder movement, the rate of swing, and the speed of the ship. This operation would be very complex when the ship is entering or leaving a channel bend. A bend or a curve in a ship channel is considered one of the danger points due to the fact that the pilot must permit the ship to yaw in the channel and thus develop the aforementioned forces and moments. If the ship, on leaving a channel bend, is off center in the channel (which, in practice, is quite often the case), the turning of the ship must be retarded, the yaw corrected, and the vessel restored to its proper position in the channel. The time that the pilot has at his disposal is a function of the extent to which the ship is off center in the channel. It is believed by the writers that, although a time study might tend to clarify conditions somewhat, the variations imposed by a channel bend are so many and so complicated that no useful information could be obtained.

Should a time study be used for a straight channel design it would still have to be based on selected rudder angles. If a range of rudder angles and off-center positions were used to compute the time factors for several channel widths and depths and ship speeds, a question still remains as to how much elapsed time, or developed force, would be considered safe. It is believed that considerable disagreement would result if such a case were presented to a group of pilots. In discussing such matters with the pilots it was found that the rate of application of the rudder changed with the pilot, and thus the rate of swing of the ship would vary. Some pilots appeared to have a tendency to apply the rudder very rapidly whereas others applied the rudder gradually. As a matter of fact the speed of the rudder control mechanism differs from ship to ship. In terms of rudder angle this is a real value that immediately registers with the pilots and it is a term they all understand.

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From Professor Davidson's discussion of forces and moments acting on the ship in an off-center position it is believed that the writers were not too clear in their description of Figs. 47 and 48. Professor Davidson states:

> "A rudder angle of 3.3° is sufficient to offset the moment, the water force on the rudder acting away from the bank. Since the rudder is about 350 ft aft of the center of gravity, the rudder force must then be about 400/350, or 1.14 tons. Fig. 47 shows, however, that in free running a rudder angle of 5° is in fact needed, corresponding presumably to a rudder force of 5/3.3 times 1.14, or 1.7 tons. The forces and moments then are not balanced, and so the ship must assume a small yaw angle."

The ship as represented by the data plotted in Fig. 47 is in equilibrium, and the ship as represented by the data plotted in Fig. 48 is not in equilibrium. Fig. 47 represents a condition for which all the forces acting on the ship are in balance, and the center of gravity of the ship will travel parallel to, and at a fixed distance from, the channel bank. Fig. 48 represents a condition under which only the moment tending to the yaw the ship is balanced by the rudder and the lateral force tending to move the ship toward the near bank is not balanced. The center of gravity of the ship would have a motion toward the near bank and the angle of yaw of the ship would be zero. To overcome the lateral force the ship must be yawed away from the near bank and a greater rudder angle applied. The forces acting on the ship are abnormal, arising from the difference in elevation of the water surface between the ship and the near bank and the ship and the far bank. The data for both Figs. 47 and 48 were obtained in the circulating water channel of the David Taylor Model Basin at Carderock, where the ship model was held stationary and the water flowed past. A three-component balance was used to obtain the balance point of equilibrium, and to measure the lateral force. The tests were conducted in an effort to obtain some critical point in the force pattern and to understand better the various forces acting on the ship. A comparison between data obtained in the towing tank and those obtained in the water channel is given in Fig. 45.

With regard to the discussion by Professor McNown, proof of his treatment of the sinkage problem is given in the official Taylor Model Basin report on these experiments. Certainly this analysis is the proper one for this part of the problem. It was hoped at the beginning of the experiments that a similar solution might be developed for estimating the conditions of force when the ship is off center, which thus would shorten the test time. Although such a solution does not appear impossible, the experimental technique gave more immediate and visible proof of the ship motions.

J. E. Reeves,[150] M. ASCE, and E. H. Bourquard,[151] Assoc. M. ASCE.—The scope of the paper was limited to the design of channel and the selection of channel dimensions. Although it is believed that the ship model studies and the investigation and analysis of the existing waterways were entirely adequate for the purposes of selecting adequate channel dimensions, it is realized that the approach is new and that many of the factors involved are deserving of much more extensive investigation and study. Consequently, the limited discussion that this subject attracted was a disappointment.

From the dimensions of tunnel and channel sections proposed by Professor Hobbs, it would appear that he considered the proposed channel overdesigned. The evidence compiled in the studies was such as to prove beyond any doubt that the sections as proposed by Professor Hobbs would be woefully inadequate, both from a navigation standpoint and a security standpoint.

Using the method presented in the Symposium, a determination of channel dimensions adequate for single-direction traffic was made. With an existing ore ship (78-ft beam) as the design ship, the required bottom width is 295 ft at 60-ft depth with side slopes of 1 on 1. Although the ore ship was also used as a design ship in the report for determining the channel width, based on the passing of a second ship, it is not the largest ship that must be able to transit a new canal safely. The 600-ft channel, recommended in the Governor's report, is more than adequate for single-direction traffic of the largest existing naval or commercial vessel, and probably for the largest conceivable ship (vessel requiring lock width of 200 ft). A single-direction channel would have to have dimensions adequate for the safe transit of this large vessel. Based on the largest existing naval vessel, making due allowances for the increased controllability of the navy ship, the required bottom width is 350 ft—just twice the width proposed by Professor Hobbs.

This proposal of two independent channels has other definite disadvantages. It would not be as safe a waterway as the channel proposed in the Symposium, since in that channel each vessel has full use of the entire 600-ft width throughout practically its entire transit. Passings of other vessels would represent a relatively short interval of the transiting time. Only 4% of Panama Canal accidents have been collisions in the relatively narrow channel at Gaillard Cut (minimum width 300 ft); on the other hand, many of the bank-striking accidents could be attributed to loss of control of one vessel following the passing of a second ship. In a channel 600 ft wide, this danger would be greatly reduced because of the increased width of canal available for recovery following the passing maneuver.

The possibility of closure of one or both channels by a ship grounding across the channel would be much greater in narrow channels. Records show that 73.5% of the ships transiting the Panama Canal for the period from 1921 to 1940 were longer than 300 ft and that only 0.33% were shorter than 600 ft. The narrow channels, particularly in the tunnel, would be a serious handicap to salvage operations.

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The economy of two channels, as compared to the single channel recommended in the report, is very questionable. At relatively shallow depths of cut, the quantities in two channels would exceed the quantity in the single channel depending on the bottom widths and side slopes required. The channel section in the tunnel as indicated in Fig. 106 has an area of approximately 9,600 sq ft. The larger existing naval vessels have submerged areas of as much as 4,100 sq ft. The ratio of channel section to ship area for these larger vessels would be approximately 2.3 to 1. If the vessel were to transit at a speed above a crawl, the "squat" of the ship would be extremely large, and the power to tow and hold the vessel in position would be tremendous. The tunnel section considered in the report investigations had a cross section of 18,000 sq ft, which is believed to be a minimum requirement. The proposal to have passing stations in the tunnel offers tantalizing problems in the handling of ships during the maneuver, particularly when both ships would have to be under the control of towing locomotives from each bank. The tunnel would be an ideal "target" for sabotage. The damage that would be created in the tunnel by the explosion of a munitions ship carrying 10,000 tons or so highly explosive material or the equivalent in atomic explosives can well be imagined. The salvage problems in this confined space would be magnified manyfold over those for a similar explosion in an open channel. An atomic explosion in the tunnel might well make the tunnel useless indefinitely.

Mr. Cole can be assured that careful and thorough consideration was given to all conceivable methods of improving the existing lock canal even though it was obvious, from the security studies, that such methods would not meet the requirements of the law authorizing the report. Space limitations, of course, prohibited the inclusion of any discussion of these studies in the Symposium. From the frequent reference to the paper[119] by Miles P. DuVal, it is concluded that several of the discussers, including Mr. Cole, consider the Pacific Terminal Lake Plan the ideal solution to the canal problem. The advantages of this plan as compared to those of the existing canal are presented in that paper. Readers can rest assured that all merits of the Terminal Lake Plan, with its numerous modifications, were thoroughly considered prior to completion of the report. In fact, probably more so than was indicated by the wording of Public Law No. 280. The lock canal developed by the investigations as most nearly meeting the requirements of the act incorporated the Terminal Lake Plan. It was ruled out as not meeting the requirements of security. Several of the discussers state or imply that the admittedly large expenditures of providing a canal, either sea level or lock type, to assure dependability of operating during times of national emergency, are not justified. This subject is believed adequately covered in the Symposium papers and in the discussions. However, it is considered that a few comments on the economics of modifying the present canal to the Terminal Lake Plan, without benefit of security, are in order.

The advantages of the Terminal Lake Plan can be condensed to the following:

(*a*) Increased capacity;
(*b*) Increased operational efficiency (reduced transit time);
(*c*) Increased ease of operation;
(*d*) Increased safety of transit; and
(*e*) Decreased operational cost.

It is difficult to establish a true value for the first three of these advantages. However, by certain modifications of the existing canal, equivalent increases in capacity and operational efficiency can be obtained. These modifications are estimated to cost $130,000,000, approximately $32,000,000 of which is chargeable to Pedro Miguel Locks and could be saved and used toward the modification of Miraflores Locks to provide a Terminal Lake Plan.

It is impossible to place a monetary value on the increased ease of operation. This advantage would be reflected only in reduced strain on pilots and simplification of dispatching problems. The modifications of the existing locks, as mentioned, would also increase the ease of operation.

Cost of past accidents is a direct measure of the relative safety of various canals. The cost of lock accidents in the existing canal has averaged approximately $1.86 per transit. The maximum capacity of the modified existing canal, or the minimum conversion to a Terminal Lake Plan, would be about 15,700 transits per yr at $1.86 per transit, or an annual cost less than $30,000 per yr. Detailed accident records show that 33% of lock accidents occur at Pedro Miguel. The value of the increased safety of transit that might be credited to the Terminal Lake Plan would be less than $250,000 (33% of $30,000 per yr capitalized at 4%), which is insignificant when considering the cost of modification of the present canal to a Terminal Lake Plan.

From a purely economic consideration, the reduction in cost of operation and maintenance expense is the best argument in favor of the Terminal Lake Plan. The capitalized value of this saving, when combined with the aforementioned savings, is a relatively small part of the cost of providing the minimum Terminal Lake Plan, even without provisions for passing larger vessels. In other words, the construction of a Pacific Terminal Lake Plan is not economically justified at this time, or in the immediate future.

A word of caution is in order for possible users of the channel design methods presented in this paper. In working with the limited data available, many attempts were made to extrapolate and interpolate both model and prototype test data. The authors found that serious inconsistencies resulted in many cases, particularly when changes in ship sizes were involved.

The designer should be familiar with the ships likely to utilize the channel that he is designing. After having traveled on many ships in the Panama Canal, the writers are almost ready to accept the pilots' belief that each ship is an individual and acts accordingly. Frequent discussions of new ideas with licensed pilots will help keep a designer from "going off on the deep end."

As many pilots as possible should be consulted on an idea as they, themselves, are not always in agreement.

The writers consider the design methods presented in the paper as steps in the right direction and not as final design procedures to be accepted without question. Further study and testing will undoubtedly develop desirable modifications in the methods.

WILSON V. BINGER,[152] ASSOC. M. ASCE, AND THOMAS F. THOMPSON,[153] AFFILIATE, ASCE.—The recapitulation, by General Kramer, of the conservative principles used in the treatment of the Cucaracha slope problem is a welcome addition to the Symposium. The confidence in a successful job, expressed by General Kramer, should reassure many who may have noted the doubts which some have tried to cast on the conclusions of this paper.

Mr. Hill is also satisfied that the new and deeper excavation required by a sea-level canal at Panama can be undertaken with assurance despite the great difficulties encountered in the original construction. He properly emphasizes the advances made in the study of the causes and prevention of slides since the tremendous Culebra and Cucaracha failures. These great slides were, indeed, among the most important factors giving impetus to the development of soil mechanics in the United States, and demonstrated the need of a thorough understanding of geologic conditions at any location considered for a large engineering project.

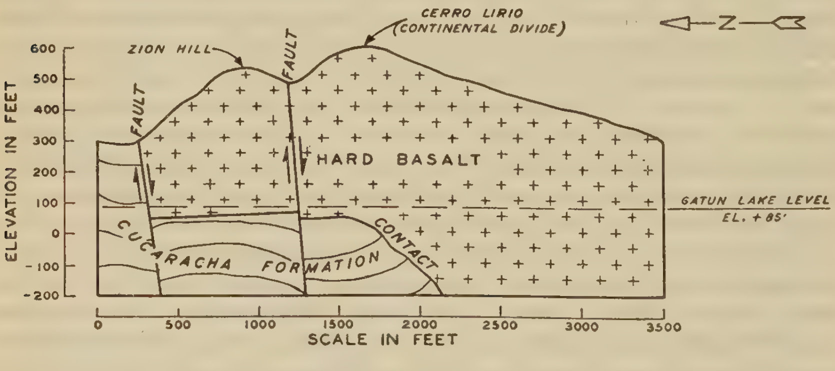

FIG. 119.—IDEALIZED SECTION THROUGH CERRO LIRIO AND ZION HILL

Mr. Justin correctly describes and illustrates the structural relationships of certain of the hills of the Canal Zone in which strong igneous rocks occur in "mushroom-like" structures (laccoliths) with heavy overhanging masses resting on weak sedimentary formations. This feature locally has played an important rôle in the development of Panama Canal slides. However, Zion Hill and the northern half of Lirio Hill at the Continental Divide are of slightly different geologic constitution. Here the igneous rock occurs as a relatively flat offshoot from a deep-seated igneous dike to the south that was later cut by a number of minor faults, thereby dividing it into huge blocks that rest on Cucaracha shale (Fig. 119). This condition is even more critical than that described by Mr. Justin in that there is no laccolith neck ("mushroom stem") to act as a pedestal and partly support the weight of the basalt cap. Cross faulting here has eliminated the possibility of a transference of some of the load to the deep-seated igneous base by cantilever action.

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Mr. Fucik believes it

> "* * * entirely possible that * * * [slopes for the Cucaracha formation] set by the use of the curve shown in Fig. 64, are too conservative, and that a material steepening of the slopes could be effected without any great risk of incurring catastrophic failures of the banks."

He suggests a curve (Fig. 110) based on the same strength values for the Cucaracha, but using a factor of safety of 1.0 instead of one of 1.3. He is not critical of the use of Fig. 64 to arrive at estimating quantities for the excavation; but he does feel that, if the sea-level canal becomes an actuality, careful and thorough study should be given to the possibility of using steeper slopes in the Cucaracha formation than were presented in the writers' paper.

The possibility of saving 65,000,000 cu yd of excavation is an attractive one, of course. The taking of some calculated risk might thus appear to be desirable; but the degree of risk that is justified in any part of a project is not a matter for final decision by engineering specialists. In a problem such as this one, geologists and soils engineers must determine to the best of their ability the distribution and the structural relationships of the rock formations and the strength characteristics of the various materials, so that curves such as those in Fig. 110 can be developed. On the basis of these investigations and with an awareness of the unknowns to be covered by the factor of safety, those responsible for the project must decide, as a matter of important policy, the appropriate factor of safety. This procedure was actually followed during the Isthmian Canal Studies (1947).

After Congressional authorization of the sea-level canal project, a number of years would be available for further study on the important problem of excavation slopes in the Cucaracha formation. This time should be used for intensive study and research to check the curve of Fig. 64 or to develop a new curve. This work should be done, not only to determine whether the slopes set by the writers are too conservative, but also to be as sure as possible that these slopes were sufficiently conservative. The writers believe that the greatest hope for reducing the excavation quantities lies in the detailed geologic exploration of the critical areas, so that slopes can be modified to take advantage of stronger phases of some formations.

Mr. Sheldon reviews briefly but interestingly the history of the difficulties experienced in building railroad embankments on Atlantic muck. He describes the rugged nature of the rock surface buried under muck at one location with which he is familiar. He then advances the theory that previous failures of embankments on muck have been caused by the material sliding on the steep rock slopes, rather than by failure in the muck.

Detailed studies made of one of the railroad embankments (that just south of Gatun crossing the area known as Stilsons Pond) tend to disprove Mr.

Sheldon's theory. Although a sloping (1 on 9) rock surface underlies the muck on which the fill was built, the history of the construction of the embankment reveals that one of the several slides of this fill was in the uphill direction and that a berm or counterweighting fill had to be built on the uphill side.[154] Furthermore, nothing in the writers' experience confirms the conclusion that the shear strength of the muck-rock contact is lower than the shear strength of the muck itself. The evidence is to the contrary. If the depth of the potential shear surface in an embankment foundation is limited by a rock surface, the stability of the embankment is normally greater than would be the case if the shear surface were free to go as deep as it wished, assuming a constant shear strength for the foundation material. When it is so limited, the shearing still takes place in the muck itself, although possibly very close to the contact.

In any case it may be reassuring to point out that the conditions in the Quebrancha-Monte Lirio-Gatun River area where the Monte Lirio Dam would be built are unlike those at Mount Hope where the Marine Railway was built. At Mount Hope the buried rock surface beneath the muck is steep and highly cut up by canyons running toward the near-by channel excavation, whereas the buried topography at the Monte Lirio site is gently rolling. Lines of borings across the Monte Lirio axis and at the Gatun Dam axis have established the bottom gradient of the old buried channel to be between 5 ft and 6 ft per mile. For all practical purposes, this is flat.

Mr. Cedergren refers to the inconsistencies present in formations in the hilly regions of the Pacific Northwest and mentions intrusions of weak materials. Such conditions do prevail in the Canal Zone along the selected route for a sea-level canal. In fact, although slope curves must necessarily be prepared for each formation to be encountered along the route, it will be necessary to treat each hill as a separate slope design problem.

The writers are grateful to Dean Marston for his kind mention of their paper. His description of the Panama Canal as a "* * * great research laboratory to solve the important problems of safe slopes for a sea-level Panama Canal" is a reminder that in no other branch of civil engineering can engineers learn more from failures than in the field of soil mechanics.

There are those who would ignore any advances made in soil mechanics and engineering geology since 1914. Mr. Kirkpatrick states that deepening the canal cuts by 100 ft is "daringly unprecedented even to the point of recklessness." Had such counsel prevailed in 1905–1906, the present canal would never have been built, for it was certainly a far more unprecedented project than any deepening and widening can now be. It will be a dark day for all human endeavor when men cease to attempt the "unprecedented."

In answer to Mr. Kirkpatrick's unfounded assertion that "Nothing is known, except geological conjectures" of the probable bank and bottom stabilities of the 20 miles of proposed canal now hidden by Gatun Lake, the writers wish to state that investigations were made during the study in sufficient detail for a satisfactory preliminary evaluation of the problems that would be involved in construction. The investigation of the Gatun Lake area included ninety-five holes drilled from the lake surface to explore the muck and to recover samples for testing. It was the opinion of the Board of Consulting Engineers that the designs proposed for the slopes through this area were on the conservative side.

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[154] "Annual Report of the Isthmian Canal Commission for Fiscal Year Ended June 30, 1909," Rept. of the Eng. Dept. of the Panama Railroad, Appendix E, U. S. Govt. Printing Office, Washington, D. C., 1909.

In commenting on the proposed dams in the Gatun Lake area, Mr. Kirkpatrick seems unduly alarmed. Gatun Dam was built on Atlantic muck, extending at the deepest point to El. −204.9. The deepest muck located elsewhere to date is at Monte Lirio (El. −163.6).

The possibility raised by Professor Casagrande that "* * * long-time effects may reduce the shear strength of the Cucaracha as a result of very slow swelling caused by reduction in load" cannot be lightly dismissed. The writers would not wish to close the door to the results of future investigations, but they do wish to record something in defense of the curve for safe excavation slopes in the Cucaracha. The evidence that the Cucaracha may lose considerable shear strength with time is too important to overlook, but it is not yet conclusive. This evidence points to a hypothesis such as that developed by Professor Casagrande, and such a hypothesis must be proved or disproved as far as possible before the slopes are actually cut. The writers are in full agreement, therefore, with the recommendations for long-range field and laboratory studies, telltale devices to detect movements in Cucaracha slopes, and the like. They are not convinced, however, that the drop in effective shear strength shown by Fig. 118 is necessarily caused principally by a long-time swelling or weakening effect.

The explanation used and accepted by the writers up to this time has been that successive sliding failures reduce the effective shear strength of the Cucaracha. In opposing this theory, Professor Casagrande states:

> "* * * However, the writer believes that in 1912 at least one half of the mass bounded by the underlying Culebra and on the east and the west by faults, as shown in Fig. 118, was not yet disturbed by the slides and that intermittently additional parts of the mass became involved in the movement. If the assumption is made that the strength characteristics of the undisturbed material have not considerably changed with time and that they are equal only to those governing the slopes in January, 1912, then it can be shown that the mass should have come to rest with much steeper slopes than those existing today."

Although it is true that in 1912 about one half of the material had not yet been disturbed by slides, between that year and 1915 almost all the remaining material did become involved in slides, but the effective shear strengths were reduced, on the east slope, only to 79%, and on the west slope, only to 62%. Since 1915, both slides have been essentially in continuous, although at times almost imperceptible, movement, and during this period the effective shear strength dropped to the very low values cited by Professor Casagrande.

It may well be, of course, that the great reduction of shear strength experienced between 1912 and the present is due to both: (1) The disturbance of slides resulting from oversteep slopes; and (2) the long-time weakening effect deduced by Professor Casagrande.

(Readers who may wish to study the Culebra slide further will be interested in a separate paper presented during 1948 in which the cross section of Fig. 118 and two other sections have been analyzed.[155])

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[155] "Analytical Studies of Panama Canal Slides," by Wilson V. Binger, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Vol. II, 1948, pp. 54–60.

Professor Casagrande suggests the possibility that the sequence of various layers of the Cucaracha formation in the model slope (on which such reliance has necessarily been placed in the development of new criteria for the design of excavation slopes) is not identical to that in the slopes which failed in the Culebra slides. Exact similarity cannot be proved by available information; however, old geologic sections and recent borings show that both the model slope and the horizon that failed in the East and West Culebra slides are of similar stratigraphic position as indicated by the position of the ash flow marker bed. Both appear to have been equally weakened by minor faults. Since the two locations are close by, it is reasonable to assume that they were originally about equal in strength.

Little information is available on the 100-ft cut that failed in 1907. The late Col. George W. Goethals, M. ASCE, described this slide as follows:[156]

> "In October 1907, a crack developed 50 feet or more from the face of the bank on the west side of the Canal near the old railroad station at Culebra. Subsequent to the cracking, the ground to the west of it showed perceptible settlement, though the elevation of the portion between the crack and the prism remained unchanged; this settlement was accompanied by a bulging up of the bottom of the Canal sufficient to raise the shovel. At the time this occurred the shovel was at elevation 175, and the subsiding portion of the upper bank was at elevation 275. Ultimately the portion between the crack and the prism settled down, sliding into the prism. Subsequently new cracks developed farther back, followed by similar settlements and movements into the cut, involving large masses of material; the same condition developed on the east side."

The writers agree definitely with Professor Casagrande's conclusion that:

> "* * * the strength characteristics of the Cucaracha clay-shale are not similar to those that can be expressed by the Coulomb equation with constant c-values and ϕ-values."

In this case, however, the only alternative to using a c-value and a ϕ-value would have been to use a constant shear strength (that is, a constant value of c and a ϕ equal to zero) determined from the model slope or from the Culebra slides. Such a course would have meant the adoption of impossibly flat slopes for the higher cuts. On the other hand, a constant shear strength would not be a better description of the strength characteristics of the Cucaracha than would the c-value and ϕ-value.

Professor Casagrande's critical review of the methods employed in arriving at the Cucaracha slope design criteria is a valuable contribution to this paper. Although he recognizes that the proposed design curve is the "* * * best that could be accomplished with the tools and the knowledge [now] available," he makes recommendations for further investigations at the time of project authorization which the writers fully endorse.

Acknowledgments.—The soils work of the Third Locks program was first under the direction of Evan W. Vaughan, M. ASCE, and later under that of E. Montford Fucik, Assoc. M. ASCE. Principal assistants in the soil mechanics section at that time were C. K. Smith, Assoc. M. ASCE; A. C. Flach, Jr.; Francis J. Woodsmall, Assoc. M. ASCE; and L. D. Collins. Geologic investigations were under the direction of H. G. Martin from June, 1938, until November, 1941, when he was succeeded by Mr. Thompson. D. F. MacDonald served as consulting geologist for the Third Locks investigations from June, 1938, until his death in May, 1942.

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[156] "The Panama Canal," by George W. Goethals, McGraw-Hill Book Co., Inc., New York, N. Y., 1916, pp. 372–373.

During the Isthmian Canal Studies (1947), important soil mechanics studies were conducted by E. J. Zegarra, Assoc. M. ASCE; H. G. Smith, Jr.; David C. Hall, Jun. ASCE; and R. L. Behrend, Jun. ASCE. C. F. Vincent was in charge of the Soil and Rock Testing Laboratory, assisted by R. L. McLees and George E. Lewis; and W. H. Bussey supervised the sampling of soil and rock. L. H. Henderson, T. G. Moran, J. R. Schultz, J. R. Townsend, S. M. Jones, and others contributed to the geologic studies.

A. Casagrande,[157] M. ASCE, and W. L. Shannon,[158] Assoc. M. ASCE.—Since the publication of the writers' paper, the investigations sponsored by the Panama Canal on the dynamic stress-deformation and strength characteristics of soils and soft rocks were completed and a detailed report was published.[159]

The discussions on the Symposium contained only a few comments relating to the subject of the writers' paper.

Mr. Hill comments on the various other engineering applications where transient loading is important. A more detailed discussion of this question, with particular attention to the stability of slopes subjected to earthquakes, was included by the senior writer in chapter 12 of the final report.[159] Some of the observations made in connection with the traffic tests on airfield pavements, conducted by the Corps of Engineers, are presented in Mr. Cedergren's discussion.

Professor Tschebotarioff points to the results of his tests which show that continuous vibrations cause much greater volume decrease of sand than does static loading. This finding corroborates similar results obtained by other investigators. Perhaps the first to describe this phenomenon was Karl Terzaghi, Hon. M. ASCE, who in one of his early publications termed it the "conservative structure" of sand.

Professor Tschebotarioff criticizes the fact that the transient tests were so conducted that a specimen was first exposed to a hydrostatic stress condition and then the axial load was increased to failure. This method of loading was used for the following two reasons: (1) It is believed that this type of transient test at one end, and the purely static test at the other end, limit the range in which the properties of the material will vary; and (2) tests as proposed by Professor Tschebotarioff complicate an already very difficult type of test. Not until certain fundamental characteristics are fully explored (still far from being accomplished) can one afford to investigate more complicated combinations of load application.

Professor Tschebotarioff cites the case of a bank of varved clay which failed during pile driving operations. As he suggests, it is probably the rock flour in this material that liquefied as the result of the vibrations from pile driving. The problem of liquefaction of a soil due to collapse of its structure was beyond the scope of the writers' paper; but some information on this point is contained in chapter 12 of the final report,[159] and an instructive example is described by Mr. Cedergren in his discussion.

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[158] Director of Div. Laboratory, U. S. Engr. Office, Portland, Ore.; formerly Research Associate in Soil Mechanics, Graduate School of Eng., Harvard Univ., Cambridge, Mass.

[159] "Research on Stress-Deformation and Strength Characteristics of Soils and Soft Rocks Under Transient Loading," *Soil Mechanics Series No. 31*, Graduate School of Eng., Harvard Univ., Cambridge, Mass., June, 1948.

J. J. ROSE,[160] ESQ., F. L. DYE,[161] M. ASCE, W. B. WATSON,[162] ESQ., AND L. T. CROOK,[163] M. ASCE.—Mr. McAlpine is correct in his statement that a considerable part of the material below El. 270 could be disposed of economically by truck, belt conveyer, or other land transportation. El. 270 was taken arbitrarily as time did not permit the detailed studies necessary to determine the most economical method for spoiling each yard of the millions of cubic yards. The economical elevation would be dependent on the transverse location of the material, the availability of dry spoil areas, the elevation of ground adjacent to the top of the slope, and the location of the wet spoil areas. Mr. McAlpine objects to dropping material directly into scows from the higher elevations. The writers agree that such a procedure would be both dangerous and infeasible and it was not proposed. The material at the higher elevations would be rehandled to reach the scow loading equipment at scow level.

Perhaps the statement that the average cost of disposal by truck haul would be approximately twice as much as by scow haul should have been qualified by the clause "based on the methods used in computing the cost in the studies." The truck haul costs were based on quotations on first cost of the equipment using equipment ownership schedules advocated by the Associated General Contractors of America to determine the plant rentals. The barge haul costs were based on quotations in which Panama Canal experience was used to determine the plant rentals. It is proposed to allow the contractors to determine the most economical method of hauling by making both land and water disposal areas available under each contract.

Mr. Massey's discussion is extremely interesting. To obtain additional production and cost data concerning the economical handling of rock excavation at the depths required by the bucket-ladder dredge, it is proposed to purchase only one dredge of this type at the beginning of operations and to experiment with the deep dredging rather than to start this phase on a large scale. Only a small percentage of the hard rock is beyond the reach of the dipper dredge and can easily be handled with a clamshell or dragline in the time allotted. The costs of the ladder dredge excavation used in the studies are ample to cover excavation of this hard rock by clamshell in the event its use might be considered advisable.

Mr. Massey states:

> "The bucket-ladder dredge purchased by the Panama Canal Authorities about 1912 in Scotland utterly failed to dig blasted rock in the Culebra Cut

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[162] Engr. in Chg. of Constr., McNary Dam, Corps of Engrs., U. S. Army, Umatilla, Ore.; formerly Chf., Dry Excavation Section, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

[163] Engr., Board of Engrs. for Rivers and Harbors, Dept. of the Army, Washington, D. C.; formerly Chf., Estimates Section, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

> and was replaced as soon as possible by dipper dredges, which were ideal for the work because of the absence of any space or size restriction on the pieces of rock to be handled."

As a matter of fact, the bucket-ladder dredge to which Mr. Massey refers (the *Corozal*) generally performed very creditably. Over a 3-year period from July 1, 1912, to June 30, 1915, material dredged by this machine consisted of 71% rocks and 29% earth, excavated and placed in dumps at an average cost of $0.514 per cu yd. No difficulty was experienced in dredging properly blasted rock. Large dipper dredges were constructed and assigned to work in Culebra (renamed Gaillard) Cut principally to handle (without blasting) large pieces of rock being brought into the cut as slide material. With reference to photographs showing these dipper dredges raising to the water surface pieces of rock too large to pass through the dipper, it should be borne in mind that such oversize rock is the exception rather than the rule. One of the writers has observed an 8-cu-ft bucket-ladder dredge operating under severe conditions in South America bringing boulders to the surface as large as any brought up by the 15-cu-yd dipper dredges at the Panama Canal. Where the dipper dredge could not free such a boulder without dropping or blasting it, the bucket-ladder dredge could raise its ladder clear of the water, reverse its bucket chain, and dump the boulder either on the bank ahead or on a barge for final disposal.

W. G. Comber, who was in charge of Panama Canal dredging at the time the *Corozal* was purchased, has stated:

> "In order to avoid the delay and expense of breaking or blasting, it was thought that a high powered modern dredge would cut the softer rocks in the section from Balboa Wharf to Miraflores. On this assumption, the *Corozal* was purchased, and proved amply able to handle this work with an output varying from 3,000 to 10,000 yards of rock per 24-hr day; the harder parts of the soft rocks were handled by cutting 6-inch layers, the softer as much as four feet at one passing of the dredge. The dredge was also very efficient in the rock of the harder shoals after blasting by the '*Toredo*,' but not so efficient in the Culebra Cut work, where large pieces of rock were encountered in places where there was not sufficient depth of water for the dredge to get directly over the rock."

It is true that buckets must be made with a high factor of safety but this practice has been followed for many years. As a result of improved designs, metals, and maintenance methods, present-day dredges very rarely have trouble with bucket-line breakage. In many cases, bucket-ladder dredges obtain 80% or more effective dredging time, year in and year out.

Both the clamshell and dragline types of dredges received full consideration during the course of the Isthmian Canal Studies. Design of a deep-digging dragline dredge, similar to that mentioned by Mr. Massey, was available to those engaged on the Isthmian Canal Studies some time prior to publication of the Symposium.

The advisability of excavating with the special dredges to an elevation satisfactory to pass traffic and to complete the excavation with regular dredges was considered and left for decision during actual construction. There would be no particular advantage in performing the common excavation in this manner.

Mr. Erickson states that the 40-in. hydraulic dredge seems to be too small and that consideration should be given to dredges having, say, 60-in. discharge lines. Actually, consideration was given to dredges having discharge lines up to 60 in. in diameter. However, on the basis of the quantity of common and soft rock excavation set up for hydraulic dredging, it can be estimated that one 40-in. deep-digging dredge would have adequate capacity to meet the 10-year construction schedule. A construction program requiring an increase in common and soft rock quantities in wet excavation would warrant further consideration of dredges larger than 40 in.

In discussing the Panama parallel sea-level canal, Mr. Jarvis refers to "* * * the unavoidable interference with normal canal traffic during the period of construction for the Panama sea-level conversion route * * *." The matter of possible interference was given careful consideration in the studies and the work was so planned that interference with traffic would be negligible. The work can be arranged so that, at the minimum, navigable widths not less than those now existing in Gaillard Cut (300 ft) would be available at all times.

Mr. Edwards questions the need for the somewhat careful cost estimates of canals at other locations and the detail of the sea-level canal estimate. In retrospect, the estimates for alternate canals might be more detailed than required, but, as they were developed concurrently with defense, traffic, geologic, structural, and other studies for all routes, it was impossible to determine in advance just what detail would be required. Forecasting probable future actions of Congress, which would determine when the data assembled would be used, was beyond the scope of these studies.

Mr. Kirkpatrick suggests lowering the channel in Gaillard Cut from El. 40 to El. 37⅓. The lock canal plans studied contemplated channels excavated to El. 37.

The possibilities of using hydraulic giants, breakage of rock by hydraulic methods, and elimination of the necessity of conveying all material to Gatun Lake were discussed by Mr. Dohn. Hydraulic giants are in use on the Panama Canal and have been used for a considerable time, flattening slopes in the areas subject to slides. The material excavated by these machines is washed into the canal for subsequent removal by dredges and it is very small in amount when compared with the project now proposed. The terrain in the vicinity of Gaillard Cut is such that there are no low areas available for disposal of large quantities of spoil by use of hydraulic giants. The only reasonable spoil disposal areas are in Gatun Lake where the water level is at the same elevation as in Gaillard Cut and where spoil disposal requires no lockages.

Breakage of rock by hydraulic methods is not practicable, for one reason, because of the existence of numerous fissures in the rock. Considerable difficulty has been experienced in drilling operations because of the loss of drill water through these fissures. Furthermore, any rock breakage that could be obtained by hydraulic methods would obviously occur along the seams or fissures, leaving large rock fragments that would require secondary blasting at a much higher cost than if the original rock were blasted.

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TRANSACTIONS

Paper No. 2379

VALUATION AND DEPRECIATION OF PUBLIC UTILITY PROPERTY

By Maurice R. Scharff,[1] M. ASCE

With Discussion by Messrs. J. Kappeyne, W. V. Burnell, Donald Gunn, Henry F. Lippitt, II, Albert P. Learned, Anson Marston, James P. McKean, and Maurice R. Scharff.

Synopsis

This paper reviews the status of the "fair value" and depreciation problems as they have been left by decisions of the United States Supreme Court in cases arising under the Natural Gas Act (315 US 575; 320 US 591; and 324 US 636), and concludes that the confusion in which they are enmeshed is deeper than ever. It is suggested that because of this confusion, and because of the inadequacy of currently accepted theories for the adjustments that will be required if the prospects of revolutionary developments in the field of power generation are realized, a general reconsideration of these problems by the engineering profession is appropriate.

A solution is suggested based on the conceptions that "fair value" and "service value" are identical and that reasonable rates are no more than those required to cover the cost of rendering the existing service if provided by facilities of the most modern and efficient kind available as a result of progress of the arts, and of capacity no greater than that required to meet the demand. It is proposed that the equal annual payment annuity, at a reasonable rate of return, on the cost of such facilities be included in such costs; and that the "fair value" or "service value" of the existing property is the present worth at a reasonable rate of return, for a reasonable future capitalization period, of the amount available from such reasonable rates after deducting the operating expenses, other than depreciation, of the existing facilities.

If this solution were adopted, it is further proposed that the difference between the cost of reproduction new of the existing property and its "fair value" or "service value," as referred to previously, be considered as a measure of accrued depreciation; and that the ratio of such accrued depreciation to the cost of reproduction new of the existing property be taken as a measure of the

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Note.—Published in June, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., New York, N. Y.

existing percentage of accrued depreciation. This percentage could reasonably be used as a measure of the "depreciation reserve requirement" under those classifications of accounts which define depreciation as "the net loss in service value not restored by current maintenance." It is also suggested that the proposed solution would permit the reasonable annual depreciation expense for the purpose of such classifications of accounts, as previously referred to, to be determined by taking the difference between the "depreciation reserve requirement," at the beginning and the end of each year, measured in the manner previously referred to, and adding the net debit to reserve during the year on account of retirements.

An example is given which illustrates the application of the suggested solution of the problem to data assumed for a hypothetical manufactured gas plant.

INTRODUCTION

This paper is not intended as a detailed review of the past discussions on this subject in Society publications,[2,3,4,5] or of the voluminous literature that has grown up in court and commission decisions and in textbooks and periodicals. Nevertheless, these references, together with others to which they will refer the reader desiring a more complete knowledge of the literature, reflect the confusions and contradictions in which the whole subject of the valuation and depreciation of public utility property had become enmeshed prior to the decisions of the United States Supreme Court in the Natural Gas Pipeline Company Case (315 US 575),[6] the Hope Natural Gas Case (320 US 591),[7] and the Panhandle Eastern Case (324 US 636).[8] In these cases the Supreme Court upheld orders of the United States Federal Power Commission (FPC), fixing just and reasonable rates under the provisions of the Natural Gas Act on a basis which is perhaps best stated in the Natural Gas Pipeline Company Case opinion[6] as follows:

> "The Constitution does not bind rate-making bodies to the service of any single formula or combination of formulas. Agencies to whom this legislative power has been delegated are free, within the ambit of their statutory authority, to make the pragmatic adjustments which may be called for by particular circumstances. Once a fair hearing has been given, proper findings made and other statutory requirements satisfied, the courts cannot intervene in the absence of a clear showing that the limits of due process have been overstepped. If the Commission's order, as applied to the facts before it and viewed in its entirety, produces no arbitrary result, our inquiry is at an end."

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[2] *Transactions*, ASCE, Vol. LXXXI, 1917, pp. 1311–1581.

[3] *Ibid.*, Vol. 104, 1939, pp. 1127–1138 and 1149–1209.

[4] *Ibid.*, Vol. 108, 1943, pp. 1235–1342.

[5] *Ibid.*, Vol. 111, 1946, pp. 1343–1400.

[6] *Public Utilities Reports*, New Series, Vol. 42, 1942, p. 129.

[7] *Ibid.*, Vol. 51, 1944, p. 193.

[8] *Ibid.*, Vol. 58, 1945, p. 100.

In the same case (in which, curiously enough, the FPC used a "cost of reproduction" rate base), Justices Hugo L. Black, William O. Douglas, and Frank Murphy wrote a concurring opinion in which they expressed the view that this was

> "* * * an appropriate occasion to lay the ghost of Smyth v. Ames (1898) 169 US 466, 42 L ed 819, 18 S Ct 418, which has haunted utility regulation since 1898. That is especially desirable lest the reference by the majority to 'constitutional requirements' and to 'limits of due process' be deemed to perpetuate the fallacious 'fair value' theory of rate making in the limited judicial review provided by the act. * * * As we read the opinion of the court, the Commission is now freed from the compulsion of admitting evidence on reproduction cost or of giving any weight to that element of 'fair value.' The Commission may now adopt, if it chooses, prudent investment as a rate base—the base long advocated by Mr. Justice [Louis D.] Brandeis. And for the reasons stated by Mr. Justice Brandeis in the Southwestern Bell Telephone Case there could be no constitutional objection if the Commission adhered to that formula and rejected all others."

In the Hope Natural Gas Company Case the Supreme Court upheld a rate order based upon a fair return on "actual legitimate cost" less "accrued depletion and depreciation" on an "economic service life basis," whereas the FPC had disregarded the testimony submitted in that case on reproduction cost less "observed depreciation" as of no probative value. In its opinion the Court stated:

> "When we sustained the constitutionality of the Natural Gas Act in the Natural Gas Pipeline Company Case, supra, we stated that the 'authority of Congress to regulate the prices of commodities in interstate commerce is at least as great under the Fifth Amendment as is that of the states under the Fourteenth to regulate the prices of commodities in intrastate commerce.' 315 US at p. 582, 42 PUR (NS) at p. 135. Rate making is indeed but one species of price fixing. Munn v. Illinois (1877) 94 US 113, 134, 24 L ed 77. The fixing of prices, like other applications of the police power, may reduce the value of the property which is being regulated. But the fact that the value is reduced does not mean that the regulation is invalid. Block v. Hirsh (1921) 256 US 135, 155–157, 65 L ed 865, 41 S Ct 458; Nebbia v. New York (1934) 291 US 502, 523–539, 78 L ed 940, 2 PUR (NS) 337, 54 S Ct 505, 89 ALR 1469 and cases cited. It does, however, indicate that 'fair value' is the end product of the process of rate making, not the starting point as the circuit court of appeals held. The heart of the matter is that rates cannot be made to depend upon 'fair value' when the value of the going enterprise depends on earnings under whatever rates may be anticipated.
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> "We held in Federal Power Commission v. Natural Gas Pipeline Co., supra, that the Commission was not bound to the use of any single formula or combination of formulae in determining rates. Its rate-making function, moreover, involves the making of 'pragmatic adjustments.' Id. at p. 586. And when the Commission's order is challenged in the courts, the question is whether that order 'viewed in its entirety' meets the requirements of the act. Id. at p. 586. Under the statutory standard of 'just and reasonable' it is the result reached, not the method employed, which is controlling. Cf. Los Angeles Gas & E. Corp. v. California R. Commission, 289 US 287, 304, 305, 314, 77 L ed 1180, PUR 1933 C 229, 53 S Ct 637; West Ohio Gas Co. v. Ohio Pub. Utilities Commission (1935) 294 US 63, 70, 79 L ed 761, 6 PUR

> (NS) 449, 55 S Ct 316; West v. Chesapeake & P. Teleph. Co. 295 US 662, 692, 693, 79 L ed 1640, 8 PUR (NS) 433, 55 S Ct 894 (dissenting opinion). It is not theory but the impact of the rate order which counts. If the total effect of the rate order cannot be said to be unjust and unreasonable, judicial inquiry under the act is at an end. The fact that the method employed to reach that result may contain infirmities is not then important."

In the Panhandle Eastern Case the court upheld a rate order of the FPC based on "actual legitimate cost" less straight-line depreciation, and on a refusal even to accept testimony on reproduction cost less observed depreciation. In this case, the Court stated in its opinion:

> "* * * The question on review is not the method of valuation which was used but the end result obtained since the issue is whether the rate fixed is 'just and reasonable' * * *."

These expressions of the United States Supreme Court have been hailed by the FPC and by other advocates of the prudent investment or original cost principle of rate making as destroying the fair value principle and giving blanket approval to rate orders designed to yield a fair return on original cost less straight-line age-life depreciation. Examples are the opinion of the Utah Supreme Court in the Utah Power and Light Company versus Public Service Commission Case,[9] where the Court stated as follows:

> "From the foregoing it becomes clear that the Supreme Court is now committed to the view that there is no constitutional requirement that utilities be permitted to earn a 'fair return' on the 'fair value' of the property devoted to the public use. The Hope Case, supra, stands squarely for the doctrine that it is the final impact of the rate order which is controlling in so far as Federal constitutlional imitations are concerned. So long as the rate set does not confiscate the property devoted to public service, the rate order will not be held to violate substantive constitutional principles. The legislature is free to determine its own economic policy in regard to the fixing of rates. Its power to set rates is, however, still circumscribed by two constitutional limitations: (1) substantive constitutional law requires that the rates finally set shall not be confiscatory; and (2) the requirements of procedural due process must still be followed."

Another example of this interpretation of the decisions of the Supreme Court is the opinion of the Louisiana Public Service Commission in its proceeding relative to the rates of the Louisiana Power and Light Company,[10] from which the following sentences are quoted:

> "* * * finally in 1944, the United States Supreme Court in the Hope Natural Gas Company Case laid the ghost of Smyth vs. Ames to rest and left regulatory commissions free to build the rate regulation process on 'Prudent Investment,' as far as the Federal Constitution is concerned. Commissions are left free to make pragmatic adjustments within the statutory (and judicial) ambit of their authority.
>
> "Although this Commission has in the past followed the 'Fair Value' theory in rate making proceedings, and although this process has been approved by the Supreme Court of this State, this approval was required by the former U. S. Supreme Court jurisprudence on Federal due process.

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[9] *Public Utilities Reports*, New Series, Vol. 56, 1944, p. 136.

[10] *Ibid.*, Vol. 65, 1946, pp. 18 and 22.

> Since there are no Louisiana decisions requiring use of the 'Fair Value' theory in public utility rate making to meet State requirements of due process, and since there are no state statutes directing use of the 'Fair Value' theory, it is the opinion of this Commission that the Supreme Court of Louisiana is free to allow the use of the 'Prudent Investment' theory in public utility rate making by this Commission. Therefore, being desirous of simplifying the rate making process by eliminating the necessity for and the use of the cumbersome, expensive, time wasting and inconclusive reproduction appraisals and studies used in the past, we herewith adopt the 'Prudent Investment' theory of rate making for public utilities and set out below our application of this theory."

These attempted extensions of the opinions of the United States Supreme Court have not gone without contradiction by members of the Court itself, and by other competent students of the valuation problem. Mr. Justice Robert H. Jackson (with whose dissent Mr. Justice Felix Frankfurter expressed general agreement) pointed out in his opinion in the Hope Case that all that the Court had done in the earlier Pipeline Company Case had been to release the Commission:

> "* * * from subservience to any single formula or combination of formulas provided its order, 'viewed in its entirety produces no arbitrary result' 315 US at p. 586."

—and he expressly disclaimed approval of the contrary view of the minority concurring opinion in the Pipeline Case, using the following language:

> "The minority opinion I understood to advocate the 'prudent investment' theory as a sufficient guide in a natural gas case. The view was expressed in the court below that since this opinion was not expressly controverted it must have been approved. I disclaim this imputed approval with some particularity, because I attach importance at the very beginning of Federal regulation of the natural gas industry to approaching it as the performance of economic functions, not as the performance of legalistic rituals."

Mr. Justice Jackson also stated in his dissenting opinion in the Hope Case:

> "I must admit that I possess no instinct by which to know the 'reasonable' from the 'unreasonable' in prices and must seek some conscious design for decision. The court sustains this order as reasonable, but what makes it so or what could possibly make it otherwise, I cannot learn."

Mr. Justice Stanley F. Reed, also, dissented in the Hope Case in an opinion, in which he, too, denied that the opinion of the Court approved the use of prudent investment alone in determining the rate base, adding:

> "This leaves the Commission free, as I understand it, to use any available evidence for its finding of fair value, including both prudent investment and the cost of installing at the present time an efficient system for furnishing the needed utility service."

In the margin he also stated,

> "'Reproduction cost' has been variously defined, but for rate-making purposes the most useful sense seems to be, the minimum amount necessary

> to create at the time of the inquiry a modern plant capable of rendering equivalent service. See I. Bonbright, Valuation of Property (1937) 152. Reproduction cost as the cost of building a replica of an obsolescent plant is not of real significance."

Mr. Justice Jackson again returned to his contentions in a concurring opinion in the Colorado Interstate Gas Company Case (325 US 581),[11] in which he stated:

> "I do not recede from the views therein stated that Hope provides no workable basis of judicial review, no key by which Commissions can anticipate what rule, if any, will control our review, and no guidance to counsel as to what issues they should try or how they should try them."

Robert L. Hale concludes,[12] apparently with regret, that the Natural Gas Pipeline Case left the Supreme Court still committed to what he considered the totally irreconcilable rate criteria of that famous case, "* * * the ghost of Smyth v. Ames still walks, and utility regulation continues to be enveloped in its fog."

The result has, therefore, been not clarification and clear decision, but confusion worse confounded. Consideration of this situation, after nearly 4 years of complete detachment from the subject while in military service, and review of the previous discussions in Society publications to which reference has been made (which seem to contain promise of possible real progress if they could be continued) have led the writer to propose the subject for further consideration and discussion by the engineering profession.

An additional reason why such reconsideration appears timely just now is the widespread scientific and engineering activity in the field of electrical power production. The gas turbine, the powdered fuel turbine, the application of atomic energy, either as a source of heat or as a direct source of electrical energy —any one of these may lead to a revolutionary development in this field within the next five to twenty-five years. It is, of course, not possible to predict at this stage of development that any one of them actually will reduce substantially the cost of power production. Neither is it possible to predict with assurance that no one of them will do so.

If such a development should occur, it is not to be expected that regulatory commissions will undertake to assure that existing steam and hydroelectric power plants will be permitted to earn a return even on their original cost less straight-line depreciation, without giving effect to the economies which such a development in the progress of the arts would make possible.

It is hoped that the suggested consideration and discussion by the engineering profession may make a useful contribution toward the relief of the present unhappy situation in this field, and may suggest the direction in which an answer can be sought that will appeal to judgment and conscience.

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[11] *Public Utilities Reports*, New Series, Vol. 58, 1945, p. 83.

[12] "Does the Ghost of Smyth v. Ames Still Walk?" by Robert L. Hale, *Harvard Law Review*, Vol. 55, 1942, p. 1116.

Discussion

As a basis for the suggested reconsideration by the engineering profession, it is proposed to go back to Smyth versus Ames (169 US 466) and, passing over the catalog of matters for consideration in determining fair value, which have been the source of so much discussion and confusion, to concentrate on two sentences from that often quoted opinion as follows:

> "What the Company is entitled to ask is a fair return upon the value of that which it employs for the public convenience. On the other hand, what the public is entitled to demand is that no more be exacted from it for the use of a public highway than the services rendered by it are reasonably worth."

Are these two criteria for the determination of reasonable public utility rates necessarily irreconcilable, as so many students of that opinion have asserted? Also, is "* * * the fog in which speculations based on Smyth v. Ames have enveloped the practical task of administering systems of utility regulation," as referred to by Mr. Justice Frankfurter (307 US 104, 123)[13] in Driscoll versus Edison Light and Power Company, and which was discussed in the introduction to this paper, a necessary result of the Court's decision; or has this confusion arisen in large measure from the failure of the engineering profession to provide an application of these criteria through a solution of the valuation and depreciation problems which reconcile them in accordance with an independent principle? Furthermore, is such an independent principle available for this purpose?

The attempt to apply these criteria by deriving reasonable rates from consideration of a fair return upon either reproduction cost or original cost of the existing property less observed or age-life depreciation, assumed to represent fair value is, as has so often been pointed out, reasoning in a circle. Neither do the methods of estimating depreciation, to which reference has been made, bear any known relation to the effect of obsolescence and changes in demand which are generally agreed to be the major causes of retirement of utility property.

It is suggested that the circle of reasoning, in attempting to base rates on value and value on rates, might be broken by recognizing that the fundamental purpose of public utility rate regulation is to give effect to the public policy of avoiding the waste that would result from leaving price fixing for public utility service to the ordinary forces of competition, and to subject such rates to control based upon simulating competition. As stated in one report,[14]

> "The purpose of regulatory procedure in the protection which it is designed to afford the consumer is to simulate and substitute the effects of competition and give the consumer the benefits which he would derive from a system of competition."

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[13] *Public Utilities Reports*, New Series, Vol. 28, 1939, p. 65.

[14] Report of the Committee on Progress in Public Utility Regulation, National Assn. of R. R. and Utilities Commrs., 1941, *Proceedings*, 53d Annual Convention of the National Assn. of R. R. and Utilities Commrs., Washington, D. C., p. 366.

To state the matter another way, the reasonable rates which are aimed at by rate regulation should be such as will produce a gross revenue which is no more than sufficient to pay the cost of operating expenses and fixed charges necessary to provide the service if furnished by facilities of the most modern, efficient, and economical kind available in the market as a result of progress in the arts, and of capacity adapted to the requirements of the service. The "fair value" or the "service value" of existing equipment is, then, no more than the capital sum corresponding with the present worth of the amounts expected to be available for fixed charges out of such reasonable gross revenues, after deducting the operating expenses (other than depreciation) of the existing facilities. This formula for "fair value" or "service value" can be restated, for ease of computation, as the present worth of the equal annual payment annuity on the estimated cost of the facilities required to furnish the service most economically, reduced by the excess of the operating expenses of the existing facilities over the operating expenses of the most economical facilities.

It is suggested that this approach would actually reconcile the two criteria of Smyth versus Ames and, in providing an independent principle for the determination of reasonable rates, would at the same time provide an economically sound solution of the valuation and depreciation problem. Under this solution, fair value would be the amount upon which a reasonable fixed charge for depreciation and return would remain after deducting from revenue at reasonable rates the operating expenses of the existing facilities; and accrued depreciation would be the difference between such fair value and the cost of reproduction of the existing facilities.

This solution of the problem is also in accord with the classical economic theory of price fixing in a free market, inasmuch as the equating of the depreciation and return on fair value with the value of the service, as measured by the cost of providing it most economically, corresponds with the determination of the price at which a willing buyer might reasonably puchase and a willing seller might reasonably sell a needed utility service. This approach also reconciles the "fair value" principle with the "yardstick" principle, and if applied in accordance with sound engineering judgment would make unnecessary the construction of "yardstick" plants to secure the benefit of true "yardstick" rates.

This concept is not novel. It is suggested herein that it is implied in any attempt at reconciliation of the two criteria of Smyth versus Ames; and Mr. Justice John M. Harlan, who wrote the Smyth versus Ames opinion, referred to it even more specifically in San Diego Land and Town Company versus National City Case (174 US 739,757), in which he referred to the original cost basis of valuation as "defective in not requiring the real value of the property and the fair value of the services rendered to be taken into consideration." It has been applied in the valuation of particular items of property in a number of commission cases, as follows: Michigan State Telephone Company,[15] Hill versus Antigo Water Company,[16] Somerville Water Company,[17] Wood versus

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[15] *Public Utilities Reports*, Vol. 1921C, p. 545.

[16] Wisconsin R. R. Comm. Repts., Madison, Wis., 1909, Vol. 3, p. 623.

[17] *Public Utilities Reports*, Vol. 1922C, p. 309.

Elmira Water, Light and Railroad Company,[18] and Department of Public Service versus Pacific Telephone and Telegraph Company.[19]

The same thought has been discussed at length by J. C. Bonbright. To select one statement from his discussion,[20]

> "* * * it would seem * * * that, if reproduction cost is used as a basis of valuation, the only significant cost is that of constructing a modern substitute plant. The cost of producing a substantial replica would then be ignored except as an indirect means of estimating the former figure. Even the cost of substitute plant could not be used directly as the basis of valuation; for its greater efficiency would mean savings in operating expenses. The operating disadvantage of the present plant as compared to the hypothetical new plant should be estimated by a capitalization of the difference in annual expenses, which should be deducted from the cost of the new plant in arriving at the old one."

It was also directly referred to in Mr. Justice Reed's marginal note in his dissenting opinion in the Hope Case, previously quoted. However, as far as the writer knows, this approach has never been completely applied in any valuation and rate case.

Possibly the reason for failure to adopt this approach is its obvious difficulty; as Mr. Bonbright states:[21] "A very nice appraisal problem—and one which calls for great engineering, accounting, and mathematical skill." On the other hand, millions of dollars are invested every year in new construction, replacement, and improvements, based upon engineering estimates and forecasts such as are standard in engineering economics and which, as explained by Eugene L. Grant,[22] M. ASCE, are different in no way from the engineering estimates required to measure reasonable rates and fair value in the manner herein suggested.

Also, it is not believed that the adoption of this suggestion would be in conflict with the rejection by the United States Supreme Court in the Indianapolis Water Company Case (272 US 400),[23] of the inclusion of a substitute for a portion of the property in an estimate of reproduction use. Examination of that opinion shows that a witness for the city proposed to eliminate a portion of a canal and a power plant used to generate power for pumping from the water carried in the canal, and to substitute a steam plant with a lower reproduction cost. The opinion does not make clear whether the witness gave any effect to the increased operating expenses that would have resulted from the substitution of the steam plant. In any case, the opinion indicates that the proposal was merely to substitute a different engineering judgment from that which had led to the construction of the canal and power plant—which engineering judgment had, as a matter of fact, been specifically approved by the Indiana Public Service Commission in two previous opinions quoted in the opinion of the Supreme Court. As the Supreme Court stated: "There is to be ascertained the value of the plant used to give the service and not the estimated cost of a different plant"; but the consideration of the effects of obsolescence and changes in demand upon the value of existing facilities, by use of estimates of cost of installation and operation of facilities of the most economical type and of capacity adapted to the requirements of the service, is a wholly different matter from substituting the estimated cost of reproduction of a different plant for that of an existing plant.

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[18] *Public Utilities Reports*, Vol. 1927B, p. 400.

[19] *Ibid.*, New Series, Vol. 37, 1940, p. 321.

[20] "Valuation of Property," by J. C. Bonbright, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 1124.

[21] *Ibid.*, p. 164.

[22] "Relationship to Competitive Industry," by Eugene L. Grant, in "Fundamental Aspects of the Depreciation Problem: A Symposium," *Transactions*, ASCE, Vol. 108, 1943, p. 1242.

[23] *Public Utilities Reports*, Vol. 1927A, p. 15.

There is no compelling reason, in theory, why the method used to estimate the value and depreciation of public utility property should have any necessary relation to public utility accounting. Accounting records provide a conventional method of informing managers, investors, and public authorities regarding the financial and fiscal operations of business enterprises, and are controlled more by "generally accepted accounting principles," and by prescribed standards than by any rigid technical or economic theories. Nevertheless, most public utility regulatory authorities have prescribed uniform classifications of accounts which define depreciation as "the net loss in service value not restored by current maintenance," and provide for charging in each month "the estimated amount of depreciation accrued during that month." These provisions cannot be given any literal application, since these systems also require the recording of "costs," not "service values"—except on the basis of the economically and legally unsound circular reasoning assumption that "cost" and "service value" are identical.

However, if the regulatory commissions believe, as the New York Public Service Commission stated in its memorandum on unification plans,[24] that regulation would be facilitated if "depreciation reserves represented even approximately the amount of depreciation actually existing in the property," then a helpful meaning can be given to the language of these requirements by maintaining depreciation reserves in the accounts which are, as nearly as possible, the same proportion of the original cost of the depreciable property as the accrued depreciation in the property is of its cost of reproduction new. If this interpretation is adopted, the difference between the cost of reproduction new of the existing property and its "fair value" or "service value" can be considered as a measure of the accrued depreciation; the ratio of such accrued depreciation to the cost of reproduction new of the existing property can be considered as a measure of the percentage of accrued depreciation; and this ratio can be applied to the original cost of the depreciable property to arrive at a measure of the desirable depreciation reserve requirement for accounting purposes. Furthermore, to preserve this relationship, the depreciation reserve requirement can be determined at the beginning and end of each year, and the net difference, plus the net debit to reserve during the year on account of retirements, can be taken as a corresponding measure of annual depreciation expense for accounting purposes.

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[24] Annual Report, New York Public Service Comm., Albany, N. Y., November 10, 1936, Vol. 1, p. 724.

Illustrative Example

Application of this suggestion to the determination of the reasonable gross revenue, valuation, and depreciation of a specific utility is illustrated by an example of a hypothetical manufactured gas plant, based upon data selected from a number of different manufactured gas appraisals made by the writer prior to 1942. Any resemblance of this hypothetical plant to an existing plant is purely coincidental.

In developing this example it should be pointed out that a practical solution is aimed at, and not a solution of the theoretical problem. The latter could only be solved if the gift of prophecy were available as to the quantity of service to be required, the competitive conditions under which it would have to be sold, the cost of installation of the most economical facilities for supplying the service, the operating expenses (both of this most economical installation and of the existing facilities), the mortality characteristics of each successive group of units of property, the fair rate of return, and various other factors for each year of an indefinite period in the future. Instead, the solution sought is one analogous to the solution of such problems as are encountered in the daily activities of business men, financiers, and entrepreneurs. Such solutions often seem to be reached without taking into account any quantitative considerations whatever. Sometimes the business man estimates the future earning power of a facility on the basis of past experience by application of a rule of thumb, rather than by calculation of present worths. Sometimes he has technical advice which takes into account the best possible estimates of relative costs of installation and operation of various alternatives and the present worths of prospective returns as a basis for judgment.

An estimate of this kind is illustrated by the following example: It is suggested that such a computation, based upon the present worth of the equal annual payment annuity on the estimated cost of the facilities of the most economical type available as the result of the progress in the arts and of capacity no more than is required to provide service performed by the existing facilities, less estimated average annual savings in operating expenses that could be accomplished by utilization of these most economical types of facilities, comes as close to the solution of the theoretical problem as is practically possible, and provides a practical solution that can be accepted as the most reasonable one available.

For purposes of the example, a manufactured gas plant has been assumed with \$50,000,000 of actual original cost, and \$60,000,000 of reproduction cost new of coke ovens and water gas sets; pumping stations and gas holders; cast iron, wrought iron, and steel mains; wrought iron and steel services; meters; general structures; and general equipment. It is assumed that the retirement experience of the various classes of property has been analyzed sufficiently to indicate what the mortality characteristics have been; that reasonable estimates have been made, both of total capitalization periods, corresponding with the average service lives resulting from these studies, and of future capitalization periods, corresponding with the indicated life expectancies; and that 6% has been adopted as a reasonable rate of return.

It is also assumed that: (1) A complete engineering study has been made of each of the groups of property comprising the existing plant; (2) estimates have been made of the increase or decrease in cost at the current price level, as compared with the estimated cost of reproduction of the existing facilities, that would result from the utilization of the most economical equipment of each type available as a result of progress in the arts and of capacity no greater than sufficient to meet the requirements of the service; and (3) corresponding estimates have been made in each case of the savings in operating expenses that could be accomplished if this most economical equipment were utilized.

For example, it is assumed that it has been estimated that the same service as is rendered by the existing coke ovens and related facilities could be performed by a modern installation costing $2,500,000 as compared with an estimated cost of reproduction of $2,900,000 for the existing installation, and that at the same time a saving in operating expense of $45,000 per yr could be accomplished. Similarly, it is assumed that it has been determined that the service rendered by the existing cast iron mains, 12 in. or less in diameter,and by all existing wrought iron mains could be furnished more economically by the use of steel mains, and that single mains of adequate size could be substituted for parallel mains in some locations, with reduction in large main capacity in certain areas where the population and consumption have declined —all at a cost for distribution mains of, approximately, $14,200,000 compared with an estimated reproduction cost new of the existing mains of $18,600,000, but without any substantial difference in operating expenses.

It is also assumed that: (*a*) Among other modernizations, installation costs would be increased by $235,000 and operating expenses decreased by $58,000 if modern water gas sets equipped with reversed air blast and waste heat boilers, and, to the extent that it would be economical, with mechanical grates, were substituted for existing installations of older types without these improvements; (*b*) installation costs would be decreased by $110,000 and operating expenses decreased by $24,000 through the substitution of liquid purification for dry box purification; (*c*) the use of a modern positive displacement type station meter in place of several old type wet meters would reduce installation cost by $62,000, and operating expense by $500; and (*d*) the installation of modern electric motor driven and gas engine driven booster units in place of certain old steam engine driven units, with gas fired boilers for holder heating, and related changes at holder stations, would reduce installation costs by $600,000 and operating expenses by $16,000.

It is also assumed that similar engineering determinations have been made for every other group of items of property considered, after investigation, to have been affected by obsolescence (progress in the arts) or changes in demand; and that future capitalization periods have been adjusted to take into account any known requirement for retirement of property in advance of dates corresponding with mortality experience because of requirements of public authority or desirability of improving the quality or reliability of service. In utilizing these modern types of equipment as a basis for estimating the cost of providing the existing service economically, the attempt has been made to use only types of equipment that had records of proved use and that were generally accepted as representing the current standard of design practice in modern plants. It is not believed that effect should be given in such estimates to recent scientific discoveries that have not passed beyond the laboratory or the pilot plant stage, or to initial installations of equipment for the operation of which no substantial amount of experience is yet available.

For the purpose of concentrating attention upon the subject of this paper, the writer has intentionally refrained from discussing the reasonableness of any of the assumptions referred to previously, or any of the numerous interesting collateral subjects that suggest themselves, such as the price level applied in estimates of cost, intangible values (including going concern value), and limitations applicable to enterprises subject to industry obsolescence where competition of alternative services or alternative objects of consumer expenditure would prevent the earning of a return on even the most economical facilites.

The resultant estimates of reproduction cost new of the existing facilities, the estimated cost new of the facilities required to render the service most economically, the reasonable total capitalization periods, the amount of the annuity on the cost of the facilities required to render the service most economically, the estimated annual savings in operating expenses allocated to the various classes of property involved, the amount available for capitalization to measure the value of the service and the service value of the existing facilities, the future capitalization period of the existing facilities, the corresponding capitalization factor at 6% return, the estimated service value of the existing facilities, the amount of accrued depreciation in the existing facilities, and the percentage of depreciation are shown in Table 1, in accordance with the New York State Public Service Commission classification of accounts, adopted for purposes of illustration. In Table 1, attention is invited particularly to Cols. 8 and 9. Where obsolescence or change in demand can be measured by comparison with the fixed charges and operating expenses of equipment of the most economical type available as a result of progress of the arts, and of capacity adapted to the requirements of the service—which is of such a nature that it would be classified in a single account identical with that in which the existing property is classified—no problem of classification arises and service value can be determined by the direct application of the suggested formula. Where, however, the most economical facilities of appropriate capacity are of such a nature that they would be classified in a number of different accounts and where the reduction in the total cost of fixed charges and operating expenses is attributable to the entire group of property, the suggested formula for service value is directly applicable only to the group as a whole, and no strictly logical basis exists for distributing the resultant combined annuity and savings in operating expenses among the individual accounts in which the most economical property is classified. It is desirable, however, that such distribution be made for the purposes of both contributing toward the utilization of the results for accounting purposes, as referred to hereafter, and of giving an indication of the service value of the subdivisions of the property for any purpose.

It has been the writer's practice in studies made in this field to combine groups of property, which are affected as a group by obsolescence or change in

TABLE 1.—SUMMARY OF COMPUTATION OF SERVICE VALUE AND ACCRUED

ACCOUNT		Cost of reproduction new of existing plant[a] (thousands of dollars)	Cost of most economical facilities to furnish service[a] (thousands of dollars)	Capitalization period for most economical facilities[b] (years)	Annuity factor at 6% for period in Col. 5	Annuity on cost of most economical facilities[c] (thousands of dollars)
No.	Designation					
(1)	(2)	(3)	(4)	(5)	(6)	(7)
311	Land	2,500	2,500	Indefinite	0.06	150
312	Structures and improvements	4,000	3,500	60	0.06187	216.5
313	Boiler plant equipment	900	1,000	35	0.06897	69
314	Other power equipment	300	400	50	0.06344	25.4
316	Coke ovens	2,000	1,800	35	0.06897	124.1
317	Producer gas equipment	400	400	38	0.060736	26.9
318	Water gas generating equipment	1,300	1,500	40	0.06646	99.7
321	Coal, coke, and ash handling equipment	2,200	2,200	55	0.06254	137.6
322	Gas reforming equipment	60	60	40	0.06646	4
323	Purification equipment	1,600	1,500	34	0.06960	104.4
324	Residual refining equipment	600	550	48	0.06390	35.1
325	Other production equipment	1,440	1,390	34	0.06960	96.7
	Total production plant	17,300	16,800			1,089.4
341	Land	500	500	Indefinite	0.06	30
342	Structures and improvements	4,500	4,200	70	0.06103	256.3
	Total storage plant	5,000	4,700			286.3
361	Land	300	300	Indefinite	0.06	18
362	Structures and improvements	900	800	60	0.06187	49.5
363	Mains	18,600	14,200	65	0.06139	871.7
364	Pumping and regulating equipment	1,000	800	35	0.06897	55.2
365	Services	6,800	6,200	48	0.06390	396.2
366.1	Meters	4,800	4,700	30	0.07265	341.5
366.2	House regulators	8	8	48	0.06390	0.5
366.3	Meter installations	1,792	1,792	50	0.06344	113.7
	Total distribution plant	34,200	28,800			1,846.3
370	Land	300	300	Indefinite	0.06	18
371	Structures and improvements	2,200	2,000	60	0.06187	123.7
372	Office furniture and equipment	500	500	20	0.08718	43.6
373	Transportation equipment	200	200	10	0.13587	27.2
374	Stores equipment	15	15	25	0.07823	1.2
375	Shop equipment	25	25	28	0.07459	1.9
376	Laboratory equipment	20	20	22	0.08305	1.7
377	Tools and work equipment	220	220	12	0.11928	26.2
379	Miscellaneous equipment	20	20	22	0.08305	1.7
	Total general plant	3,500	3,300			245.2
	Total plant in service	60,000	53,600			3,467.2

[a] Estimated. [b] Assumed same as for existing facilities except for account 363, for which total capitalization land excluded this value is 0.27218. [f] With land excluded this value is 0.11162. [g] With land excluded this

demand, in "study projects" for which total annuities, total savings in operating expenses, total service values, and total loss in service value have been computed. These total annuities and total savings in operating expenses have then been apportioned among the several accounts affected in proportion to the equal annual payment annuity on the estimated cost of reproduction new of the corresponding existing property. The results of such "study project" computations have then been combined with the results of similar computations for cases in which the most economical property of appropriate capacity fell in a single account identical with that in which the existing property was

DEPRECIATION FOR HYPOTHETICAL MANUFACTURED GAS COMPANY

Redistribution of Col. 7 (thousands of dollars)	Annual operating expense savings by most economical facilities[a] (thousands of dollars)	Amount available for capitalization,[d] Col. 8 − Col. 9 (thousands of dollars)	Future capitalization period for existing facilities (years)	Present worth factor at 6% for period in Col. 11	Service value of existing facilities Col. 12 × Col. 10 (thousands of dollars)	ACCRUED DEPRECIATION	
						Col. 3 − Col. 13 (thousands of dollars)	Col. 14 / Col. 3 (%)
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
150	....	150	Indefinite	16.667	2,500	....	....
231.1	20	211.1	45	15.456	3,262.8	737.2	0.18430
64.8	20	44.8	21	11.764	527	373	0.41444
20.3	10	10.3	39	14.949	154	146	0.48667
121.2	30	91.2	25	12.783	1,165.8	834.2	0.41710
26.9	....	26.9	26	13.003	349.8	50.2	0.12550
99	50	49	29	13.591	666	634	0.48769
136.3	2	134.3	42	15.225	2,044.7	155.3	0.07059
4	....	4	29	13.591	54.4	5.6	0.09333
98.1	30	68.1	21	11.764	801.1	798.9	0.49931
35.8	3	32.8	39	14.949	490.3	109.7	0.18283
97.4	5	92.4	29	13.591	1,225.8	184.2	0.12792
1,084.9	170	914.9			13,271.7	4,028.3	0.23285[e]
30	....	30	Indefinite	16.667	500	....	....
257.5	3	254.5	49	15.708	3,997.7	502.3	0.11162
287.5	3	284.5			4,497.7	502.3	0.10046[f]
18	....	18	Indefinite	16.667	300	....	....
50.1	2	48.1	43	15.306	736.2	163.8	0.18200
871.7	....	871.7	57	16.065	14,003.9	4,596.1	0.24710
56.6	12	44.6	28	13.406	597.9	402.1	0.40210
396.2	....	396.2	33	14.230	5,637.9	1,162.1	0.17089
341.5	....	341.5	17	10.477	3,577.9	1,222.1	0.25460
0.5	....	0.5	39	14.949	7.5	0.5	0.06250
113.7	....	113.7	31	13.929	1,583.7	208.3	0.11624
1,848.3	14	1,834.3			26,445.0	7,755.0	0.22675[g]
18	....	18	Indefinite	16.667	300	....	....
125	3	122	35	14.498	1,768.8	431.2	0.19600
43.6	....	43.6	12	8.384	365.5	134.5	0.26900
27.2	....	27.2	6	4.917	133.7	66.3	0.33150
1.2	....	1.2	16	10.106	12.1	2.9	0.19333
1.9	....	1.9	14	9.295	17.7	7.3	0.29200
1.7	....	1.7	12	8.384	14.3	5.7	0.28500
26.2	....	26.2	6	4.917	128.8	91.2	0.41450
1.7	....	1.7	12	8.384	14.3	5.7	0.28500
246.5	3	243.5			2,755.2	744.8	0.21280[h]
3,467.2	190	3,277.2	. .		46,969.6	13,080.4	0.21717[i]

period of existing facilities is 80 years. [c] Col. 6 times Col. 4. [d] As service value of existing facilities. [e] With
value is 0.22876. [h] With land excluded this value is 0.23275. [i] With land excluded this value is 0.23104.

classified, and with those of corresponding computations on the portions of the property assumed, after study, not to be affected by obsolescence or change in demand. This procedure has been followed in the example presented herein and is the basis of the classifications in Cols. 8 and 9, Table 1. It is hoped that criticisms, comments, and suggestions regarding this convention and procedure will be included by those who discuss this paper.

In Table 2 is shown, for the same example, the application of the percentages of accrued depreciation arrived at in Table 1 to the assumed original cost of the property for the purpose of estimating the depreciation reserve requirement for

TABLE 2.—SUMMARY OF COMPUTATION OF DEPRECIATION RESERVE
PURPOSES OF HYPOTHETICAL

Account		Original cost of existing plant (thousands of dollars)	Accrued depreciation
No. (1)	Designation (2)	(3)	(4)
311	Land	2,500	....
312	Structures and improvements	6,000	18.430
313	Boiler plant equipment	900	41.444
314	Other power equipment	500	48.667
316	Coke ovens	2,300	41.710
317	Producer gas equipment	300	12.550
318	Water gas generating equipment	1,500	48.769
321	Coal, coke, and ash handling equipment	2,500	7.059
322	Gas reforming equipment	40	9.333
323	Purification equipment	1,600	49.931
324	Residual refining equipment	600	18.283
325	Other production equipment	1,460	12.792
	Total production plant	20,200	23.397
			26.701[d]
341	Land	500	....
342	Structures and improvements	3,400	11.162
	Total storage plant	3,900	9.731
			11.162[d]
361	Land	300	....
362	Structures and improvements	800	18.200
363	Mains	11,600	24.710
364	Pumping and regulating equipment	700	40.210
365	Services	4,600	17.089
366.1	Meters	3,800	25.460
366.2	House regulators	8	6.250
366.3	Meter installations	992	11.624
	Total distribution plant	22,800	22.644
			22.946[d]
370	Land	300	....
371	Structures and improvements	1,900	19.600
372	Office furniture and equipment	450	26.900
373	Transportation equipment	200	33.150
374	Stores equipment	15	19.333
375	Shop equipment	25	29.200
376	Laboratory equipment	20	28.500
377	Tools and work equipment	175	41.450
379	Miscellaneous equipment	15	28.500
	Total general plant	3,100	21.048
			23.304[d]
	Total plant in service	50,000	21.842
			23.537[d]

[a] Col. 4 times Col. 3. [b] Col. 7 times Col. 6

accounting purposes. Also shown, based on assumptions as to the corresponding percentages of accrued depreciation at the end of the following year, and the net debit to retirements during the year, are the amount of the depreciation reserve requirement at the end of the following year and the corresponding annual depreciation expense during such year. Table 2 is included for the

REQUIREMENT AND ANNUAL DEPRECIATION EXPENSE FOR ACCOUNTING MANUFACTURED GAS COMPANY

Depreciation reserve requirement[a] (thousands of dollars) (5)	Original cost of existing plant one year later (thousands of dollars) (6)	Accrued depreciation one year later (%) (7)	Depreciation reserve requirement one year later[b] (thousands of dollars) (8)	Net debit to depreciation reserve (thousands of dollars) (9)	Annual depreciation expense[c] (thousands of dollars) (10)
....	2,500	....	....	....	....
1,105.8	6,100	19.2	1,171.2	5	70.4
373	920	41.8	384.6	2	13.6
243.3	510	48.6	247.9	2	6.6
959.3	2,350	42.2	991.7	2	34.4
37.6	310	12.6	39.1	1	2.5
731.5	1,520	49.3	749.4	1	18.9
176.5	2,540	7.6	193	10	26.5
3.7	40	10.7	4.3	....	0.6
798.9	1,620	50.6	819.7	5	25.8
109.7	610	18.5	112.8	7	10.1
186.8	1,490	13.7	204.1	30	47.3
4,726.1	20,510	24.0	4,917.8	65	256.7
		27.3[d]			
....	500	....	....	....	....
379.5	3,460	11.0	380.6	10	11.1
379.5	3,960	9.6	380.6	10	11.1
		11.0[d]			
....	300	....	....	....	....
145.6	810	18.1	146.6	3	4
2,866.4	12,000	23.9	2,868	150	151.6
281.5	720	39.8	286.6	7	12.1
786.1	4,700	16.5	775.5	200	189.4
967.5	3,850	26.8	1,031.8	70	134.3
0.5	8	7.5	0.6	....	0.1
115.3	1,012	11.3	114.4	10	9.1
5,162.9	23,400	22.3	5,223.5	440	500.6
		22.6[d]			
....	300	....	....	....	....
372.4	1,920	19.9	382.1	5	14.7
121.1	460	28.4	130.6	20	29.5
66.3	210	31.5	66.2	40	39.9
2.9	15	20.0	3	1	1.1
7.3	25	30.8	7.7	1	1.4
5.7	20	30.5	6.1	1	1.4
72.5	165	48.2	79.5	16	23
4.3	15	30.0	4.5	1	1
652.5	3,130	21.7	679.7	85	112.2
		24.0[d]			
10,921	51,000	22.0	11,201.6	600	880.6
		23.6[d]			

[c] Col. 8 − Col. 5 + Col. 9. [d] With land excluded.

purpose of illustrating the application of the suggested approach to the establishment of the depreciation reserve requirement and annual depreciation expense in case the existing classifications of accounts and the desire to have depreciation reserves represent "approximately the amount of depreciation actually existing in the property" are continued without change. However, both the regulatory commissions and the public utility industry would be well advised to separate completely the accounting requirements from the methods of estimating service value. This separation would permit the adoption of some relatively simple accounting convention such as straight-line age-life depreciation for accounting purposes without any implication that the resultant depreciation reserves, original cost less such reserves, and depreciation expense charges had any effect on valuation or rate making.

CLOSING COMMENT

Attention is called to the fact that the result of the calculations shown in Table 1 can be stated either as a determination of the reasonable annuity for depreciation and return which, in combination with the operating expenses, other than depreciation, of the most economical property establish a reasonable level of gross revenues without reference to the valuation or operating expenses of the existing property; or as a determination of the service value of the existing property for use in the conventional formula in which reasonable rates are based upon a return on the fair value of the property. The result will be identical provided annual depreciation expense is treated in a corresponding manner in the two statements, and may be illustrated as follows:

A. Gross Revenue Based Upon Value of Service.—

Operating expenses of existing plant, exclusive of depreciation	$ 8,500,000
Assumed saving by most economical facilities	190,000
Operating expenses of most economical facilities	$ 8,310,000
Annuity on cost of most economical facilities	3,467,200
Total gross revenue	$11,777,200

B. Gross Revenue Based Upon Return on Fair Value.—

Operating expenses of existing facilities, exclusive of depreciation	$ 8,500,000
Annual depreciation and return on existing facilities	3,277,200
Total gross revenue	$11,777,200

This illustration, together with Tables 1 and 2, also calls attention to the effect on income statements that could result from the adoption of this suggestion as a result of the lack of correspondence between the actual incidence of retirements and the incidence of retirements corresponding with the theoretical depreciation expense included in the estimated reasonable gross revenue. Thus, in the example, where net debits to retirement reserve have purposely been assumed as substantially in excess of what would correspond with the theoretical depreciation expense, the operating income for the year would be reduced to $2,397,200, or 4.9% or the assumed fair value of $46,969,600, instead of the 6% assumed as a fair rate of return (still a little more than 6% of the average original cost less depreciation reserve requirement). In other words, this procedure would have the disadvantage, if it is such, of introducing fluctuations into the operating net income (as retirement expense accounting did) as the actual retirements varied above or below those corresponding with the theoretical depreciation. It would also introduce further irregularities with the irregular occurrence of new developments in progress in the arts and changes in demand.

On the other hand, it would have advantages over other procedures of reflecting proportionate losses in value more nearly when they occur, and tending to maintain reserves more nearly at the same proportion of original cost as accrued depreciation was of reproduction cost new. Whatever may be the advantages or disadvantages of the adoption of this suggestion from the accounting and financial points of view, it is believed that it offers a solution of the valuation and rate problems which is worthy of further study and development.

The point of view expressed herein has been presented in the form of testimony in the United States District Court for the Eastern District of New York, in a case involving the valuation of certain gas and electric distribution property taken through condemnation by the federal government. In this final opinion of the District Court in this proceeding, after stating that:

> "* * * [the writer's] understanding of the basic purpose of rate regulation is to give the consuming public the benefit of competition based upon the value of the service if it were to be furnished according to operating expenses and fixed charges as resulting from the functioning of such facilities of the most modern and economical type."

the Court stated: "That would seem to provide a fair standard of value for present purposes" (65 Federal Supplement 333 and 71 Federal Supplement 248). The Attorney General of the United States has filed notice of appeal in this proceeding.

DISCUSSION

J. Kappeyne,[25] M. ASCE.—During the half century that has elapsed between the decision of the United States Supreme Court in the Smyth versus Ames case (169 US 466) and its decision in the Hope Natural Gas Company case (320 US 591), the economic and legal status of public utilities has undergone considerable change.

In the latter part of the nineteenth century public utilities were competing with private enterprises, having all the attributes of unrestrictive private property. Today public utilities are regulated monoplies from which the right to speculative gains inherent in unrestrictive private property has been taken. Regulation and speculation are inconsistent, regulation being merely a substitute for competition.

It may well be that the Supreme Court had this change in legal status in mind when it made its decisions in the Natural Gas companies cases. In these cases the Court merely held that, if the rate fixed is just and reasonable, it is not compulsory to consider reproduction cost in arriving at the end result. It would seem that, if this were given some further consideration, the "confusion worse confounded" may be somewhat clarified.

As the writer [26] has stated previously, it is unfortunate that several uniform systems of accounts in effect today include the terms "value" and "depreciation." The meaning of these two words is defined in the classification itself and is different from their usual meaning in the economic sense. The fact that this is so, however, makes it pertinent for the engineering profession to make clear what meaning is intended in each given instance and not to confuse conclusions based upon different meanings.

In the economic sense "service value" means the value to the consumer, of the service rendered; "depreciation" means a loss in value, and not a loss in service capacity resulting in the ultimate retirement of property. Both terms relate to value.

In these uniform systems of accounts these terms relate to cost and not to value. "Service value" is the difference between original cost and the net amount received for property retired, less the cost of removal. "Depreciation" is the loss in "service value" (as defined herein) incurred in connection with the prospective retirement of the plant. Merely to recite these accounting definitions shows their artificiality and the confusion of tongues that their use can and has induced.

Apparently Mr. Scharff has in mind the economic meaning of the terms "value" and "depreciation." However, when he recommends using a certain ratio relating to value as a measure of the "depreciation reserve requirement" (which is an accounting concept), that confusion previously mentioned as occurring between economic and accounting concepts seems to be present.

The theory of determining future rates based on the cost of rendering service by a substitute plant of the most modern and efficient kind available is interesting from an economic viewpoint. However, it is feared that such a process is too complicated to be applied by regulatory authorities in that it is solely dependent upon estimates, which undoubtedly will vary with the views as submitted by various experts, not only as to applicable costs, but also as to basic design. Such complications may be far worse than those attendant upon the fixing of the old-fashioned "reproduction cost," and thus are of little value to the fundamental problem to be answered by the regulatory authority.

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[25] Cons. Engr., Brooklyn, N. Y.

[26] *Proceedings*, ASCE, September, 1944, p. 1171.

The owners of the utility—that is, the investors who furnished the required capital—not only are entitled to the protection of their investment, but also are entitled to the opportunity of earning a return thereon that they had good reason to expect at the time the investment was made. Once an investment is made in a monopolistic, public utility enterprise, the investors are compelled to keep the facilities extended in advance of those needed to serve their customers adequately.

The present accounting methods, prescribed by the regulatory authorities, permit the determination, by years, of the cost of the surviving facilities, including elements of working capital. The historical rates of money for such years may then be weighted, resulting in an average rate of return that the investors had reason to expect when they had to make their investments. One would reach substantially the same end by multiplying the present average rate, at which money for utility investments can be obtained, by the weighted average purchasing power of the dollar over the years, in which the investment was actually made. This would seem to be the easier and more practical solution for determining the rate of return which is equitable to both the investors and the customers, subject to possible adjustments as warranted by circumstances in the particular instance.

The more consistent and simple the regulatory method of determining the rate base, the better able will the utility be to prepare for the future. The fluctuation in the value of the dollar, the realized return on funds invested that do not take the full risk of the enterprise, the efficiency of management, and the other economic factors that will require special consideration may then be compensated for in the rate of return allowed rather than by a revised method of determining the rate base. If these factors are not taken into consideration, an insufficient rate of return results and thus prevents attracting new capital on favorable terms.

These comments are made for the purpose of drawing from the engineering profession further clarifying discussions of a subject which is of vital interest to the welfare of the entire nation.

W. V. BURNELL,[27] ESQ.—In the view of the writer, the author has made a significant contribution toward the clarification of a subject which has long been mired down in confusion—namely, the broad subject of fair value as related to regulated public utilities.

Governmental regulation of public utilities in the final analysis is a judicial process that calls for the formation of reasonable judgments, supported by and in harmony with acceptable evidence. The final resort in such proceedings, in the United States, is to the judicially pre-eminent Supreme Court. One looks to that tribunal, beyond all others, for a clarity of reasoning of the highest order. There is cause for discouragement, however, when that court sustains an order as reasonable but fails to disclose "what makes it so or what could possibly make it otherwise," as was charged in the dissenting opinion of Mr. Justice Robert H. Jackson in the Hope Case.

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[27] Vice-Pres., Stone and Webster Eng. Corp., New York, N. Y.

Nevertheless, that discouragement may not be so completely attributable to the shortcomings of the Court as it may appear for, after all, fairness in public utility rate making and particularly the determination of property values is fraught with many technical factors of economics as well as engineering. If some of these factors are not brought clearly to the Court's attention, it is hardly to be expected that they will be considered.

There may be ground for a belief that the engineering profession is somewhat at fault for this error of omission. It is on that account especially that Mr. Scharff's clarifying contribution deserves more than a passing word of commendation. He lays no claim to the origination of the concept of "service value," but he has presented the concept with noteworthy persuasiveness. The engineering fraternity would do well to take heed thereof. The regulatory commissions, the courts, and especially the United States Supreme Court, are entitled to better "engineering-economic" advice than they have received in the past.

The writer confesses to a definite share of blame for the failure of the engineering profession to "put across" the soundness of this cost-of-service concept of value. In a case before the Public Utilities Commission of the District of Columbia, he went to great pains to demonstrate the concept as applied to the property of the Potomac Electric Power Company, but his powers of persuasion proved to be quite inadequate.

By the very practical process suggested in that case, and now by Mr. Scharff, one arrives first at a fair price for the product—the price for which a possible or potential competitor could afford to sell the product after providing for the most efficient facilities available for its production. When that result is obtained, the answer to the question of value may be found by means of a simple arithmetical equation.

Only one question is raised as to Mr. Scharff's conclusions, and that concerns only a single factor of the equation—the "annual depreciation allowance." Referring to Table 1, the conclusion for the property as a whole is a summation of the results for each of the classified groups. For simplicity, Account No. 316, coke ovens, is selected for the purpose of analysis. For the most economical substitute plant the investment would be $1,800,000 and the annuity factor at 6% for a life expectancy of 35 years is 0.06897, the amount of the annuity being $124,100. An adjustment is made here to bring this annuity down to $121,200, which, in effect, reduces the $1,800,000 to $1,757,200. For the excess cost of operation and maintenance of the existing plant there is a deduction from the foregoing annuity of $30,000, leaving a balance of $91,200 available as an annuity on the existing plant for its remaining service life, which Mr. Scharff has taken to be 25 years. Since the present worth factor, for 25 years at 6%, is 12.783, the indicated present value of this annuity, and therefore of the existing plant, is $1,165,800.

The annuity factors are very important in the foregoing calculation. They must necessarily be based on a judgment as to the respective remaining life of each of the two plants under consideration. The writer does not undertake to question either of the adopted estimates of remaining life. He does raise a question, however, as to the consistency of either estimate in its relation to the other.

For this line of inquiry the only published precedent known to the writer is a paper[28] presented at a meeting of the Technical Valuation Society in January, 1947. The present situation is especially opportune to develop in more meticulous detail the essential point of that paper.

In this case of the coke ovens, the fundamental hypothesis is that the remaining life of the existing property is 25 years. Provision is made, as a part of the annuity, for the accumulation of a 100% depreciation reserve, during the 25-year period, to compensate for the full loss in value of $1,165,800. Consistency appears to require the assumption that this future loss in value will have been caused, as a general rule, not by the physical wearing out of the property, but by developments similar to those which have brought about the present loss in value of the existing plant. These developments, both external to the existing plant, are: (*a*) A smaller amount of investment required, and (*b*) a lower cost of operation. It follows that the prospective fall in value of $1,165,800 will be the result of a combination of these same two external factors. It also follows that the development of these factors, caused by improvements in the design of a future substitute plant, would necessarity have the same future effect on the present substitute plant as found in the case of the existing plant.

The next implication of the process is that the accumulated amount of the depreciation reserve for the present substitute plant at the end of the 25-year period, if today's assumptions of remaining life are consistent, must also be $1,165,800. What is the fact about this? The depreciation annuity factor for the substitute plant is 0.00897. Against the adjusted investment in this plant of $1,757,200 the amount of the annual depreciation charge is $15,760. From the tables it will be found that the accumulated amount of these charges will be 54.86 × $15,760, or $864,600.

It would follow from the foregoing that the remaining life assumptions of 35 years and 25 years, respectively, for the substitute and existing plants, as tested by the yardstick of prospective improvements in design, are not consistent. Assuming the 35-year assumption to be correct for the substitute plant, the calculations of the writer force the conclusion that the correct assumption for the existing plant would be slightly more than 30 years. On this basis, moreover, the economic value of the existing plant comes to $1,257,200, instead of $1,165,800. The depreciation annuity factor for 30 years (plus) is 0.01254, which produces the same $15,760 required for the substitute plant with a 35-year life. At the end of the 30 years (plus) with a multiplier of 79.77 the accumulated reserve will amount to $1,257,200 for both plants.

A similar calculation accepting the 25-year life for the existing plant results in a life expectancy for the substitute plant of approximately 29.5 years, and a value for the existing plant of the same $1,257,200. In both cases it will be seen that the difference in value between the substitute plant ($1,757,200) and the existing plant ($1,257,200) is exactly $500,000. Upon further reflection, it becomes clear that this difference in value is nothing more than the capitalization at a 6% rate of the original $30,000 annual operating expense savings.

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[28] *Technical Valuation*, April, 1947.

In arriving at estimates of the total cost of service it is clearly necessary to include an adequate allowance on both sides of the equation, for a depreciation annuity. From the viewpoint of comparative property values, however, the essential requirement would seem to be that the allowances be consistent with each other.

In the over-all result as reached by Table 1, the indicated total annual saving is $190,000. It will be observed, however, that this potential saving is related to less than half of the total cost of the substitute plant. For the remainder of the property, the impairment in value must be attributed, in its entirety, to the mere lack of newness in the existing property. The extent of the indicated impairment, in each of these remaining accounts, results only from the corresponding assumptions as to the length of the capitalization period—that is, the remaining service life.

The important point to consider, in this matter of relative age, is that no provision appears to have been made in Table 1 for any saving in the annual cost of maintenance that might be gained by having a "new" property in place of the "old" one. Unless there would be such a saving, it is difficult, with only a few special exceptions, to conceive a reasonable justification for the abandonment of the existing property for the sole reason of its physical deterioration. The exceptions relate to such items as individual lengths of pipe, wooden poles, or furnace linings, which are subject to corrosion, decay, and the ravages of intense heat, respectively.

In general it is safe to state that public utility property, or any industrial property for that matter, can be kept in physical condition for service indefinitely by adequate maintenance and replacement of parts, usually charged to the maintenance account. Does it not follow from this that the differential in value between a new property of this character and one that is "old" is, in practical reality, the capitalized value of the difference between the two in the annual cost of maintenance and physical upkeep?

Referring again to Table 1, it is suggested that Col. 9 ("Annual Operating Expense Savings by Most Economical Facilities") be amplified in its scope to include any saving in the cost of maintenance that would be gained because of the newness of the substitute property. With this amplification, the limitation in the expectancy of remaining service life would be governed entirely by external developments. The required consistency in the relative annual allowance for depreciation could thus be satisfied and the "speculative" element in the calculation, in so far as relative property value is concerned, could thus be eliminated. It is quite probable that if the author adopted the modification herein suggested his final conclusion would not be greatly changed. It is believed, however, that the conclusion would be founded on a less speculative and more provable basis.

Donald Gunn,[29] Assoc. M. ASCE.—This timely analysis of recent court decisions treating the subject of depreciation in regulated industry reveals a chaotic situation. It reveals also that, as a result of that situation, this important factor in regulation is rapidly drifting to the point where it will be treated on a purely arbitrary basis, remotely, if at all, related to facts. The writer has recently appeared three times before the Federal Power Commission and twice before the Pennsylvania Public Utility Commission as a witness on the subject, and has clearly observed this trend in full force.

The proposition that determinations of actual depreciation be used to measure the adequacy of depreciation reserves and as a basis for determining a reasonable annual allowance to the reserve was advocated in those cases. In the first two cases, which were before the Federal Power Commission, that proposition was rejected. The decision in the second case before the Federal Power Commission is now on appeal before the United States Court of Appeals for the Third Circuit.

Much confusion in discussions of this subject was caused from the use of loosely defined words such as "inadequacy," "obsolescence," "changes in the art," and the like. The writer will endeavor to avoid increasing this confusion by grouping the various causes of depreciation into the following categories: "Physical depreciation," which includes wear and tear, decay, action of the elements, and so forth; "economic disadvantage," which includes obsolescence, certain types of inadequacy, changes in the art, changes in demand, and the like, that are susceptible of determination through comparative economic studies; and "other causes," which include other types of inadequacy, requirements of public authority, and similar causes not related to economic or physical causes.

Mr. Scharff proposes to evaluate depreciation purely through present-worth calculations based on present age and assumed probable life, modified by the present worth of any potential annual savings as measured by comparison with the annual cost of a substitute plant. Thus, when the interest rate and cost estimates are considered factual, as they should be, the sole determinant of the proposed evaluation of depreciation is the assumed probable lives.

The author's proposal appears to be based on the assumption that there is an adequate volume of reliable statistics for various elements of public utility plants. It is generally recognized, however, that in the case of many important elements of property, there are no statistics whatever and there will be none for an indefinitely long time. Even such statistics as are available are of limited usefulness, either because they are not applicable to the type of property now installed, or because it cannot rationally be assumed that the causes of retirements in the past will be repeated in the future. This fact is obvious when the trend in development of steam stations is considered. Twenty-five years ago, approximately 2 lb of coal was required to generate 1 kw-hr of energy, whereas modern steam stations use less than 1 lb per kw-hr. Thus, it is obvious that technical improvement in modern steam stations has reached the point where the margin remaining for improvement, even if perfection were attainable, cannot result in a like degree of obsolescence.

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In the absence of adequate and reliable statistics, there is no basis for an estimate of service life except the actual physical and economic condition of the element of property under consideration. If this is determined, however, there is no real need for converting the condition into a service life. The general procedure for making such determination has been outlined elsewhere.[5]

Even if the estimates of time used by Mr. Scharff were construed as "reasonable capitalization periods," they would be of equally doubtful validity. It is unlikely that it would ever be proper to base a selection of property on such long periods of time as those suggested, which, it will be noted, range up to 80 years. Certainly the writer would never recommend a capital expenditure on such a basis. For such purpose, a period of more than 20 years or 30 years would be quite unrealistic, however plausible it might appear as a matter of theory. To the extent it can be appropriately used, it appears more rational to employ this same period of time for the evaluation of depreciation during the life of the property.

Certainly the evaluation of depreciation during the useful life of property must be governed by the same principles which justified its installation and by which its replacement or abandonment will be dictated. When property is new, there is no depreciation; when retirement or abandonment is justified, depreciation in that use is 100%. When property is physically worn out, it is replaced or abandoned wholly without regard to whether it also stands at much or little relative economic disadvantage. In like manner, when economies, available through alternative property, justify replacement or abandonment, physical condition—even though it were near perfect—would have no proper influence on the decision. In practice, however, such cases are seldom met. The more normal situation is where the property is at an economic disadvantage short of justifying replacement, and the total physical depreciation is also short of justifying replacement or abandonment.

In the case of most major elements of property, the physical depreciation is separable into two categories: That which can be eliminated by rebuilding or remodeling, and that which cannot be so eliminated. For example, a generator, turbine, transformer, or building may have suffered physical depreciation of 75%, of which 25% can and must be restored by rebuilding in order to realize further potential usefulness. Obviously, if there is depreciation from no other cause, it will be restored and use will continue. However, if at this time, it stands at an economic disadvantage of 80%, that—together with the imminent 25% rebuilding or renovation cost—will justify replacement. It is important to note that it is only that part of the physical depreciation which is restorable in character that enters into combination with economic disadvantage to justify replacement or abandonment and bring about 100% depreciation.

Thus, the true depreciation in property is either the total physical depreciation, or the physical depreciation that can be eliminated in combination with comparative economic disadvantage, whichever is the greater. This is a fundamental principle that cannot properly be ignored in the evaluation of partial depreciation during the useful life of property. There is no indication that

Mr. Scharff has recognized this principle in his proposal. It is not clear how he could have, when he appears to base his treatment on an assumed all-comprehensive remaining life.

Depreciation due to physical causes and that due to economic causes should be treated separately. An evaluation of physical depreciation based solely on an assumed future life is questionable as a practical matter. With the exception of certain mass types of property (poles, crossarms, meters, service connections, and the like) of minor import, there is nothing that resembles statistics which can serve as a guide in estimating remaining life. The more usual situation is where the actual physical condition is the sole indication of remaining life. This being the primary known fact, any assumption of remaining life must be a secondary factor, helpful only in evaluating the economic significance of physical condition. The separate evaluation of physical depreciation affords the opportunity to segregate the restorable from the non-restorable and thus makes possible the appropriate combination of depreciation due to physical and economic causes.

Depreciation arising out of comparative economic disadvantage should be treated separately from physical and other causes, and on the basis of total annual cost. When property is new, the total annual cost of its use is no greater than that of available alternative property and its depreciation brought about by economic causes is zero. Retirement for economic reasons is justified when the mere out-of-pocket operating cost of existing property is equal to or greater than the operating cost plus all fixed charges (return depreciation, taxes, insurance, etc.) of alternative property. Thus, it seems that the depreciation resulting from comparative economic disadvantage during the useful life of property can be expressed as a function of these annual costs in terms of capital cost.

The writer does not agree with that part of Mr. Scharff's proposal in which he suggests that depreciation can be translated from a fair value basis to an original cost basis by a simple ratio of reproduction cost to original cost. This proposition seems reasonable where the causes of depreciation are physical, but the writer sees no reason for assuming that it is generally applicable where there is depreciation due to economic causes. Indeed, technological progress is frequently accompanied by an increase of capital cost with a greater-than-offsetting decrease in operating cost. Again, a change in price level is seldom, if ever, reflected correspondingly in both capital and operating cost. In such a situation, it does not appear rational that such a general rule could ever be appropriate.

The author's basic procedure, as well as the suggested simple transition from a fair value basis to an original cost basis, inevitably carries with it a completely erroneous implication—which implication is that because depreciation is properly deductible from reproduction cost in deriving a fair value basis for regulating earnings, it is also properly deductible from original cost for the same purpose. Furthermore, this implication is little weakened by Mr. Scharff's caution to the contrary, and is one of the primary assertions on which zealots in some regulatory jurisdictions seek to confiscate regulated property through manipulation of the straight-line, age-life method of accounting for depreciation.

It can be simply demonstrated and fully proved that rate regulation on the basis of original cost is grounded on completely different principles from regulation based on fair value. For this reason, the writer does not believe that the depreciation problem can be adequately considered from the limited viewpoint of simply measuring and evaluating the effect of the causes of the phenomenon. It is urged, therefore, that the scope of any further consideration of the depreciation problem be broadened to include the proper treatment of depreciation in derivation of rate bases under the various theories of regulation. The writer urges that a comprehensive study of this problem be undertaken forthwith because he is convinced that failure to do so will seriously impair the future usefulness of the Sociey in this field.

HENRY F. LIPPITT, II,[30] ESQ.—Although Mr. Scharff's paper is a valuable and intelligent theoretical contribution to the field of public utility economics, it is doubtful whether any proposed solution such as this will ever be adopted for public utility rate regulation. For example, Mr. Scharff proposes that reasonable rates should be based on the costs of using the most efficient plant to produce the service. To the extent that costs from the existing plant exceed these "most efficient" costs that difference, capitalized, would represent the measure of accrued depreciation on the existing plant. Thus Mr. Scharff, as an example, shows (see "Closing Comment") that service by a most efficient plant (costing $53,600,000) should cost the consumers $11,777,200 a year. Since the operating expenses of existing facilities are $8,500,000 a year, the difference of $3,277,200 represents the amount available for annual depreciation and return. Capitalized at 6%, the present worth of this amount available for annual depreciation and return is $46,969,600, which Mr. Scharff considers the fair value or service value of the existing facilities. Since the existing plant is taken at $60,000,000, the $13,080,400 difference, or 21.7%, is taken as a measure of the accrued depreciation in the existing plant.

Essentially, Mr. Scharff proposes that utility rates be set on a "most efficient" replacement cost, a replacement cost in this instance not only of the existing plant, but a replacement cost of the existing expenses as well. This replacement cost he would use as a "yardstick."

The objections to the use of any such system may be summed up as follows:

1. It is difficult to determine replacement costs of a "most efficient" system, as to required investment as well as to estimated operating expenses;
2. Wide fluctuations would be introduced by such a standard; and
3. There is a serious lack of correspondence between the annual depreciation expense and the indicated accrued depreciation, with resulting unfairness to consumers or investors.

Difficulty of Determining Replacement Costs.—In taking the example which Mr. Scharff used, it was first necessary for him to assume that (*a*) a complete engineering study had been made of the existing plant; that (*b*) estimates had been made of the increase or decrease in cost at the current price level, of the most economical plant, compared to the existing plant; and that (*c*) corresponding estimates were made of the savings in operating expenses.

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[30] Attorney, New York, N. Y.

Plant Costs.—Practically a full page of the paper (see under the heading, "Illustrative Example") expresses the engineering assumptions that would have to be made in computing the cost differences between the theoretical "most efficient" plant and the existing plant (and the corollary savings in operating costs). An indication of the extent of the possible area of difference in the electrical utility field is shown by a "steam station cost survey" which appeared[31] in the technical press on July 3, 1948. The dollars per kilowatt of generating capacity, scheduled for installation in smaller stations, between 1940 and 1945, is shown to vary from at least $100 to $150. At one time a kilowatt of generating station capacity could apparently be set up for as low as $46 (outdoor boiler installation).

The difficulties of determining the replacement cost accurately (which is the very crux of Mr. Scharff's proposal) is adequately illustrated by reference to the Minority Report of the Commissioners of the Commission on the Revision of the Public Service Commission Law in New York,[32] published in 1930:

> "The first objection to the adoption of this standard lies in the extreme difficulty of estimating what it would cost for an entirely different, hypothetical new plant to produce the service. Suppose, for example, that a commission were called upon to fix the charges for electricity in New York City. It would have to determine where the power plants could be most economically located, what kind of generating units would be installed, what transmission and distribution systems would be adopted. Then it would have to estimate the capital costs of construction of this entirely new system of generation and distribution. On this estimated capital cost it would have to estimate a rate of return necessary to attract, within some assumed period, the required amount of capital into that new and 'unseasoned' enterprise. It would then need to estimate the probable operating expenses of the system, taking into account on the one hand all the economies that would result from the use of larger generating units, better located power plants, alternating current, 60 cycle equipment (as against the large amount of direct current or 25 cycle equipment now being used by the present company), and taking into account on the other hand the lack of experience of the operating and executive staffs, the large initial expenditures necessary to develop the business and to 'break in' the new equipment. Does anyone really suppose that a Commission could and would really do any such thing? Is it not almost a certainty that the doctrine of 'cost of reproducing the service' would revert eventually to the existing, meaningless doctrine of cost of reproducing a substantially identical plant? We believe that such an outcome is inevitable."

As to the legal aspects of the problem, the leading case applying this so-called "value of service" doctrine is the famous Market Street Railway Case (324 US 548) decided in 1945; but this case is considered an unusually harsh one in utility circles.

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[31] "Fifth Steam Station Cost Survey," *Electrical World*, July 3, 1948, p. 73, Chart 2(*a*).

[32] "Report of the Commission on the Revision of the Public Service Commission Law in New York," *Legislative Document No. 75*, 1930, p. 372.

Mr. Scharff stated (under the heading, "Illustrative Example") that he

> "* * * has intentionally refrained from discussing the reasonableness of any of the assumptions referred to previously, or any of the numerous interesting collateral subjects that suggest themselves, such as the price level applied in estimates of cost, intangible values (including going concern value)* * *"

and the competition of alternative services. From this statement one can assume that these other problems exist as well.

Operating Costs.—In computing the service value of the service to be rendered, Mr. Scharff makes certain deductions against operating expense to reflect some theoretical savings which might exist by using his "most efficient" plant, the difficulties of determining which are already apparent.

First, it is clear that estimating the annual savings from assumed modernizations will be equally as difficult as estimating the assumed modernizations —if not more difficult. Does the saving in operating expense of $45,000 a year due to the replacement of existing coke ovens depend on (*a*) 100% layoffs of the former employees, (*b*) the calculation of severance pay or pensions for those laid off, (*c*) the existence and use of untrained help in the initial year of the new installation, and (*d*) the effect of possible new wage and "fringe" benefits to the remaining employees? As to material costs, does the annual saving from the assumed modernization take into account the average inventory cost of coal and coke purchased at lower (or higher) prices over the past year against spot prices for substituted oil, natural gas, or liquid propanes—or are prices for the substituted fuels estimated over the coming year? To list such difficulties is only to compound the difficulty of the problem; but an indication of the answer in the electrical field may be shown by survey data[33] published on July 3, 1948, in which it is stated:

> "* * * a fair average across the country [the United States] shows production costs ranging from as low as 1.5 and as high as 14 mills, with an average value of 4.5 mills per net kilowatt-hour * * *"

Second, it may be seriously doubted whether such assumed savings could be properly deducted according to present court decisions, such as the case of of the Southern Bell Telephone and Telegraph Company versus the Tennessee Railway and Public Utility Commission, decided by the Tennessee Chancery Court on June 17, 1948 (74 PUR NS 150), and later sustained;[34] and, a similar decision by the Georgia Supreme Court.[35] In Mr. Scharff's proposals, to be sure, the excess operating costs of the present plant are not deducted directly from allowed operating expenses, but are related to the present worth of a "most efficient" plant and then deducted; but the result is the same since the total rates of approximately $11,277,200 in Mr. Scharff's example, to be set, in effect disallow some $190,000 of assumed savings.

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[33] "Fifth Steam Station Cost Survey," *Electrical World*, July 3, 1948, p. 72.

[34] *Public Utilities Fortnightly*, August 26, 1948, p. 324.

[35] *Ibid.*, September 23, 1948, p. 455.

In other words, perhaps it is difficult enough, both practically and legally, to determine the net rate base to be allowed for rate making purposes by using a theoretical replacement "yardstick" on the invested capital—to say nothing of also applying such a "yardstick" to operating expenses as well.

Wide Fluctuations.—The use of Mr. Scharff's proposed plan would introduce wide swings in the net "service value" or "fair value" rate base to be used as the result of changes in the price level. Today, there are almost no data to show what the yearly variations would be in the cost of replacing a modern gas or electric distribution system efficiently; but data covering the production systems of electric utilities are available, and these show wide fluctuations.

Thus, quoting the cost survey:[33]

> "* * * the plants of less than 60 megawatts built in 1920–1929 and not since expanded cost on the average about $150 per kilowatt. Those built in 1930–1934 averaged about $107 and those in 1935–1939 as low as $76. By 1940–1944 this unit cost had risen to $110 and by 1945–1947 had soared to about $135."

Although these data are for average plants, and an average cost of a "most efficient" plant would be less in each period, the cost of a "most efficient" plant would show similar changes.

The level of operating expenses, including fuel, labor, and maintenance—which are also an essential part of Mr. Scharff's determinations of "fair value" and "service value"—have shown similar and perhaps even larger fluctuations.

The acceptance of Mr. Scharff's proposal would therefore embark the utility business again on the roller coaster of Smyth versus Ames, although this time in the guise of replacement "service value" rather than "fair value" reproduction costs.

Correlation of Annual and Accrued Depreciation.—As a result of Mr. Scharff's proposal, he arrives at a "service value" for his "most efficient" plant of $46,969,600 compared with an original cost of existing plant of $50,000,000. His annual depreciation expense is approximately $880,600. Although he would retain original cost on the utility's books, he would not apply the annual depreciation expense toward increasing the depreciation reserve to be deducted for rate making purposes. Thus, in spite of the fact that in the 10 years prior to Mr. Scharff's example his consumers might have contributed $8,806,000 in annual depreciation charges reflected in the prevailing level of rates, no equivalent deduction would be made for this contribution. The only measure of accrued depreciation to be deducted from original cost would be the $3,080,000 difference between the "service value" computed by Mr. Scharff's formula and the original cost contributed by the investors in the utility's plant. Therefore, this would ignore, for rate making purposes, more than $5,000,000 contributed by consumers in the preceding 10 years. As Mr. Scharff states (in the paragraph preceding "Closing Comment"):

> "* * * both the regulatory commissions and the public utility industry would be well advised to separate completely the accounting requirements from the methods of estimating service value."

That this treatment runs contrary to a fair division between a public utility's consumers and its stockholders was recognized as far back as 1909 when the United States Supreme Court stated, in ruling on the case of the Louisiana Railroad Commission versus the Cumberland Telephone Company (212 US 414):

> "It was obligatory upon the [utility] to show that no part of the money raised to pay for depreciation was added to capital, upon which a return was to be made to stockholders in the way of dividends for the future. It cannot be left to conjecture, but the burden rests with the [utility] to show it. It certainly was not proper for the [utility] to take the money, or any portion of it, which it received as a result of the rates under which it was operating, and so to use it, or any part of it, as to permit the company to add it to its capital account, upon which it was paying dividends to shareholders. If that were allowable, it would be collecting money to pay for depreciation of the property, and, having collected it, to use it in another way, upon which the [utility] would obtain a return and distribute it to its stockholders. That it was right to raise more money to pay for depreciation than was actually disbursed for the particular year there can be no doubt, for a reserve is necessary in any business of this kind, and so it might accumulate; but to raise more than money enough for the purpose and place the balance to the credit of capital upon which to pay dividends cannot be proper treatment."

Similar judgments were handed down in the following cases: The Lincoln Gas Company versus Lincoln, Nebr. (223 US 349) in 1912, the Clarks Ferry Bridge Company versus the Commission (291 US 241) in 1934, Lindheimer versus the Illinois Bell Telephone Company (292 US 151) in 1934, and the St. Joseph (Mo.) Stockyards Company versus the United States (298 US 38) in 1936.

This correlation between the annual depreciation expense and the accrued depreciation on a utility's books deductible for rate making purposes is followed by perhaps most regulatory commissions today,[36] and is the position adopted by the National Association of Railway and Utilities Commissioners.[37] Mr. Scharff's proposal therefore runs counter to the well-defined trends in present day regulation.

Summary.—Although it may be possible that "the ghost of Smyth versus Ames still walks," it is increasingly clear that his ephemeral peregrinations have become greatly limited within recent years. Mr. Scharff adequately points out the signs of the times in the first few pages and this is reinforced by the comparative results of the Federal Power Commission's recent survey[37] with a similar one in 1941 which indicates that the direction of public utility regulation today is toward the acceptance of original cost less the annual depreciation accrued on the company's books (rather than any engineering estimates of "service value" or "fair value," however precise and accurate such estimates may be).

In the last paragraph of his paper, Mr. Scharff points out that an application of his theory has been accepted as a "fair standard of value" for a condemnation award for certain gas and electric distribution property taken by the federal government. Unfortunately, whatever the merits of Mr. Scharff's proposal may be in valuation for condemnation purposes, they do not appear to be likely of acceptance today in the rate making and regulatory field.

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[36] "State Commission Jurisdiction and Regulation of Electric and Gas Utilities," Federal Power Commission, U. S. Govt. Printing Office, Washington, D. C., 1948, p. 8.

[37] "Report of Committee on Depreciation," National Assn. of R. R. and Utilities Commrs., Washington, D. C., 1943, pp. 154–156.

Albert P. Learned,[38] M. ASCE.—Recent United States Supreme Court decisions are reconciled by the author with the Smyth versus Ames decision,[12] although at the same time attention is called to the different bases used in the decisions. This follows the idea contained in Justice Robert H. Jackson's comment that the federal regulation of the natural gas industry was a performance of economic rather than of legalistic rituals. Furthermore, Mr. Scharff states that the accounting method of providing for depreciation is not a fair measure of the actual depreciation, but may still be a convenient procedure in the accumulation of the reserve.

Apparently, the author is making the points that:

(1) "Fair value" and "service value" are identical;

(2) Service value is measured by the most modern and efficient equipment of capacity to meet the demand;

(3) Accrued depreciation of an existing property is the difference between its "reproduction cost new" and the "service value";

(4) A fair return to the owner, as well as payment by the user, for what the service is reasonably worth cannot be secured by use of either reproduction cost new less depreciation, or original cost less depreciation, where the observed or life method is used to determine depreciation, the result being "reasoning in a circle";

(5) Fair value is measured by capitalizing, at the selected rate of return, the difference between the cost of operation of a modern plant plus its required annual depreciation allowance, and the return on the estimated cost of a modern plant less the cost of operation and the annual depreciation allowance on the existing plant; and

(6) The substitution of the modern plant with its costs introduces the element of competition into the solution of the problem.

The writer feels that this procedure could only bring simplification if there were generally accepted bases for both the cost of construction and the operation of a plant proper to replace the present plant. If such measures are in controversy, it would add to the problem. The relative value of proper design might even be injected into the considerations.

If Mr. Scharff's method has been analyzed correctly, the accrued depreciation would be constant if there was no change in the art and there was no change in operating expense caused by lessened efficiency of equipment, costs varying with output, or added maintenance costs. The element of functional depreciation used in reproduction cost new less depreciation appraisals reflects many of the points Mr. Scharff's plan is intended to accomplish. The writer has always proceeded on the basis that obsolescence should be recognized in the observation method. The author's method seems to the writer to be even more complicated than the normal valuation procedure.

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[38] Asst. Engr., Black & Veatch, Kansas City, Mo.

Anson Marston,[39] Past-President and Hon. M. ASCE.—The part of this paper most valuable to the writer is the discussion (under the heading, "Introduction") of the indications of the present views of the United States Supreme Court on the authority of the 1898 Smyth versus Ames decision—as shown by the recent rulings in the Natural Gas Pipeline Company Case (315 US 575),[6] the Hope Natural Gas Case (320 US 591),[7] and the Panhandle Eastern Case (324 US 636).[8]

In brief, the essential principle of the Smyth versus Ames decision is simply that the "fair values" of utility properties must be determined in the same way as all other property values are determined in sales and purchases—by the judgments of the parties thereto. Every factor which should affect such judgments must be considered and these factors given their "just and right" weights in each case—as determined by sound judgment, not by any formula. Instead of Mr. Hale's [12] Does the Ghost of Smyth versus Ames Still Walk? the true present question is: Does the Soul of Smyth versus Ames Still Survive? To this the writer would answer yes—and not even the Supreme Court can destroy this true basis of every exchange value transaction, any more than it can destroy the law of gravity. Whether this long-established principle still prevails with the present Supreme Court is anybody's guess—as the author of the paper correctly concludes.

The rest of the author's discussion on this point is devoted to presenting what the writer considers to be an unsound and unwise "suggestion." It is the suggestion that, in valuations of utility properties and determinations of their accrued and annual depreciations, a return be made to the Smyth versus Ames decision, but that only two sentences in it be used, discarding its requirements that every factor affecting value must be given due consideration; and that, in adopting an interpretation of these two sentences, a valuation method be used in which everything is based on calculated estimates of the reproduction cost of existing properties and of the reproduction cost of substitute properties, designed but not built.

The Smyth versus Ames "fog" of meaning, which various economists, commission authorities, lawyers, some judges, and other writers on valuation find so blinding, is in their own minds. If in every purchase and sale of property the exchange value had to be determined in court, the same "fog" would ensue. It arises from the conflicting testimony of experts, paid by opposing parties in court proceedings. Each such party can be sure that it can hire a battery of "experts" to support its particular contentions. The "suggestion" made by the author of this paper would, if adopted, make his present "confusion worse confounded" even more confounded.

It is the writer's view that the trend of progress in valuation and the determination of depreciation is in the opposite direction from that suggested by the author. More and more, there is a departure from the estimates, and sometimes fanciful speculations, involved in experts' opinion testimony in court cases.

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[39] Dean Emeritus of Eng., Iowa State College, Ames, Iowa.

Instead, the trend is to give more and more weight to fully itemized property inventories showing the actual original costs of the individual items of the actually existing properties; and to only such overhead percentages of cost as have been actually incurred in the past; but not excluding such substitution of present prices (and the resulting cost values of individual items) as sound, conservative judgment deems certain to prevail for a considerable future period.

In the opinion of the writer, the same methods of estimating the values and the depreciations of public utility properties should be used as are applied to other properties.

JAMES P. McKEAN[40] M. ASCE.—In view of the rapidly spiraling construction costs and the increasing demand of American industry for additional plant facilities this paper is a timely one. The proposed method of valuation is presented as a tool to clear the "confusion worse confounded" in which the subject of valuation and depreciation is now enmeshed. This writer wishes to direct his discussion toward some of the underlying causes of this confusion and to suggest certain criteria that must be satisfied by any method of valuation that is to be generally accepted as the better measure of value.

Methods of Pricing.—Common methods in use for pricing manufacturing inventories include the "first in-first out," "last in-first out," "accumulated average cost," "cost or market, whichever is lower," and "standard costs." No one of these methods has universal acceptance. Pricing property inventories is a specialized application of the general pricing problem. In pricing inventories for the purpose of valuation the two more widely accepted methods are "bottomed" on either (1) original construction costs or (2) present construction costs. Both methods have their variations, and are adjusted to reflect the accrued depreciation on the units in service.

Replacement Cost Experience, Past and Present.—The desirability of using the present cost of construction as a basis for value was vigorously and forcefully presented in the famous Smyth versus Ames Case (169 US 466). Labor was scarce following the Civil War, financing was difficult to obtain, the United States was in a period of postwar inflation, working conditions on the transcontinental railroads were rugged, and the resulting original costs for these arteries of commerce were high. For three decades prior to the Smyth versus Ames litigation construction costs had been decreasing. Such was the cost setting at the time the representatives of regulation argued so vehemently for the use of the "present costs of construction" as the criterion of value. The United States Supreme Court held, however, that all factors had to be considered.

During the following two decades construction costs remained relatively constant, but with the outbreak of hostilities in Europe, heralding the beginning of World War I, construction costs again advanced. They "peaked" in 1920, dipped in 1921, and then leveled out into a plateau for the remainder of the 1920's. As these costs rose, and particularly after they had been temporarily stabilized on a new level, the proposal to use present reproduction costs as the primary, if not the only, measure of value was given favorable reception, and often accorded major weight in the "fair values" that were found.

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[40] Prof., Dept. of Gen. Eng., Iowa State College, Ames, Iowa.

The U. S. Supreme Court handed down a decision in 1930 holding that the legal depreciation base should be "present value," and not cost in the case of the United Railways and Electric Company of Baltimore versus West (280 US 234). Economic conditions changed markedly thereafter, and with the depression of the early 1930's this case was avoided by the Court in their decisions handed down in later rate cases.

By 1944 economic conditions showed another change, and in the Hope Natural Gas Case (320 US 591) the Court specifically overruled its 1930 decision declaring, "we cannot approve a contrary holding in the United Railways vs West."

With the advent of World War II, and particularly after the lifting of price controls, there was another substantial rise in construction costs with the attendant economic problems. Mr. Scharff has proposed the use of the "replacement cost of the most economical substitute plant" as the desirable measure of value. A method of estimating value and depreciation somewhat similar to this proposal is used by the United States Steel Corporation. In its publication[41] the annual depreciation charge is shown in two parts—$43,600,000, based on the original cost of the facilities, and an additional $13,100,000, covering wear and exhaustion of facilities "in view of the greatly increased present cost of facilities to be replaced." In a later edition of the same publication[42] the method is again mentioned with the explanation that it is used:

> "* * * in an endeavor to set up a reserve for wear and exhaustion of facilities which would more realistically accord with such higher current costs, an additional charge to cover more accurately such current wear and exhaustion was made."

Other current literature reflects some of the same thinking in discussing depreciation and replacement problems.

Original Costs and Price Fixing.—The original cost of construction of a property has long been used as one factor to be considered in determining value. Since before 1898 price fixing has been a recognized prerogative of the legislature (Munn versus Illinois (94 US 123) in October, 1876). Rate making is but a species of price fixing. In passing the Natural Gas Act Congress was exercising the right to fix prices when it vested the Federal Power Commission with the responsibility of determining the "lowest reasonable rate." With such a mandate it is not surprising that those concerned with administering the Natural Gas Act should use the original cost approach, when such costs are lower than the cost levels at the time of the rate controversy.

One of the most recent Supreme Court decisions which bears on the matter of price fixing is the United States versus John J. Felin and Company (68 US 1238) decided on June 14, 1948. In this case the government had requisitioned meat products belonging to a meat slaughterer under the authority of the

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[41] *U. S. Steel Quarterly,* August, 1948, p. 1.

[42] *Ibid.,* November, 1948, p. 1.

Second War Powers Act. The food distributing authority offered compensation based on the existing ceiling prices of the Office of Price Administration (OPA). The slaughterer refused to accept the offered compensation and sued for "just compensation" under the Fifth Amendment in the Court of Claims. That court decided that the replacement cost of the meat products, and not the maximum ceiling price, was the proper measure of value.

An appeal was taken to the U. S. Supreme Court which reversed the Court of Claims. Three justices (Chief Justice Fred M. Vinson and Justices Felix Frankfurter and Harold H. Burton) held that the "replacement cost" basis utilized by the Court of Claims was spurious, and that the burden was on the slaughterer to prove that the compensation offered by the government was not "just." In their view the slaughterer had failed to meet this burden. Three other justices (Justices Stanley F. Reed, Hugo L. Black, and Frank Murphy) concurred separately in the judgment on the ground that, under the circumstances of the case, the constitutionally established maximum price (OPA ceiling) should be considered "just compensation" within the meaning of the constitution.

Justice Wiley B. Rutledge concurred in the opinion. He advanced the reason that in the case of government requisition of perishable commodities the legal or ceiling price should furnish presumptively the measure of just compensation, subject to revision if the owner sustained the burden of proving he was entitled to a greater reward. Although these seven justices advanced different lines of reasoning, they united in opposing the use of "replacement cost" as the basis of value.

Two justices (Justices Robert H. Jackson and William O. Douglas) dissented, holding that:

> "* * * the constitutional guaranty of compensation * * * becomes meaningless if the government may first, under its 'war power' fix the market price and then make its controlled figure the measure of compensation."

This case is a recent pronouncement of the Court and indicates its opinion on the use of "replacement cost" as a measure of value in pricing meat confiscated by a governmental agency.

Governmental bodies have been building for themselves a place in the management of the economic affairs of the United States since the passage of the Interstate Commerce Commission Act in 1887. At first their growth was incipient; but with the legislation of the 1930's and the extensive powers vested in such agencies as the Securities and Exchange Commission, the Federal Communications Commission, and the Federal Power Commission, the relationship between government and business has ripened into a close association. Uniform systems of accounts, and property records developed on the original cost basis, have served to make precise cost records readily available. The legislative mandate in the creation of these commissions and the availability of the facts based on original cost have resulted in many decisions decidedly favorable to the original cost philosophy. In favoring the use of the original cost basis, Everett C. McKeage[43] suggests:

> "The advantage of the cost formula as contrasted with other formulas is that it is certain or can be made certain. * * * It appeals to me that the very nature of a public utility, occupying as it does a quasi-public and a quasi-trustee position, lends itself to the application of a rule of valuation in consonance with the obligations flowing from such a public status. It would appear that valuation on a cost basis fits into such a status. * * * I believe that the cost basis is preferable for valuing all property for rate fixing purposes."

The Administrative Procedure Act passed in 1946 (Public Law No. 404, Seventy-ninth Congress) has served, and will continue to serve, to clarify the position of the governmental regulatory body. The Felin decision antedates the Administrative Procedure Act and must be noted for the philosophy it develops as well as its timing. It is evident that one who proposes the use of reproduction cost or "replacement cost" must meet the original cost theory point by point, and in order to prevail must show a definite superiority.

When Is a Public Utility Property "Devoted to Public Service"?—In selecting the more desirable price basis it is necessary to determine the time at which the unit of property is "devoted to public service." The "replacement cost" theory is predicated upon the idea that the title to the units of property remains in the owners, and that as the small increments are actually "consumed" in service they are then considered to be "devoted" to the public service.

Some of the advocates of the original cost approach believe that at the time the property is purchased, installed, and placed in service it immediately is "sold" to the public. The cost at the time of construction measures the value when new, and the resulting charges made for depreciation are merely prepaid expenses for the cost of the property to be returned to the owners over the period selected for charging off such depreciation. This line of thought parallels that of the investor who places his money in the debt obligations of utility companies. This investor expects to be paid interest at fixed intervals, and to have his principal returned to him at the maturity date. The investor anticipates no adjustment in either principal or interest because of variations in the purchasing power of his money which is used to purchase the property units serving the public.

Although the Court spoke repeatedly on this point some years ago, recent clear-cut pronouncements since the appearance of some of the new regulatory agencies are lacking.

Facts and Estimates.—In addition to the problems of pricing and time when a property is devoted to public service, other elements in a valuation include: (1) A determination of operating costs, (2) a determination of the proper units of property to inventory, (3) selection of the probable lives of the units for depreciation purposes, and (4) selection of the depreciation method. Table 3 indicates the source material available, and the amount of estimating required, for original and replacement cost analyses.

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[43] "Public Utility Property: Views of Commission Counsel As to Valuation," *Journal*, Am. Bar Assn., December, 1948, p. 1156.

Information and Personnel Available for Making Intelligent Replacement Studies.—In the early days of American industry reliable records of construction costs were notable by their absence. In the early part of the twentieth century considerable weight was accorded estimated costs of reproduction because original costs were not available, nor was there substantial variation between original and reproduction costs. Fact is generally considered preferable to estimate, and where estimates have been used to influence the finding of value "proof of fair value must be full and convincing" as in the Missouri rate cases, Knott versus the Chicago, Burlington and Quincy Railroad Company (230 US 474) in June, 1913.

TABLE 3.—VALUATION BASIS FOR AN ORIGINAL AND REPLACEMENT COST ANALYSIS

Original cost	Present construction cost of the most economical substitute plant
(a) PRICING OF UNITS	
Facts from actual construction cost records of the units in service	Part: Facts from cost records for units now in service where the construction cost levels have not changed. Part: Estimated present construction costs for installed units which are considered the most modern to operate. Part: Estimated present construction costs for the estimated most economic substitute units.
(b) OPERATING COSTS	
Actual experience of units in service projected into the near future	Part: Actual experience of units now in service, projected into the near future. Part: Estimated costs on estimated substitute units not yet installed or operated by this plant.
(c) INVENTORY OF UNITS	
Actual items now installed and operating	Part: Usable units now installed and operating. Part: Estimated substitute units.
(d) DETERMINATION OF PROBABLE LIVES	
From best data of actual experience and mortality characteristics of units retired and now in service	Part: From best data of actual experience and mortality characteristics of units retired and still in service. Part: Estimated lives on estimated substitute units.
(e) WHEN IS THE PROPERTY DEVOTED TO PUBLIC SERVICE?	
Construction date is definite	Time of consumption is a matter of estimate.

Estimates will be recognized as evidence if well supported, but the Court is likely to look with a jaundiced eye on the basic assumptions as it did in the case of the United Fuel Gas Company, et al, versus the Railroad Commission of Kentucky, et al.[44] It stated (concerning some inconclusive estimates) that:

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[44] *Public Utilities Reports,* Vol. 1921A, p. 433.

> "* * * such predictions can only be made on the basis of data which are not and cannot be known, and most of which are in the highest degree speculative. Such process of estimating value is without any known sanction."

Although economic analyses can be made with reasonable precision, they involve careful study and require time, money, and data. The requirements for making estimates of the most economical substitute plant involve all the hazards of estimating the present construction costs of the present plant and add the requirement of a proper selection of the most economical substitute. Such estimates vary from rough approximations to precise estimates.[45] Eugene L. Grant,[46] M. ASCE, suggests that "the burden of proof should always be on the proposed replacement." Professor Grant also indicates[47] that obstacles such as unconscious bias, personal financial interest, errors in costs or estimates, and incorrect estimates of economic lives of proposed machines and structures are ever present. In his closing paragraph, he states:[48]

> "* * * as a general rule, an estimator cannot be expected to make a successful estimate for a structure he would not know how to build. * * * Engineers are frequently called upon to make preliminary estimates of the cost of work with which they are not entirely familiar. Although many such estimates must be made, the estimator should realize the resulting danger of errors."

Certainly it is true that contractors have their own estimators to guide them in their bids. These estimators are specialists and in relatively short supply. The writer's experience and observations, since about 1928, lead him to the conclusion that most of the concerns in the middle west would not have sufficient information or an adequate staff to make the economic studies for their entire properties. They might find it a substantial burden to recruit, train, and maintain sufficient personnel to make these estimates on a company-wide basis. Many of the operating companies do have engineering and economic staffs engaged in making economic studies; but they are limited to the older property and those sections of plant that are unique and require special analyses. Since Mr. Scharff's proposal depends on the precision of many estimates, special efforts need to be made to convince all parties interested in valuation that adequate records and sufficient technical personnel are available to do an eminently satisfactory job.

A study[49] released in 1948 is concerned with this replacement problem and reports information on depreciation and replacement policies of almost two hundred different machinery manufacturing companies whose financial well-being is vitally affected by this problem. The summarization of part of their findings is interesting, as presented in Table 4.

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[45] "Engineering Economic Analysis," by Clarence E. Bullinger, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, Chapters 6 to 8.

[46] "Principles of Engineering Economy," by Eugene L. Grant, Ronald Press, New York, N. Y., 1938, p. 206.

[47] *Ibid.*, p. 363.

[48] *Ibid.*, p. 405.

[49] "MAPI Survey of Replacement and Depreciation Policies," *Bulletin No. 2119*, Machinery and Allied Products Inst., Chicago, Ill., October 1, 1948.

It is recognized that these data are not for the utility industry; nevertheless they do indicate a paucity of trained personnel and records necessary for making intelligent replacement studies in the machinery industry.

TABLE 4.—DEPRECIATION AND REPLACEMENT POLICIES OF MACHINERY MANUFACTURING COMPANIES (PERCENTAGES OF TOTAL COMPANIES REPORTING YES OR NO)

Item	Companies having provision for:	Yes	No
1	An equipment engineer that specializes in making replacement studies	28	72
2	Making periodic reviews of their equipment policies	35	65
3	Necessary collateral records of problems affecting replacement, such as spoilage of work, machine hours per year, and lost time for repairs	34	66

Aids to Clarification.—The field of valuation touches the entire national economy; it is not the exclusive province of the civil engineer, nor of the engineering profession. Accountants, architects, business executives, economists, financiers and investment bankers, lawyers, and public administrators—as well as many others—are vitally concerned with some or all the phases of valuation. Any acceptable method must be (*a*) capable of precise determination by qualified personnel; (*b*) reliable in application so that qualified valuators, even though representing hostile financial interests, will reach substantially equivalent results; (*c*) susceptible of proof; (*d*) appealing to reason and justice; and (*e*) understandable in principle to others, not only to the professional practitioners. These other parties are also concerned as to the source, the quantity, and the quality of the available personnel to expedite any proposed method of valuation. The necessary training required for new entrants into the field, and their ability to produce consistent results at reasonable cost, are also matters of interest.

The valuation engineer can assist in clarifying the basic principles by so preparing and presenting his findings that separate decisions become mandatory on (1) the proper units to include in the valuation inventory; (2) the proper pricing basis; (3) the acceptable methods of determining probable lives to be used in depreciation calculations, especially for the proposed units to be incorporated in the substitute plant analysis; (4) the depreciation methods to be used to calculate both annual and accrued depreciation; and (5) the time when the "property is first devoted to public service."

The Society has long been in the forefront of progress in the valuation field and has had a Joint Technical Division Committee on Depreciation, as well as a Committee on Valuation Procedure in the Engineering Economics Division. When the notable and comprehensive "Report of the Committee on Depreciation" of the National Association of Railroad and Utilities Commissioners (NARUC) was published, in 1943[50] a special committee of the Board of Direction, ASCE, was appointed to prepare an immediate reply.[5] This small committee was only partly representative of the entire Society membership; it was working under severe time limitations and the report, "Principles of

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[50] "Report of Committee on Depreciation," National Assn. of R. R. and Utilities Commrs., Washington, D. C., 1943.

Depreciation," was disappointing except for some excellent dissenting discussions. It appears to be the proper province of the constituted committees of the Society to make continuing studies on this subject and present to the membership carefully considered progress reports.

For several years the writer has waited for an expression from these committees or the members thereof to clarify points developed in the NARUC report or in Mr. Scharff's paper. It is hoped that Mr. Scharff's dissertation will serve to reactivate new Society committees so selected as to be truly representative of all segments of the Society membership, so that notable solutions to some of the present problems can be had and so that ASCE can resume its proper place in the field of valuation.

Mr. Scharff has rendered a distinct service by so presenting his proposals that the issues are joined. The decision of the Supreme Court on the appeal of the Attorney General of the United States will be read with interest in the hope that it will help chart the course of valuation in the light of present economic conditions.

Maurice R. Scharff,[51] M. ASCE.—Messrs. Kappeyne, Gunn, Lippitt, Marston, and Learned all emphasize the difficulties and uncertainties involved in estimates of value of service, and corresponding value of property and depreciation, such as the writer has suggested. In addition, Dean Marston points to the contribution to such uncertainty of "* * * the conflicting testimony of experts, paid by opposing parties in court proceedings."

The writer fully agrees that there are difficulties and uncertainties involved in the procedure which he suggested. He believes, however, that three comments should be made regarding the quoted part of the discussions.

In the first place, if "Each such party can be sure that it can hire a battery of 'experts' to support its particular contentions" the professions of engineering and economics have failed to secure acceptance of professional standards of conduct by "experts" from their ranks; or to secure recognition by industry and finance of the value of genuinely professional opinion as distinguished from readiness to support particular contentions. In either case it can never be hoped that any method of proof of value within the framework of the American economic system can ever be established, and the system itself will eventually be impaired by the persistence of "experts," industrialists, and financiers in these practices. Discussions of methods of valuation are worthwhile only in terms of professional competence and conduct, and belief in the ability of a democratic capitalistic society to work out and accept sound conclusions.

In the second place, difficulty and uncertainty in establishing a sound conclusion are not sufficient grounds for its rejection and for acceptance of a conclusion that is unsound. Mr. Lippitt has made a helpful contribution in calling attention to the Minority Report of the Commission on the Revision of the Public Service Commission Law in New York on these difficulties and uncertainties, and to certain legal opinions which he cites as bars to acceptance of the writer's proposal. Nevertheless, it has been the writer's attempt to suggest that the regulatory commissions and courts have fled from these uncertainties and difficulties to unsound procedures of valuation and rate making, and the emphasis on the obstacles referred to is not a conclusive answer to the writer's contention that his proposal accords with sound engineering economic concepts.

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[51] Cons. Engr., New York, N. Y.

In the third place, it is the writer's belief that the difficulties and uncertainties referred to, great as they are, and great as they have always been in connection with all valuation procedures, are, nevertheless, exaggerated. It has been the experience of the writer that substantial agreement between qualified engineers, who adhere to professional standards, is not too difficult to reach on what constitutes modern efficient design and the cost factors that correspond with it. The real requirement is for clarification of the principles under which these engineering-economic factors are to be applied.

Messrs. Kappeyne, Gunn, and Lippitt also indicate that the writer has not succeeded in clarifying the distinction he sought to emphasize—between valuation and rate making on the one hand, and original cost and accounting on the other. It was intended to suggest that it would be advantageous if complete separation could be restored between these two groups of concepts. The only suggestion as to any continued relation between them was for the use of proportionate loss in value as a measure of depreciation reserve requirement in accounting (as a substitute for the more conventional straight-line or compound interest reserve requirement standard) in the undesirable case that the regulatory commissions continue to insist that depreciation reserves represent "approximately the amount of depreciation actually existing in the property."

Mr. Kappeyne argues that investors who furnish the capital required for a utility:

> "* * * not only are entitled to the protection of their investment, but also are entitled to the opportunity of earning a return thereon that they had good reason to expect at the time the investment was made."

Mr. McKean quotes from the very interesting article of Everett C. McKeage[43] to somewhat the same effect. These arguments (which are commonly advanced in support of the prudent investment or original cost less accrued depreciation theory of rate making) seem to the writer to be based on the erroneous assumption that the property right in public utility property is substantially different from that in other private property.

In a dissenting opinion, the late Mr. Justice Louis D. Brandeis, joined by the late Mr. Justice Harlan F. Stone, in New State Ice Company versus Liebmann (285 US 262) stated that:

> "The notion of a distinct category of business, 'affected with a public interest,' employing property 'devoted to a public use' rests upon historical error. * * * In my opinion, the true principle is that the state's power extends to every regulation of the business reasonably acquired and appropriate for the public protection."

This statement seems to the writer to be the sounder doctrine. The real question, therefore, is whether the scope and the character of the regulation applied are reasonably required and appropriate for the public protection; and it is the writer's contention that this criterion is met within the framework of a competitive capitalist system only when the concept of the value of the service, as measured by competitive cost, is applied.

Furthermore, the writer is confident that those who rely on any assumed guarantee by the public of the right to recover their investments, however prudent at the time they were made, or to continue to earn a return on such investments less some accounting reserve for depreciation, after some major technological or economic change which substantially reduces the competitive cost of service or limits the market for sales because of the competition of alternative services, are deluding themselves. A substantial decrease in the cost of generating electric energy, for example, such as might conceivably result from developments in the field of atomic power, would inevitably be followed, in the opinion of the writer, by a reduction in the earning power and value of investments in steam boiler plants—and, if the scientists succeed in converting nuclear energy to electric energy without the intervention of the heat cycle, the effect might readily extend to turbo generator plants as well. The effect of the competition of alternative services on the earning capacity and value of street railway properties, and to a lesser extent on the earning capacity and value of manufactured gas properties, is also too well known to require further comment.

The same error, as well as failure to appreciate the distinction (which the writer attempted to make between valuation and accounting, as previously referred to), is involved in Mr. Lippitt's discussion of the correlation of annual and accrued depreciation. If, as the writer contends, rate making is a species of economic price fixing unrelated to accounting, and if accounting is a conventional method of amortizing investments unrelated to valuation and rates, the entire concept that annual and accrued depreciation are "contributed by consumers" is meaningless.

Mr. Burnell, although expressing general agreement with the paper, raises one question which the writer is not sure he fully understands. To exemplify his question, Mr. Burnell selects the item of coke ovens, with a service value of $1,165,800, and a future capitalization period of 25 years. Then he states:

> "The next implication of the process is that the accumulated amount of the depreciation reserve for the present substitute plant at the end of the 25-year period, if today's assumptions of remaining life are consistent, must also be $1,165,800."

The writer does not understand why such a relation is implied. In his opinion, the only implication is that the annual depreciation on the $1,165,800, compounded for the future capitalization period of 25 years, must equal the same amount. The facts as to this relation are, that the sinking fund payment factor for 25 years is 0.0182267, which, applied to the service value of $1,165,800, gives an annual sum of $21,248.70. The present-worth factor for an annuity of one per year for 25 years is 54.8645 and when this is multiplied by $21,248.70 it gives $1,165,800.

Mr. Burnell also suggests that public utility property can be kept in physical condition for service indefinitely by adequate maintenance and replacement of parts, and that the only difference between the value of a new property and an old one is the capitalized value of the difference in their annual cost of maintenance and physical upkeep; and Mr. Learned suggests that the writer's own proposal, in the absence of changes in the arts, would correspond with no increase of accrued depreciation unless unit operating and maintenance costs increased with age. These contentions seem to leave out of account the facts that every estimate of value is implicitly, if not explicitly, an estimate of the present worth of an annual advantage from the use of a capital good for a future capitalization period; and that the future capitalization period of an old capital good which enters into such implicit or explicit present-worth estimate is less than the future capitalization period of a new capital good.

Mr. Gunn suggests that he would never recommend a capital expenditure on the basis of a capitalization period, ranging up as far as 80 years, and that for such purposes periods of more than 20 or 30 years would be unrealistic. The writer agrees with him that the evaluation of depreciation during the useful life of property must be governed by the same principles that justified its instalment and by which its replacement or abandonment will be dictated; but he ventures to doubt that Mr. Gunn really intends to suggest that the justification of the capital expenditures for the dams, involved in the cases to which he refers in his discussion, should have been based on capitalization periods of no more than 20 or 30 years.

Mr. Gunn also suggests the separate treatment of depreciation due to physical causes and that due to economic causes. This is a point of view to which the writer adhered in 1939 when he contributed his paper on "Depreciation and Obsolescence."[52] He has subsequently come to recognize more clearly that value of service, and value and depreciation of property, are indivisible economic concepts, and that the effort to measure, separately, depreciation due to physical deterioration (or to take it into account, in connection with obsolescence, by "observation," as Mr. Learned suggests) is futile.

Dean Marston suggests that the correct application of the essential principle of Smyth versus Ames is that values are determined by judgments and that:

> "Every factor which should affect such judgments must be considered and these factors given their 'just and right' weights in each case—as determined by sound judgment, not by any formula."

He then goes on to suggest that the present trend is

> "* * * to give more and more weight to fully itemized property inventories showing the actual original costs of the individual items of the

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[52] *Transactions*, ASCE, Vol. 104, 1939, pp. 1127–1138.

> actually existing properties; and to only such overhead percentages of cost as have been actually incurred in the past; but not excluding such substitution of present prices (and the resulting cost values of individual items) as sound, conservative judgment deems certain to prevail for a considerable future period."

This application of the Smyth versus Ames principle makes the determination of value entirely subjective, and it has been the writer's intention to suggest that it has led to the present confusion. Indeed, he would go further and argue that it was the failure of the engineering and economic professions to develop a sounder application of the Smyth versus Ames decision that led to the extreme development of the subjective approach which reached its climax in the case in Wisconsin of the City of Two Rivers versus the Commonwealth Telephone Company.[53] In that case the Wisconsin Public Service Commission carried the subjective approach to its logical conclusion and stated:

> "* * * after careful consideration, we have concluded that reasonable rates are not properly arrived at merely by means of a formula whereby a percentage figure (which is deemed 'a reasonable rate of return') is applied to another figure which is termed a rate base. * * *"

and they stated further:

> "The rates herein prescribed are estimated and intended to afford approximately an annual net profit of a determined number of dollars which we think it reasonable for the utility to enjoy from the operation of its business."

This determination was reversed by the Wisconsin Circuit Court of Dane County[54] in deciding the case of the Commonwealth Telephone Company versus the Public Service Commission and the reversal was affirmed by the Supreme Court of Wisconsin[55] on the ground that its inherent fallacy appeared in the statement just quoted, and that there was no way for either the Commission or a reviewing court, or a utility, or the public, to determine whether the profit which the Commission thought proper was actually proper unless the Commission made specific findings of the "relevant facts and circumstances." "It was not the intention of the legislature," the Supreme Court of Wisconsin stated, "to bestow such arbitrary powers upon the Commission, and nothing in the statutes can be so construed."

The writer called attention, at the conclusion of his paper, to a decision of the United States District Court for the Eastern District of New York in a condemnation case, holding that a procedure substantially identical with that proposed in the paper "would seem to provide a fair standard of value for present purposes" (65 Federal Supplement 333 and 71 Federal Supplement 248).

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[53] *Public Utilities Reports*, New Series, Vol. 70, 1947, p. 5.

[54] *Ibid.*, Vol. 71, 1948, p. 65.

[55] *Ibid.*, Vol. 73, 1948, p. 97.

Consequently, he feels that he should report the subsequent development to date in that proceeding (United States of America Versus 25.4 Acres of Land and 53¼ Acres of Land). The decision of the District Court was appealed to the United States Court of Appeals for the Second Circuit and in May, 1948, the Circuit Court handed down its decision, reversing the District Court and remanding the case for further hearing (168 Federal 2d; 391). The principal grounds for reversal were: First, that, in its acceptance of the "estimates of the defendants' expert, accompanied as they were by an accumulation of imprecise theory in what is considered the strong tradition of American judicial valuation," the District Court "got itself into the position of assuming a loss before one was proven and of measuring it by the extent of curtailment of the facilities"; and, second, that the trial court declined to consider evidence as to increased sales of electricity and gas to the government after the date of the condemnation. The Circuit Court concluded that:

> "* * * the value at the time of taking must be developed, in default of a sale price, largely from a consideration of past earnings and such showing of prospective earnings as can be made."

Subsequently, in February, 1949, a third hearing was held in the United States District Court and the case was resubmitted for decision in accordance with the opinion of the Circuit Court. On April 6, 1949, just as this closing discussion was in course of preparation, the District Court handed down its opinion, holding as follows:

> "Since the evidence * * * is thought to supply that which was deemed necessary to completion of a proper record, there remains but to say that, while the figures previously adopted could reasonably be increased in light of the testimony taken and exhibits filed at the third hearing, to yield to the impulse so awakened might lead to misunderstanding; accordingly the decision will be that the court hereby makes and states the following awards in this proceeding, to these claimants. * * *"

after which the identical awards made after the second hearing before the District Court are restated.

In the course of this opinion, the District Court stated:

> "At the second hearing, the expert Scharff whose qualifications were not impaired by anything shown in the record, and who impressed me as a technically informed person, expressed his opinion as to a fair measure of compensation to be paid for the taking of these properties and franchises. His testimony is somewhat discussed in the opinion which appears in 71 Fed. Supp. 248 at 252 et seq., and consequently need not be further expounded. It has not been overlooked that his estimates have been characterized as being accompanied by 'an accumulation of imprecise theory.'
>
> "That statement may not be ignored by this court, but the Government has not developed it for present purposes, i.e., the imprecise theories have not been identified or discussed in any analytical sense.
>
> "Nor have contrasting theories been advanced by any witness except Mr. Shlichta, who thought that the useful life of these properties in their

> dual aspect was from 15 to 20 years, instead of the 40 years attributed to them by Mr. Scharff. Since the court used a 20-year period because that measures what is usually called a generation, it seems, with all deference, that Scharff's opinion has indeed not been discredited. Since it was an opinion and could be nothing more, and since it was offered by one whose training and experience invite respectful attention, I am aware of no reason why it should not be consulted by this court in reaching a conclusion."

At the close of the third hearing in the District Court, counsel for the United States stated that he would appeal again if an award were made, so that this case may yet become something of a "cause célèbre."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2380

UNUSUAL DESIGN PROBLEMS—SECOND TACOMA NARROWS BRIDGE

By Charles E. Andrew,[1] M. ASCE

With Discussion by Messrs. O. H. Ammann, Charles A. Ellis, F. H. Frankland, and Charles E. Andrew.

Synopsis

The purpose of this paper is to outline the unusual problems that confronted the designing engineers and consulting board in developing a safe, yet economical, design to replace the first Tacoma Narrows Bridge in the State of Washington which had failed under dynamic wind forces on November 7, 1940. Some of these are problems applicable to the specific problem at hand and result from physical conditions of the former structure as they remained after the failure. Although many of the solutions will be of interest to engineers, they will not be useful in future bridge design unless a similar failure should occur to make them so. Other solutions should have important bearing on the design of future suspension bridges, especially those which involve long spans and are situated where traffic is relatively light, requiring extreme economy. The first Tacoma bridge was such a structure. The designer was limited by a relatively low estimated income from tolls and consequently extreme economy. In spite of these limitations, the first structure did meet all the requirements of modern suspension bridge design practice and was amply designed to withstand all possible live loads, dead loads, temperatures, and direct wind forces as they had been applied in other bridges.

To meet the economies required, the designer of the first Tacoma Narrows Bridge reduced the ratio of width to span length far below any former value and adopted the principle—which had been widely advanced by several prominent bridge designers—that deep stiffening trusses were unnecessary and that a proper limiting of grade change (accomplished in this case by a flat cable curve) was all that was necessary. Following this design principle, a shallow plate girder 8 ft deep was substituted for the conventional and much deeper stiffening truss.

Note.—Published in December, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., Washington Toll Bridge Authority, Tacoma Narrows Bridge, Tacoma, Wash.

The completed structure, because of its extreme freedom of motion and faulty form, became a ready prey to aerodynamic forces (the importance of which were not recognized at that time), and failure resulted. This bridge, however, withstood serious oscillatory motions for a period of 4 months although traffic over the bridge was 60% greater than had formerly been estimated as a basis of financing. Thus, a considerably higher cost of the original bridge and a more rigid, conventional design would have been justified. This experience demonstrates the importance of careful traffic estimates and the danger of extending design precedents too far in order to meet limited appropriations.

Corrective Measures Applied To The First Bridge

The first problem to engage the attention of engineers in charge of construction on the first bridge was one of attempting corrective measures to reduce the motions so that they would not be detrimental to the structure. It was quite evident that the first bridge, with its extreme freedom of action, was subject to dynamic forces which had not been considered in the design and had fallen into the category of a machine rather than a static structure. Energy was being fed into that structure by some form of pulsating force derived from the wind, separate and apart from the normal wind pressure forces considered in structural design practice. The first bridge was unable to dissipate this energy until large amplitudes had developed. Two courses of corrective action were feasible:

(1) To increase the dissipating capacity or damping of the structure by adding structural members or mechanical devices; or

(2) To reduce the amount of energy being fed into the structure by changing its form.

A corrective measure suggested by the designer of the first bridge came under classification (1). It consisted of diagonal cable ties connected at the top to the central cable bands of the main span and at the bottom to the top flange of the distributing girders at points one panel distant from the center. The purpose of these ties was to eliminate the differential horizontal motion that occurs between the main cable and the girders under single-noded vertical or torsional motion. To prevent the horizontal motions of girders from becoming synchronized with those of the cables under single-noded vertical motion (which was the most harmful), hydraulic jacks were installed; but they were poorly designed and in addition were connected to the towers near the center of the floor-level cross strut, allowing considerable side deflection. The jacks became ineffective as soon as the cylinder packing failed and free end movements, as great as 2 in. or more, occurred often when the bridge was oscillating.

There is little doubt, however, that in spite of the jack failure these central diagonal ties limited the amplitude of vertical motion and prevented single-noded torsional motion. In resisting these motions they were subject to continual reversal of stress and finally on the day of failure either one of the cable bands slipped, or the ties broke, permitting the bridge to go into single-noded torsion resulting in failure.

In an attempt to add further resistance to motion, vertical cable ties were installed between the girders and ground anchors at points seven panels out from the "hold-downs" in the side spans. These ties almost entirely eliminated vertical motion in the side spans but had only negligible effect on motion in the main span. Various systems of diagonal ties from the tower tops to various points on the main and side spans, and from the towers at roadway level to points on the cables, were considered and tested with models. Attempts to stabilize other suspension bridges of similar type, but of much shorter span by similar tie systems, had been only partly effective and model tests were not promising. To be effective such systems must be set at high initial tension which distorts the bridge from its natural dead-load position and, to a considerable extent, defeats the action of a suspension system. They are subject to high live load and temperature stresses. In consequence, they were not used.

Just prior to the opening of the first bridge, and four months before its failure, studies were started along the second possible course of action—that is, to reduce the amount of energy being fed into the structure. This was a problem in aerodynamics and the highly developed science of airplane design had established the fact that stability depended largely on the matter of geometric shape or form. If the amount of energy being fed into the structure could be reduced materially, the existing capacity of the structure to decrease amplitude could be increased correspondingly, and the life of the structure could be prolonged.

It was impossible to make drastic alterations in the general form of the structure, of course; but there were several methods by which the character of air flow could be changed materially at a minimum expense. Several types of streamlining, consisting of curved fairing of different forms attached to the face of girders and flat plates extending the girder widths, were tested by 20-scale sectional models at the University of Washington in Seattle. In other model tests various arrangements and sizes of holes were cut out of the girder webs. These tests were made on three-dimensional, static models in a high-speed wind tunnel using the slope of the lift line as a criterion for stability. It was found that this criterion indicated extreme instability in the original bridge form for horizontal and small angles of wind attack.

Several forms of streamlining, and one pattern of holes cut in the girder webs, indicated a very decided improvement in stability and encouraged the engineers to such an extent that material had been ordered and it was planned to proceed with the most likely and feasible scheme approximately one week before failure.

These early experiences and tests have been mentioned at some length because they did have a considerable bearing on the procedures that followed.

THE USEFUL REMAINS OF THE FIRST STRUCTURE

The very complex problem of removing the cables, towers, and what remained of the suspended floor arose after the collapse and lasted for more than a year. The methods used can be considered unusual, largely because conditions due to World War II required that all possible material, including cable wire, be salvaged. Probably no other major parallel-wire cable had ever been taken down under similar circumstances and difficulties.

Actual traffic over the bridge during its brief existence had greatly exceeded all preliminary estimates; and this was conclusive proof that a bridge at this location was a public necessity. Thus, a considerably higher cost would have been justified. As a result the Washington Toll Bridge Authority decided to rebuild the structure as soon as possible—if a safe structure could be designed which would fall within a cost that could be amortized as a toll bridge by probable traffic. A consulting board was appointed in the fall of 1941, a design department was organized, and intensive studies were begun.

In addition to the usual problems involved in suspension bridge design, the engineers were confronted with two which were decidedly unusual. The first one was to fit a new, wider, and heavier superstructure to the former piers, anchorages, and approaches which had been uninjured by the superstructure failure. These parts of the former structure have a value of approximately \$3,000,000 and therefore must be utilized as far as possible in the second bridge. The other unusual problem (which was without precedent) was to design a superstructure that would be free from the oscillations inherent in the several bridges similar to the Tacoma bridge which had been completed at approximately the same time. The solution of these two unusual problems are dependent, one on the other.

First, the usual problem of fixing roadway width and capacity was established after a re-estimate of traffic had indicated that a four-lane structure might readily be required within 20 years—fixing the width between cables at 60 ft. Next, attention was given to the main piers. Would they accommodate and sustain the wider, heavier superstructure with safety? Fortunately, the design of the main piers was controlled largely by forces resulting from the extremely swift tidal flow, possible ship collision, and the dead weight of the great, deep piers themselves. The weight of the first superstructure constituted only approximately 6% of the total foundation pressure, and, in consequence, increasing the superstructure weight by from 50% to 60% was of no consequence.

It was also fortunate that the towers of the first bridge were constructed with battered legs—the legs being 50 ft on centers at the pier tops and 39 ft at the cable shoes. The towers of the second bridge were designed with vertical legs 60 ft apart. Thus, it was only necessary to remove the present pedestals above the main pier tops and to design new and larger pedestals 60 ft apart. Actually, the arrangement of walls in the cellular body of the piers was such as to give a better distribution of load than before. There was the problem, however, of securing proper anchorage between the main body of the pier and towers to resist overturning during the free standing state. The development of sufficient anchorage, by anchor rods, to the body of the piers would have required extensive changes and some underwater work. The necessary temporary anchorage was developed, however, by the use of outrigger braces bearing on the outer walls of the cellular pier and connected to the tower approximately 50 ft above the pier top. These outriggers are made up largely from chord members and stringers to be used in the suspended structure after the cables are spun.

Another rather unusual problem was presented at the anchorages. They remained intact after the failure with embedded eyebar anchors 39 ft apart, whereas the anchors for the new design are 60 ft apart. The design department spent considerable time and study in an attempt to use the former eyebars in their original position. To do so the new cables would have to be deflected together back of the "hold-downs" by tension and compression struts Spinning would be greatly complicated. Also, the heavier bridge would increase the horizontal pull of the cables about 25% with the sag ratio increased to one tenth instead of one twelfth as in the former structure. This fact necessitated not only adding to the existing eyebars in some manner but also enlarging the anchor blocks to increase their weight. The studies led to the conclusion that it was better design to abandon the former anchor bars, disposing of only those within the volume of concrete which would have to be removed, to provide space for a complete new set of anchor bars spaced 60 ft center to center. This procedure will require the removal of approximately 12,000 cu yd of concrete from the existing anchors to accommodate the new bars. Then approximately 20,000 cu yd will have to be placed to embed these bars and increase the weight of the blocks. The new concrete, heavily reinforced, will be poured in blocks to take care of shrinkage and will take the form of a horseshoe around the sides and back of the remaining core of the former anchorage. In this way approximately two thirds of the former anchor blocks are utilized.

FORMULATING A DESIGN CRITERION

Experiences with, and study of, the first bridge furnished convincing evidence that development of a design which would possess stability and yet be economically possible would be most difficult. The problem was without precedent and was one which, in a major part, fell outside the structural field and into the aeronautical engineering field.

A few relatively small and poorly designed suspension bridges had failed from similar causes and several, modern, major bridges of similar design were experiencing undesirable, but not necessarily detrimental, movement. Nevertheless no successful corrective measures or methods in design had been made. The oscillatory motions could be caused only by alternating forces derived from the wind. Long experience in the design of airplanes was proof that forces on a wing are governed largely by its shape. There are only two logical ways to stop a moving automobile: (*a*) Removal of the driving power; and (*b*) application of the brakes. If the power of the engine is reduced, the speed will be less and the braking power required to stop the car will be decreased in proportion.

Common sense leads to the design criterion set up at the beginning:

> "The first effort of the Designer should be to reduce the forces acting to a minimum and, having done so, then supply sufficient energy dissipation to reduce movement to a point which will not be injurious to the structure or noticeable to the traveling public."

In general, this is the rule followed by the engineers in devloping an apparently stable design, and many new problems were encountered and much original research was made in the process.

At the outset, the engineers had a fairly complete and relatively accurate record of the movements in the first bridge; also, they knew that practically all the existing suspension bridges of similar form were subject to undesirable motion of more or less amplitude. By their inherent freedom of motion, all suspension bridges are subject to large live-load deflections and, therefore, could not be harmed by small oscillatory motions, which probably do occur in all suspension bridges but which must be limited to unnoticeable amplitudes.

With the exception of the observations on the first Tacoma bridge, there were no dependable suspension bridge records to draw from in determining the answers to questions such as:

a. What movements have occurred?

b. What were the character, amplitude, and frequency of the movements?

c. What were the accompanying angle of attack and the velocity of the wind?

d. Has the bridge ever been subjected to heavy windstorms and from what horizontal direction?

Without answers to these questions, there was no assurance, except hearsay, that many, or any, of the existing bridges had been subjected to their critical storm; and, for this simple reason, it is fallacy to fix design criteria on the supposedly observed performance of existing suspension bridges until accurately measured answers to questions *a* to *d* are found for many bridges under severe wind conditions. There was quite conclusive evidence, however, that the existing girder type of suspended systems had proved to be much more susceptible to aerodynamic forces than the truss stiffened systems. This evidence eliminated any consideration of the girder type in the redesign.

Aerodynamic Considerations

During many years of experimentation, the "aerodynamicist" has sought out almost perfect forms for airplane wings and, in so doing, has reduced to practically zero the forces that cause "wing flutter" (one of the most common causes of airplane failure). "Wing flutter" is analogous in many ways to oscillation in a bridge. However, the type of air flow around a perfectly formed wing is entirely different from that to be expected around blunt or rectangular forms such as those that must be used in a practical bridge form. One is a smooth, clinging, nonturbulent flow; the other is an exceedingly turbulent flow forming whorls or so-called vortices, and the designer is at once confronted with the probability that the forces causing the two vastly different flow patterns are widely separated in their action. Furthermore, the highly developed knowledge of aeronautics, as applied to the airplane, cannot be relied on as applying to a bridge without extensive original research involving turbulent flow. This research followed along much the same lines as that for the airplane—that is, preliminary designs were made in the design department using all available data and knowledge which might improve stability and still preserve economy. These designs were then tested by the use of aerodynamic models in a specially built wind tunnel.

Following the aforementioned design criterion, a general bridge form was adopted which presented the least resistance to the passage of wind through and around the structure and still constituted a practical bridge form. In general, this form included or excluded the following structural characteristics (see Fig. 1):

1. Deep open trusses were preferred to plate girder stiffening members.
2. Open trussed floor beams were used instead of the conventional plate girder type.

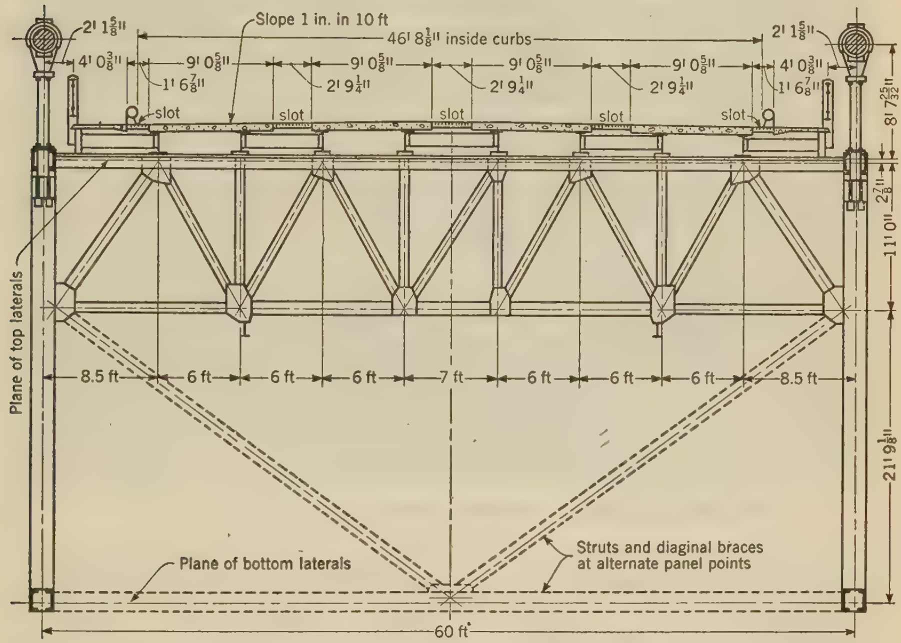
2' 1⅝"
Slope 1 in. in 10 ft
46' 8⅛" inside curbs
2' 1⅝"
4' 0⅜"
9' 0⅝"
9' 0⅝"
9' 0⅝"
9' 0⅝"
4' 0⅜"
1' 6⅞"
2' 9¼"
2' 9¼"
2' 9¼"
1' 6⅞"
slot
slot
slot
slot
slot
8' 7 25/32"
2 7/8"
11' 0"
21' 9⅛"
Plane of top laterals
8.5 ft
6 ft
6 ft
6 ft
7 ft
6 ft
6 ft
6 ft
8.5 ft
Plane of bottom laterals
Struts and diaginal braces at alternate panel points
60 ft

Fig. 1.—Section at the Center Line

3. Frontal area was reduced, as far as possible, by using relatively shallow chord sections, as well as narrow faced members for verticals, web members, and floor beams. Thus, by breaking up the frontal area into many small parts instead of one large commanding area, as in the former bridge, it was hoped to set up a condition in which the aerodynamic forces would tend to fight or nullify each other and, at the same time, reduce drag.
4. A streamlined rail section was designed.

These four general characteristics were incorporated in an attempt to reduce the aerodynamic forces. It was considered impossible, however, to reduce these forces by a sufficient amount to insure desired stability and a further effort was made to increase the capacity of the structure to resist motion. This, in general, could be effected by: (*a*) Increased weight, (*b*) increased stiffness, and (*c*) external mechanical absorbing devices.

Additional weight is expensive as the cost of foundations, cables, towers, and anchorages increases in proportion. Thus, sufficient weight increase might readily exceed the economic limit. Additional stiffness, which could be accomplished most readily by a flat cable sag, seemed more economical. However, a careful study had determined the most economic sag ratio to be one tenth, and added stiffness was sought through an increase in moment of inertia of the stiffening trusses by making their depth a maximum, at the same time, using a minimum chord section. The minimum chord section was determined by lateral wind and distance between cables, both of which were fixed and the depth of the truss was determined by live moments and shears.

Under the live loading adopted for the design of stiffening trusses, the maximum depth of truss within the limiting stresses was 33 ft. Cables and towers were designed by conventional methods.

Model Studies

The preliminary design plans were then sent to the laboratory where a wind tunnel had been constructed for model tests. Many unusual problems were encountered in performing these tests, and it is possible to enumerate only a few of the important ones.

To make model tests of a bridge, it is first necessary to have a design; then a three-dimensional aerodynamic model must be constructed to a scale that truly simulates the prototype, both in form and elastic properties. This scale must be relatively small because of the size of the bridge and the limitations of a suitable wind tunnel. A wind tunnel must be built that will be capable of projecting a wind stream of scaled velocity over a model of the entire bridge at different angles of attack.

The aerodynamicist has important problems such as:

A. When tested in a wind tunnel, will a small-scale model of a bridge perform in the same general manner, in so far as stability is concerned, as the prototype?

B. How small can that scale be?

C. Can a wind tunnel be constructed which will simulate the action of wind in nature?

Technique and instruments must be devised to measure low wind velocities, angles or wind, amplitudes, and frequencies of motion.

Since complete models are extremely expensive and since, in all probability, changes in form will be necessary before satisfactory stability can be realized, is it possible to obtain results from section models (Fig. 2) comparable with those obtained through complete models? Will the wind tunnel tests yield results that are on the side of safety? These, and many other questions, face the aerodynamicist. Since no aerodynamic model of a bridge had been made or tested, no precedent existed which could be referred to in answering the foregoing questions.

More or less accurate records of movements existed in only one bridge—the first Tacoma Narrows Bridge. These records now became valuable as a source of checking model action in a wind tunnel.

Soon after the completion of the wind tunnel and as the first experiment, a 1:50 scale complete model of the former bridge was built and tested. The test results showed quite remarkable similarity of motion and gave gratifying proof that models tested in this manner could be relied on as a basis for predicting the probable action of a prototype.

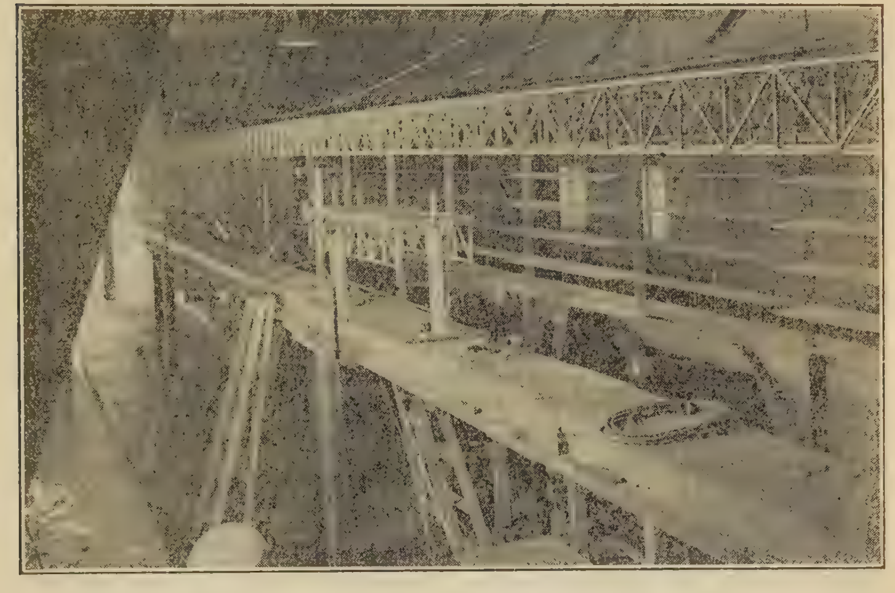

Fig. 2.—Full Model and Section Model of the Proposed Design

The next step was to build a 1:50 scale complete model of the proposed design and run tests. It was found that stability had been greatly increased over that of the former bridge. Complete stability had been established in all angles of attack between 2° upward and 5° downward. This degree of stability might have been considered sufficient as smoke tests in the field indicated that, in general, winds are horizontal at the site. However, it was feared that cross winds blowing toward the high bluff on the west shore might deflect the wind stream upward as much as 6° or 8°; and a search was started, through modification of form, to establish stability at higher angles. Several changes at the leading edge which had a marked effect toward added stability were suggested by the design department and tested in the laboratory. Continued study in the design department resulted in a suggestion that the solid concrete deck, since it had been used on all tests to that date, be fitted with open steel grid slots of varying widths between each of the four traffic lanes and at the curb (see Fig. 3). Sketches were sent to the laboratory and tests were run on varying widths of slots with quite remarkable results.

Complete stability was established over all downward angles of attack between 0° and 15° with only very slight limited instability occurring under upward angles of attack between 3° and 7°. Wind velocities in all tests cover the range of velocities between 0 and 120 miles per hr. Movements were now noncatastrophic between critical angles whereas, in all tests prior to this time, all motions beyond the critical angle were catastrophic. The maximum double amplitude that could be excited over the limited range of noncatastrophic motion was less than 1° in torsion. No purely vertical motion was observed. Translated to the prototype, this amplitude represents an approximate total movement of 6 in. at the chords which is considered of no importance and one which is to be expected in any suspension bridge.

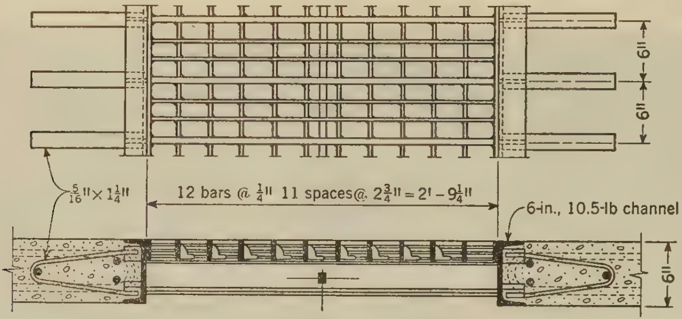
6''
6''
$\frac{5}{16}$'' × $1\frac{1}{4}$''
12 bars @ $\frac{1}{4}$'' 11 spaces @ $2\frac{3}{4}$'' = 2' – $9\frac{1}{4}$''
6-in., 10.5-lb channel
6''

Fig. 3.—Plan and Section through a Slot

Major Problems

Throughout these design studies and model tests there were at least four major problems that were troublesome. No definite answer to them has been established and their significance in bridge design must remain a matter of judgment of the part of the engineer, at least until much more research work is done.

The first one is the matter of damping in a suspension bridge. There is no actual, measured value for the damping in a suspension bridge. Since damping (either structural or mechanical) is the only method by which dynamic energy is fed into the bridge and by which resulting amplitudes can be dissipated and controlled, it becomes of utmost importance.

Because the damping capacity of the first bridge is unknown, the adopted design for the second bridge, or for that matter that for any suspension bridge, cannot be truly simulated in a model. A full model, when once built, has a fixed damping capacity which cannot be reduced; and, therefore, the model is constructed at the lowest possible value. Actual measurements of damping are available for trussed bridges, however, and the values for the logarithmic decrement (a measure of damping) vary from a maximum of 0.2 to a minimum of approximately 0.02. The actual total logarithmic decrement, in still air, of the full model of the final design was 0.048. This value, in the judgment of the engineers, was considered a maximum, with 0.02 a minimum.

By the use of section models in which the decrement could be changed through a range of from 0.2 to less than 0.02, the effect on amplitude of a change in decrement was obtained. Using the graph so determined, it was found that the 1° maximum double amplitude determined for a decrement of 0.048 on the full model would probably be increased to a maximum of 3° in torsion if the decrement was reduced to 0.02. Since it seems that only engineering judgment can be used to estimate the actual damping of a long span bridge, it is vitally necessary that further research be undertaken to establish the decrements of several suspension bridges by actual measurements in the field.

The second important problem, which has been solved only partly, is the question of scale effect involving the Reynolds number when turbulent flow is involved. Does the small-scale model truly represent the prototype aerodynamically?

Numerous hydraulic experiments, as well as high-velocity wind-tunnel tests on a 1:20 scale section model at different velocities, seem to indicate that the Reynolds number may be ignored in cases where the flow pattern is extremely turbulent. Although reliable authorities agree with that conclusion, further proof should be established.

The third problem involves a thorough study of winds as they exist in nature. Such studies may well become extremely important in the preliminary design studies of every suspension bridge and may have considerable influence in regard to the type of structure to be used. A full knowledge of the natural wind including maximum velocity, direction (both horizontal and vertical), the effect of terrain on angle of attack, and probable duration of periods of nearly constant velocity has a bearing on design and model tests. The sufficiency and safety of existing bridges cannot be judged unless considerable knowledge of the actual storms to which they have been exposed is available.

In model studies it is not possible to design a wind tunnel that will produce a wind stream simulating the natural wind. In the wind tunnel the air stream is constant in direction and velocity and can be maintained in that manner for an indefinite period. In nature the wind is constantly changing in these respects and a given constant velocity and direction exist for only a few seconds.

In tests on the first bridge, model motion began very quickly, and change in velocity or direction would damp out previous motion and immediately start some other type. Of course, this bridge was extremely unstable and possessed little damping capacity. This same rapid building and changing of modes and motion occurred on the prototype, apparently following the changing natural wind.

In the model of the final design which possesses very high stability, comparatively long periods of time under constant velocity and direction of wind were required to reach the small maximum amplitudes. Thus, there is a great possibility that the much more rapid change in the natural wind will result in damping effects and movements will not be as severe as the tests indicate.

The fourth problem involves the coordination of section model tests and full model tests. Although tests for the Tacoma Narrows Bridge have been most useful along these lines, not enough has been accomplished to insure reliance upon their results without the proof of a full model. Section models cost only a fraction of the cost of the full model and, for less expensive wind tunnels, they are necessary. Further research is required.

Engineers in charge believe that the design from which the final full model was made will be entirely satisfactory and free from noticeable vibration. In fact, it is entirely probable that the preliminary design without the slotted deck would have been satisfactory also. There was some shadow of doubt concerning the question of damping and scale effect, however, and two methods of increasing the damping effect were added, namely: (1) A double lateral system, and (2) mechanical energy absorbing devices.

The double lateral system (see Fig. 1) greatly increases the frequency of motion in torsion and, likewise, the torsional stiffness. It has no value in resisting vertical motion, but is very beneficial in coupled motion should such motion occur. Mechanical energy absorbing devices consisting of hydraulic cylinders were incorporated in rigid diagonal ties at the center of the main span (see Fig. 4), and between the top main span and side span stiffening truss chords. They were also placed between deep outriggers extending downward from the bottom chords and the towers. It is not anticipated that these jacks will be particularly effective when small amplitudes of motion occur. They are installed as a matter of insurance against motions greater than those anticipated and are effective in all forms of torsion or vertical motion. The design of both the double lateral system and jacks is an unusual problem in itself.

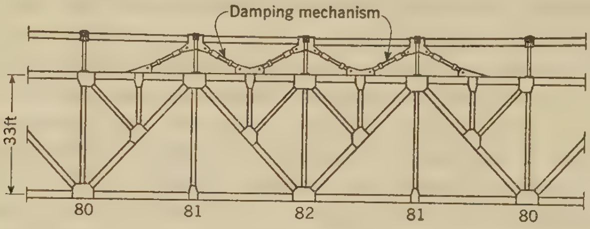

Fig. 4.—Part Elevation at the Center Line of the Bridge

Pertinent Literature

The problems that confronted the engineers charged with the design of the second Tacoma Narrows Bridge have attracted world-wide attention among engineers and scientists generally, with the result that much has been written on the subject. An incomplete list of the literature published prior to the failure is presented in Section I, and the literature published subsequent to the failure is included in Section II.

Acknowledgment

Personnel who directed and developed the design are: The writer, as chairman of the consulting board and principal engineer in general charge; Glenn B. Woodruff and John I. Parcel, Members, ASCE, members of the board; Theodor von Kármán, M. ASCE, aerodynamicist; Dexter R. Smith, designing engineer; and F. B. Farquharson, M. ASCE, in charge of model tests. The United States Public Roads Administration is cooperating in the tests which are under the general direction of E. F. Kelley, and in direct charge of George S. Vincent, M. ASCE.

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(31) "Two Recent Bridges Stabilized by Cable Stays," *ibid.*, December 5, 1940, p. 56.

(32) "Earthquake Stresses in San Francisco-Oakland Bay Bridge," by Norman C. Raab and Howard C. Wood, *Transactions*, ASCE, Vol. 106, 1941, p. 1363.

(33) **"On Method of Complimentary Energy,"** by H. M. Westergaard, *Transactions*, ASCE, Vol. 107, 1942, p. 765.

(34) **"Observed Vibrations of Bridges,"** by Dion S. Corder, *Bulletin*, Seismological Soc. of America, October, 1940.

(35) **"The Failure of the Suspension Bridge Over Tacoma Narrows,"** a report to Paul Carew, chairman, Narrows Bridge Loss Committee, June 2, 1941.

(36) **"Report of the Board of Investigation, Tacoma Narrows Bridge,"** to James A. Davis, Acting Director of Highways, February 17 and June 26, 1941.

(37) **"The Failure of Tacoma Narrows Bridge,"** a report to the Hon. John M. Carmody, Administrator, Federal Works Agency, Washington, D. C., March 28, 1941. (*a*) Appendix IX.

(38) **"The Failure of the Tacoma Narrows Bridge,"** *Bulletin No. 78*, Texas Eng. Experiment Station, College Station, Tex., 1944.

(39) **"Report on Behavior of the Bronx-Whitestone Bridge and on Provisions Made to Stiffen the Bridge,"** by O. H. Ammann, July 23, 1945.

(40) **"Rigidity and Aerodynamic Stability of Suspension Bridges,"** by D. B. Steinman, *Transactions*, ASCE, Vol. 110, 1945, p. 439.

(41) **"Redesign of Tacoma Narrows Bridge,"** by Charles E. Andrew, *Engineering News-Record*, November 29, 1945, p. 64.

(42) **"Design of Tacoma Narrows Bridge,"** by C. E. Andrew, *Pacific Builder and Engineer*, October, 1945, and November, 1945.

(43) **"Construction Features of the Tacoma Narrows Bridge,"** by C. E. Andrew, *ibid.*, January, 1946.

(44) **"Failure of the Tacoma Narrows Bridge,"** Report of the Special Committee of the Board of Direction, *Proceedings*, ASCE, December, 1943, p. 1555. (a) Appendix V. Bibliography of Suspension Bridges, p. 1581.

(45) *Transactions*, ASCE, Vol. 100, 1935, p. 1175.

(46) *Ibid.*, Vol. 98, 1933, p. 1104.

DISCUSSION

O. H. Ammann,[2] M. ASCE.—Credit is due the author and his associates responsible for the development of the redesign of the Tacoma Narrows Bridge in the State of Washington for their efforts to explore the elements which influence the dynamic motions in suspension bridges under wind action, and to produce a design that can be judged adequate and within reasonable economic limits on the basis of present knowledge.

In these efforts the designers were fortunate in having at their disposal two essentials for research work not usually available to engineers planning a bridge of this type—namely, ample time and sufficient funds. With construction funds partly available, and with important parts of the old structure utilizable, in addition to proved economic justification, the eventual reconstruction of the bridge was a foregone conclusion. However, due to wartime conditions it had to be delayed for several years, which offered an opportunity for extended research. The research expenditures to which the federal government liberally contributed could be justified on the ground that the findings were expected to be of value for the design of suspension bridges in general. Such expenditures could not be economically justified for individual projects, even if funds could be raised as a preliminary to financing a project (which, as a rule, is not the case). It is very doubtful, in the opinion of the writer, whether the results of the research work so far obtained and applied in the form of new features in the redesign of the Tacoma bridge may be utilized by the profession in designing other suspension bridges without further extended research.

The author properly points out that, for a more complete exploration of all the complex elements contributing to the behavior of suspension bridges under wind action, more research is needed. Among others, he mentions two most important elements on which so far there has been only very inadequate exploration—namely, the damping characteristics of the structure, and the character of the winds as they exist in nature, which it is impossible to reproduce in a wind tunnel. These and other important factors are so complex, so widely variable in different structures and localities, and so difficult to explore adequately, that one may well doubt whether it will ever be possible to arrive at generally applicable design directives based on scientific research such as so far has been applied in the case of the Tacoma bridge.

The author confines himself to a general qualitative outline of the redesign of the Tacoma bridge. Although he points out the major features that contribute to the adequacy of the design, with respect to wind action, he does not attempt even a rough appraisal of their effectiveness and economic merits, which are of prime importance to the design engineer. Unfortunately, the paper does not contain all the essential design data—such as dimensions of members, sectional areas of members, and weight of structure—on which the reader can base such an appraisal. In an article published elsewhere the author gives some of these data (41).[3] It may be assumed that they have not been materially changed, except in one important feature—namely, the addition of a second lateral system in the plane of the bottom chords of the stiffening trusses.

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[2] Cons. Engr., New York, N. Y.

[3] Numerals in parentheses, thus: (41), refer to corresponding items in the Bibliography, in the Appendix of the paper.

One who has thoroughly familiarized himself with the nature of the problem of designing suspension bridges against dynamic wind action, and has closely followed the research work in this respect, cannot but get the impression that the effect of what may be called streamlining provisions in the design of the Tacoma bridge have been grossly everemphasized in this paper. It cannot be doubted that these provisions will contribute to the margin of safety of the structure, more particularly against the extremely remote possibility of dangerous torsional oscillations under winds of high velocity. On the other hand, there is nothing to indicate that they contribute any major share to that margin, or that the structure, as otherwise designed, would not be adequate without them.

If, as sound judgement would seem to permit, the streamlining of the rail section is disregarded as a wholly negligible factor, major reliance for streamlining must be placed on the grid covered, open slots in the roadway slab, which, according to the wind tunnel tests on the model, appear to influence the behavior of the structure favorably. Aside from the question whether such open slots may prove to be objectionable for traffic, or as a feature to be permanently maintained on the bridge, there is still the unsolved question of whether, in the actual structure under actual wind conditions, this provision can be relied on to furnish any appreciable share of safety, not only against dangerous torsional oscillations, but also particularly against such relatively moderate vertical motions as would be noticeable to the traveling public. The author expresses the opinion that it is entirely probable the design without the slotted deck also would be satisfactory. The writer agrees with this opinion, even to the extent of expressing full confidence that the bridge would prove adequate without the slots, as the latter merely represent an additional safeguard of uncertain and remote effectiveness.

The elements which can be (and are) evidently relied on to secure the necessary safety are those which contribute to the mass and stiffness of the structure and, therefore, to its resistance against the building up of dynamic deformations. Principal among these elements are the following:

1. Dead Weight.—In the new design the dead weight appears to be at least 50% greater than that of the original bridge. It is now well recognized that even such a substantial addition to its weight would not have stabilized the old bridge, and that the addition of weight not otherwise required would have been wasteful. Nevertheless, in the new design the additional weight required, because of the greater width and traffic capacity of the bridge, adds substantially to its resistance to dynamic wind forces as they actually occur. In the case of the George Washington Bridge (New York, N. Y.) it is practically weight alone that insures adequate stability.

2. Great Depth and Stiffness of the Stiffening Girders.—The author emphasizes, as an improvement in form, the substitution of trusses for the solid plate girders of the old design. He does not, however, bring out the important fact that these trusses, four times deeper than the girders of the old bridge,

have been given an elastic stiffness vertically, as measured by their moment of inertia, thirty-seven times greater.

3. Location of the Deck and Its Resistance to Vertical Oscillations.—In the old design the floor stringers, the concrete slab, and the single lateral truss were located very close to the neutral axis of the stiffening girders, and, therefore, could have added only a negligible amount to the extremely low resistance of the latter against vertical oscillations. In the new design the stringers, the upper lateral system, and the deck are located practically in the plane of the top chords of the stiffening trusses and, in addition, a lateral system is provided in the plane of the bottom chords. Although expansion joints are probably to be provided in the deck, the frictional and elastic resistances of the stringers and laterals and also the damping effect of the concrete deck against the building up of vertical oscillations must be almost equivalent to the stiffening effect of the stiffening trusses. In other words, the resistance against vertical motions by reason of the deep trusses and the location of the deck and lateral systems in the plane of the chords of the trusses must be of the order of seventy times that of the old design.

4. Double Lateral System and Its Effect on the Resistance Against Torsional Motions.—As already mentioned, the old bridge had only a single lateral system approximately in the plane of the neutral axis of the stiffening girders. This, together with the relatively narrow width between girders, made the suspended structure extremely flexible in torsion and consequently low in its resistance against the building up of the fatal torsional motions. In the new design the width is more than 50% greater, and two lateral systems 33 ft apart are provided. This has the effect of increasing the torsional resistance of the suspended structure in the new design, compared to that of the old, in a ratio even greater than that of the respective resistances against vertical motions. It is safe to say, without exact analysis, that the new design possesses ample resistance against objectionable and dangerous torsional oscillations, whether the open slots in the deck are provided or not.

The effects of these various stiffening elements have as yet been insufficiently explored by model tests in wind tunnels. It is questionable whether they can be reliably explored by such tests, because of the difficulty of adequately reproducing on a model in a wind tunnel the wind action on actual bridges and the internal damping and frictional resistances in the structure.

The author deprecates the value of experience gained from the behavior of actual suspension bridges on the grounds that it is allegedly not known what movements have occurred, or whether the bridges have been subjected to heavy wind storms and from what direction. Nevertheless, this is the most important and reliable source of information presently available to the profession for the purpose of deriving safe design criteria. There are or have been many bridges in existence which have experienced observable and more or less severe motions or failure. Their physical properties essential for an analysis of their resistance to wind are known. It is not necessary to have accurate measurements of the motions they have experienced as long as it is known under what resistance limitations of the structure failure or objectionable or dangerous motions have occurred. The author's argument, that it is not known whether existing bridges have been subjected to severest wind action, is unquestionably valid when applied to bridges of comparatively recent construction. There are, however, hundreds of suspension bridges which have existed, and whose behavior in wind has been known, for periods up to more than a century. Among them the great majority is known not to have developed objectionable motions. Of some twenty bridges that are known to have developed observable motions, several have suffered more or less serious damage to the structure and about half have failed.

It would be a serious reflection on the profession if it had to be admitted that adequate design limitations could not be derived from such a wealth of practical experience. The problem of deriving adequate and, within reasonable limits, economical design limitations for suspension bridges against wind action is, like many other design problems, not one which can be solved with mathematical accuracy. The influencing factors are too complex and too varied. Some of them are extremely difficult to determine. However, on the basis of the behavior of actual structures, it is as possible to derive design criteria within the same degree of safety and economy as it is to design individual members or composite structures subject to the action of static forces. Studies made by the writer and others show that design criteria can be developed, based on the weight and stiffness characteristics of the suspended structure, which are reasonably consistent with the behavior of the many suspension bridges whose behavior is known, notwithstanding the great variety of structural forms represented by them.

Until much more information is derived by research (such as experiments with large models in wind tunnels to determine the effect of form, and studies of the wind action on existing structures), the profession must rely on design features that assure adequate mass and adequate elastic and frictional resistances of the structure. This is, in the present state of knowledge, what had to be done in the case of the new Tacoma Narrows Bridge design.

CHARLES A. ELLIS,[4] M. ASCE.—The failure of the Tacoma Narrows Bridge made newspaper headlines for various reasons, one of which is particularly significant. It is common knowledge that medicine is by no means an exact science, and untimely deaths are commonplace. However, most everyone (including some engineering professors and authors) believes, or at least infers, that engineering is an exact science. It may be stated without fear of successful contradiction that no engineer ever correctly answered any engineering problem, unless his mistakes in computation exactly compensated for the errors inherent in all engineering theory. Consequently, when an engineering structure fails, the man has bitten the dog and headlines appear.

Jonathan Jones, M. ASCE, was a true prophet when he wrote (45) the following paragraph:

> "The design of other types of bridges is based upon their carrying certain loads at certain unit stresses; but a suspension bridge design can-

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[4] Evanston, Ill.

> not even be begun until there are stipulated, in addition to the loads and unit stresses, the criteria for maximum admissible flexibility. To a certain extent these criteria must be psychological, possibly as much so as mechanical. Whether such criteria of flexibility as have been and are being chosen are valid, and to what extent deviations therefrom would be regretted, is the most important remaining question in the field of suspension bridge design."

The maximum admissible flexibility was rapidly being approached in several suspension spans being built at about this time. It was reached and surpassed on the Tacoma bridge.

Just what are the criteria for maximum admissible flexibility? A comparison of the original Tacoma design with the writer's Golden Gate Bridge (San Francisco, Calif.) design may throw some light on this question.

The Golden Gate span is 50% longer; but the stiffening trusses, which are 90 ft apart, 25 ft deep, and have from 160 sq in. to 200 sq in. of silicon steel in each chord, have vertical and lateral moments of inertia many times larger than do the 8-ft plate girders, 39 ft apart, of the Tacoma bridge. The concrete deck located about halfway between the girder flanges gave little resistance to torsional movement. On the Golden Gate Bridge the sway bracing of diagonals from the bottom chords to the floor beams was intended to hold the top and bottom chords of each truss in a vertical plane, making it feasible to include the moment of inertia of the bottom chord with the lateral system of the top chord in resisting horizontal wind pressure. These diagonals act as a deterrent to torsional movements, although none were anticipated at the time the design was made. The maximum computed horizontal deflection at midspan due to a 30-lb wind load is about 23 ft, the maximum bending movement being at about each quarter point of the span (46).

Russell G. Cone, M. ASCE (37*a*), has reported that on February 9, 1938, when a wind of very high velocity was blowing through the Golden Gate Bridge and normal to the structure, with a force so great that he could not stand erect, the span was deflected an estimated 8 ft or 10 ft, and was held in this position. Vertical undulations were obvious, but no mention is made of torsional movements. Assuming that the wind velocity was 80 miles per hr, the pressure would be from around 20 lb to 25 lb per sq ft. The horizontal deflection was less than one half of the computed deflection for a 30-lb wind. This may be accounted for in part by the omission of the concrete deck and stringers in computing the lateral moment of inertia.

The ratio of cable sag to span length was considerably less in the Tacoma design than in the Golden Gate span. Any comparison should take cognizance of this fact because the smaller the sag, the greater is the stiffness of the system.

The computed maximum vertical deflections due to live load and temperature are nearly equal in the two bridges; so also are the computed horizontal deflections due to wind. Nevertheless, examination of the two designs and the performance of the two structures makes it difficult to escape the conclusion that the Golden Gate design is many times stiffer than the original Tacoma design. This poses a very important question. Can the criteria for maximum admissible flexibility be established, and, if so, what is the modus operandi for approaching the solution?

An important question, which unfortunately can never be answered, concerns the stay ropes installed on the Tacoma bridge, extending from the central cable band to the stiffening girder on each side. The structure was destroyed by torsional oscillations caused by the slipping of one cable band which threw the cables out of phase. Would the bridge still be standing had the cable band not slipped, or had the stay ropes never been installed?

F. H. Frankland,[5] M. ASCE.—The problems described by Mr. Andrew applying to the design of the second Tacoma Narrows Bridge are naturally affected in their solution by considerable data recently developed relative to the effect of aerodynamic forces on suspension bridges.

The author states (under the heading, "Major Problems"),

> "* * * There is no actual, measured value for the damping in a suspension bridge. * * *
>
> "Because the damping capacity of the first bridge is unknown, the adopted design for the second bridge, or for that matter that for any suspension bridge, cannot be truly simulated in a model."

The Advisory Board on the Investigation of Suspension Bridges, in considering the effects of structural damping, realized that knowledge regarding frictional damping was so incomplete that an attempt must be made to approach the problem by rational methods in order to obtain an insight into the effects of dry friction acting between structural parts of suspension bridges. Consequently, Friedrich Bleich, M. ASCE, in connection with analytical mathematical studies for the Advisory Board, prepared a report on the effects of structural damping which covers comprehensive analytical studies of the aerodynamic behavior of suspension bridges. The results of these damping studies furnish analytical methods for arriving at the damping factor of any suspension bridge design.

From the aforementioned theoretical methods it is possible to assign a definite damping factor to both the original and the second design of the Tacoma Narrows Bridge if the theory is checked by tests. A series of damping tests on simple beams and trusses was made for the Advisory Board by the Division of Physical Research, United States Public Roads Administration, at the Arlington (Va.) laboratory. These tests disclosed that the internal damping capacity of modern steel structures (that is, damping capacity inherent in the elastic hysteresis of the material, resulting from the plastic yielding and friction in the riveted joints) is comparatively small. From these tests the conclusion could be drawn that a considerable part of the damping effect in steel bridges is the result of energy dissipation by internal and external dry friction.

Further tests at the Arlington laboratory are in progress for the purpose of examining the effect of friction between roadway stringers and floor beams and the effect of special friction devices on the damping capacity of suspension bridges. These tests, it is hoped, may afford an opportunity to check the reference made to theoretical studies.

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[5] Cons. Engr., New York, N. Y.

The various reports on the collapse of the Tacoma Narrows Bridge failed to take into account the effects of the failure of the side span, tie down ropes installed about a month before the collapse on the initiative of the bridge authority. These side span ties consisted of $1\frac{9}{16}$-in. wire ropes fastened to the stiffening girders at points 300 ft from the anchorages, held down by concrete ground anchor blocks of 50 cu yd. When the first of these four ropes failed (November 7, 1940), a violent torsional motion was induced in the main span; with the vertical motions of the main cables thrown out of phase with one another, a severe stressing of the center diagonal ties resulted on the north side and a consequent slipping of the cable band and a rupture of the ties. A trigger action was thus created which immediately threw the main span roadway into violent torsion, one side of the span being freed from the restraint of the tie, while the other (south) side was still restrained by its diagonal ties. A careful study of the motion pictures taken during the collapse clearly shows this sequence of events.

The cable bands, to which the center diagonal ties were connected in the first bridge, were deficient in contact area with the cables, thus providing the possibility for an extremely high unit frictional stress, and a consequent slipping of the bands under the conditions existing at the time of failure. This deficiency is evidently recognized and provided for in the new design by incorporating three sets of diagonal ties in each truss at the center of the main span.

It is extremely doubtful whether the streamlined handrail will provide any beneficial effect. The vortex shedding from the comparatively large upper chord area dominates the airflow characteristic over the bridge deck and any effect from streamlining the handrails must be negligible, especially when the angle of attack of the wind is rarely normal to the plane of the roadway, and is widely variable.

Mr. Andrew does not refer to the cable sag at midspan used in the design of the second bridge. Is it to be assumed that the center sag of 232 ft (a sag-to-span length ratio of 1:12) used in the first bridge is to be maintained, thus realizing a high stiffening factor from the main cables? The main cables of a suspension bridge are by far the most important stiffening elements in such a system, and, for flat sags, the cables exercise a dominant effect on resistance to torsion and in resisting vertical and lateral movements of the suspended structure. The principal contributing factor to the stiffness of the George Washington Bridge (New York, N. Y.) is not the weight of the suspended span, but the use of four cables, two of which are at present in excess, but which are necessary to carry a future increase in dead and live loads in the event that a lower deck is added to the structure.

Charles E. Andrew,[6] M. ASCE.—As written, the paper had as a primary object the enumeration of problems encountered at Tacoma which are not ordinarily present in bridge design and construction, such as fitting a new, heavier, and wider bridge to existing piers and abutments and, of course, attempting to improve aerodynamic stability.

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[6] Cons. Engr., Washington Toll Bridge Authority, Tacoma Narrows Bridge, Tacoma, Wash.

The failure of the former bridge attracted world-wide attention and, to say the least, created grave doubt in the public mind, if not in all engineers' minds, as to the safety of certain types of suspension bridges. In consequence, engineers responsible for the new design felt that all possible safeguards must be incorporated to be doubly sure that the new bridge would be a safe structure. These safeguards were the result of much study and extensive and costly experiments, and represent the best judgment of those responsible. Some of these safeguards may be unnecessary but, it is believed, they must be considered further insurance at a relatively low cost. Their significance was manifest in the importance which bond investors attached to them and in the writer's opinion the extensive studies and the safeguards that resulted were largely responsible for the successful consummation of an otherwise difficult financing problem.

In any original research, such as the aerodynamic problem in suspension bridges, the investigators encounter difficulty and practical failures in many of their first guesses, abandon them, and try some other course of action. The problem involving the second Tacoma Bridge was no different. The very first preliminary design based on available knowledge of aerodynamics, coupled with judgment when tested by model, proved to be a great improvement over the design for the original bridge. This design included the same greatly increased stiffening truss and practically the same weight as does the final design; yet it showed decided instability in certain velocities and angles of wind. As a next step in seeking added stability, sundry modifications of the rail and leading edges were tried. Some of these developed a relatively small amount of added stability in certain winds. The streamlined rail was incorporated in the final design but none of the leading edge modifications were. Finally, the original rail was changed to one less streamlined but retaining the same percentage of openings and practically the same aerodynamic properties. Not until the roadway slots were incorporated did the tests indicate a very decided improvement in stability. In fact, almost complete stability resulted and the very minor movement that occurred became noncatastrophic whereas all former motions were catastrophic. Surely such an improvement is worthy of consideration and incorporation in the structure.

As to placing too much reliance on past experience in suspension bridges and setting up design criteria from former performance, it should be called to the designer's attention that the first Tacoma Narrows Bridge was designed by the best talent of the day, guided by accepted past practice; yet the design, unfortunately, was inadequate. Thus the writer contends that each suspension bridge is an individual case that must be designed to suit its site and the traffic that it may be called on to carry. Many may need no consideration of aerodynamic problems, others need it badly. Borderline cases will be more numerous in the future—hence, the necessity for caution.

The writer wishes to correct Mr. Frankland's impression that a hold-down cable parted on November 7, 1940, the day of failure. One of these ropes did part at an included weak link about one week prior to the failure under pure vertical oscillation, but on the day of failure, which was the first time that torsion occurred, all hold-down cables were intact. The cable sag of the new design is 1/10.

The writer did not go into the many and various ramifications of why all features of the design were adopted, simply because the limit of such a paper would not allow the vast amount of data required. Final reports (five volumes) are now (1949) being written on the subject.

The writer wishes to thank Messrs. Ammann, Ellis, and Frankland for their interest and discussion.

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HIGHWAY BRIDGE FLOORS
A SYMPOSIUM

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LABORATORY RESEARCH ON CONCRETE BRIDGE FLOORS

By F. E. Richart,[1] M. ASCE

1. Purpose and Scope

A program of systematic research, which will be designated the "Concrete Slab Investigation" in this Symposium, was begun in September, 1936, as a cooperative project between the Engineering Experiment Station, University of Illinois (at Urbana), the Illinois Division of Highways, and the United States Bureau of Public Roads (later the United States Public Roads Administration).

The purpose of the project, as set forth at the outset was

> "* * * to make investigations and tests of reinforced concrete slabs to determine the behavior thereof under varying conditions and to develop information which will advance the art of bridge building."

The main problem envisaged was that of the theoretical analysis and tests of bridge slabs under the concentrated loads applied by heavy trucks, but the work has necessarily been extended to the action of the supporting beams, so that the investigation has actually been applied to the complete floor systems of highway bridges.

2. General Procedure

Throughout the work, a combination of analytical and experimental study has been employed. In so far as possible, it has been the practice, on any problem, first to derive analytical solutions and then, by testing structures selected for their practical importance, to study the applicability of the analysis and the deviation of actual conditions in the structure from those assumed in the analysis. The experimental program has been largely confined to laboratory tests, although some field studies and observations have been made. The laboratory structures tested have necessarily been models, generally with scales of 1:2 to 1:5. Most of these models were first designed in the full-sized prototype, using standard design methods and loadings specified by the American Association of State Highway Officials (AASHO). Great care was taken to keep the materials and dimensions of the models in true proportion to those of the prototype, and many studies were made to determine whether any scale effect existed between large-scale and small-scale models.

Obviously, the major difficulty in the analysis of the action of highway bridge slabs arises from the fact that they are subject to truck loadings. In tests, such loadings can be simulated by the application of concentrated loads, applied to represent wheel loads of trucks. In analyses, certain assumptions must be made, such as those used by H. M. Westergaard,[2] M. ASCE, in his classic study of the effect of loads concentrated on a small finite area of a simple span slab of infinite width. Many strain measurements were made in the vicinity of such load points in the test slabs, to study the validity of the assumptions.

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[1] Research Prof., Eng. Materials, Univ. of Illinois, Urbana, Ill.

[2] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, March, 1930, p. 1.

In the experimental work, a feature common to all the large model tests has been the study of the behavior of slabs in both the uncracked condition and the cracked condition. The former is of interest in comparing the action of the slab at low loads, before cracks have formed and when the structure most nearly approaches the elastic, homogeneous slab assumed in the theoretical analysis. At these low loads, the procedure has generally been to apply a series of loads, moved from point to point to simulate a moving vehicular load, and to make strain measurements, using sensitive gages, at points of critical stress.

It is necessary also to determine the behavior of the structure at design loads. To this end, a series of design loads was frequently applied over most of the surface of the slab—simulating the effect of continued traffic which might produce general cracking of the slab. The slab was then subjected to a series of design loads at selected points, with extensive strain measurements, to furnish load-strain data for the cracked structure. This procedure affords a picture of the expected behavior of the slab in a bridge in actual service.

A third feature of the experimental tests, is a study of conditions at or near failure, in which loads representing wheel loads were applied at critical sections of the slab to produce failure. Such loads generally reached several times the design load, and the manner of failure was observed carefully. In many cases, with multiple-lane slabs, a group of concentrated loads representing the rear wheel loads of two passing trucks was employed to produce the maximum load condition.

Since the ultimate object of the entire program is to improve the design of bridge floors, the published reports have been planned to cover three definite phases of the study: (1) The mathematical analysis, along conventional lines; (2) the experimental research on the structure, with results analyzed and correlated with the theoretical findings; and (3) the development of simplified design procedures for easy and rapid use. Phase (3) may be accomplished in many cases by fitting empirical equations to both computed and observed results, but always with the primary object of producing a simple and usable procedure for the designing engineer. This last step in the program was completed for the right, simple-span, solid-slab bridge in 1946. By the beginning of 1947, thirty-six scale-model bridges, ranging in length from 5 ft to 30 ft, and more than two hundred smaller specimens consisting of concrete or mortar slabs, composite T-beams, etc., had been tested.

3. Testing Methods

Since most of the test slabs were too large to be placed in the ordinary testing machine, some very adaptable testing rigs were built. Structural steel loading frames were erected and anchored on the heavy reinforced concrete floor of the laboratory. These frames enclosed the test slab and, in conjunction with jacks, dynamometers, and distributing beams, furnished the loading apparatus. The frames were movable and permitted load applications at several points. The dynamometers were built and calibrated in the laboratory.

Strain measuring apparatus varied considerably over the 10-year period. The type depended on the precision required and included attached Huggenberger and graphic gages, special interferometer and level bubble gages, and the well-known Berry and Whittemore gages. On continuous I-beam bridges tested recently a large number of SR-4 electric gages were used. Deflections were generally measured by deflectometers equipped with dial micrometer gages.

4. Fundamental Studies

Several studies were made which have very general application and are not limited in application to highway bridge slabs. One of these is the analytical solution by V. P. Jensen, M. ASCE, of a large number of slab problems, involving many combinations of slabs supported on rigid or flexible beams with different degrees of continuity.[3] Moments were derived for the supporting beams as well as for the slabs. Among other cases investigated was the ordinary single-span slab with curbs.

Another extremely useful contribution to slab theory is particularly applicable to the slab supported on parallel flexible beams, as in the slab-and-stringer bridge. In this ingenious method, derived by Professor Newmark (and described in the second Symposium paper), an extension of the moment distribution method for continuous beams is made applicable to slab problems.[4] The method is equivalent to the solution of a number of analogous continuous beams, corresponding to the number of terms in the infinite series applied to the slab which are needed to give the required precision. The analysis of a slab continuous in one direction is therefore more laborious than the solution of a beam by moment distribution but the same basic principles are used. When the supporting beams are flexible, an added step in the analysis is required, somewhat like that for sidesway in a rigid frame.

Another fundamental study by Professor Jensen is concerned with the so-called plastic theory of failure of reinforced concrete beams. In the study of most of the slab results, rather large discrepancies have been found between the stresses computed by the conventional straight-line theory, assuming no tension in the concrete, and the stresses determined from the measured strains. Part of the discrepancy is doubtless due to the existence of tension in concrete, and part to a departure from straight-line distribution of compressive stress. Professor Jensen introduced the concept of a plasticity ratio in concrete, directly related to the compressive strength, and derived general expressions for determining the ultimate load in flexure on reinforced concrete beams.

The use of the plasticity ratio and of idealized stress-strain diagrams for both concrete and steel brings about the use of a trapezoidal compressive stress diagram for concrete in flexural compression. The analysis has been verified by reference to the results of a large number of beam tests, and seems to furnish a satisfactory explanation of the phenomena of beam failure.[5]

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[3] "Solutions for Certain Rectangular Slabs Continuous Over Flexible Supports," by V. P. Jensen, *Bulletin No. 303*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1938.

[4] "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," by N. M. Newmark, *Bulletin No. 304*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1938.

Other studies on fundamental problems include an unpublished analysis, by Professor Newmark and W. A. Renner, of slabs composed of a nonisotropic material. The application to slabs having distinctly different reinforcement in the two directions is obvious.

A similar study is concerned with the effectiveness of reinforcing steel placed in various directions in a slab. The most general application is in the skew-slab bridge, where the direction of principal stress can only be discovered by analysis or test, and where this direction varies at different parts of the slab. In such skew slabs, it is common practice to place the steel so that it makes a fairly large angle with the direction of the major principal stress. Furthermore, in such bridges, the reinforcement is frequently placed in two directions which are not at right angles to each other.

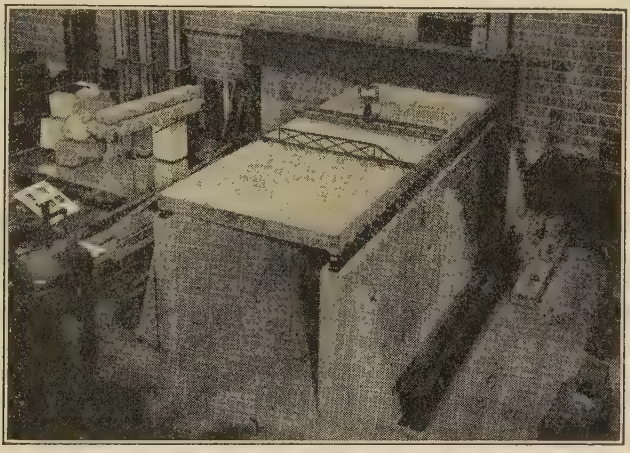

FIG. 1.—TEST SETUP OF RECTANGULAR SLAB

The effectiveness of reinforcing bars making various angles with the direction of applied moment was studied in a series of slab tests (unpublished) by Professor Newmark and John Houbolt and the results correlate fairly well with those determined by analysis.

5. EXPLORATORY TESTS

a. The Rectangular Slab, of Simple Span.—The first tests made in the concrete slab investigation were on two rectangular slabs, simply supported on two edges and subject to a single movable concentrated load.[6] The slabs were 20 ft wide, had a span of 6 ft 8 in., were 6.5 in. thick, and were loaded through a 6-in. circular disk. The width-span ratio of 3 was chosen because studies indicated that Professor Westergaard's analysis of the slab of infinite width would apply, although, for slabs of narrower width, the theory would require modifying.

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[5] "Ultimate Strength of Reinforced Concrete Beams, as Related to the Plasticity Ratio of Concrete," by V. P. Jensen, *Bulletin No. 345*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1943.

[6] "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," by F. E. Richart and R. W. Kluge, *Bulletin No. 314*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939.

The principal observations made in these tests were fairly extensive measurements of strains, deflections, and slab reactions in the region of the loads. Fig. 1 is a view of one of the slabs under test and illustrates the loading frame, dynamometer, and jack used in applying load, a series of reaction weighing rings, and a portable deflectometer bridge. Strains were measured in two directions along both center lines, and under nine different load points in the central part of the slab. The reaction rings were used only on one slab. Despite the fact that they were designed to give a deflection of only 0.002 in. at maximum load, this movement was sufficient to disturb the normal distribution of reactions and their use was omitted on later tests.

The strains at low loads, before cracking had occurred, may be studied by referring to Fig. 2. The test results for the two slabs were brought to a comparable basis by introducing the term

$$N = \frac{E\,I}{1 - \mu^2} \quad \text{(1)}$$

which involves any variation in the quality of concrete or dimensions of the slab used in the two tests. In Eq. 1, E is the modulus of elasticity; I is the moment of inertia; and μ is Poisson's ratio. Values of N were determined experimentally by a special test of the slab, using a live load along the midspan center line of the slab. The values of N times the strain ϵ may thus be compared directly with the values computed by theory using Professor Westergaard's analysis. The four curves show the strains in top and bottom of the slab, in the x-direction and in the y-direction, on both the x-axis and y-axis of the slab. Compressive strains at the top surface were adjusted so as to apply to an element below the top which was at the same distance from the computed neutral surface as was the bottom of the slab. The adjustment was made by multiplying the measured strains by the ratio 3.13 : 3.37. In general there is a fair correspondence between experimental values, with the latter somewhat greater in the vicinity of the load point. The distribution of strains along each axis gives a diagram quite similar in shape to the theoretical distribution.

Another comparison between measured and computed strains at higher loads, after cracking has occurred, is shown in Figs. 3 and 4. Fig. 3 shows strains in the concrete at the top of the slab, whereas Fig. 4 shows corresponding values for the steel in the bottom of the slab. The reinforcement consists of $\frac{1}{2}$-in. square bars at 4-in. spacing in both directions, with an effective depth of $5\frac{1}{2}$ in. for the short bars and 5 in. for the long ones. This comparison is of interest, since it shows the relation between the measured concrete strains and values computed both considering and neglecting the tension in the concrete. At the 16-kip load the measured strains fall between the two computed curves, but at the 48-kip load the measured values agree best with the curve based on no tension in concrete. A somewhat different effect is noted from examination of the steel strains in Fig. 4 in which the steel strains, even at the highest load shown, are generally no more than 80% of the computed values, and are less

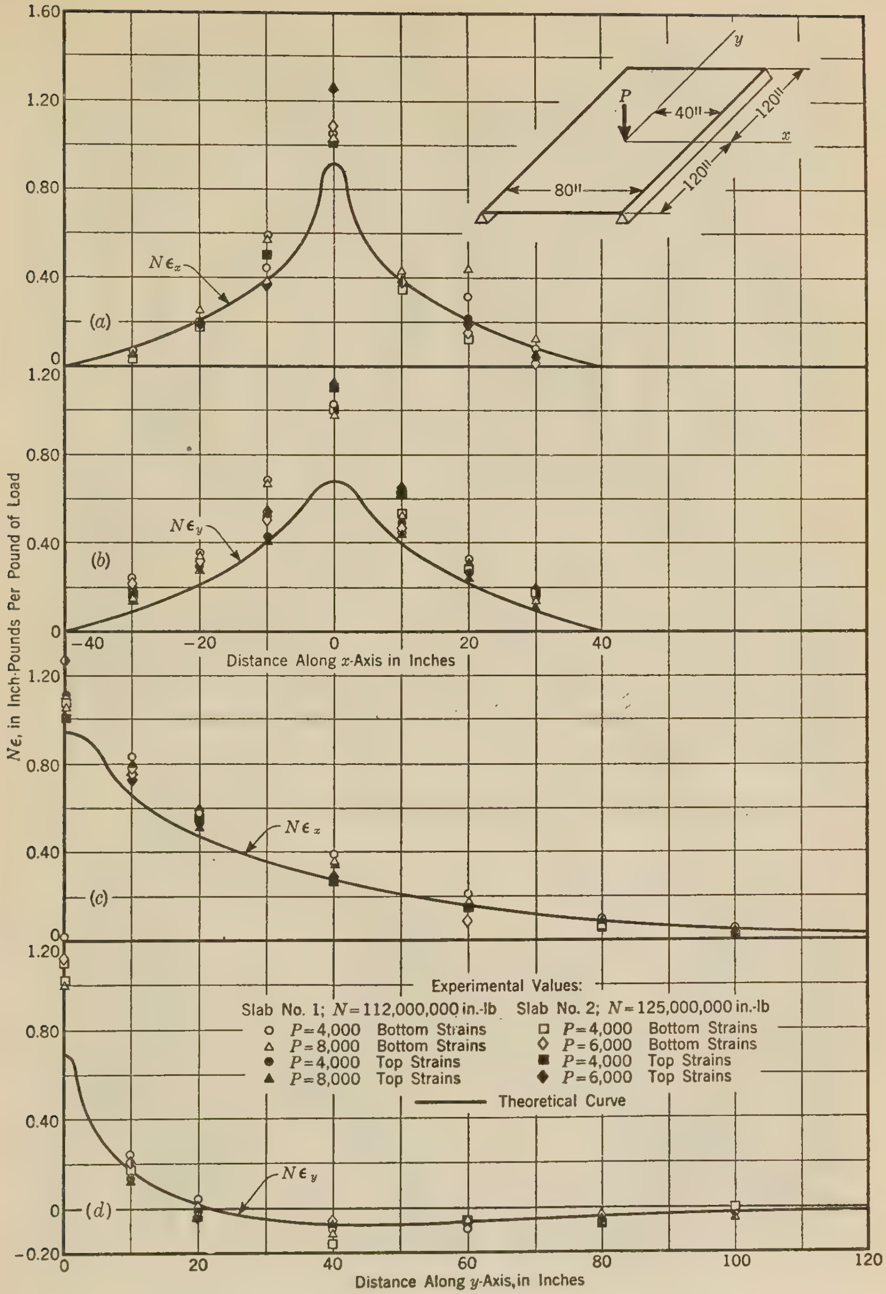


FIG. 2.—WEIGHTED STRAINS ($N\ \epsilon$) RESULTING FROM LOW LOAD AT MIDSPAN

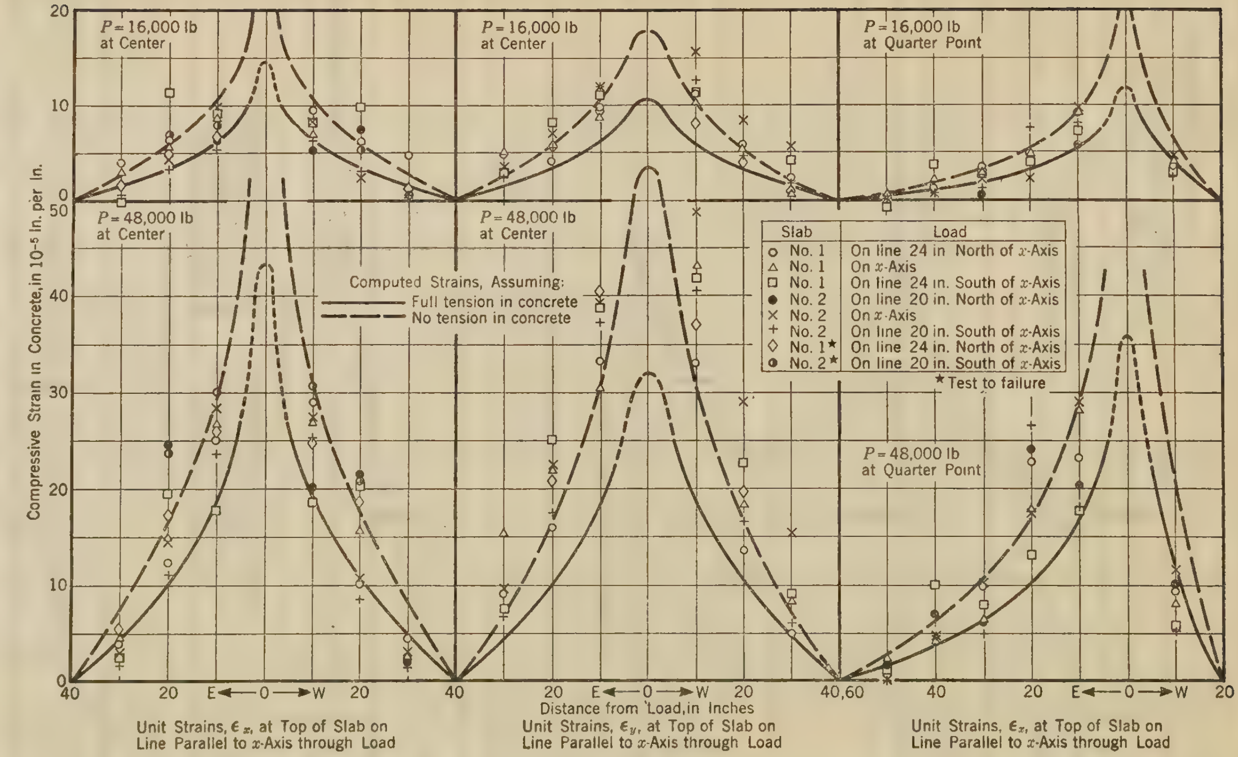
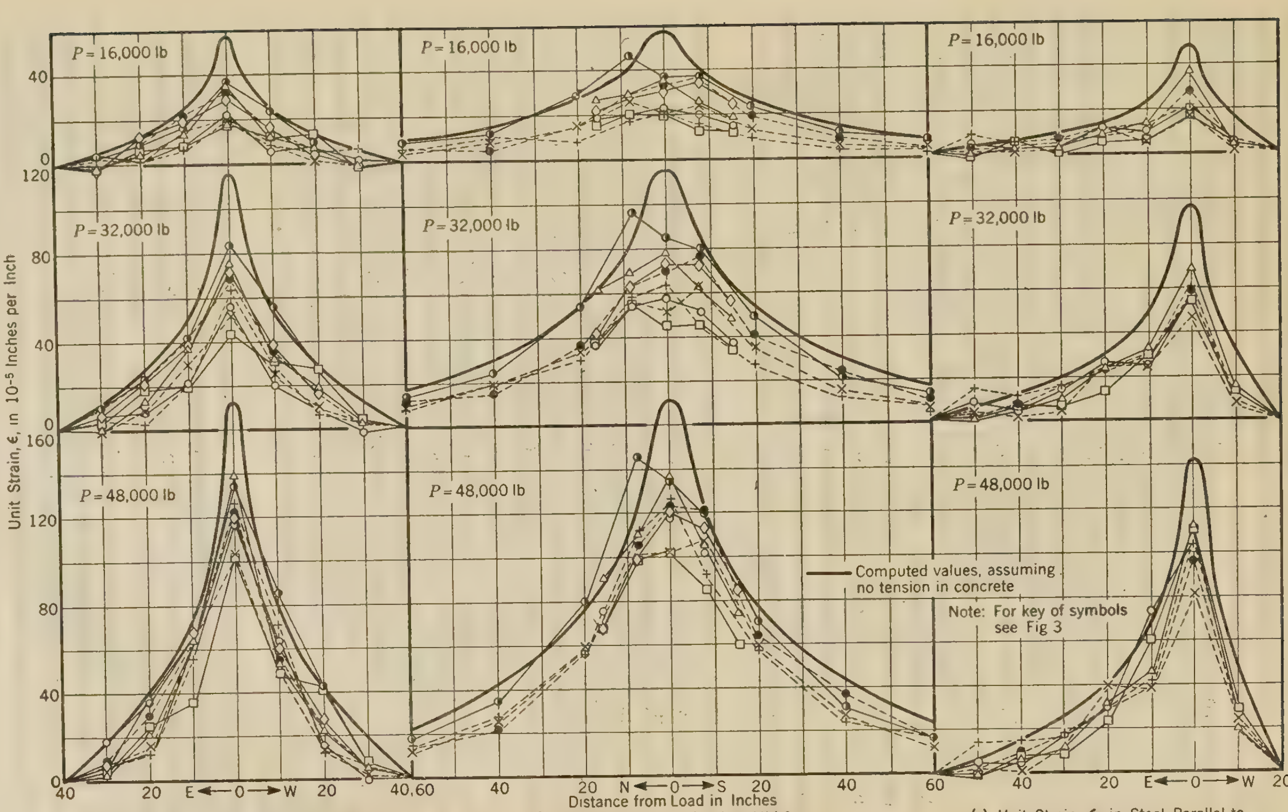


FIG. 3.—CONCRETE STRAINS IN SLAB DUE TO LARGE LOADS



(*a*) Unit Strain, ϵ_x, in Steel Parallel to Span, Under a Central Load

(*b*) Unit Strain, ϵ_x, in Steel at Mid-Span Parallel to Span

(*c*) Unit Strain, ϵ_x, in Steel Parallel to Span, Under a Quarter-Point Load

FIG. 4.—STEEL STRAINS IN SLAB DUE TO LARGE LOADS

than this directly under the load. These comparisons involve an inconsistency, since the moments in the slab were computed for a homogeneous slab, whereas the strains in the steel are based on the conventional assumption that no tension exists in the concrete. However, as an exploratory test, these studies served to indicate the kind of deviations between measured and computed values that have been found in many of the later tests.

In tests to failure, a final punching through of the slab occurred under the loading disk, but only well after the yield point of the steel had been passed and general cracking of the slab had developed. This action was also observed in practically all the subsequent tests of slabs under concentrated loads. By applying the load to widely separated points on the slab, several failures were produced, with average values of 83.5 kips for slab No. 1 and of 79.6 kips for slab No. 2. The location of the load point apparently had little effect on the value of this final punching load.

It may be of interest to note that quarter-scale models of the two large slabs were made and tested. These were mortar slabs, with $\frac{1}{8}$-in. bars, fairly true to scale in all respects. Because of their small size, a precise strain gage was needed and an interferometer extensometer was built for the purpose. The results of these model tests compared remarkably closely to those from the large slabs.

b. Square Slabs, with Concentrated Load Applied Over Various Bearing Areas.—When these tests were made, there was much speculation over the proper application of concentrated loads to simulate the effect of truck wheel loads. Since that time several studies have been reported on the shape of the tire contact and the pressure distribution within that bearing area.

A series of twenty square slabs, 5 ft wide and 5 ft in simple span, was tested, using central bearing areas as follows: Circular disks, 2 in., 6 in., 10 in., and 14 in. in diameter; rings, 1 in. thick and 6 in. and 14 in. in outside diameter; and pairs of circular disks, 2 in. and 6 in. in diameter.[7] The 6-in. disk was used with a layer of either sponge rubber or plaster of Paris between disk and slab; for the other tests, the sponge rubber was used.

Because of the small size of the slabs, they were tested in a large testing machine, although jacks and dynamometers were used to apply the load. Compressive strains were measured with the graphic strain gage, by providing a suitable groove in the loading disk, and tensile strains were obtained with a Berry gage.

Both the compressive and tensile strains under the load point of these slabs showed almost constant values for the five smaller bearing areas, whereas those for the 14-in. disk, 14-in. ring, and the pair of 6-in. disks, at 12-in. center spacing, gave definitely lower values for a given load. This condition is shown for the steel strains in Fig. 5 where the two definite groups of load-strain curves are evident. With so little difference resulting from disks ranging from 2 in. to 10 in. in diameter, it was concluded that the use of the circular disk, of a size properly related to the size of the truck tire, was justified for subsequent tests.

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[7] "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," by F. E. Richart and R. W. Kluge, *Bulletin No. 314*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939, p. 51.

As in the previous series of tests, these tests on loading areas were repeated in quarter-scale models of reinforced mortar. The results corroborated those from the larger slabs.

c. Tests of Plaster Models.—As one of the early studies of slab action under concentrated loads, a fairly large number of plaster-of-Paris model slabs were made and tested.[8] Since the ordinary theory of flexure is not valid at or near the load point, and since the stress-strain relation for pottery plaster is a fairly straight line up to the ultimate load, it was reasoned that test slabs of this material would furnish a good indication of the stresses existing in an elastic slab and thus would supplement the theoretical analysis.

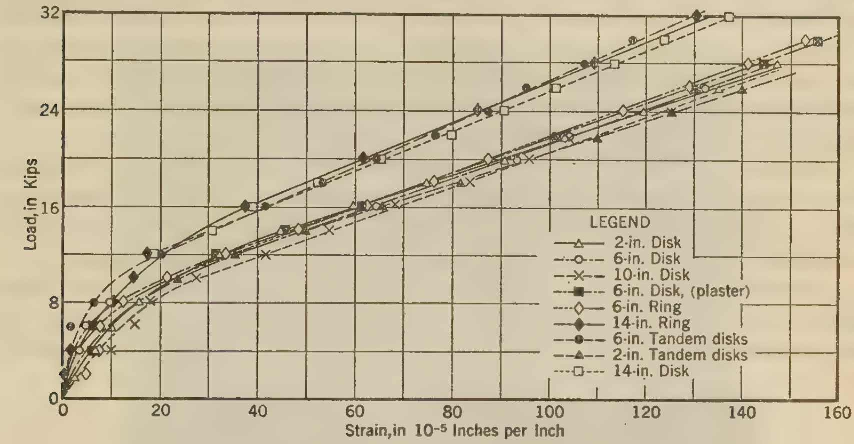

FIG. 5.—LOAD-STRAIN CURVES FOR REINFORCEMENT DIRECTLY UNDER THE LOAD

Four groups of slabs were made, generally 1 in. thick and 12 in. in span. These included (1) circular slabs, supported around the periphery, with variable loading area and variable thickness; (2) rectangular slabs, supported on two edges, with variable width and load position; (3) rectangular slabs, supported on four edges, with variable load position and restraint of corners; and (4) miscellaneous square slabs, with support at corners. The results of these tests were very useful. Where the stresses were computed by the theory of elasticity, the measured results showed fairly good agreement. There was some indication that, although control beams showed a straight-line stress-strain curve, some plastic adjustment occurred in the slabs under the load concentration. In general, for slabs of comparable size, the material failed according to a limiting tensile flexural stress.

The tests gave valuable information on the effect of size of bearing area, and corroborated other findings that the exact shape of the loaded area is not of major importance. The ratio of bearing area to slab thickness is much more significant. The type of support at the edge of a slab does not have much effect on the stresses near a central load, and a load near a support is of greater importance in a slab than it would be in a beam.

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[8] "Tests of Plaster-Model Slabs Subjected to Concentrated Load," by N. M. Newmark and H. A. Lepper, Jr., *Bulletin No. 313*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939.

6. SIMPLE-SPAN RIGHT-SLAB BRIDGES WITH CURBS

The simple slab used in highway bridges is provided with a curb and handrail which considerably modify the stress distribution that exists in a slab of uniform thickness. This type of bridge finds such widespread use that it was one of the first to be studied in the concrete slab investigation. The mathematical solution of the slab with stiffened edges has been presented by Professor Jensen.[3,9] On the basis of this analytical work, laboratory tests were made on two half-scale and seven quarter-scale models. The principal variables studied were the span length and the relative stiffness of curb and slab. In one test, handrails were included to determine their effect.

Except for curb details, these models were geometrically similar to full-sized bridges, designed by AASHO specifications, for H-20 truck loadings. The concrete used had a compressive strength varying from 3,500 lb per sq in. to 4,000 lb per sq in. and the reinforcing steel was of intermediate grade.

As in other slab tests, the strains measured in these models agreed quantitatively in some of the tests and only qualitatively in others. In general, the strains measured in the principal (longitudinal) reinforcement were a little smaller than the computed values, whereas the strains in the transverse reinforcement were very much less than those computed. In several cases, the compressive strains at midspan of curb were unduly high in spite of the fact that much more compressive reinforcement had been used in the curbs than is common design practice.

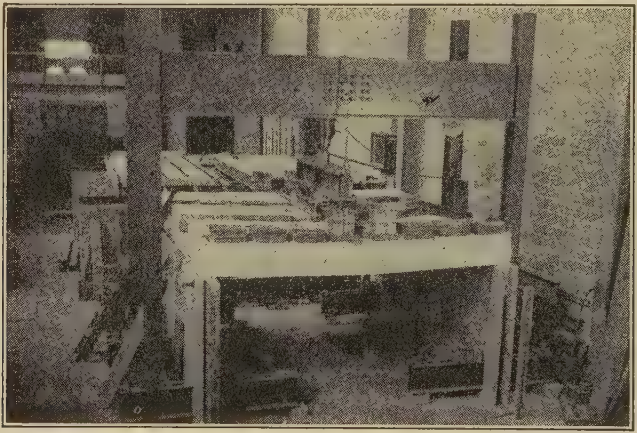

FIG. 6.—VIEW OF HALF-SCALE RIGHT SLAB BRIDGE WITH CURBS

A striking result of the tests was the high strength of the slab, as related to the design load. In most of the tests, the critical load (where it was evident that yielding of the longitudinal steel had occurred near midspan) was equal to the dead load plus from 4 to 5 live loads. Beyond this point there was a

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[9] "Moments in Simple Span Bridge Slab with Stiffened Edges," by V. P. Jensen, *Bulletin No. 315*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939.

further reserve of strength of about 25% beyond the critical load. Such bridges evidently have a high factor of safety.

Fig. 6 shows one of these large models under test, with apparatus for applying four concentrated loads, representing two sets of rear truck wheels.

The results of these studies have been presented elsewhere by Professor Jensen, with R. W. Kluge, Assoc. M. ASCE, and C. B. Williams, Jr., Jun. ASCE. Their study[14] presents the final objective of the work, a simplified method of design. The procedure is intended to cover the usual bridges of this type, with spans of 30 ft or less, but also treats other cases, such as bridges of longer spans or those with unusual types of curb.

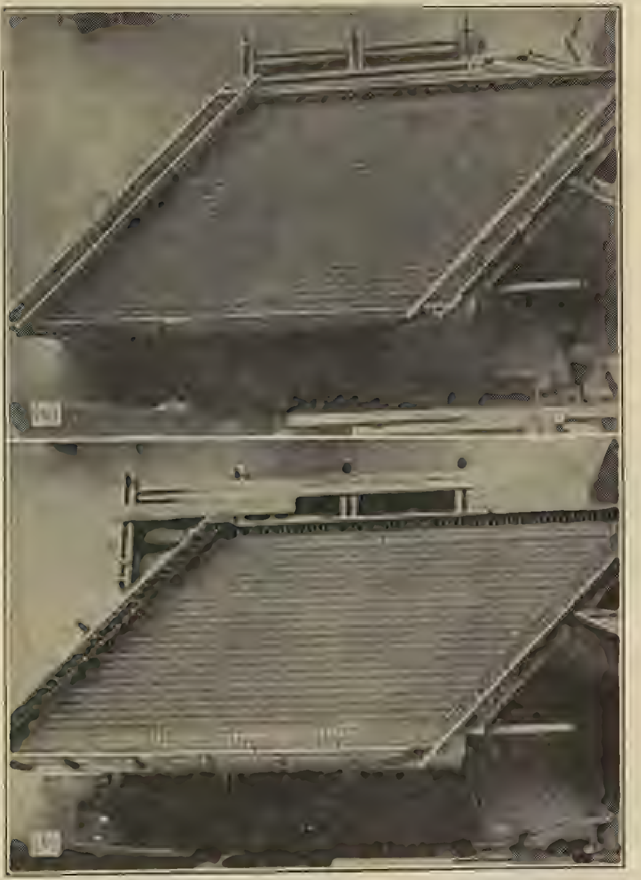

FIG. 7.—REINFORCEMENT OF SIMPLE-SPAN SKEW-SLAB BRIDGES

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[14] "Highway Slab-Bridges with Curbs: Laboratory Tests and Proposed Design Methods," by V. P. Jensen, R. W. Kluge, and C. B. Williams, Jr., *Bulletin No. 345*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1943.

7. Simple-Span Skew-Slab Bridges with Curbs

Since many slab bridges are built as skew spans, it was necessary to extend the studies of Section 6 to the skew slab. The mathematical analysis of the skew slab under concentrated loads is fairly difficult, but Professor Jensen developed a practical procedure involving the solution of numerical cases by the use of difference equations.[11] Although the solutions involved some degree of approximation, they are believed to be the only answer to the problem available. In Professor Jensen's report a careful study is made of the boundary conditions and of the effectiveness of reinforcing steel when placed in two directions generally not at right angles. Applying these methods to the present study, solutions have been made for a number of typical bridges of varying proportions and skew angles. Influence surfaces for moments in these bridges have been determined and, on the basis of the results found, curves have been plotted which may be used in the design of bridges within the range of the proportions studied. The method, as used thus far, has not reached the stage of simplification for design practice that has been developed for right bridges.

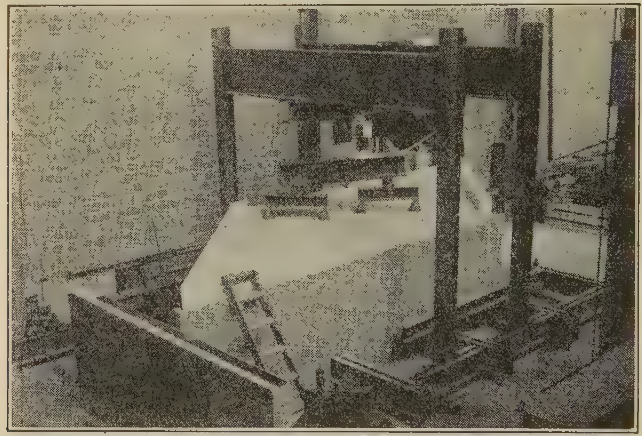

Fig. 8.—View of Half-Scale 45° Skew Slab with Curbs

The foregoing analysis was used, however, to design three half-scale models of 45° skew bridges and one one-fifth scale model of a 60° skew span. The three half-scale models present an interesting study of the arrangement of the reinforcement. In model No. 1, the main or longitudinal reinforcement is placed parallel to the roadway (see Fig. 7(*a*)) despite the fact that analysis indicates that the principal stresses in the central part of the bridge are in a direction normal to the abutments.

Because of the ineffective angle, the amount of this main reinforcement is twice as great as is required in model No. 2 (see Fig. 7(*b*)), in which the main steel is placed normal to the abutments. In model No. 3, the steel is placed in the same direction as in model No. 1, but in reduced amount, with a view to determining whether such a design would produce a satisfactory factor of safety. A view of one of these bridges under test is shown in Fig. 8.

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[11] "Analysis of Skew Slabs," by V. P. Jensen, *Bulletin No. 332*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1941.

As in the case of the right bridge slabs, the factors of safety for models No. 1 and No. 2 were quite large. The load carried at first yielding of the reinforcement, in addition to the dead load, was 3.5 live loads for model No. 1, 4.0 live loads for model No. 2, and 2.4 live loads for model No. 3. Corresponding numbers of live loads at failure were 7.5, 7.25, and 4.7, for the three models, respectively.

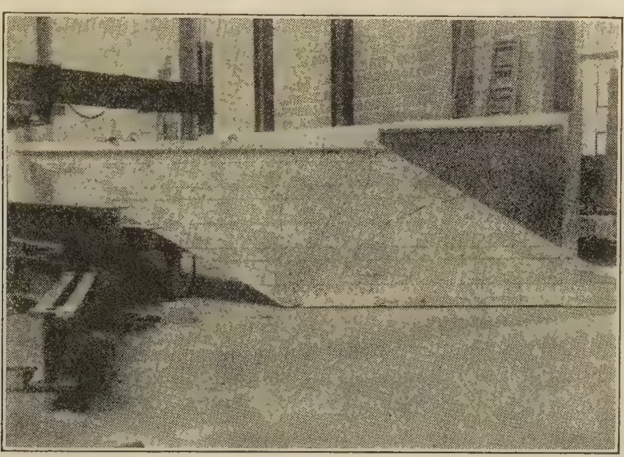

FIG. 9.—CRACKS IN THE TOP SURFACE OF A 60° SKEW SLAB

An interesting feature of a test of the one-fifth scale bridge with 60° skew (shown in Fig. 9) is the large crack that developed roughly on a line joining the obtuse corners of the slab, as produced by simulated truck loadings. Although such cracking on the top surface of the slab would generally not be anticipated, a little study indicates that its occurrence is perfectly natural.

8. THE SLAB-AND-STRINGER BRIDGE

One of the most common types of highway bridge, for spans of from 20 ft to 80 ft, is the stringer bridge, consisting of a slab supported on several steel or reinforced concrete stringers. Probably the most common case is that with five stringers, although three or four are sometimes used.

The distribution procedure developed by Professor Newmark[4] formed the basis of an extensive series of computations of beam and slab moments in some fifty bridge designs, in which variations in span and in relative stiffness of stringer and slab were systematically determined. This study covers only the bridge having five stringers. These results were analyzed and plotted, and as a final step the significant design factors were expressed in the form of empirical equations applying to the entire range of designs covered.[12]

A series of tests of I-beam bridges was made on the basis of this analytical work. These tests were all on quarter-scale models, with five I-beam stringers, supporting a reinforced slab continuous in the direction normal to the beams.[13] In six of fifteen right simple-span models, the slabs were mechanically anchored to the beams to provide composite action. Two spans of the prototype were used—20 ft and 60 ft—and variations were studied in the amount of reinforcement and in the effect of shear connectors. The principal objectives of the tests were to compare computed and measured strains in beams and slab, to determine the ultimate capacity of such bridges, and to aid in the development of improved design procedures.

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[12] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942.

A view of the stringers used in two of these bridges is given in Fig. 10. The shear connections used, consisting of short sections of bar channel welded to the top flanges of the I-beams, are shown in the right-hand bridge. The method of applying and measuring the load on these bridges was similar to that used on the slab bridges.

The skew **I**-beam bridge was also studied. However, the skew span with flexible stringers is so difficult to analyze that the test program was planned without having any analytical treatment as a guide to the experimental work. Five quarter-scale models of 60-ft **I**-beam bridges, with skews of 30° and 60°, were tested. Three of these bridges were designed to produce composite action of beam and slab.

Besides the tests of right and skew simple spans, three tests on two-span continuous stringer bridges have been made. Of these quarter-scale bridges, one was built with no shear connectors, the second with shear connectors throughout, and the third with the shear connectors used only in the region of positive moment.

A view of one of these bridges under test is seen in Fig. 11. It shows the application of load near the middle of each span, in position to produce a maximum negative moment at the middle support. The wiring for electric gages is seen at the top of the slab, at the middle pier, and at midspan, and leads to the switching apparatus and portable strain indicator at the side of the structure. More than three hundred electric gages were used on each of these test structures. The spans illustrated were each of 15 ft.

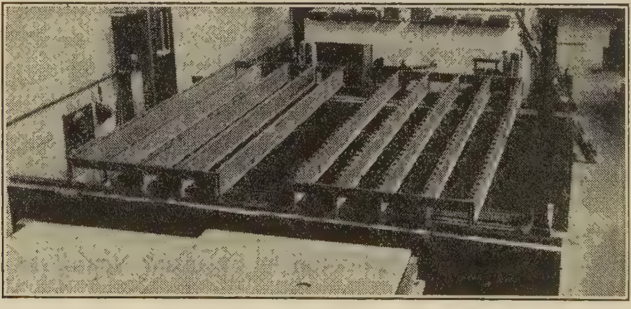

Fig. 10.—Stringers Used in Right I-Beam Bridges

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[13] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Seiss, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946.

9. GENERAL COMMENTS

In addition to the main results cited in this paper, the concrete slab investigation has produced many valuable by-products, including the development of many measuring instruments and procedures.

Not the least of the benefits from the program has been the training of young men in structural research methods and techniques. During the period of its existence the investigation has had the services of ten full-time investigators and fifteen research graduate assistants, the latter being graduate students employed half time for a 2-year period. There have also been a number of laboratory assistants, mechanics, and student and office helpers. Although some progress has been made in solving important problems of highway bridge design, it is evident that far more remains to be done than can ever be accomplished by any single organization. It is hoped that this Symposium will serve to stimulate interest and activity in the highway bridge field which will lead to definite improvements in design practice.

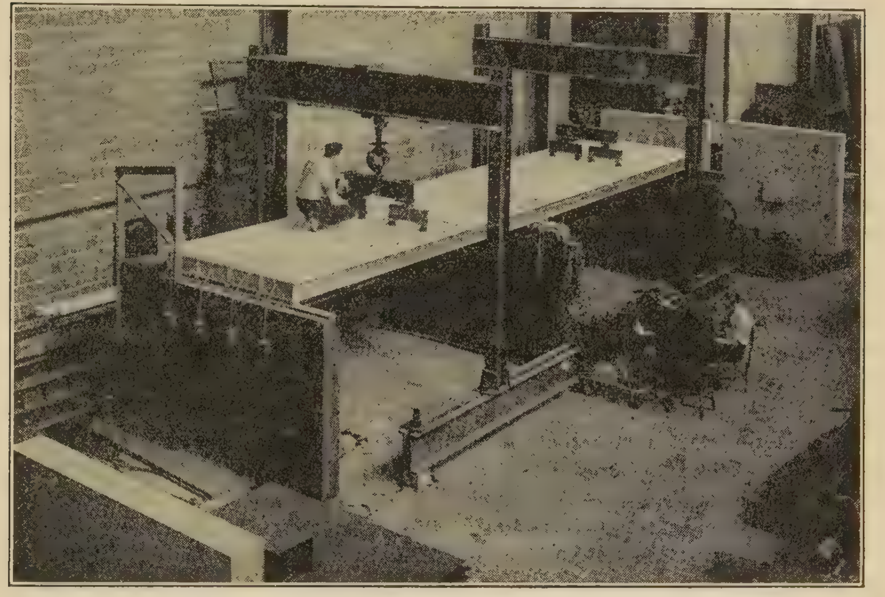

FIG. 11.—VIEW OF TWO-SPAN CONTINUOUS I-BEAM BRIDGE

10. ACKNOWLEDGMENTS

Major credit for initiating the program belongs to Alfred Benesch, then engineer of grade separations, Illinois Division of Highways, who realized the need for more information on many features of bridge design. Others, including E. F. Kelley and A. L. Gemeny of the United States Public Roads Administration and W. M. Wilson, H. M. Westergaard, and Hardy Cross, Members, ASCE, made important contributions to the initial outline of the program. Throughout the investigation, the planning of the various projects undertaken has been under the direction of an advisory committee, composed, in 1947, as follows:

U. S. Public Roads Administration—E. F. Kelley and Raymond Archibald, M. ASCE.
Illinois Division of Highways—George F. Burch, M. ASCE, and L. E. Philbrook, Assoc. M. ASCE.
University of Illinois—F. E. Richart and N. M. Newmark, Members, ASCE, with W. M. Wilson and T. C. Shedd, Members, ASCE, as consultants to the committee.

In addition to Professor Newmark and the writer, four technical investigators are engaged on this project at the University of Illinois: C. P. Siess, Assoc. M. ASCE, special research assistant professor; M. L. Gossard, Jun. ASCE, and W. E. Johnson, special research associates; and L. W. Jones, research graduate assistant. Two laboratory mechanics and student and clerical assistants also compose an essential part of the project.

The investigation is under the administrative direction of M. L. Enger, M. ASCE, dean of the College of Engineering and director of the Engineering Experiment Station, and F. B. Seely and W. C. Huntington, M. ASCE, respective heads of the departments of Theoretical and Applied Mechanics and Civil Engineering at the University of Illinois.

DESIGN OF I-BEAM BRIDGES

By N. M. Newmark,[14] M. ASCE

1. Introduction

Recommendations are presented herein for the design of the slab and the beams of I-beam highway bridges. Consideration is given to simple and continuous spans, to right and skew bridges, and to bridges in which T-beam action is obtained by the use of either shear connectors or concrete beams. The treatment is based on both analyses and tests. The practical significance of the design recommendations is discussed in the light of current practice and current specifications.

The I-beam bridge, consisting of a concrete roadway slab continuous over steel stringers, has found widespread application for highway bridges because of its simplicity of design and ease of construction. This paper discusses the behavior of the slab-and-stringer bridge having beams in the direction of traffic. The slab may be mechanically bonded to the beams by shear connectors. The bridges considered, in general, are supported by four or more parallel beams of equal stiffness at a constant spacing of the beams between the limits of 5 ft and 8 ft.

The terminology used in the paper is illustrated in Fig. 12. The span of the bridge is denoted by a, the spacing of the beams by b, and the thickness of the slab by h. The main reinforcement in the slab is in the direction transverse to the beams; the longitudinal reinforcement is secondary reinforcement.

The analytical background for this work was presented by the writer in 1938[4] and in 1942.[12] The experimental background is based mainly on the writer's tests of quarter-scale models at the University of Illinois.[13,15] A paper on the same general subject, but with more conservative recommendations based almost entirely on analyses, appeared in 1943.[16] The analytical studies[4,12] involve the usual assumptions of the ordinary theory of flexure of slabs. The detailed calculations are long and tedious. However, influence coefficients for moments in the slab and in the beams at various points for a number of structures of different proportions are summarized in the 1942 report.[12] Simple empirical equations expressing the moments resulting from truck loads in both the slab and the beams are also given in this report.

As stated, the experimental work[13,15] has been entirely on quarter-scale models. Initial studies indicated that duplication of the behavior of full-scale prototype structures could be obtained in quarter-scale models if attention were paid to the proper scaling of the aggregate and reinforcement. Extensive studies indicated good correlation, over the entire range of action, from elastic behavior at low loads, the subsequent action at working loads, and the behavior after considerable yielding of the reinforcement, to and including the failure of the slab by punching. The reinforcement used in the models consisted of bars, $\frac{1}{8}$ in. square, specially heat treated to have stress-strain characteristics corresponding to the full-sized reinforcement used in an actual structure.

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[14] Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[15] "Studies of Slab and Beam Highway Bridges: Part II. Tests of Simple Span Skew I-Beam Bridges," by N. M. Newmark, C. P. Siess, and W. M. Peckham, *Bulletin No. 375*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1948.

[16] "Design of Slab and Stringer Highway Bridges," by N. M. Newmark and C. P. Siess, *Public Roads*, January–February–March, 1943, pp. 157–164.

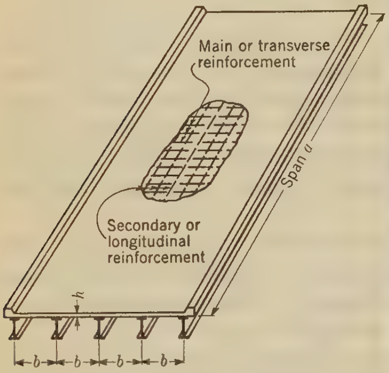
Main or transverse reinforcement
Span a
Secondary or longitudinal reinforcement
h
b
b
b
b
b

Fig. 12.—Typical I-Beam Bridge

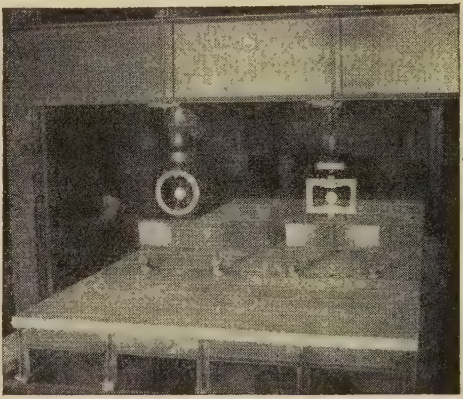

Fig. 13.—Quarter-Scale Model of I-Beam Bridge

Fig. 13 shows a completed model bridge subjected to tests in the laboratory.[17] The loading represents the rear wheel loads of two trucks in adjacent lanes. Tests have been made on both right and skew bridges, with and without shear connectors, and on continuous bridges.

2. Distribution of Wheel Loads to Slab and Stringers in Simple-Span Right I-Beam Bridges

a. Fundamental Concepts.—The moments in the slab and in the beams of an I-beam bridge are dependent on the geometric proportions of the bridge and on the stiffness of the beams relative to the stiffness of the elements of the slab in a direction transverse to the beams. The "effective width" of the slab, for the purpose of computing relative stiffness, is dependent on the type and arrangement of loading and on the beam spacing. In previous analytical work it was convenient to define the stiffness of the beams relative to that of a width of slab equal to the span of the bridge, although the latter is greater than the "effective width" for most practical conditions. In order to keep the same definition in this paper, the relative stiffness of the beams, denoted by the symbol H, is defined as the dimensionless ratio of the product of the modulus of elasticity, E_b, and the moment of inertia, I_b, of the beam to the corresponding product for a width of slab equal to the span of the beams:

$$H = \frac{E_b\, I_b}{a\, E\, I} \qquad \text{(2)}$$

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[17] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 25, Fig. 5.

In Eq. 2, E is the modulus of elasticity of the slab; and I is the moment of inertia per unit of width of the slab, which can be taken as $h^3/12$.

The quantity of greatest influence on the action of the slab in distributing loads to the beams is the stiffness of the slab in the direction transverse to the beams. However, the slab also has stiffness in the longitudinal direction and consequently carries part of the longitudinal moment. The part of the longitudinal moment carried by the slab is small, and at any section through the bridge where the slab may be cracked the beams carry a relatively larger share. It is on the safe side, and not unreasonable, to assume that the beams carry all the longitudinal moment.

The effect of the relative stiffness of the beams on the moments resulting from a single concentrated load is large; but the effect on the moments caused by truck loads (which control the design of the I-beam bridge) is small. This is fortunate because the magnitude of H is subject to considerable uncertainty, since its value depends on the modulus of elasticity and on the moment of inertia of the concrete slab.

In general, the value of H for representative designs of a 60-ft span may range from 3 to 8 for noncomposite beams, from 5 to 15 for composite I-beams, and from 15 to 30 for concrete T-beams. For other span lengths, the values of H are roughly proportional to the span, increasing slightly less rapidly than the span increases.

When the slab cracks from moments in the transverse direction, the relative flexural stiffness of the slab at the crack is reduced. However, the average stiffness of the slab determines the distribution of the loads to the beams, and the average transverse stiffness is only slightly reduced by cracking since there are usually fairly large sections of the slab that remain whole between cracks. For this reason, the value of I used in defining H can be taken as the value corresponding to a plain concrete slab of full depth h, without cracks. The slight increase in stiffness resulting from the presence of the transverse steel is balanced to some extent by the slight decrease caused by cracking. The results of tests justify this choice of I for the slab in the determination of H.

Since H varies inversely as the cube of the depth of the slab, the depth of the slab has great influence on the action of the bridge. For example, changing the depth from 6.5 in. to 7 in., other factors being equal, will reduce H by very nearly 20%. For a given type of bridge, however, the range in values of H is generally small enough that formulas for the moments controlling the design can usually be made independent of H.

The wheel loads considered herein are those for the standard H-truck loading in the AASHO Standard Specifications for Highway Bridges.[18]

The standard H-truck has the wheels of each axle spaced 6 ft apart, with front and rear axles spaced 14 ft apart. Each of the rear wheels carries a weight of four tenths of the total weight of the truck, and each of the front wheels carries a weight of one tenth of the total weight of the truck, or one fourth of the rear wheel weight. The weight of the truck, in tons, is designated by a numeral; thus: H-20. The rear wheel load P, in terms of which moment coefficients are stated, is the weight on a rear wheel increased by the impact factor.

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[18] "Standard Specifications for Highway Bridges," AASHO, Washington, D. C., 4th Ed., 1944.

Each truck is considered to occupy the central part of a 10-ft traffic lane; therefore, the minimum distance between the center of a wheel and the face of a curb is taken as 2 ft, and the minimum distance between centers of wheels of trucks in adjacent lanes is taken as 4 ft.

A subsequent revision in the specifications has the effect of considering a single maximum wheel-load concentration of 12,000 lb, corresponding to an H-15 truck, in the design of the floor slab for a bridge designed for H-20 loading.

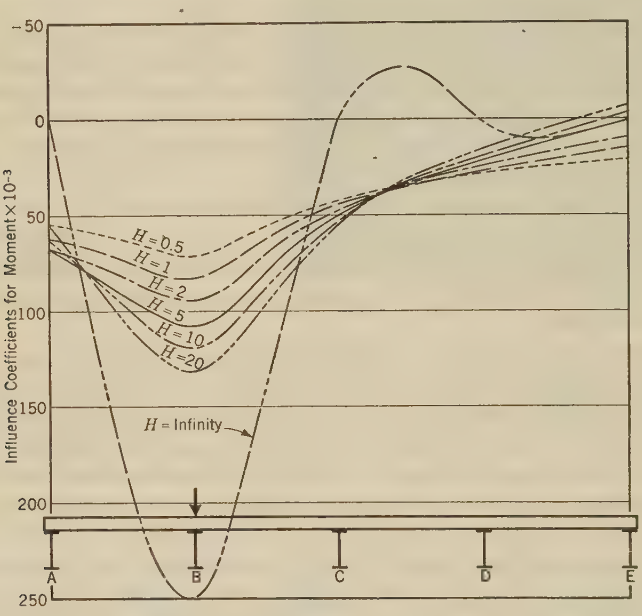
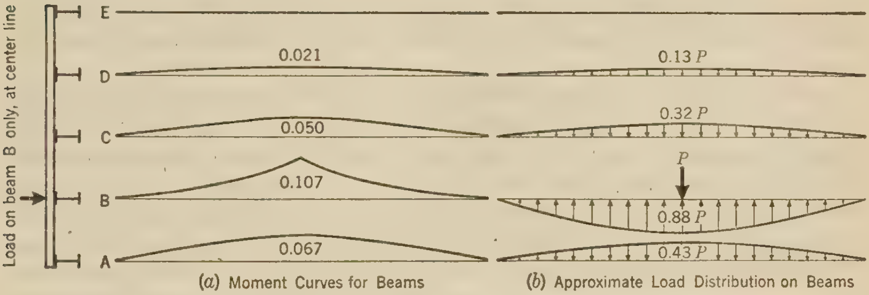


Fig. 14.—Effect of Slab Stiffness on the Moments in Beams, for a Single Load in the Bridge

b. Action of Slab in Distributing Loads to Beams.—The principal load-carrying elements of an I-beam bridge are the beams themselves; the slab is merely the roadway, and can be repaired without serious difficulty if it is locally damaged. However, the loads reach the beams through the slab, and the magnitude of the loads carried by the beams is affected to an important extent by the stiffness of the slab. As an indication of the role the slab plays in distributing loads to the beams, consider Fig. 14 which shows influence lines for moments at midspan of beam B, an intermediate beam in a 5-beam bridge, in which $b/a = 0.1$. By reason of a reciprocal relation, the ordinates to the diagram at points under the beams show also the moments in all beams for a single concentrated load at beam B.

The stiffness of the slab is measured by the reciprocal of the value of H. For a slab of zero stiffness, $H = \infty$, and the moments in all beams, except beam B, are zero, whereas the moment in beam B is $P\,a/4$. On the other hand, for $H = 0.5$ the moment in beam B is only 0.071 $P a$, and the moments in the adjacent beams are not much less. This fact emphasizes the importance of the slab thickness in distributing the loads to the beams.

To a marked extent the slab acts in the same manner as a diaphragm or a transverse truss. The major difference between the action of the slab and that of a diaphragm is in the nature of the loading transferred to the beams. For a diaphragm, the loads carried to the beams are essentially concentrated loads at the points where the diaphragm crosses the beams. The nature of the loading transferred to the beams by the slab is shown in Fig. 15. The complete moment diagram for each beam is indicated for a unit concentrated load on beam B. The nature of the moment diagrams is noteworthy: For beams A, C, D, and E, the curve is roughly a sine curve; but, for beam B, it is concave upward and has a cusp at the load point. Also shown in Fig. 15 are the loading curves that correspond to the moment diagrams. The loadings are approximately sine curves and are downward for all beams except B. For beam B the loading consists of two parts—a downward concentration equal to P, and an upward sine curve of distributed load. The total distributed load is indicated on each curve.



(a) Moment Curves for Beams (b) Approximate Load Distribution on Beams

FIG. 15.—ACTION OF SLAB IN DISTRIBUTING CONCENTRATED LOAD, AND THE RESULTING MOMENTS IN BEAMS ($H = 5$ AND $b/a = 0.1$)

The fact that tensions between the beam and the slab arise in the I-beam bridge results from the difference in the nature of the deflections of a slab and a beam subjected to concentrated loads. The deflection curve is more localized for a slab, but is smoother for a beam. Hence, the two elements tend to separate. Separation is furthermore caused by the tendency of the slab to curl because of shrinkage and temperature variations.

The action of the slab as a diaphragm or as a transverse beam can be considered as follows: For a line load distributed over the length of the bridge approximately as a sine curve, the slab acts as a transverse beam with an effective width practically equal to the span length. For a concentrated load at midspan, the first term in the Fourier sine-series expansion accounts for about 81% of the moment at midspan. The remaining terms in the series, accounting for the remaining 19% of the moments, are distributed in different ways with different effective widths of the slab. The net effect is practically the same as if the slab were considered as a diaphragm having an effective width somewhat less than the length of the span, but more than half of the span. The exact proportion does not make much difference as the distributions of loading to the beams are not greatly sensitive to minor variations in stiffness of the slab or of transverse diaphragms. One possible diaphragm analogy, in which the effective width of the slab is taken as half the span for 81% of the load, and the remaining 19% is considered as being carried by the loaded beam, gives results which are on the safe side, and in excellent agreement with more elaborate analyses.

Since the slab acts as a very effective diaphragm, it is unnecessary to provide additional diaphragms, except for construction purposes, if the slab can perform its function of distributing loads to the beams at the same time that it provides a roadway for the wheels to roll over. Where it is expedient or desirable to make the slab thin, and therefore flexible, some additional transverse bridging is desirable. In general, however, such bridging is not particularly effective except for loads at or close to the section where the transverse frames are located.

As the loading on an I-beam bridge approaches in magnitude the capacity of the bridge, both the beams and the reinforcement in the slab start to yield. Although one element may yield before another, the over-all effect is to maintain approximately the same relative stiffness of the beams; consequently, even near capacity loads, there are differences in the moments carried by the various beams. It is not necessarily safe, therefore, to use as a basis for design of the beams the so-called plastic theory of limit design. Such a procedure would be applicable and justified only when the slab or other transverse framing remained stiff enough at the higher loads to distribute the loads equally to all the beams.

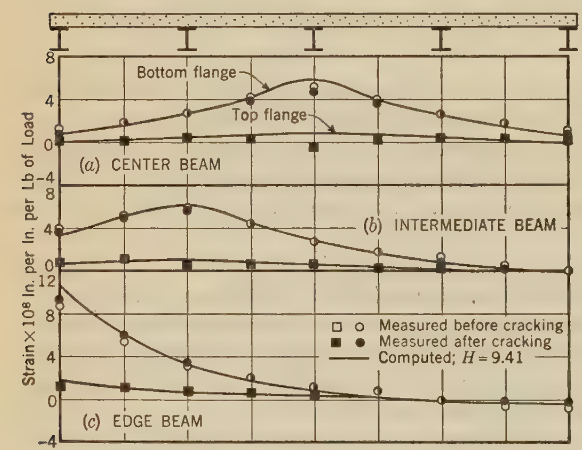


FIG. 16.—INFLUENCE ORDINATES FOR STRAINS IN BEAMS AT MIDSPAN FOR A LOAD MOVING ACROSS THE BRIDGE AT MIDSPAN; QUARTER-SCALE MODELS OF BRIDGES OF 60-FT SPAN, WITH COMPOSITE ACTION

(Top Flange Strain Plotted with Compression Above the Base Line)

c. Moments in Beams.—Test results indicate good agreement between the measured and computed values of moments in the beams. As an example of the agreement, experimentally determined influence lines for flexural strains in the beams of a composite I-beam bridge are shown in Fig. 16, where comparison is made with the theoretical values. Incidentally, Fig. 16 illustrates good agreement with the theory for both top and bottom flange strains, and shows little difference in the results before and after cracking of the concrete.

When several loads are on the bridge, corresponding to trucks in adjacent lanes, the moments in the beams are much more uniform than those for a single load on the bridge, even when the relative stiffness of the slab is fairly small. Nevertheless, there is still some difference in the moments for a given arrangement of the loads. This condition is shown by Fig. 17, which is a comparison of measured and calculated beam strains in a model bridge (60-ft span without composite action) for a condition of loading corresponding to maximum moment in beams B and C. A slightly altered arrangement of loads would be required to produce maximum moment in beam A unless the curb were directly over beam A. The close agreement between the measured and calculated values is to be noted.[19] For this loading, beams B and C yielded first at a panel load of about 5 kips, whereas beam A did not yield until a load of about 6 kips was reached. Beam D carried a larger proportion of load after the other beams yielded, and began to yield at a load of about 7 kips, whereas beam E did not yield at all, although the remaining beams had developed large permanent deflections and were unserviceable when the test was stopped.

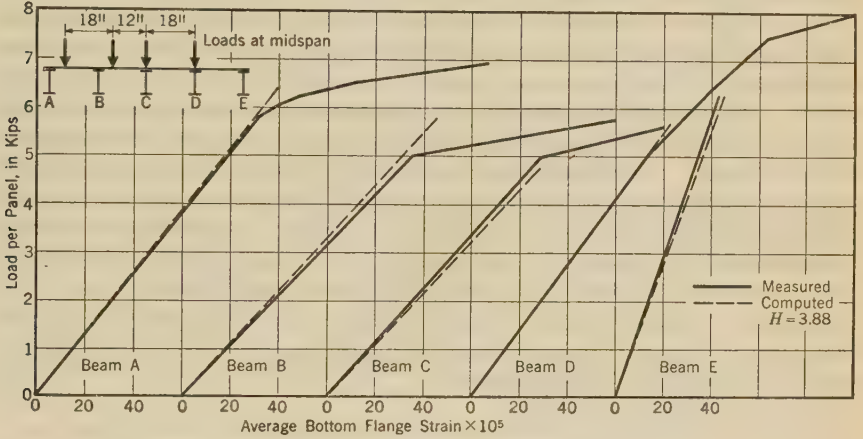


FIG. 17.—LOAD-STRAIN CURVES FOR BEAMS; QUARTER-SCALE MODELS OF 60-FT SPAN, WITHOUT COMPOSITE ACTION

The close agreement between the measured and computed beam strains is typical of the test results on long-span bridges for all arrangements of loading.[13] However, slight discrepancies were obtained in some of the tests of models of prototype bridges 20 ft in span length. The differences are easily explained. In short-span bridges, from about 20 ft to 30 ft long, the relative longitudinal stiffness of the slab is fairly high and an appreciable part of the total longitudinal moment is carried by the slab. In general, after the slab is cracked, the agreement between measured and computed maximum strains in the beams is not good unless allowance is made for the failure of the slab to carry the theoretical proportion of the total longitudinal moment. In other words, after appreciable cracking takes place, the slab acts much less effectively as a longitudinal beam than before, even though its effectiveness in carrying loads laterally is relatively unimpaired.

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[19] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 94, Fig. 40*a*.

A safe assumption for the beams is that the slab is capable of carrying no part of the static moment. If the computed beam moments are increased arbitrarily so as to account for the total static moment the computed strains are in close agreement with the measured strains. It can be concluded, therefore, that the theoretical analysis can be used as a guide in apportioning the total static moment among the beams for the purpose of design.

d. Concept of Proportion of Wheel Load Carried by Beams.—Although the actual load distribution to the beams is fairly complicated, a fictitious distribution of loading to the beams can be selected to account for the moments in the beams. The maximum moments in the various beams as a result of truck loading are nearly equal, and are approximated closely by assigning part of a wheel load, k, to each beam. This proportion depends on a, b, and H; but within practical limits, for bridges of a given type, the proportion is equal to the spacing of the beams divided by a constant of between 5 ft and 6 ft, depending on whether the stiffness of the beams relative to that of the slab is high or moderate. The proportion of a wheel load "carried" by each beam must be greater than the average obtained from the number of wheel loads on the bridge divided by the number of beams, since different load positions are required to produce maximum moments in various beams.

It is expedient to express k as

$$k = \frac{b}{s} \qquad (3)$$

in which s is some length which depends on the characteristics of the structure.

Live-load moments are reported[12] for standard truck loads on more than 50 I-beam bridges of various span, beam spacing, and relative stiffness of beams to slab. Some of the structures analyzed represented extreme conditions but were investigated to determine trends in the moment coefficients beyond the range of practical applications. Values of s to be used in Eq. 3, in computing maximum moments at midspan of interior beams due to rear wheels of standard trucks, were determined from tabulated moment coefficients.[12] A value that fits practical designs is given by the relation:

$$s = 4.6\text{ (feet)} + 0.04\frac{a}{\sqrt{H}} \qquad (4)$$

Similar studies for the effect of rear wheels or of additional trucks in a lane indicate that the same factor k can be used. Consequently, for any truck loading, the proportion of a wheel load to be used in the design will be given by Eqs. 3 and 4.

e. Deflections of Beams.—The longitudinal distribution of loading on the beams is not the same shape for all beams. Therefore, the moments in the beams and the deflections of the beams in general are not proportional, especially for single concentrated loads. This condition is revealed most clearly in Fig. 18 which shows the moments and deflections at the center of a bridge relative to the maximum values, and to the averages for the five beams, when a single load is placed at the center ($H = 5$ and $b/a = 0.1$). The moments are divided among the beams in a much less uniform manner than the deflections. Consequently, it is necessary to measure strains rather than deflections to obtain an accurate picture of the action of the structure. However, the differences between the patterns of moment and deflection are much smaller for truck loadings than for single concentrations. It is on the safe side but not overly conservative to compute maximum deflections for the same proportion k of a wheel load that is used to determine moments.

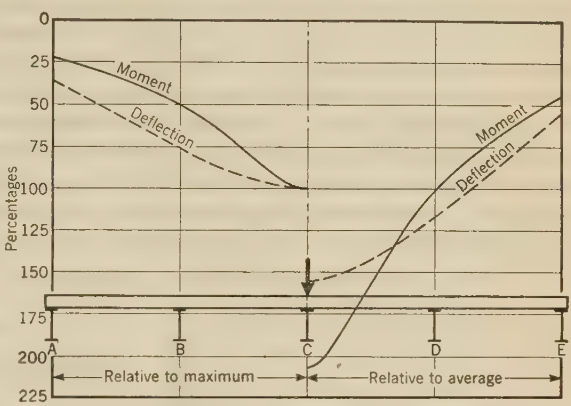

FIG. 18.—RELATIVE MOMENTS AND DEFLECTIONS OF BEAMS FOR A SINGLE LOAD ON THE CENTER LINE OF SPAN

($H = 5$; $b/a = 0.1$)

For more accurate determination of deflections, prepared tables of coefficients[12] may be used. Justification for computing deflections by the theory is apparent from the test results published in 1946, which demonstrate[20] the close agreement between the theoretical values and the test results.

f. Factors Affecting Moments in Slab Due to Concentrated Wheel Loads.—The behavior of the slab may be considered as divided into several component actions, as follows:

(1) The behavior of a single panel acting as a simply-supported rectangular slab. Moments corresponding to such action have been given by Professor Westergaard.[2] (His moments are given for Poisson's ratio = 0.15 instead of zero, as assumed in this paper.)

(2) The effect of continuity with adjacent panels, assuming nondeflecting beams. The maximum moments resulting from wheel loads in action (1) are reduced by continuity, the reduction for representative structures being of the order of 20%.

(3) The effect of the deflections of the supporting beams. The effect of this action is to increase the maximum positive moments, and the increase is of the same general order of magnitude as is the decrease caused by action (2).

The relative magnitudes of these effects, determined from the ordinary theory of flexure of slabs, are shown in Fig. 19 for a particular type of bridge.

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[20] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 97, Fig. 42.

The single-panel moment is determined for Poisson's ratio = 0 and is therefore somewhat less than the value given by Professor Westergaard. The curves in Fig. 19 give influence ordinates for moment at the center of a panel of the slab. The magnitude of the moment at this point, when the load is applied at the point, is determined by a correction to the ordinary theory of flexure of slabs in a manner devised by Professor Westergaard.[21] The effects of continuity on the single-panel moment are shown by the curve for $H = \infty$—that is, for nondeflecting beams. The differences between the curves for $H = \infty$ and for the other values of H are due to deflection of the beams. For a bridge of the proportions indicated, the value of H in a reasonable design is approximately 5. In any case, the greatest part of the maximum positive moment at the center of a panel of the slab, resulting from the combined actions, is the moment directly beneath the load point caused by the action of the slab as a single panel.

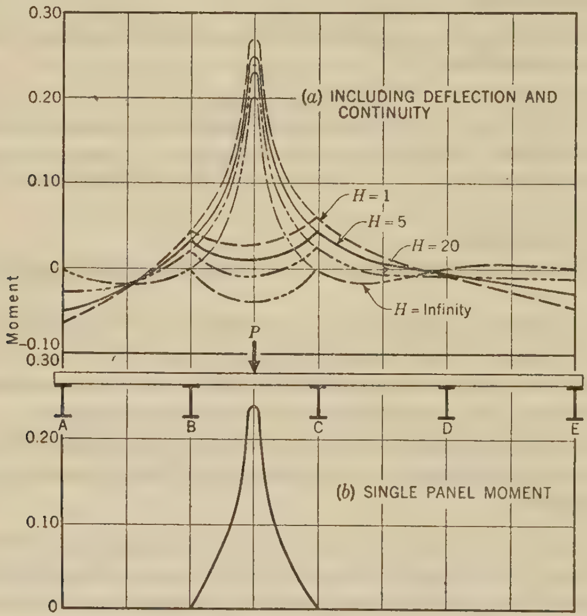

FIG. 19.—INFLUENCE LINES FOR TRANSVERSE MOMENT IN SLAB AT BC AT MIDSPAN (Load on Center Line ($b/a = 0.1$))

The theoretical values of the transverse moment and the longitudinal moment (respectively, M_t and M_l) for a single panel of a slab at a point directly beneath a load P are given approximately by the following formulas:

$$M_t = \frac{1.16\,P}{3 + 10\,\dfrac{c}{b}} \qquad (5a)$$

and

$$M_l = M_t - 0.080\,P \qquad (5b)$$

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[21] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942, pp. 14–15.

In Eqs. 5, c is the diameter of the equivalent circular area over which the wheel load is assumed to be distributed. For H-15 and H-20 loadings the quantity c is ordinarily taken as 15 in. or 1.25 ft.

To keep in mind the relative magnitudes of the moments due to various causes, it is desirable to remember that for ordinary proportions of I-beam bridges M_t may vary between 0.20 P and 0.28 P, whereas M_l may vary between 0.12 P and 0.20 P.

g. Major Design Moments in Slab.—As a result of tests of both single-slab panels and I-beam bridges, it was found that in a reinforced concrete slab the stress in the reinforcing steel immediately beneath a single concentrated load is always considerably less than the stress computed by the theory. This fact is not unreasonable since even the modified theory applying to the immediate region of the load is not properly applicable to reinforced concrete after cracking.

It is impossible to separate into the component actions (1), (2), and (3) the stresses in the slab determined from tests, and it is therefore difficult to state whether the effects of deflection and continuity are indicated reasonably well by the theory. However, test results do indicate good agreement with theoretical beam moments; consequently, it seems reasonable that at least the qualitative values of these effects must be as indicated by the theory.

Typical quarter-scale model bridges were tested in the laboratory. Load-strain curves for slab reinforcement in a number of these bridges, summarized from the test data, demonstrate[13] the discrepancies between the theory and the test results which require some explanation.

A number of factors influence the magnitude of the strains in the steel at the center of a panel of the slab. A major uncertainty is the extent and degree of cracking, for steel stresses are generally low until the concrete cracks. In addition to the direct effect of cracking, there is an indirect effect produced by redistribution of moments on various sections as the relative stiffness of different parts of the structure changes due to cracking—that is, the amount of transverse steel over the beams may have an appreciable influence on the strains in the reinforcement at the center of a panel. If inadequate transverse reinforcement is provided over the beams, the effect of continuity is reduced by cracking of the slab as the result of negative moments over the beams. The result may be an appreciable increase in the positive moments at the centers of the panels. On the other hand, if enough transverse reinforcement is provided over the beams so that the slab does not crack there, an increased part of the moment is attracted to the part of the panel over the beams, and the slab acts to some extent as if it were haunched. The increase in negative moments is accompanied by a decrease in positive moments at the center of the panels.

The effect of torsional resistance of the beams, which is especially marked when there are shear connectors, is to increase the effect of continuity and to reduce the effect of deflections of the beams, resulting in an appreciable net decrease in the positive moments at the centers of the panels.

The test results can be summarized by the statement that the measured steel strains in the reinforcement at the center of a panel of the slab are always considerably less than is indicated by the theory, both for simply-supported panels and for I-beam bridges, with the exception of very short-span bridges. For the latter, the steel strains after extensive cracking are about equal to the values indicated by the theory.

For short spans, the relative longitudinal stiffness of the slab is fairly high and an appreciable part of the total longitudinal moment is carried by the slab. A rough measure of the proportion of the static moment carried by the slab is the ratio of the total stiffness of the slab to the total stiffness of the beams and the slab on any cross section. With four slab panels and five beams, this ratio, r, becomes

$$r = \frac{4\,b\,E\,I}{4\,b\,E\,I + 5\,E_b\,I_b} = \frac{1}{1 + \frac{5}{4}\frac{a\,H}{b}} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

For the short-span bridges tested, the value of r is between 0.23 and 0.07, whereas for the long-span bridges the value of r is 0.02 or less. When the slab cracks in the short-span bridges, the beams must carry a larger part of the total moment; consequently, the slab must distribute laterally a greater part of the load, and in doing so the transverse slab moments are increased. This behavior (at least in part) is an explanation of the fact that the test results indicate measured moments more nearly equal to the theoretical moments for the bridges with large ratios of spacing to span and low values of H.

It is apparent from test results[13] that the slab probably did not crack as the result of negative transverse moments over the beams—at least not until relatively high loads were applied.

For a long-span bridge the effects of front wheels on the moments become of importance, and the effect of deflection of the beams increases relative to the effect of continuity of the panel. Previous design procedures given by the writer have provided for increased design moments for longer spans in agreement with the increase computed from the theory. The nature of the effect of increase in span length on the transverse moment at the center of a panel can be inferred from an examination of typical influence surfaces for transverse moment.[22] As more loads, including the front wheels of the truck with rear wheels at the point under consideration, come on the span, the moment tends to increase because the relative deflections of the beams increase.

The general results of the tests indicate that the theory is considerably on the conservative side. Furthermore, there is a large reserve of strength in the slab beyond the point at which the reinforcement first begins to yield. Final and complete failure does not occur until yielding is general in the entire region near the load. The failure takes place by a kind of punching action in which a cone of concrete is pushed out of the slab. This type of failure occurs at loads of from 50% to 100%, or more, greater than the loads producing first yielding.

Therefore, it appears that a reduction in the theoretical moment in the slab can safely be made for the purposes of design. It is common in floor slabs for buildings, subjected to uniform loads, to reduce the theoretical moments by about 30%. A similar reduction is suggested herein. The design moments recommended in Section 7 of this paper are about 70% of the theoretical moments for the longer span bridges, and are slightly higher for short-span bridges—which is in line with the indications from the test results.

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[22] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942, Appendix B.

h. Secondary Moments in Slab.—Empirical equations giving the approximate magnitude of the maximum negative moment at the center of a panel of the slab, the maximum positive and negative transverse moments in the slab over a beam, and the maximum longitudinal moments in the slab were reported in 1942.[12] At the center of a panel the maximum negative transverse moment is less than one fourth as much as the positive transverse moment, and the proportion decreases with greater flexibility of the beams. Over the beams both positive and negative transverse moments can be produced by different loadings. The negative moments are generally about three fourths as great as the positive moments near the centers of the panels except when the beams are very stiff. The positive moments vary from about 50% of the negative moments when the beams are very flexible to practically nothing for stiff beams.

The relative magnitudes of the secondary transverse moments in the slab, compared with the major design moment, vary at different points along the span of the bridge. For this reason, and because the test results indicate significantly lower stresses than does the theory, the transverse moments to be provided for in a design can be stated most conveniently in terms of a fixed proportion of the steel used in the transverse direction at the centers of the panels. Detailed recommendations are given subsequently in Section 7.

In the region of a concentrated load on a single-panel slab there is a moment in the longitudinal direction about two thirds as great as the moment in the transverse direction. The theoretical longitudinal moment is appreciably increased by the effect of deflection of the beams, and is only slightly reduced by continuity. The theory alone indicates that almost as much longitudinal reinforcement as transverse reinforcement is required for short-span bridges, with somewhat less for long-span bridges. However, both tests and judgment indicate that the structure cannot be seriously harmed if the required theoretical amount of longitudinal steel is not furnished.

An illustration of the effect on the strains in the longitudinal steel caused by varying the amount of longitudinal steel is shown in Fig. 20, where strains are plotted for three quarter-scale models of 20-ft span bridges similar in all respects except in the amount of longitudinal steel. The longitudinal steel for the various bridges was as follows: For WL5b, 1.06%; for W5 and N5, 0.59%; and, for W05, 0.20%. It can be noted that the strains are not increased at all in proportion to the amount of decrease in reinforcing steel. The relationship is typical of secondary "participation" stresses in which the deformations are practically fixed, and the stresses are relatively unchanged by removing or adding material.

However, the tests[13] indicate that the action of the structure is very unsatisfactory if practically no longitudinal steel is furnished, since the action of the slab in distributing loads to the beams is impaired. The cracks in the bridge with only 0.20% of longitudinal reinforcement[23] are practically all in the transverse direction, and opened up fairly wide, whereas the cracks in the bridge with 1.06% longitudinal reinforcement were more numerous but relatively small and were in a more random pattern. The appearance of the bridge with only nominal longitudinal reinforcement was distinctly unsatisfactory.

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[23] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 54, Fig. 17.

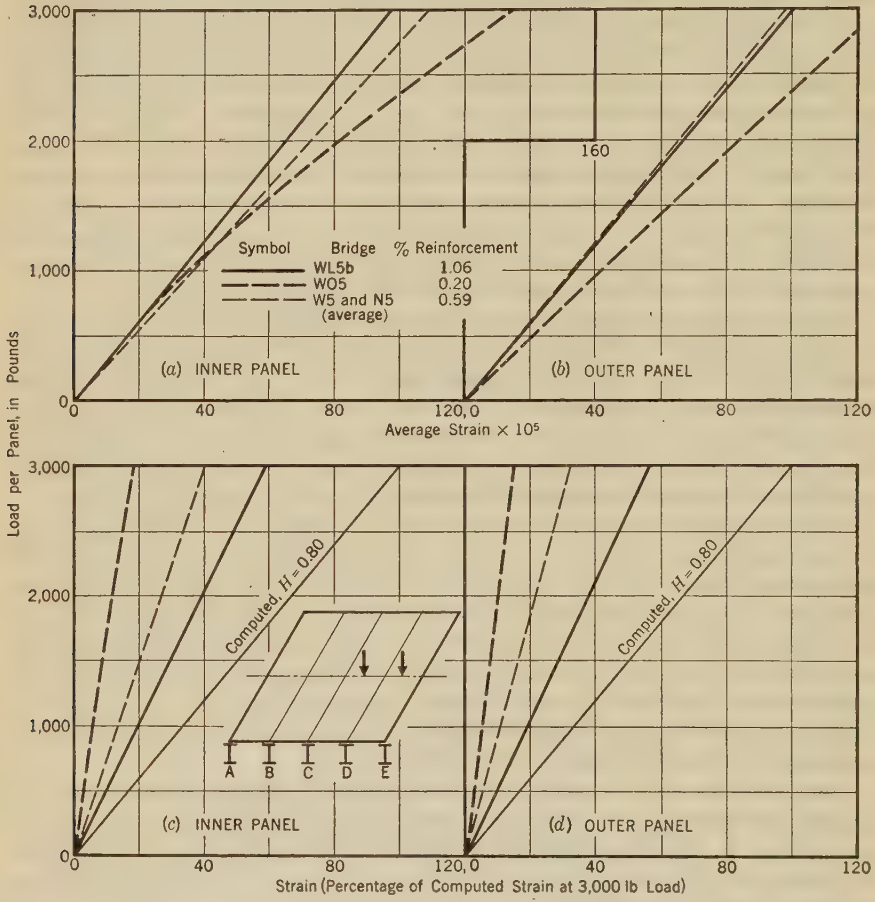

FIG. 20.—LOAD-STRAIN CURVES FOR LONGITUDINAL REINFORCEMENT AT CENTERS OF PANELS; QUARTER-SCALE MODELS OF BRIDGES OF 20-FT SPAN, WITHOUT COMPOSITE ACTION

This comparison illustrates the desirability of furnishing an adequate amount of longitudinal reinforcement. Moreover, the longitudinal steel serves the additional purpose of tying the slab together and resisting to some extent the action of temperature changes and shrinkage.

To prevent disintegration of the concrete from cracking due to other causes than loads, it is desirable to provide a reasonable amount of longitudinal reinforcement. In structures where longitudinal continuity of the slab is necessary for the safety of the structure, as in a composite I-beam bridge, an amount of longitudinal steel is desirable sufficient to prevent shrinkage cracks from opening up and impairing the safety of the structure—in spite of the fact that the analysis indicates much lower requirements in the amount of reinforcement than those for noncomposite bridges.

3. DISTRIBUTION OF DEAD LOAD TO SLAB AND STRINGERS IN SIMPLE-SPAN RIGHT I-BEAM BRIDGES

a. Moments in Beams.—The dead-load moments in the beams may be computed by dividing the total dead-load moment at the center of the bridge equally among the beams. In general, this procedure is the same as considering each beam to carry its own weight, the weight of one panel of the slab, and whatever paving allowance is considered. The added weight of curbs, sidewalk, and handrails will compensate for the fact that only a half panel of the slab is carried by an edge beam. Even when the edge beam is more heavily loaded than the interior beams, the action of the slab transmits a part of the load to the interior beams, and all the beams share to some extent in carrying the additional load applied to the edge beam. The relative proportions of dead-load and live-load moments in the beams may vary over a considerable range depending primarily on the slab thickness and on the span of the bridge. For designs made according to the provisions recommended herein, dead-load and live-load moments are approximately equal for spans of about 65 ft; dead-load moments exceed live-load moments by some 20% for spans of about 85 ft; and live-load moments exceed dead-load moments by perhaps 50% for spans of about 45 ft. However, an increase in slab thickness of 1 in. in 6½ in. will increase the dead-load moment about 15%. Therefore, the design of the beam is fairly sensitive to changes in slab thickness and to provisions for wearing surface or added pavement.

b. Moments in Slab.—Dead-load moments in the slab are affected by the manner of construction and by the sequence of casting the concrete.[12] Therefore, they are subject to considerable uncertainty. Fortunately, the dead-load moments in the slab are small—ordinarily only about 10% of the live-load moments.

In a bridge constructed without shoring, the weight of the slab is transferred to the beams by the formwork immediately after casting. When the formwork is removed, the reactions of the slab on the beams are changed from simple-span reactions to continuous beam reactions. The consequent deflections of the beams change the relative proportions of positive and negative moments in the slab. In general, the positive moments are increased and the negative moments reduced from the continuous beam moments. This change is greater the more flexible the beams.

When a uniform load is placed over the entire structure, as an added pavement applied subsequent to removal of the forms, large positive moments are introduced over the entire structure. On the other hand, a loading applied to the outer beams, such as a curb or handrail cast after the roadway forms are removed, produces appreciable negative moments in the transverse direction over the entire structure. For these reasons it is felt that, unless the construction sequence is controlled, provision should be made in the design for a positive dead-load moment in the transverse direction at the centers of panels of the slab of magnitude $\frac{1}{8}\, w\, b^2$, in which w is the intensity of the dead load per unit of area. When the beams are unusually stiff, or where composite construction is used, this value may be reduced.

4. Composite I-Beam Bridges

a. Advantages of Composite Action.—Composite action of the slab with the beams may exist to a limited extent, or even almost completely, although no provision for such action is made in the design. On the other hand, it is impossible to depend on composite action unless positive provision is made for adequate shear connectors, since temperature changes, shrinkage, and the effects of wheel loads may sooner or later destroy the adhesive bond between the slab and the beams.

In a composite I-beam bridge a panel of the slab acts as the upper flange of the composite beam. Tests indicate that, for usual proportions, the full width of the slab is effective. The moment of inertia of the composite section may be from two to three times as great as that for a design that does not utilize composite action. Therefore, the composite bridge has a greater value of H and the beams have relatively greater maximum moments compared with the average. Consequently, in the design of a composite I-beam bridge, a somewhat greater proportion of a wheel load must be assumed to be carried by the beams than in a noncomposite bridge.

The distance to the bottom flange from the neutral axis of the composite section is considerably greater than in a noncomposite design. Therefore, the stress controlling the design of a composite beam is decreased much less proportionately than the moment of inertia is increased. In general, there will be a reduction in depth and a slight saving in weight of the beams if rolled sections are used as the beams in the composite construction; a greater saving in weight will result from the use of cover plates on the bottom flanges; and a considerable saving is obtained by use of a section with a heavier bottom flange than top flange.

The increased strength and stiffness of a composite beam compared with those of an I-beam acting alone are obtained without impairing the ability of the slab to perform its primary function in distributing the loads laterally to the beams as a roadway slab. This conclusion was indicated by the theory and was shown by the tests.

In the negative moment regions of continuous bridges the advantages of composite action are not so marked. Indeed, it is questionable whether there is a disadvantage in having the slab subjected to appreciable tensile stresses as the upper flange of a composite beam. However, even though it may not be desired in continuous bridges, there may be partial or complete composite action because of the bond between the concrete slab and the steel beams. Although this bond cannot be depended on in design, it may exist as a disturbing and complicating element.

b. Shear Connectors.—Shear connectors to insure composite action may be composed of channels, plates, angles, or other structural elements, welded or riveted to the top flange of the beam, and embedded in the concrete of the slab. To prevent uplift there should be a mechanical anchorage of the connector in the concrete. The load to be carried by the shear connector may be computed by the ordinary formulas of elementary mechanics from the shear on the composite T-beam section.

When the composite structure is constructed in the usual manner, without falsework supporting the beams during construction of the slab, the dead load from both the slab and the beam is carried by the steel beam section alone, but the live load is carried by the composite T-beam section. Where the dead-load shear is carried by the steel beam section alone, it should be noted that the shear connectors do not have to provide for the dead-load action.

The dead-load shear varies linearly from a value equal to the reaction at the end of a beam to zero at the center. It is recommended that the live-load shear in a beam be computed for either truck or truck train loadings by the use of the same proportion, *k*, of a wheel load that is used in determining the moment in the beam. The total force acting on any shear connector may then be computed from the longitudinal shear per unit of length between the beam and the slab, multiplied by the spacing between the shear connectors.

When the shear connector is capable of resisting forces tending to separate the slab from the beam, a greatly increased effectiveness of the shear connector is obtained. The tying-down action of the connector serves to make the natural adhesive bond between the concrete slab and the steel beam effective also in resisting shearing forces.

For design purposes, the tentative recommendation is made that the distribution of load over the face of the shear connector be considered linear, from zero at the top of the connector to a maximum at the junction of the connector and the top flange of the beam. The maximum stress in bearing on the concrete may be taken equal to the usual compressive working stress for concrete, and the maximum flexural stress in the shear connector may be taken as the usual working stress for the steel. The spacing of the shear connectors should be not more than from three to four times the depth of the slab. The usual criteria should cover the design of the connection between the shear connector and the beam.

The model tests and small-scale tests on composite beams indicate that, when the shear connector is a channel having one flange attached to the beam, the foregoing working stresses may safely be doubled, or the load that must be provided for by the shear connector may be reduced by one half. Tests of quarter-scale models of I-beam bridges with such shear connectors indicated that the strength of the slab and of the beams may be developed without any evidence of failure of the shear connectors, even with a less conservative design than that proposed.

In the bridges tested, no evidence of inadequate action of shear connectors was detected even at loads causing failure of the slab and beams. Corresponding to the shear connectors used in several of the structures, the prototype shear connectors in a full-sized bridge would be 6-in. lengths of 4-in. channels with a web thickness of $\frac{1}{2}$ in., spaced 25 in. apart.

Somewhat less conservative recommendations are under consideration and the entire problem is the subject of an extensive series of tests which are described more fully in Professor Siess' paper in this Symposium.

c. Effect of Shoring Beams During Construction.—For bridges constructed without falsework, the dead-load stresses in the bottom flanges of the beams are a fairly large proportion of the total stress. The dead-load stresses can be reduced by use of temporary shores during construction so as to permit the dead load to be carried by the composite section instead of by the steel beams alone. By jacking up the beams even more, a favorable pre-stress in the beams can be introduced.

Studies indicate that the use of from one to three shores under each beam does not produce quite the same effect as a continuous rigid support. When the shores are released, the maximum effective dead-load moment is slightly different from the value corresponding to release of a continuous support. However, for practical purposes the shoring used may be considered as fully effective. Then the dead-load and live-load stresses will be approximately proportional to the magnitudes of the dead-load and the live-load moments.

In a design made for the usual working stresses for a loading consisting of one dead load plus one live load applied to the structure, the effect of shoring in composite design is to permit the use of a lighter beam than that corresponding to composite design without shores.

The decrease in weight, in a series of representative designs made according to the procedures described herein, amounted to about 15% for rolled beams, and ranged from 15% to 30% for welded built-up sections.

However, it is believed that these data are misleading since the resulting structure has a reduced ultimate strength and a reduced usable maximum capacity compared with designs in which shoring is not contemplated. This conclusion is entirely aside from any considerations of economy of construction, although the major reasons for the popularity of I-beam bridges are the ease and simplicity of the construction procedure.

d. Capacity of Composite I-*Beam Bridges.*—Properly designed shear connectors will develop the ultimate strength of the slab and beams before showing any evidences of failure to function properly. Weaker shear connectors may also develop almost the same ultimate strength of the structure, but may show considerable slip in doing so; and at maximum loads the structure will have considerably larger deflections. However, extremely weak shear connectors may fail at loads such that the noncomposite structure resulting from the failure of the shear connectors will not have sufficient resistance to carry the load on the bridge. Such a failure can be catastrophic, and is not to be tolerated; consequently, it is reasonable to be extremely conservative in the design of shear connectors.

In general, the dead-load stresses and deflections are larger, and the live-load stresses and deflections are smaller, for a composite bridge than for a noncomposite bridge of the same span length. Then the capacity, measured in number of live loads, will usually be greater for the composite bridge. To take advantage of the increased strength of the composite structure, not only must the shear connectors be conservatively designed, but the concrete slab must be adequately reinforced in the longitudinal direction to prevent serious opening of shrinkage cracks. Where the continuity of the slab is interrupted too greatly the upper flange of the composite section is materially weakened.

Consequently, although live-load stresses in the slab do not indicate a need for very much longitudinal steel in the slab, sufficient amounts of such steel must nevertheless be provided. It does not seem desirable to use an amount of steel less than about 0.5% of the effective concrete area, to provide for temperature and shrinkage effects.

The results of the tests appear to indicate that the action of bridges with composite beams agrees with the theoretical analysis in about the same way as does the action of bridges with noncomposite beams. The effect of composite action is to increase appreciably the strength of the roadway slab. There appears to be a reduction in the transverse stresses in the reinforcement, possibly due to the increased torsional resistance of the composite I-beams. There is also an increase in the loads required to produce failure of the slab by punching.

5. Skew I-Beam Bridges

a. General Considerations.—No analyses of skew I-beam bridges are available, but laboratory tests have been made on five skew bridges having angles of skew of 30° and 60°. In all respects, except for angle of skew, the bridges were similar to corresponding right I-beam bridges previously tested. The results of the test indicated that for a 30° angle of skew the action of the skew bridge was not greatly different from the action of a corresponding right bridge. However, for the 60° angle of skew, there was a considerable difference, with appreciably greater relative deflections of the structure at a given cross section and with markedly higher strains in the slab reinforcement.

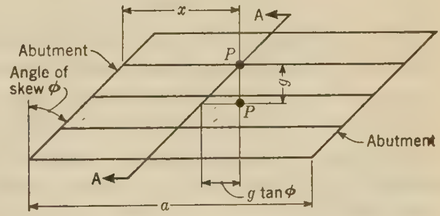

Fig. 21.—Skew Section Through a Skew Bridge, Showing the Location of Rear Wheel Loads

Consider the plan of a skew bridge shown in Fig. 21 with a section AA parallel to the abutments. For any loading the total bending moment normal to the section can be computed by the laws of statics. For the loading shown in Fig. 21, with one wheel of a truck at the section considered, the total moment M_n normal to the section is given by the expression:

$$M_n = 2\,P\,x \cos\phi\left(1 - \frac{x}{a} - \frac{g}{2\,a}\tan\phi\right) \qquad (7a)$$

in which ϕ is the angle of skew; g is the spacing of the wheels on the axle; a is the span of I-beams; and x is the distance in the direction of the span from a support to the section. In addition, there may be a twisting moment in the plane of the section, but the twisting moment is not statically determinate.

b. Effect of Skew on Moments in Beams.—An approximate value of the sum of the bending moments in the beams can be derived by making the assumptions that the slab carries only a negligible part of the total moment on the section and that the twisting moment in each beam on the section normal to the axis of the beam is negligible. With this procedure the sum of the moments in the beams, ΣM, on section AA in Fig. 21, is as follows:

$$\Sigma M = 2\,P\,x\left(1 - \frac{x}{a} - \frac{g}{2\,a}\tan\phi\right) \qquad (7b)$$

By inserting the proper values in Eq. 7*b*, the moments on the skew center line (or the maximum value of ΣM which is slightly greater than the value for the skew center line) can be obtained. For a right bridge, $\phi = 0$ and the maximum value is the same as that on the center line—namely $\frac{1}{2}\,P\,a$. The values of the average beam moments in the skew bridge, M_s, relative to those in a right bridge, M_r, are therefore given by the following formulas: For skew center line, $x = \frac{a}{2}$—

$$\frac{M_s}{M_r} = 1 - \frac{g}{a}\tan\phi \qquad (8a)$$

and, for maximum ratio, M_s/M_r—

$$\frac{M_s}{M_r} = \left(1 - \frac{g}{2\,a}\tan\phi\right)^2 \qquad (8b)$$

In Eqs. 8, g should be taken as 6 ft.

In other words, there is a reduction in the average moments in the beams in a skew bridge from the average in a right bridge because of the fact that the axles of the truck loads on the bridge are not parallel to the abutment; and, consequently, some of the heavy loads are closer to the abutments than in the right bridge. The theoretical reduction in average beam moments for a bridge of 60-ft span with 30° skew is about 6%, and for one with 60° skew about 17%.

The reduction in average moment on a section of a skew bridge does not necessarily mean that there is a reduction in moment in all the individual beams at a section. The test results indicate a more nonuniform distribution of strain in the beams at a given section than in a right bridge. However, some reduction in strain in the beams for bridges with large angles of skew is indicated, but practically no reduction is indicated for bridges with 30° skew.

Load-strain curves for the beams of one of the 60° skew bridges are shown in Fig. 22. The ratio of the measured maximum beam strains for the skew bridge to those in a corresponding right bridge is about 0.80. However, for the loading considered, the average moment on the skew section where the average is greatest is only about 71% of the moment in a right bridge for the same type of loading. Although there is some indication from the tests that shifting the loads to produce a greater moment on a skew section would not greatly increase the maximum measured strains, such a test unfortunately was not actually performed. Consequently, to be conservative, a possible increase must be inferred in the ratio of maximum measured strains from 0.80 to 0.94 if the average moment is increased from 71% to 83% of the corresponding moment in a right bridge. This argument suggests that the reduction in maximum moments in a skew bridge is perhaps about half as much as the reduction in average moment on a skew section; but, in view of the uncertainties involved, it is recommended that the beams of skew bridges be designed for the same proportion of a wheel load that applies in the case of a right bridge of the same beam span.

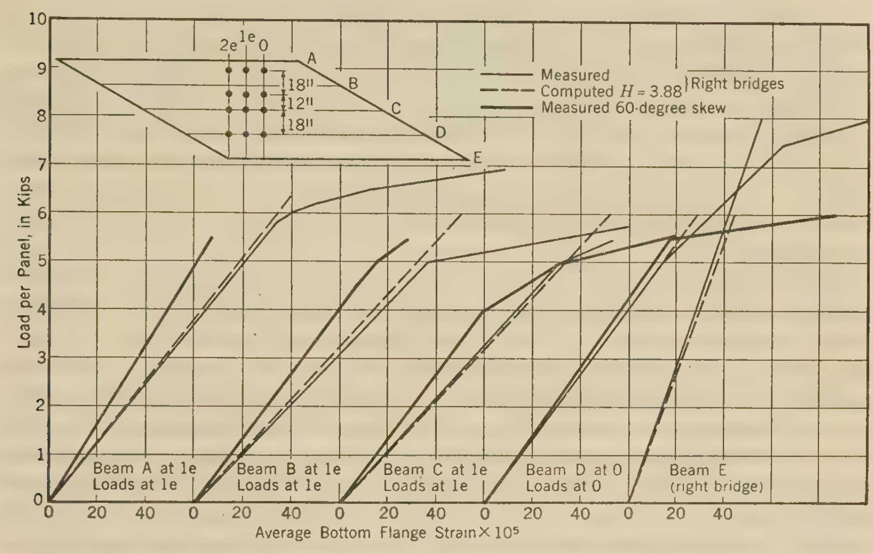

Fig. 22.—Load-Strain Curves for Beams; Quarter-Scale Models of Bridges of 60-Ft Span, Without Composite Action

c. *Effect of Skew on Moments in Slab.*—It is difficult to generalize about the moments in the slab in skew bridges from the results of the tests. First yielding of the reinforcement occurred at lower loads for the skew bridges and the loads were smaller with greater angles of skew. However, the ultimate capacity of the slab, as measured by the loads producing punching failures, appeared to be independent of the angle of skew—at least for skews up to 60°. A small but significant increase in strain in the transverse reinforcement for skew bridges over corresponding right bridges was observed in the tests.

The results of the skew bridge tests indicated increased strains in the longitudinal as well as in the transverse reinforcement in the slab. The increase arises from the more nonuniform deflections of the beams in a skew bridge. However, there is an additional reason for the increased strain in the longitudinal steel. The slab tends to carry the load directly to the abutments by the shortest route, which is at an angle to the beams; consequently, the principal moments in the slab tend to deviate from the directions of the beams. For this reason it is unwise to reduce the amount of longitudinal steel in the slab of skew bridges. To do so might invite cracking of a nature and magnitude that would be undesirable.

In view of the fact that the design recommendations for slabs are conservative in any case and furthermore since the ultimate strength of the slab does not seem to be affected by skew, it is tentatively suggested that no change in the provisions for the design of the slab be made in skew bridges, except to emphasize the requirement of longitudinal reinforcement.

6. CONTINUOUS I-BEAM BRIDGES

a. General Considerations.—Although analyses have been made of several two-span continuous right I-beam bridges, it was unfortunately necessary to consider relatively short spans compared with the beam spacing. Tests have been made on quarter-scale models of a two-span continuous bridge where the prototype spans are 60 ft. In the first test structure, the beams were not attached to the slab by shear connectors. In the second test structure, shear connectors were used to provide composite action over the entire length of the bridge. The results were in general agreement with those reported for simple-span bridges.

There seems to be no particular advantage in providing for composite action in the region of negative beam moments in a continuous bridge. Whether shear connectors are used or not, in the regions where the slab may be in tension at the upper surface, adequate reinforcing must be provided. In either case the slab is subjected to deformation stresses rather than to load stresses.

b. Moments in Beams and Slab.—The results of the analyses and the tests that have been made indicate that the beams can be designed for the same proportion of a wheel load as is suggested for simple-span bridges with a span length equal to the distance between points of inflection of the continuous bridge. In other words, the moments in the beams can be determined by the usual procedure for a continuous beam subjected to a particular proportion of a wheel load and to the proper dead load.

The indications are also that the slab may be designed as for a simple-span bridge with a span of the beams, *a*, equal to the distance between the "points of inflection" for the particular span of the continuous bridge.

In the central part of the spans of the continuous bridge, the secondary moments in the slab are approximately the same as those in a simple-span bridge. However, over the supports, the secondary moments are practically the same as those in a bridge having infinitely rigid or nondeflecting beams. The major change is in the negative transverse moments over the beams. These are likely to be somewhat larger over the supports than in the central parts of the spans. Nevertheless, the model structures that have been tested indicate that it is not necessary to have as much steel provided for negative moment over the beams as for positive moment at the centers of the panels. The test structures had only two thirds as much transverse steel at the top of the slab as at the bottom, but no evidence of serious weakness or of incipient failure was observed. The situation might be worse in a skew-span continuous bridge, however, and it seems desirable to use more steel over the beams in the regions of negative beam moments, than in the regions of positive beam moments.

7. DESIGN RECOMMENDATIONS FOR SLAB

a. Major Design Moment.—The test results that have been cited indicate that the maximum positive transverse moment in the slab, as given by the theory, corresponds to considerably larger strains than have been measured in the tests except for a few instances. Moreover, there is a factor of safety beyond the first yielding of slab reinforcement of at least 50% before punching failure occurs. Consequently, it appears permissible to design the slab for a moment smaller than the theoretical moment. A reduction of about 30% appears to be reasonable, and is the basis of the following recommendations.

The maximum positive live-load transverse moment in the central part of panels of the slab, expressed in foot-pounds, for a 1-ft width, is to be computed from the following formulas: For spans of beams less than 60 ft—

$$M_t = \frac{P_m}{3 + 10\frac{c}{b}} \qquad (9a)$$

and, for spans of beams greater than 60 ft—

$$M_t = \frac{P_m}{3 + 10\frac{c}{b}} + \frac{P_m\,(a - 60)}{1{,}000} \qquad (9b)$$

In continuous spans, a is to be taken as the distance between points of contraflexure in the region considered. In Eqs. 9, P_m is the maximum wheel load, including impact, in pounds; c is the width of loaded area over which P is distributed, ordinarily 1.25 ft; b is the center-to-center spacing of beams, in feet; and a is the span of beams, in feet.

The positive transverse dead-load moment M_w is to be taken as

$$M_w = \tfrac{1}{8}\,w\,b^2 \qquad (10)$$

in which w is the intensity of uniform dead load of slab and paving surface, in pounds per square foot. In bridges in which composite action is provided for by shear connectors, or in concrete T-beam bridges, the moments given by Eqs. 9 and 10 may be reduced 10%.

TABLE 1.—RELATIVE AMOUNTS OF REINFORCEMENT REQUIRED IN A SLAB

Place	TRANSVERSE		LONGITUDINAL	Notes
	Center of panel	Over the beams	Over the entire panel	
Bottom	1.0	0.4	0.5[b]	[a] This value applies to simple-span bridges and to the positive moment regions of continuous bridges. In the negative moment regions the value should be 0.8. [b] For composite bridges, and for all skew bridges, the reinforcement should be not less than 0.5% of the gross concrete area.
Top	0.2	0.7[a]	0.2	

b. Arrangement of Reinforcement.—The areas of steel per unit of width provided at other points and in various directions in the slab shall be at least as great as those given in Table 1 in terms of the area of steel per unit of width in the transverse direction at the bottom of the central parts of panels. Where bottom bars are bent up to provide reinforcement over the beams, the points of bend at the bottom should not be greater than 0.25 b from the center of the beams.

8. DESIGN RECOMMENDATIONS FOR BEAMS

a. Dependence of Moments in Beams on Depth of Slab.—The design of the beams of an I-beam bridge is dependent on the depth of the slab in two respects: (1) The depth of the slab affects the relative stiffness of the beams and, consequently, the magnitude of the live-load moment in the beams; and (2) the depth of the slab influences the dead load on the beams and thereby the magnitude of the dead-load moment in the beams. For example, in a bridge of 65-ft span and 6.75-ft beam spacing, a 7.25-in. slab (which is approximately that required for an H-20 single wheel concentration of 16 kips plus impact) gives a dead-load moment in the beams of 420 ft-kips, whereas a 6.25-in. slab, which is approximately that required for an H-15 single wheel concentration of 12 kips plus impact, gives a dead-load moment of 368 ft-kips. If the beams in these bridges are to be designed for H-20 truck loading, the proportion of the wheel load assigned to each beam is less for the bridge in which the slab is thicker. The values recommended in Table 2 give (for the live-load moments in the two bridges) the quantities 471 ft-kips and 488 ft-kips, respectively, with consequent total moments of 891 ft-kips in the structure with the thicker slab and 856 ft-kips in the structure with the thinner slab. The structure in which the slab is designed for a heavier wheel load requires a heavier beam. It appears that any loss in less effective distribution by the thinner slab is more than regained by the material reduction in dead load.

TABLE 2.—VALUES OF s (IN FEET) TO BE USED IN THE EQUATION $k = b/s$

Type of structure	Noncomposite I-beam bridges	Composite I-beam bridges
Bridges designed for a standard H-truck loading in which the slab is designed for a single wheel concentration of 0.4 times the total weight of the truck plus impact..........................	5.7	5.3
Bridges designed for an H-20 truck loading in which the slab is designed for a single wheel concentration of only 12,000 lb plus impact[a]..	5.5	5.2

[a] The values given in the first line may be used if the depth of the slab is made as great as would be required for a 16,000-lb wheel concentration, plus impact. This provision is made to insure that the slab will have adequate stiffness to distribute the loads to the beams, in order to take advantage of the reduced live-load moments.

It is worth mention, in this regard, that, if the design is based on a so-called "limit load" producing yielding of the beams, with an appropriate factor of safety, there will be less difference in the two structures. For example, a design based on dead load plus 2.5 live loads at a stress of 33,000 lb per sq in. would require a 36-in., 170-lb wide-flange beam in either case. It is easily noted from the values quoted that the structure with the heavier slab has a somewhat greater live-load rating than does the structure with the thinner slab.

The design coefficients discussed herein are developed for the working stresses specified in the AASHO specifications—namely, 18,000 lb per sq in. for structural steel and reinforcing bars, and 1,000 lb per sq in. for concrete. The modulus of elasticity of concrete is assumed to be 3,000,000 lb per sq in. for live load, and 1,000,000 lb per sq in. for sustained loads, to include the effect of plastic flow. Other working stresses may affect slightly the ranges in values of relative stiffness of the beams, H, and may require slight changes in Table 2. However, Eqs. 2, 3, and 4 would remain valid for determining beam moments for any case considered.

b. Dead-Load and Live-Load Moments in Beams.—The provisions herein apply only to bridges with four or more parallel beams of the same size, with uniform spacings between beams of 5 ft to 8 ft. A wheel load may pass over an edge beam, but may not extend outside the edge beams. The live-load moments in all beams are to be computed for a proportion, k, of a wheel load, according to Eq. 3—namely, $k = \frac{b}{s}$, in which b is the center-to-center spacing of the beams, in feet, and s may be computed from Eq. 4 or taken from Table 2. However, in no case is k to be less than the number of wheel loads that may come on any cross section of the bridge, divided by the total number of beams.

The dead-load moments in all beams are to be computed by dividing the total dead load of the structure equally among the beams; but the dead load apportioned to any beam should not be less than the weight of the beam, the weight of one panel of the slab, and the weight of any wearing surface over a width of one panel of the slab.

The same factors should be used for continuous bridges, and skew bridges as for simple-span bridges.

c. Composite I-*Beams.*—For composite design, the width of the slab that may be considered as effective as the upper flange of the composite beam may be taken as the panel width. The modular ratio, n, should be taken as 10 for live loads and for loads of short-time duration; and as 30 for dead loads or for long-time loads, such as the weight of added pavement or wearing surface.

The local longitudinal flexural stress in compression at the top of the slab under the wheel load, to be added to the compressive stress in the concrete acting as a part of the composite beam section, should be 30% of the transverse compressive stress in the slab at the same point. The total compressive stress should be limited to the value of the usual working stress for concrete in compression. The foregoing provisions may be applied without modification to concrete T-beam bridges. However, for such bridges, the value of s to be used in Eq. 3 should be 5.0 ft.

9. Practical Significance of Design Recommendations

The moments recommended herein for design of the slab differ somewhat from the provisions of the 1944 AASHO specifications. The value of live-load moment given by Eq. 9*a* is numerically the same as that in the specifications except for the use of a center-to-center span in the equation. The difference is slight and makes the equation easier to use since it is not necessary to design the beams to find the flange width before computing the slab moments. The dead-load moment given by Eq. 10 is greater than would be computed from the AASHO specifications. The reason for the increased dead-load moment is explained in Section 3. The provision for increase in slab moments for spans longer than 60 ft, in Eq. 9*b*, differs also from the specifications, and is included to take account of the increase in theoretical moments for longer spans arising from an increased effect of deflection of the beams and from the effect of front wheels.

The negative moments over the beams provided for in this paper are considerably less than those provided for by the specifications, but they are consistent with the results of analyses and tests. The procedure outlined herein offers a more rational guide to the proper reinforcement of the slab of an I-beam bridge than do previous studies. The numerical values, based on the results of tests as well as analyses, are somewhat less conservative than the values previously presented by the writer. The design moments in the beams are less conservative than those given by the specifications, but the results of analyses and tests justify the departure from previous standards.

In this regard, it must be emphasized that the proportion of a wheel load, k, used in the design of the beams is a function of the relative stiffness of the beams to the slab and consequently of the thickness of the slab. Where changes in specifications and working stresses, or other causes, lead to decreased slab thickness, the values of k must increase.

The provisions for the design of the slab and the beams, although less conservative than present practice in some respects, are justified by the extensive analyses and tests that have been made on this type of structure. The recommendations have been tried on actual designs, and comparisons have been made with standard practice in many cases. It is felt that the provisions given herein are not only reasonable, but practicable, and should lead to more efficient utilization of steel and concrete in I-beam highway bridges.

COMPOSITE CONSTRUCTION FOR I-BEAM BRIDGES

By C. P. Siess,[24] Assoc. M. ASCE

Synopsis

The results of analytical and experimental studies of composite construction for I-beam highway bridges are presented in this paper. Comparative design studies were made to determine the savings in weight possible from the use of the following types of composite beams: Rolled wide-flange beams, rolled beams with welded cover plates, welded built-up beams, beams consisting of two structural T-sections welded together, and rolled unsymmetrical beams. The savings in weight as compared to noncomposite construction ranged from 8% for the ordinary rolled beams to 30% or more for welded built-up beams.

A number of problems encountered in the design of composite beams are discussed. Included are the effect of slip between slab and beams, the distribution of compressive stress across the width of the slab, and questions regarding the proper value of the modular ratio *n* to be used in design. Empirical formulas for use in proportioning composite beams are also given.

Tests of shear connectors, particularly of the channel type, are described. The effects of the following variables were investigated: Width of connector, thickness of channel web, and compressive strength of the concrete in the slab. Certain of the results obtained in these tests may be explained by the concept of dowel-like action of flexible shear connectors. According to this concept, the nature of the distribution of bearing pressure on the connector is a function of the relative stiffness of the connector and the concrete.

Introduction

The several problems associated with the design of composite I-beam highway bridges which are discussed in this paper may be divided into three groups:

(1) Comparative design studies of I-beam bridges to determine the savings in weight possible from the use of various types of composite beams;

(2) Experimental and analytical studies of the behavior of composite beams and composite bridges; and

(3) Studies of the behavior of shear connectors for use in composite I-beam bridges.

Although composite construction is not, in itself, a new development, its application to highway bridges is of comparatively recent origin. The I-beam bridge is almost ideally suited to the effective use of composite construction, and the number of such structures has shown a marked increase. An ordinary

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I-beam bridge consists essentially of a reinforced concrete floor slab supported on a number of parallel steel I-beam stringers extending in the direction of traffic. In a composite I-beam bridge, some type of shear connection is provided to transmit horizontal shear between the slab and the beams. The longitudinal load-carrying member is thus transformed from a simple steel I-beam to a composite steel and concrete T-beam in which the concrete floor slab acts as the top flange. In this type of construction the slab serves two functions: First, it transmits load laterally to the beams and, second, it assists the steel stringer in carrying the load longitudinally to the piers. In its first function the slab acts in bending; in its second, it acts chiefly in compression. Tests have demonstrated that the capacity of the slab to perform its primary function of acting as a roadway deck is not impaired by its action as part of the composite beam.

PART A. SAVINGS IN WEIGHT WITH COMPOSITE CONSTRUCTION

1. General Considerations

Both the strength and the stiffness of a composite beam are appreciably greater than the corresponding properties of the steel beam acting alone. The increase in strength resulting from the use of composite construction permits the use of a lighter beam section than would otherwise be required, and, in many instances, permits the use of a shallower beam. Moreover, even with a lighter steel section, the stiffness of the composite beam will be from two to three times as great as the stiffness of the original noncomposite beam. In most cases, the increased stiffness is effective only for live load, since bridges of this type are commonly constructed without shoring beneath the beams, and dead load is carried by the steel beam acting alone. Since the size of the steel beam may be decreased in composite construction, there is likely to be some increase in dead-load deflection for bridges of this type. However, since dead-load deflection can easily be provided for by cambering the beams, this increase is probably not significant. Of greater importance is the considerable decrease in live-load deflection resulting from the increased stiffness of the composite beam. Although present knowledge concerning impact on highway bridges is too meager to permit any definite statements, it seems reasonable to believe that from the standpoint of impact and vibration the stiffer structure is to be desired.

2. Outline of Studies of Weight Savings

Studies have been made of the savings in weight resulting from the use of composite construction utilizing several different types of beams. Comparative designs have been made for simple-span I-beam bridges having span lengths of 30 ft to 90 ft and beam spacings of 5 ft to 7 ft, and designed for both H-15 and H-20 loadings.

Both analytical studies and laboratory tests have indicated that the beams in a composite bridge should be designed for a greater live-load moment than those in a noncomposite structure. The reason for this is that, because of the greater stiffness of the beams relative to that of the slab, there is less lateral distribution of the wheel-load concentrations. In a previous publication[16] it was recommended that the proportion of a wheel load used in the design of a single beam be taken as $\frac{b}{6}$ and $\frac{b}{5.5}$, respectively, for noncomposite and composite structures. The quantity b is the spacing of the beams, in feet. This recommendation, providing for a 9% greater moment in the beams of the composite bridges, was followed in the designs reported herein. Professor Newmark has proposed a modification of the foregoing recommendation in which the terms $\frac{b}{5.7}$ and $\frac{b}{5.3}$ are substituted for the previous values. For these latter coefficients, the moment for the composite beams is only 7.5% greater than that for the noncomposite beams. This discrepancy between the specifications used and those recommended by Professor Newmark should have no appreciable effect on the comparisons.

Except for the designs given subsequently in Section 6, it was assumed in all cases that dead-load moments are resisted by the steel beam acting alone, and that only live-load moments are carried by the composite section. The results of the various studies are summarized in Table 3, in which the relative weights of the steel beams are compared for the various types of structures considered. In making these comparisons, an amount equal to the weight of the shear connectors has been added to the weights of the composite beams. Three sets of relative values are given to facilitate comparisons of rolled beams, both composite and noncomposite, with the other types of beams considered.

TABLE 3.—RELATIVE WEIGHTS OF STEEL BEAMS IN COMPOSITE I-BEAM BRIDGES

Noncomposite rolled beams	COMPOSITE BEAMS							
	SYMMETRICAL ROLLED BEAMS				Unsymmetrical rolled beam, without temporary supports	Double T-section without temporary supports	WELDED BEAMS[b]	
	WITHOUT COVER PLATE		WITH COVER PLATE[a]				Without temporary supports	With temporary supports
	Without temporary supports	With temporary supports	Without temporary supports	With temporary supports				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
100	92	77	76	64	82	82	69	40 to 60
	100	84	83	70	89	89	74	45 to 70
		100	99	83	106	106	88	53 to 75

[a] Average weight per foot based on cover plate 0.6 of the length of beam. [b] Maximum weight per foot.

3. ROLLED WIDE-FLANGE BEAMS IN COMPOSITE BRIDGES

The average decrease in weight of the beams resulting from the use of composite construction with rolled beams was 8%, as indicated in Col. 2, Table 3. In half of the designs made, the depth of the beams could be reduced by 3 in.

If the most economical depth of beam cannot be used because of clearance requirements, the savings in weight arising from the use of composite construction may be more than twice as great as those quoted, and the shallower depth need not be accompanied by increased flexibility of the bridge in so far as live loads are concerned.

4. Unsymmetrical Beams for Composite Bridges

Because the slab acts as a very heavy cover plate, the neutral axis of a composite beam is raised some distance above mid-depth of the steel beam, and the stresses in the top flange of a symmetrical beam are appreciably smaller than those in the bottom flange. This condition suggests that greater reductions in weight might be obtained with the use of beams having larger bottom flanges than top flanges.

The economies permitted by the use of unsymmetrical sections have been investigated by a limited number of comparative designs for three types of beams: (*a*) Rolled wide-flange beams with cover plates welded to the bottom flanges; (*b*) built-up welded beams; and (*c*) beams consisting of two structural T-sections welded together.

(*a*) *Rolled Beams with Cover Plates.*—As indicated by the values in Col. 4, Table 3, the use of rolled wide-flange beams with cover plates in composite bridges was found to result in an average saving in weight of 24% as compared to bridges without composite action. Furthermore, the depths of the composite beams were from 3 in. to 6 in. less, except in those cases where the heavy series of 36-in. beams were required in the noncomposite design.

(*b*) *Built-Up Welded Beams.*—A more efficient section from the standpoint of weight reduction may be obtained by welding together two flange plates and a web plate to form an unsymmetrical I-beam. In making comparative designs for economy studies of this type of beam, two conditions were imposed which have an appreciable effect on the results obtained: (1) The webs were made not less than 1/87 times the clear depth, a condition which it is believed provides adequate safety against buckling and obviates the need for stiffeners; and (2) a minimum top flange section, 6 in. by $\frac{3}{8}$ in., was specified to provide space for the attachment of shear connectors.

At first the proportioning of these welded beams was a cut-and-try operation since the depth of the beam as well as the areas of both flanges could be varied. However, as a result of these studies, it was found that the economical depth of beam was very nearly equal to the depth of the rolled wide-flange section required in a composite design. Furthermore, after a great many designs had been made, a simple semi-empirical procedure was developed for proportioning the flanges in such a manner that the total dead-load plus live-load stresses would be equal in the two flanges.

The results of these studies are summarized in Col. 8, Table 3. It was found that the weight of welded beams was on the average about 10% less than that of composite rolled beams with cover plates, 26% less than that of composite rolled beams without cover plates, and 31% less than that of noncomposite rolled beams. If the flanges of the welded beams are made smaller in regions of reduced moment, somewhat greater savings may be obtained.

(c) *Beams Made from Structural Tees.*—A third method of obtaining an unsymmetrical section is by welding together the webs of two structural tees of different sizes, and thus producing an I-shaped beam with one flange heavier than the other. Several beams of this type were designed using structural tees cut from wide-flange sections, and the average saving in weight as compared to composite rolled beams was 11%. This is less than one half as much saving as can be obtained with all-welded beams. Comparisons with other types of beams may be made by reference to Col. 7, Table 3.

5. Rolled Unsymmetrical Beams

The comparatively large saving in weight which may be obtained with the use of unsymmetrical beams in composite construction suggests the desirability of rolling unsymmetrical beams having one flange heavier than the other. A brief study has been made of the optimum dimensions of such beams and of the economies that might be obtained from their use in composite construction.

An unsymmetrical rolled beam permitting the greatest saving in weight would have dimensions and proportions similar to those of the aforementioned built-up all-welded beams. The saving in weight possible from the use of such welded beams in place of ordinary rolled beams was about 26%. The saving in weight for a series of unsymmetrical rolled beams patterned after the welded beams would be somewhat less because of the lack of flexibility in the choice of a beam to provide a given section modulus. There is some question, however, as to whether beams of these proportions could be rolled because of the relatively large difference in thickness of the web and the flanges.

A more practical unsymmetrical rolled beam suitable for use in composite construction may be obtained by a relatively simple modification of the ordinary wide-flange beam, as follows: The area of the top flange is reduced by one half and the remaining dimensions are unchanged. The weight of a beam so modified will be reduced by approximately 14%. However, because the series of beams is not continuous, the average savings in weight arising from the use of such beams in place of ordinary rolled beams will be about 11%, as indicated in Col. 6, Table 3.

6. Composite Bridges with Temporary Supports

Since the section modulus of a composite beam is always appreciably greater than that for a rolled beam alone, further economies in the weight of the beam may be obtained by constructing the bridge in such a manner that the dead load as well as the live load is carried by the composite section. This result can be obtained by placing temporary supports or shores beneath the beams and removing them only after the concrete of the slab has set.

The additional saving in weight resulting from the use of temporary supports averaged about 16% for rolled beams, with or without cover plates, as indicated in Cols. 3 and 5, Table 3. For built-up welded beams the additional saving ranged from 16% for short spans to as much as 40% for long spans where appreciable reductions in the area of the top flange could be made. These data are given in Col. 9, Table 3.

In obtaining the foregoing results, it was assumed that the beams were supported along their entire length although in practice the beams would probably be supported by falsework bents at only two or three points. The error resulting from this assumption was investigated for one, two, or three equally spaced temporary supports. For one support, the maximum dead-load stress in the bottom flange of the beam was 3% to 6% less than that computed for the assumption of continuous support, while for two or three supports, the corresponding stress was 1% to 3% greater.

PART B. PROBLEMS IN THE DESIGN OF COMPOSITE BEAMS FOR I-BEAM BRIDGES

A number of studies, both experimental and analytical, have been made of various problems encountered in the design of composite beams and composite I-beam bridges. Since frequent reference is made to experimental data, it is necessary to give a brief description of certain of the laboratory tests that have been made.

1. Tests of Composite I-Beam Bridges

Laboratory tests have been made on twenty quarter-scale models of simple-span I-beam bridges, nine of which were of composite construction utilizing channel shear connectors. Of these nine, six were right bridges with prototype spans of 20 ft and 60 ft, and three were skew bridges with prototype spans of 60 ft and angles of skew of 30° and 60°.

In these tests,[13] the model bridges were loaded with either two or four concentrated loads simulating the rear wheels of one or two trucks. Measurements made included strains in the beams and in the slab reinforcement, deflections of the beams, and slip between the slab and the beams. The results of the tests for the various bridges were compared with each other and with the analysis for the simple-span right bridge.

2. Tests of Composite T-Beams

Tests were made on a number of small composite T-beams of the type illustrated in Fig. 23. These T-beams consisted of an I-beam and a mortar slab, tied together with channel shear connectors, 1 in. by $\frac{3}{8}$ in. by 1 in. long, welded to the top flange of the I-beam and embedded in the slab. The variables studied included the size and spacing of the shear connectors and the compressive strength of the mortar slab. The beams were loaded with a single concentrated load, usually at midspan, and measurement was made of deflection, slip between the slab and I-beam, and strains in both the I-beam and the slab.

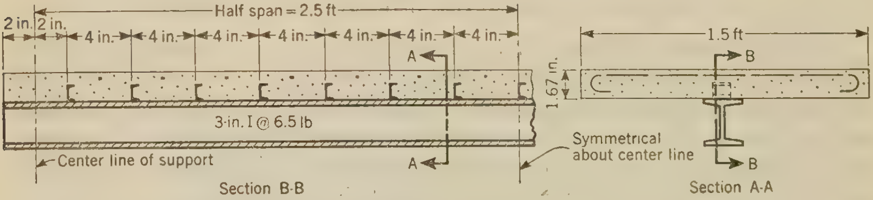

Fig. 23.—Details of Beam Specimen, T-Beam Tests of Shear Connectors

3. SLIP BETWEEN SLAB AND BEAMS IN I-BEAM BRIDGES

A fundamental question in the design of composite beams is whether or not the transformed area method may safely be used in the calculation of stresses. The principal condition for the use of this method is that there be no slip between the two elements making up the beam. The degree to which this condition was obtained in the tests of composite I-beam bridge models with channel shear connectors is illustrated by the typical curves in Fig. 24.[25] The slip between the slab and the center beam at the ends of the beam is shown in Fig. 24 for both composite and noncomposite model bridges having span lengths of 5 ft; the results, however, are typical of all the bridges tested. The maximum slip obtained in the bridges with composite construction was less than 0.0005 in., even at loads several times greater than those ordinarily considered in design.

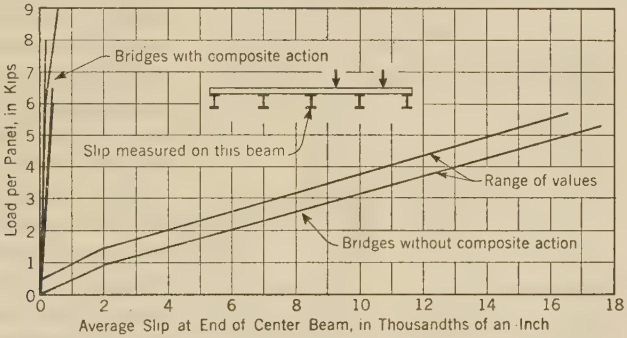

FIG. 24.—LOAD-SLIP CURVES FOR TESTS WITH SIMULATED WHEEL LOADS

4. EFFECT OF SLIP ON STRAIN IN BEAMS

The effect of small amounts of slip on the distribution of strain in a composite beam is shown in Fig. 25, which is based on data obtained in tests of the small composite T-beams which were similar to those used in the 5-ft I-beam bridges. The distribution of strains throughout the depth of the composite beam is shown in Fig. 25. The dashed line represents the theoretical straight-line distribution for an ideal composite beam with no slip between the two elements. The solid circles represent measured strains, and the solid line drawn through them represents the strain distribution existing in these beams when some slip was present. The amount of slip at the location of the measurements is also shown.

The slip for beam T1 was approximately the same as that indicated in Fig. 24 for the model bridges. The agreement with the theoretical distribution is quite good, and the increase in strain in the bottom flange—the most important value—is negligible.

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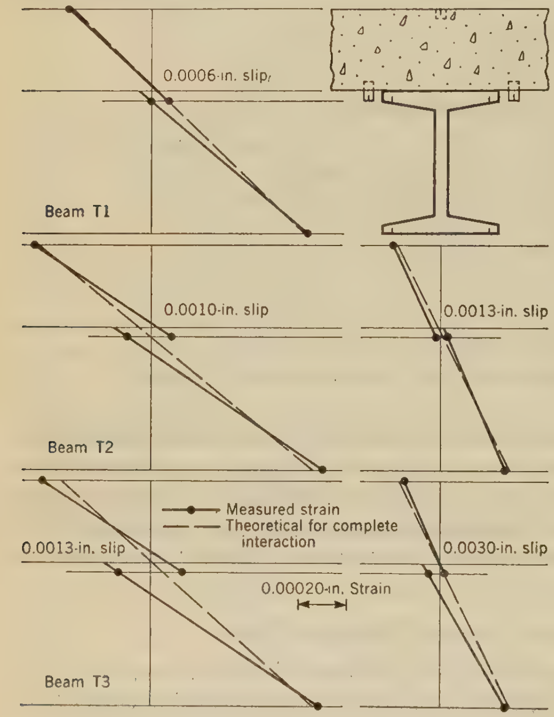
[25] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 43, Fig. 13.

Beam T2 was similar to beam T1 except that the compressive strength of the mortar slab was considerably less. As a result, there was a greater amount of slip in this beam, and its effect can be noted in the jog in the strain-distribution curve at the junction of the slab and beam. However, the increase in bottom-flange strain is still quite small.

The shear connectors in beam T3 were considerably weaker than those used in beams T1 and T2, and appreciably greater slips were obtained. The effect of slip on the strains, however, is greatest at the junction of the beam and slab, and is still not important at the bottom of the beam where the greatest strains occur.



(*a*) AT 3¾ IN. FROM MIDSPAN (*b*) AT 19 IN. FROM MIDSPAN

Fig. 25.—Comparison of Measured and Theoretical Strain Distribution for a Load of 5.2 Kips at Midspan

The data plotted in Fig. 25(*a*) are for a section only 3¾ in. from the point of load application at midspan. Those in Fig. 25(*b*) are for a section near the quarter point of the span. In these tests, the slip was greatest at the ends of the beam and decreased in magnitude for points closer to the location of the load at midspan where, of course, it was zero. It may be noted in Fig. 25 that the slips at a section 19 in. from midspan were greater than those at the section near the load point. On the other hand, it may also be noted that the strains at the location some distance from midspan are affected only slightly by these greater slips. This behavior is in agreement with that predicted from the analysis of a composite beam in which small amounts of slip are present. The analysis indicates that the effect of slip on the distribution of strain is a relatively localized effect confined to the region extending a short distance on either side of the point of application of a concentrated load.

The conclusion can be drawn from the foregoing results that a slab and beam tied together by suitable shear connectors may be considered to act as a single composite section, and may therefore be analyzed on the basis of the transformed section.

5. Effective Width of Slab as T-Beam Flange

For composite I-beam bridges of ordinary dimensions, it has been recommended that a section of the slab having a width equal to the beam spacing be assumed to act as part of the composite beam. If this assumption is to be valid, the distribution of compressive stress in the slab acting as a flange of the T-beam should be uniform over the entire width. The distribution of this stress on the top surface of the slab of an isolated composite T-beam, shown in Fig. 26, is substantially uniform, both at a section near the load and at a section some distance removed.

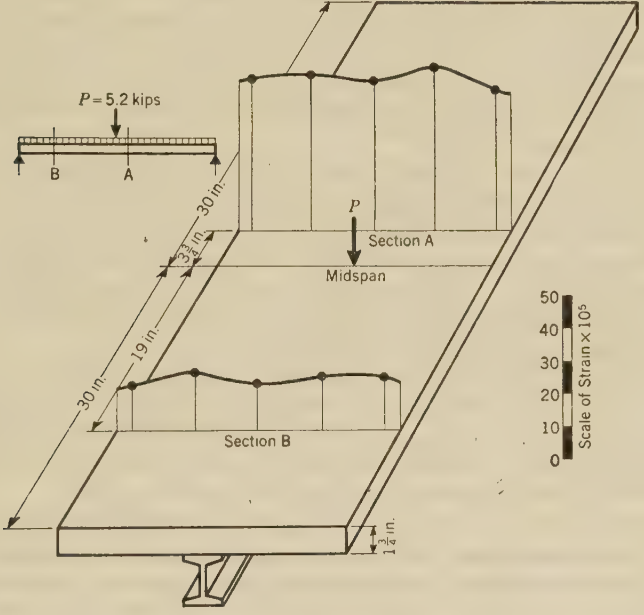


Fig. 26.—Distribution of Strain Across a Slab

6. Selection of the Value of the Modular Ratio

The transformed area of the concrete slab serving as the T-beam flange is a direct function of the value of n, the ratio of the modulus of elasticity of steel to that of concrete. It is perhaps fortunate, therefore, that the properties of the transformed section of the composite beam are not appreciably affected by moderately large variations in the value of this ratio. For example, a decrease of 28% in the value of n (from 10 to 7.15) produces a decrease of only 3% in the section modulus of the bottom flange of the composite beam, a decrease of 7% to 9% in the moment of inertia, and a decrease of 11% in the magnitude of the horizontal shear between the slab and the beam for a given value of vertical shear. These changes are negligible and a value of n equal to 10 may safely be used in computing the properties of composite beams, except in cases where it is desirable to take into account the effects of plastic flow of the slab.

7. Plastic Flow of Slab

If a composite bridge is constructed without the use of shoring beneath the beams, the dead load is carried by the steel beams alone and the concrete slab acts only for live loads. Consequently, there should be no significant plastic flow in the slab. On the other hand, if shoring is used and the composite beam carries both dead and live load, the slab is subjected to a sustained compressive stress, and plastic flow is likely to occur. The magnitude of this flow will not be very great since the maximum compressive stress in the slab will seldom exceed 600 lb per sq in. or 700 lb per sq in. However, if it is desired to make allowance for the effects of such flow, it may be done by computing the contribution of the slab on the basis of an increased value of n. A commonly used value is $n = 30$.

The effect of considering $n = 30$ rather than $n = 10$ is to decrease the section modulus of the bottom flange by about 10%. A much greater decrease in the top flange section modulus will be produced, but in general this will be of little importance since the stresses in the beam at that location are quite small if shoring is used. The decrease in moment of inertia of the composite section is about 25%, and for this reason it may be desirable to consider plastic flow when computing dead-load deflections.

A decrease of about 25% in the magnitude of the horizontal shear between the beam and the slab will also be obtained as the result of using $n = 30$ instead of $n = 10$. However, since full plastic flow will not be realized for some time, and possibly not at all, the shear connectors should be designed for the greater horizontal shear computed on the basis of $n = 10$.

A further possibility for plastic flow occurs in a bridge constructed without shoring if a wearing surface is added at some time after construction is completed. The weight of this added material is carried by the composite section and thus produces sustained stresses in the slab. The effect of possible plastic flow in such a case, however, is quite small and may usually be neglected in design.

8. Methods of Proportioning Composite Beams

In most cases in the design of I-beam bridges, the spacing of the beams and the thickness of the slab will be known when the design of the beam is undertaken. The problem of design then reduces to the choice of a steel beam section which, in combination with the slab, will be able to carry the known moments at the specified allowable stresses. The procedure for doing this, however, is usually neither simple nor direct, especially in cases where temporary supports are not used during construction. Although, in general, a cut-and-try procedure is required, in certain cases that have been studied in connection with this investigation, design aids have been developed which materially reduce the labor required. Two types of such aids are: (*a*) Those facilitating the design of composite rolled beams without cover plates; and (*b*) those for use in the design of built-up welded beams.

(*a*) *Designs with Rolled Beams.*—For composite beams utilizing rolled wide-flange beam sections without cover plates, the problem of choosing a section may be simplified by the use of tables giving the properties of composite beams Such tables have been prepared for composite beams consisting of various wide-flange beam sections acting in combination with a section of slab varying in width from 5 ft to 7 ft, and in thickness from 6 in. to 8 in. The properties tabulated are: (1) Section modulus of the bottom flange; (2) moment of inertia of the beam; and (3) ratio of horizontal shear at the junction of slab and beam to the vertical shear at any given location.

(*b*) *Designs with Built-Up Welded Beams.*—Part A of this paper demonstrated that the built-up welded beam with unsymmetrical flanges is a particularly economical type for use in composite construction. The most economical proportions for such a beam will be those for which the total dead-load plus live-load stresses are equal for the two flanges. If temporary supports are not used, the adjustment of flange areas to give equal stresses is rather complicated, since dead load is carried by the steel beam alone and live load is carried by the composite section. However, for a particular set of conditions, the process of design may be simplified by the use of an empirical procedure based on the results of a number of designs made by trial and error. In these designs, the webs were made of such a thickness that stiffeners were not required. Computations indicated that a thickness of not less than 1/87 times the clear depth would provide adequate safety against buckling. Any appreciable change in this ratio will require alteration of the following empirical formulas for the trial flange areas.

The procedure developed consists of a preliminary design, an analysis, and a revision. Usually a second analysis will not be necessary unless unusually high precision is required.

(1) Preliminary Design.—The area of the bottom flange, A_b, and the area of the top flange, A_t, in square inches, are given by the expressions:

$$A_b = A_o - 2.5 \quad \text{(11}a\text{)}$$

and

$$A_t = 0.8\,A_o - 6.0 \quad \text{(11}b\text{)}$$

in which

$$A_o = \frac{M_D + M_L}{f_s \times d_e} \quad \text{(12)}$$

and $M_D + M_L$ is the sum of dead-load and live-load moments, in inch-pounds; f_s is the allowable stress in steel, in pounds per square inch; and d_e is the distance between centers of gravity of the flanges, in inches, plus the height of any fillet between the slab and the beam.

(2) Analysis.—The stresses are computed for the section obtained in the preliminary design, step (1). It is assumed that the dead-load moment is carried by the steel beam alone and that the live-load moment is carried by the composite section.

(3) Revision.—If the stress in either or both flanges is different from f_s, the flange area may be adjusted in the following manner: Let f_b be the desired stress in the flange; f'_b be the actual stress in the flange; A_b be the desired area of flange to give a stress of f_b; A'_b be the present area of the flange giving a

stress of f'_b; and A_w be the total area of the web. Then

$$\frac{A_b + 0.4\,A_w}{A'_b + 0.4\,A_w} = \frac{f'_b}{f_b} \qquad (13)$$

This adjustment can be made simultaneously and independently for both top and bottom flanges.

As an illustration of the effect of changing the thickness-depth ratio for the web, the following expressions for trial flange areas have been found to hold approximately if this ratio is 1/50 instead of 1/87 as before:

$$A_b = 0.95\,A_o - 2.5 \qquad (14a)$$

and

$$A_t = 0.80\,A_o - 7.5 \qquad (14b)$$

PART C. BEHAVIOR OF SHEAR CONNECTORS FOR COMPOSITE I-BEAM BRIDGES

1. Requirements for Shear Connectors

Composite action is produced in an I-beam bridge by the introduction of shear connectors whose function is to tie together the concrete slab and the steel I-beam in such a manner that they act as a single element. To do this, two requirements must be met by the shear connectors: First, they must effectively prevent slip between the slab and the beam; and, second, they must be strong enough to withstand the shearing forces with an adequate factor of safety.

The capacity of shear connectors to prevent slip has been demonstrated by the results of tests on quarter-scale model bridges discussed in Part B.

When slip is prevented, the shear connectors are subjected to loads resulting from the shearing forces acting between the slab and the beam. The total force acting on each connector may easily be computed from the horizontal shear and the spacing of the connectors. However, the stresses in the connector cannot be obtained unless the distribution of force over the height and width of the connector is known. A primary objective of this investigation was the determination of the nature of this distribution and of the factors that may cause changes in it.

Another factor which must be considered is that the shear connectors in many bridges are subjected to repeated loading under conditions favorable to failure in fatigue. In a composite bridge constructed without shoring beneath the beams, the shear connectors are stressed as a result of live loads only, since dead load is carried by the steel stringers acting alone. At the center of the span where a complete reversal of shear occurs for moving loads, the stress on the connectors also undergoes complete reversal. At the ends of the span, the stress varies from zero to a maximum with each passage of a load. This large range of stress, plus the possible presence of stress raisers at or near welds or fillets, makes it desirable to consider the possibility of failure under repeated loading.

Consideration must also be given to the stresses produced in the concrete slab in bearing against the shear connectors. It is apparent that the function of the connector may be impaired or destroyed by the failure of the concrete adjacent to it, as well as by the failure of the connector itself.

A third requirement of shear connectors which is desirable, although not necessarily essential, is that they be able to resist uplift forces tending to pull the slab vertically away from the beam. This tendency for separation may be the result of loading, or it may be caused by warping of the slab resulting from shrinkage, unequal expansion due to radiant heat, or other effects.

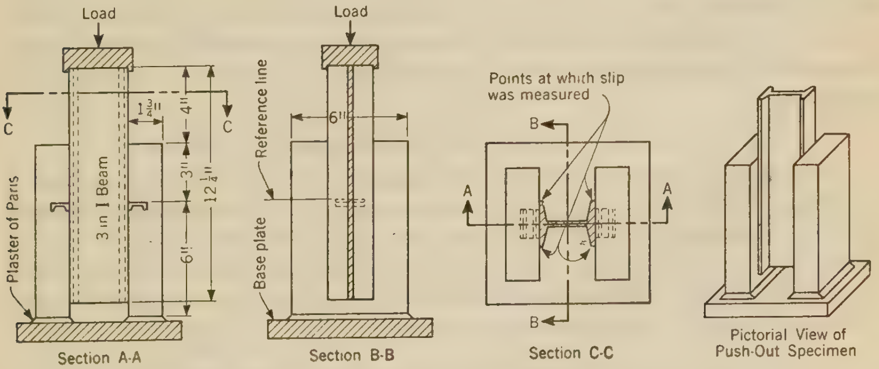
2. Bond As Shear Connection

Both laboratory tests and field observations have shown that bond between the steel beam and the concrete slab may provide a fairly effective shear connection under certain conditions. However, once this bond is broken it cannot be restored, and its function as a shear connection is lost forever. Although the bond stresses resulting from loads may be fairly low, there is always the possibility that the bond may be destroyed as a result of movements caused by shrinkage, temperature, vibration, etc. In particular, bond offers practically no resistance to vertical separation of the slab and beam, and such separation destroys entirely its ability to transfer shear.

There can be no question that bond alone should not be relied on as a shear connection for beams which are designed for composite action. A positive mechanical method for transferring shear and preventing slip and uplift should be provided. The use of a positive shear connection also helps to preserve the bond intact by holding the slab and beam in contact.

3. Push-Out Tests of Shear Connectors

To compare the behavior of several different types of shear connectors, a series of simple shear tests was made, utilizing the push-out type of specimen illustrated in Fig. 27. One shear connector was welded to each flange of a short length of 3-in. I-beam, and each of the connectors was embedded in a mortar slab cast against the flange of the beam. The connectors were subjected to shear by applying load to the beam and forcing it to move relative to the slabs. Slip between the I-beam and the slabs was measured with dial



Section A-A

Section B-B

Section C-C

Pictorial View of Push-Out Specimen

Fig. 27.—Details of Push-Out Specimen

indicators at the four corners of the I-beam as indicated by the arrows on section C–C, Fig. 27.

These tests included only the types of shear connectors that transmit shearing force primarily by bearing of the connector on the concrete. For some of these types, the width and the thickness of the connector were varied, and in another series of tests, the compressive strength of the mortar in the slabs was varied.

4. Flexible and Rigid Connectors

The various types of connectors used in the push-out tests may be divided into two general classes, rigid connectors and flexible connectors. The types considered as rigid consisted of short lengths of channel, angle, or tee section placed on end with their longitudinal axis perpendicular to the beam flange. Several connectors of the flexible type were tested; the only one considered herein, however, is the channel placed with the web vertical and having one flange welded to the beam.

Typical load-slip curves for both rigid and flexible connectors are shown in Fig. 28(*a*). It is evident from these curves that the rigid type of connector is superior to the flexible-type channel, from the standpoints of both ultimate strength and the amount of slip permitted at low loads. It is significant, however, that the differences in behavior of the two types are much less than might be expected from the very large differences in strength and stiffness of the connectors themselves. The reason for this is that both the ultimate strength of the test specimens and their load-slip characteristics were dependent to a very considerable extent on the stresses and deformations in the concrete, and were affected to a much smaller degree by the properties of the shear connectors.

Although the rigid type of connector has certain advantages, as illustrated by the results of these tests, this type also possesses certain disadvantages, the most important of which is its failure to resist uplift forces tending to separate the slab and the beam. Another disadvantage results from the relatively large width of the connectors in the direction parallel to the beam axis, which in many cases will offer serious interference to placing the transverse reinforcement in the slab.

5. Effect of Variations in Thickness of Connector

The effect of varying the web thickness of channel shear connectors is illustrated in Fig. 28(*b*). Load-slip curves are given both for ordinary channels and for channels with the top flange removed. Two web thicknesses were used: 0.125 in. and 0.097 in. It may be concluded from the data in Fig. 28(*b*) that a decrease of 22% in the thickness of the web did not produce any significant changes in the behavior of the connectors.

6. Effect of Variations in Compressive Strength of Slab

The load-slip curves in Fig. 28(*c*) are for specimens with three different values of compressive strength. It is evident from the curves that the effectiveness of the shear connectors was increased considerably by an increase in the compressive strength of the slabs. This increase, however, was proportionately less for the higher strengths than for the lower ones.

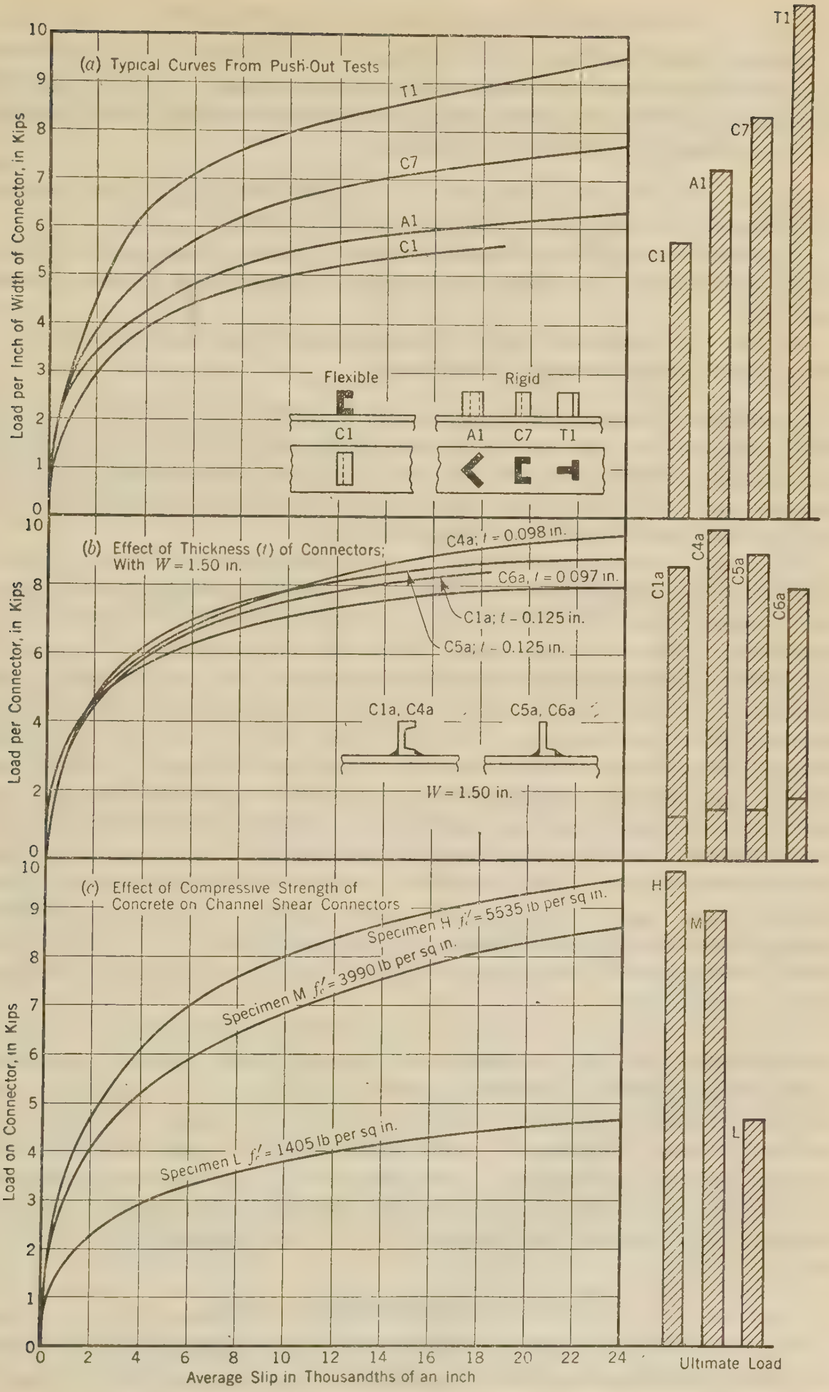


FIG. 28.—COMPARISON OF LOAD-SLIP CURVES

7. SUMMARY OF RESULTS OF PUSH-OUT TESTS

Typical results obtained in the push-out tests are given in Fig. 28. From these results, and from other data not given in this paper, the following conclusions may be drawn regarding the effect of certain variables on the behavior of shear connectors loaded in simple shear:

(*a*) The effectiveness of the shear connectors considered in these tests is approximately proportional to the width of the connector;

(*b*) A decrease of 22% in the web thickness of channel shear connectors had no appreciable effect on either the load-slip characteristics or the ultimate load; and

(*c*) The effectiveness of channel shear connectors was increased by an increase in compressive strength of the slab.

In addition to these conclusions, certain characteristics common to all the shear connectors may be noted: (1) Slip begins with the first application of load; (2) the load-slip relation is not linear at any load, but is roughly parabolic in shape; and (3) the ultimate loads are accompanied by such large slips that they have little application as a design criterion.

As a result of the push-out tests, it was decided that the flexible type of shear connector, consisting of a channel with one flange welded to the beam, was the most promising type, and that all future tests would be made on such connectors. This choice was based on several considerations. The channel was preferred to the rigid type of connector because of the disadvantages of the latter which have been mentioned. Moreover, the performance of the channel, although not as good as that of the rigid types, was still believed to be satisfactory. Other types of flexible connectors were also considered. A Z-section, although not included in these tests, would probably behave in a manner very similar to a channel. However, this type of section is not so easily procured as the channel. A plate placed vertically and welded to the beam is a lighter section than a channel but it provides no resistance to uplift and, in addition, is not so easily placed in position and welded in place. The latter objection applies also to an angle placed with the vertical leg down and welded to the beam. This type of connector resists uplift forces, but the welding operation is likely to be awkward.

The push-out type of specimen was chosen primarily because it furnished a fairly simple and rapid method of comparing several types of connectors. It was realized, however, that the conditions existing in such a test were not necessarily similar in all respects to those in a composite beam. Consequently, the tests of small composite T-beams which have previously been described (see Fig. 23) were undertaken. The effects of two variables were studied—the thickness of the channel web and the compressive strength of the slab. In general, the results of these tests agreed with those obtained in the push-out tests.

8. CONCEPT OF DOWEL-LIKE ACTION

The most interesting results from the tests on channel shear connectors were those indicating the relative importance of variations in web thickness of the channel and variations in compressive strength of the slab. A study of these results led to the conclusion that the action of this type of connector is similar to that of a dowel embedded in an elastic medium. The type of behavior associated with this concept of dowel-like action is illustrated in Fig. 29. A possible distribution of pressure on the dowel or connector is shown in Fig. 29(*a*). It is markedly nonuniform and may change sign as indicated.

The exact shape of the pressure distribution depends on the relative stiffnesses of the connector and the concrete. The stiffness of the channel depends primarily on its web thickness; the stiffness of the concrete depends on its modulus of elasticity and thus on its compressive strength. Variation of either the web thickness of the channel or the strength of the concrete changes the relative stiffness of the dowel and matrix and thereby changes the pressure distribution.

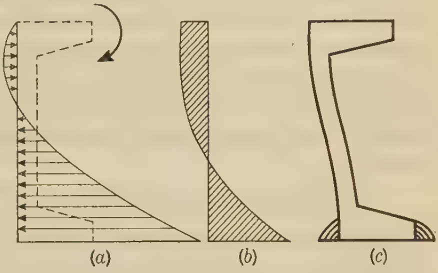


FIG. 29

Both the maximum stress in the concrete and the moment in the connector at the critical section are functions of the shape of the pressure distribution curve. For example, if the thickness of the channel web is reduced by one half, the section modulus of the channel is reduced to one fourth of its original value. However, its stiffness is only one eighth of its original value, and the relative stiffness of the connector as compared to that of the concrete is greatly decreased. This decrease, in turn, is accompanied by an increase in nonuniformity of the pressure distribution and a lowering of the center of pressure which decreases the moment on the channel at the critical section. Thus, a reduction in web thickness decreases the moment as well as the section modulus, and the corresponding decrease in effectiveness of the connector may be relatively small.

The effect of variations in concrete strength may also be considered. If the concrete strength is increased, there is a corresponding increase in modulus of elasticity which has the effect of decreasing the stiffness of the connector as compared to that of the concrete. As before, this change results in a more nonuniform pressure distribution, and a consequent increase in the maximum concrete bearing stress. Thus an increase in concrete strength is accompanied by an increase in concrete stress, and the net result is an increase in effectiveness of the connector somewhat less proportionately than the increase in strength.

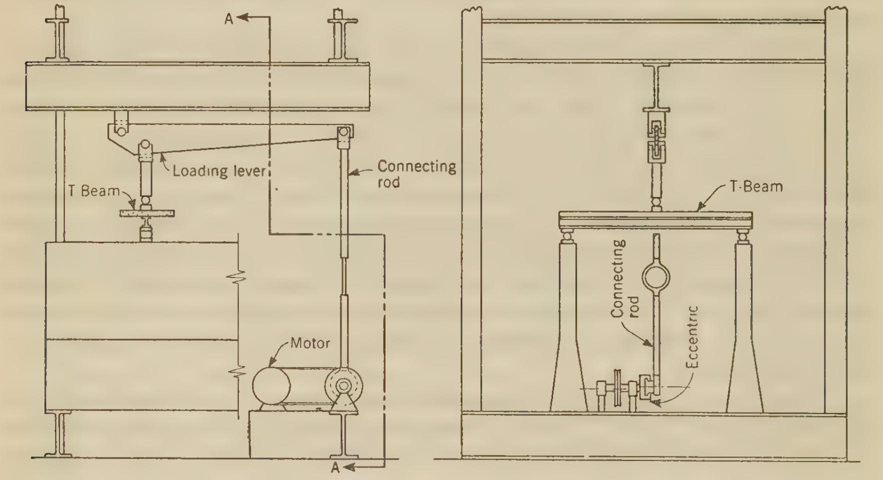
A curve of moment in the connector corresponding to the assumed pressure distribution is shown in Fig. 29(*b*). The moment indicated at the upper end of the connector is a result of the restraint to rotation offered by the flange of the channel. To apply the dowel analogy, the moment is that which would exist if the flange were bent 90° and extended as a continuation of the web.

Fig. 29(*c*) represents a type of deflection consistent with the pressure and moment curves shown. Deflections of a typical channel shear connector, after being subjected to shear in a push-out test, are in substantial agreement with the approximate deflection shown in Fig. 29(*c*).

The hypothesis of dowel-like action of channel shear connectors serves to explain qualitatively the observed behavior of such connectors in both the push-out tests and the static tests of composite beams. It also suggests that rather high compressive stresses are produced in the concrete, and probably also high bending stresses in the channel web. These potentially high stresses, together with the fact that the shear connectors in an I-beam bridge are subjected to repeated loads, indicated the desirability of fatigue tests on channel shear connectors. Moreover, it was felt that such tests might provide a quantitative basis for evaluating the effects of variation in concrete strength and web thickness.

9. Repeated-Load Tests of Channel Shear Connectors

It was decided that fatigue tests of shear connectors could be made most conveniently by applying repeated loads to small composite T-beams of the type illustrated in Fig. 23. Such tests have been made in the testing machine illustrated in Fig. 30. Load is applied at midspan of the T-beam by a lever actuated by a connecting rod attached to a variable-throw eccentric. The load is measured by an elastic-ring dynamometer inserted in the connecting rod. The speed of operation is 190 cycles per min. Measurements made during the repeated-load test include slip between the slab and the I-beam at the ends of the beam, and deflection of the beam at midspan. Static tests in which strains in the beam are also measured are made on each beam both before and after it is tested in fatigue.



Elevation Section A–A

Fig. 30.—Machine for Applying Repeated Load to a Composite Beam

The principal variables considered in these tests were the compressive strength and modulus of elasticity of the mortar slab and the web thickness of the channel shear connectors.

There is little information available regarding the number of load applications that should be considered for highway bridges. However, a preliminary study of traffic survey and "loadometer" data indicated that the number of heavy truck loads may easily approach 1,000,000 or 2,000,000 during the life of a bridge. Consequently, these tests were planned to obtain data for load-cycle diagrams with a maximum of 2,000,000 cycles.

10. MANNER OF FAILURE IN FATIGUE

A rather detailed description of the manner of failure of the composite T-beams in the repeated-load tests is desirable so that the results of these tests might be interpreted properly. There was no well-defined point at which it could be stated that failure had occurred. Instead there was a gradual, progressive breakdown of composite action accompanied by increases in slip and deflection. Finally, the slip became so great that it was obvious that interaction between the beam and the slab was seriously impaired. In these tests, this stage of failure was usually reached when the slip at the end of the beam attained a value of 0.010 in. If the test was stopped at this point, it was found in every case that several shear connectors at one end of the beam had fractured.

In these tests, all the shear connectors were faced in the same direction, as indicated in Fig. 31(*a*). As a result, the direction of the load in relation to the direction in which the channel flanges projected was different for the two ends of the beam. With few exceptions, the fractured connectors were those loaded in the manner shown in Fig. 31(*b*). It is probable that the conditions at the other end of the beam were as illustrated in Fig. 31(*c*)—that is, the channel was strengthened by the resistance to compression of the block of mortar enclosed between its flanges.

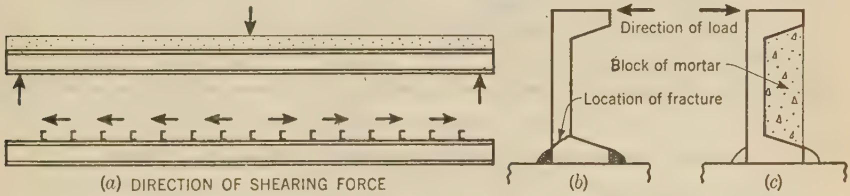


FIG. 31.—FAILURE OF CHANNELS IN REPEATED-LOAD TESTS

In all cases, the final failure of the beam was by fracture of the shear connectors at the location shown in Fig. 31(*b*). There is some question, however, as to whether this fracture was always the primary cause of failure. If the concrete adjacent to the connector fails by crushing, there will be a redistribution of pressure on the connector in such a manner as to raise the center of pressure and consequently to increase the moment on the channel. This increased moment, in turn, may produce a fatigue failure of the connector. There is no way of knowing conclusively, from the data thus far obtained, whether the primary cause of failure in any given case is crushing of the concrete or fracture of the connector.

11. Effect of Variations in Concrete Strength

A series of repeated-load tests was made on twenty composite T-beams for which the compressive strength of the mortar slab ranged from 1,000 lb per sq in. to 6,500 lb per sq in. The results of those tests are given in Fig. 32. In Fig. 32(*a*) the load on the T-beam (not corrected for compressive strength) is plotted against the logarithm of the number of cycles required to produce failure. The beams are divided into three groups on the basis of their compressive strengths, as indicated by different symbols. The data contained in Fig. 32(*a*) were studied, and the variation of load with compressive strength was found to be approximately as shown in Fig. 33. The quantity κ is simply a correction factor which may be used to reduce all the results to those corresponding to a compressive strength of 3,000 lb per sq in. Data so reduced are given in Fig. 32(*b*), in which the corrected load, P/κ, is plotted against the number of cycles.

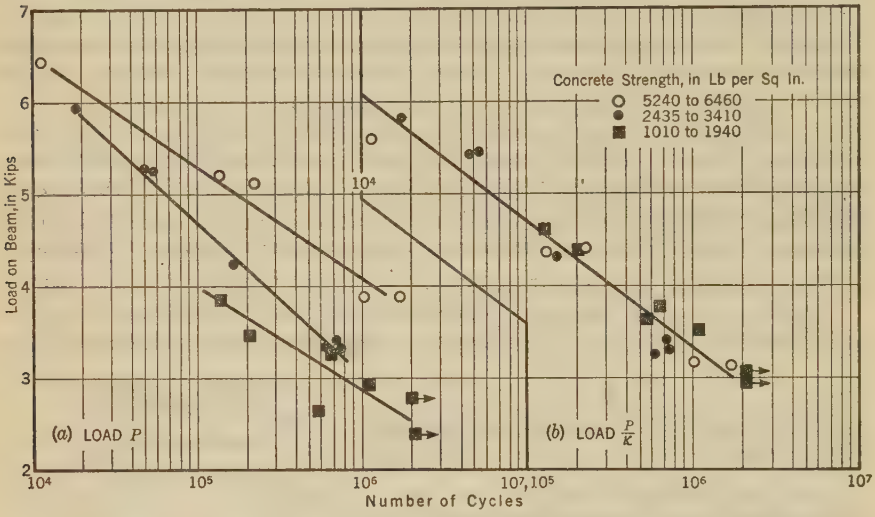


Fig. 32.—Load-Cycle Curves for Repeated-Load Tests with Variable Values of Concrete Strength

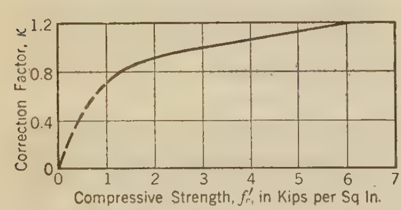


Fig. 33.—Basis of Correction for Compressive Strength

The effect of compressive strength on the effectiveness of channel shear connectors is defined quantitatively by the curve in Fig. 33. Precisely why this relation takes the form shown cannot be determined conclusively from the available evidence. However, a possible explanation may be that at low compressive strengths the primary cause of failure is crushing of the concrete, whereas at higher compressive strengths the primary cause is fracture of the steel without prior failure of the concrete. Since the strength of the concrete affects its stiffness and thus affects the shape of the stress distribution, there would still be some variation in load with concrete strength for the higher strengths even though primary failure was by fracture.

12. Effect of Variations in Web Thickness

Repeated-load tests were also made on thirty-three beams having connectors with different web thicknesses. Two values of web thickness were used—$\frac{1}{8}$ in. and $\frac{1}{16}$ in.— and for each thickness two concrete strengths—approximately 2,000 lb per sq in. and 4,000 lb per sq in.—were used. The results are presented in Fig. 34 in the form of load-cycle curves for the connectors of each web thickness. The variations in compressive strength have been eliminated by a relation similar to that given in Fig. 33.

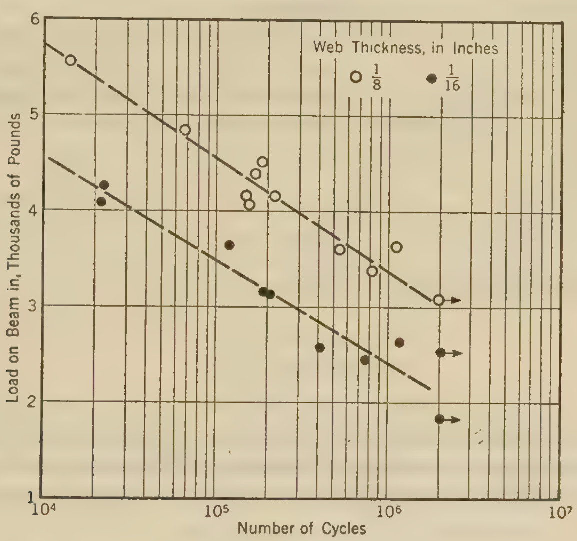


Fig. 34.—Effect of Web Thickness on Effectiveness of Channel Shear Connectors

It may be determined from the diagram that a reduction in web thickness from $\frac{1}{8}$ in. to $\frac{1}{16}$ in. reduced the effectiveness of the connectors by only about 25%. Since the reduction in section modulus of the channel web is 75%, the decrease in thickness must have been accompanied by an appreciable reduction in moment on the connector. In accordance with the dowel concept, such a reduction would result from the change in pressure distribution caused by the reduction in stiffness of the connector.

13. Distribution of Pressure on Channel

It has been shown that the assumption of dowel-like behavior of channel shear connectors is in reasonably good agreement with the results of tests, and this hypothesis serves to explain the influence of certain variables on the effectiveness of the connectors. The principal assumption of this hypothesis is that the distribution of pressure on the connector is a function of the relative stiffness of the connector and the concrete. The distribution of this pressure is quite complex, and it is doubtful if its true nature can be determined by tests similar to those which have been described. Nevertheless, it is possible to draw a greatly simplified and idealized pressure diagram which has certain properties similar to those of the actual distribution. Such a pressure diagram is given in Fig. 35. Its chief characteristics are the uniform distribution of pressure over the very stiff area of the channel opposite the flange, and the expression of the distance h_w as a function of the web thickness t.

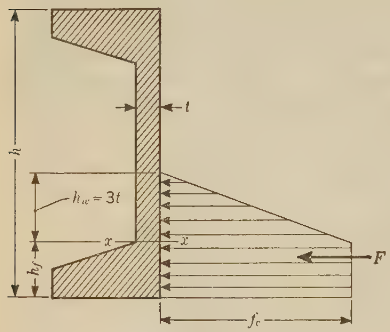


FIG. 35.—IDEALIZED PRESSURE DISTRIBUTION

Let F be the total load, in pounds per inch of width. Then, for the stress distribution of Fig. 35, the maximum concrete stress is expressed by

$$f_c = \frac{F}{h_f + \frac{3t}{2}} \quad \text{(15a)}$$

and the maximum steel stress on section x–x by

$$f_s = \frac{9F}{h_f + \frac{3t}{2}} \quad \text{(15b)}$$

This type of pressure distribution is being studied as a possible basis for the design of channel shear connectors. However, sufficient information is not yet available to permit recommendations regarding a definite design procedure to be made.

SUMMARY

This paper has presented the results of analytical studies and laboratory tests of composite construction for I-beam highway bridges. The principal conclusions can be summarized as follows:

a. Savings in Weight with Composite Construction.—Composite construction permits the use of lighter beams and at the same time provides a much stiffer structure. The saving in weight which may be obtained by the use of various types of construction ranges from 8% for ordinary rolled beams to 30% or more for built-up welded beams.

b. Design of Composite Beams.—A composite beam consisting of a slab and beam tied together by a suitable shear connector may be analyzed and designed on the basis of the transformed section. In I-beam bridges of ordinary proportions, a section of the slab having a width equal to the beam spacing may be considered effective as part of the composite beam.

Variations in the value of n, the ratio of modulus of elasticity of steel to that of concrete, have only a negligible effect on the properties of a composite beam. Except where it is desired to allow for effects of plastic flow, it is sufficiently accurate to assume a value of $n = 10$.

c. Shear Connectors.—This investigation was concerned chiefly with the behavior of channel shear connectors. The effects of the following variables were investigated—width of connectors, thickness of channel web, and compressive strength of the concrete in the slab. The principal results were as follows:

(1) The effectiveness of channel shear connectors was approximately proportional to the width of the connector;

(2) Variations in web thickness produced corresponding variations in the effectiveness of the connectors, but the magnitudes of these changes were far less than would be expected from the changes in section modulus of the channel web; and

(3) The effectiveness of channel shear connectors was increased by an increase in compressive strength of the concrete, but by a proportionately smaller amount for the higher compressive strengths.

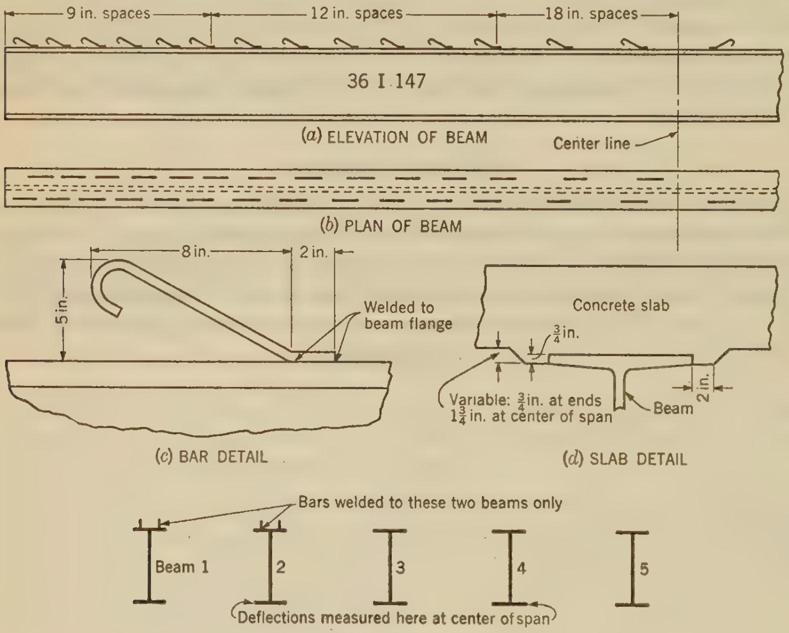
The effects of changes in web thickness and concrete strength on the behavior of channel shear connectors may be explained by the concept of dowel-like action. According to this concept, the nature of the distribution of bearing pressure on the connector is a function of the relative stiffness of the channel and the concrete.

ACKNOWLEDGMENTS

The comparative design studies in Part A were made by C. A. Erzen and N. Y. Yakovljevitch, graduate students in civil engineering, and M. N. Tokay, special research assistant in theoretical and applied mechanics. The repeated-load tests of shear connectors were made in part by R. F. Mosher, Jun. ASCE, and R. A. Bennitt, Jr., graduate students in civil engineering, and W. E. Johnson, special research associate in civil engineering.

DISCUSSION

Searcy B. Slack,[26] M. ASCE.—Publication of this group of papers has made available an excellent summary of results of tests and analyses of steel beam bridge spans with concrete floors. With the introduction of the deep beam and girder sections and wider roadways, the design of highway bridges with spans ranging from 40 ft to 100 ft or more changed very quickly to this simple type and the problem of load distribution and interaction of beam and floor slab became more acute. The Bridge Committee of the American Association of State Highway Officials spent endless hours in discussions but E. F. Kelley, then chairman of that committee, did the basic work of formulating the design specifications for loads and distribution which, with modifications due to later developments, are now in general use.[18] With the meager information then available, Mr. Kelley did an excellent piece of work.


9 in. spaces
12 in. spaces
18 in. spaces
36 I 147
Center line
8 in.
2 in.
5 in.
Welded to beam flange
Concrete slab
$\frac{3}{4}$ in.
Variable: $\frac{3}{4}$ in. at ends $1\frac{3}{4}$ in. at center of span
Beam
2 in.
Bars welded to these two beams only
Beam 1
2
3
4
5
Deflections measured here at center of span

(a) ELEVATION OF BEAM

(b) PLAN OF BEAM

(c) BAR DETAIL

(d) SLAB DETAIL

(e) SECTION OF BEAMS

Fig. 36.

In 1930, 1931, and 1932, quite a number of measurements and tests were made in Georgia under the direction of the writer on steel beam spans with concrete slab floors. These measurements showed quite clearly that the spans were stiffer than expected. The measured deflections were only from 50% to 75% of the computed deflections. This result indicated some interaction of the beams and slab and a wider distribution of the live, or added, load than expected. As no shear connectors between the concrete slab floor and the top of the beams had been provided, the interaction must have been due to friction or to partial bond.

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To compare the stiffness of beams with shear connectors between the beam and floor slab with beams with no shear connectors, a test was carried out in 1931 on a simple 62-ft-span beam bridge. The roadway of this bridge was a 22-ft-wide concrete floor slab supported by five beams, 36 in. deep, spaced 5 ft apart. The slab was $7\frac{1}{2}$ in. thick. The location, spacing, and detail of the $\frac{3}{4}$-in.-round-steel shear ties are shown in Fig. 36. The deflections were measured from constant tension wires and the strain measurements were made with a set of ten electric strain gages. The strain measurements in the beam flanges were checked with a mechanical strain gage.

TABLE 4.—MEASURED AND COMPUTED STRESSES IN TEST SPAN SHOWN IN FIG. 36

Location	MEASURED STRESS[a] (LB PER SQ IN.)		COMPUTED STRESS[a,b] (LB PER SQ IN.)	
	Dead load only	Dead plus live load	Dead load only	Dead plus live load
Slab over beam 2	− 468	− 567	0	0
Slab over beam 4	− 394	− 458	0	0
Top flange, beam 2	−1,080	−1,680	−7,930	−9,930
Top flange, beam 4	−2,550	−4,080	−7,930	−9,930
Bottom flange, beam 2	+6,720	+8,340	+7,930	+9,930
Bottom flange, beam 4	+5,490	+6,970	+7,930	+9,930

[a] − denotes compression, + denotes tension. [b] Assuming no interaction between slab and beams.

TABLE 5.—TOTAL MEASURED AND COMPUTED DEFLECTION AT CENTER OF TEST SPAN SHOWN IN FIG. 36

Location	MEASURED DEFLECTION (IN.)		COMPUTED DEFLECTION (IN.)	
	Dead load only	Dead plus live load	Dead load only	Dead plus live load
Beam 1	0.425	0.475	0.85	1.02
Beam 2	0.500	0.575	0.85	1.02
Beam 4	0.550	0.675	0.85	1.02
Beam 5	0.575	0.725	0.85	1.02

Temporary intermediate pile bent supports were driven at the quarter points and the beams were wedged until the loads on these bents were about equal. The floor slab formwork was built and the floor poured of concrete with 1.58 bbl of cement per cubic yard of concrete, with a slump of $3\frac{1}{2}$ in. Test cylinders of the concrete indicated a strength of 3,620 lb per sq in. at 28 days. When the slab was 28 days old, the forms were removed, the strain gages were set, and the initial readings were taken. The wedges were then driven out from the intermediate supports and the readings were taken for dead-load strains and deflections. A load of 15,000 lb was then placed on the span concentrated at the center of beams 2 and 4. This added load is referred to as the live load in

Tables 4 and 5, which present the test results. Stresses were computed on the basis of $E = 4{,}000{,}000$ for concrete and $E = 30{,}000{,}000$ for steel.

When the tests were planned, it was not realized how stiff the slab and cross walls would be. It is evident from the measurements that beams 1 and 2, which were tied to the slab with shear ties, were much stiffer than the other three beams and carried much more than their share of the dead and live load.

At the time the measurements were made on this bridge span, it was not known how much changes in temperature would affect beam and slab bridges. Measurements of deflection of a concrete deck girder bridge showed that sudden changes in air temperature would cause more deflection of the span than would be caused by a live load of two 12-ton trucks. Because of the exposed steel under the concrete floor slab, it might be expected that changes in the temperature would greatly affect deflection in a composite or semicomposite beam and slab span.

In 1932, measurements were made of deflections of two spans, each 60 ft ong, for a period of about 2 weeks after the spans were completed. Measurements were made early in the morning when the temperature was near the daily minimum, and at 2 p.m. when the temperature was near the maximum. The greatest change in temperature in a single day during the time of measurement was from a low of 57° F at 6 a.m. to 80° F at 2 p.m. The deflection for span 21 at 57° F was 0.706 in.; for span 22 it was 0.673 in. At 80° F the deflections were 0.739 in. for span 21 and 0.712 in. for span 22. This was a change of about 5% in the deflection for a 23° F change in temperature. Incidentally, the computed deflection for these two spans was 0.94 in. for dead load only. Under two 12-ton trucks the total measured deflection for dead load and live load was 0.86 in. as compared with a computed deflection for this combined load of 1.28 in. With no shear connectors these spans had been greatly stiffened by composite action of the beam and slab. Friction and partial bond must serve to transfer some of the shear between the top of the beam and the bottom of the slab.

These measurements and observations on full-size bridge spans give interesting support to the laboratory scale tests reported in the Symposium. The authors are to be congratulated on suggesting rather simple design methods. With the large number of variables and uncertain factors in the information available, refinement in design is scarcely warranted. The methods they suggest do not take into account the stiffening brought about by friction or partial bond between slab and beam. In the laboratory scale model tests it was evidently possible to eliminate this factor, but field measurements on full-size bridges show that computations of stress or deflections are from 20% to 40% in error.

A series of tests and measurements was made late in 1932 on both full-size bridges in the field and laboratory models. Unfortunately, it was found that the electric strain gages used would not hold calibration and the information gathered was of value only qualitatively. The measurements of deflection, however, were direct and consistent. As the sets of measurements reported in this discussion are typical and in line with the later work, little would be accomplished by repetition in detail.

Neglect of the factor of interaction due to friction or partial bond is a conservative reaction and possibly is proper until further information is available on both the curling and the longitudinal effect of a sudden change in temperature. The effect of plastic flow over a long period on a composite beam and slab span built with temporary intermediate supports is another question worthy of consideration. As this type of bridge floor will probably continue to be widely used, it is to be hoped that these tests will be continued and extended.

E. W. Wendell,[27] M. ASCE.—A coordinated, forward step in the matter of determining the theory to be applied in the design of composite bridges is represented by this Symposium. In the writer's opinion, results obtained in this manner, from actual tests, have a far better chance for acceptance by practicing engineers than does any purely theoretical approach. Regardless of any theory which can be utilized as a vehicle of approach to a problem, most working engineers react favorably to model tests and full-size tests when carried out under the type of supervision extended by the authors.

Interest in the design of composite beam and girder bridges has been quickened for various reasons. The chief reason for this interest is that this type of structure provides a satisfactory answer to the problem of combining economy, simplicity, and beauty. In the matter of economy, however, it is well not to become overly enthusiastic about the savings in weight of the composite design as compared with the noncomposite design. A saving in weight alone in this day of high labor costs does not necessarily mean a saving in dollars. At present there must be at least a saving in weight of 20% in order to compensate for the additional costs of the spiral type of shear connectors which have been generally adopted for use, but in which there is not sufficient interest to exclude other types.

Mr. Siess indicates that from his investigation of welded steel beams a limiting web thickness of not less than one eighty seventh of clear depth is desirable. This is stated to provide adequate safety against buckling and to obviate the need for intermediate web stiffeners. Specifications[18] of the American Association of State Highway Officials (AASHO) require that the web thickness shall be not less than one sixtieth of the clear depth before intermediate stiffeners can be eliminated. Picking at random a structure from the design table and assuming that one eighty seventh of clear depth would be satisfactory, the case of a 115-ft all-welded beam may be investigated The web plate would be 57 in. by $\frac{5}{8}$ in. and would weigh 14,000 lb for one 115-ft span. By careful analysis, it is possible to use a web plate 57 in. by $\frac{3}{8}$ in., with a total of fifty-four intermediate plate stiffeners, properly spaced. The weight of this web plate plus stiffeners is 10,000 lb for one 115-ft span. This clearly shows that the use of the thinner web plate, properly stiffened, saves 28% in the weight of steel. Of course, there might well be a desire on the part of the designer to avoid the use of stiffeners for esthetic reasons, in which case economy would not be one of the controlling factors. The cost of the additional welding must also be considered, as previously mentioned in the matter of shear connectors.

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[27] Deputy Chf. Engr., State Dept. of Public Works, Albany, N. Y.

It is indeed gratifying to engineers to read the conclusions which Mr. Siess has indicated with regard to the greater live-load moment taken by a properly proportioned composite bridge than by a noncomposite structure. It has frequently been suspected that the individual beams in a composite bridge take more live loads than those in a noncomposite structure, and Mr. Siess gives an acceptable reason for this phenomenon. Certainly it is reasonable to conclude that composite beams having a greater stiffness produce less lateral distribution of superimposed loads to adjacent beams. The findings of Mr. Siess as to the value of unsymmetrical sections as compared to symmetrical sections, when utilized in composite designs, are again in conformity with what has been recognized for some time. In Table 3 there are listed the relative weights of steel beams in composite and noncomposite I-beam bridges.

The comparison clearly indicates an over-all saving of 24% for composite sections, using symmetrical rolled beams and cover plates. It would seem as if the additional savings of 7% for all-welded beams, as compared to rolled beams having cover plates, would be more than overcome by the additional welding costs. Of course, there are many instances when the depth ratio required by the design specifications for a particular span eliminates the consideration of rolled beams with cover plates. When this condition prevails, the all-welded beam must be used. Under present market conditions, it is also frequently necessary to recognize the availability of materials. It might well be that plates can be procured more readily than rolled beams. When this occurs the all-welded beam, of course, should be specified. As a matter of fact, from experience in the writer's office and as a recognition in Mr. Siess' paper, it is thought that rolled beams with attached cover plates can be specified for spans up to about 90 ft. For spans greater than 90 ft an all-welded beam should be used. When it comes to rolling unsymmetrical beams, one is entering a field from which the steel companies have retreated consistently since 1938. The variations in spans, loadings, and spacings, of beams would require too many different types of rolled, unsymmetrical sections. Steel companies are reducing the number of symmetrical sections, and unless there is a marked change in this trend the suggestion of producing unsymmetrical sections would meet with little enthusiasm.

The use of temporary supports for composite bridges does not produce the advantages that have been indicated many times in the past. It seems to the writer that the cost of providing temporary supports beneath the beams until the concrete slab has set will, in the vast majority of structures, more than offset any possible additional savings in the weight of steel effected by having the composite section take the dead load as well as the live load. There may be exceptional cases, of course, where temporary supports can be used to effect a saving but, in observing the design of several thousand structures, the writer fails to recognize many locations where this saving could be clearly established.

The records of the results of the tests made to ascertain the slip between concrete and steel, the ensuing effect of this slip on strain in the composite section, the effective width of concrete as T-beam flange, the selection of modular ratio, and the plastic flow of the slab constitute interesting features in the paper. The tests clearly indicate that by using shear connectors the concrete and steel actually do act together as a single composite section, thereby verifying the transformed section method as a basis of design. The tests also prove that the distribution of compressive stress in concrete is substantially uniform, both at the point of maximum moment and at other sections some distances from the location of maximum moment.

Although the writer is inclined to believe that the investigators used various types of shear connectors, Mr. Siess reports on only one type, which is clearly indicated as satisfactory. The tests show that the compressive strength of concrete is important in its control of the effectiveness of shear connectors. The New York State Department of Public Works has assumed 3,000-lb concrete in its designs with satisfactory results. The AASHO specifications covering shear devices to be used in composite construction recognize the importance of both the concrete and the shear connectors in the functioning of a composite beam. These specifications in part state:

> "The shear devices shall be of such construction as will permit a thorough compaction of the concrete mass and will insure entire surfaces of shear devices being in contact with the surrounding concrete. The nature of the shear devices shall be such as to prevent a vertical separation of the slab and beam or else additional means shall be provided for this purpose."

The writer contends that various types of shear connectors can give satisfactory results. The spiral used in designs in the writer's office complies with the requirements of the AASHO specifications, and has the additional advantage that the placing of the transverse reinforcement is not difficult to accomplish. In addition the steel reinforcing bars from which the spirals are made are more easily obtained than small channels. From the practical standpoint, the ideal accomplishment would be to develop a composite type of construction which utilizes no special shear connectors. It might well be that the transference of horizontal shear between the steel beam and the concrete slab could be effected by using the transverse reinforcing bars in the slab for this purpose. By threading the lower slab bars through the beam web and by welding the top slab bars on top of the top flange, an effective composite action could be effected.

In conclusion, the writer is unable to become concerned with the plastic flow of the slab. Quite naturally, $n = 30$ is used for added dead loads because they are required in the specifications. This is an unnecessary requirement; the small additional load of the wearing surface, sidewalks, and railings could be provided for with the usual value of $n = 10$ rather than $n = 30$.

It would be well to analyze carefully the "methods of proportioning composite beams," as indicated through the use of empirical formulas. To an experienced designer, the cut-and-try method of ascertaining the proportions of a composite beam is not as laborious as has been indicated. Two trials at the most, with a recognition of proper adjustments, almost always result in an efficient and economical distribution of flange areas. Experience proves that it is no more difficult to ascertain a proper distribution in this type of beam than in the case of ordinary plate girders.

Certainly engineers who are interested in composite construction should be extremely grateful to the men who have so carefully and so competently advanced the investigations. It should be, of course (and undoubtedly is), recognized that their efforts would not have been possible if it were not for those who desired the investigations and provided the necessary finances and the physical plant.

H. TACHAU,[28] JUN. ASCE.—In view of the increased activities in highway bridge construction and design, this paper is timely and especially valuable. Large sums of public funds are spent on highway bridges every year, and it is only fitting that a certain part of the engineer's efforts should go into research and development of new methods of design, such as those presented in this paper.

The purpose of this discussion is to compare the new method for proportioning beams with the 1944 specifications[18] of the American Association of State Highway Officials (AASHO) and to elaborate somewhat on the selection of the beam spacing.

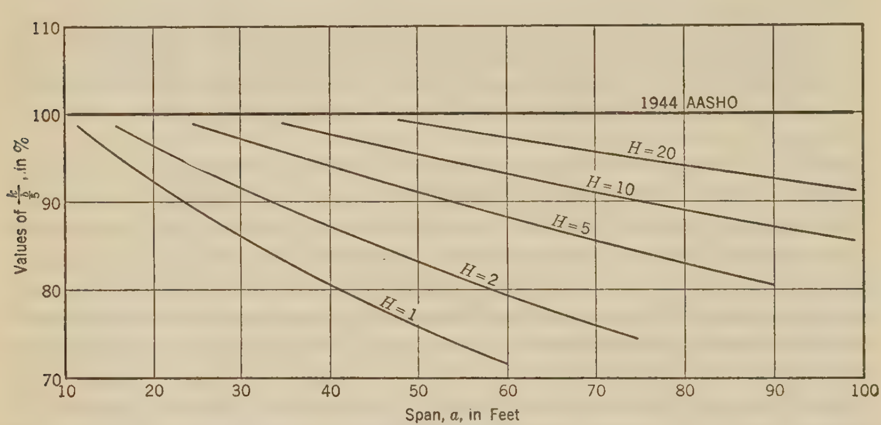

FIG. 37.—COMPARISON BETWEEN RECOMMENDED LIVE-LOAD DESIGN MOMENTS FOR BEAMS AND 1944 SPECIFICATIONS OF THE AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS

Fig. 37 shows at a glance how the recommended live-load design moments for the I-beams are related to values computed according to the 1944 AASHO specifications. By plotting the ratio of k to $b/5$ versus the span length a, a comparison is obtained which holds true for any beam spacing. The quantity $b/5$ gives the proportion of the wheel loads carried by one beam, according to AASHO specifications. In other words, it corresponds to k as used in the paper. Therefore, the AASHO value is shown as 100% in Fig. 37, regardless of the span. The recommended moments are lower, depending not only on the span, a, but also on the relative stiffness of slab and beam, H.

Inasmuch as the new method is based on extensive model tests, as well as on theoretical analyses, it appears that in some cases the AASHO method results in overdesign of the I-beams. The difference is most pronounced for small values of H and for longer spans—that is, when the stiffness of the slab is high compared to the stiffness of the beams.

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[28] Chicago, Ill.

Very little has been written about an important phase of the structural design—the selection of the beam spacing. Except when the spacing is set by functional requirements or by architectural considerations, the designer has a wide choice. Proper selection of the beam spacing is a highly complex matter requiring good judgment. Closer spacing means more rockers, rollers, and bearing plates, and more steel will be needed for the diaphragm connections. These disadvantages are balanced by saving concrete in the deck and by using shallower beams, usually accompanied by a reduction of earthwork in the approaches. Some designers believe that the saving thus effected may, at times, offset the additional cost of steel details. Wider spacing, on the other hand, requires a heavier slab, but the cost of steel fabrication and concrete forms will be reduced.

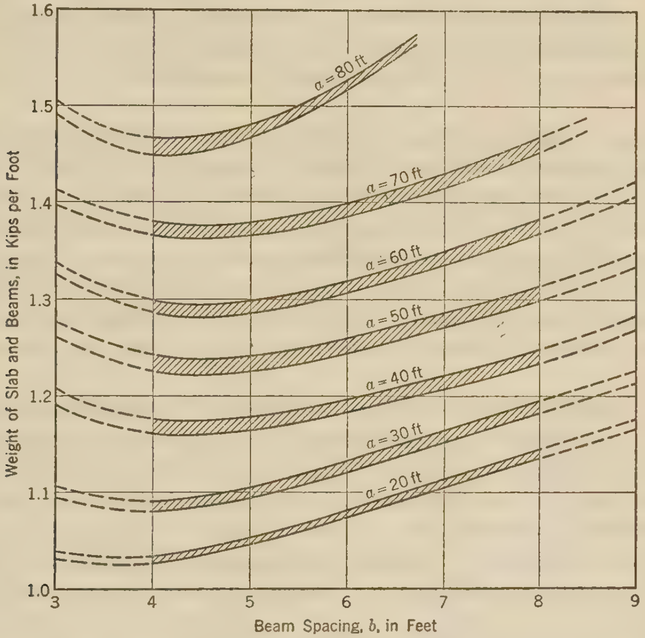

FIG. 38.—VARIATION IN WEIGHT OF SUPERSTRUCTURE (SLAB AND BEAMS FOR A 10-FT LANE FOR VARIOUS SPANS AND BEAM SPACINGS

A study of the dead loads of several designs using different beam spacings may prove helpful for making a good selection. The smallest weight per traffic lane would indicate the optimum beam spacing. Such a comparison has been made in Fig. 38.

The calculations for Fig. 38 are based on a typical cross section of the concrete slab and beams, not including sidewalks. The loading is H20-S16-44, by AASHO specifications. All beams are wide-flange beams without cover plates, designed according to the procedure presented in the paper. Composite action is not considered. For n, the ratio of the modulus of elasticity of steel to that of concrete, a value of 8 is used. The slab is of uniform thickness except over the beams, the thickness varying from a minimum of $6\frac{1}{2}$ in. for

3-ft and 4-ft spacings to 7¾ in. for 9-ft spacing, regardless of the span. These thicknesses include the wearing surface. To take account of concrete "fillets" all concrete weights are increased by 10%. The weights of the steel beams are increased by 20% to account for diaphragms, connections, splices, and other miscellaneous details.

It may be of interest to point out that usually the pavement has camber and crown; that is, it is curved in the transverse and longitudinal directions. The beams are ordinarily straight between splice points, and the space between the top of the beam and the bottom of the slab is taken up by so-called concrete "fillets," cast monolithic with the pavement slab.

Of course, the curves of Fig. 38 apply strictly only to the previously assumed values. However, they will retain their general shape if other design values are used. A change in the value of n, for instance, implies a larger H and a larger k; consequently, the curves will be shifted upward. A set of curves such as those shown in Fig. 38 may easily be constructed for any given conditions.

Possibly, it might be presumed that, for longer spans, larger beam spacings and thicker pavement slabs are in order. Nevertheless, the curves of Fig. 38 indicate that, for the assumptions stated, spacings of from 4 ft to 5 ft are best. Closer spacing seems to be preferable for short spans, also. Similar curves based on the 1944 AASHO specifications would show the same trend. As the possible saving of material appears to be less important for shorter spans, other considerations might govern. In many cases the relative cost of concrete forms and steel fabrication may furnish a relevant basis for the proper selection.

Reto Furrer,[29] Esq.—Part B of the paper by Professor Siesss hows some strain diagrams (Figs. 25 and 26) obtained from load tests on composite beams. Using a novel method of analysis,[30] these test results have been evaluated in the Department of Civil Engineering at the Swiss Federal Institute of Technology in Zurich, Switzerland. It was possible, for example, to take into account the deformations of the shear connectors in computing the stresses for the composite section, differing in this respect from the classical theory of bending.

Fig. 39 is symbolic of a composite beam, the upper concrete part (designated by subscripts c) being separated from the lower steel part (designated by subscripts s), for analytical purposes.

The unknown shearing forces at a section A–A resolved into the horizontal component X_i and the vertical component Y_i. The symbol S_i represents the sum of all forces X_i from the left support to section A–A. The vertical components Y_i are expressed by their moments M_{yi} about the centroidal axis of both beams. In the simple system with $X = 0$ and $Y = 0$, the external moment M is distributed to the two beams as follows (assuming that the deflections of the two parts are equal):

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[30] "Composed Beams," by F. Stüssi, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Switzerland, Vol. 8, 1947, p. 249.

$$M_{0c} = M \frac{n_c I_c}{I_{n0}} \quad \text{and} \quad M_{0s} = M \frac{n_s I_s}{I_{n0}} \ldots\ldots\ldots\ldots (16)$$

$$n_c = \frac{E_c}{E} \quad \text{and} \quad n_s = \frac{E_s}{E} \ldots\ldots\ldots\ldots (17a)$$

in which E is an arbitrary value of the modulus of elasticity and

$$I_{n0} = n_c I_c + n_s I_s \ldots\ldots\ldots\ldots (17b)$$

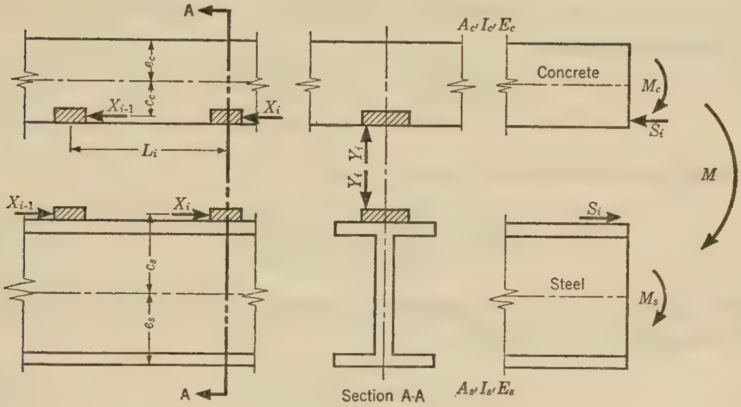
A
e_c
c_c
X_{i-1}
X_i
L_i
A_c, I_c, E_c
Concrete
M_c
S_i
Y_i
Y_i
M
X_{i-1}
X_i
S_i
c_s
e_s
Steel
M_s
A
Section A-A
A_s, I_s, E_s

FIG. 39

The moments acting at the center of gravity of the two beams are

$$M_c = M_{0c} + M_y + S_i c_c \ldots\ldots\ldots\ldots (18a)$$

and

$$M_s = M_{0s} - M_y + S_i c_s \ldots\ldots\ldots\ldots (18b)$$

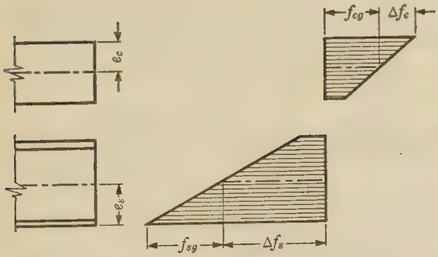
f_{cg}
Δf_c
e_c
e_s
f_{sg}
Δf_s

FIG. 40

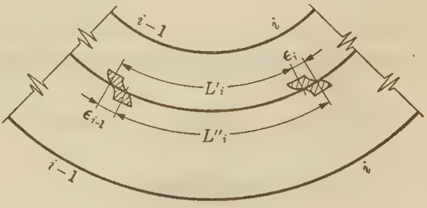
i-1
i
ϵ_i
L'_i
ϵ_{i-1}
L''_i
i-1
i

FIG. 41

In Fig. 40, let f_{cg} and f_{sg} be, respectively, the stresses at the centroids of the upper beam and of the lower beam; $f_{cg} + \Delta f_c$ and $f_{sg} + \Delta f_s$ be the stresses at the upper edge of the upper beam and at the lower edge of the lower beam; and e_c and e_s be the distances from the upper centroid to the upper edge and from the lower centroid to the lower edge.

Fig. 41 shows the displacements resulting from the deflection of the composite beam. Let L_i be the spacing of shear connectors before deflection; ϵ_{i-1} and ϵ_i represent the deformation of shear connectors; and

$$L'_i = L_i + \Delta L'_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

and

$$L''_i = L_i + \Delta L''_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

are the changed spacings, respectively, at the lower edge of the upper beam and at the upper edge of the lower beam. From the relation $\frac{\Delta L}{L} = \epsilon = \frac{f}{E}$ the changes in length are determined with the aid of Fig. 40 as: The bottom fiber of the upper beam—

$$\Delta L'_i = \frac{L_i}{E_c}\left(-f_{cg} + \frac{\Delta f_c}{e_c} c_c\right) \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

and the top fiber of the lower beam—

$$\Delta L''_i = \frac{L_i}{E_s}\left(f_{sg} - \frac{\Delta f_s}{e_s} c_s\right) \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

The deformation of the shear connector is assumed proportional to the force on the connector, X_i; therefore,

$$\epsilon_{i-1} = \frac{X_{i-1}}{C_{i-1}} = \frac{S_i - S_{i-1}}{C_{i-1}} \ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

and

$$\epsilon_i = \frac{X_i}{C_i} = \frac{S_{i+1} - S_i}{C_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

in which the quantity C represents the resistance of the shear connector to deformation.

The geometrical conditions of elasticity require that

$$\Delta L'_i + \epsilon_i = \Delta L''_i + \epsilon_{i-1} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Substituting Eqs. 20 and 21 in Eq. 22,

$$\frac{L_i}{E_c}\left(-f_{cg} + \frac{\Delta f_c}{e_c} c_c\right) + \frac{S_{i+1} - S_i}{C_i} = \frac{L_i}{E_s}\left(f_{sg} - \frac{\Delta f_s}{e_s} c_s\right) + \frac{S_i - S_{i-1}}{C_{i-1}} \ldots (23)$$

In Eqs. 18 substitute

$$M_{0c} = \frac{\Delta f_c}{e_c} I_c \quad \text{and} \quad M_{0s} = \frac{\Delta f_s}{e_s} I_s \ldots\ldots\ldots\ldots (24)$$

The equation of equilibrium $M = M_c + M_s$ then becomes

$$M = \frac{\Delta f_c}{e_c} I_c + \frac{\Delta f_s}{e_s} I_s + S_i(c_c + c_s) \ldots\ldots\ldots\ldots (25)$$

In Eq. 25, replace $(c_c + c_s)$ by d, the distance between the two centroidal axes.

The elasticity condition of "equal deflection of both individual beams" may be replaced in the middle of the panel by the condition that the strain distribution lines for each beam have the same slope. According to Fig. 40,

$$\frac{\Delta f_c}{E_c\, e_c} = \frac{\Delta f_s}{E_s\, e_s} \qquad (26)$$

Substituting Eq. 26 in Eq. 25 and simplifying,

$$\frac{\Delta f_c}{E_c\, e_c} = \frac{M - S\, d}{E_c\, I_c + E_s\, I_s} E_c \qquad (27a)$$

and

$$\frac{\Delta f_s}{E_s\, e_s} = \frac{M - S\, d}{E_c\, I_c + E_s\, I_s} E_s \qquad (27b)$$

The total force S can be expressed as

$$S = f_{cg}\, A_c = f_{sg}\, A_s \qquad (28)$$

Furthermore, the following abbreviations can be introduced:

$$I_{n0} = n_c\, I_c + n_s\, I_s \qquad (29a)$$

$$A_n = 2\,\frac{n_c\, A_c \times n_s\, A_s}{n_c\, A_c + n_s\, A_s} \qquad (29b)$$

and

$$I_n = I_{n0} + A_n\,\frac{d^2}{2} \qquad (29c)$$

Eq. 27 is then substituted in Eq. 23, and the following fundamental equation of the problem is obtained:

$$-\frac{S_{i-1}}{C_{i-1}} + S_i\left(\frac{1}{C_{i-1}} + \frac{1}{C_i} + 2\,\frac{L_i\, I_n}{E\, I_{n0}\, A_n}\right) - \frac{S_{i+1}}{C_i} = \frac{L_i\, d}{E\, I_{n0}} M_{im} \qquad (30)$$

in which M_{im} is the average of the external moments over the distance between two shear connectors.

Eq. 30, set up as a system of equations, yields the unknown quantity S_i. The stresses in the middle of the panel can be computed using

$$f_{cg} = \frac{S}{A_c} \quad \text{and} \quad f_{sg} = \frac{S}{A_s} \qquad (31)$$

Thus,

$$\Delta f_c = \frac{M - S\, d}{I_{n0}} n_c\, e_c \quad \text{and} \quad \Delta f_s = \frac{M - S\, d}{I_{n0}} n_s\, e_s \qquad (32)$$

By taking into account the moment M_v resulting from the vertical shearing forces, the distribution of stresses over the entire composite beam can be determined.

The strain diagrams in Fig. 25 were evaluated by these theoretical considerations. First, the following values of $n = \frac{E_s}{E_c}$ were determined from the relative values of the strains at the centroidal axis: For beam T1, $n = 9.4$;

for beam T2, $n = 14.0$; and, for beam T3, $n = 12.5$. It is evident from these values that the compressive strength of the concrete slab for beam T2 was less than that for beam T1.

To determine the deformation resistance factor, C, for shear connectors, Eq. 30 was solved for various values of C. Assuming that the C-values for beams T1 and T2 must be equal (since the shear connectors were the same) and that the C-value for beam T3 must be smaller, and also based on test data from the Swiss Federal Testing Laboratory, the following C-values (in tons per centimeter) were obtained: For beams T1 and T2, $C = 400$; and, for beam T3, $C = 200$.

The stresses computed by the foregoing procedure are compared with those computed by the Symposium tests and with these computed by the classical theory of bending in Table 6. (The classical theory of bending does not consider the deformation of the shear connectors.)

TABLE 6.—COMPARISON OF STRESSES (KILOGRAMS PER SQUARE CENTIMETER = 14.23 LB PER SQ IN.) IN A COMPOSITE BEAM FOR A LOAD OF 5,200 LB AT MIDSPAN

Method	SECTION A, 3.75 IN. FROM MIDSPAN				SECTION B, 19 IN. FROM MIDSPAN			
	Upper Edge		Lower Edge		Upper Edge		Lower Edge	
	Stress	Diff.[a]	Stress	Diff.[a]	Stress	Diff.[a]	Stress	Diff.[a]
BEAM T1, FIG. 25 (n = 9.4 AND C = 400 TONS PER CM)								
Symposium tests	85.5	....	1,505	....	....	....	....	....
Stüssi method[b]	90.8	+ 6.2	1,489	− 1.1	....	....	....	....
Classical theory of bending	86.3	+ 0.9	1,421	− 5.6	....	....	....	....
BEAM T2, FIG. 25 (n = 14.0 AND C = 400 TONS PER CM)								
Symposium tests	82.6	....	1,690	....	32.2	....	627	....
Stüssi method[b]	78.0	− 5.6	1,606	− 5.0	30.5	−5.3	633	+ 1.0
Classical theory of bending	74.7	− 9.6	1,486	−12.1	29.1	−9.6	573	− 8.6
BEAM T3, FIG. 25 (n = 12.5 AND C = 200 TONS PER CM)								
Symposium tests	96.0	....	1,640	....	29.4	....	628	....
Stüssi method[b]	81.8	−14.8	1,596	− 2.7	31.4	+6.8	611	− 2.7
Classical theory of bending	73.0	−24.0	1,375	−16.2	30.4	+3.4	563	−10.4

[a] Percentage difference from values observed in the Symposium tests. [b] "Composed Beams," by F. Stüssi, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Switzerland, Vol. 8, 1947, p. 249.

It may be noted that the Stüssi method of analysis results in a stress distribution which approximates very closely the actual stress distribution obtained in the tests reported by the Symposium authors. Therefore, it offers a good basis for the further evaluation of tests and presents the civil engineer with the possibility of an improved method of calculation.

Acknowledgment.—The basic method of evaluating Professor Siess' laboratory results was developed by F. Stüssi, professor of steel construction at the Swiss Federal Institute of Technology.[34]

LOUIS BALOG,[31] ESQ.—In general, in the design of a bridge, it is not necessary to consider the framing effect of the slab alone. Actually, a great variety of efficient structures can be designed by disregarding the framing effect of the slab entirely and providing load distributing diaphragms. In this connection Professor Newmark states (Section 2*b*):

> "Since the slab acts as a very effective diaphragm, it is unnecessary to provide additional diaphragms, except for construction purposes, if the slab can perform its function of distributing loads to the beams at the same time that it provides a roadway for the wheels to roll over. Where it is expedient or desirable to make the slab thin, and therefore flexible, some additional transverse bridging is desirable. In general, however, such bridging is not particularly effective except for loads at or close to the section where the transverse frames are located."

The foregoing statement applies only to small bridges of the type investigated by the author. When properly designed, framed bridges are an advantage because diaphragms assure that the structure will behave as contemplated in the design for its entire life and, if desired, for the entire range of loading, up to capacity loads. The cost of efficient diaphragms is insignificant as compared with the great advantages that result from their use.

In addition to their definite and permanent action, diaphragms facilitate the application of new ideas in floor construction. High quality pre-cast floor units of low weight can be manufactured at an efficient plant with a strength and durability far superior to those attained in the field. Such units can be pre-stressed; they can be provided with special connecting elements; and they can be used as forms for the part of the slab that is cast in place. In long-span work, particularly, the wide range of possibilities for controlling the weight of the slab is of great significance. All these advantages are possible only by the use of diaphragms.

The writer questions, especially, the part of the foregoing quotation in which Professor Newmark states:

> "* * * such bridging [transverse bridging or diaphragms] is not particularly effective except for loads at or close to the section where the transverse frames are located."

Any rigid transverse connection between longitudinal girders (or trusses) will influence the deflections of the connected girders. An applied load will be distributed to all members of the framework in proportion to the magnitude of the deformation resulting from this load. Fig. 42 will serve to illustrate the writer's point of view as applied to a 60-ft span, the bridge having three girders connected by a center diaphragm. The flooring consists of a two-way slab with bays 10 ft square. The diaphragm causes the outside beams to

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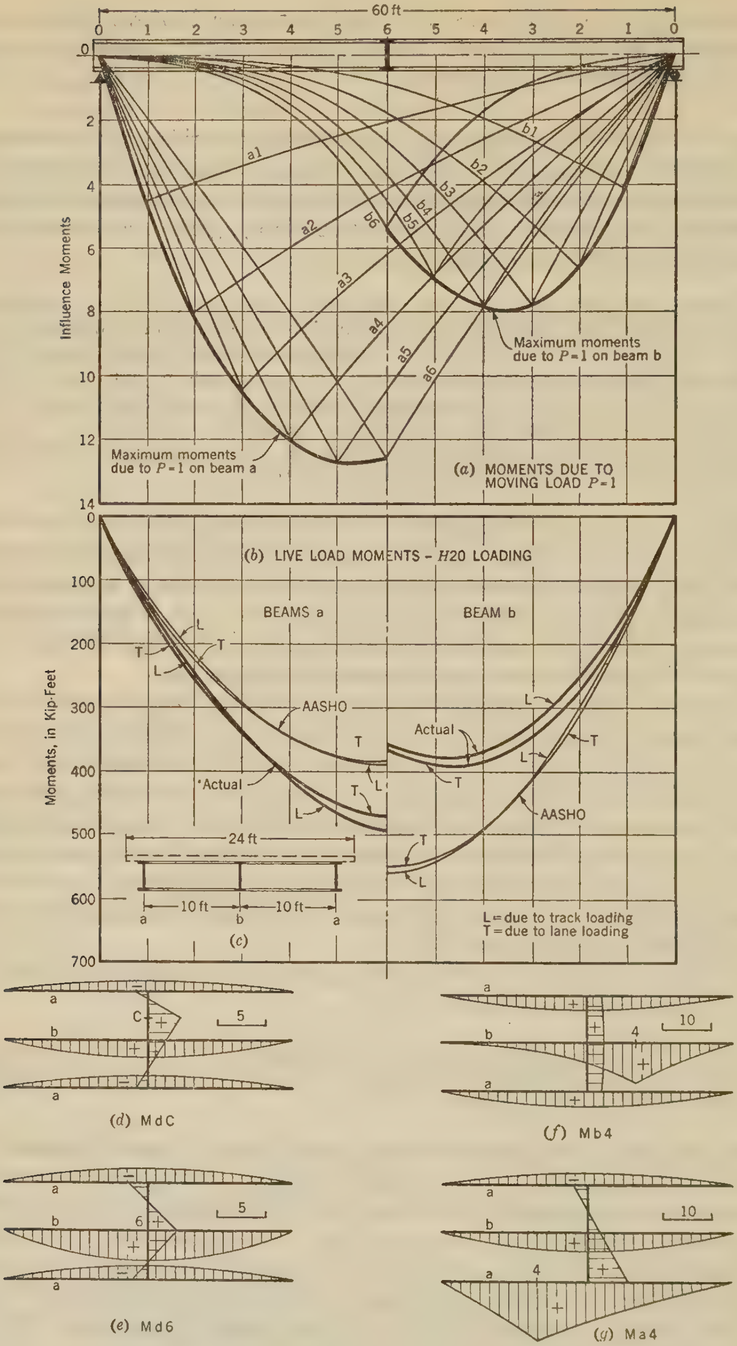

FIG. 42.—EFFECT OF A CENTER DIAPHRAGM ON THE MOMENTS IN A 60-FT SPAN

support the center beam at the point of connection, which means that the center beam acts as a continuous beam on an elastic center support when it is loaded. When an outside beam is loaded, the unloaded beams behave in a similar manner but to a lesser degree. A moving unit load on an unframed beam creates a maximum moment of 15, whereas the introduction of the diaphragm results in a comparable maximum moment of 8 in the center beam (beam b, Fig. 42), the point of maximum moment being shifted toward the quarter point of the span. If the moving unit load is on an outside beam (beam a, Fig. 42) it causes a maximum moment of 12.7 by the introduction of the diaphragm, and the point of maximum moment moves about only one twentieth of the span length from the center.

Evaluation of fourteen influence lines of this framing (four of which, MdC, Md6, Mb4, and Ma4 are shown) for H-20 track and lane loadings (see Fig. 42) demonstrates: (1) That the diaphragm is a very effective load distributor in the entire length of the bridge; and (2) that the maximum values of the moments in the outside beams are 23.2% and 25.8% larger, whereas in the center beam they are 47.9% and 57.4% smaller, than the American Association of State Highway Officials (AASHO) specifies.[18] The maximum stress in the diaphragm is 96% larger than that obtained according to the specification. Load distribution, such as that indicated in Fig. 42, is characteristic for bridges that have been built whether the girders are framed by diaphragm or are framed predominantly by floor slab. The latter is the case—for instance, in the Little Miami River bridge at Xenia, Ohio.[32] Evaluation of the deflection measurements[33] of this 5-beam, two-span continuous bridge, for which H is approximately 5 and b/a, approximately 0.1, reveals that the AASHO specifies 32.6% smaller loads and 38.8% and 50.9% larger loads for the outside and interior beams, respectively, than the actual values. In this case also, the deviations of the specifications from the actual are of the same order as for the bridge with a center diaphragm shown in Fig. 42. The recommendations of Professor Newmark (Section 6*b*, also Eqs. 3 and 4, and Table 2) deviate from 13.4% to 32.7% from the values measured at Xenia. They constitute, therefore, some improvement of the specifications, but they fail to approach reality to a satisfactory degree.

A case in point is the 1,068-ft-long, continuous plate-girder bridge over the Elbe River at Stillhorn-Neuland, Germany (1938). The spans are 193 ft, 341 ft, 341 ft, and 193 ft; the girder spacing is 20.34 ft; the floor beams, 5.3 ft deep, extend 1.5 ft above the girders and their triangular brackets extend to the bottom flanges of the girders. Measurements[34] of the load distribution among the four girders of this bridge disclosed behavior similar to that indicated by Fig. 42. Many other cases of this nature could be cited to support the writer's contention concerning the action of diaphragms. The Xenia measurements indicated that the slab acts as a diaphragm, which knowledge is not new. Comparative calculations show, however, that with a properly connected diaphragm of insignificant cost the same effect can be achieved reliably.

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[32] "Vibration Studies of Continuous Span Bridges," by L. E. Vandegrift, *Bulletin No. 119*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio, July, 1944.

[33] "Load Distribution Over Continuous Deck Type Bridge Floor Systems," by W. S. Hindman and L. E. Vandegrift, *Bulletin No. 133*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio, May, 1945.

[34] "Brücke über die Süderelbe im Zuge der RAB Hamburg-Hannover-Bremen," by Max Roloff, *Die Bautechnik*, Vol. 17, 1939, pp. 481–488 and 513–517.

Fig. 43 affords a comparison between three sets of values for the transverse distribution of a load at midspan of beam B, in a 5-beam bridge: A computed value assuming one diaphragm with $I_d/I_g = 0.2$ (subscript d denotes "diaphragm" and subscript g denotes "girder"); values reported by Professor Newmark (Figs. 14 and 15); and values measured at Xenia. The latter values and the computed effect of the center diaphragm are practically identical, whereas the Newmark values (especially the value of 0.428 P determined for beam B) are considerably different from the value of 0.309 P, as measured.

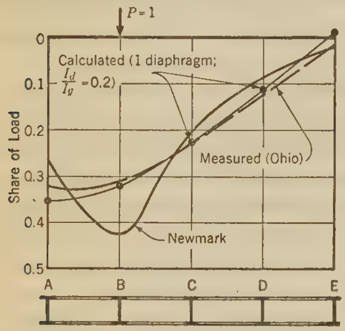

FIG. 43.—DISTRIBUTION OF LOAD $P = 1$ ON BEAM B IN A 5-BEAM BRIDGE

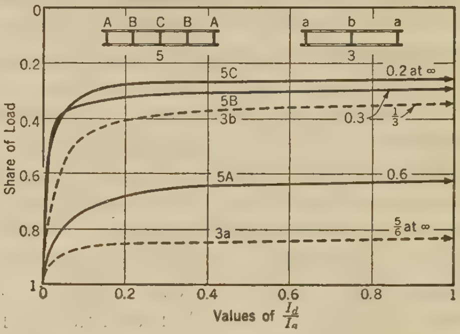

FIG. 44.—SHARE OF LOAD $P = 1$ ACTING ON THE BEAM FOR VARIOUS I_d/I_g RATIOS IN A 5-BEAM AND A 3-BEAM BRIDGE

Field measurements on other bridges revealed distribution lines similar in character to those obtained at Xenia, whereas measurements on a model of a bridge with an 125-ft span and five girders framed by four evenly spaced diaphragms, but without the deck slab[35] ($I_d/I_g = 0.21$ and $b/a = 0.129$), produced lines similar in character to those in Fig. 14. The effect of this change in the load-distribution diagram due to the action of four diaphragms as compared to that distribution produced by a center diaphragm was found to be small on the stresses in the connected girders, but not for the diaphragm stresses which were considerably reduced. It can be stated generally that the stresses in girders remain practically the same regardless of the number of load-distributing diaphragms greater than three, even in spans longer than 300 ft.

Fig. 44 shows the effect of the diaphragm and girder stiffness ratios on the distribution of load placed on one of the beams and carried by that beam. The number of beams are, respectively, five ($b/a = 0.1$) and three ($b/a = 1/6$). This diagram demonstrates that comparatively flexible but properly connected diaphragms are efficient load distributors between closely spaced beams. Furthermore, with an increase in the spacing of the beams the stiffness of the diaphragm gains in importance. In the last sentence of Section 2*b* Professor Newmark states:

> "Such a procedure [plastic theory of limit design] would be applicable and justified only when the slab or other transverse framing remained stiff enough at the higher loads to distribute the loads equally to all the beams."

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[35] "Anleitung für die vereinfachte Trägerrostberechnung," by Fritz Leonhardt, Wilhelm Ernst & Sohn, Berlin, 1940, p. 91.

Fig. 44 shows that this criterion for the safety of the application of limit design principles is impossible because, even by the use of infinitely rigid transverse framing, the loads on all the beams cannot be made equal. (The fractions along the right side of Fig. 44 are the values of I_d/I_g at infinity for each of the curves 3a to 5C.)

The use of rigid floor beams, however, is advantageous if such beams (diaphragms) are justified structurally. They unite the girders with the lateral systems into an efficient space structure. The deflections of such floor beams are negligibly small as compared with those of the girders they connect, and therefore the load distribution among the girders and the stresses in the floor beams can be determined from the vertical and torsional rigidity of the connected system by comparatively simple calculations. The 3,095-ft-long, 90.55-ft-wide "Arpád" bridge over the Danube River in Budapest, Hungary (1938–1947), is such a structure.[36] It contains three continuous girder units in which 197 ft and 338 ft are the shortest and longest spans, respectively. The four girders are spaced at 23.8 ft, 20.7 ft, and 23.8 ft. Floor beams of the Vierendeel type extend the full depth of the girders. It was found that the positive area of the load-distribution diagram of the outside girders is 24.15% larger than that of the interior girders; the outside girders carry 52.2% of a load uniformly distributed between the interior girders; and they carry 53.05% of the load on the two electric railway tracks located between the two interior girders. The extent to which the AASHO specifications are faulty, in preventing the outside girders from sharing the load placed between the two interior girders, is apparent from these data.

The specifications, and the opinions of Professor Newmark concerning the action of diaphragms, will also prevent the proper design of a system comprising widely spaced girders and widely spaced, load-distributing, rigid floor beams (diaphragms), between which grids consisting of shallow floor beams and stringers are provided. If such a system is well proportioned the economies that are attainable are quite considerable. By 7,800 individual measurements [37] on two tranverse sections of seven double-track railway truss bridges, ranging from a 138-ft span to a 620-ft span, it was found that such an insignificant transverse connection as a shallow top strut changed the distribution of the load on one track between two 66-ft-deep trusses 16%.

In his paper, Professor Siess reports the comparative behavior of small scale models of smooth flanged beams without, and with, special provisions for hindering the movement of a mortar slab at the top of these beams. In the "Summary" he cites savings in metal from 8% to 30% in favor of the latter arrangements. In appraising the practical value of this conclusion the behavior of bridges under their design loads cannot be disregarded. Measurements of the deflections of actual bridges having concrete decks resting on the tops of the girders, whether riveted or welded, were found to be from 30% to 65% smaller than those determined theoretically under the assumptions made during the design.[38] Considerable deviations between the calculated and actual deflections and the strains in bridges of this type were observed consistently, irrespective of the age of the structure. What happens under ideal laboratory conditions, therefore, does not indicate the actual behavior of bridges built without special shear connectors.

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[36] "Construction du Pont Arpád sur le Danube à Budapest," by Ch. Széchy, *L'Ossature Métallique*, October, 1947, p. 413.

[37] "Über die Verwindungssteifigkeit von zweigleisigen Eisenbahnfachwerkbrücken," by Rudolf Bernhard, *Der Stahlbau*, Vol. 3, pp. 85–92.

Measurements [39] on a 557.74-ft-long, two-span, continuous, through, plate-girder, bridge having a concrete deck on five roadway stringers and on two sidewalk stringers, and a lateral system at the bottom of the floor beams, 5 ft deep, disclosed that the strains in the extreme top and bottom fibers of the girders were from 9% to 21% smaller than calculated despite the fact that the loads were large enough to cause 3% remaining deflections. The reduction of the strains was evidently the result of the composite action of the floor, the laterals, and the girders. It is a significant fact that—even in this type of girder—the stress was 21% smaller than that stress which would be used in deciding the relative economy of this structure.

The economic comparisons by Professor Siess give undue advantage to a type of bridge over other types whose actual behavior he did not report. Therefore, in structures of different quality these comparisons depict girders with shear connectors in which the allowed stresses are reached at ordinary working conditions; and they depict others in which the stresses are much lower, permanently. Considering further the behavior of framed structures during times of overload, the comparative quality of bridges that rely entirely on the effectiveness of the connectors is also lower at the higher loads.

Two-way slabs are bonded into transverse elements that are framed rigidly into the main girders. The economy of this type of construction is enhanced by the triple action of the transverse beams which support and bond the slab and frame the girders. If the beam flanges are extended into the slab large rectangular rims or plugs resist sliding. Even if the bond could be destroyed completely, such a slab would not lose "its ability to transfer shear" (see Part C, Section 2). There are several structural arrangements by which effective composite action accrues as a by-product and in which additional resistance to slab sliding can be created by simple details. The significant fact is, however, that composite action of all the parts of bridges occurs in actual structures regardless of the conscious effort made by designers to guarantee it. In order to produce bridges of uniform strength and quality and to compare the economy of various types of bridges on a fair basis it should be recognized that comparison of the complete action of one type with the action of other types, without considering the true composite action of their floors, framings, and lateral systems, cannot yield consistent results.

On the basis of their observations, the Symposium authors declare transverse framing to be ineffective, and define a rule for load distribution. In the opinion of the writer, these observations appear too limited in scope to support general conclusions. Load distribution is a function of the geometric and stiffness characteristics of the girders and the framing, and therefore cannot be generalized or fixed by standard specifications. A bridge or a floor system can be laid out so as to be the most advantageous solution of a given problem by regulating the load distribution in the design. Such regulation can be accomplished by varying: (*a*) The geometry of the framing; (*b*) the stiffness ratio of the longitudinal and transverse elements; and (*c*) the stiffness ratio of the girders. Three properly designed intermediate floor beams or cross frames will distribute bridge loads practically as effectively as any larger number of frames. Therefore, floor beams or cross frames required for purposes other than load distribution need not be of the same section as the frames provided specifically for the purpose of load distribution.

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[38] "Composite Girder Bridges," by Louis Balog, *Engineering News-Record*, Vol. 140, 1948, p. 64.

[39] "Neubauten von Strassenbrücken," by Arthur Lämmlein, *Die Bautechnik*, Vol. 18, 1940, pp. 25–29.

The behavior of the actual structure is the integrated result of the composite action of all its parts. Research can help to advance design procedures by segregating, quantitatively, the component effects of the actual behavior of the structures, thus making possible the simple calculation of the true simultaneous action of such effects in any practical combination by which the designer wishes to achieve a desired structural performance.

N. M. NEWMARK[40] AND F. E. RICHART,[41] MEMBERS, ASCE, AND C. P. SIESS,[42] ASSOC. M. ASCE.—The entire discussion appears to have centered on the problems of design of I-beam bridges and of the behavior of composite beams. Several discussers have made important contributions to the knowledge of these problems. Their comments and suggestions are appreciated.

An interesting summary of his early work is given by Mr. Slack. He calls attention to the partial interaction shown by field measurements, as indicated by the fact that measured deflections were only from 50% to 75% of the "computed" deflections. Part of the discrepancy, however, can be ascribed to incorrect assumptions in the computation of deflection. The design specifications for highway bridges, even to the present time, do not take full account of the lateral distribution of the loading produced by the roadway slab. This deficiency is shown by the values of s in Table 2, compared with the corresponding value of 5 ft in the "Standard Specifications for Highway Bridges"[18] of the American Association of State Highway Officials. These factors are for truck loading or lane loading. The lateral distribution of a single concentration, sometimes used in tests, is even greater.

Nevertheless, good evidence supports the belief that in actual structures there is some partial interaction between the steel beams and the concrete slab, even if it is due to friction alone. Unfortunately, other evidence also indicates that such partial interaction can cease to exist, under unfavorable conditions, unless positive methods are provided to hold the two elements of the structure in contact. The data contributed by Mr. Slack agree with this view. For example, from Table 4, one can compute the increments in measured strains due to the application of the live load. The results are given in Table 7. The strains are determined by dividing the increments in stresses by the values of the modulus of elasticity given by Mr. Slack.

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[41] Research Prof., Eng. Materials, Univ. of Illinois, Urbana, Ill.

[42] Special Research Asst. Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

It is apparent from the data in Table 7 that in the composite beam the neutral axis is above the center at roughly the place where one would expect it, and that the slab acts with the beam as an additional top flange with a greater strain than the top flange of the beam. However, in the other beam the neutral axis is about in the middle of the depth, and the strains show evidence of considerable slip between the concrete and the steel.

TABLE 7.—MEASURED STRAINS FOR LIVE LOAD ONLY COMPUTED FROM TABLE 4

Strains, in microinches per inch	Composite beam; beam 2	Noncomposite beam; beam 4
Top of slab	−25	−16
Top flange of beam	−20	−51
Bottom flange of beam	+54	+49

Because of such experiences, the writers do not feel that bond alone can be relied on to insure composite action unless the entire upper flange and possibly part of the web of the beam are encased in the concrete of the slab.

It appears to be common sense and good engineering to accept composite action only as a bonus or as an added factor of safety unless some positive mechanical method of transferring shear is provided. The procedure would depend on circumstances and judgment. Many expedients are available and some have been described in the papers and in the discussions.

The writers concur with Mr. Slack that simple design methods are preferable, and that unusual refinements may be unwarranted in view of the uncertainties and the large number of variables that must be considered.

Mr. Wendell raises a number of questions of great practical importance. His comments on the economic factors involved in the design of composite bridges are particularly valuable. He points out that a considerable additional saving in weight is possible by using a thin web with stiffeners in place of the thick web, one eighty-seventh the depth, used in the studies reported in the third Symposium paper. However, such additional savings are possible only for beams deeper than those considered in that paper. The studies reported were limited in scope to a maximum simple-span length of 90 ft, a maximum beam spacing of 7 ft, and a maximum live loading of H-20. These limitations were necessary to obtain bridges in which rolled wide-flange beams could be used. Otherwise, the comparisons in Table 3 for the several types of beams could not have been made. The maximum over-all depth of welded beam considered was 42 in.; and, for depths not greater than this, the savings in weight from the use of a thin web with stiffeners will probably not be enough to offset the cost of additional welding. Actually, the built-up welded beam is not limited to this depth; and, for longer spans and consequently greater depths, the thin webs with stiffeners may be more economical, as has been indicated by Mr. Wendell.

Mr. Wendell expresses some skepticism regarding the possibilities of the steel companies rolling unsymmetrical beams, and he may well be right. It is of interest to note, however, that the studies made to determine the optimum proportions for such beams were undertaken only after a tentative and unofficial request for such information had been received from a steel company.

The effect of plastic flow on the concrete slab was mentioned by Messrs. Slack and Wendell. Mr. Slack raised the question of plastic flow of composite bridges built with temporary supports in which the dead-load moments are resisted by the composite section. The action of such a structure was discussed in the third Symposium paper, by simply assuming that the effects of plastic flow could be estimated by using a value of $n = 30$ rather than one of $n = 10$. This approach is purely analytical, of course; but it does not seem likely that much evidence concerning plastic flow can be obtained from laboratory tests on small-scale models. Observations on actual structures would be preferable in this case, but even these may not be too fruitful because of the complications introduced by deformations caused by shrinkage of the concrete, changes in ambient temperature, and radiant heating.

Mr. Wendell is not concerned about plastic flow in the slab. His opinion is no doubt associated with the fact that he does not approve of the use of temporary supports. Where only live load is carried by the composite section, plastic flow occurs only as the result of any dead load added after the structure is built; and Mr. Wendell does not think that this condition requires serious consideration. In this opinion he is in agreement with the opinions expressed by the writers.

The lack of real economy in the use of temporary supports or shores for composite bridges is cited by Mr. Wendell. He calls attention to the fact that the cost of providing temporary supports will offset, in general, any saving in weight in the structure. This reason is not the only one why the use of shores may be inadvisable. It was stated in the second Symposium paper (Section 4*c*) that a structure designed to be constructed with temporary supports will have a reduced ultimate strength and a lower usable capacity, and therefore the saving in weight is gained at the expense of a loss in the real factor of safety.

The contributions to the subject made by Mr. Wendell emphasize the fact that experience as well as analysis and tests must be used in approaching the solution of an engineering problem. One of the major reasons for the success of the program of study of highway bridge floors at the University of Illinois in Urbana has been the cooperation of highway bridge engineers and others who have contributed freely from their background of practical knowledge.

Two interesting studies have been presented by Mr. Tachau. It might be helpful to note that the relative beam stiffness H is dependent on the span length as well as on the type of construction. In general, the longer the span the greater is the value of H. If reasonable limits are set on H in Fig. 37, a roughly horizontal band, covering the range from about 85% to 95%, is found to represent the proposed design moments. This result is essentially the basis for the values reported in Table 2.

The selection of the proper beam spacing is an important point, which has been treated by Mr. Tachau. It should not be concluded from Fig. 38 that the most economical beam spacing is between 4 ft and 5 ft, however. It might have been more useful to compare the weights of the steel beams and of the concrete slab separately, as a function of beam spacing. Part of the trend obtained by Mr. Tachau is due to the fact that the slab, which is the heaviest part of the structure, increases in weight with the spacing of the beams, according to Mr. Tachau's assumptions. The weight of steel would probably be a minimum for somewhat wider spacings, and the minimum cost would occur at intermediate spacings. It is only fair to assume that the minimum cost lies somewhere in the range in spacing from 5 ft to 8 ft because most I-beam bridges are constructed in this range of dimensions.

An interesting attempt to account for the measured strain distribution in composite beams having small amounts of slip is presented by Mr. Furrer. He uses an analytical procedure developed by F. Stüssi, and applies it to the beam shown in Fig. 23 to explain the results indicated in Fig. 25. It is unfortunate, however, that the greatly condensed data given in the paper were not sufficient for his purpose. For example, the beam shown in Fig. 23 is typical only of beam T3; the shear connectors used in beams T1 and T2 were spaced at varying distances, ranging from 3¾ in. at midspan to 2 in. at the ends. Furthermore, the correct values of the modular ratio, *n*, were 7.3, 10.5, and 8.7 for beams T1, T2, and T3, respectively, as compared to the values of 9.4, 14.0, and 12.4 assumed by Mr. Furrer. His values of the shear connector stiffness, *C*, are likewise in error. As a result, the comparisons presented in Table 6 are not too reliable. In addition, the values given in Table 6 for the strains or stresses observed in the test and computed according to the classical theory for full composite action do not agree with those given in Fig. 25.

It was mentioned in Part B, Section 4, of the third Symposium paper that an analysis had been made of the behavior of a composite beam in which slip can take place. This analysis was developed by Professor Newmark and was described in an unpublished progress report in 1943. It is somewhat more approximate in its assumptions than is the method of Professor Stüssi. In it, the shear connection is assumed to be continuous along the length of the beam, and the resulting behavior of the beam is characterized by a differential equation for which a general solution was obtained in terms of an infinite trigonometric series. For certain specific cases the solution may be expressed in finite form. Comparative analyses have been made with both Professor Newmark's and Professor Stüssi's methods, and in both cases using the correct values of *n* and *C* for the three T-beams tested. The differences between the stresses computed by either method and those observed in the tests were usually less than the experimental errors of observation. In general, though, Professor Newmark's procedure seemed to give results in somewhat better agreement with the trends of the tests.

The foregoing discussion of methods of analysis should not obscure a much more important principle:

> If the shear connectors in an I-beam highway bridge are properly designed, the difference between the actual behavior of the structure and that computed for the assumption of full composite action will be so small that it need not be considered in design.

Consequently, little of a practical nature is to be learned from analyses of the type just discussed; and, for this reason, the analysis developed by Professor Newmark was never prepared for publication. The foregoing statements do not diminish in any way the importance of these analytical tools as applied to other types of composite structures.

The major part of the discussion by Mr. Balog does not appear to be pertinent to the papers. The second Symposium paper was specifically limited to recommendations for the design of I-beam highway bridges, consisting of a concrete roadway slab continuous over steel stringers in the direction of traffic. In the second paragraph of the paper, it was stated that:

> "The bridges considered, in general, are supported by four or more parallel beams of equal stiffness at a constant spacing of the beams between the limits of 5 ft and 8 ft."

In setting these limitations, Professor Newmark had in mind the necessity for fixing on a specific type of bridge if a design specification were to be possible. It was not intended that all highway bridges should be constructed in this fashion or that they should be designed by the recommendations if they fell outside these specific limitations. Other types of framing may certainly be used, and possibly under certain conditions may be more economical. The I-beam bridge has certain advantages, however, that make it now one of the most widely used types of highway bridge in the United States. It seemed, therefore, worthwhile to study this type of bridge in great detail.

It is impossible to consider the Stillhorn-Neuland (Germany) bridge with 20.34-ft girder spacing, or the "Arpád" bridge (over the Danube River in Budapest, Hungary) with comparable spacing, as an I-beam bridge. These bridges are so far from the proportions considered that there is no point in applying to them the results of the Illinois tests and analyses, even if specific limitations excluding such bridges had not been set in the paper. The writers are entirely at a loss to understand Mr. Balog's purpose in discussing such additional extraneous systems as

> "* * * widely spaced girders and widely spaced, load-distributing, rigid floor beams (diaphragms), between which grids consisting of shallow floor beams and stringers are provided."

Of course the recommendations stated in the Symposium papers do not apply; they were never intended to apply to anything but the simple ordinary I-beam bridge which Mr. Balog apparently considers only as a trivial structure.

Since Mr. Balog agrees that Professor Newmark's statement (which he quotes in his first paragraph), regarding diaphragms in I-beam bridges, "* * * applies only to small bridges of the type investigated by the author," there seems to be no point in discussing the matter of transverse framing in general, since such a discussion is outside the stated purpose of the Symposium, and would warrant a separate paper or a series of papers.

However, in his presentation of the data for the Xenia (Ohio) bridge, Fig. 43, Mr. Balog makes an unfortunate and serious error to which attention must be called. The error results from confusion regarding the so-called "distribution" of a concentrated load on a bridge. This matter is discussed in some detail in Sections 2*b*, 2*d*, and 2*e* of the second Symposium paper. In Fig. 18, there are shown the different types of "distributions" that one obtains from deflections and from moments in the beams of an I-beam bridge. It is entirely unsafe to use the "distribution" computed from deflections in determining stresses or moments.

In the Xenia bridge tests to which Mr. Balog refers, only deflections of the beams were measured. These measured deflections are actually in good agreement with computed deflections for this bridge. However, the moments in the beams are not proportional to the deflections of the beams. The difference is virtually that shown in Fig. 43 between the measured (deflection) values, and the curve marked "Newmark" which was obtained from Fig. 15, and which applies to the moments.

TABLE 8.—RELATIVE PROPORTIONS OF LOAD ON BEAM B OF XENIA (OHIO) BRIDGE TO ACCOUNT FOR DEFLECTIONS AND MOMENTS PRODUCED BY A CONCENTRATED LOAD

Quantity	PROPORTION OF LOAD DETERMINED FROM	
	Computations	Measurements
Moment......	0.428	....
Deflection......	0.339	0.309

To illustrate the difference more clearly, in Table 8 a summary is given of the relative proportions of the load applied to beam B, in Fig. 43, required to account for: (1) The computed moments; (2) the computed deflections; and (3) the measured deflections. The last value is taken from Mr. Balog's discussion. The first may be obtained from Fig. 15, and the second value, for $b/a = 0.1$, $H = 5$, may be determined from a report published in 1942.[43]

It can be noted from Table 8 that the agreement between the proportions of load carried by the loaded beam, on the basis of computed and measured deflections, is quite good. The small discrepancy can be attributed to the fact that the exact proportions of the structure were not used in the calculations. However, for the same structure, the computations indicate a 26% greater load to account for the moment in the beam than is required to account for the deflection.

As a matter of fact, if one makes proper allowance for the difference in results obtained from deflection measurement rather than from strain measurement, the Xenia bridge tests verify rather than contradict the studies and recommendations given in the second Symposium paper.

In his discussion of the Xenia tests, Mr. Balog is quite correct in stating that the measurements indicate that the slab acts as a diaphragm. He is not correct in stating that one properly connected diaphragm can achieve the same effect. However, several properly connected diaphragms can achieve the same effect—at least three, and possibly more, are required to replace a slab so as to keep the maximum moment, for any position of the loads (on or off the diaphragm), at the same value. However, since the slab has to be there, why put the other diaphragms in? There is no need to do so except (see Section 2*b*) "Where it is expedient or desirable to make the slab thin, and therefore flexible * * *."

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[43] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment station, Urbana, Ill., 1942, p. 121, Table 100.

However, it should not be inferred that the use of diaphragms is to be discouraged. They can help somewhat in any case, even when they are not necessary; and, certainly if pre-cast floor systems or steel grid floors are to be used, transverse framing is necessary to make the structure act as an integral unit. The writers are glad that Mr. Balog called attention to this point.

Since the paper was written, some additional data have become available regarding the action of channel shear connectors. A full-size composite I-beam has been tested, which was designed as a composite beam of 37.5-ft span, with one H-20 wheel load at midspan, including 30% impact. It was designed for 6/5.3 wheel loads. The steel beam was a 24-in. WF 76-lb-per-ft beam and the slab was 6 in. thick and 6 ft wide.

Shear connectors were 6-in. lengths of 4-in., 5.4-lb channels, uniformly spaced throughout the length of the span at 18 in. The top flange of the beam was covered with a heavy coat of white lead paint before pouring the slab to insure that there would be no bond between the slab and beam. This coating seemed to be very effective, because slip between the slab and beam was measured from the start of tests. The sharp break, that is usually found when adhesive bond is broken, was not observed.

Strains were measured at five points on several shear connectors with type A-7, SR-4 electrical strain gages. Four gages were placed on the back of the connector and one in the fillet. The gages were waterproofed with an asphaltic compound ("Petrolastic").

In the test to failure, the highest load reached was 111,000 lb or 4.7 live loads. No indication of shear connector failure was observed other than a slight yielding in the fillet of the connector. The shear connectors were continuing to take load, and there was no break in the total load-slip curves at this load. At the maximum load there was absolutely no indication of inadequate shear connection except a slight amount of slip.

TABLE 9.—MAXIMUM STRESS IN SHEAR CONNECTOR COMPUTED BY VARIOUS METHODS

Load on connector (lb)	Computed stress (lb per sq in.)	Method of computation
31,300	32,400	Based on measured strains.
	61,000	Computed by Eq. 15*b*.
	144,000	Computed on the basis of linear pressure distribution on the back of the channel, from zero at the top to a maximum at the bottom.

A comparison of the measured strains with various methods of computing stress in the connectors is given in Table 9. The comparison is made at a load of 80,000 lb, or 3.4 live loads, at midspan. This point of comparison was chosen because it represents a load at which the beam was just beginning to yield in the lower flange at midspan. The connector load X was computed by the usual formula, $X = \frac{V\,Q}{I}$ times connector spacing. The connector selected was one near a support and the one in which the measured strain was largest.

Obviously, the last method of computing stress in the connector is far too wasteful and conservative, when used as a basis for design. Even the formula in the third paper, Eq. 15*b*, is extremely conservative. Full composite action can be attained with even less adequate shear devices when the natural bond of the concrete with the steel is not destroyed. In any case the design procedure given in the third Symposium paper is amply conservative for channel shear connectors.

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Paper No. 2382

A CENTROIDAL METHOD OF RIGID-FRAME ANALYSIS

By S. J. Bell,[1] Assoc. M. ASCE

With Discussion by Messrs. L. E. Grinter, Alexis L. Gluckmann, Yi-Mai Yao, Phil M. Ferguson, Leroy A. Beaufoy, William A. Conwell, and S. J. Bell.

Synopsis

The object of this paper is to present a method for the analysis of rigid frames. Rotations and translations of the ends are represented by a resultant vector at the centroid. By use of this vector and the circle of inertia the amount and direction of the thrust are found. The distance from the centroid to the line of action of the thrust is shown to be a function of the relative rotation of the ends. In a structure with intermediate loads which has been made determinate by release of joints the rotation and the displacement at the centroid determine the thrust required to return it to its original indeterminate position. The moments in the structure are the algebraic sum of the determinate and indeterminate moments. A method of finding the deflection and rotation of any point on the structure is shown. The method is developed analytically and illustrative examples are solved graphically.

1. Introduction

The method of analysis presented in this paper is the result of an effort to shorten the time required for the determination of stresses in vapor lines connecting petroleum refinery fractionating towers to stationary heat exchangers. Application of Castigliano's theorem to the mathematical equations of the shape of the pipe led to the development of an analytical method of solution based on displacements along the principal axes. The solution presented herein is a method of finding the thrust directly from the resultant of these component displacements by use of the circle of inertia.

Some of the problems in which this method will be found useful are the analysis of arches, rigid-frame bridges, pipe expansion bends, rings, box cul-

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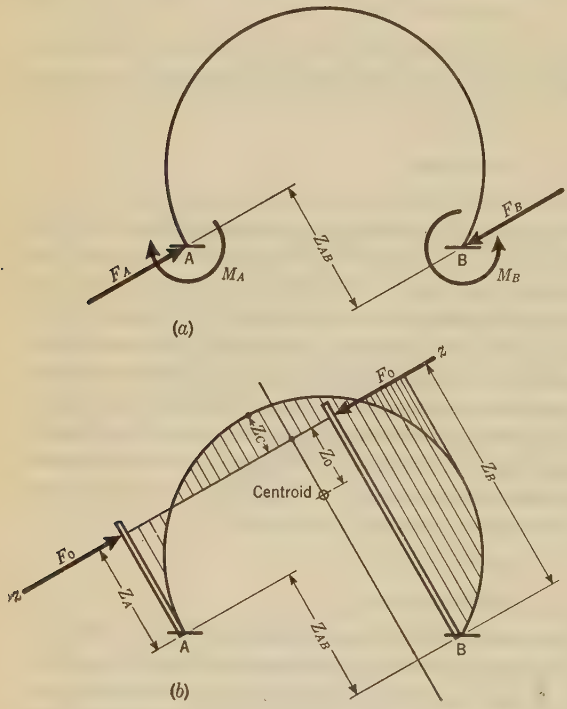
Note.—Published in November, 1947, *Proceedings.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Refinery Supt., Am. Republics Corp., Houston, Tex.

verts, and other similar structures. It will also be found useful in determining influence lines, secondary stresses, and moment-distribution factors. It is applicable to any planar structure which is subjected to bending stresses and which has no restraint against free flexure except at the two ends. The examples given are graphical illustrations of the application of the method and are not exact solutions.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

2. Moments Produced by End Movements



(a) (b)

FIG. 1

The displacement of one end of a frame with respect to the other induces a state of internal stress which resists the movement. External forces must be applied to the two ends to maintain equilibrium. In Fig. 1, the forces, F_A and F_B, and the moments, M_A and M_B, are shown applied to ends A and B, which have been displaced with respect to each other. In order to fulfil the laws of statics, F_A must be equal to, parallel to, and opposite in direction to, F_B, and the moment at B must be equal to and opposite in direction to the sum of the moment at A and the moment of F_A about B. By designating the equal forces by F_o and letting them act along a parallel line z–z at a distance Z_A from end A, so that

$$Z_A = \frac{M_A}{F_o} \qquad \text{.....(1)}$$

the moment at any point C on the member is given by

$$M_C = F_o Z_C \qquad \text{.......................(2)}$$

The internal work in the member due to this displacement is

$$W = \frac{1}{2}\int_A^B \frac{M^2\,ds}{E I} \qquad \text{.......................(3)}$$

in which M is the moment; ds is an element of length; E is the modulus of elasticity; and I is the moment of inertia of the cross section. If ϕ is the angular displacement of end A with reference to end B, by Castigliano's theorem,

$$\phi = \frac{\partial W}{\partial M} = \frac{1}{E\,I}\int_A^B M\,ds \dots\dots (4)$$

Substituting Eq. 2 in Eq. 4,

$$\phi = \frac{F_o}{E\,I}\int_A^B Z\,ds = \frac{F_o\,Z_o\,L_o}{E\,I} \dots\dots (5)$$

or

$$Z_o = \frac{\phi\,E\,I}{F_o\,L_o} \dots\dots (6)$$

in which Z_o is the distance from the line of action of F_o to the centroid and L_o is the length of the member.

For $F_o = 0$, that is, where equilibrium is maintained by equal and opposite moments applied at points A and B,

$$\phi = \frac{1}{E\,I}\int_A^B M\,ds = \frac{M\,L_o}{E\,I} \dots (7)$$

the moment being constant at all points. In this case $Z_o = \infty$.

The expression for internal work may be written

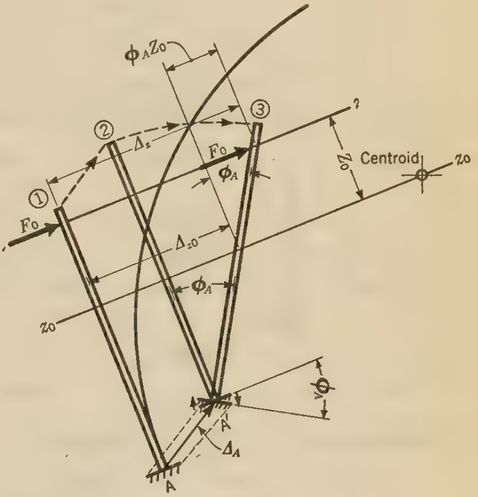


FIG. 2

$$W = \frac{1}{2}\int_A^B \frac{(F_o\,Z)^2\,ds}{E\,I} = \frac{F^2_o}{2\,E\,I}\int_A^B Z^2\,ds \dots\dots (8)$$

However,

$$\int_A^B Z^2\,ds = I_z = I_{zo} + Z^2_o\,L_o \dots\dots (9)$$

in which I_z is the moment of inertia of the member about line z–z and I_{zo} is the moment of inertia about an axis through the centroid parallel to z–z.

Therefore, the internal work is

$$W = \frac{F^2_o}{2\,E\,I}(I_{zo} + Z^2_o\,L_o) \dots\dots (10)$$

The distance through which the force F_o moves may be found, by Castigliano's theorem, to be

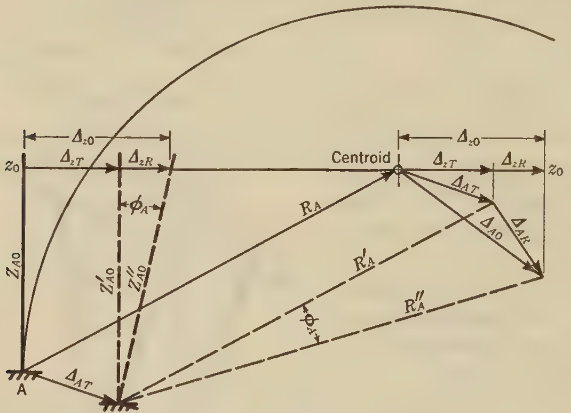
$$\Delta_{ZFo} = \frac{\partial W}{\partial F_o} = \frac{F_o}{E\,I}(I_{zo} + Z^2_o\,L_o) \dots\dots (11)$$

Substitution of Eq. 5 in Eq. 11 gives

$$\Delta_{ZFo} = \frac{F_o\,I_{zo}}{E\,I} + \phi\,Z_o \dots\dots (12)$$

In Fig. 2, end A is represented as being displaced through a linear distance, Δ_A, and an angle, ϕ_A with respect to end B. Force F_o acts through the distance Δ_{ZF_o} along line z–z against an imaginary lever arm, which has an intercept Δ_{zo} on the centroidal axis parallel to z–z. The following relationship is seen to be true:

$$\Delta_{ZF_o} = \Delta_{zo} + \phi_A Z_o \qquad (13)$$

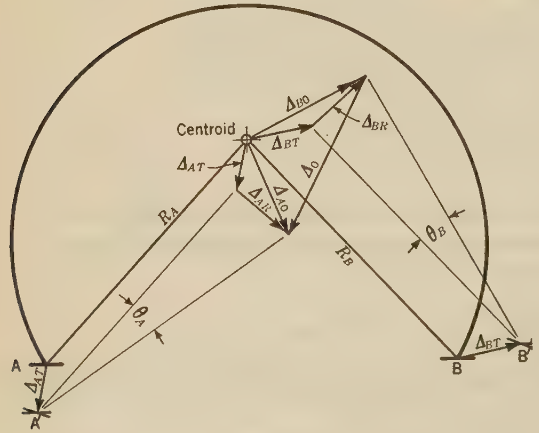


Δ_{zo}
Δ_{zT}
Δ_{zR}
z_0
ϕ_A
Z_{AO}
Z'_{AO}
Z''_{AO}
Centroid
Δ_{zo}
Δ_{zT}
Δ_{zR}
z_0
Δ_{AT}
Δ_{AO}
Δ_{AR}
R_A
R'_A
R''_A
ϕ_A
Δ_{AT}
A
A'

FIG. 3

Substitution of Eq. 13 in Eq. 12 gives

$$\frac{F_o I_{zo}}{E I} = \Delta_{zo} \qquad (14a)$$



Centroid
Δ_{BO}
Δ_{BR}
Δ_{BT}
Δ_{AT}
Δ_O
Δ_{AR}
Δ_{AO}
R_A
R_B
θ_B
θ_A
A
Δ_{AT}
A'
B
Δ_{BT}
B'

FIG. 4

from which

$$F_o = \frac{\Delta_{zo} E I}{I_{zo}} \qquad (14b)$$

In Eq. 14b, the force F_o is a function of the movement Δ_{zo} (of the imaginary lever arm) along the centroidal axis parallel to the force, as well as of the moment of inertia about this axis. It is independent of the manner of translation and rotation of end A that causes this intercept Δ_{zo}.

In Fig. 3 it is shown that the imaginary lever arm, R_A, may be assumed to extend from end A to the centroid, and the distance Δ_{zo} in Eq. 14b is the projection of the movement, Δ_{Ao}, of the end of R_A on the centroidal axis parallel to F_o. This holds true for both ends. If R_A and R_B are lever arms from points A and B to the centroid, both rotation and translation may be shown as displacements of R_A and R_B away from the centroid. The relative displacement, Δ_o, of end A with respect to end B is the line drawn from the end of R_B to the end of R_A as shown in Fig. 4. The projection of Δ_o on the line of action of F_o is equal to Δ_{zo} as given in Eq. 14*b*. The projections of Δ_o on the principal centroidal axes (x–x and y–y) give components Δ_{xo} and Δ_{yo}, from which F_x and F_y, the components of F_o parallel to the principal axes may be found by

$$F_x = \frac{\Delta_{xo} E I}{I_x} \ldots\ldots (15a)$$

$$F_y = \frac{\Delta_{yo} E I}{I_y} \ldots\ldots (15b)$$

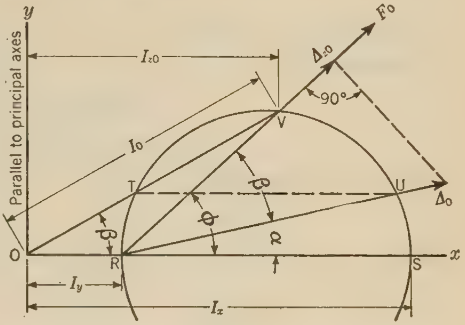


FIG. 5

Knowing the length and direction of Δ_o, the direction and value of the force F_o may be found by use of Mohr's circle as shown in Fig. 5. The construction is as follows:

(1) Draw Δ_o through point R (where OR is equal to I_y if parallel to principal axis x–x and to I_x if parallel to principal axis y–y), cutting the circle at point U.
(2) Through point U draw a line parallel to OR cutting the circle at point T.
(3) Draw a line through points O and T cutting the circle at point V.
(4) A line drawn through points R and V is parallel to F_o.

This may be shown to be true because the slope, ϕ, of F_o with respect to the x–x axis is

$$\tan \phi = \frac{F_y}{F_x} = \frac{\dfrac{\Delta_y E I}{I_y}}{\dfrac{\Delta_x E I}{I_x}} = \frac{\Delta_y I_x}{\Delta_x I_y} \ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

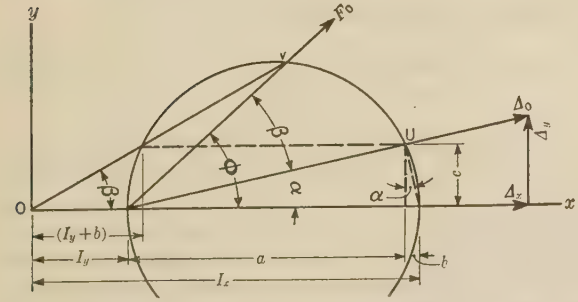


FIG. 6

which, from Fig. 6, will be seen to be

$$\tan \phi = \frac{c}{a}\frac{I_x}{I_y} \ldots (16b)$$

The slope, α, of Δ_o with respect to the axis x–x is

$$\tan \alpha = \frac{\Delta_y}{\Delta_x} = \frac{c}{a} \ldots (17)$$

Furthermore, the angle of obliquity, β, of the force F_o with respect to the movement Δ_o is

$$\beta = \phi - \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and may be found by

$$\tan \beta = \tan (\phi - \alpha) = \frac{\tan \phi - \tan \alpha}{1 + \tan \phi \tan \alpha} \quad \text{(18b)}$$

By substitution of Eqs. 16*b* and 17, as well as linear values from Fig. 6, Eq. 18*b* becomes

$$\tan \beta = \frac{c(a + b)}{a\, I_y + \frac{c^2}{a} I_x} \quad \text{(18c)}$$

However, by construction—

$$I_x = I_y + a + b \quad \text{(18d)}$$

and

$$\frac{c^2}{a} = b \quad \text{(18e)}$$

Substitution of Eqs. 18*d* and 18*e* in Eq. 18*c* gives

$$\tan \beta = \frac{c}{I_y + b} \quad \text{(18f)}$$

In Fig. 6, $\frac{c}{I_y + b}$ is the tangent of the angle at the origin, which in turn is equal to the angle of obliquity of force F_o with respect to Δ_o, both of these angles being measured by one half the arc VU.

The value of F_o may be found by use of Eq. 14*b*, Δ_{zo} and I_{zo} being found as shown in Fig. 5. However, the value of F_o may be found without finding Δ_{zo}, the component of Δ_o along the line of action of the force, by

$$F_o = \frac{\Delta_o\, E\, I}{I_o} \quad \text{(19)}$$

in which I_o is a "relative" value of the moment of inertia to be used with the value of the centroidal displacement Δ_o. The value of I_o may be found as shown in Fig. 5.

Eq. 19 may be obtained from Eq. 14*b* by substituting

$$\cos \beta = \frac{\Delta_{zo}}{\Delta_o} = \frac{I_{zo}}{I_o} \quad \text{(20a)}$$

or

$$\frac{\Delta_o}{I_o} = \frac{\Delta_{zo}}{I_{zo}} \quad \text{(20b)}$$

It should be noted that the equations given in this paper are for structures with constant values of E and I. If either is variable, it must be used in locating the centroid and in finding the moments of inertia. In the frame shown in Fig. 7 the sectional moment of inertia is variable and the structural length must be written

$$L_o = \int \frac{ds}{I} \quad \text{(21a)}$$

—its dimension being L^{-3}. In like manner, the structural moment of inertia I_z must be written

$$I_z = \int Z^2 \frac{ds}{I} \qquad (21b)$$

and its dimension is L^{-1}.

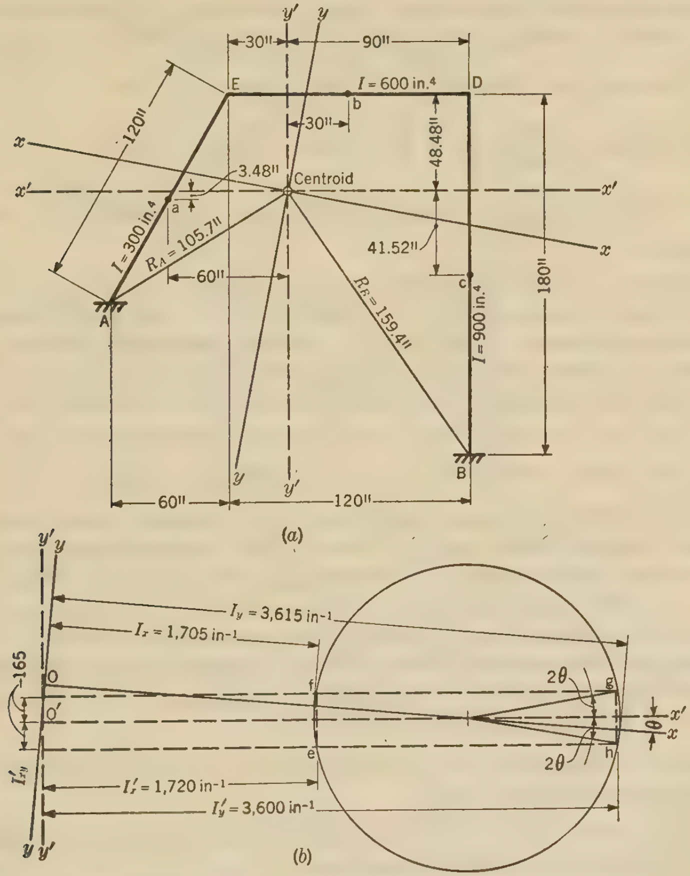

y'
y
30"
90"
E
I = 600 in.⁴
D
b
120"
x
30"
48.48"
3.48"
Centroid
x'
x'
a
I = 300 in.⁴
R_A = 105.7"
41.52"
x
60"
c
180"
R_B = 159.4"
I = 900 in.⁴
A
B
y
y'
60"
120"
(a)
y' y
I_y = 3,615 in⁻¹
I_x = 1,705 in⁻¹
-165
O
f
2θ
g
O'
x'
θ
x
e
2θ
h
I'_xy
I'_x = 1,720 in⁻¹
I'_y = 3,600 in⁻¹
y y'
(b)

FIG. 7

In using these values in the equations given, I must be omitted since it has been taken into account. For example, Eqs. 5, 10, and 19 become, respectively,

$$Z_o = \frac{\phi E}{F_o L_o} \qquad (21c)$$

$$W = \frac{F^2_o}{2E}(I_{zo} + Z^2_o L_o) \qquad (21d)$$

and

$$F_o = \frac{\Delta_o E}{I_o} \qquad (21e)$$

Example 1.—Locate the centroid and the principal axes, and find the principal moments of inertia in the frame shown in Fig. 7. For all members, $E = 30 \times 10^6$. (This example is not an essential part of the method. Its purpose is to find the principal axes and principal moments of inertia to be used in the examples which are to follow.)

(*a*) Location of the Centroid.—Taking moments about the right leg of the bent, $\bar{x} = 90$ in. Taking moments about the top member, $\bar{y} = 48.48$ in.

(*b*) Moments and Products of Inertia About x'–x' and y'–y'.—$I'_x = \frac{1}{12} \times (120 \times 0.866)^2 \times \frac{120}{300} + 0 + \frac{1}{12} \times 180^2 \times \frac{180}{900} + 3.48^2 \times \frac{120}{300} + 48.48^2 \times \frac{120}{600} + 41.52^2 \times \frac{180}{900} = 1{,}720$ in.$^{-1}$ Similarly, $I'_y = 3{,}600$ in.$^{-1}$, and $I'_{xy} = -165$ in.$^{-1}$

(*c*) Location of Principal Axes.—Lay off I'_x and I'_y on a line parallel to one of the axes as shown in Fig. 7. Locate points e, f, g, and h above and below this line a distance equal to I'_{xy}. A circle passed through these points is the circle of inertia. Radii drawn from the center of the circle to points g and h each make an angle equal to $2\,\theta$ with the line. A principal axis bisects one of these two angles, the direction of rotation from the base line to this principal axis being determined by the sign of the criterion, $\frac{I'_{xy}}{I'_x - I'_y}$. A positive sign indicates clockwise rotation and a negative sign, counterclockwise.

By this construction the angle of rotation, θ, is always less than 45°, that is, the axes are always rotated to the nearest principal axes. There is no interchange of the x-axis and y-axis—for example, if I'_y is larger than I'_x, I_y will be the maximum moment of inertia and I_x will be the minimum.

(*d*) Principal Moments of Inertia.—The principal moments of inertia are measured along the base line to the points cut by the circle of inertia. In this example, $I_x = 1{,}705$ in.$^{-1}$ and $I_y = 3{,}615$ in.$^{-1}$

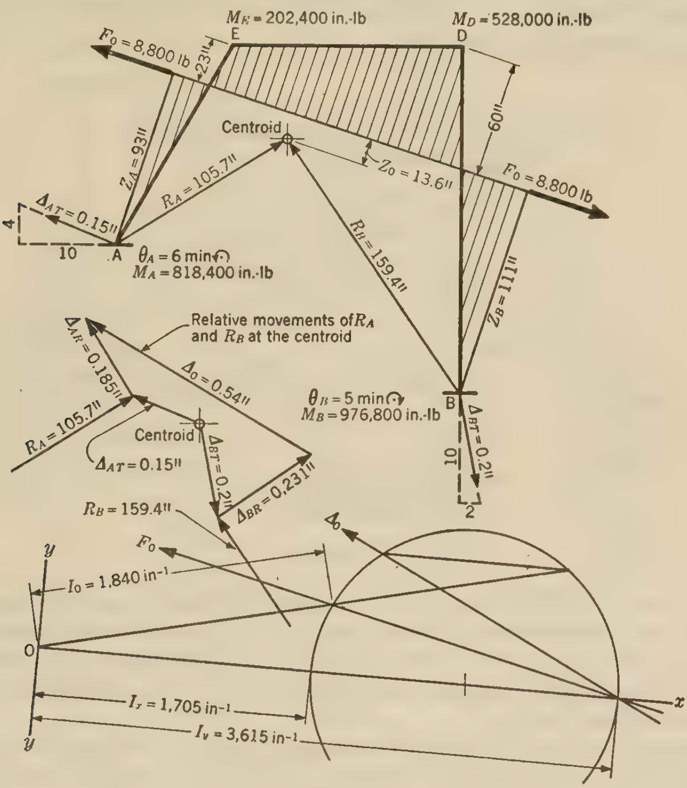
Example 2.—Determine the moments produced in the bent shown in Fig. 7 by the following end movements:

1. End A moves 0.15 in. to the left and up, the direction of movement making the angle $\tan^{-1} 0.4$ with the horizontal.
2. End A rotates counterclockwise through an angle of 6 min.
3. End B moves 0.2 in. down and to the right, making the angle $\tan^{-1} 0.2$ with the vertical.
4. End B rotates clockwise through an angle of 5 min.

(*a*) Direction and Value of Δ_o.—In Fig. 8, using any convenient scale, lay off the linear movements, $\Delta_{AT} = 0.15$ in. and $\Delta_{BT} = 0.2$ in., in their true directions. From the end of Δ_{AT} draw $\Delta_{AR} = R_A\,\theta_A$ perpendicular to R_A and in the direction of the movement of the end of R_A caused by the rotation θ_A. In like manner, draw Δ_{BR} from the end of Δ_{BT}. Vector Δ_o is drawn from the end of Δ_{BR} to the end of Δ_{AR}. Then, since $\theta_A = 6$ min $= 0.00175$ radian, $\Delta_{AR} = 105.7 \times 0.00175 = 0.185$ in., and since $\theta_B = 5$ min $= 0.00145$ radian,

$\Delta_{BR} = 159.4 \times 0.00145 = 0.231$ in. The value of Δ_o is 0.54 in. and the direction is as shown.

(*b*) Line of Action, Direction, and Value of F_o.—Since the base line of the circle of inertia is parallel to the axis x–x, draw a line parallel to Δ_o through the point where the circle cuts the base line corresponding to I_y. Locate F_o and I_o on the inertia diagram as shown. Then, by Eq. 21*e*, $F_o = \dfrac{0.54 \times 30 \times 10^6}{1{,}840}$ $= 8{,}800$ lb. The distance of the line of action of F_o from the centroid is found by Eq. 5, in which $\phi = (\theta_A - \theta_B) = -0.00175 - 0.00145 = -0.0032$ radian; therefore, $Z_o = -\dfrac{0.0032 \times 30 \times 10^6}{8{,}800 \times 0.8} = -13.64$ in. The negative sign indicates that the moment $F_o Z_o$ of the force acting at the left tends to cause counterclockwise rotation about the centroid. The line of action of F_o serves as a base line for a moment diagram as shown.



M_E = 202,400 in.-lb
E
M_D = 528,000 in.-lb
D
F_o = 8,800 lb
23"
60"
Centroid
Z_A = 93"
R_A = 105.7"
Z_o = 13.6"
F_o = 8,800 lb
Δ_{AT} = 0.15"
4
10
A
θ_A = 6 min
M_A = 818,400 in.-lb
R_B = 159.4"
Z_B = 111"
Relative movements of R_A and R_B at the centroid
Δ_{AR} = 0.185"
Δ_o = 0.54"
θ_B = 5 min
M_B = 976,800 in.-lb
B
Δ_{BT} = 0.2"
10
2
R_A = 105.7"
Centroid
Δ_{AT} = 0.15"
Δ_{BT} = 0.2"
Δ_{BR} = 0.231"
R_B = 159.4"
Δ_o
F_o
y
I_o = 1,840 in^{-1}
O
I_x = 1,705 in^{-1}
I_y = 3,615 in^{-1}
y
x

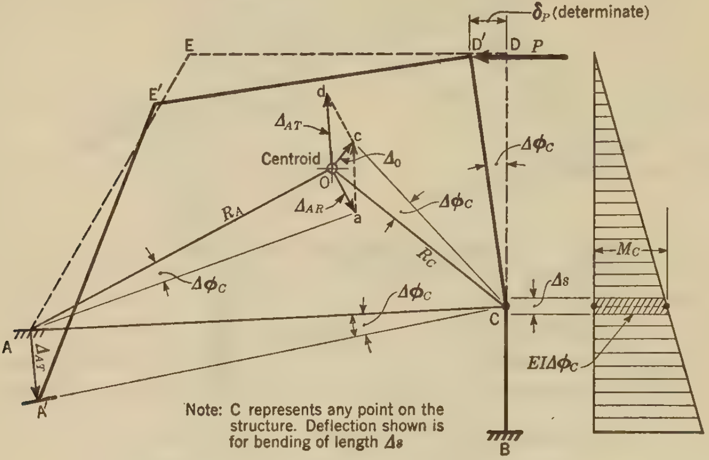
FIG. 8

3. APPLICATION TO LOADED STRUCTURES

The loaded bent in Fig. 9 is made determinate by releasing end A, leaving end B fixed. The angular change between the ends of any element Δs (as at C) is

$$\Delta\phi_C = \frac{M_C\,\Delta s}{E\,I} \qquad (22)$$

in which M_C is the determinate moment at C. This causes end A to move to A′, a linear distance $\Delta_{AT} = r\,\Delta\phi_C$ and to rotate through the angle $\Delta\phi_C$ with reference to its original position.



Note: C represents any point on the structure. Deflection shown is for bending of length Δs

FIG. 9

In order to return this end to its original position it must be moved from A′ to A and rotated through angle $\Delta\phi_C$ in an opposite direction to the first rotation. The movement of the end of R_A due to this translation and rotation is shown by vectors, Δ_{AT} and $\Delta_{AR} = R_A\,\Delta\phi_C$, at the centroid, the resultant of these vectors being Δ_o.

Triangle Oac formed by these vectors is similar to triangle OAC, Oa and ac being perpendicular to, and equal to, $\Delta\phi_C$ times the sides OA and AC, respectively. For this reason Oc is perpendicular to OC and equal to $\Delta\phi_C$ times OC; that is,

$$\Delta_o = R_C\,\Delta\phi_C \qquad (23)$$

Thus, the resultant displacement, Δ_o, of the end of the lever arm, R_A, caused by returning end A to its original position, is equal in value and opposite in direction to the displacement of lever arm, R_C, away from the centroid due to the rotation of R_C about point C through the angle $\Delta\phi_C$.

The center of gravity of all the rotations or angular changes is the center of rotation. The total angular change, $\Sigma\,\frac{M\,\Delta s}{E\,I}$, multiplied by the lever arm from this center of rotation to the centroid, will give the resultant movement, Δ_o, of R_A away from the centroid due to bending of all the elements from points A to B. The moment at any point on the fixed bent is equal to the algebraic sum of the determinate moment and the moment set up in the structure as a result of returning ends A and B to their original positions.

Example 3.—Find the moments in the bent shown in Fig. 7 produced by a load of 10,000 lb acting horizontally to the left at the top of the right leg (Fig. 10).

(*a*) Determinate Moments and Displacement Δ_o at the Centroid.—Assume the bent to be made determinate by releasing end A. The total angular change in the determinate structure is $\phi = \frac{1}{2} \times 1{,}800{,}000 \times \dfrac{180}{900\,E} = 0.006$ radian.

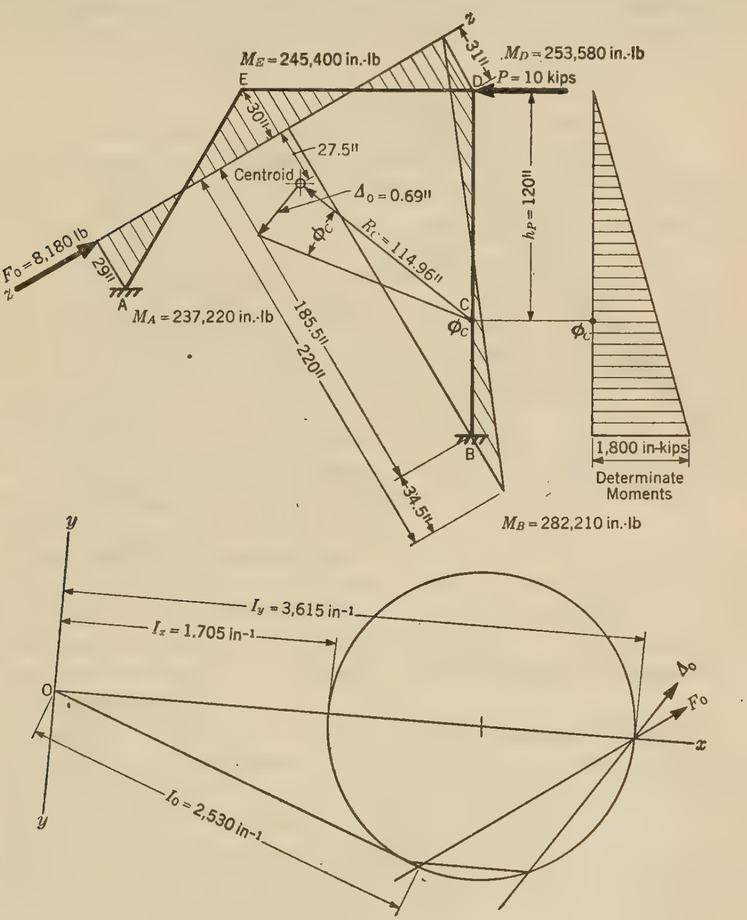
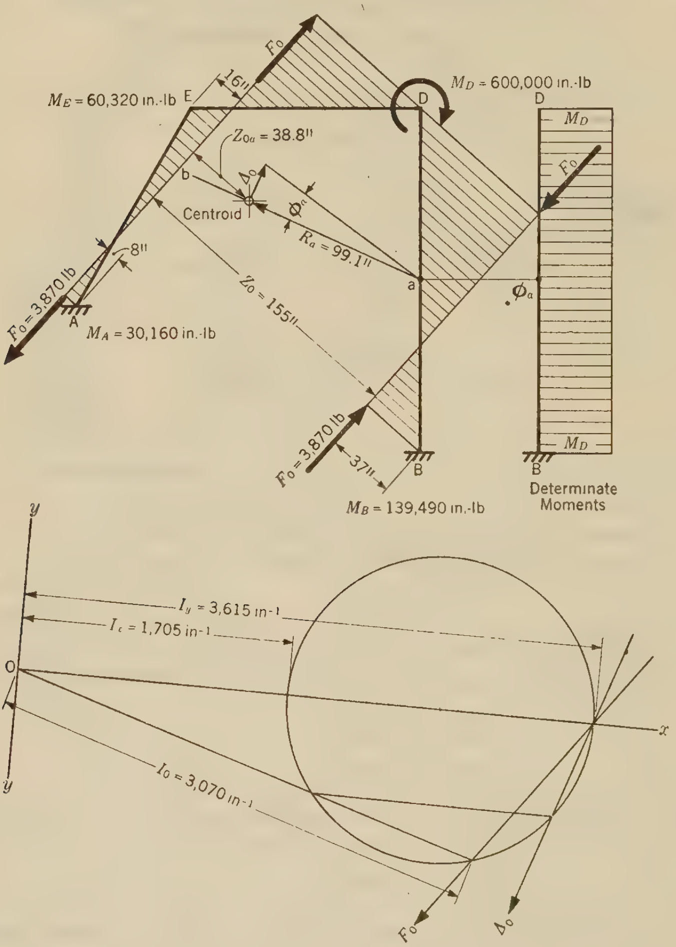

M_E = 245,400 in.-lb
E
M_D = 253,580 in.-lb
D
P = 10 kips
z
31"
30"
27.5"
Centroid
Δ_0 = 0.69"
ϕ_C
R_C = 114.96"
h_P = 120"
C
ϕ_C
ϕ_C
F_0 = 8,180 lb
z
29"
A
M_A = 237,220 in.-lb
185.5"
220"
34.5"
B
1,800 in-kips
Determinate Moments
M_B = 282,210 in.-lb
y
I_y = 3,615 in-1
I_x = 1,705 in-1
O
y
I_0 = 2,530 in-1
Δ_0
F_0
x

FIG. 10

The center of rotation, C, is on the leg BD at a point corresponding to the center of gravity of the $\dfrac{M}{EI}$ -diagram. Its distance from the centroid is $R_C = 114.96$ in. The angular displacement ϕ_C is counterclockwise and causes centroidal displacement $\Delta_o = 114.96 \times 0.006 = 0.69$ in.

(*b*) Indeterminate Moments Produced by Return of the Ends to Their Original Positions.—The return of the ends to their original positions will cause a relative displacement of R_A away from R_B at the centroid equal and opposite in direction to Δ_o, as previously determined. By use of the circle of inertia, as shown in Fig. 10, the direction of F_o is determined and the value of I_o is found to be 2,530 in.$^{-1}$ Substituting this value in Eq. 19 gives $F_o = \dfrac{0.69 \times 30 \times 10^6}{2{,}530} = 8{,}180$ lb. Also $Z_o = \dfrac{180{,}000}{8{,}180 \times 0.8} = 27.5$ in. The rotation of end A in returning to its original position is clockwise (being opposite to the rotation ϕ_C). The force F_o at the left acting on lever arm Z_o will cause a similar rotation about the centroid which determines the direction of Z_o.



F_o
16"
M_E = 60,320 in.-lb
E
M_D = 600,000 in.-lb
D
D
M_D
Z_{Oa} = 38.8"
b
Δ_o
Centroid
ϕ_a
R_a = 99.1"
F_o
a
ϕ_a
Z_o = 155"
8"
F_o = 3,870 lb
A
M_A = 30,160 in.-lb
F_o = 3,870 lb
37"
B
M_B = 139,490 in.-lb
M_D
B
Determinate Moments
y
I_y = 3,615 in-1
I_c = 1,705 in-1
O
x
y
I_O = 3,070 in-1
F_o
Δ_o

FIG. 11

(c) Final Moment Diagram.—The moment at any point on the bent is equal to the algebraic sum of the determinate and indeterminate moments. Using the line of action of F_o as the base line for plotting both moments, a diagram of the combined moments may be drawn as shown.

4. Moments Produced by Intermediate External Moments

An external moment applied to any point on an indeterminate structure may be assumed to act on either part of the structure as a determinate load. Combination of the resultant moment with the indeterminate moment induced by returning the ends to their original positions will give the actual moment. Solution of a problem of this type is shown in Example 4.

Example 4.—In the bent shown in Fig. 11, the thrust and its lever arm due to an external clockwise moment of 600,000 in.-lb at point D are found to be: $\phi_a = \sum_{B}^{D} \frac{M\,\Delta s}{E\,I} = \frac{600{,}000 \times 180}{30 \times 10^6 \times 900} = +\,0.004$ radian; $\Delta_o = \phi_a R_a = 0.004 \times 99.1 = 0.396$ in.; $F_o = \frac{\Delta_o E}{I_o} = \frac{0.396 \times 30 \times 10^6}{3{,}070} = 3{,}870$ lb; and $Z_{oa} = \frac{\phi_a E}{F_o L_o} = \frac{0.004 \times 30 \times 10^6}{3{,}870 \times 0.8} = 38.76$ in. By combining the determinate and indeterminate moments a moment diagram may be drawn as shown in Fig. 11.

5. Application to Structures with One End Hinged

The direction and amount of the force F_o acting on the hinged end produced by any linear movement of the hinge with respect to the other end may be found by use of the circle of inertia for the structure at the hinge. The moments and product of inertia for the structure about any pair of rectangular axes through the hinge may be found by transfer from parallel axes through the centroid. The principal moments of inertia are found and the base line is rotated to a position parallel to one of the principal axes at the hinge, in the same manner as previously shown.

The position of the hinge when the structure is in an unstressed condition is treated as the centroid, and the relative displacement of the hinged end away from the end of a rigid lever arm from the other end to this centroid is used as the value and direction of Δ_o to be used in finding F_o. Structures with intermediate loads are treated in the same manner as shown for loaded structures with both ends fixed.

Knowing the direction of F_o and that it acts through the hinge, Z_o (the distance from its line of action to the centroid of the fixed-end structure) is determined. By substitution of this value and the value of F_o in Eq. 5, the rotation of the hinged end with respect to the other end may be found.

Analysis of an unloaded structure for movement of the hinged end is illustrated in Example 5, which is based on Fig. 12. The construction in Fig. 13 shows the analysis of the same structure with an intermediate load and no linear movement of the ends. For loaded structures the rotation of the hinge is the algebraic sum of its rotation for the determinate condition and the rotation due to the action of F_o in bringing the hinged end back to its original position.

Example 5.—Find the moments produced in the bent shown in Fig. 12 by moving end A 1 in. down and to the right at an angle of 45° with the horioztnal.

(*a*) Circle of Inertia at Point A.—From Example 1, the moments of inertia and product of inertia about horizontal and vertical axes are $I_{xo} = 1{,}720$ in.$^{-1}$;

$I_{yo} = 3{,}600$ in.$^{-1}$; and $I_{xyo} = -165$ in.$^{-1}$ The distances of the centroid from the horizontal and vertical axes at point A are $\bar{x} = +90$ in. and $\bar{y} = +55.44$ in. The moments of inertia and product of inertia about parallel axes at point A are: $I'_{xA} = 1{,}720 + 55.44^2 \times 0.8 = 4{,}179$ in.$^{-1}$; and $I'_{yA} = 3{,}600 + 90^2 \times 0.8 = 10{,}080$ in.$^{-1}$; and $I'_{xyA} = -165 + 55.44 \times 90 \times 0.8 = +3{,}827$ in.$^{-1}$. Using these values, the principal moments of inertia and the product of inertia are $I_{xA} = 2{,}300$ in.$^{-1}$; $I_{yA} = 11{,}950$ in.$^{-1}$; and $I_{xyA} = 0$. The rotation of the base line is counterclockwise since the criterion, $\frac{I'_{xy}}{I'_x - I'_y}$, is negative.

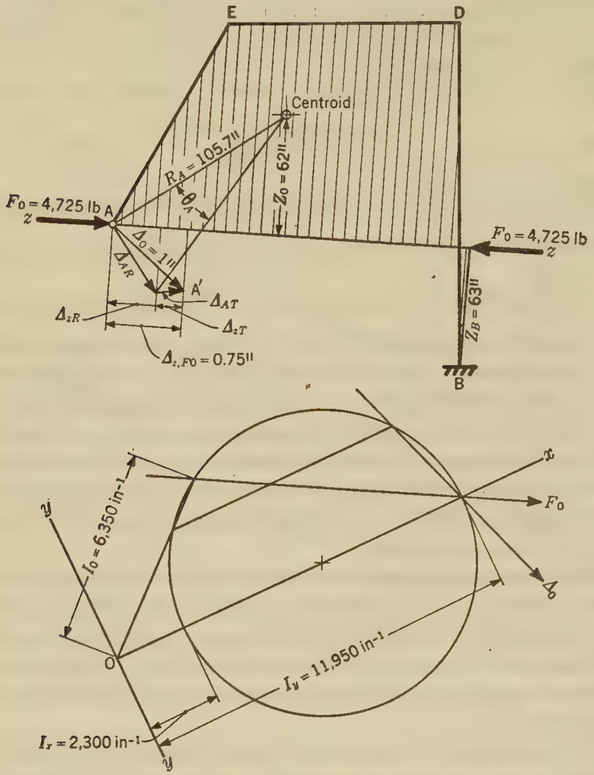
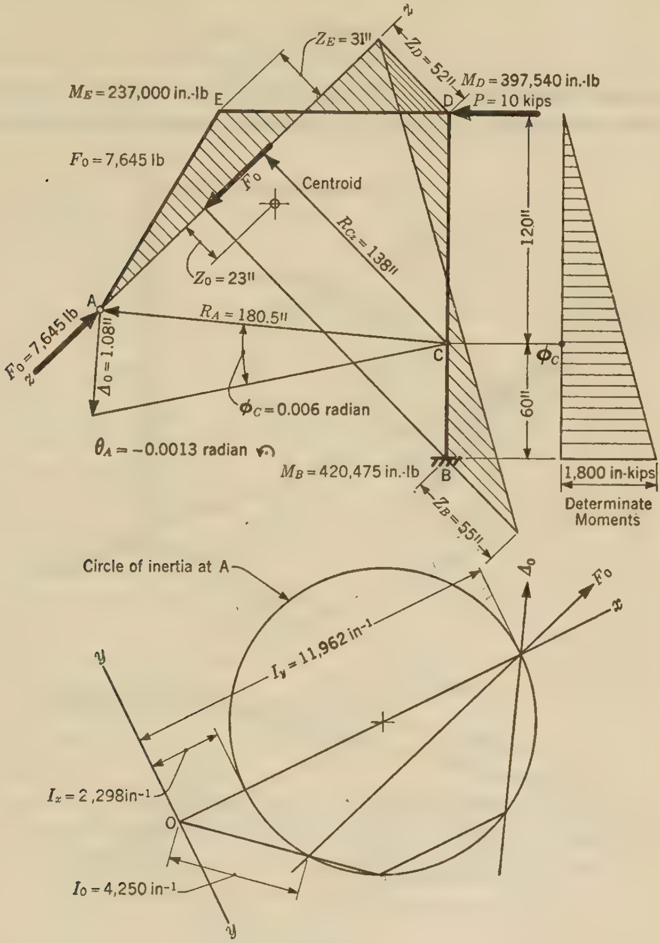

E
D
Centroid
$R_A = 105.7''$
θ_A
$Z_o = 62''$
$F_o = 4{,}725$ lb
z
A
$\Delta_o = 1''$
Δ_{AR}
A'
Δ_{AT}
Δ_{zR}
Δ_{zT}
$\Delta_{z,F0} = 0.75''$
$F_o = 4{,}725$ lb
z
$Z_B = 63''$
B
x
y
F_o
z
$I_o = 6{,}350$ in^{-1}
O
$I_y = 11{,}950$ in^{-1}
$I_x = 2{,}300$ in^{-1}
y

FIG. 12

(b) Line of Action and Value of F_o.—By use of the circle of inertia at point A, the direction of F_o is found and drawn through the hinge, and the value of I_o is found to be 6,350 in.$^{-1}$ Substituting in Eq. 19, $F_o = \frac{1 \times 30 \times 10^6}{6{,}350} = 4{,}725$ lb. Knowing the value and line of action of F_o, the moments in the bent can be determined.

(c) Rotation of the Hinged End.—The value of Z_o is found by construction to be 62 in. Substituting in Eq. 5, $\theta_A = \frac{4{,}725 \times 62 \times 0.8}{30 \times 10^6} = 0.0078$ radian.

Rotation θ_A is counterclockwise, since the moment of F_o at the left about the centroid is counterclockwise. The translational and rotational components of the end movement, Δ_o, may be found graphically as shown in Fig. 12.



z
$Z_E = 31''$
$Z_D = 52''$
$M_D = 397{,}540$ in.-lb
$P = 10$ kips
$M_E = 237{,}000$ in.-lb
E
D
$F_0 = 7{,}645$ lb
F_o
Centroid
$R_C = 138''$
120''
$Z_0 = 23''$
A
$R_A = 180.5''$
$F_0 = 7{,}645$ lb
z
$\Delta_0 = 1.08''$
C
ϕ_C
$\phi_C = 0.006$ radian
60''
$\theta_A = -0.0013$ radian
$M_B = 420{,}475$ in.-lb
B
1,800 in-kips
Determinate Moments
$Z_B = 55''$
Circle of inertia at A
Δ_o
F_o
x
y
$I_y = 11{,}962$ in^{-1}
$I_x = 2{,}298$ in^{-1}
O
$I_0 = 4{,}250$ in^{-1}
y

FIG. 13

6. APPLICATION TO STRUCTURES WITH BOTH ENDS HINGED

For structures with both ends hinged, the line of action of F_o passes through the two hinges. The inertia about this line may be found by transfer from the circle of inertia at the centroid. Using the value thus determined as I_z, F_o may be found by Eq. 11. The relative rotation of the two ends is determined for any value of F_o since Z_o is known by construction.

Part of Δ_z is due to the rotation of the ends about the centroid, the remainder being due to translation without rotation; that is,

$$\Delta_z = \Delta_{zR} + \Delta_{zT} = \frac{F_o Z^2_o L_o}{EI} + \frac{F_o I_{zo}}{EI} \qquad (24)$$

from which

$$\Delta_{zR} = \frac{Z^2_o\, L_o}{I_{zo} + Z^2_o\, L_o} \times \Delta_z \qquad (25a)$$

and

$$\Delta_{zT} = \frac{I_{zo}}{I_{zo} + Z^2_o\, L_o} \times \Delta_z \qquad (25b)$$

Knowing these components of the translational and rotational movements the actual movements of the ends may be found by construction as shown in Fig. 14.

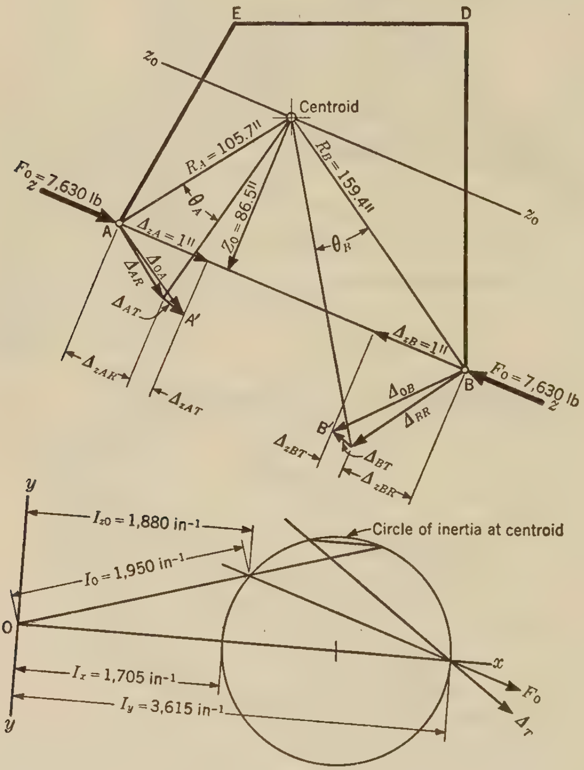

E
D
z_o
Centroid
$R_A = 105.7''$
$R_B = 159.4''$
F_o z = 7,630 lb
A
θ_A
$\Delta_{zA} = 1''$
$Z_o = 86.5''$
θ_B
z_o
Δ_{AR}
Δ_{OA}
Δ_{AT}
A'
Δ_{zAR}
Δ_{zAT}
$\Delta_{zB} = 1''$
$F_o = 7{,}630$ lb z
B
Δ_{OB}
Δ_{BR}
B'
Δ_{zBT}
Δ_{BT}
Δ_{zBR}
y
$I_{zo} = 1{,}880$ in^{-1}
$I_o = 1{,}950$ in^{-1}
Circle of inertia at centroid
O
$I_x = 1{,}705$ in^{-1}
$I_y = 3{,}615$ in^{-1}
x
F_o
Δ_r
y

FIG. 14

Example 6.—Find the movements produced in the bent shown in Fig. 14 by moving the hinges 2 in. closer together. Assuming this movement equally divided between the two ends, find the rotation and actual movement of each hinge.

(*a*) Evaluation of I_z and F_o.—The moment of inertia about a line, z_o–z_o, through the centroid parallel to the line of action of F_o is found from the circle of inertia at the centroid to be $I_{zo} = 1{,}880$ in.$^{-1}$ From construction, $Z_o = 86.5$ in., and the inertia about the axis z–z is $I_z = 7{,}866$ in.$^{-1}$ Using this value of I_z in Eq. 11, $F_o = 7{,}630$ lb.

(*b*) Rotations and Movements of the Ends.—Assuming that point A moves 1 in. toward point B, the rotational component of this movement is $\Delta_{zAR} = 0.76$ in., and the translational component is $\Delta_{zAT} = 0.24$ in. Similarly, for a 1-in. movement of point B toward point A, $\Delta_{zBR} = 0.76$ in. and $\Delta_{zBT} = 0.24$ in.

The movement Δ_{AR} is perpendicular to R_A and the movement Δ_{BR} is perpendicular to R_B. Knowing the components of these movements along the axis z–z, the amount of each movement may be found graphically as shown at points A and B in Fig. 14. The direction of the translational movements Δ_{AT} and Δ_{BT} is the same as Δ_T found by the circle of inertia at the centroid. The actual movements of the ends A and B are Δ_{oA} and Δ_{oB}, and the rotations are found to be $\theta_A = 0.0088$ radian, and $\theta_B = 0.0088$ radian, respectively.

Example 7.—Find the moments produced in the bent by the load shown in Fig. 10 if both ends are hinged.

(*a*) Determinate Moments.—Assuming the reaction at point A to act on the hinge in the direction AE and taking moments about points B and D, the determinate reactions are found to be 9,280 lb and 9,660 lb, respectively. The moment diagram for determinate moments is shown in Fig. 15. The centers of rotation for BD and DE are shown at a and b, respectively, and their combined center of rotation is shown at C. The rotation, ϕ_C, which is equal to the sum of the values $\sum \dfrac{M\,\Delta s}{E\,I}$ for BD and DE, equals 0.0064 radian.

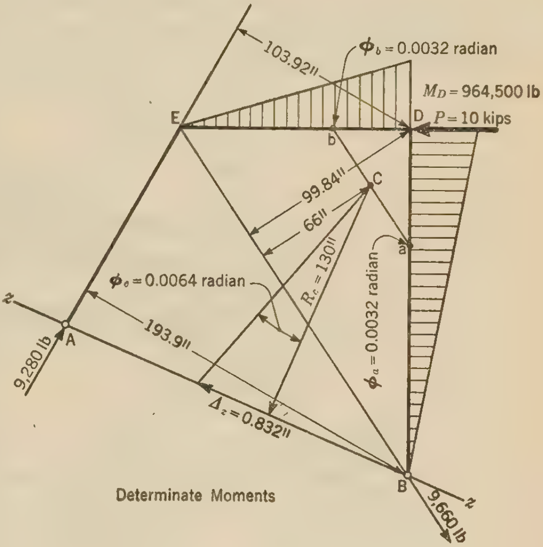


Fig. 15

(*b*) Value of F_o and of Combined Moments and Reactions.—The distance Δ_z through which F_o must act to bring the ends of the deflected determinate structure back to their original positions at points A and B is given by $\Delta_z = 0.0064 \times 130 = 0.832$ in. Substituting this value in Eq. 11, $F_o = 3{,}175$ lb. With F_o known, the diagram of moments may be drawn as shown in Fig. 16, in which the reactions at points A and B are the resultants of the determinate reactions and the forces F_o.

7. Deflections

At the point of application of a load, P, the displacement in the direction of its action is found by application of Castigliano's theorem to be

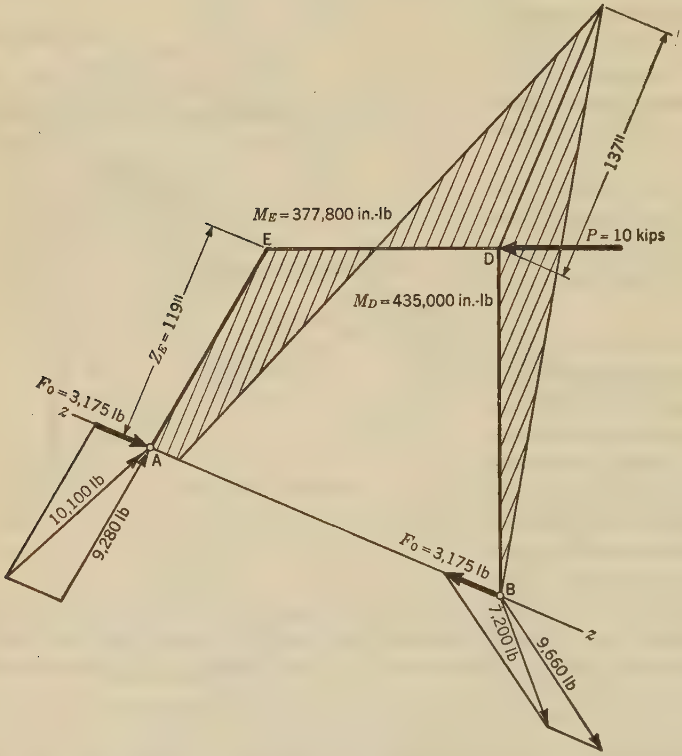
$$\delta_p = \frac{\partial W}{\partial P} = \int M \frac{\partial M}{\partial P} \frac{ds}{E\,I} \qquad \text{(26a)}$$

A constant unit force ($P = 1$) acting at any point and in the direction of the desired deflection during the loading of a structure gives

$$\frac{\partial M}{\partial P} = \frac{\partial M}{1} = m \quad \text{(26b)}$$

in which m is the moment produced by the unit force at any point in the unloaded structure. Substituting Eq. 26b in Eq. 26a gives

$$\delta_p = \int M\, m \frac{ds}{E\,I} \quad \text{(26c)}$$



M_E = 377,800 in.-lb
E
D
P = 10 kips
M_D = 435,000 in.-lb
137"
Z_E = 119"
F_0 = 3,175 lb
z
A
10,100 lb
9,280 lb
F_0 = 3,175 lb
B
7,200 lb
9,660 lb
z

FIG. 16

The moment in a structure caused by movement of its ends is

$$M = F_o Z \quad \text{(26d)}$$

and the deflection of the point due to the action of F_o is

$$\delta_p = \int F_o Z \frac{m\, ds}{E\,I} = F_o \int Z\, d\phi_1 \quad \text{(26e)}$$

in which $d\phi_1$ is the rotation of any element of length ds caused by moment m. By the assumption that the angular total change in the unloaded structure caused by the unit force acts at its center of rotation, the deflection becomes

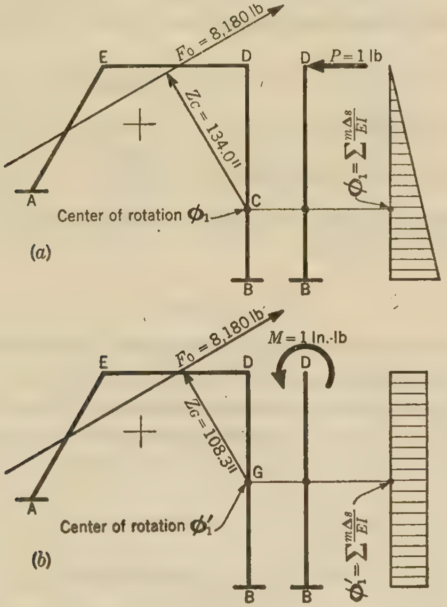
$$\delta_p = F_o Z_1 \phi_1 \qquad (27)$$

in which Z_1 is the distance from the center of rotation of ϕ_1 to the line of action of F_o.

If m represents the moment at any point due to a unit moment at point p, the rotation at this point may be found, in the same manner, by

$$\alpha_p = F_o Z_1 \phi_1 \qquad (28)$$

in which ϕ_1 is the total angular change in the structure produced by the unit moment and Z_1 is the distance from the center of rotation ϕ_1 to the line of action of F_o. The deflection or rotation found by the foregoing equations is the displacement caused by the action of thrust F_o. This is the total displacement for structures having no intermediate loads. The total displacement for structures with intermediate loads is found by combining the determinate displacement with the indeterminate displacement as previously found.



$F_o = 8{,}180$ lb
E
D
D
$P = 1$ lb
$Z_C = 134.0''$
$\phi_1 = \sum \frac{m\Delta s}{EI}$
A
C
Center of rotation ϕ_1
(a)
B
B
$F_o = 8{,}180$ lb
$M = 1$ in.-lb
E
D
D
$Z_C = 108.3''$
G
A
Center of rotation ϕ_1'
(b)
$\phi_1' = \sum \frac{m\Delta s}{EI}$
B
B

FIG. 17

Example 8.—Find the horizontal deflection and the angular rotation of point D for the loading shown in Fig. 10.

(*a*) Determinate Deflections.—Assume point A free to move and point B to be fixed. From the area of the determinate moment diagram ϕ_C is found to be 0.006 radian and the determinate deflection of point D is $\delta_1 = 0.006 \times 120 = -0.72$ in.

(*b*) Indeterminate Deflections.—With point A free to move and B fixed (see Fig. (17*a*)), a unit horizontal load acting to the left is applied at point D. The center of rotation is at C and $\phi_1 = \frac{1}{2} \times \frac{180 \times 180}{30 \times 10^6 \times 900} = \frac{6}{10^7}$ radian. The distance from the center of rotation to the line of action of F_o is $Z_C = 134.0$ in. and the horizontal deflection of point D caused by the action of F_o is $\delta_2 = 8{,}180 \times 134.0 \times 6 \times 10^{-7} = +\,0.658$ in.

(*c*) Total Deflection.—The total deflection of point D is $\delta_o = \delta_1 + \delta_2 = -\,0.062$ in.

(*d*) Determinate Rotation.—The determinate rotation of point D is, as found in Example 8(*a*), $-\,0.006$ radian.

(*e*) Indeterminate Rotation.—Assuming point A free to move and point B to be fixed in Fig. 17(*b*), the total angular change in DB caused by a unit moment applied at point D and acting on DB is: $\phi'_1 = \Sigma \frac{m\,\Delta s}{E\,I} = \frac{1 \times 180}{30 \times 10^6 \times 900} = \frac{6.667}{10^9}$ radian. The center of rotation is at point G, the midpoint of BD, and its distance from the line of action of F_o is $Z_G = 108.3$ in. The indeterminate rotation is $\alpha_1 = 8{,}180 \times 108.3 \times \frac{6.667}{10^9} = +\,0.005906$ radian.

(*f*) Total Rotation.—The rotation of D with reference to end B is $\alpha_o = \alpha_2 + \alpha_1 = -\,0.000094$ radian.

8. CONCLUSION

The method of solution herein presented offers a simple procedure for the analysis of rigid frames which can be quickly applied, either for the determination of stresses or as a check on other methods. It is offered with the hope that its viewpoint may be of interest and that it may be found useful as a method of analysis.

APPENDIX. NOTATION

The following letter symbols conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards Association, with Society representation and approved by the Association in 1932:

C = center of rotation; also, an intermediate point on a structure;
E = modulus of elasticity;
F = force:
    F_A, F_B = force acting at ends A and B, respectively;
    F_o = thrust;

I = moment of inertia:

I_o = a value for I which will give the value of F_o in $F_o = \dfrac{\Delta_o E I}{I_o}$;

I_x, I_y = about the axes x–x and y–y, respectively;

I_z = about z–z, the line of action of F_o;

I_{zo} = about centroidal axis z_o–z_o, parallel to F_o;

L = length: L_o = value of $\int_A^B ds$;

M = moment: M_A, M_B, M_C = at points A, B, and C, respectively;

P = external force;

R_A, R_B = vectors from ends A and B, respectively, to the centroid;

R_C = vector from center of rotation C to centroid;

s = a distance along the center line of the structure;

W = work;

x–x, y–y = rectangular coordinate axes;

Z = a moment arm:

Z_A, Z_C = about points A and C, respectively;

Z_o = of F_o about the centroid;

z–z = line of action of thrust F_o;

z_o–z_o = centroidal axis parallel to the line of action of thrust F_o;

α = angle;

β = angle;

Δ = linear displacement:

Δ_{AR} = linear displacement of end A caused by rotation about the centroid;

Δ_{AT} = linear displacement of end A caused by translation at the centroid;

Δ_o = resultant centroidal displacement;

Δ_z = linear movement of F_o along its line of action;

Δ_{zo} = component of Δ_o along centroidal axis z_o–z_o;

δ = deflection;

θ_A = angular change of end A; and

ϕ = angular change, usually total angular change for part or all of a structure.

DISCUSSION

L. E. GRINTER,[2] M. ASCE.—The many methods of frame analysis should not be looked upon as so many unrelated theories. Actually, the basic principles of ring analysis are few. By comparing the centroidal procedure with other methods, such as that involving the use of Maxwell's equations and the neutral-point method, one sees that the author's procedure involves the use of common principles.

The objective of avoiding the need for solving Maxwell's simultaneous equations is inherent in all modern tools of ring analysis. The concept of applying the reactions at the elastic center and of carrying them down to the abutments by one or more rigid brackets was the first method used for eliminating simultaneous equations. To accomplish this result, the reactions are applied at the centroid of the $\frac{ds}{E\,I}$-values of the several parts or divisions of the structure. This centroid has been termed the neutral point as well as the elastic center.

The principal variation of the neutral-point method has been the concept of the column analogy introduced by Hardy Cross,[3,4] M. ASCE. The column analogy offers nothing not possible through the neutral-point method, since the two have identical root derivations—that is, both simplify Maxwell's equations by choosing such coordinate axes that all except two terms in each of the three simultaneous equations become zero and the three simultaneous equations reduce to three independent equations. The usefulness of the column analogy is its helpfulness in organizing calculations, eliminating confusion over signs, and reducing the intermediate consideration of reaction forces when moments are needed for design. It is possible, therefore, to state categorically that the author's centroidal method can accomplish no more than the neutral-point method or the column analogy. However, its usefulness is to be judged on the basis of such important considerations as clarity or ease of understanding, convenience in making routine calculations, and visual characterstics that aid one both to understand and to remember the procedures. For clarity of the physical picture, the writer has always considered the use of neutral-point forces to be desirable, but anyone who has used the column analogy knows that it saves much time and reduces the probabilities of clerical errors.

The author's procedure is more closely akin to the older neutral-point method than to the column analogy. In fact, it is a neutral-point method in which use is made of one resultant reactive force which takes the place of the thrust, shear, and moment reactions at the neutral point. In Fig. 9, for example, the eccentric force F_o of 8,180 lb produces a moment of 8,180 × 27.5 in. or 225,000 in.-lb about the neutral point (centroid of the $\frac{ds}{E\,I}$-values).

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[2] Research Prof. of Civ. Eng. and Mechanics, Illinois Inst. of Technology, Chicago, Ill.

[3] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1930.

[4] "Theory of Modern Steel Structures," by L. E. Grinter, The Macmillan Co., New York, N. Y., 1937, Vol. 2, pp. 206 and 216.

The same neutral-point moment is obtained by dividing the rotation at the neutral point for the static structure, $\int M_s \frac{ds}{EI}$, by the rotation at the neutral point due to unit neutral-point moment, $\int \frac{ds}{EI}$. Hence,

$$M_o = \frac{\int M_s \frac{ds}{EI}}{\int \frac{ds}{EI}} = \frac{0.5 \times 1{,}800{,}000 \times 180 \div 900}{\frac{180}{900} + \frac{120}{600} + \frac{120}{300}} = 225{,}000 \text{ in.-lb.}$$

Evidently, as soon as M_o, H_o, and V_o are computed by the neutral-point method, the value of F_o may be obtained as $\sqrt{H^2_o + V^2_o}$, its inclination may be found from $\tan \theta = V_o/H_o$, and its eccentricity from the neutral point is M_o/F_o.

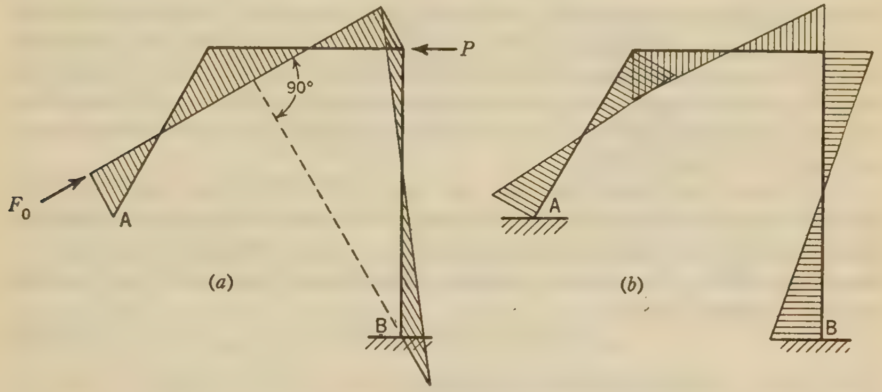

FIG. 18.—MOMENT DIAGRAMS

(a) Plotted on F_o, Ordinates Normal to F_o
(b) Plotted on Compressive Side, Normal to Each Member

The question is in reality whether it is more effective to use a resultant eccentric force, F_o, than its neutral-point components H_o, V_o, and M_o. As may be noted from the author's illustrations, one of which is reproduced in part as Fig. 18, there is a certain pictorial quality to the moment diagram drawn on the force F_o as a base line. It is a device that may be used qualitatively in an effective manner. However, even in Fig. 18(*a*), one notices that the device does not add clarity but rather, in fact, is somewhat confusing when applied to drawing the moment diagram for the right-hand column. The ordinary type of bending moment diagram shown in Fig. 18(*b*) seems preferable.

It appears that the method described is in all ways a direct alternative to the common procedure of the neutral-point method. Its distinguishing characteristic is that it gives directly the resultant eccentric force in place of its neutral-point components. This force may then be used to determine graphically a moment diagram plotted on the eccentric force as a base line, all ordinates being normal thereto. The author's procedure might conceivably replace the neutral-point method, but not the column analogy, which offers quite a different set of advantages. The time needed to analyze any ring structure by the author's centroidal procedure with a single reactive force will not be appreciably different from that required for previously known variations of the neutral-point method.

ALEXIS L. GLUCKMANN,[5] ESQ.—The methods of analysis of rigid frames exposed in this interesting paper result directly from the properties of the ellipse of elasticity as shown by M. Marbec, in 1905.[6] For frames of uniform cross section his results are similar to those of Mr. Bell. Among other helpful short cuts, Mr. Marbec gave the following very practical rule for computing the centers of gravity, the statical moments, the moments of inertia, and the centrifugal moments of straight lines and of circular arcs with a central angle $\alpha \leqq \frac{\pi}{2}$:

Assume 1/6 of the total mass of the straight line (or of the circular arc) to be concentrated at each end; and assume the remainder, or four sixths of the total mass to be concentrated at the middle point of the straight line (or of the arc). These assumptions, used for computing the centers of gravity, the statical moments, and the moments of inertia, give an exact result for the straight line and an approximate result for the circular arc; the approximations are sufficient for slide rule computations.

The writer has found that, for this type of problem, and especially for frames with given displacements of both supports A and B, it is often convenient to use the deformation equations of curved beams as established by J. A. C. Bresse in 1854.[7] For a frame without directly applied loads, the bending moment can be written

$$M = a\,x + b\,y + m \quad \ldots\ldots(29)$$

To compute the three unknowns m, a, and b, the following formulas, deduced from those of Mr. Bresse, may be used

$$-m \int_A^B \frac{ds}{EI} = \theta_B - \theta_A \quad \ldots\ldots(30)$$

$$b \int_A^B y^2 \frac{ds}{EI} = \Delta^B{}_x - \Delta^A{}_x - \theta_B\, y_B + \theta_A\, y_A \quad \ldots\ldots(31)$$

and

$$-a \int_A^B x^2 \frac{ds}{EI} = \Delta^B{}_y - \Delta^A{}_y + \theta_B\, x_B - \theta_A\, x_A \quad \ldots\ldots(32)$$

in which x and y are the coordinates of the center line of the frame with respect to its principal axes; x_A, y_A, x_B, and y_B are the coordinates of the supports A and B with respect to the same axes; and θ_A, θ_B, $\Delta^A{}_x$, $\Delta^B{}_x$, $\Delta^A{}_y$, and $\Delta^B{}_y$ are the given displacements of the supports. The integrals of Eqs. 30, 31, and 32 have an obvious meaning and, in all cases, can be computed easily with the help of Marbec's rule previously given.

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[5] Asst. Designing Engr., Gibbs and Hill, Inc., New York, N. Y.

[6] "Étude sur la Dilatation des Tuyautages," by M. Marbec, Ministère de la Marine, Memorial du Génie Maritime, 3me Serie, Fascicule VII, Paris, 1905.

[7] "Recherches Analytiques sur la Flexion et la Résistance des Pièces Courbes," by Jacques Antoine Charles Bresse, Mallet-Bachelier, Paris, 1854.

YI-MAI YAO,[8] ESQ.—The theory of the semigraphical method, offered by Mr. Bell, is sound and is similar in character to the column analogy method.[3] The equivalent items are shown in Table 1.

TABLE 1.—COMPARISON OF EQUIVALENT ITEMS

Item	CENTROIDAL METHOD		COLUMN ANALOGY METHOD	
	Name	Notation	Name	Notation
(1)	(2)	(3)	(4)	(5)
Moment	Moment	M	Stress	$f = M\,i$
Angular displacement...	Angular displacement	$\phi = \Sigma\theta$	Total load	$P = \Sigma\theta$
$\int \frac{ds}{I}$	Structural length	$L_o = \int \frac{ds}{I}$	Area	$A = \int \frac{ds}{I}$
Linear displacement....	Component of Δ_o parallel to x-axis Component of Δ_o parallel to y-axis	Δ_{xo} Δ_{yo}	Moment with respect to x-axis Moment with respect to y-axis	M_{x-x} M_{y-y}
Moment of inertia......	Moment of inertia with respect to x-axis Moment of inertia with respect to y-axis	I_x I_y	Moment of inertia with respect to x-axis Moment of inertia with respect to y-axis	I_x I_y
Thrust line............	Thrust line	....	Neutral axis	....

There is no advantage in locating the thrust line by use of Mohr's circle as, in many instances, I_x and I_y are almost equal and the "circle" will give an inaccurate result. The additional equations are derived as follows:

For a rigid frame with variable moment of inertia,

$$F_o = \sqrt{F^2{}_x + F^2{}_y} \qquad (33)$$

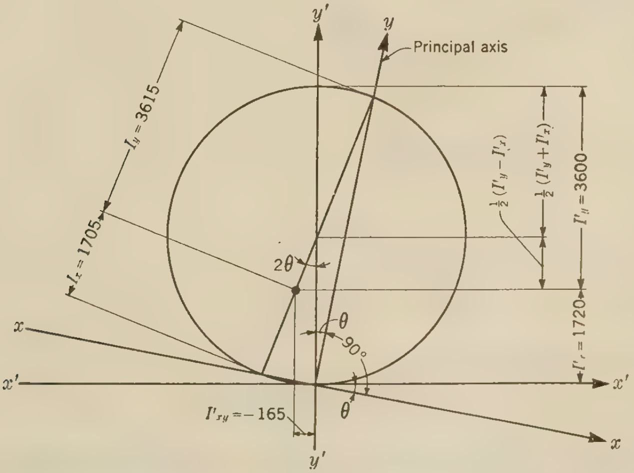

FIG. 19

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[8] Junior Engr., TangKu New Harbor Construction Bureau, TangKu, Tientsin, China.

in which

$$F_x = \frac{\Delta_{xo} E}{I_x} \quad \text{(34a)}$$

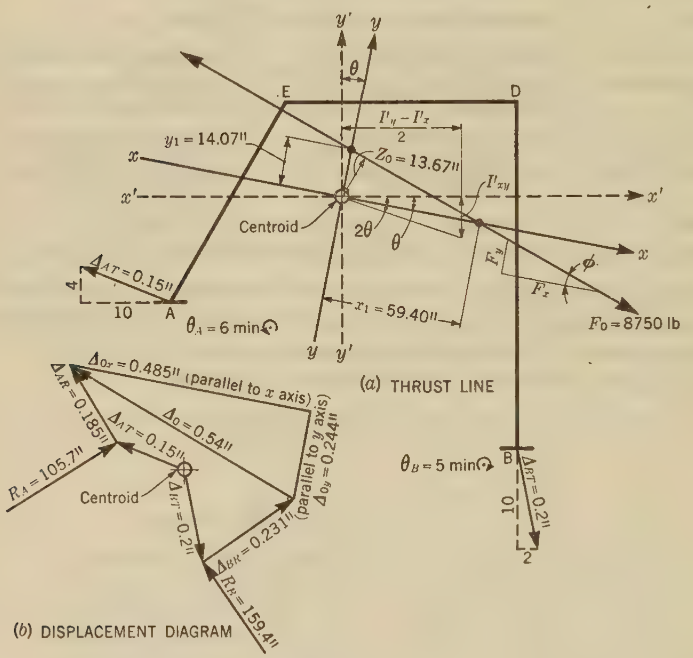
and

$$F_y = \frac{\Delta_{yo} E}{I_y} \quad \text{(34b)}$$

Eqs. 34 are similar in form to Eqs. 15.

In the column analogy method, the stress at any point on the analogous column section can be computed from the simple equation for an eccentrically loaded column—

$$f = \frac{P}{A} \pm \frac{M_{x-x}\, y}{I_x} \pm \frac{M_{y-y}\, x}{I_y} \quad \text{(35)}$$



y'
y
θ
E
D
$\frac{I'_y - I'_x}{2}$
y_1 = 14.07"
Z_0 = 13.67"
x
x'
x'
I'_{xy}
Centroid
2θ
θ
x
F_y
φ
Δ_{AT} = 0.15"
4
10
A
F_x
x_1 = 59.40"
F_0 = 8750 lb
θ_A = 6 min
y
y'
Δ_{0x} = 0.485" (parallel to x axis)
Δ_{AR} = 0.185"
Δ_{AT} = 0.15"
Δ_0 = 0.54"
(parallel to y axis) Δ_{0y} = 0.244"
R_A = 105.7"
Centroid
Δ_{BT} = 0.2"
Δ_{BR} = 0.231"
R_B = 159.4"
θ_B = 5 min
B
Δ_{BT} = 0.2"
10
2

(a) THRUST LINE

(b) DISPLACEMENT DIAGRAM

FIG. 20

From Eq. 35, if $f = 0$ and $y = 0$,

$$x_1 = \frac{P\, I_y}{A\, M_{y-y}} \quad \text{(36a)}$$

and, if $f = 0$ and $x = 0$,

$$y_1 = \frac{P\, I_x}{A\, M_{x-x}} \quad \text{(36b)}$$

By substitution of Eqs. 34a and 34b in Eqs. 36a and 36b, respectively, then

$$x_1 = \frac{\phi\, I_y}{L_o\, \Delta_{yo}} = \frac{\phi\, E}{L_o\, F_y} \quad \text{(37a)}$$

and

$$y_1 = \frac{\phi\, I_x}{L_o\, \Delta_{xo}} = \frac{\phi\, E}{L_o\, F_x} \qquad (37b)$$

Such equations can be derived independently, of course, as was Eq. 6.

The distance from the centroid to the thrust line can be easily calculated by geometry; that is,

$$Z_o = \frac{x_1\, y_1}{\sqrt{x^2{}_1 + y^2{}_1}} \qquad (38)$$

By use of Eqs. 37, the thrust line can be located easily and accurately.

For Example 2, Mohr's circle as shown in Fig. 5 may be omitted. The quantities I_x, I_y, and the rotation angle θ to the principal axes can be obtained by another form of Mohr's circle,[9] as shown in Fig. 19.

From Fig. 19 and the displacement diagram (Fig. 20(b)), $\Delta_{xo} = 0.485$ in., $\Delta_{yo} = 0.244$ in., $I_x = 1{,}705$ in.^{4}, and $I_y = 3{,}615$ in.^{4} Substituting these values in Eqs. 34a, 34b, 37a, and 37b gives $F_x = \frac{(0.485)\,(30 \times 10^6)}{1{,}705} = 8{,}520$ lb, $F_y = \frac{(0.244)\,(30 \times 10^6)}{3{,}615} = 2{,}022$ lb, $x_1 = \frac{(0.0032)\,(30 \times 10^6)}{(0.8)\,(2{,}022)} = 59.40$ in., and $y_1 = \frac{(0.0032)\,(30 \times 10^6)}{(0.8)\,(8{,}520)} = 14.07$ in., from which the thrust F_o can be located as shown in Fig. 20 and is found to be $F_o = \sqrt{8{,}520^2 + 2{,}022^2} = 8{,}750$ lb. The distance from the centroid to the thrust is $Z_o = \frac{(59.40)\,(14.07)}{\sqrt{59.40^2 + 14.07^2}} = 13.67$ in. The value Z_o is useless in analysis, and is only used here as a check on the value computed by the method of the paper.

PHIL M. FERGUSON,[10] M. ASCE.—Engineering literature is enriched and the viewpoint of engineers is widened by papers of this type, which indicate fresh theories and methods derived from particular problems that have faced the author. The majority of structural engineers have regarded actual movements of supports of rigid frames as more or less incidental problems, often limited to relatively simple temperature changes. Their primary problem has been an analysis for external loads. Hence, they have adopted methods particularly suited to external loading and have managed to apply these methods to such temperature deformations or settlements as they wished to assume. Mr. Bell's method comes out of a situation where movements of supports are the basic data and lack of symmetry is common. It is the writer's opinion that the proposed method is most useful in this area, although it can be extended, as the author shows, to external loads.

The ellipse of elasticity and the column analogy also lend themselves to this type of calculation. The ellipse of elasticity is seriously handicapped by its reliance on the geometry of the ellipse, this geometry not being generally familiar to American engineers. On the other hand, the column analogy also labors under handicaps: First, the concept whereby a couple acting on the analogous column represents a linear movement of one end of the frame is thoroughly artificial; and, second, the signs of the couples representing linear displacements become particularly confusing when they must be related to movements of both supports. Those who use the column analogy frequently will not find this handicap serious, but signs seem less of a problem with Mr. Bell's method. The two methods seem about equally usable for stiffness and carry-over determinations.

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[9] "Über die Bestimmung und die graphische Darestellung von Trägheitsmomenten ebener Flächen," by Otto Mohr, *Zivilingenieur*, 1887, Vol. 33, p. 43.

[10] Prof. of Civ. Eng., Univ. of Texas, Austin, Tex.

For those who particularly like a graphical as compared to an analytical solution, Mr. Bell's method should have a wider range of appeal because analytical work is reduced to a minimum. The combination of the classical circle of inertia method for determining the principal axes and the further use of this same circle in the solution for F_o leads to economy of time and reduction in errors.

Although those to whom Mr. Bell's method appeals may wish to apply it to frames with hinges at the ends, it does not seem particularly well adapted to such usage. A two-hinged frame is statically indeterminate only to the first degree, and very simple methods are available for such a problem.

In summary, it is believed that this method offers certain advantages in cases where end movements are complex. These advantages are emphasized for engineers who prefer graphical constructions. Those who have mastered the method may want to use it for more general loadings, even though it seems to hold no great advantage in that respect over the column analogy method.

LEROY A. BEAUFOY,[11] ESQ.—It is difficult to follow this paper because of the variability of dimensions in the quantities employed. Thus, whereas in Eq. 3 it will be found that the dimensions of I are L^4, in Eq. 9 both I_z and I_{zo} have the dimensions L^3; yet in Eqs. 21*b* and 21*d* the dimensions of I_z and I_{zo}, respectively, are $1/L$. Similarly, in Eqs. 15*a* and 15*b*, respectively, the dimensions of I_x and I_y are L^3, but in Eq. 18*c* these quantities are given the dimension L. (It may be claimed that, as Eq. 18*c* is the algebraic expression of a graphical construction in which I_x and I_y are represented vectorially, the dimension L is rational; however, in the construction relating to Eq. 19, I_o is given the dimensions L^3 and not L.) Again, in Eq. 5 the quantity L_o is assigned the dimension L, but in Eq. 21*a* the dimensions become $1/L^3$; in Eq. 21*e*, I_o has the dimensions $1/L$ and not L^3 (compare Eq. 19). All this makes the interpretation of the equations in the paper anything but a straightforward matter, since the same symbols have different meanings in different places.

Section 6, "Application to Structures with Both Ends Hinged," seems to be based on a fundamental error. It may easily be demonstrated—for example, by virtual work methods—that Eq. 24 applies to a structure with one end fixed and the other end free, and not to a structure with both ends hinged. Thus, referring to Fig. 21, in which any continuous bent AB is fixed at point B and free at point A, it may be shown that the displacements at point A due to a force F_o acting in the direction AB are as given in Fig. 21. The elastic centroid conception enables one to distinguish between displacements due to rotation and those due to translation. The coordinate axes through points

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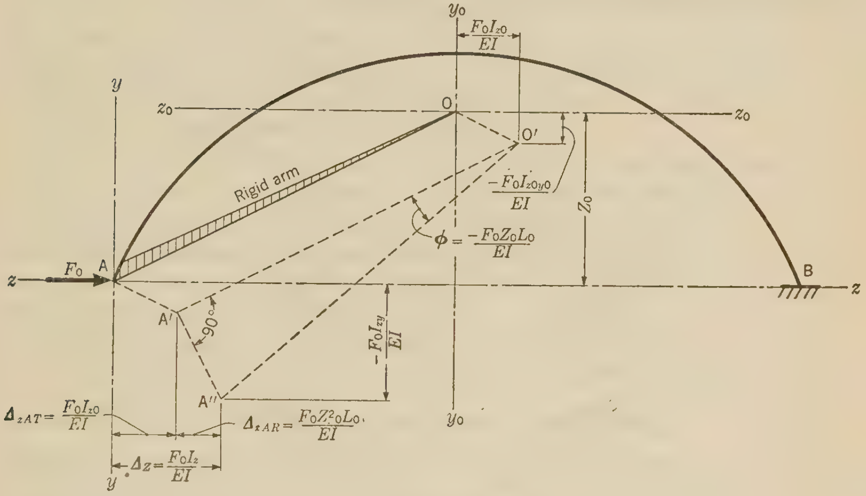
A and O, respectively, are (z, y) and (z_o, y_o) as used in the paper. Because of translation without rotation, a rigid arm AO would move to A′O′ and because of rotation without translation, it would move about O′ from O′A′ to O′A″. Then the horizontal projection of AA′ is

$$\Delta_{zAT} = \frac{F_o\, I_{zo}}{E\, I} \quad \text{.........................} (39)$$

and the horizontal projection of A′A″ is

$$\Delta_{zAR} = \frac{F_o\, Z^2{}_o\, L_o}{E\, I} \quad \text{.........................} (40)$$



y_0
$\frac{F_0 I_{z0}}{EI}$
y
z_0
O
z_0
O′
Rigid arm
$\frac{-F_0 I_{z0y0}}{EI}$
Z_0
$\phi = \frac{-F_0 Z_0 L_0}{EI}$
z
F_0
A
B
z
A′
90°
$-\frac{F_0 I_{zy}}{EI}$
A″
$\Delta_{zAT} = \frac{F_0 I_{z0}}{EI}$
$\Delta_{zAR} = \frac{F_0 Z^2{}_0 L_0}{EI}$
y_0
$\Delta z = \frac{F_0 I_z}{EI}$
y

FIG. 21

It will also be noted that the total horizontal displacement of point A is

$$\Delta_z = \frac{F_o\, I_z}{E\, I} \quad \text{.........................} (41a)$$

and the total vertical displacement is

$$\Delta_y = -\frac{F_o\, I_{zy}}{E\, I} \quad \text{.........................} (41b)$$

Fig. 21 also shows values of the rotation ϕ, and of the horizontal and vertical displacements of the elastic centroid.

Thus, Eq. 24 would not appear to be correct for a bent with both ends hinged. Incidentally, it should be noted that Fig. 3 gives the symbols Δ_{zT} and Δ_{zR} different meanings from those used in Eq. 24. In Fig. 3, these movements refer to the centroid end of the rigid arm R_A; in Eq. 24, to the free end of the bent. In Fig. 21, therefore, the writer has used the symbols Δ_{zAT} and Δ_{zAR}.

The numerical examples are not very convincing; rather, they are the reverse. For instance, in Example 6 the author misleads himself into regarding Δ_{oA} as the "actual movement" of point A. In fact, the relative displacement of points A and B requires a force F_o in the line AB and this relative displacement will be achieved by separate movements of points A and B in the line AB, without lateral movement as suggested in Fig. 14, by the displacement Δ_{oA}, which has a component perpendicular to AB. These separate end translations can have any actual values—in Example 6 only the relative value is fixed at 2 in. Thus, for example, points A and B may both move 1 in. (each toward the other), or point A may move ½ in. and point B 1½ in., etc. The end rotations are readily obtained by the conjugate beam method. For $F_o = 1$ lb, the $\frac{M}{EI}$-diagram is as shown in Fig. 22, from which the end reactions on the conjugate beam are determined as presented in Table 2.

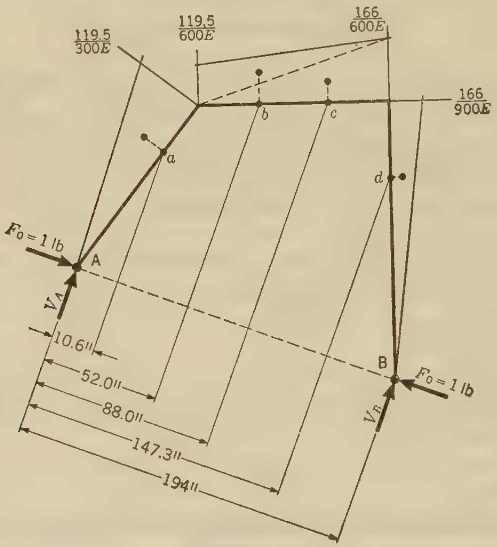


FIG. 22

TABLE 2.—DETERMINATION OF END REACTIONS ON THE CONJUGATE BEAM (FIG. 22)

Elastic weight (1)	Area of $\frac{M}{EI}$-diagram (2)	Lever arm from A (in.) (3)	Col. 2 × Col. 3 × E (4)
a........	$60 \times 119.5/300\,E = 23.9/E$	10.6	253.5
b........	$60 \times 119.5/600\,E = 11.95/E$	52.0	621.5
c........	$60 \times 166\ /600\,E = 16.6/E$	88.0	1,460
d........	$90 \times 166\ /900\,E = 16.6/E$	147.3	2,450
Total......	$69.1/E$	....	4,785.0

Hence,

$$V_B = \frac{4,785}{194\,E} = +\frac{24.6}{E} \qquad (42a)$$

and

$$V_A = -\frac{(69.1 - 24.6)}{E} = -\frac{44.5}{E} \qquad (42b)$$

in which V_B and V_A are the reactions at ends B and A of the conjugate beam, respectively.

The end reactions are the slopes of the beam. Using the author's value of 7,630 lb for F_o, $\theta_B = \frac{+ 24.6 \times 7{,}630}{30 \times 10^6} = + 0.00625$ radian; and $\theta_A = \frac{- 44.6 \times 7{,}630}{30 \times 10^6} = - 0.01135$ radian. The total change of slope is, therefore, 0.0176 radian, as obtained by the author, but the actual slopes at the ends are very different from the values in the paper. According to Mr. Bell's solution, this total change of slope is equally divided between the two ends. It will be noticed that in Example 6 the author states: "* * * Assuming this movement [2 in.] equally divided between the two ends, find the rotation* * *." Whether the relative displacement is equally or unequally divided will not affect the end rotations, and in the present case it is not possible for these to have equal values.

The writer is mystified by the author's use of the elastic centroid. If Figs. 11, 13, and 14 are compared, it would appear that the elastic centroid is the same in all cases—that is, irrespective of end-fixing conditions. Evidently, the author does not give the term "elastic centroid" its usual meaning, which would, for example, call for the location of the elastic centroid in the case of Fig. 14 to be at the midpoint of line AB.

In Fig. 10, inasmuch as the total change of slope between points A and B is zero due to the fixity of the ends, a simple check on the continuity of the frame is provided by the fact that the total area of the $\frac{M}{E\,I}$-diagram must be zero. Applying this check, and omitting E, as it is constant throughout, the total area of the $\frac{M}{E\,I}$-diagram $= \frac{60}{300}$ (237,220 − 245,400) + $\frac{60}{600}$ (253,580 − 245,400) + $\frac{90}{900}$ (255,580 − 282,220) = − 3,681—and not zero. It follows that the moments are in error. In fact, it will be found that there are discrepancies of the order of 5%. It would probably be agreed that a check on the self-consistency of the results of any graphical construction is always desirable, particularly so when it may easily be applied as in the present instance.

Finally, the procedure described in Section 7 of the paper for determining deflections suffers from the inherent drawback that it involves differences of quantities that are of the same order of magnitude. Thus, very small percentage errors in either of the quantities (the determinate and the indeterminate quantities) may lead to gross errors in the value for the deflection. This point is well illustrated by Example 8, in which an increase of 1% in the value of the determinate deflection combined with a reduction of 1% in the value of the indeterminate deflection leads to an increase of nearly 23% in the value of the total deflection. If the values for rotation are considered, the comparison is even more striking. Thus, changes of 1% in the determinate and indeterminate rotations give rise to a change of more than 125% in the value of the total rotation. As both of these quantities are determined graphically, it becomes questionable whether the graphical construction permits the attainment of a sufficient degree of accuracy.

Using the moment distribution method, the writer found in the present case that the moments at points A, E, D, and B were +242,080, −243,730, +257,270, and −267,410, respectively, as compared with the values shown in Fig. 10. These moments are readily checked, as ends A and B are fixed, and therefore the total change of slope is zero—that is, the total area of the $\frac{M}{E\,I}$-diagram is zero. Applying this check, $\frac{60}{300}(+\,242{,}080 - 243{,}730) + \frac{60}{600}(257{,}270 - 243{,}730) + \frac{90}{900}(+\,257{,}270 - 267{,}410) = +\,10$, which indicates a small residual error due to use of the slide rule. Hence, by the moment area method the value of the rotation at point D may at once be obtained as $\frac{1}{30 \times 10^6 \times 900}(257{,}270 \times 90 - 267{,}410 \times 90) = -\,0.0000338$ radian; and the value of the translation at point D as $\frac{1}{30 \times 10^6 \times 900}(257{,}270 \times 90 \times 60 - 267{,}410 \times 90 \times 120) = -\,0.05551$ in. These values should be compared with the author's values of −0.000094 radian and −0.062 in., respectively.

Elimination of all such errors in the paper as those mentioned would enable the reader better to appraise the value of this paper.

WILLIAM A. CONWELL,[12] M. ASCE.—The value of this paper is felt by the writer to lie in two characteristics: First, it is of more than academic interest; and, second, it assists in visualizing the action of a structure under loads. The first point is borne out by the fact that the author's method was developed to shorten the time required for solution of such problems in the field. This implies that the method has been successfully used in design offices and, therefore, has proved its worth in competition with other methods. The visualization of structural action obtained from this method is good. When these actions are referred to the centroid, however, there creeps into the method an element which is extraneous to the structure itself and is, therefore, somewhat of a departure. Mr. Bell relies on the methods of Castigliano in developing his thesis. In line with a visualization of a structural action, it might have been advantageous to use simply the geometry of flexure. It should be pointed out that the method is, of course, limited in application to bents of a single aisle or frames of one span.

The writer believes that, since the paper deals with a method of analysis which the author expects to compete successfully with other established methods, it should have contained some comparisons with the older methods and should have listed the advantages and disadvantages of each. He will attempt to make a few such comparisons with the thought that the author may like to comment on them in his closing discussion.

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[12] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

The moment-distribution method,[13] although not generally applicable to the curved structures adopted by the author in demonstrating the theory, is definitely applicable to the numerical examples illustrating his method. Moment distribution has, of course, been proved in the field. Since it relies on geometry, it is always intimately associated with the deformation of the structure and does not require the reference of deformations to any point outside the structure. Furthermore, in its application it is not limited to single spans. The writer was struck by the parallel between Mr. Bell's method and the column analogy[3] in the type of data employed. Inasmuch as all the data required for a solution of the problem by the column analogy are available before the author resorts to his circle of inertia, the point is raised as to whether its application might not be desirable. It is true that the column analogy in its working offers no information on the deformation of the structure; that knowledge must be obtained after the solution by a consideration of the bending moments developed. If, however, it is required to obtain quick values of the bending moments, the column analogy, which enables the computer to arrange computations in easily tabulated form, may prove an expeditious method of obtaining such results.

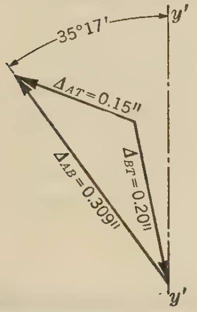

FIG. 23

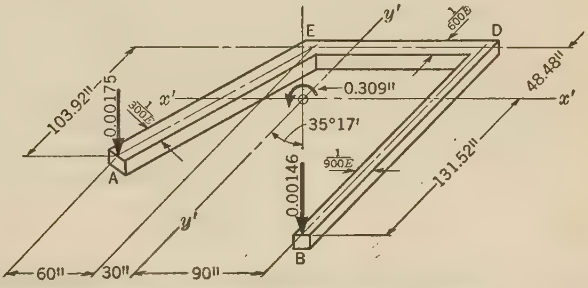

FIG. 24

Example 2 is a natural one for the application of the column analogy. Fig. 23 shows the vector triangle required to obtain the relative deflections of ends A and B of the frame. Fig. 24 shows an isometric view of the analogous column for the frame, to which the angular rotations have been applied as loads and the lateral movement has been applied as a moment about the line of the movement. Values for the analogous column shown in Fig. 24 are as follows:

$$A = \frac{0.8}{E} \text{ (lb}^{-1}\text{ in.}^{-1}) \quad \text{(43a)}$$

$$I'_x = \frac{1{,}720}{E} \text{ (lb}^{-1}\text{ in.)} \quad \text{(43b)}$$

$$I'_y = \frac{3{,}600}{E} \text{ (lb}^{-1}\text{ in.)} \quad \text{(43c)}$$

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[13] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

and

$$I'_{xy} = -\frac{165}{E}\,(\text{lb}^{-1}\ \text{in.}) \ldots\ldots\ldots\ldots\ldots\ldots (43d)$$

Fig. 25 indicates the unit stresses produced in the analogous column by the application of the deflections as loads. It is seen that such stresses in the column correspond to the bending moments in the frame for which the solution is sought. Incidentally, the values of the bending moments so obtained indicate the degree of difference which might be expected between the semigraphical solution of the author and the writer's analytical slide rule work.

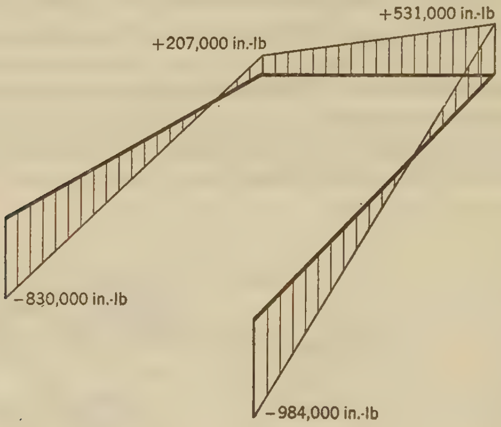

FIG. 25

Mr. Bell's review of a comparison of the centroidal method with the methods treated in this discussion and with others in regular use would, it is believed, form a valuable addition to the paper.

S. J. BELL,[14] Assoc. M. ASCE.—The discussions of the paper are appreciated by the writer. No claim was made that the method would replace all other methods of ring analysis; however, the hope that it would prove to be an interesting addition to them seems to be justified.

Professor Grinter points out that the centroidal method "* * * involves the use of common principles" and that the "* * * principles of ring analysis are few." These statements are true because all the methods of ring analysis are attempts to simplify the application of the same fundamental principles. Professor Grinter's comparisons of the centroidal method with the neutral-point method and with the column analogy are interesting. The writer agrees that the methods should be judged by

> "* * * clarity or ease of understanding, convenience in making routine calculations, and visual characteristics that aid one both to understand and to remember the procedures."

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[14] Supt., Refining Dept., Am. Republics Corp., Houston, Tex.

Mr. Gluckmann notes that results obtained by the centroidal method are similar to those obtained by the ellipse of elasticity which arrives at the amount and direction of the thrust by using the properties of the ellipse. The centroidal method obtains the same result by using Mohr's circle of inertia which is a graphical method of solving elliptical equations.

Mr. Yao shows that the components F_x and F_y of the thrust F_o may be found by using displacements parallel to the principal axes. There are many variations in the application of part or all of the method.

Professor Ferguson believes that the method will be found more useful for situations where "* * * movements of supports are the basic data and lack of symmetry is common." The writer has found this statement to be true. Some of the applications shown in the paper (for instance, applications to structures with both ends hinged) were included to give a more complete picture of how the procedure would apply to all types of end conditions.

Mr. Beaufoy finds it difficult to follow the paper because of the variability of dimensions in the quantities employed. The method was developed for a structure with constant values of E and I, the latter without subscript being the sectional moment of inertia. An explanation of how to apply the equations to structures with variable values of E and I is given in the paragraphs following Eqs. 20. If desired, Eqs. 21 may all be written for variable values of both E and I in which case the structural moment of inertia would be (compare with Eq. 21*b*)

$$I_s = Z^2 \frac{ds}{E\,I} \qquad \text{(44}a\text{)}$$

and the structural length would be (compare with Eq. 21*a*)

$$L_o = \frac{ds}{E\,I} \qquad \text{(44}b\text{)}$$

The writer has found it convenient to factor out the product $E\,I$ when it is constant throughout the structure.

The writer believes that there is nothing fundamentally wrong with Section 6, "Application to Structures with Both Ends Hinged." Moving the hinges A and B 2 in. closer together is a relative movement and any or all of the movement may be assumed to take place at either end. In any case the total force F_o through the ends will be the same, but the actual movement of the ends will be different. The movements in Fig. 14 are shown correctly for the assumption that the relative movement is equally divided between the two ends. The actual rotations of ends A and B in moving to positions A′ and B′ are equal to 0.0088 radian each. The rotations of the ends with reference to a line through the final positions A′ and B′ will be those shown in the text following Eqs. 42 of Mr. Beaufoy's discussion.

In Section 7, point D was chosen for the determination of both deflection and rotation because it was easy to illustrate the determinate moments, as may be noted in Fig. 17. The final displacements of the point are very small and the values found in Example 8 are not exact. The examples shown in the paper are for the purpose of illustrating the application of the method and no attempt has been made to check exact solutions. All the original illustrations were drawn on 8½-in. by 11-in. paper. If greater accuracy is desired it may be obtained by using larger scales.

Mr. Conwell suggests that it might have been advantageous to use the geometry of flexure in developing the method. The writer found that the development in the paper was shorter and took less explanation than any other he could devise.

The centroidal method was developed soon after the column analogy was published and before it had come to the attention of the writer. Had he been familiar with the column analogy at that time, in all probability, the centroidal method would not have been developed. The writer believes that both methods have points in their favor, and that individual experience and preference in each case will determine which will be used.

The writer wishes to thank all those who discussed the paper and is pleased that most of them considered it of interest.

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Paper No. 2383

A SIMPLIFIED METHOD OF ANALYZING SUSPENSION BRIDGES

By LING-HI TSIEN,[1] ESQ.

WITH DISCUSSION BY MESSRS. PEDRO B. J. GRAVINA, G. S. RAMASWAMY, HUI-SIEN LIU, AND LING-HI TSIEN.

SYNOPSIS

A rational method of analysis for the design of suspension bridges is presented in this paper. The proposed method gives simple and explicit equations of the horizontal tension in a cable and a direct method of determining maximum moments, shears, deflections, and slopes in the stiffening trusses. A slide rule may be used in all computations. The tiresome cut-and-try processes inherent in the existing methods are avoided.

Typical computations for the Mount Hope suspension bridge in Rhode Island illustrate the method, proving that the accuracy is high and that the saving in labor is considerable.

INTRODUCTION

1. *Description of Method.*—Many investigators have shown that the so-called "elastic theory" is not adequate for the exact design of a suspension bridge. The deflection theory is more accurate and is used extensively; but it involves many cut-and-try processes and a computing machine or a table of logarithms must be used to avoid possibilities of serious errors. The "trigonometric series" method developed by S. Timoshenko[2] has distinct advantages over the deflection theory method in several respects; but the expressions are not completely explicit, and the computations are lengthy and involved. Moreover, the location and magnitude of maximum bending moments and shears in a stiffening truss for various loading conditions cannot be determined directly without trial and error.

The method proposed herein eliminates all these inconveniences. It will be presented in three parts: Part I deals with the computation of horizontal tension in the cable; Part II with the moment equations in the stiffening truss;

NOTE.—Published in September, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof., Civ. Eng., National Univ. of Chekiang, Hangchow, Chekiang, China.

[2] "The Stiffness of Suspension Bridges," by S. Timoshenko, *Transactions*, ASCE, Vol. 94, 1930, p. 377.

and Part III with the design of the stiffening truss (determination of maximum moments, shears, deflections, and slopes).

Two important conclusions are reached as follows:

(*a*) When the live load moves on the bridge, the horizontal tension in a cable varies very closely according to certain circular functions irrespective of both the ratio between the dead load and the live load, and the degree of stiffness of the truss; and

(*b*) The equations of moments and shears can be reduced to explicit and simple forms in hyperbolic functions which are suitable for the direct determination of maxima by mathematics, with certain acceptable approximations.

In Parts I and III, some minor approximations are introduced in the derivation, but the maximum possible error is within a few percentage points and the calculated result is probably more accurate than that obtained by the cut-and-try method. The approximations are certainly justified by the considerable saving in labor and the mistakes that are avoided because the formulas are clear and explicit. Because of its simplicity and accuracy, the method can be used for preliminary design purposes as well as for the final analysis of a suspension bridge. To illustrate the design procedure, the paper includes a complete analysis of the Mount Hope suspension bridge between Bristol and Portsmouth, R. I.

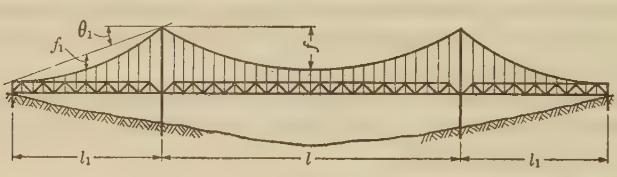

Fig. 1

Fig. 1 is a line diagram of a two-hinged suspension bridge of the type discussed in this paper. The theory applies equally well to the bridge with straight backstays.

2. Notation.—The letter symbols in this paper are defined where they first appear, in the text or by illustrations. They conform essentially to American Standard Letter Symbols for Structural Analysis, prepared by a Committee of the American Standards Association, with Society participation.

PART I. VARIATION OF THE HORIZONTAL COMPONENT OF CABLE TENSION

Live Load Moving Symmetrically Both Ways from the Center Line of the Span

3. Extreme Types of Suspension Bridge.—First, it is the writer's purpose to study the variation of H (the horizontal component of the tension in the cable due to live load) by analyzing two extreme cases applied to suspension bridges, namely:

Case *a*. The cable is not stiffened (moment of inertia of stiffening truss $I = 0$), and the ratio of live load to dead load, $q = p/w$, ranges from 0 to 1. (In other words, p varies from 0 to w.)

Case *b*. The cable is perfectly stiffened ($I \rightarrow \infty$), and q also ranges from 0 to 1.

It will be assumed first that the span length and cable length do not change. The effects of side span interaction, cable stretch, and temperature effect are discussed in sections 14 to 20.

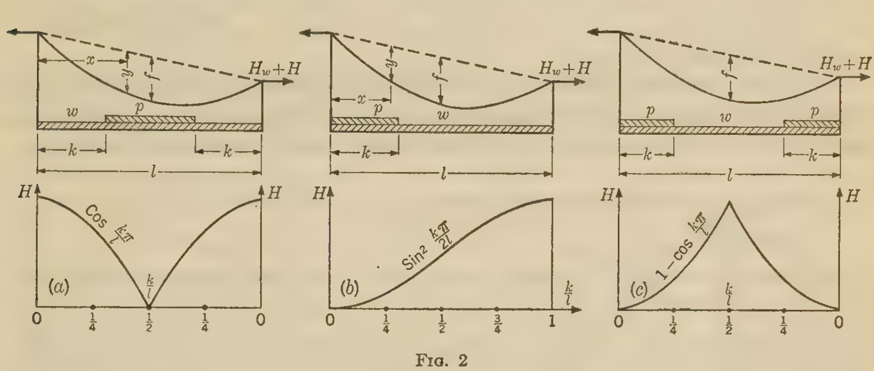
x
y
f
H_w+H
w
p
k
k
l
H
H
Cos $\frac{k\pi}{l}$
(a)
$\frac{k}{l}$
0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{1}{4}$
0
y
f
x
H_w+H
p
w
k
l
H
Sin² $\frac{k\pi}{2l}$
(b)
$\frac{k}{l}$
0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{3}{4}$
1
f
H_w+H
p
w
p
k
k
l
H
H
1 − cos $\frac{k\pi}{l}$
(c)
$\frac{k}{l}$
0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{1}{4}$
0

FIG. 2

4. *Case a: I = 0.*—Consider the inclined parabolic cable as shown in Fig. 2(*a*). The original ordinate of the cable[3] is

$$y = \frac{4\,f}{l^2}\,(l\,x - x^2) \quad \ldots\ldots(1)$$

When a center partial live load is applied, the cable will assume the shape of three parabolas with common tangents at their junctions, which occur at the two ends of the live load (see Fig. 2(*a*)). The new ordinates of the cable can be obtained as follows: When $x < k$, considering $\Sigma M = 0$ at section x,

$$(H_w + H)\,y_1 = \frac{w}{2}\,(l\,x - x^2) + \frac{p\,(l - 2\,k)}{2}\,x \quad \ldots\ldots(2a)$$

or

$$y_1 = \frac{w\,(l\,x - x^2) + p\,(l - 2\,k)\,x}{2\,(H_w + H)} \quad \ldots\ldots(2b)$$

When $l - k > x > k$,

$$(H_w + H)\,y_2 = \frac{w}{2}\,(l\,x - x^2) + \frac{p\,(l - 2\,k)}{2}\,x - \frac{p\,(x - k)^2}{2} \quad \ldots\ldots(3a)$$

or

$$y_2 = \frac{w\,(l\,x - x^2) + p\,(l\,x - k^2 - x^2)}{2\,(H_w + H)} \quad \ldots\ldots(3b)$$

If the cable length does not change, the external work performed in deflecting the cable must be equal to zero; hence,

$$\int_0^k w\,(y_1 - y)\,dx + \int_k^{l/2} \left(w + \frac{p}{2}\right)(y_2 - y)\,dx = 0 \quad \ldots\ldots(4)$$

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[3] "Preliminary Design of Suspension Bridges," by Shortridge Hardesty and Harold E. Wessman, *Transactions*, ASCE, Vol. 104, 1939, p. 582, Eq. 2.

Substituting Eqs. 1, 2*b*, and 3*b* in Eq. 4, performing the integrations and combining terms,

$$H = \frac{p\,l^2}{8\,f}\left[\frac{2 - 4\,\frac{k^2}{l^2}\left(3 - 2\,\frac{k}{l}\right) + q\left(1 - 2\,\frac{k}{l}\right)^2\left(1 + 4\,\frac{k}{l}\right)}{2 + q - 2\,q\,\frac{k^2}{l^2}\left(3 - 2\,\frac{k}{l}\right)}\right]$$

$$= \frac{p\,l^2}{8\,f}\,F_1(k,\,q)\,..(5)$$

in which q is the ratio p/w.

Therefore, when the live load, p, moves symmetrically both ways from the center of the span, k/l varies from 0.5 to 0. The horizontal component of cable tension H due to live load will be a function of k and q, as defined by Eq. 5.

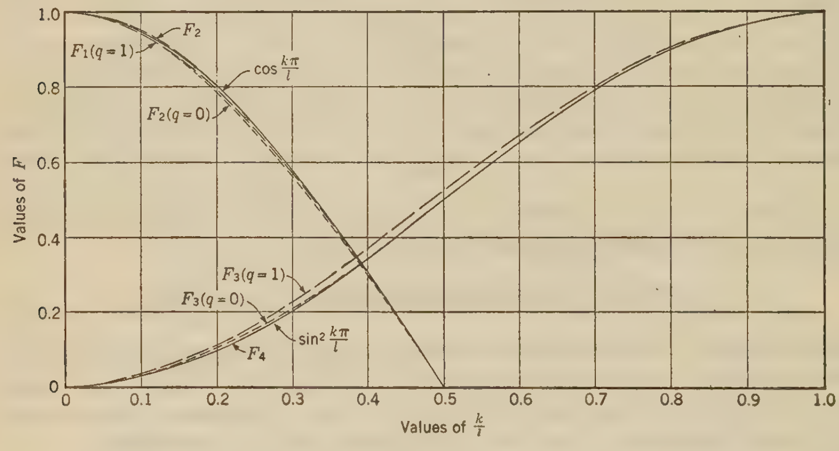

FIG. 3

5. *Case b:* $I \rightarrow \infty$.—When the stiffness of bridge increases to a certain extent, the cable will remain practically undistorted, so that the so-called "elastic theory" becomes almost true and will give exact solution. When a central partial live load is applied, the "elastic theory" gives the equation for H as follows:[4]

$$H = \frac{\frac{1}{15}\,p\,f\,l^3\left[1 - \frac{k^2}{l^2}\left(5 - 5\,\frac{k^2}{l^2} + 2\,\frac{k^3}{l^3}\right)\right]}{\frac{8}{15}\,f^2\,l + \frac{I\,L_s}{A}}\,\ldots\ldots\ldots\ldots(6a)$$

Neglecting the effect of cable stretch, $\frac{I\,L_s}{A}$, it can be reduced to

$$H = \frac{p\,l^2}{8\,f}\left[1 - \frac{k^2}{l^2}\left(5 - 5\,\frac{k^2}{l^2} + 2\,\frac{k^3}{l^3}\right)\right] = \frac{p\,l^2}{8\,f}F_2(k)\,\ldots\ldots\ldots(6b)$$

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[4] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 9th Ed., 1916, Pt. II, p. 219, Eq. 15.

Fig. 3 shows that the two functions F_1 and F_2 are practically the same when q varies from 0 to 1, and it can be very closely represented by the trigonometric function $\cos \frac{k\pi}{l}$ for $\frac{k}{l}$ from 0.5 to 0. The difference between $\cos \frac{k\pi}{l}$ with F_2 is practically indistinguishable. Thus, it may be concluded that:

Theorem 1.—

> When live load moves symmetrically from each side of the center of the span of a suspension bridge (with parabolic cable and a horizontal or inclined chord), the horizontal component of the induced cable tension, H, increases (very closely) in proportion to the function $\cos \frac{k\pi}{l}$ irrespective of both the degree of truss stiffness and the ratio of live load to dead load, within limits $q = 0$ to $q = 1$.

This may be expressed by

$$H = \frac{p\, l^2}{8\, f\, D} \cos \frac{k\pi}{l} \quad \text{(7)}$$

in which k varies from $\frac{1}{2}l$ to 0; and D is a correction factor for cable stretch and side span effects, being unity in the present case. To estimate these effects, one needs only to consider the particular case when the span is fully covered by a uniform live load ($k = 0$). Factor D is a function of the physical properties of the bridge and will remain nearly constant for other cases of loading.

LIVE LOAD ADVANCING FROM ONE END OF SPAN

6. *Case a: $I = 0$.*—As in the case of symmetrical live loading, the original ordinates of the cable are determined by Eq. 1. When $x < k$, the equation $\Sigma M = 0$ gives (Fig. 2(*b*))

$$(H_w + H)\, y_1 = \frac{w}{2}(l\,x - x^2) + \frac{p\,k\left(l - \frac{k}{2}\right)}{l}\, x - \frac{p\,x^2}{2} \quad \text{(8a)}$$

or

$$y_1 = \frac{w\,(l\,x - x^2) + p\left(2\,k\,x - \frac{k^2\,x}{l} - x^2\right)}{2\,(H_w + H)} \quad \text{(8b)}$$

When $x > k$,

$$(H_w + H)\, y_2 = \frac{w}{2}(l\,x - x^2) + \frac{p\,k\left(l - \frac{k}{2}\right)}{l}\, x - p\,k\left(x - \frac{k}{2}\right) \quad \text{(9a)}$$

or

$$y_2 = \frac{w\,(l\,x - x^2) + p\left(-\frac{k^2\,x}{l} + k^2\right)}{2\,(H_w + H)} \quad \text{(9b)}$$

Substituting the values of y_1 and y_2 in Eqs. 8 and 9 in the work equation,

$$\int_0^k \left(w + \frac{p}{2}\right)(y_1 - y)\, dx + \int_k^l w\,(y_2 - y)\, dx = 0 \quad \text{(10)}$$

and performing integrations,

$$H = \frac{p\,l^2}{8\,f}\left[\frac{2\,\frac{k^2}{l^2}\left(3 - 2\,\frac{k}{l}\right) + q\,\frac{k^3}{l^3}\left(4 - 3\,\frac{k}{l}\right)}{2 + q\,\frac{k^2}{l^2}\left(3 - 2\,\frac{k}{l}\right)}\right] \quad \text{.........(11)}$$

7. *Case b: I* → ∞.—For the reason stated in section 6, the "elastic theory" gives, for this case of loading, the equation of H as follows:[5]

$$H = \frac{\frac{1}{30}\,p\,f\,l^3\left[\frac{k^2}{l^2}\left(2\,\frac{k^3}{l^3} - 5\,\frac{k^2}{l^2} + 5\right)\right]}{\frac{8}{15}\,f^2\,l + \frac{I\,L_s}{A}} \quad \text{............(12)}$$

Again neglecting the effect of cable stretch, $\frac{I\,L_s}{A}$, Eq. 12 becomes

$$H = \frac{p\,l^2}{8\,f}\left[\frac{k^2}{2\,l^2}\left(2\,\frac{k^3}{l^3} - 5\,\frac{k^2}{l^2} + 5\right)\right] = \frac{p\,l^2}{8\,f}\,F_4(k) \quad \text{..........(13)}$$

Fig. 3 shows that the two functions F_3 and F_4 are practically the same when q varies from 0 to 1 and that they both vary very closely according to the trigonometric function $\sin^2 \frac{k\,\pi}{2\,l}$ for $\frac{k}{l}$ from 0 to 1. The curve of $\sin^2 \frac{k\,\pi}{2\,l}$ almost coincides with F_4 and F_3 ($q = 0$). A conclusion can then be made:

Theorem 2.—

> When the live load advances from one end of the span of a suspension bridge (with a horizontal or inclined chord), the horizontal component of induced cable tension varies (very closely) in proportion to the function $\sin^2 \frac{k\,\pi}{2\,l}$, irrespective of both the degree of truss stiffness and the ratio of live load to dead load (from $q = 0$ to $q = 1$).

Theorem 2 may be expressed algebraically as follows:

$$H = \frac{p\,l^2}{8\,f\,D}\sin^2 \frac{k\,\pi}{2\,l} \quad \text{......................(14)}$$

in which k varies from 0 to l. Factor D is the same as in Eq. 7. Eq. 14 also can be written as follows:

$$H = \frac{p\,l^2}{8\,f\,D}\,\frac{1}{2}\left(1 - \cos \frac{k\,\pi}{l}\right) \quad \text{..................(15)}$$

In other words, the horizontal tension H due to a partial loading from one end (Fig. 2(*b*)) is equal to the half of the difference between H due to full loading and H due to a corresponding center loading (Fig. 2(*a*)). Consequently, one may write Theorem 3, as follows:

Theorem 3.—

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[5] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 9th Ed., 1916, Pt. II, p. 218, Eq. 13.

In suspension bridges, for different loading conditions, the cable tension can be computed by superposition, with sufficient accuracy for all practical purposes.

LIVE LOAD ADVANCING FROM BOTH ENDS OF SPAN

8. Horizontal Tension H for a Live Load Advancing from Both Ends.—When the live load advances symmetrically from both ends of the span (Fig. 2(*c*)), superposition yields

$$H = \frac{p\,l^2}{8\,f\,D}\left(1 - \cos\frac{k\,\pi}{l}\right) \qquad (16)$$

in which k varies from 0 to $\frac{1}{2}\,l$. Factor D is the same as in Eq. 7.

A MOVING CONCENTRATED LOAD

9. Horizontal Tension H for a Moving Concentrated Load.—A concentrated load P can be considered analogous to a uniform load covering an infinitesimal length—that is, $p \times dk$. Therefore,

$$H = \frac{p\,l^2}{8\,f\,D}\left[\sin^2\frac{\pi}{2\,l}\,(k + dk) - \sin^2\frac{\pi\,k}{2\,l}\right] = \frac{p\,dk\,l^2}{8\,f\,D}\,\frac{d}{dk}\sin^2\frac{k\,\pi}{2\,l} \qquad (17a)$$

or

$$H = \frac{P\,l\,\pi}{16\,f\,D}\sin\frac{k\,\pi}{l} \qquad (17b)$$

GENERAL EXPRESSION FOR H WHEN THE MAIN SPAN IS LOADED

10. Sag Changes.—When a live load moves on a bridge, or when a temperature change occurs, stress is induced in the cable and in the truss. Any change of cable tension or temperature causes a change in the length of the cable and affects the shape of the curve, increasing or decreasing the center sag. A change in cable stress also causes a change in the side span sag. This in turn causes a movement of the tower tops, which affects the main span length and its sag.

11. Moment in Stiffening Truss When the Sag Changes.—Whenever the center sag changes by an amount Δf, the stiffening truss is raised or lowered by the same amount at the center. If the suspender forces (s) are uniformly distributed, the center deflection of the truss will be given by

$$\Delta f = \frac{5}{384}\,\frac{s\,l^4}{E\,I} \qquad (18a)$$

and the center moment by

$$M = \frac{s\,l^2}{8} = \frac{9.6\,E\,I}{l^2}\,\Delta f \qquad (18b)$$

In other words, a change in center sag, Δf, will produce a positive or negative moment at the center of the stiffening truss as determined by Eq. 18*b*. A positive Δf (downward) produces a positive moment (compression in the top chord).

12. Span Fully Loaded and Subjected to a Temperature Change.—At mean temperature and when there is no live load on the bridge, the lengths of the suspender rods are so adjusted that the stiffening truss is relieved of all stresses and the dead load is uniformly supported by the cable, so that

$$H_w = \frac{w\,l^2}{8\,f} \quad \text{(19)}$$

The loading conditions and the temperature are now changed as follows:

Description	Condition
Total uniform load per unit length	$w + p$
Temperature change	$\pm \Delta t$
Horizontal component of cable tension	$H_w + H$
Cable sag in main span	$f + \Delta f_p \pm \Delta f_t$

Considering the static equation, $\Sigma M = 0$, the formula for the center section of the main span can be written as follows:

$$(H_w + H)\,(f + \Delta f_p \pm \Delta f_t) + \frac{9.6\,E\,I}{l^2}\,(\Delta f_p \pm \Delta f_t) = \frac{(w + p)\,l^2}{8} \quad \text{(20a)}$$

or

$$H\left\{1 + \frac{\Delta f_p}{f}\left[1 + \frac{H_w}{H}\left(1 + \frac{9.6\,E\,I}{H_w\,l^2}\right)\right] \pm \frac{\Delta f_t}{f}\right\} = \frac{p\,l^2}{8\,f} \mp H_w\left(1 + \frac{9.6\,E\,I}{H_w\,l^2}\right)\frac{\Delta f_t}{f} \quad \text{(20b)}$$

Let $q = \frac{p}{w}$; $c^2{}_0 = \frac{H_w}{E\,I}$; $B = 1 + \frac{9.6\,E\,I}{H_w\,l^2} = 1 + \frac{9.6}{c^2{}_0\,l^2}$. Then

$$H = \frac{p\,l^2}{8\,f}\,\frac{1 \mp T}{1 + N \pm N_t} = \frac{p\,l^2}{8\,f\,D_{\pm t}}\,(1 \mp T) \quad \text{(21)}$$

This is the expression of H for the case of a full span live load, in which

$$T = \frac{B}{q}\,\frac{\Delta f_t}{f} \quad \text{(22a)}$$

$$N = \frac{\Delta f_p}{f}\left(1 + \frac{H_w}{H}\,B\right) \quad \text{(22b)}$$

$$N_t = \frac{\Delta f_t}{f} \quad \text{(22c)}$$

and

$$D_{\pm t} = 1 + N \pm N_t \quad \text{(22d)}$$

13. Span Arbitrarily Loaded.—For the case of partial loading (see sections 5, 7, and 8), the general expression of the horizontal component of cable tension is as follows (see subsequently in Table 1):

$$H = \frac{p\,l^2}{8\,f\,D_{\pm t}}\left[F\left(\frac{k}{l}\right) \mp T\right] \quad \text{(23)}$$

in which $F\left(\frac{k}{l}\right)$ represents certain trigonometrical functions that vary according to the loading distribution (see sections 5 to 8), and B, N, N_t, T, and $D_{\pm t}$ are the constants defined in section 12. The upper signs of T, N_t, and $D_{\pm t}$ correspond to a rise of temperature, and the lower ones to a drop of temperature.

Determination of Design Constants

14. Sag Changes Δf_t and Δf_p.—In Eqs. 22*a*, 22*b*, and 22*c*, Δf_t is the sag change due to change in temperature, and the sag change Δf_p is partly due to the effect of side span and partly due to the stretch of the cable. These two factors, respectively, give the two components N_s and N_c of the constant N.

15. Factor N_s to Reflect the Effect of Side Spans.—An increase of H in cable produces a change in sag, Δf_1, in the side span which can be evaluated as follows: Apply the static equation, $\Sigma M = 0$, at the center of the side span:

$$(H_w + H)(f_1 + \Delta f_1) + \frac{9.6\,E\,I_1}{l^2_1}\Delta f_1 = \frac{w_1\,l^2_1}{8} = H_w\,f_1 \ldots\ldots (24a)$$

from which

$$\Delta f_1 = \frac{-1}{1 + \frac{H_w}{H}B_1}f_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

In Eq. 24*b*,

$$B_1 = 1 + \frac{9.6\,E\,I_1}{H_w\,l^2_1} = 1 + \frac{9.6}{c^2_{01}\,l^2_1} \ldots\ldots\ldots\ldots\ldots\ldots (25a)$$

and, in Eq. 25*a*,

$$c^2_{01} = \frac{H_w}{E\,I_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

Because of Δf_1, the length of cable in the side span will change by an amount equal to[6]

$$\Delta l_1 = -\frac{16\,n_1}{3\sec^2\alpha_1}\Delta f_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

in which $n_1 = f_1/l_1$ is the sag ratio of the side span, and α_1 is the inclination of the chord of side span cable. When the saddle moves on the tower tops, a change occurs in the length of the main span, equal to

$$\Delta l = -2\,\Delta l_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26b)$$

This, in turn, affects the sag of main span by an amount Δf equal to[6]

$$\Delta f = -\frac{15 - 40\,n^2 + 288\,n^4}{16\,n\,(5 - 24\,n^2)}\Delta l \doteqdot -\frac{3\,\Delta l}{16\,n(1 - 2.35\,n^2)} \ldots\ldots (27)$$

Substituting the values from Eqs. 24*b* and 26 in Eq. 27,

$$\frac{\Delta f}{f} = \frac{2\,r\,n^2_1}{\sec^2\alpha_1\,n^2\,(1 - 2.35\,n^2)} \times \frac{1}{1 + \frac{H_w}{H}B_1} \ldots\ldots\ldots\ldots (28)$$

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[6] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1929, p. 11, Eq. (52).

in which $n = f/l$, the sag ratio of the main span, and $r = l_1/l$. Then the value of N due to side span effect will be

$$N_s = \frac{2\, r\, n_1^2}{\sec^2 \alpha_1\, n^2\, (1 - 2.35\, n^2)} \times \frac{1 + \frac{H_w}{H} B}{1 + \frac{H_w}{H} B_1} \qquad (29a)$$

Neglecting temperature change, $H = \frac{p\, l^2}{8\, f\, (1 + N)}$ and $\frac{H_w}{H} = \frac{1 + N}{q}$, which may be assumed to be $\frac{1.2}{q}$ as a first approximation. In a suspension bridge, with the center span fully loaded, the value of $(1 + N)$ usually varies between 1.10 to 1.30. As $\frac{\Delta f}{f}$ is small compared to unity, an error of 10% in the value of $\frac{\Delta f}{f}$ will involve an error of only 1% or 2% in the final value of H. Moreover, the error more or less compensates for itself since, in Eq. 29*a*, it appears both in the numerator and in the denominator. Therefore, the value of N_s can be written with sufficient accuracy as follows:

$$N_s = \frac{2\, r}{\sec^2 \alpha_1\, (1 - 2.35\, n^2)}\, \frac{n_1^2}{n^2}\, \frac{q + 1.2\, B}{q + 1.2\, B_1} \qquad (29b)$$

16. Factor N_c to Reflect Cable Stretch.—The total virtual work (W_1) due to the vertical displacement η of the suspender forces (s) plus the cable weight (g) must be equal to the total work (W_2) done by the cable tension ($H_w + H$) in stretching the cable. These work quantities W_1 and W_2 are expressed, approximately,[7] as the integrated products of the forces and their respective displacements, as follows:

$$W_1 = \int_l \left(g + \frac{s}{2} \right) \eta\, dx = \frac{8\, f}{l^2} \left(H_w + \frac{H}{2} \right) \int_l \eta\, dx \qquad (30a)$$

and

$$W_2 = \sum \left(H_w + \frac{H}{2} \right) \left(\frac{H}{E_c\, A_0} \int_l \frac{A_0}{A} \frac{ds^3}{dx^2} \right) \qquad (30b)$$

The symbol $\sum$ (in Eq. 30*b*) denotes the summation of work done in stretching the cable in all spans and segments of cable. Eq. 30*a* deals only with the main span, since the side span effect is considered separately. In Eq. 30*b*, A denotes the cable section at any point, and A_0 denotes the cable section at midspan.

When the main span is fully loaded, the cable deforms from the initial parabola (sag $= f$) to another parabola with a bigger sag equal to $f + \Delta f$; therefore, the deflection η may be expressed by

$$\eta = \frac{4\, \Delta f}{l^2} (l\, x - x^2) \qquad (31)$$

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[7] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1929, p. 250.

Substituting in Eq. 30*a* and integrating,

$$W_1 = \frac{8f}{l^2}\left(H_w + \frac{H}{2}\right)\tfrac{2}{3}\,\Delta f\, l \ldots\ldots\ldots\ldots (32a)$$

Let $L_s = \sum \int_l \frac{A_0}{A}\frac{ds^3}{dx^2}$, then Eq. 30*b* becomes

$$W_2 = \left(H_w + \frac{H}{2}\right)\frac{H}{E_c A_0} L_s \ldots\ldots\ldots\ldots (32b)$$

For a wire cable having a uniform section A, one may write, with sufficient accuracy[8]

$$L_s = \sum l\,(\sec^3 \alpha + 8\, n^2) \ldots\ldots\ldots\ldots (33a)$$

For an eyebar cable with variable A proportional to the slope secant $\frac{ds}{dx}$, one may write, with sufficient accuracy,[8]

$$L_s = \sum l\,\left(\sec^2 \alpha + \frac{16}{3} n^2\right) \ldots\ldots\ldots\ldots (33b)$$

in which $n = f/l$ and α is the inclination of chord in any span and segment of cable. The symbol $\sum$ denotes the summation of similar expressions for all spans, and segments of cable. Equating W_1 and W_2,

$$\frac{\Delta f}{f} = \frac{3}{16}\frac{H}{E_c A_0}\frac{L_s}{n^2 l} \ldots\ldots\ldots\ldots (34)$$

The value of N due to cable stretch will be

$$N_c = \frac{3}{16}\frac{H}{E_c A_0}\frac{L_s}{n^2 l}\left(1 + \frac{H_w}{H} B\right) \ldots\ldots\ldots\ldots (35a)$$

or, using $\frac{H_w}{H} = \frac{1.2}{q}$ as before,

$$N_c = \frac{0.187}{n^2}\frac{H_w}{E_c A_0}\frac{L_s}{l}\left(\frac{q}{1.2} + B\right) \ldots\ldots\ldots\ldots (35b)$$

17. Factors N_t and T in Relation to Temperature Change.—When the temperature changes, the cable lengthens or shortens by a distance which changes the sag, f, an amount Δf_t. The extent of the deformation can be evaluated in a manner similar to that for determining the effect of cable stretch due to live load. The virtual work, W_1, done in the vertical displacement is equal to

$$W_1 = \frac{8f}{l^2}\left(H_w + \frac{H}{2}\right)\tfrac{2}{3}\,\Delta f_t\, l \ldots\ldots\ldots\ldots (36a)$$

The work, W_2, done by the cable tension $H_w + H$ is equal to

$$W_2 = \left(H_w + \frac{H}{2}\right)\epsilon\,\Delta_t\, L_t \ldots\ldots\ldots\ldots (36b)$$

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[8] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1929, p. 251.

in which ϵ is the temperature coefficient of expansion of cable; Δ_t is the temperature change; and[8]

$$L_t = \sum \int \frac{ds^2}{dx} = \sum l \left(\sec^2 \alpha + \frac{16}{3} n^2 \right) \qquad (37)$$

Equating W_1 and W_2,

$$\frac{\Delta f_t}{f} = \frac{3}{16} \frac{\epsilon \Delta_t L_t}{n^2 l} \qquad (38a)$$

Then

$$N_t = \frac{0.187 \epsilon \Delta_t L_t}{n^2 l} \qquad (38b)$$

and

$$T = \frac{B}{q} N_t = \frac{0.187 \epsilon \Delta t L_t B}{n^2 l q} \qquad (38c)$$

GENERAL EXPRESSION OF H WHEN THE SIDE SPAN IS LOADED

18. Both Side Spans Fully Loaded.—Let H be the horizontal component of cable tension induced by a live load, p, which fully covers both side spans. The sag of the main span will be changed by an amount $\Delta f'$.

Applying the condition $\sum M = 0$ at the center of the main span,

$$(H_w + H)(f + \Delta f') + \frac{9.6 E I}{l^2} \Delta f' = \frac{w l^2}{8} = H_w f \qquad (39a)$$

Hence,

$$H = \frac{p l^2}{8 f} \frac{B}{q} \frac{-\frac{\Delta f'}{f}}{1 + \frac{\Delta f'}{f}} \qquad (39b)$$

As $\frac{\Delta f'}{f}$ is very small compared to unity, the value of H due to side span loads is small as compared to that when the main span is fully loaded. Considering the cable stretch and side span effects, the equation for H may be written, with sufficient accuracy, as follows:

$$H = \frac{p l^2}{8 f D_{st}} (2 S) \qquad (40a)$$

in which

$$S = -\tfrac{1}{2} \frac{B}{q} \frac{\Delta f'}{f} \qquad (40b)$$

19. One Side Span Arbitrarily Loaded.—According to the conclusions of sections 5, 7, and 8 and the results given in section 18, when a side span is subject to partial loading, the general expression for H will be

$$H = \frac{p l^2}{8 f D} \left[S F \left(\frac{k}{l_1} \right) \right] \qquad (41)$$

in which $F\left(\frac{k}{l_1}\right)$ is a certain trigonometrical function according to the loading distribution (see subsequently in Table 1).

20. Computation of Factor S.—According to Eq. 40*b*,

$$2\,S = -\frac{B}{q}\,\frac{\Delta f'}{f}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

in Eq. 42, $\Delta f'$ is the change of center span sag due to full loading of both side spans. Both $\frac{\Delta f'}{f}$ and H are relatively small in this case. The cable stretch, which depends only upon the magnitude of H, can therefore be neglected. Only the value of $\Delta f'$ due to side span effect need be considered.

By a similar process as given in section 15,

$$\frac{\Delta f'}{f} = \frac{-\,2}{\sec^2\alpha_1\,(1 - 2.35\,n^2)}\,\frac{n_1}{n}\,\frac{f_1}{f}\,\frac{q_1}{B_1}\ldots\ldots\ldots\ldots\ldots(43)$$

in which $q_1 = \frac{p}{w_1}$. Since, by Eq. 19, $H_w = \frac{w\,l^2}{8\,f} = \frac{w_1\,l^2{}_1}{8\,f_1}$, the same intensity of live load (p) yields $\frac{q_1}{q} = \frac{w}{w_1} = \frac{l^2{}_1}{l^2}\,\frac{f}{f_1}$ and, using $r = l_1/l$,

$$S = -\,\tfrac{1}{2}\,\frac{B}{q}\,\frac{\Delta f'}{f} = \frac{r^2}{\sec^2\alpha_1\,(1 - 2.35\,n^2)}\,\frac{n_1}{n}\,\frac{B}{B_1}\ldots\ldots\ldots\ldots(44)$$

Computation of H for a Moving Concentrated Load

21. H-Equation for a Moving Concentrated Load.—Section 9 permits writing the following general expression for H due to a moving concentrated load P, applied at a distance k from the left support:

$$H = \frac{P\,l\,\pi}{16\,f\,D_{\pm t}}\left(\sin\frac{k\,\pi}{l} \mp T\right)\ldots\ldots\ldots\ldots\ldots\ldots(45)$$

By reasoning similar to that in sections 10 to 17, the constants $D_{\pm t}$ and the factors T are the same as in the case of uniform live load, except that the live load ratio, q, must be replaced by $q' = \frac{P\,\pi}{2\,w\,l}$ in the formulas.

Summary, Part I

22. Design Formulas of H.—The equations of H for various loading conditions useful in design are presented in Table 1. They are exceedingly simple in form and are as accurate as the results computed by the deflection theory. These values are used to compute the force H for several existing bridges (such as the Mount Hope Bridge, in Rhode Island, Manhattan Bridge in New York, N. Y., and in a numerical example solved[9] by J. I. Parcel and the late G. A. Maney, Members, ASCE), and the results for different loading conditions check very closely with those calculated by the more exact equations of the deflection theory.

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[9] "An Elementary Treatise on Statically Indeterminate Stresses," by J. I. Parcel and G. A. Maney, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1936.

TABLE 1.—EQUATIONS OF H FOR VARIOUS LOADING CONDITIONS

Constants

$$D_{\pm t} = 1 + N_s + N_e \pm N_t$$

$$T = \frac{B}{q} N_t$$

$$S = \frac{r^2}{\sec^2 \alpha_1 \, (1 - 2.35\, n^2)} \frac{n_1}{n} \frac{B}{B_1}$$

$$B = 1 + \frac{9.6}{c^2_0 \, l^2}$$

$$B_1 = 1 + \frac{9.6}{c^2_{01} \, l^2_1}$$

$$N_s = \frac{2\, r}{\sec^2 \alpha_1 \, (1 - 2.35\, n^2)} \frac{n^2_1}{n^2} \frac{q + 1.2\, B}{q + 1.2\, B_1}$$

$$N_e = \frac{0.187}{A_0 \, E_e} \frac{H_w}{n^2} \frac{L_s}{l} \left(\frac{q}{1.2} + B \right)$$

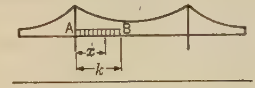
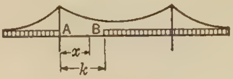
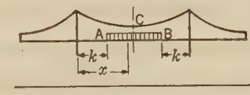
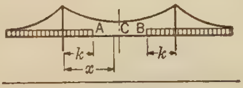
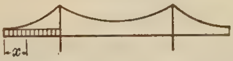
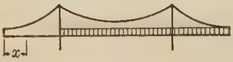
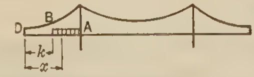
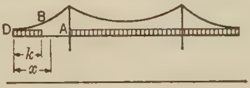
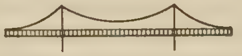
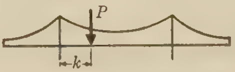
$$N_t = \frac{0.187\, \epsilon\, \Delta t\, L_t}{n^2\, l}$$

$$r = \frac{l_1}{l};\ n = \frac{f}{l};\ n_1 = \frac{f_1}{l_1}$$

$$c^2_0 = \frac{H_w}{E\, I};\ c^2_{01} = \frac{H_w}{E\, I_1}$$

$$q = \frac{p}{w}, \text{ for uniform live load}$$

$$q = \frac{P\, \pi}{2\, w\, l}, \text{ for a concentrated load}$$

Case	Loading condition	Tempera-ture	Equation
I		Highest	$H_{\text{I}} = \frac{p\, l^2}{8\, f\, D_{+t}} \left(\sin^2 \frac{k\, \pi}{2\, l} - T \right)$
II		Lowest	$H_{\text{II}} = \frac{p\, l^2}{8\, f\, D_{-t}} \left(\cos^2 \frac{k\, \pi}{2\, l} + 2\, S + T \right)$
III		Highest	$H_{\text{III}} = \frac{p\, l^2}{8\, f\, D_{+t}} \left(\cos \frac{k\, \pi}{l} - T \right)$
IV		Lowest	$H_{\text{IV}} = \frac{p\, l^2}{8\, f\, D_{-t}} \left(1 - \cos \frac{k\, \pi}{l} + 2\, S + T \right)$
V		Highest	$H_{\text{V}} = \frac{p\, l^2}{8\, f\, D_{+t}} (S - T)$
VI		Lowest	$H_{\text{VI}} = \frac{p\, l^2}{8\, f\, D_{-t}} (1 + S + T)$
VII		Highest	$H_{\text{VII}} = \frac{p\, l^2}{8\, f\, D_{+t}} \left(S \cos^2 \frac{k\, \pi}{2\, l_1} - T \right)$
VIII		Lowest	$H_{\text{VIII}} = \frac{p\, l^2}{8\, f\, D_{-t}} \left(1 + S + S \sin^2 \frac{k\, \pi}{2\, l_1} + T \right)$
IX		Lowest	$H = \frac{p\, l^2}{8\, f\, D_{-t}} (1 + 2\, S + T)$
X		Highest or lowest	$H = \frac{P\, l\, \pi}{16\, f\, D_{\pm t}} \left(\sin \frac{k\, \pi}{l} \mp T \right)$

PART II. EQUATIONS FOR MOMENTS M IN A STIFFENING TRUSS

INTRODUCTION

23. Assumptions.—The following derivation of the moment equations involves the same assumptions as are used in the deflection theory. The equations are the same as those obtained by the deflection theory, but they are condensed in forms more suitable for further mathematical treatments. The equations of shear, slope, and deflection, of the stiffening truss, can be obtained easily by differentiating or integrating the M-equations.

A symbol Q is introduced in these equations; thus:

$$Q = H\frac{8f}{p\,l^2} \qquad \text{(46)}$$

It will be shown that the major and direct effect of H on the truss appears in this term.

THE STIFFENING TRUSS UNDER VARIOUS LOADING CONDITIONS

24. Derivation of M-Equations.—When the bridge is subject only to dead load (w), and at mean temperature, the suspender force is intended to be equal to w, and the differential equation of the cable curve is

$$\frac{d^2y}{dx^2} = \frac{-w}{H_w} \qquad \text{(47)}$$

When an additional suspender force, s, due to live load or temperature change, acts upon the cable, the cable ordinates become $y + \eta$, and the differential equation becomes

$$\frac{d^2(y+\eta)}{dx^2} = -\frac{w+s}{H_w + H} \qquad \text{(48a)}$$

or

$$-w - \frac{H}{H_w}w + (H_w + H)\frac{d^2\eta}{dx^2} = -w - s \qquad \text{(48b)}$$

Since the differential equation of curvature for the stiffening truss is

$$\frac{d^2\eta}{dx^2} = -\frac{M}{E\,I} \qquad \text{(49)}$$

therefore Eq. 48b gives

$$s = p\,Q + c^2\,M \qquad \text{(50)}$$

in which

$$Q = H\frac{8f}{p\,l^2} \qquad \text{(51a)}$$

and

$$p\,Q = \frac{H}{H_w}w \qquad \text{(51b)}$$

and

$$c^2 = \frac{H_w + H}{E\,I} \qquad \text{(51c)}$$

For the loaded and unloaded segments of truss, respectively,

$$\frac{d^2M}{dx^2} = s - p \quad \ldots\ldots (52a)$$

and

$$\frac{d^2M}{dx^2} = s \quad \ldots\ldots (52b)$$

Substituting Eqs. 52 in Eq. 50,

$$\frac{d^2M}{dx^2} - c^2 M = -p(1 - Q) \quad \ldots\ldots (53a)$$

and

$$\frac{d^2M}{dx^2} - c^2 M = p\,Q \quad \ldots\ldots (53b)$$

By solving these differential equations, the moment equation for the loaded segment of truss becomes

$$M = C' \cosh (c\,x) + C'' \sinh (c\,x) + \frac{p}{c^2}(1 - Q) \quad \ldots\ldots (54a)$$

and that, for the unloaded segment,

$$M = C' \cosh (c\,x) + C'' \sinh (c\,x) - \frac{p}{c^2}Q \quad \ldots\ldots (54b)$$

in which C' and C'' are constants of integration which may be determined when the loading conditions are fixed.

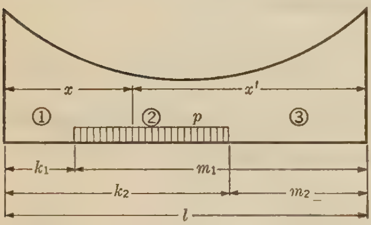


FIG. 4

25. Evaluation of C' and C''.—Consider the case of the loading shown in Fig. 4. It is evident that the M-curve will have three different forms, characterized by three different pairs of constants, corresponding to the segments (1), (2), and (3). The six equations of conditions are:

At $x = 0$—

$$M_1 = 0 \quad \ldots\ldots (55a)$$

at $x = k_1$—

$$M_1 = M_2; \quad \text{and} \quad \frac{dM_1}{dx} = \frac{dM_2}{dx} \quad \ldots\ldots (55b)$$

at $x = k_2$—

$$M_2 = M_3; \quad \text{and} \quad \frac{dM_2}{dx} = \frac{dM_3}{dx} \quad \ldots\ldots (55c)$$

and at $x = l$—

$$M_3 = 0 \quad \ldots\ldots (55d)$$

Substituting the moment M from Eqs. 54 in Eqs. 55 and solving the latter, the resulting three pairs of constants are as follows:

$$\left.\begin{aligned} C'_1 &= \frac{p}{c^2}Q \\ C''_1 &= \frac{p}{c^2}\left[-Q \tanh \frac{c\,l}{2} + \frac{\cosh (c\,m_1) - \cosh (c\,m_2)}{\sinh (c\,l)} \right] \end{aligned}\right\} \quad \ldots\ldots (56a)$$

$$\left.\begin{aligned} C'_2 &= \frac{p}{c^2}\{-(1-Q)-[\cosh(c\,k_1)-1]\} \\ C''_2 &= \frac{p}{c^2}\left[(1-Q)\tanh\frac{c\,l}{2}+\frac{\cosh(c\,k_1)-1}{\tanh(c\,l)}-\frac{\cosh(c\,m_2)-1}{\sinh(c\,l)}\right]\end{aligned}\right\}\ldots(56b)$$

$$\left.\begin{aligned} C'_3 &= \frac{p}{c^2}\{Q-[\cosh(c\,k_1)-\cosh(c\,k_2)]\} \\ C''_3 &= \frac{p}{c^2}\left[-Q\tanh\frac{c\,l}{2}+\frac{\cosh(c\,k_1)-\cosh(c\,k_2)}{\tanh(c\,l)}\right]\end{aligned}\right\}\ldots\ldots(56c)$$

Let $x' = l - x$ and

$$A_x = \frac{\cosh c\left(\frac{l}{2}-x\right)}{\cosh\frac{c\,l}{2}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(57)$$

Substituting the resulting values of C' and C'' in Eqs. 54, the moment in the three segments of Fig. 4 will be: Segment (1)—

$$M = \frac{p\,l^2}{c^2\,l^2}\left[-Q(1-A_x)+\frac{\cosh(c\,m_1)-\cosh(c\,m_2)}{\sinh(c\,l)}\sinh(c\,x)\right]\ldots(58a)$$

segment (2)—

$$M = \frac{p\,l^2}{c^2\,l^2}\left[(1-Q)(1-A_x)-\frac{\cosh(c\,k_1)-1}{\sinh(c\,l)}\sinh(c\,x') - \frac{\cosh(c\,m_2)-1}{\sinh(c\,l)}\sinh(c\,x)\right]\ldots(58b)$$

and segment (3)—

$$M = \frac{p\,l^2}{c^2\,l^2}\left[-Q(1-A_x)+\frac{\cosh(c\,k_2)-\cosh(c\,k_1)}{\sinh(c\,l)}\sinh(c\,x')\right]\ldots(58c)$$

Eqs. 58 will suffice for all conditions of uniform live load ordinarily met in practice. Different formulas for particular cases are condensed in Table 2. In the case of a single concentrated load, P, items 13 and 14, Table 2, the moment equations for the two segments (at the right and at the left of the load) may be derived similarly and will be as follows:

$$M_1 = -\frac{P\,l}{c\,l}\left[Q'(1-A_x)-\frac{\sinh(c\,m)}{\sinh(c\,l)}\sinh(c\,x)\right]\ldots\ldots(59a)$$

and

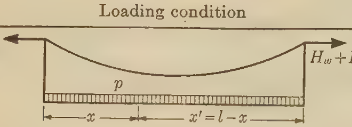
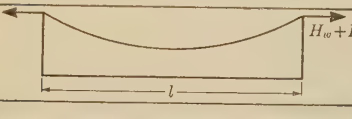
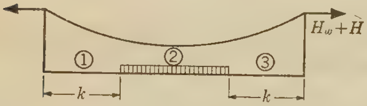
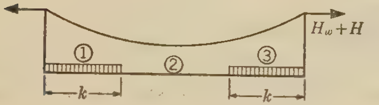
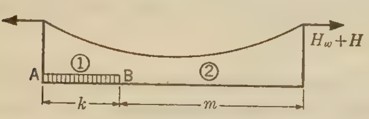
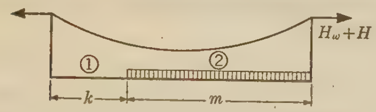
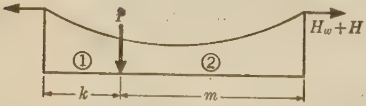
$$M_2 = -\frac{P\,l}{c\,l}\left[Q'(1-A_x)-\frac{\sinh(c\,k)}{\sinh(c\,l)}\sinh(c\,x')\right]\ldots\ldots(59b)$$

in which

$$Q' = H\frac{8\,f}{P\,c\,l^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(60)$$

TABLE 2.—MOMENT FORMULAS FOR TYPICAL LIVE LOADINGS

VARIABLES	CONSTANTS
$x' = l - x$	$Q = H\frac{8f}{pl^2}$; $m = l - k$; $B_m = \frac{\cosh(cm) - 1}{\sinh(cl)}$
$A_x = \frac{\cosh c\left(\frac{l}{2} - x\right)}{\cosh\frac{cl}{2}}$	$Q' = H\frac{8f}{Pcl^2}$; $A_k = \frac{\sinh c\left(\frac{l}{2} - k\right)}{\cosh\frac{cl}{2}}$; $B_k = \frac{\cosh(ck) - 1}{\sinh(cl)}$

Loading condition	Item	Bending moments
	1	$M = \frac{pl^2}{c^2l^2}(1 - Q)(1 - A_x)$
	2	$M = \frac{-pl^2}{c^2l^2}Q(1 - A_x)$; or $M = -H\frac{8f}{c^2l^2}(1 - A_x)$
	3 4 5	$M_1 = \frac{-pl^2}{c^2l^2}[Q(1 - A_x) - A_k\sinh(cx)]$ $M_2 = \frac{pl^2}{c^2l^2}\{(1 - Q)(1 - A_x) - A_x[\cosh(ck) - 1]\}$ $M_3 = \frac{-pl^2}{c^2l^2}[Q(1 - A_x) - A_k\sinh(cx')]$
	6 7 8	$M_1 = \frac{pl^2}{c^2l^2}[(1 - Q)(1 - A_x) - A_k\sinh(cx)]$ $M_2 = \frac{-pl^2}{c^2l^2}\{Q(1 - A_x) - A_x[\cosh(ck) - 1]\}$ $M_3 = \frac{pl^2}{c^2l^2}[(1 - Q)(1 - A_x) - A_k\sinh(cx')]$
	9 10	$M_1 = \frac{pl^2}{c^2l^2}[(1 - Q)(1 - A_x) - B_m\sinh(cx)]$ $M_2 = \frac{-pl^2}{c^2l^2}[Q(1 - A_x) - B_k\sinh(cx')]$
	11 12	$M_1 = \frac{-pl^2}{c^2l^2}[Q(1 - A_x) - B_m\sinh(cx)]$ $M_2 = \frac{pl^2}{c^2l^2}[(1 - Q)(1 - A_x) - B_k\sinh(cx')$
	13 14	$M_1 = \frac{-Pl}{cl}\left[Q'(1 - A_x) - \frac{\sinh(cm)}{\sinh(cl)}\sinh(cx)\right]$ $M_2 = \frac{-Pl}{cl}\left[Q'(1 - A_x) - \frac{\sinh(ck)}{\sinh(cl)}\sinh(cx')\right]$

STIFFENING TRUSS ENTIRELY LOADED

26. Deflection, Slope, Moment, Shear, and Truss Load.—An interesting illustration of the structural behaviors of a stiffening truss is offered by the case in which the span is completely covered by live load (Fig. 5).

Integrating and differentiating the moment equation in Table 2, item 1, the different expressions for truss functions are as follows: For deflection, the expression is

$$\eta = \frac{p\,l^4}{c^4\,l^4\,E\,I}(1-Q)\left[\frac{c^2\,l^2}{2}\left(\frac{x}{l}-\frac{x^2}{l^2}\right) - \left(1-\frac{\cosh c\,l\left(\frac{1}{2}-\frac{x}{l}\right)}{\cosh\frac{c\,l}{2}}\right)\right] \quad ..(61a)$$

which reveals that the deflection diagram is a superposition of a parabola and a (1 − cosh)-curve as shown in Fig. 5(*f*).

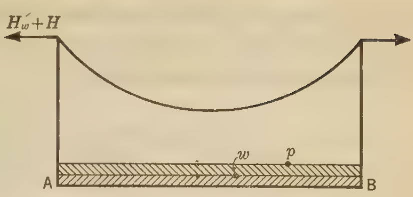
For slope, the expression is

$$\theta = \frac{-\,p\,l^3}{c^3\,l^3\,E\,I}(1-Q)\left[c\,l\left(\frac{1}{2}-\frac{x}{l}\right) - \frac{\sinh c\,l\left(\frac{1}{2}-\frac{x}{l}\right)}{\cosh\frac{c\,l}{2}}\right] \quad ..(61b)$$

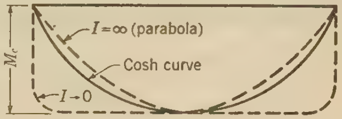
which reveals that the slope diagram is a superposition of a straight line and a (− sinh)-curve as shown in Fig. 5(*e*).

For moment, the expression is

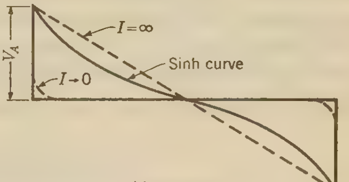
$$M = \frac{p\,l^2}{c^2\,l^2}(1-Q) \times \left[1-\frac{\cosh c\,l\left(\frac{1}{2}-\frac{x}{l}\right)}{\cosh\frac{c\,l}{2}}\right] \quad ..(61c)$$



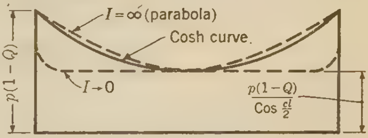
(*a*)


M_c
$I=\infty$ (parabola)
Cosh curve
$I\to 0$

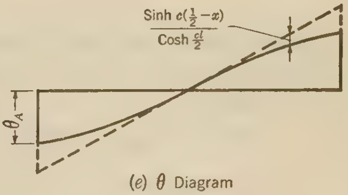
(*b*) *M* Diagram


V_A
$I=\infty$
Sinh curve
$I\to 0$

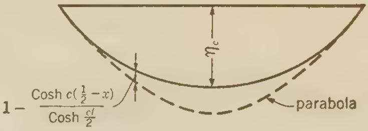
(*c*) *V* Diagram


$p(1-Q)$
$I=\infty$ (parabola)
Cosh curve
$I\to 0$
$\frac{p(1-Q)}{\mathrm{Cos}\,\frac{cl}{2}}$

(*d*) Truss Load: p_t


$\frac{\mathrm{Sinh}\,c(\frac{1}{2}-x)}{\mathrm{Cosh}\,\frac{cl}{2}}$
θ_A

(*e*) θ Diagram


$1-\frac{\mathrm{Cosh}\,c(\frac{1}{2}-x)}{\mathrm{Cosh}\,\frac{cl}{2}}$
η_c
parabola

(*f*) η Diagram

FIG. 5

which reveals that the moment diagram is a hyperbolic cosine curve symmetrical about the center of span as shown in Fig. 5(*b*).

For shear, the expression is

$$V = \frac{p\,l}{c\,l}(1 - Q)\left[\frac{\sinh c\,l\left(\frac{1}{2} - \frac{x}{l}\right)}{\cosh \frac{c\,l}{2}}\right] \qquad (61d)$$

which reveals that the shear diagram is a hyperbolic sine curve as shown in Fig. 5(*c*).

For truss load, the expression is

$$p_t = p\,(1 - Q)\,\frac{\cosh c\,l\left(\frac{1}{2} - \frac{x}{l}\right)}{\cosh \frac{c\,l}{2}} \qquad (61e)$$

which reveals that the diagram of truss load is a hyperbolic cosine curve, maximum at both ends and minimum at center as shown in Fig. 5(*d*).

27. Limit Case, $c\,l \to 0$.—If the dimensionless number $c\,l$ approaches zero, $I \to \infty$, and Q tends to be zero, since H approaches zero, Eqs. 61 have the following limiting values, respectively,

$$\eta \to \frac{p\,l^4}{24\,E\,I}\left(\frac{x}{l} - 2\frac{x^3}{l^3} + \frac{x^4}{l^4}\right) \qquad (62a)$$

$$\theta \to \frac{p\,l^3}{24\,E\,I}\left(1 - 6\frac{x^2}{l^2} + 4\frac{x^3}{l^3}\right) \qquad (62b)$$

$$M \to \frac{p\,l^2}{2}\left(\frac{x}{l} - \frac{x^2}{l^2}\right) \qquad (62c)$$

$$V \to p\,l\left(\frac{1}{2} - \frac{x}{l}\right) \qquad (62d)$$

and

$$p_t \to p \qquad (62e)$$

which correspond to the case of a simply supported beam without suspension.

28. Limit Case, $c\,l \to \infty$.—If $c\,l$ approaches infinity, the limiting values of Eqs. 61 will be

$$\eta \to \frac{p\,l^2}{H_w + H}(1 - Q)\,\frac{1}{2}\left(\frac{x}{l} - \frac{x^2}{l^2}\right) \qquad (63a)$$

$$\theta \to \frac{p\,l}{H_w + H}(1 - Q)\left(\frac{1}{2} - \frac{x}{l}\right) \qquad (63b)$$

$$M \to 0 \qquad (63c)$$

$$V \to 0 \qquad (63d)$$

and

$$p_t \to 0 \qquad (63e)$$

—which correspond to the case of an unstiffened cable span.

29. Flexibility Factor, $c\,l$.—Sections 27 and 28 reveal that the dimensionless term $c\,l$ characterizes the flexibility of a suspension bridge: The more flexible the bridge, the larger the value of $c\,l$. It can be designated as the "flexibility factor" of a suspension bridge.

From an investigation of the existing stiffened suspension bridges, it is found that the usual range of $c\,l$ is between 0 and 10.

30. Maximum Values of η, θ, M, V, *and* p_t.—The maximum deflection occurs at the center of span; it will be obtained in putting $x = l/2$ in Eq. 61*a* and is equal to

$$\eta_c = \frac{p\,l^4}{c^4\,l^4} \times \frac{1-Q}{E\,I}\left(\frac{c^2\,l^2}{8} - 1 - \frac{1}{\cosh\frac{c\,l}{2}}\right) \doteqdot \frac{5}{384}\frac{p\,l^4}{E\,I}\left[\frac{1-Q}{1+\frac{c^2\,l^2}{9.7}}\right] \quad (64)$$

The maximum slope occurs at both ends of span. In putting $x = 0$ and $x = l$ in Eq. 61*b*, it will be

$$\theta_{A,B} = \mp\frac{p\,l^4}{c^3\,l^3} \times \frac{1-Q}{E\,I}\left(\frac{c\,l}{2} - \tanh\frac{c\,l}{2}\right) \doteqdot \mp\frac{p\,l^3}{24\,E\,I}\left[\frac{1-Q}{1+\frac{c^2\,l^2}{10.5}}\right] \quad (65)$$

The maximum moment occurs at the center of span; it will be obtained in putting $x = \frac{l}{2}$ in Eq. 61*c* and is equal to

$$M_c = \frac{p\,l^2}{8}\left[\frac{8\,(1-Q)}{c^2\,l^2}\tanh\frac{c\,l}{2}\tanh\frac{c\,l}{4}\right] \quad (66)$$

The maximum shear occurs at both ends of span. In putting $x = 0$ and $x = l$ in Eq. 61*d*—

$$V_{A,B} = \pm\frac{p\,l}{2}\left[\frac{2\,(1-Q)}{c\,l}\tanh\frac{c\,l}{2}\right] \quad (67)$$

—and the maximum truss load occurs at both ends of span. In putting $x = 0$ and $x = l$ in Eq. 61*e*,

$$p_t = p\,[1-Q] \quad (68)$$

The last members of Eqs. 64 and 65 are found empirically; they are simple in form and fairly exact for the usual range of $c\,l$ (from 0 to 10). The values outside the brackets of Eqs. 64 to 68 are those corresponding to the case of a simply supported girder without suspension. It is obvious, then, that the terms within the bracket may be considered as coefficients reflecting the effect of suspension by a cable for different functions.

31. Effect of Variable Suspender Force on Value of H.—As $\eta_c = \Delta f$, Eq. 64 may be transformed into the following form:

$$H\,f + \frac{H_w + H}{1.01}\,\Delta f + \frac{9.6\,E\,I}{l^2}\,\Delta f = \frac{p\,l^2}{8} \quad (69)$$

Eq. 69 is the static equation $\Sigma M = 0$, at the center of the span, as used in Part I, with the assumption that the suspender forces are uniformly distributed. The coefficient of the quantity $(H_w + H)$, 1/1.01 rather than unity, is a measure of the error induced by this assumption. Its effect on the resulting cable stress is only about 1%, but its effect on moment is much greater.

32. Elastic Theory.—In Eqs. 64 to 68, if $I = \infty$ or $c\,l = 0$, but $Q \neq 0$ (that is, $H \neq 0$), all coefficients due to suspension reduce to $(1 - Q)$, and (since the fundamental assumption in this theory is that the stressed cable curve remains undistorted, when loaded) these equations become equal to those obtained by the so-called "elastic theory." It is shown then that the more flexible the truss (the larger the value of $c\,l$) the greater will be the error of the elastic theory.

PART III. DESIGN PROCEDURE, MAXIMUM MOMENTS, SHEARS, SLOPES, AND DEFLECTIONS IN STIFFENING TRUSS

Introduction

33. Cases of Loading and Temperature.—To determine the maximum bending moment, shear, slope, and deflection for a given section of the stiffening truss, one must refer to the appropriate loading condition. There are eight cases of loading and temperature to be considered in the design computations,[10,11] but, in customary practice, the extent of the live load (k) for maximum values of these variables must be found by trial and error, and the calculation is rather tiresome. Moreover, the location of the maximum positive and negative moments in the truss cannot be determined without drawing the complete maximum N-curve, although it is known to occur approximately at the quarter point of the span.

With the help of the simplified formulas for H and M in this paper, one may undertake a direct analytical determination of the maximum conditions.

The different cases of loading and temperature will be presented in the same order (Cases I to VIII) as is done elsewhere by D. B. Steinman,[11] M. ASCE. The H-formulas corresponding to each of these cases (H_{I} to H_{VIII}) may be found in Table 1, and the corresponding M-formulas in Table 2.

Case I

34. Maximum Positive Moment in Main Span.—In this case (see Table 1) the main span is loaded uniformly a distance x from the left end, the side spans are not loaded, and the highest probable temperature is assumed.

From Table 2, line 9, for segment AB:

$$M = \frac{p\,l^2}{c^2\,l^2}\left\{(1 - Q)\left[1 - \frac{\cosh c\left(\frac{l}{2} - x\right)}{\cosh \frac{c\,l}{2}}\right] - \frac{\cosh c\,(l - k) - 1}{\sinh (c\,l)} \sinh (c\,x)\right\} \ldots (70)$$

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[10] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., Pt. II, 9th Ed., 1916.

[11] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons, Inc., New York, N Y., 2d Ed., 1929.

in which

$$Q = H_{\mathrm{I}} \frac{8f}{p\,l^2} = \frac{1}{D_{+t}} \left(\sin^2 \frac{k\,\pi}{2\,l} - T \right) \ldots\ldots\ldots\ldots\ldots (71)$$

The moment varies from section to section and also varies at any given section as the live load moves along the span. The maximum value occurs when the conditions $\frac{dM}{dx} = 0$ and $\frac{dM}{dk} = 0$ are satisfied simultaneously. The differentiation with respect to x is easily performed; but, as c and Q both vary with k, the differentiation with respect to k gives a result too involved for exact solution.

For a given load (k, c, and Q all being constant), the condition $\frac{dM}{dx} = 0$ gives the position x_{+m} of the maximum moment, as follows:

$$\tanh\,(c\,x_{+m}) = \tanh \frac{c\,l}{2} - \frac{\cosh c\,(l - k) - 1}{(1 - Q)\sinh\,(c\,l)} \ldots\ldots\ldots\ldots (72)$$

and the maximum M becomes

$$(+)\ M\ (\max) = \frac{p\,l^2}{c^2\,l^2}\,(1 - Q) \tanh\,(c\,x_{+m}) \tanh \frac{c\,x_{+m}}{2} \ldots\ldots (73)$$

both of which can be easily calculated.

It is still necessary, however, to determine the load length k_{+m} which will give the highest of all possible maxima. The differentiation will be greatly simplified by the assumption that c is nearly constant as k varies. This is almost true when the live load ratio $q = \frac{p}{w}$ is small, and is only slightly in error for ordinary values of loading. A numerical investigation of the effect of variations in the value of c shows that it can be largely compensated for by using a higher value of the constant c, in the equation $\frac{dM}{dk} = 0$, which corresponds to assuming full span loading irrespective of the actual length of k; that is,

$$c = c_f = \sqrt{\frac{H_w + H_f}{E\,I}} = (\text{constant}) \ldots\ldots\ldots\ldots\ldots (74)$$

In Eq. 74, H_f is the horizontal force in the cable when the main span is fully loaded and the highest possible temperature is assumed (that is, $H_f = H_{\mathrm{I}}$ at $k = l$). Therefore, the condition $\frac{dM}{dk} = 0$ gives, approximately,

$$\frac{\sinh c_f\,(l - k_{+m})}{\sin \frac{k_{+m}\,\pi}{l}} = \frac{\pi}{2\,c_f\,l\,D_{+t}} \sinh c_f\,l \left(\tanh \frac{c_f\,l}{2} - \tanh \frac{c_f\,x_{+m}}{2} \right) \ldots (75)$$

The simultaneous equations required to solve for x_{+m} and k_{+m} are then Eqs. 72 and 75. The computations involve assuming two or three successive values of k (from 0.35 l to 0.45 l), simultaneously, in Eqs. 72 and 75 and obtaining two curves of $x = f(k)$, whose intersection gives the final values of k_{+m} and x_{+m} (use $c = c_f$ in both equations).

An extensive numerical investigation of existing bridges shows that it is also possible to proceed more easily as follows: Assume $x_{+m} = 0.22\ l$ in Eq. 75

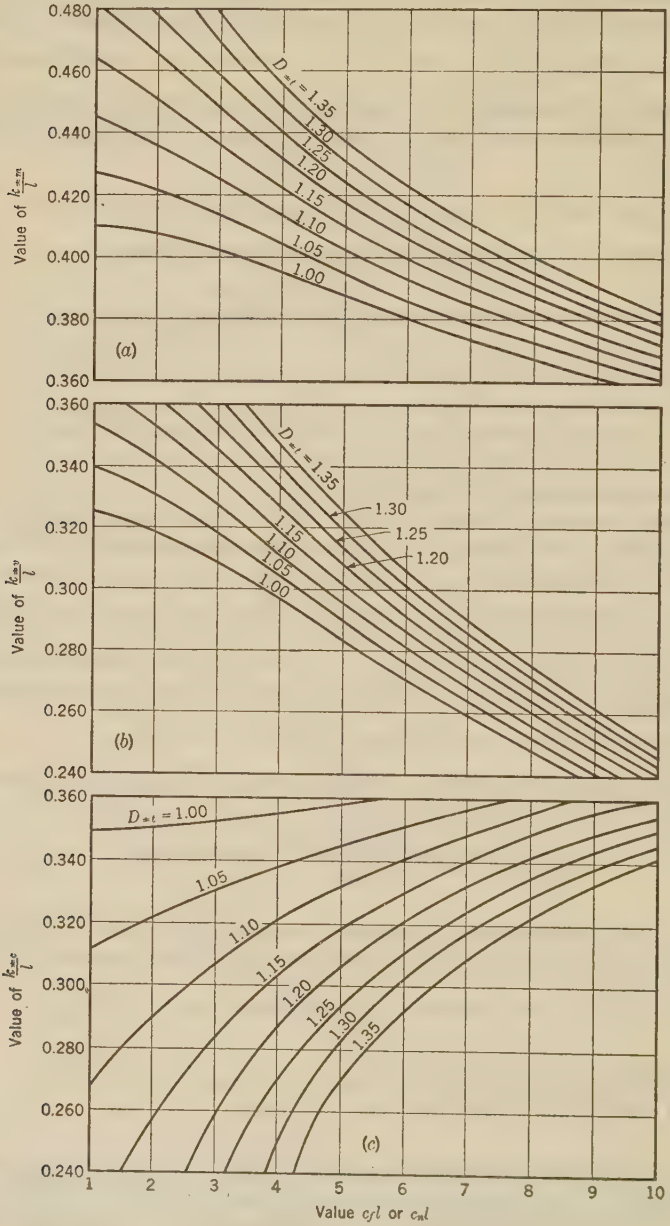

FIG. 6.—DESIGN CHARTS

and solve for k_{+m} which will be very nearly the actual load length k_{+m}. Then compute the corresponding values of H, c, and Q, substitute them into Eqs. 72 and 73, and derive x_{+m} and the maximum value of $+M$ in the truss. If the value of x_{+m} thus found differs too much from the assumed value $0.22\,l$, a second calculation can be made by using the new value of x_{+m} in Eq. 75, and solving again for k_{+m}; but the correction would be small (especially for the magnitude of maximum $+M$) and not worthwhile in usual cases.

To facilitate the solution of Eq. 75 (in which $x_{+m} = 0.22\,l$ is assumed), Fig. 6(a) is given to read directly when D_{+t} and $c_f\,l$ are known.

35. Maximum Positive Shear + V at Left End.—The shear equation is as follows:

$$V = \frac{dM}{dx} = \frac{p\,l}{c\,l}\left[(1 - Q)\frac{\sinh c\left(\frac{l}{2} - x\right)}{\cosh\frac{c\,l}{2}} - \frac{\cosh c\,(l - k) - 1}{\sinh\,(c\,l)}\cosh\,(c\,x)\right] \qquad (76)$$

At the end, $x = 0$, the end shear equation is

$$V_A = \frac{p\,l}{c\,l}\left[(1 - Q)\tanh\frac{c\,l}{2} - \frac{\cosh c\,(l - k) - 1}{\sinh\,(c\,l)}\right] \qquad (77)$$

The shear will be maximum when the load length k satisfies the condition $\frac{dV_A}{dk} = 0$. By the same procedure as in section 34, the differentiation is simplified by assuming a constant value of $c = c_f$. Therefore, the condition $\frac{dV_A}{dk} = 0$ gives

$$\frac{\sinh c_f\,(l - k_{+v})}{\sin\frac{k_{+v}\,\pi}{l}} = \frac{\pi}{c_f\,l\,D_{+t}}\sinh^2\frac{c_f\,l}{2} \qquad (78)$$

Fig. 6(b) contains the curve necessary to read k_{+v} directly when both D_{+t} and $c_f\,l$ are known.

When k_{+v} has been found, c and Q can be computed accordingly. Substituting these values in Eq. 78, the maximum positive shear at the left end can easily be determined.

36. Maximum Positive Shear at Center with Span Reversed (That Is, with the Live Load Starting from the Right End of the Span).—To find the maximum center shear, one end of the live load must be at the center section so that $k = 0.5\,l$, substituting the values of k and $x = 0.5\,l$ in Eq. 77, the maximum positive shear at the center is equal to

$$+V_c = \frac{p\,l}{2\,c\,l}\tanh\frac{c\,l}{4} \qquad (79a)$$

in which

$$c = \sqrt{\frac{H_w + H_{\text{I}(k=0.5l)}}{E\,I}} \quad \text{.......................(79}b\text{)}$$

CASE II

37. Maximum Negative Moment in Main Span.—In this case (see Table 1) the loading is the reverse of Case I; that is, the side spans are both loaded and the main span is partly loaded from the right-hand tower, a distance $l - k$. The lowest probable temperature is assumed.

From Table 2, item 11, for segment AB,

$$M = -\frac{p\,l^2}{c^2\,l^2}\left\{ Q\left[1 - \frac{\cosh c\left(\frac{l}{2} - x\right)}{\cosh \frac{c\,l}{2}}\right] - \frac{\cosh c\,(l - k) - 1}{\sinh\,(c\,l)} \sinh\,(c\,x)\right\} \quad \text{..(80}a\text{)}$$

in which

$$Q = H_{\text{II}}\frac{8\,f}{p\,l^2} = \frac{1}{D_{-t}}\left(\cos^2\frac{k\,\pi}{2\,l} + 2\,s + T\right) \quad \text{...........(80}b\text{)}$$

The condition $\frac{dM}{dx} = 0$ gives

$$\tanh\,(c\,x_{-m}) = \tanh\frac{c\,l}{2} - \frac{\cosh c\,(l - k) - 1}{Q\sinh\,(c\,l)} \quad \text{...........(81)}$$

and

$$(-)\,M\,(\max) = -\frac{p\,l^2}{c^2\,l^2}\,Q\tanh\,(c\,x_{-m})\tanh\frac{c\,x_{-m}}{2} \quad \text{.........(82)}$$

The assumption, that c remains nearly constant as k varies, is also made here in order to differentiate $\frac{dM}{dk} = 0$; but a smaller value of c, corresponding to no load in the main span, is used to compensate for the effect of variation of c:

$$c = c_n = \sqrt{\frac{H_w + H_n}{E\,I}} = (\text{constant}) \quad \text{...............(83)}$$

in which H_n is the horizontal force in the cable when the live load covers both side spans, when the main span is not loaded, and when the temperature is assumed at its lowest (that is, $H_n = H_{\text{II}}$ at $k = l$). Approximately, therefore, the condition $\frac{dM}{dk} = 0$ gives

$$\frac{\sinh c_n\,(l - k_{-m})}{\sin\frac{k_{-m}\,\pi}{l}} = \frac{\pi}{2\,c_n\,l\,D_{-t}}\left(\tanh\frac{c_n\,l}{2} - \tanh\frac{c_n\,x_{-m}}{2}\right) \quad \text{......(84)}$$

The two simultaneous equations necessary to solve for x_{-m} and k_{-m} are then Eqs. 81 and 84. The solution can be determined in a manner similar to that used in Case I, section 34. Fig. 6(*a*) can also be used to read k_{-m} when D_{-t} and $c_n\, l$ are calculated.

38. Maximum Negative Shear at the Left End.—In a manner similar to that used in Case I, section 35, the end shear equation is found to be

$$V_A = -\frac{p\,l}{c\,l}\left[Q \tanh\frac{c\,l}{2} - \frac{\cosh c\,(l-k) - 1}{\sinh\,(c\,l)}\right] \quad \text{......(85a)}$$

Assuming also that c is nearly constant and equal to c_n, the condition $\dfrac{dV_A}{dk} = 0$ gives

$$\frac{\sinh c_n\,(l - k_{-v})}{\sin\dfrac{k_{-v}\,\pi}{l}} = \frac{\pi}{c_n\,l\,D_{-t}} \sinh^2\frac{c_n\,l}{2} \quad \text{..............(85b)}$$

Fig. 6(*b*) can also be used to read k_{-v} directly when D_{-t} and $c_n\,l$ are known.

39. Maximum Negative Shear at Center (with Span Reversed).—Similar to Case I, section 36, the maximum center shear is found by Eq. 79*a*, in which c, in terms of H_{II} is found by Eq. 79*b*.

CASE III

40. Maximum Positive Moment at the Center of Main Span.—In this case (see Table 1) the main span is loaded symmetrically at each side of the center, the side spans are not loaded, and the highest probable temperature is assumed.

From Table 2, item 4, for segment AB,

$$M = \frac{p\,l^2}{c^2\,l^2}\left\{(1-Q)\left[1 - \frac{\cosh c\left(\dfrac{l}{2} - x\right)}{\cosh\dfrac{c\,l}{2}}\right] - \frac{\cosh c\left(\dfrac{l}{2} - x\right)}{\cosh\dfrac{c\,l}{2}}\left[\cosh\,(c\,k) - 1\right]\right\} \quad \text{..(86a)}$$

in which

$$Q = H_{III}\frac{8\,f}{p\,l^2} = \frac{1}{D_{+t}}\left(\cos\frac{k\,\pi}{l} - T\right) \quad \text{............(86b)}$$

At the center, $x = \dfrac{l}{2}$, and the moment equation (Eq. 86*a*) will be

$$M_c = \frac{p\,l^2}{c^2\,l^2}\left[(1-Q)\left\{1 - \frac{1}{\cosh\dfrac{c\,l}{2}}\right\} - \frac{\cosh\,(c\,k) - 1}{\cosh\dfrac{c\,l}{2}}\right] \quad \text{.....(86c)}$$

It will be maximum when k satisfies the condition $\dfrac{dM_c}{dk} = 0$. Assuming that

c remains nearly constant and equal to c_f (corresponding to full loading of main span, the same as in Case I), the condition $\frac{dM}{dk} = 0$ gives

$$\frac{\sinh (c_f\, k_{+c})}{\sin \frac{k_{+c}\, \pi}{l}} = \frac{\pi}{c_f\, l\, D_{+t}} \left(\cosh \frac{c_f\, l}{2} - 1 \right) \ldots\ldots\ldots\ldots (87)$$

Fig. 6(c) is given to obtain k_{+c} directly as a function of D_{+t} and $c_f\, l$. With k_{+c} found, H_{III}, c, and Q can be calculated, and the maximum positive moment $+M$ at the center will be determined easily from Eq. 86c.

CASE IV

41. Maximum Negative Moment at the Center of Main Span.—Both side spans are loaded for this case (see Table 1) and the center span is loaded symmetrically from the towers a distance k toward the center. The lowest probable temperature is assumed.

In a manner, similar to that used in Case III, the center moment equation is

$$M_c = -\frac{p\, l^2}{c^2\, l^2} \left[Q \left(1 - \frac{1}{\cosh \frac{c\, l}{2}} \right) - \frac{\cosh (c\, k) - 1}{\cosh \frac{c\, l}{2}} \right] \ldots\ldots (88a)$$

in which

$$Q = H_{\mathrm{IV}} \frac{8\, f}{p\, l^2} = \frac{1}{D_{-t}} \left(1 - \cos \frac{k\, \pi}{l} + 2\, S + T \right) \ldots\ldots\ldots (88b)$$

Assuming $c = c_n$ corresponding to a no-load condition on the main span as in Case II, the condition $\frac{dM_c}{dk} = 0$ gives

$$\frac{\sinh (c_n\, k_{-c})}{\sin \frac{k_{-c}\, \pi}{l}} = \frac{\pi}{c_n\, l\, D_{-t}} \left(\cosh \frac{c_n\, l}{2} - 1 \right) \ldots\ldots\ldots\ldots (89)$$

Eq. 89 can also be solved by reference to Fig. 6(c), first obtaining k_{-c} as a function of D_{-t} and $c_n\, l$.

CASE V

42. Maximum Positive Moment in Side Span.—With only one side span fully loaded and assuming the highest probable temperature, Table 2, item 1, shows that the moment for the loaded side span is equal to

$$M = \frac{p\, l^2_1}{c^2_1\, l^2_1} (1 - Q) \left[1 - \frac{\cosh c_1 \left(\frac{l_1}{2} - x \right)}{\cosh \frac{c_1\, l_1}{2}} \right] \ldots\ldots\ldots (90a)$$

in which

$$Q = H_{\mathrm{V}} \frac{8\, f_1}{p\, l^2_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (90b)$$

The moment reaches a maximum simultaneously in every section of the loaded span. The highest maximum positive moment, $+M$, occurs at the center section, since the maximum M-curve is a hyperbolic cosine curve symmetrical about the center of span. It is equal to

$$M_{c1} = \frac{p\,l^2_1}{c^2_1\,l^2_1}(1 - Q)\left(1 - \frac{1}{\cosh \frac{c_1\,l_1}{2}}\right); \text{ or}$$

$$M_{c1} = \frac{p\,l^2_1}{c^2_1\,l^2_1}(1 - Q)\tanh\frac{c_1\,l_1}{2}\tanh\frac{c_1\,l_1}{4} \qquad (91)$$

CASE VI

43. Maximum Negative Moment in Side Span.—The loading for this case is the reverse of Case V; that is, the live load covers the entire bridge except one side span. The lowest probable temperature is assumed.

In a manner similar to Case V, section 42, the equation for maximum negative moment in the unloaded side span is

$$M = -\frac{p\,l^2_1}{c^2_1\,l^2_1}Q\left[1 - \frac{\cosh c_1\left(\frac{l_1}{2} - x\right)}{\cosh\frac{c_1\,l_1}{2}}\right] \qquad (92a)$$

in which

$$Q = H_{\text{VI}}\frac{8\,f_1}{p\,l^2_1} \qquad (92b)$$

The maximum negative moment at the center is

$$M_{c1} = -\frac{p\,l^2_1}{c^2_1\,l^2_1}Q\left(1 - \frac{1}{\cosh\frac{c_1\,l_1}{2}}\right); \text{ or}$$

$$M_{c1} = \frac{p\,l^2_1}{c^2_1\,l^2_1}Q\tanh\frac{c_1\,l_1}{2}\tanh\frac{c_1\,l_1}{4} \qquad (93)$$

CASE VII

44. Maximum Positive Shear at Any Section in Side Span.—The left side span is partly loaded, from the tower toward the anchorage, as shown in Table 1. There is no live load on the remainder of the bridge; and the highest probable temperature is assumed. From Table 2, by differentiating the M-equation of line 12,

$$V = \frac{dM}{dx} = \frac{p\,l_1}{c_1\,l_1}\left[(1 - Q)\frac{\sinh c_1\left(\frac{l_1}{2} - x\right)}{\cosh\frac{c_1\,l_1}{2}} + \frac{\cosh(c_1\,k) - 1}{\sinh(c_1\,l_1)}\cosh c_1(l_1 - x)\right] \qquad (94a)$$

in which

$$Q = H_{\text{VII}} \frac{8 f_1}{p l^2_1} \qquad (94b)$$

As the one end of partial loading must extend to the given section where the maximum shear is sought, it is possible to substitute $k = x$ to obtain the maximum shear at any section x, so that

$$+V = \frac{p l_1}{c_1 l_1} \left[(1 - Q) \frac{\sinh c_1 \left(\frac{l_1}{2} - x \right)}{\cosh \frac{c_1 l_1}{2}} + \frac{\cosh (c_1 x) - 1}{\sinh (c_1 l_1)} \cosh c_1 (l_1 - x) \right] \qquad (95)$$

At the center of the span where $x = \frac{l_1}{2} (= k)$, the maximum positive shear will be

$$V_{c1} = \frac{p l_1}{2 c_1 l_1} \tanh \frac{c_1 l_1}{4} \qquad (96)$$

In Eq. 96,

$$c_1 = \sqrt{\frac{H_w + H_{\text{VII}(k=0.5l_1)}}{E I_1}} \qquad (97)$$

At the left end, $x = 0$, the maximum positive shear will be

$$V = \frac{p l_1}{c_1 l_1} (1 - Q) \tanh \frac{c_1 l_1}{2} \qquad (98)$$

in which Q and $c_1 l_1$ correspond to full loading condition on this side span.

Case VIII

45. Maximum Negative Shear at Any Section in Side Span.—In Case VIII, the loading is the reverse of that in Case VII; that is, the loaded part of the left side span in Case VII is the only part left unloaded in Case VIII (see Table 1). The lowest probable temperature is assumed.

In a manner similar to that used in Case VII, the equation for maximum negative shear is found to be

$$V = -\frac{p l_1}{c_1 l_1} \left[Q \frac{\sinh c_1 \left(\frac{l_1}{2} - x \right)}{\cosh \frac{c_1 l_1}{2}} + \frac{\cosh c_1 (l_1 - x) - 1}{\sinh (c_1 l_1)} \cosh (c_1 x) \right] \qquad (99a)$$

in which

$$Q = H_{\text{VIII}} \frac{8 f_1}{p l^2_1} \qquad (99b)$$

At the center of span, $x = \frac{l_1}{2}$, the maximum negative shear will be

$$V_{c1} = -\frac{p l_1}{2 c_1 l_1} \tanh \frac{c_1 l_1}{4} \qquad (100a)$$

in which

$$c_1 = \sqrt{\frac{H_w + H_{\mathrm{VIII}(k=0.5l_1)}}{E I_1}} \qquad (100b)$$

At the left end, $x = 0$, the maximum negative shear will be

$$V = -\frac{p l_1}{c_1 l_1} Q \tanh \frac{c_1 l_1}{2} \qquad (100c)$$

in which Q and $c_1 l_1$ correspond to the condition of no load on this side span.

CURVES OF MAXIMUM MOMENT IN THE MAIN SPAN

46. Simplified Method.—To obtain the maximum positive, or maximum negative, moment at each section of the main span, it is necessary to substitute different values of x in Eqs. 75 and 84; and it is also necessary to solve for k, the corresponding load length that will produce maximum positive, or maximum negative, moment at the section x considered. With k determined, substituting in Eq. 70 or Eq. 80a, the maximum positive, or maximum negative, moment at x can be calculated.

In sections 34 to 41, a procedure has been given for determining: (a) The values of x_{+m} and x_{-m}, which are the locations of the points of maximum moments (positive and negative); (b) the maximum values of positive moment and negative moment at points x_{+m} and x_{-m}; (c) the maximum values of positive moment and negative moment at the center of the main span; and (d) the maximum positive and negative shears at the ends of the span (the slopes of the maximum moment curves ($\pm M$) at the ends of the span).

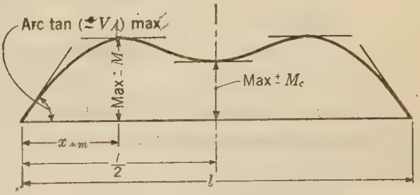

FIG. 7

These four items give five points (two zero points, two maxima points, and one minimum point) and five tangents of each curve of maximum moment (Fig. 7), and the intermediate parts can be sketched in readily by free hand. The most important information about these curves having been determined exactly, the resulting moment curves are sufficiently accurate for practical purposes.

MAXIMUM CENTER DEFLECTION IN THE STIFFENING TRUSS

47. Main Span and Side Span.—It is known that the maximum downward deflection at center in any span is produced under the probable highest temperature with full live load on the span alone. Then, the maximum value is expressed by Eq. 64 in which Q and $c\,l$ correspond to the temperature and loading condition previously cited.

MAXIMUM INCREASE OF GRADE IN THE STIFFENING TRUSS

48. Increase in Grade.—Many investigators[12] have suggested the increase of grade as a criterion to define the rigidity of a suspension bridge. For a vehicle traveling across a suspension bridge the point at which the maximum grade occurs is near the towers for a hinged truss. As in most suspension bridges, if there is a camber of several feet in the main span, and the side spans have a grade leading upward to the main span, the resultant grade will be maximum when the upward slope is maximum at the end of the main span, or when the downward slope is maximum at the tower end of the side span.

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[12] "A Generalized Deflection Theory for Suspension Bridges," by D. B. Steinman, *Transactions*, ASCE, Vol. 100, 1935, pp. 1132–1234.

49. End of Main Span.—The maximum upward slope at the left end evidently occurs at the loading and temperature conditions in Case II, Table 1. The equations of slope can be obtained by integrating the M-equation $\left(\theta = \int \frac{M}{EI}\,dx\right)$. Making $x = 0$ in the θ-equation of the left (unloaded) segment, the equation of end slope will be

$$\theta_A = \frac{p\,l^3}{E\,I\,c^3\,l^3}\left(\frac{c\,l}{2} - \tanh\frac{c\,l}{2}\right)[Q - f(k)] \quad \text{(101a)}$$

or, by Eq. 65,

$$\theta_A = \frac{p\,l^3}{24\,E\,I}\,\frac{Q - f(k)}{1 + \frac{c^2\,l^2}{10.5}} \quad \text{(101b)}$$

in which

$$Q = H_{\text{II}}\,\frac{8\,f}{p\,l^2} \quad \text{(102)}$$

and

$$f(k) = \frac{c^2\,(l-k)^2 \sinh(c\,l) - 2\,c\,l\tanh\frac{c\,(l-k)}{2}\sinh c\,(l-k)}{c^2\,l^2\sinh(c\,l) - 2\,c\,l\tanh\frac{c\,l}{2}\sinh(c\,l)} \quad \text{(103)}$$

It seems necessary to try several values of k in Eq. 101a to find which one will give the maximum value of θ_A. However, a numerical study leads one to believe that

Theorem 4.—

> The maximum upward grade change at the end of main span will be produced under the probable lowest temperature with the full live load on both side spans and approximately the far half of the main span.

Thus, with $k = \frac{1}{2}\,l$, Eq. 103 becomes

$$f(k)_{k=0.5l} = \frac{\frac{1}{4}\,c\,l - \tanh\frac{c\,l}{4}\,\frac{1}{\cosh\frac{c\,l}{2}}}{c\,l - 2\tanh\frac{c\,l}{2}} \quad \text{(104)}$$

With H_{II}, Q, and $c\,l$ computed for $k = \frac{l}{2}$, Eq. 104 gives the value of $f(k)$ and Eq. 101b will give the maximum increase of grade at end of main span. The result is approximate, but the error is slight.

50. Tower End of Side Span.—The maximum increase of grade at tower end occurs at the loading and for the assumed temperature condition of Case V,

Table 1. It is equal to

$$\theta_A = \frac{p\,l^3_1}{24\,E\,I_1} \times \frac{1 - Q}{1 + \dfrac{c^2_1\,l^2_1}{10.5}} \qquad (105)$$

in which Q and $c_1\,l_1$ correspond to the same values in Case V, Table 1, only one side span is fully loaded and the highest probable temperature is assumed.

Illustrative Example, Mount Hope Bridge

51. Data.—The Mount Hope suspension bridge will be used to illustrate the typical design computations. The results check almost exactly with those obtained by the deflection theory, but a complete analysis requires only from about 6 hours to 8 hours. All numerical values are calculated on a 12-in. slide rule. The following data are used:

Description	Main span	Side span
Length, l, in feet	1,188.33	498.33
Sag, f, in feet	118.795	20.891
Sag ratio $f/l = n$	0.100	0.042
$E\,I$, in (Feet)^2-Kips—		
Main span	123,511,000	
Side span	120,408,000	
Cable section, A, in square inches	75.92	
Sec α_1	1.043	
Live load, p, in kips per foot of bridge	0.750	
Dead load, w, in kips per foot of bridge	2.650	
Coefficient of expansion, ϵ	65×10^{-6}	
Temperature change, Δt, in degrees Centigrade	±60°	
Cable Length Functions, in Feet—		
For cable stretch, L_s	3,133	
For temperature change, L_t	2,996	

52. Constants.—By reference to Table 1, design constants are determined as follows:

$r = l_1/l = 0.42$; $n_1/n = 0.42$; $q = p/w = 0.283$,

$N_s = 0.0805$ (Eq. 29*b*); $N_c = 0.1280$ (Eq. 35*b*); $N_t = 0.0185$ (Eq. 38*b*);

$\dfrac{p\,l^2}{8\,f} = 1{,}113$; $\dfrac{p\,l^2}{8\,f\,D_{+t}} = 907$; $\dfrac{p\,l^2}{8\,f\,D_{-t}} = 936$;

$$H_w = \frac{w\,l^2}{8\,f} = \frac{w_1\,l^2_1}{8\,f_1} = 3{,}940 \text{ kips};$$

$$S = 0.0372 \text{ (Eq. 44)};$$

$$T = \frac{B}{q} N_t = 0.0795;$$

$c^2_0\,l^2 = \dfrac{H_w\,l^2}{E\,I} = 45$; $c^2_{01}\,l^2_1 = \dfrac{H_w\,l^2_1}{E\,I_1} = 8.1$;

$B = 1 + \dfrac{9.6}{c^2_0\,l^2} = 1.213$; $B_1 = 1 + \dfrac{9.6}{c^2_{01}\,l^2_1} = 2.182$;

$D_{+t} = 1 + N_s + N_c + N_t = 1.227$; $D_{-t} = 1 + N_s + N_c - N_t = 1.190$.

53. H-Formulas.—Maximum H occurs when all three spans are fully loaded and when the temperature is at its lowest. This is Case IX in Table 1, from which H (max) $= 936\ (1 + 2 \times 0.0372 + 0.0795) = 1{,}080$ kips. The deflection theory, by comparison, yields 1,079 kips. By similar use of Table 1, the force H, in kips, for different loading conditions is, as follows:

Case	Horizontal tension, H, in kips
I	$H_{\text{I}} = 907\left(\sin^2 \frac{k\,\pi}{2\,l} - 0.0795\right)$
II	$H_{\text{II}} = 936\left(\cos^2 \frac{k\,\pi}{2\,l} + 0.1539\right)$
III	$H_{\text{III}} = 907\left(\cos \frac{k\,\pi}{l} - 0.0795\right)$
IV	$H_{\text{IV}} = 936\left(1 - \cos \frac{k\,\pi}{l} + 0.1539\right)$
V	$H_{\text{V}} = -38$
VI	$H_{\text{VI}} = 1{,}045$
VII	$H_{\text{VII}} = 907\left(0.0372 \cos^2 \frac{k\,\pi}{2\,l_1} - 0.0795\right)$
VIII	$H_{\text{VIII}} = 936\left(0.0372 \sin^2 \frac{k\,\pi}{2\,l_1} + 1.1167\right)$

Fig. 8 shows the curves of H_{I}, H_{II}, H_{III}, and H_{IV} as functions of k/l.

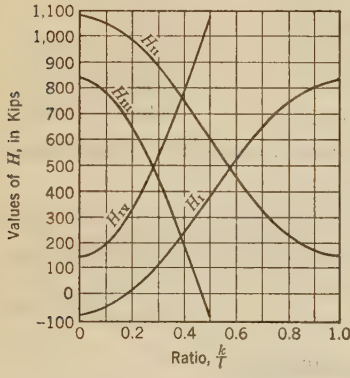

FIG. 8

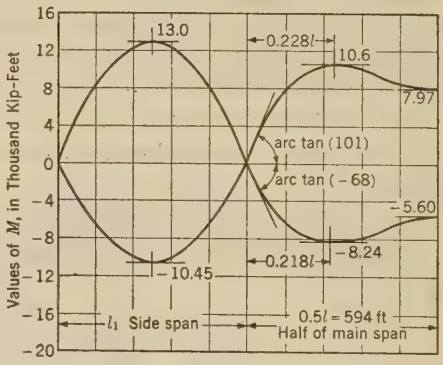

FIG. 9

54. Maximum Deflections.—Case III, Table 1, defines the maximum deflection at the center of the main span. For this case: $k = 0$; $H_{\text{III}} = 835$; $Q = \frac{835}{1{,}113} = 0.75$; and $c^2/l^2 = 45\left(1 + \frac{835}{3{,}940}\right) = 54.6$. By Eq. 64, $\eta_c = \frac{5}{384} \times \frac{0.75 \times 1{,}188.33^4}{123{,}511{,}000} \times \frac{1 - 0.75}{1 + \frac{54.6}{9.7}} = 5.98$ ft. This value of η_c compares with 6.10 ft determined by the deflection theory.

Case V, Table 1, defines the maximum deflection at the center of the side spans. Again, $H_{\mathrm{V}} = -38$; $Q = -0.034$; $c^2_1 l^2_1 = 8.1 \left[1 + \frac{(-38)}{3{,}940} \right] = 8.1$; and, inserting appropriate values in Eq. 64, $\eta_{c1} = \frac{5}{384} \times \frac{0.750 \times 498.33^4}{120{,}408{,}000} \times \frac{1 - (-0.034)}{1 + \frac{8.1}{9.7}} = 2.84$ ft. This value of η_{c1} checks the value computed by the deflection theory.

55. Maximum Increase of Grade.—Case II, Table 1, defines the maximum slope at the end of the main span. For this case, $k = \frac{l}{2}$; $H = 612$; $Q = 0.559$; and $c^2 l^2 = 52.0$. By Eq. 104, $f(k) = 0.336$. By Eq. 101*b*, $\theta_A = \frac{0.750 \times 1{,}188.33^3}{24 \times 123{,}511{,}000} \times \frac{0.55 - 0.336}{1 + \frac{52.0}{10.5}} = 0.0153$, which checks against 0.0152 by the deflection theory.

For the maximum slope at the tower end of the side span $H = -38$; $Q = -0.034$; and $c^2_1 l^2_1 = 8.1$. By Eq. 105, $\theta_A = \frac{0.750 \times 498.33^3}{24 \times 120{,}408{,}000} \times \frac{1 - (-0.034)}{1 + \frac{8.1}{10.5}} = 0.0191$, as against 0.0194 by the deflection theory.

56. Maximum Positive Moment.—Values of H in the problem of determining the location and magnitude of the maximum moment in the main span are given under Case I, Table 1. With $c_f l = \sqrt{54.6} = 7.4$ and $D_{+t} = 1.227$, entering Fig. 6(*a*), $k_{+m} = 0.395\,l$. Therefore, $H_{\mathrm{I}} = 234$; $Q = 0.21$; and $c\,l = 6.90$. By making appropriate substitutions in Eq. 72, solve for $\tanh (c\,x_{+m}) = 0.9175$, from which $c\,x_{+m} + 1.57$ and $x_{+m} = 0.228\,l$. Finally, Eq. 73 yields: $+M$ (max) $= 10{,}600$ ft-kips, compared with 10,500 ft-kips by the deflection theory.

The loading conditions of the maximum positive moment at the center of the main span correspond with Case III, Table 1. For this case, $c_f l = 7.4$ and $D_{+t} = 1.227$; entering Fig. 6(*c*) to determine k: $k_{+c} = 0.332\,l$. Therefore, $H_{\mathrm{III}} = 380$; $Q = 0.341$; and $c\,l = 7.03$; and, by Eq. 86*c*, $M_c = 7{,}970$ ft-kips, as compared with 7,960 ft-kips by the deflection theory.

Finally, Case V, Table 1, represents the conditions for maximum positive moment in the center of the side span. Referring to section 53, $H_{\mathrm{V}} = -38$; $Q = -0.034$; and $c_1 l_1 = 2.83$. Proper insertion of values in Eq. 91 yields: $M_c = 13{,}000$ ft-kips, as compared with 12,970 ft-kips by the deflection theory.

57. Maximum Negative Moment.—Similar to section 56, the location and magnitude of the maximum negative moment of the main span are represented by Case II, Table 1. With $c_n l = \sqrt{46.7} = 6.84$ and $D_{-t} = 1.19$, Fig. 6(*a*) yields $k_{-m} = 0.396\,l$. Therefore, $H = 760$; $Q = 0.682$; and $c\,l = 7.34$. By Eq. 81 $\tanh (c\,x_{-m}) = 0.9214$; $c\,x_{-m} = 1.6$; and $x_{-m} = 0.218\,l$. Then, by Eq. 82, the maximum negative moment is $-8{,}240$ ft-kips ($-8{,}180$ ft-kips by the deflection theory).

The loading condition of the maximum negative moment at the center of the main span corresponds with Case IV, Table 1. With $c_n l = 6.84$ and $D_{-t} = 1.19$, Fig. 6(*c*) yields $k_{-c} = 0.331 l$. Therefore, $H_{IV} = 610$; $Q = 0.548$; and $c l = 7.21$. With these values in Eq. 88*a*, $M_c = -5{,}600$ ft-kips ($-5{,}580$ ft-kips by the deflection theory).

For the center of the side span (Case VI, Table 1), $H_{VI} = 1{,}045$; $Q = 0.938$; $c_1 l_1 = 3.2$; and, by Eq. 93, $M_c = -10{,}450$ ft-kips ($-10{,}190$ ft-kips by the deflection theory).

58. Maximum Positive Shear.—At the end of the main span, the condition for maximum positive shear is defined by Case I, Table 1, with $c_f l = 7.4$ and $D_{+t} = 1.227$; and Fig. 6(*b*) yields $k_{+v} = 0.275 l$. Therefore, $H_I = 86$; $Q = 0.077$; $c l = 6.78$; and, by substitution in Eq. 77, $V_A = 101$ kips, which checks the deflection theory.

At the center of the main span (Case I, Table 1), $k = 0.5 l$; $H_I = 381$; $c l = 7.03$; and, by Eq. 79*a*, the maximum positive shear, V_c, is 60 kips, which again agrees with the deflection theory.

At the end of the side spans (Case VII, Table 1), $k = 0$; $H_{VII} = -38$; $Q = -0.034$; $c_1 l_1 = 2.83$; and, by substitution in Eq. 98, the maximum positive shear V is 122 kips (121 kips by the deflection theory).

At the center of the side spans (Case VII, Table 1), $k = 0.5 l$; $H = -55$; $c_1 l_1 = 2.83$; and, by Eq. 96, the maximum positive shear V is 40 kips, which agrees with the deflection theory.

59. Maximum Negative Shear.—At the end of the main span the condition for maximum negative shear V is defined by Case II, Table 1, in which, with $c_n l = 6.84$, and $D_{-t} = 1.19$, Fig. 6(*b*) yields $k_{-v} = 0.279 l$. Therefore, $H_{II} = 912$; $Q = 0.819$; $c l = 7.45$; and, by substitutions in Eq. 85*a*, $V = -83$ kips, which is a check on the deflection theory.

At the center of the main span (Case II, Table 1), $k = 0.5 l$; $H_{II} = 612$; $c l = 7.21$; and, in Eq. 79*a*, $V = -59$ kips, which again checks the deflection theory.

At the end of the side span (see Case VIII, Table 1), $k = 0$; $H_{VIII} = 1{,}045$; $Q = 0.938$; $c_1 l_1 = 3.2$; and, by Eq. 100*c*, the maximum negative shear V is -101 (also -101 by the deflection theory).

Finally, for maximum negative shear at the center of a side span (Case VIII, Table 1), $k = 0.5 l$; $H_{VIII} = 1{,}065$; $c_1 l_1 = 3.21$; and, by Eq. 100*a*, $V = -39$ kips (deflection theory -38 kips).

60. Curves of Maximum Moment.—As explained in Part III, section 46, a fairly accurate curve of maximum moment for the main span of a suspension bridge can be drawn free hand between the five points and five tangents obtained by the foregoing computations. The curves of maximum moment for the side spans (see Fig. 9) are hyperbolic cosine curves which are maximum at the center and zero at both ends.

61. Acknowledgments.—The writer is indebted to C. H. Tsien (professor of mechanical engineering, National University of Chekiang, China) for many of his helpful and valuable suggestions during the course of this study.

In view of the extreme scarcity of technical literature in China during World War II, the writer counted himself doubly fortunate in having available the work of Mr. Steinman,[11] as a guide in the preparation of this paper.

DISCUSSION

PEDRO B. J. GRAVINA,[13] ESQ.—An interesting method of analysis, based on the deflection theory contrived to simplify the design of suspension bridges, is presented in this paper. Instead of employing the cut-and-try method, it is possible, by means of the author's formula, to make a direct analysis, partly sacrificing exactness in favor of greater simplicity in design.

It is the opinion of the author (see section 1) that, when employing the trigonometric series, "the computations are lengthy and involved." This statement is partly true for deflections, moments, and shears in the stiffening trusses; but it is not true in the case of horizontal tension in the cable, especially when the live load is uniformly distributed, as the convergence of the series is then rapid.

In order to evaluate the approximation of the formulas presented in the first part of the paper, it will be convenient to compare them with those derived directly from the deflection theory. In connection with some studies of the convergence of the series that appear through the method of the trigonometric series,[14] the writer has demonstrated the possibility of computing the value of H for a uniformly distributed live load by means of a simplified formula. In the special case of the symmetrical bridge with three variable spans, and with the load distributed between any two sections, distant k' and k'' from the left support (Fig. 4), the formula follows:

$$H = \left[p \left(\frac{8\,f}{l} \right) \frac{4\,l^2}{\pi^4} \frac{\bar{F}\left(\frac{k}{l}\right)}{1 + \frac{\pi^2}{c^2\,l^2}} + p \left(\frac{8\,f_1}{l_1} \right) \frac{4\,l^2_1}{\pi^4} \frac{\bar{F}_{1l}\left(\frac{k}{l_1}\right)}{1 + \frac{\pi^2}{c^2_1\,l^2_1}} \right.$$

$$\left. + p \left(\frac{8\,f_1}{l_1} \right) \frac{4\,l^2_1}{\pi^4} \frac{\bar{F}_{1r}\left(\frac{k}{l_1}\right)}{1 + \frac{\pi^2}{c^2_1\,l^2_1}} \right] \frac{1}{1 + N'_c + \bar{N}'_t} \div l\,G + 2\,l_1\,G_1 \ldots . (106)$$

in which, for the main span,

$$\bar{F}\left(\frac{k}{l}\right) = \cos\frac{k'\,\pi}{l} - \cos\frac{k''\,\pi}{l}; \qquad (k' < k'') \ldots\ldots\ldots (107a)$$

for the left side span,

$$\bar{F}_{1l}\left(\frac{k}{l_1}\right) = \cos\frac{k'\,\pi}{l_1} - \cos\frac{k''\,\pi}{l_1}; \qquad (k' < k'') \ldots\ldots\ldots (107b)$$

and, for the right side span,

$$\bar{F}_{1r}\left(\frac{k}{l_1}\right) = \cos\frac{k'\,\pi}{l_1} - \cos\frac{k''\,\pi}{l_1}; \qquad (k' < k'') \ldots\ldots\ldots (107c)$$

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[14] "Teoria das Pontes Pênseis," Pedro B. J. Gravina, editor, Polytechnic School of São Paulo Univ., São Paulo, Brazil, 1948, pp. 95–109.

Also,

$$N'_c = \frac{H_w + H}{l\,G + 2\,l_1\,G_1}\,\frac{L_s}{E_c\,A_0} \qquad \text{(108}a\text{)}$$

$$\bar{N}'_t = \frac{H_w + H}{H\,(l\,G + 2\,l_1\,G_1)}\,\epsilon\,\Delta_t\,L_t \qquad \text{(108}b\text{)}$$

and

$$G = \frac{512}{\pi^4\left(1 + \dfrac{\pi^2}{c^2\,l^2}\right)}\left(\frac{f}{l}\right)^2 \qquad \text{(109}a\text{)}$$

and

$$G_1 = \frac{512}{\pi^4\left(1 + \dfrac{\pi^2}{c^2_1\,l^2_1}\right)}\left(\frac{f_1}{l_1}\right)^2 \qquad \text{(109}b\text{)}$$

In Eq. 106 the effect of temperature changes is represented only in the term $\bar{N}'_t$, as a correction to the approximate value of H. The functions $\bar{F}\left(\frac{k}{l}\right)$, $\bar{F}_{1l}\left(\frac{k}{l_1}\right)$, and $\bar{F}_{1r}\left(\frac{k}{l_1}\right)$, defined by Eqs. 107, are general and are valid for any loading condition, including, therefore, as special cases, the ones examined separately in sections 3 to 8, inclusive, of the paper. As the length $\Delta l = k'' - k'$ decreases, the convergence of the series decreases, so that in the extreme case of a concentrated load it is no longer possible to consider only the first term of the series, as was done in the case of the distributed load; if this were done, the resulting error would be considerable. Consequently, Eq. 17*b* appears to be in much greater error than the other formulas in the paper that were used for the computation of the horizontal tension of the cable in the case of the distributed load.

Let

$$1 + \frac{\pi^2}{c^2\,l^2} = \bar{B} \qquad \text{(110}a\text{)}$$

and

$$1 + \frac{\pi^2}{c^2_1\,l^2_1} = \bar{B}_1 \qquad \text{(110}b\text{)}$$

and assume that only the main span is loaded, Eq. 106 can be written

$$H = \frac{p\left(\dfrac{8\,f}{l}\right)\dfrac{4\,l^2}{\pi^4}\dfrac{\bar{F}\left(\dfrac{k}{l}\right)}{\bar{B}}}{\dfrac{512}{\pi^4}\left(\dfrac{f}{l}\right)^2\dfrac{l}{\bar{B}}}\,\frac{1}{1 + N_c + \bar{N}_t + N_s} \qquad \text{(111)}$$

in which

$$N_c = \frac{H_w + H}{l\,G}\,\frac{L_s}{E_c\,A_0} \qquad \text{(112}a\text{)}$$

$$\bar{N}_t = \frac{H_w + H}{H\,l\,G}\,\epsilon\,\Delta_t\,L_t \qquad \text{(112}b\text{)}$$

and

$$N_s = 2\,\frac{l_1\,G_1}{l\,G} \dots\dots\dots\dots\dots\dots\dots (112c)$$

If

$$C = N_c + \bar{N}_t + N_s \dots\dots\dots\dots\dots\dots\dots (113)$$

the value of H by Eqs. 111 and 112 will be

$$H = \frac{p\,l^2}{8\,f}\,\frac{\bar{F}\left(\frac{k}{l}\right)}{2}\,\frac{1}{1 + C} \dots\dots\dots\dots\dots\dots\dots (114)$$

Substituting the values of G and G_1 from Eqs. 109 in Eqs. 112,

$$N_c = \frac{0.189}{n^2}\,\frac{H_w}{E_c\,A_0}\,\frac{L_s}{l}\left(1 + \frac{H}{H_w}\right)\bar{B} \dots\dots\dots\dots\dots (115a)$$

$$\bar{N}_t = \frac{0.189}{n^2}\,\frac{\epsilon\,\Delta_t\,L_t}{l}\left(1 + \frac{H_w}{H}\right)\bar{B} \dots\dots\dots\dots\dots (115b)$$

and

$$N_s = 2\,r\,\frac{n^2{}_1}{n^2}\,\frac{\bar{B}}{\bar{B}_1} \dots\dots\dots\dots\dots\dots\dots (115c)$$

If the load occurs only on one of the side spans,

$$H = \frac{p\,l^2}{8\,f}\,\frac{n_1}{n}\,r^2\,\frac{\bar{B}}{\bar{B}_1}\,\frac{\bar{F}_1\left(\frac{k}{l_1}\right)}{2}\,\frac{1}{1 + C} \dots\dots\dots\dots\dots (116)$$

Let

$$B = 1 + \frac{\pi^2}{c^2{}_0\,l^2} \dots\dots\dots\dots\dots\dots\dots (117a)$$

and

$$B_1 = 1 + \frac{\pi^2}{c^2{}_{01}\,l^2{}_1} \dots\dots\dots\dots\dots\dots\dots (117b)$$

—it follows that

$$\bar{B} = \frac{1 + \frac{H_w}{H}\,B}{1 + \frac{H_w}{H}} \dots\dots\dots\dots\dots\dots\dots (118a)$$

and

$$\bar{B}_1 = \frac{1 + \frac{H_w}{H}\,B_1}{1 + \frac{H_w}{H}} \dots\dots\dots\dots\dots\dots\dots (118b)$$

Substituting Eqs. 118 in Eqs. 115 and 116,

$$N_c = \frac{0.189}{n^2}\,\frac{H}{E_c\,A_0}\,\frac{L_s}{l}\left(1 + \frac{H_w}{H}\,B\right) \dots\dots\dots\dots\dots (119a)$$

$$\bar{N}_t = \frac{0.189}{n^2}\,\frac{\epsilon\,\Delta_t\,L_t}{l}\left(1 + \frac{H_w}{H}\,B\right) \dots\dots\dots\dots\dots (119b)$$

$$N_s = 2\,r\,\frac{n^2{}_1}{n^2}\,\frac{1 + \frac{H_w}{H}\,B}{1 + \frac{H_w}{H}\,B_1} \dots\dots\dots\dots\dots\dots\dots (119c)$$

and

$$H = \frac{p\,l^2}{8\,f}\frac{n_1}{n}\,r^2\,\frac{\bar{F}_1\left(\frac{k}{l_1}\right)}{2}\,\frac{1+\frac{H_w}{H}B}{1+\frac{H_w}{H}B_1}\,\frac{1}{1+C}\ldots\ldots\ldots\ldots(120)$$

By simple algebraic transformation, the following formulas can be derived from Eqs. 114 and 120:

$$H = \frac{p\,l^2}{8\,f\,D}\left[\frac{\bar{F}\left(\frac{k}{l}\right)}{2} - T\right]\ldots\ldots\ldots\ldots(121a)$$

and

$$H = \frac{p\,l^2}{8\,f\,D}\left[\frac{S\,\bar{F}_1\left(\frac{k}{l_1}\right)}{2} - T\right]\ldots\ldots\ldots\ldots(121b)$$

in which

$$S = r^2\,\frac{n_1}{n}\,\frac{1+\frac{H_w}{H}B}{1+\frac{H_w}{H}B_1}\ldots\ldots\ldots\ldots(122a)$$

$$T = \frac{0.189\,\epsilon\,\Delta_t\,L_t\,B}{n^2\,l\,q}\ldots\ldots\ldots\ldots(122b)$$

$$D = 1 + N_c + N_s + N_t\ldots\ldots\ldots\ldots(122c)$$

and

$$N_t = \frac{0.189}{n^2}\,\frac{\epsilon\,\Delta_t\,L_t}{l}\ldots\ldots\ldots\ldots(122d)$$

Eqs. 119*a*, 119*c*, 121, and 122 are derived directly from the deflection theory and, therefore, afford an opportunity to appraise the equations for the variation of the horizontal component of the cable tension, and of the design constants, proposed by the author.

G. S. RAMASWAMY,[15] JUN. ASCE.—The already impressive array of literature on suspension bridges has been enhanced by the paper. The author's conclusion that the horizontal tension in the cable is practically independent of the ratio of the live load to the dead load and the degree of stiffness of the truss is provocative of further thought.

The writer wishes to confine his remarks to Part I of the paper, in which the author selects, as his point of departure, certain equations for the horizontal component of the cable tension which he has quoted from another source.[4] These equations were derived by energy methods; but they can be obtained more elegantly by the application of the Müller-Breslau principle.

The object is to determine the influence line for the horizontal component *H* of the cable tension for varying positions of a unit concentrated load traversing the span. The varying position of the unit concentrated load will be denoted by $n\,l$, in which n has values ranging from 0 to 1. The procedure is to give a unit deformation along the line of action of H. The resulting deflected shape of the stiffening truss will then give the influence line for H. The structure is thus made to draw its own influence line. If a unit horizontal tension is applied to the cable it will induce a uniform vertical load on the cable of intensity $\frac{8\,f}{l^2}$. The vertical uniform load, in turn, will exert a uniform pull on the stiffening truss, which will deflect as a hinged beam with a uniformly distributed load. The deflection curve of a hinged beam with a uniformly distributed load of intensity $\frac{8\,f}{l^2}$ is given by

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[15] Instr., College of Eng., Kakinada, India.

$$\Delta = \frac{f\,l^2}{3\,E\,I}\,(n - 2\,n^3 + n^4) \qquad (123)$$

in which Δ is the deflection. The ordinates of the curve defined by Eq. 123 are not the influence line ordinates themselves, but are proportional to them, since a unit load was applied, instead of a unit deformation, along the line of action of H. The actual influence line curve can be obtained from Eq. 123 by the following artifice. Write the equation for the influence line as

$$H = A\,(n - 2\,n^3 + n^4) \qquad (124)$$

in which A is a constant to be determined by some known condition. When the entire span is covered by a uniformly distributed load of unit intensity, the horizontal tension can be obtained by integrating Eq. 124 and placing $n = 1$. Performing the integration and substitution,

$$H_1 = \frac{A}{5} \qquad (125a)$$

Since $H = \frac{l^2}{8\,f}$ for this particular condition of loading,

$$A = \frac{5\,l^2}{8\,f} \qquad (125b)$$

Equating Eqs. 125*a* and 125*b*, the formula of the influence line for H is

$$H = \frac{5\,l^2}{8\,f}\,(n - 2\,n^3 + n^4) \qquad (126)$$

Professor Tsien approximates the form of the influence line contained in Eq. 126 by a sine curve. Let the desired sine curve be

$$H = B \sin n\,\pi \qquad (127a)$$

The area under this sine curve should be equal to $\frac{l^2}{8\,f}$. By applying this condition one obtains

$$H = \frac{l\,\pi}{16\,f} \sin n\,\pi \qquad (127b)$$

Hence, the horizontal tension H created by a load P at a distance $n\,l$ from the left support is given by

$$H = \frac{P\,l\,\pi}{16\,f}\sin n\,\pi \qquad (127c)$$

If $n\,l$ is replaced by k in Eq. 127c,

$$H = \frac{P\,l\,\pi}{16\,f}\sin\frac{k\,\pi}{l} \qquad (127d)$$

Eq. 127d is the same as Eq. 17b, with the exception of a correction factor $\frac{1}{D}$.

The other cases considered by Professor Tsien can easily be deduced as corollaries by integrating either Eq. 127b or Eq. 127d and inserting appropriate limits.

The writer feels that the case of the concentrated load traversing the span is the fundamental one and should therefore be treated first; having once obtained the influence line for H, any distribution of live load can be solved conveniently.

Hui-Sien Liu,[16] Esq.—Stresses in suspension bridges under static load could be analyzed with accuracy after the deflection theory was introduced. Further effort has been aimed at two opposite ends:

(1) To eliminate the assumptions made in the deflection theory for a more accurate analysis and

(2) To make more approximations for the simplification of formulas used in practical design. The author is to be congratulated in having achieved the latter end successfully.

The author finds that the relation between the horizontal component of cable stress and the length of an advancing load under extremely simplified conditions can be fitted by a sine curve; then he concludes that the same sinusoidal relation holds for all conditions. This is expedient but not rigorous. The magnitude of error inherent in the formulas thus derived is not established unless results are compared with those produced by other accurate methods; and, in particular, the author does not prove the applicability of the principle of superposition although he has repeatedly used it.

To demonstrate this point,[17] consider a general case of loading such as that in Fig. 10. The load on the cable is equal to the tension in the suspenders which consists of two parts: The dead load, w, and the increment, s, created by the live load and change of temperature. The deflection of the cable is usually considered to be equal to the deflection of the stiffening truss. Consequently, the suspender tension corresponding to a given cable deflection, η, is defined as follows:

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[16] Formerly Prof., Structural Eng., Tsinhua Univ., Peiping, China.

[17] "On the Theory and Design of Suspension Bridges," by Hui-Sien Liu, *Research Bulletin*, Tsinhua Univ., Peiping, China, 1946.

Span (see Fig. 10)	Cable deflection	Load on cable
Main	η	$w + s$
Side 1	η_1	$w_1 + s_1$
Side 2	η_2	$w_1 + s_2$

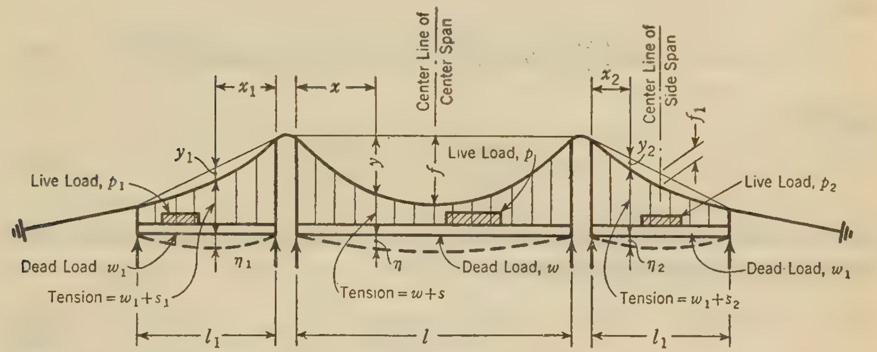
x_1
x
Center Line of Center Span
x_2
Center Line of Side Span
f_1
y_1
y
f
Live Load, p
y_2
Live Load, p_1
Live Load, p_2
Dead Load w_1
η_1
η
Dead Load, w
η_2
Dead Load, w_1
Tension $= w_1+s_1$
Tension $= w+s$
Tension $= w_1+s_2$
l_1
l
l_1

FIG. 10

Isolating the cable as a free body, the increase of strain energy in the cable, resulting from live load and temperature change, must be equal to the work done by the loads on the cable. Hence an equation of energy can be written as

$$\left(H_w + \frac{H}{2}\right)\frac{H\,L_s}{E_c\,A_o} \pm \left(H_w + \frac{H}{2}\right)\epsilon\,\Delta t\,L_t = \int_0^l \left(w + \frac{s}{2}\right)\eta\,dx$$

$$+ \int_0^{l_1}\left(w_1 + \frac{s_1}{2}\right)\eta_1\,dx + \int_0^{l_1}\left(w_1 + \frac{s_2}{2}\right)\eta_2\,dx \quad\text{........}(128a)$$

or

$$\left(1 + \frac{\beta}{2}\right)\beta\,\frac{H^2_w\,L_s}{E_c\,A_o} \pm \left(1 + \frac{\beta}{2}\right)H_w\,\epsilon\,\Delta t\,L_t = \int_0^l \left(w + \frac{s}{2}\right)\eta\,dx$$

$$+ \int_0^{l_1}\left(w_1 + \frac{s_1}{2}\right)\eta_1\,dx + \int_0^{l_1}\left(w_1 + \frac{s_2}{2}\right)\eta_2\,dx \quad\text{........}(128b)$$

In the second term of the left-hand member, the plus sign (+) is for an increase, and the minus sign (−) for a decrease, in temperature.

To generalize, let the live load, p, be any kind of load which can be represented by a Fourier series as

$$p = \sum_m \frac{4\,w}{m\,\pi}\,K_m \sin\frac{m\,\pi\,x}{l} \quad\text{................}(129a)$$

in which

$$K_m = \frac{m\,\pi}{4\,w}\,\frac{2}{l}\int_0^l p \sin\frac{m\,\pi\,x}{l}\,dx \quad\text{................}(129b)$$

Then, by methods[17] described elsewhere, the following series for η and s can be written

$$\eta = \frac{32}{\pi^3} f \sum_m \frac{K_m - \beta \sin^2 \frac{1}{2} m \pi}{m^3 (1 + \beta + m^2 \mu)} \sin \frac{m \pi x}{l} = \frac{32}{\pi^3} f \sum_m A_m \sin \frac{m \pi x}{l} \quad \text{..(130a)}$$

and

$$s = p - E I \frac{d^4 \eta}{dx^4} = \beta w + \frac{4 w}{\pi} (1 + \beta) \sum m^2 A_m \sin \frac{m \pi x}{l} \quad \text{...(130b)}$$

in which

$$A_m = \frac{K_m - \beta \sin^2 \frac{1}{2} m \pi}{m^3 (1 + \beta + m^2 \mu)} \quad \text{.................(131)}$$

In Eqs. 130 and 131, β is the ratio between the additional and the initial horizontal components of cable stress:[2]

$$\beta = \frac{H}{H_w} \quad \text{.......................(132a)}$$

and μ is the "stiffening factor" of main span, a dimensionless constant adopted to denote the degree of stiffening:

$$\mu = \frac{E I}{H_w} \left(\frac{\pi}{l} \right)^2 \quad \text{.......................(132b)}$$

An expression, μ_1, for the stiffening factor of side spans can be written as

$$\mu_1 = \frac{E I_1}{H_w} \left(\frac{\pi}{l_1} \right)^2 \quad \text{.......................(132c)}$$

The integrals on the right-hand side of Eq. 128b can then be evaluated as follows:

$$\int_0^l \left(w + \frac{s}{2} \right) \eta \, dx$$

$$= \int_0^l \left(1 + \frac{\beta}{2} \right) w \, \eta \, dx + \int_0^l \frac{2 w}{\pi} (1 + \beta) \, \eta \sum_m m^2 A_m \sin \frac{m \pi x}{l} \, dx$$

$$= \frac{64}{\pi^4} \left(1 + \frac{\beta}{2} \right) w \, l \, f \sum_{1,3,5,\ldots} \frac{A_m}{m} + \frac{32}{\pi^4} (1 + \beta) \, w \, l \, f \sum_{1,2,3,\ldots} m^2 A^2_m$$

$$= \frac{512}{\pi^4} \left(1 + \frac{\beta}{2} \right) n \, H_w \, f \sum_{1,3,5,\ldots} \frac{A_m}{m} + \frac{256}{\pi^4} (1 + \beta) \, n \, H_w \, f \sum_{1,2,3,\ldots} m^2 A^2_m$$

$$= \frac{512}{\pi^4} \left(1 + \frac{\beta}{2} \right) n \, H_w \, f \left(\sum_{1,3,5,\ldots} \frac{A_m}{m} + \frac{1 + \beta}{2 + \beta} \sum_{1,2,3,\ldots} m^2 A^2_m \right) \quad \text{.......(133)}$$

Writing similar expressions for the side span integrals and substituting into the energy equation of the cable,

$$\begin{aligned}
&\beta \frac{H_w L_s}{E_c A_o} + \epsilon \Delta t L_t \\
&= \frac{512}{\pi^4} n f \left(\sum_{1,3,5,\dots} \frac{A_m}{m} + \frac{1+\beta}{2+\beta} \sum_{1,2,3,\dots} m^2 A^2_m \right)_{\text{main}} \\
&+ \frac{512}{\pi^4} n_1 f_1 \left(\sum_{1,3,5,\dots} \frac{A_m}{m} + \frac{1+\beta}{2+\beta} \sum_{1,2,3,\dots} m^2 A^2_m \right)_{\text{side 1}} \\
&+ \frac{512}{\pi^4} n_1 f_1 \left(\sum_{1,3,5,\dots} \frac{A_m}{m} + \frac{1+\beta}{2+\beta} \sum_{1,2,3,\dots} m^2 A^2_m \right)_{\text{side 2}} \cdots\cdots(134)
\end{aligned}$$

The series, $\sum\limits_{1,2,3,\dots} m^2 A^2_m$ represents the effect of variations in the tension in the suspenders. This effect is sufficiently small to be neglected in the deflection theory. The series, $\sum\limits_{1,3,5,\dots} \frac{A_m}{m}$ converges rapidly—that is, all terms $m > 1$ can be omitted without sacrificing much accuracy. Therefore,

$$\begin{aligned}
&\beta \frac{H_w L_s}{E_c A_o} \pm \epsilon \Delta t L_t \\
&= \frac{512}{\pi^4} n f (A_1)_{\text{main}} + \frac{512}{\pi^4} n_1 f_1 [(A_1)_{\text{side 1}} + (A_1)_{\text{side 2}}] \\
&= \frac{512}{\pi^4} n f \frac{(K_1)_{\text{main}} - \beta}{1+\beta+\mu} + \frac{512}{\pi^4} n_1 f_1 \left[\frac{(K_1)_{\text{side 1}} - \beta}{1+\beta+\mu_1} + \frac{(K_1)_{\text{side 2}} - \beta}{1+\beta+\mu_1} \right] \\
&= \frac{512}{\pi^4} \frac{n f}{1+\beta+\mu} \left\{ (K_1)_{\text{main}} + \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} [(K_1)_{\text{side 1}} + (K_1)_{\text{side 2}}] \right\} \\
&\qquad - \frac{512}{\pi^4} \frac{n f}{1+\beta+\mu} \left(1 + 2 \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} \right) \beta \cdots\cdots(135)
\end{aligned}$$

Multiplying through by $\frac{\pi^4}{512} \frac{n f}{1+\beta+\mu}$ and rearranging,

$$\begin{aligned}
&\beta \left(1 + 2 \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} + \frac{\pi^4}{512} \frac{H_w L_s}{E_c A_o} \frac{1+\beta+\mu}{n f} \pm \frac{\pi^4}{512} \frac{\epsilon \Delta t L_t}{n f} \right) \\
&= (K_1)_{\text{main}} + \frac{n_1 f_1}{n f} [(K_1)_{\text{side 1}} + (K_1)_{\text{side 2}}] \mp (1+\mu) \frac{\pi^4}{512} \frac{\epsilon \Delta t L_t}{n f} \cdots(136)
\end{aligned}$$

Let:

$$N_s = 2 \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} \cdots\cdots(137a)$$

$$N_e = \frac{\pi^4}{512} \frac{H_w L_s}{E_c A_o} \frac{1+\beta+\mu}{n f} \cdots\cdots(137b)$$

$$N_t = \frac{\pi^4}{512} \frac{\epsilon \Delta t L_t}{n f} \cdots\cdots(137c)$$

and

$$D_{\pm} = 1 + N_s + N_c \pm N_t \quad \dots\dots\dots\dots\dots\dots (137d)$$

Then,

$$\beta = \frac{(K_1)_{\text{main}} + \frac{N_s}{2}(K_1)_{\text{side 1}} + \frac{N_s}{2}(K_1)_{\text{side 2}} \mp (1 + \mu) N_t}{D_{\pm}} \quad \dots\dots (138)$$

Eq. 138 demonstrates that the principle of superposition applies to β for different load combinations only when N_s, N_t, and N_c are constants. The quantity N_t is a constant, as shown by Eq. 137c; but, although N_s and N_c involve β they are small in comparison to unity and a wide variation of β does not affect their values materially. It follows that the principle of superposition is applicable, in practice, in determining the horizontal component of cable stress for different load combinations.

For ordinary bridges, β varies roughly from zero for no live load to 1.0 g for full live load. An average value of $\beta = 0.5\,g$ can be assumed in Eqs . 137a and 137b without causing any serious error, thus producing the simple formulas:

$$N_s = 2\,\frac{n_1 f_1}{n f}\,\frac{1 + 0.5\,g + \mu}{1 + 0.5\,g + \mu_1} \quad \dots\dots\dots\dots\dots\dots (139a)$$

and

$$N_c = 0.190\,\frac{H_w L_s}{E_s A_c}\,\frac{1 + 0.5\,g + \mu}{n f} \quad \dots\dots\dots\dots\dots\dots (139b)$$

The expression for K_m defined by Eq. 129b can be determined easily for any loading. Referring to Fig. 4, let k_1 be expressed as $k'\,l$ and $k_2 - k_1$ as $k\,l$. For the general case of uniform load shown in Fig. 4,

$$K_m = \frac{m\,\pi}{4\,w}\,\frac{2}{l}\int_{k'l}^{(k+k')l} p \sin\frac{m\,\pi\,x}{l}\,dx = \frac{m\,\pi}{4\,w}\,\frac{2}{l}\left(-\,p\,\frac{l}{m\,\pi}\cos\frac{m\,\pi\,x}{l}\right)$$

$$\times \int_{k'l}^{(k+k')l} = \frac{p}{w}\sin\tfrac{1}{2}\,m\,(k + 2\,k')\,\pi\,\sin\tfrac{1}{2}\,m\,k\,\pi$$

$$= g\sin\tfrac{1}{2}\,m\,(k + 2\,k')\,\pi\,\sin\tfrac{1}{2}\,m\,k\,\pi \quad \dots\dots\dots\dots\dots (140)$$

Using $k\,l$ to denote length of load and $k'\,l$ to denote position of load, K_m for concentrated loads and special cases of uniform loading can be readily deduced from Eq. 140. It is evident from Eq. 129b that K_m for the combination of a number of loads is equal to the sum of the K_m-values for the individual loads. Once again, the applicability of the principle of superposition is demonstrated.

Application of the general formula, Eq. 138, for H to the author's ten cases of loading results in a series of equations that can be compared with the author's formulas in Table 1. Doing so, the writer found that only the discrepancies for N_s and S are significant. The more accurate of the two procedures remains to be tested by checking each against the exact formula (Eq. 134) for various bridges. However, the writer believes either set of formulas will yield results sufficiently accurate for ordinary design purposes.

The author expresses the characteristics of stiffening trusses in hyperbolic functions (Part II). With a good table of hyperbolic functions at hand, the computation required by the author's method is probably the least among all the methods. With the simplified formulas for H, he is able to formulate the conditions of maximum values for different functions which are usually found by trial. These achievements, demonstrated in the paper, will certainly facilitate the analysis considerably. However, the writer does not believe that it is justifiable to determine the section of maximum moment and the loading for it simultaneously because the equations (see Eqs. 72 and 75) are not exact and their solution is not simple. It would be more simple and dependable to determine the maximum moments at a few points near the quarter point and then to pass a curve through them.

The method of trigonometrical series as applied to the functions of the stiffening truss has an advantage over the deflection theory—that is, the general nature of the formulas. A single formula can be written for a function at any section of stiffening truss for all cases of loading. Conditions of maximum can be established in the method of trigonometrical series[17] too.

Summarizing the foregoing discussion, conclusions can be drawn as follows:

1. To derive the simple formulas for H, the writer's suggestion of using the trigonometric series[17] is more general and rigorous.
2. The author's formulas for truss functions require least computation whereas the formulas in trigonometrical series have the advantage of generality.
3. Contrary to the general impression of engineers, the analysis of suspension bridges is no longer more complicated than for other continuous structures.

Ling-hi Tsien,[18] Esq.—There being little disagreement in the discussions with the method proposed in the paper, few closing comments appear to be needed. The writer wishes to express his indebtedness for the clarifying effect of the discussion.

Professor Gravina, Mr. Ramaswamy, and Professor Liu each offer an approach to the derivation of formulas for the horizontal components of cable tension. They derive the same results as those presented in Table 1. Professors Gravina and Liu both use the trigonometrical series and are rigorous in their procedures. Mr. Ramaswamy suggests that the case of a moving concentrated load on the span should be treated first, and that the other cases of loading could easily be deduced by integration. The idea of influence lines could be used only after the applicability of the principle of superposition is demonstrated. Although superposition is proved to be applicable, approximately, for distributed loadings by Theorem 3 (a conclusion drawn from Theorems 1 and 2), Eqs. 17, thus deduced for the case of concentrated load, should be used with care. As pointed out by Professor Gravina:

> "* * * in the extreme case of a concentrated load it is no longer possible to consider only the first term of the series, as was done in the case of the distributed load; if this were done, the resulting error would be considerable."

Eqs. 17 could be considered only as corollaries to the cases of distributed loading, rather than as fundamental formulas as Mr. Ramaswamy proposes.

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[18] Prof., Civ. Eng., National Univ. of Chekiang, Hangchow, Chekiang, China.

In fact, there are other approaches to the derivation of simplified formulas for H. For instance, at first the writer was preoccupied by the realization that the classical but implicit and lengthy formulas from the deflection theory[19] may be greatly simplified and reduced to the same form as those given in the paper. This result can be achieved by expanding terms in series, and by introducing some minor approximations. However, the writer prefers the demonstration in the paper for the following reasons: By comparing the two extreme types of suspension bridges (namely, the unstiffened and the very rigidly stiffened types), it reveals immediately the fact that, although the horizontal component of cable tension may be appreciably different in magnitude, its law of variation remains practically the same for all types of suspension bridges when the live load moves arbitrarily on the span. The writer emphasizes the fundamental fact that this structural behavior of cable tension should be the key point of the analysis, and demonstrates his point by a simple comparison. The remaining task is to determine the correction factor D for the effects of side span interaction, cable stretch, and temperature changes. Each step furnishes its own physical meaning and reveals its own relative weight, or importance.

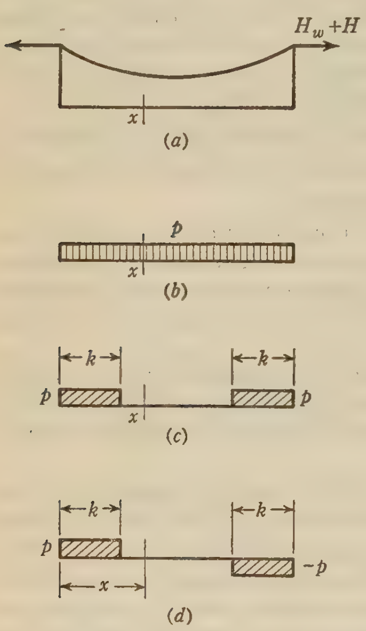

FIG. 11

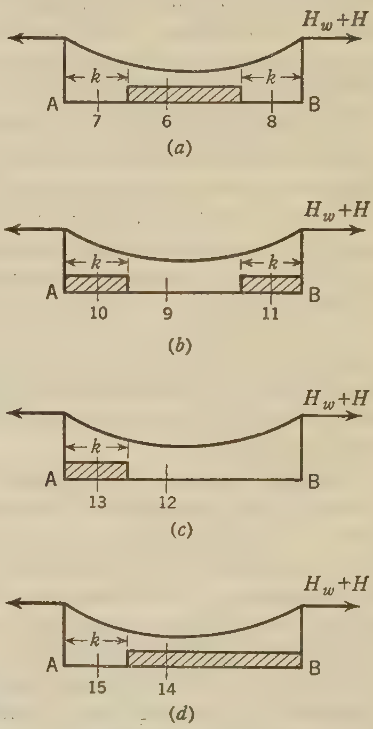

FIG. 12

Professor Liu's discussion is especially valuable for its precise point of view on the problem of suspension bridges. His general equation (Eq. 138) for the horizontal component of cable tension reveals the applicability of the principle of superposition, although this is partly demonstrated in the paper by Theorem 3.

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[19] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1929, pp. 264–267.

Incidentally, a possible mode of superposition can be also found even for the bending moment in a stiffening truss. By inspecting the formulas in Table 2, it will be shown that the M-equation for different loadings can be obtained by summation of five moment components. Let

$$m = M \frac{c^2 l^2}{p l^2} \qquad \text{(141a)}$$

$$Q = \frac{H}{H_w} \qquad \text{(141b)}$$

and

$$A_x = \frac{\cosh c \left(\frac{l}{2} - x \right)}{\cosh \frac{c l}{2}} \qquad \text{(141c)}$$

In Eq. 141c, A_x is a symmetrical function of x which varies from 0 to l; and it remains the same for section x whether x is measured from one end or the other of the span. Five moment components are as follows:

1. m_1 due to the direct effect of cable tension (Fig. 11(a))—

$$m_1 = - Q (1 - A_x) \qquad \text{(142a)}$$

2. m_2 due to full uniform live load without considering the direct effect of cable tension (Fig. 11(b))—

$$m_2 = 1 - A_x \qquad \text{(142b)}$$

3. m_3 due to symmetrical partial loading from both ends without considering the direct effect of cable tension (Fig. 11(c))—

$$m_3 = A_x (\cosh c k - 1) \qquad \text{(142c)}$$

(Section x is in the unloaded length measured from either end of the span.)

4. m_4 due to pure couple loading (Fig. 11(d))—

$$m_4 = \frac{\sinh c (\frac{1}{2} l - x)}{\sinh \frac{1}{2} c l} (\cosh c k - 1) \qquad \text{(142d)}$$

(Section x is in the unloaded length measured from the left end of the span.)

5. Whenever section x is shifted from an unloaded part (m) to a loaded part (m'), the value of m is reduced by a component m_5, and vice versa—

$$m_5 = m - m' = \cosh c (k - x) - 1 \qquad \text{(142e)}$$

(To shift section x from a central region to a side (Figs. 12(a) and 12(b)) the distance x must be measured from the nearer ends of the span to this side.)

The m-value at sections noted in Fig. 12 can be obtained by superposition as follows:

$$m_6 = m_1 + m_2 - m_3 \text{ (} x \text{ measured from either end A or end B)} \qquad (143a)$$

$$m_7 = m_6 + m_5 \text{ (} x \text{ measured from end A)} \qquad (143b)$$

$$m_8 = m_6 + m_5 \text{ (} x \text{ measured from end B)} \qquad (143c)$$

$$m_9 = m_1 + m_3 \text{ (} x \text{ measured from either end A or end B)} \qquad (143d)$$

$$m_{10} = m_9 - m_5 \text{ (} x \text{ measured from end A)} \qquad (143e)$$

$$m_{11} = m_9 - m_5 \text{ (} x \text{ measured from end B)} \qquad (143f)$$

$$m_{12} = m_1 + \tfrac{1}{2}(m_3 + m_4) \text{ (} x \text{ measured from end A)} \qquad (143g)$$

$$m_{13} = m_{12} - m_5 \text{ (} x \text{ measured from end A)} \qquad (143h)$$

$$m_{14} = m_1 + m_2 - \tfrac{1}{2}(m_3 + m_4) \text{ (} x \text{ measured from end A)} \qquad (143i)$$

$$m_{15} = m_{14} + m_5 \text{ (} x \text{ measured from end A)} \qquad (143j)$$

The principle of superposition can be applied only to the derivation of expressions of m, not to the numerical values of m.

Professor Liu declares that the determination of the section of maximum moment is not justifiable because Eqs. 72 and 75 are not rigorously exact. Eq. 72 is exact, but Eq. 75 is obtained with the assumption of a constant c, and the method of solving these two simultaneous equations is also approximate. By checking several existing bridges, it has proved to be quite accurate for practical purposes. Since the variation of moments near the maximum point is small, the procedure given in the paper for determining the maximum moment can be judged justifiable for its simplicity and its directness.

It is hoped that the theory and the design procedure presented in the paper will be tested in practical designs.

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EXPANSION JOINT PRACTICE IN HIGHWAY CONSTRUCTION

By A. A. Anderson,[1] Assoc. M. ASCE

With Discussion by Messrs. Frank H. Gardner, K. B. Woods, J. K. Knoerle, William A. Conwell, and A. A. Anderson.

Synopsis

This paper points out that in concrete highway pavement construction the trend is toward the elimination of expansion joints. It explains why properly maintained modern pavements do not "blow up" even though little or no provision for expansion has been made.

Information taken from the reports on Michigan and Minnesota experimental jointing projects is cited to show typical slab end movements at expansion and contraction joints and how these are affected by different jointing arrangements. Using actual strain gage readings and other data from a 20-ft section of the Minnesota experimental project where the expansion joints were 5,260 ft apart, the compressive stress that developed due to an 83° F rise in the temperature of the concrete is shown to be only 628 lb per sq in. Based on this and the other data presented, it is concluded that in pavements built with properly spaced and maintained contraction joints, expansion joints may be eliminated, except for unusual conditions of construction.

Introduction

In the early days the amount of concrete pavement that could be placed in one day was small, so that even in pavements constructed without intermediate jointing the construction joints were at relatively short intervals. As these early pavements aged, they developed transverse cracking at irregular intervals. Early expansion joints were mastic-filled openings placed at relatively short intervals, ranging from 30 ft to about 150 ft, but these joints did not prevent the formation of cracks at random intervals between the expansion joints. Therefore, some engineers decided that it was unnecessary and uneconomical to install joints except at the end of each day's run. They would allow the pavement to form its own joints by cracking as needed.

Note.—Published in September, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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Many thousands of miles of pavement were built without transverse joints, other than the construction joints at the end of each day's run. These pavements were simple to construct, lower in first cost, and have given good service. Their major difficulty, which did not develop until after the pavement had been in service for five years or more, was a type of expansion failure commonly known as a "blowup."

These blowups were considered evidence that concrete pavements needed expansion joints to protect them from compressive stresses. Therefore, the use of expansion joints was generally resumed, with considerable variation in joint spacing adopted by the various state highway departments. In 1934 the United States Bureau of Public Roads (predecessor of the Public Roads Administration (PRA)) issued regulations which required ¾-in. expansion joints at intervals not to exceed 100 ft in concrete pavement on all federal aid roads. A further requirement was that contraction joints be used at intervals not exceeding 30 ft, unless the pavement was adequately reinforced for greater slab length.

For several years jointing practices were in general conformance with this ruling. Later experience indicated the desirability of modification in jointing requirements. Accordingly, the PRA invited a number of state highway departments to cooperate in the construction of experimental projects in which different jointing arrangements would be tested. Six states—California, Kentucky, Michigan, Minnesota, Missouri, and Oregon—joined in the cooperative studies. Progress reports on the experimental projects have been published.[2,3,4,5,6,7,8,9,10,11,12] The results from these experimental projects and the data obtained from surveys of other pavements in service have led to some modification in jointing regulations governing federal aid projects. Considerable freedom in jointing of concrete pavement is permitted—provided that the states are able to support their proposed jointing scheme by sound data from condition surveys or other sources.

Recent changes in state highway department practices in expansion joint spacings reflect the change in regulation. At least six state highway departments now construct their rural pavements without expansion joints, or use such joints only where the pavements join structures—that is, at bridges or at intersections with other pavements. Six state highway departments place their expansion joints at intervals ranging from 300 ft to 600 ft, twelve states use an interval ranging from 100 ft to 120 ft, and the remainder of the state highway departments from which data were available place their expansion

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[2] "Investigational Concrete Pavement in Kentucky," by F. P. Anderson, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 20, 1940, p. 337.

[3] "Investigational Concrete Pavement in Michigan," by J. W. Kushing, *ibid.*, p. 339.

[4] "Investigational Concrete Pavement in Minnesota," by F. C. Lang, *ibid.*, p. 348.

[5] "Investigational Concrete Pavement in Oregon," by G. S. Paxson, *ibid.*, Vol. 21, 1941, p. 147.

[6] "Investigational Concrete Pavement in Missouri," by F. V. Reagel, *ibid.*, p. 150.

[7] "Investigational Concrete Pavement in California," by T. E. Stanton, *Research Reports No. 3B*, Highway Research Board, National Research Council, Washington, D. C., Vol. 25, 1945, p. 1.

[8] "Investigational Concrete Pavement in Kentucky," by T. R. Thomas, *ibid.*, p. 28.

[9] "Investigational Concrete Pavement in Michigan," by H. C. Coons, *ibid.*, p. 35.

[10] "Investigational Concrete Pavement in Minnesota," by F. C. Lang, *ibid.*, p. 58.

[11] "Investigational Concrete Pavement in Missouri," by F. V. Reagel, *ibid.*, p. 72.

[12] "Investigational Concrete Pavement in Oregon," by G. S. Paxson, *ibid.*, p. 77.

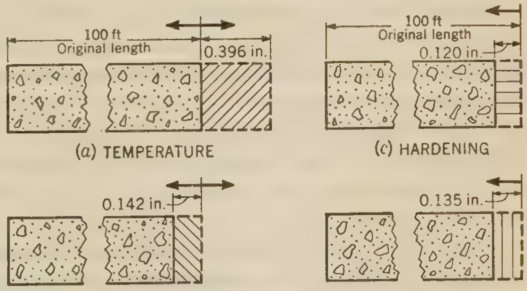
joints at intervals of less than 100 ft. It is believed that time will eliminate some of the wide variations now in existence, but it is obvious that there still remains considerable difference of opinion on expansion joint spacing. Doubtless many engineers will disagree with the conclusions drawn hereinafter, but their study of the supporting data is respectfully invited.

A discussion of jointing practices in concrete pavement cannot be limited to expansion joints alone. The decision to include or to omit expansion joints in concrete pavement depends, more than any other single factor, on whether contraction joints are installed at intervals that will control cracking. A discussion of expansion joints must include a discussion of contraction joints.

Contraction joint spacing practices of state highway departments are also at variance, but the degree is not as marked as for expansion joint practices. At least thirty-one states place contraction joints at intervals varying from 15 ft to 30 ft. Only one state uses contraction joints spaced less than 15 ft apart, whereas the remainder for which data were available use spacings in excess of 30 ft. Generally speaking, those states, which employ contraction joint spacings in excess of 30 ft, employ a relatively short spacing of expansion joints and place but one or, at most, two contraction joints between successive expansion joints. Of the twelve states which place their expansion joints at intervals of 300 ft or more, all employ a contraction joint spacing ranging from between 15 ft and 30 ft.

ORIGINS OF SLAB STRESSES

Stresses in a concrete highway slab come from two principal sources. Traffic using the pavement causes load stresses which are fairly well understood. Theoretical calculations of load stresses have been checked by actual stress measurements made both in the laboratory and in the field. Load stresses have a bearing on the jointing of concrete pavements. Of greater importance to the joint arrangement, however, are those stresses which are not caused by loads but rather by volume changes that take place within the concrete during and after the hardening period.



(a) TEMPERATURE (c) HARDENING

(b) MOISTURE (d) PLASTIC FLOW

FIG. 1.—PRINCIPAL CAUSES OF VOLUME CHANGES IN AN UNLOADED CONCRETE SLAB

Typical volume changes that may take place in a 100-ft length of concrete pavement are shown graphically in Fig. 1. It is assumed that the pavement is constructed with normal materials and that it is resting on a frictionless subgrade. The greatest volume change is caused by temperature fluctuations. Assuming that the temperature of the concrete at the time of placing was 70° F, and that some time after hardening the temperature of the concrete was raised to 130° F,a 0.396-in. increase in the length of the 100-ft slab could be expected. This is based on a linear coefficient of thermal expansion of 0.0000055, an average value for normal concrete, and also on the assumption that other factors affecting the volume change have been inactive during the period of temperature rise. If the temperature decreased by 60° F, instead of rising, the pavement would have shortened by an equal amount.

A second factor which will cause change in the length of the slab is a change in the moisture content of the concrete. The concrete tends to expand and contract as its moisture content increases and decreases. The moisture content of pavement concrete changes very slowly, and the effects on volume are not as great as those due to changes in temperature. Nevertheless, in most regions there is a change in average moisture content of pavement concrete from one season to the next and, as would be expected, these seasonal variations will differ from one locality to another. The Minnesota State Highway Department has determined that, for the conditions existing on their experimental project, the unit linear variation in concrete due to seasonal changes in moisture amounts to 0.000118. Using this factor and again assuming that other factors remain inactive, a seasonal change in the length of a 100-ft slab would amount to 0.142 in. Seldom, if ever, will concrete again contain as much moisture as it contained at the time of hardening. Therefore, the over-all effect of moisture changes will always result in a net shortening from the original placement length. Also, in most areas subsequent maximum moisture contents will occur during the season of low temperatures. Thus the volume changes due to moisture and temperature fluctuations tend to offset rather than to augment each other.

It is a well-recognized fact that as concrete hardens shrinkage takes place. Tests reported by various authorities indicate the unit change in length due to shrinkage to be at least 0.0001. Some authorities report considerably higher figures. Using this conservative unit rate, the change in length of the 100-ft slab would be approximately 0.120 in.

When concrete is subjected to a sustained stress, a plastic deformation, commonly called plastic flow, takes place. Data from laboratory and field studies show that the amount of plastic deformation or flow varies with the intensity of the stress and the length of time the stress is maintained. If the 100-ft pavement slab is assumed to be under a compressive stress of 680 lb per sq in. 5 hours per day for a period of 10 days, a plastic flow of about 0.135 in. could be expected. As indicated in Fig. 1, this plastic flow would cause a shortening of the slab.

Each one of the foregoing factors acting alone to cause volume change is fairly easy to apply. For pavements in service, however, these factors do not act singly and any movements which take place in a pavement are due to a combination of two or more of the factors. As temperature causes the greatest and most spectacular changes in the volume of a concrete slab, this factor alone was considered in the past, with little or no consideration being given to other factors. This often led to erroneous conclusions.

If all the factors shown in Fig. 1 acted simultaneously, the slab would be shorter than its 100-ft original length so that even for the assumed high temperature of 130° F there would be no compressive stress. However, all these changes will not be effective at any one time. If it is assumed that only the change due to temperature and that due to the initial hardening of the concrete have occurred, the slab, at the high temperature of 130° F, would be 0.276 in. longer than the 100-ft initial length. If no space were provided for expansion, the resulting restraint will cause a unit strain of $\frac{0.276}{100 \times 12} = 0.00023$. For concrete of pavement quality and for strains which develop slowly, a suitable value for the modulus of elasticity, E, is 4,000,000 lb per sq in., and the calculated stress corresponding to the strain would be $0.00023 \times 4{,}000{,}000 = 920$ lb per sq in. Plastic flow and the loss of moisture in the slab at the high temperature of 130° F would undoubtedly result in an actual stress below this amount, which may be considered the maximum.

Pavement concrete usually has a compressive strength of more than 4,000 lb per sq in. Even when load stresses are added to the maximum of 920 lb per sq in., the total compressive stress will be less than half the ultimate compressive strength of the concrete. Since studies have shown that stresses up to 50% of the ultimate strength of concrete may be repeated indefinitely without causing fatigue or leading to rupture, it is apparent that compressive stresses which will result from the omission of expansion joints will not be critical in themselves.

What then causes compression failures, commonly known as blowups? Although high stresses undoubtedly do develop at points where blowups occur, they seldom, if ever, develop in newly placed pavements. Rather, they occur after the pavements have been in service for 4 or 5 years. This question has been the subject of much study and all evidence indicates that blowups are caused by a gradual loss of expansion space provided by crack or joint openings because of infiltration of incompressible material. This leads to the conclusion that blowups can be prevented by eliminating infiltration or reducing it to an insignificant amount. Much evidence shows this conclusion to be true.

It is extremely difficult to maintain an effective sealing which will avoid infiltration of foreign material into haphazard cracks. Therefore, an effective method of preventing blowups is to joint the pavement so that intermediate cracking will be controlled. This can be done economically and effectively by installing contraction joints of the "dummy groove" type at a suitable spacing. The short slab lengths required for crack control will reduce joint openings, and dummy groove joints are more readily maintained than cracks. Experience shows that under average maintenance infiltration is practically prevented.

The primary function of contraction joints is to form slab lengths that will reduce tensile stresses and prevent the formation of intermediate cracks. These cracks are caused by a combination of tensile stresses resulting from wheel loads, curling, and differential movement of the subgrade, or by subgrade restraint to contraction during hardening of the concrete, contraction due to drops in temperature, or reduction in moisture content. Some of these stresses decrease as individual slabs become shorter. By reducing slab lengths some of the stresses will be kept small and the combined tensile stresses will not be great enough to cause the formation of intermediate cracks.

Experience from roads in service is the best guide as to spacing of contraction joints which will control cracking. In most instances about 20 ft will be satisfactory. The type of aggregate used in the concrete has a major influence, slag and small size siliceous gravel requiring a spacing of about 15 ft. Some aggregates, such as crushed granite or limestone, may permit satisfactory control of cracking by contraction joints spaced at 25 ft or more, but spacings in excess of 25 ft are generally to be avoided as they will be productive of excessive opening and closing of the joint and thus increase the difficulty of maintaining a satisfactory seal and make the joint less effective for load transference. Experience with pavements in service indicates that these joint spacings will prevent intermediate cracks and will also keep the average joint opening small enough so that it can be kept sealed without difficulty, providing the distance between expansion joints is relatively large or that expansion joints are completely eliminated.

Experimental Results

It is of interest to compare the foregoing discussion on jointing with the data from the experimental projects which were constructed by the six state highway departments in cooperation with the PRA during 1940 and 1941. All six projects were constructed with a series of paving sections in which numerous combinations of expansion and contraction joint spacings were used, thus making it possible to study the effect of different jointing arrangements. The information presented herein was obtained from the Michigan and Minnesota projects, both constructed in 1940, after 4 years of service. These data were considered typical and were compiled from reports[2,3,4,5,6,7,8,9,10,11,12] published in 1945. Fig. 2 shows the annual and progressive changes in expansion joint openings that have taken place on various sections of these two projects. Referring to the Michigan project during the warmest time of its first year (1941) the joint closed 1.30 in. As the pavement subsequently cooled the joint did not return to its original opening, but there was a residual closure of

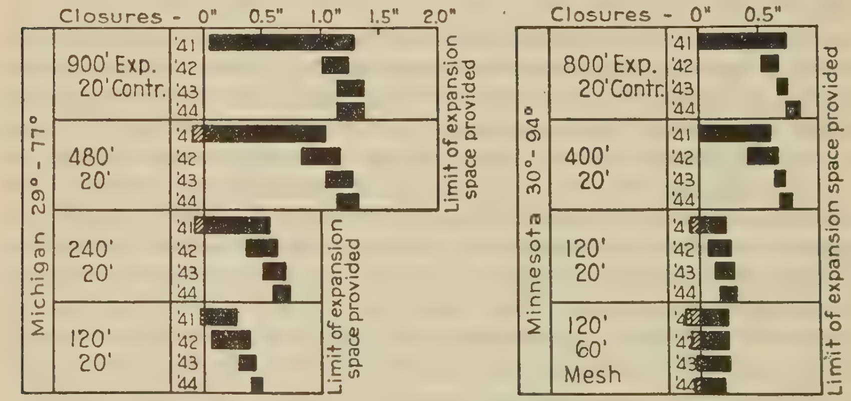


Fig. 2.—Typical Annual and Progressive Changes in Expansion Joint Widths

about 1 in. During the warm period of the second year the joint closure was about 1.25 in. from the original, but the total movement that took place in that joint during 1942 amounted to only 0.25 in. In 1943 a residual closure of about 1.15 in. remained; the total movement in that year amounted to about 0.17 in. In 1944, when the pavement was in its fourth year, the total amount of joint movement was again less than 0.20 in., indicating that the movement at that particular joint tended to become stabilized.

In following through the joint spacings of 480 ft, 240 ft, and 120 ft, it is seen that the movements taking place at each joint follow the pattern of the one previously outlined—namely, that the greatest movement at any expansion joint takes place the first year and that the amount of recovery is relatively small. After a period of from two to three years, the movement that takes place within any expansion joint has become stabilized and is small. As would be expected, the closer the spacing of the expansion joints, the less is the movement that takes place the first year. However, even an expansion joint spacing of 120 ft does not prevent the gradual closing of the joint.

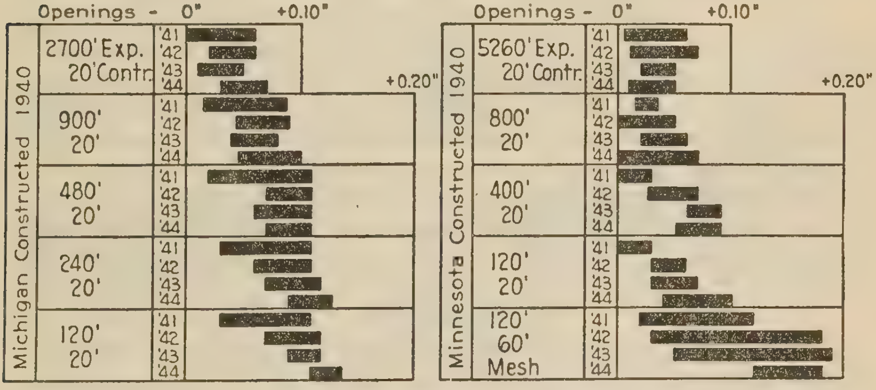


Fig. 3.—Typical Annual and Progressive Changes in Contraction Joint Widths

By referring to the Minnesota data, it can be seen that they follow the same trend as those from the Michigan project. Results similar to those shown herein have been noted by other observers. It has often been said that due to the rapid closure of expansion joints and the small amount of recovery that takes place, it is but a short time until expansion joints in reality serve only as contraction joints.

Fig. 3 shows the contraction joint movements which accompanied the variations in expansion joint openings shown in Fig. 2 and reveals some interesting facts. The annual average movement that takes place at any contraction joint is relatively small where the pavement is divided into the short panels used on the two projects for which data are given and is not greatly affected by the spacing of the expansion joints. However, as the distance between expansion joints becomes less, the maximum opening of contraction joints which takes place during each successive year becomes greater, and the annual residual joint opening likewise becomes larger. The extremes of this pattern can be seen in the action of the Minnesota contraction joints for expansion joint spacings of 120 ft and 5,260 ft. In the 120-ft expansion joint spacing, there is a progressive increase in both maximum and residual opening at the contraction joints with no apparent sign of stability even after 4 years. However, in the 5,260-ft section these are more or less constant after the first year. By the third year the maximum joint openings, as well as the residual joint openings, are about half of those occurring where the expansion joints are spaced at 120 ft. After studying Fig. 3 it is apparent that the smaller maximum and residual contraction joint openings are found where the expansion joints are spaced at longer intervals.

Fig. 4 shows the average seasonal changes in contraction joint openings resulting from the use of various combinations of expansion and contraction joint spacings. Almost without exception the average maximum contraction joint opening for any one spacing of contraction joints varies inversely with the distance between expansion joints. It can also be seen, taking corresponding data from each group, that for any particular expansion joint spacing both the opening and the annual movement at contraction joints materially decrease as the distance between contraction joints becomes shorter. The joint arrangements on the Minnesota project varied somewhat from those used in Michigan; however, the same trends are apparent.

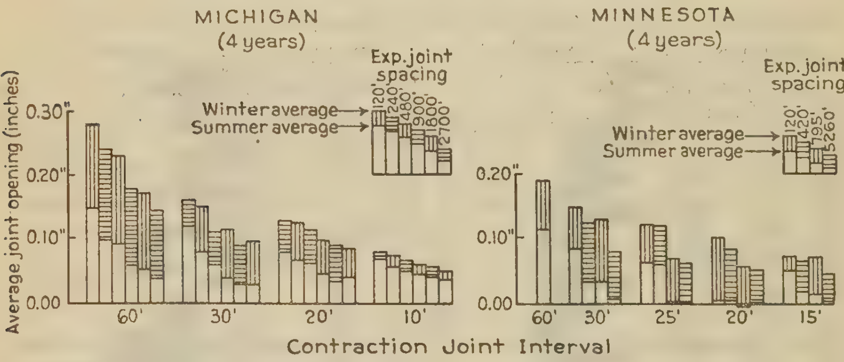


Fig. 4.—Average Seasonal Changes in Contraction Joint Widths as Related to Spacings of Expansion and Contraction Joints

In reading the magnitude of joint opening for any particular section, it must be remembered that the values shown are average for all contraction joints within that section. Within any section, especially one having expansion joints spaced in excess of 1,000 ft, the contraction joints near the center of the section generally have smaller annual movement than those near the expansion joints. Two or three contraction joints on each side of the expansion joints may be expected to open more than indicated herein, and this fact must be given consideration in the design of the slab.

Actual measurements on two contraction joints (adjacent to a 20-ft panel approximately in the center of the section of the Minnesota project where the expansion joints were spaced 1 mile apart) show the joint openings measured in February, when the concrete had a temperature of 29° F, to be 0.056 in. and 0.052 in., respectively. Similar values obtained in August, when the temperature was 112° F, were 0.004 in. and 0.012., respectively. With this information, and knowing the actual coefficient of expansion and modulus of elasticity, it is possible to make a theoretical determination of the probable stress that existed in the pavement when the temperature was the highest recorded on the project. The temperature rise is 83° F. Were this 20 ft of pavement completely unrestrained, this rise in temperature would have caused a lengthening of the slab equal to 0.109 in. However, the actual increase in slab length was only 0.046 in., the difference between the average joint openings measured in February and August. Between this winter and summer period, data from the project show the moisture content of the pavement decreased and produced a net shortening between winter and summer of 0.028 in. The restrained movement thus amounts to 0.035 in., which represents the amount of elongation which would have occurred in a slab completely free of restraint, but which was prevented from occurring in this restrained slab and was absorbed in the 20-ft length of pavement, producing a unit strain of 0.000146.

Using the actual measured value of 4,300,000 lb per sq in. for the modulus of elasticity, the computed stress developed in this slab amounts to only 628 lb per sq in., which is approximately $\frac{1}{6}$ of the actual 28-day compressive strength of this concrete and cannot be considered critical. Strain measurements made at plugs installed in the immediate vicinity of this slab show that for the same temperature of 112° F, the stress was 684 lb per sq in., only 56 lb per sq in. higher than that obtained in the foregoing computation. This is undoubtedly very close to the maximum stress which has developed on this project. It is of interest to note that strain measurements at quarter points in the mile showed substantially the same compressive stress.

These data from the experimental projects agree with the previously presented analysis of stresses which may develop in pavements built without expansion joints but with contraction joints at short intervals. Both the data and the analysis explain why pavements constructed with little or no provisions for expansion, but with well-constructed and maintained contraction joints at relatively close intervals, do not rupture during periods of high temperature.

Conclusions

Based on the information and data presented, which is substantiated by other experimental projects and studies that have been made on pavements in service, the following conclusions may be drawn:

During the construction of concrete pavement, little or no expansion space need be provided if—

(1) The pavement is constructed of materials that have normal expansion characteristics;

(2) The pavement is constructed during those periods of the year when the temperatures are well above freezing;

(3) The pavement is divided into relatively short panels by satisfactory contraction joints so spaced as to prevent the formation of intermediate cracks; and

(4) The contraction joints are properly maintained to prevent the infiltration of incompressible material.

Thus, with infiltration controlled by the installation and maintenance of properly spaced dummy contraction joints, it appears that concrete pavements do not need expansion joints. Only in concrete pavement built during cold weather does there seem to be any reason for using expansion joints, and even then they are needed only at long spacing, perhaps from 600 ft to 800 ft. In fact, concrete pavement built without excess expansion space may be expected to have increased structural capacity and to give better performance than pavement having expansion joints at short intervals. In addition it is simpler and less expensive to construct, easier to maintain, and has better riding qualities. It can be stated that the current trend in jointing practices on modern American highways is in the directions indicated.

DISCUSSION

Frank H. Gardner,[13] **Assoc. M. ASCE.**—This excellent treatment of spacing of expansion joints in concrete pavements stops short, however, of the total range of the subject in that the effect of subgrade friction on slab movement; reinforcing; formation of joints; load transmission; joint materials; prevention of infiltration; and other factors are not mentioned. It is hoped that in his closure the author will discuss these fields so that the paper will fully cover this important subject.

The writer agrees with the author's principal conclusions and predicts that the trends, in addition to those set out in the paper, will include:

1. Development of machines for cutting contraction joints after the concrete has set, or other methods to avoid interrupting the finishing operation at joints;
2. Increase of the tie across contraction joints by lapping the reinforcement at joints or, preferably, by additional corrosion resisting metal;
3. Omission of load transfer devices at joints so tied;
4. Improvement of joint filling compounds, possibly by the addition or use of recently developed plastics;
5. More extensive subsealing in areas adjacent to joints; and
6. Lengthening slabs to or beyond the upper limits suggested by the author.

The author is to be commended for his excellent treatment of the part of the subject covered.

K. B. Woods,[14] **M. ASCE.**—The subject of expansion joints in highway pavements has been treated in a timely and interesting manner by Mr. Anderson. The idea of restricting the use of expansion joints is a good one, and the writer is in general agreement with the conclusions drawn. However, the statement (see "Origins of Slab Stresses") that blowups are caused by infiltration of incompressible material in joints and cracks of concrete pavements is open to question.

In regard to pavements built without expansion joints, Mr. Anderson states, in the second paragraph under the heading, "Introduction," that:

> "Their major difficulty, which did not develop until after the pavement had been in service for five years or more, was a type of expansion failure commonly known as a 'blowup.'"

Also, under the heading, "Origins of Slab Stresses," he asks a question and answers it, thus:

> "What then causes compression failures, commonly known as blowups? * * * all evidence indicates that blowups are caused by a gradual loss of expansion space provided by crack or joint openings because of infiltration

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[13] Highway Engr., U. S. Public Roads Administration, Washington, D. C.

[14] Associate Director, Joint Highway Research Project, and Prof. of Highway Eng., Purdue Univ., Lafayette, Ind.

> of incompressible material. This leads to the conclusion that blowups can be prevented by eliminating infiltration or reducing it to an insignificant amount. Much evidence shows this conclusion to be true."

Although infiltration is, no doubt, an important factor in causing blowups, as are certain combinations of temperature and moisture, it does not follow that infiltration of extraneous material is always a fundamental cause of blowups. Neither does it follow that all, old, nonjointed pavements blow up even though the crack interval may be long and may thus provide many large openings into which extraneous materials can infiltrate. Work done at Purdue University (Lafayette, Ind.) in connection with extensive performance surveys throughout Indiana has shown that blowups should be correlated with the source of coarse aggregates used, at least for conditions prevailing in the State of Indiana.[15]

In this same work, field data were obtained from district offices covering the occurrence of blowups including time of day, age of pavement, prevailing weather conditions, location of the road with the failure, the location on the road with reference to grade and alinement, and materials incorporated in the pavement. Data were obtained and analyzed for a total of 517 blowups which occurred in the summers of 1940, 1941, and 1942. Precipitation and maximum temperature data were collected from the weather stations nearest to the road which contained the blowup. It was concluded that these blowups occurred predominantly in midafternoon at a temperature above 90° F—during a period which was usually preceded by varying amounts of precipitation. This analysis confirmed a report[16] published in 1925 to the effect that high temperatures following some precipitation appear to be the immediate causal factor in the occurrence of blowups. The infiltration of dirt into cracks, of course, would accentuate the difficulty by eliminating some of the otherwise available room for expansion.

The climatic and subgrade soil factors are further emphasized, in connection with an analysis of the causes of blowups, in a series of surveys made on three long road projects in Indiana, each of which was built on soils with a wide range of drainage characteristics.[17] These projects were all constructed with coarse aggregate materials of a less than average quality. However, detailed surveys showed that "mapcracking," which had previously been correlated with blowup susceptibility,[15] prevailed on those sections of each of the three projects that were constructed on relatively impervious soils. In contrast, those sections of all three projects which were built on well-drained sands, and, in one instance, on 18 in. of rock-flour silt, were relatively free from blowups and associated mapcracking. The susceptibility of certain types of pavements to blowups under certain conditions of temperature and moisture, and the correlation between blowups and mapcracking with materials used, as well as the drainage characteristics of the subgrade soils used, emphasize the materials-moisture-temperature-subgrade-soil variables rather than the infiltration-of-dirt-in-cracks-and-joints variable as causal factors. Furthermore, the importance of these variables raises the question in regard to the possibility of standardization of pavement design.

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[15] "Pavement Blowups Correlated with Source of Coarse Aggregate," by K. B. Woods, H. S. Sweet, and T. E. Shelburne, *Proceedings*, Highway Research Board National Research Council, January, 1946, pp. 147–168.

[16] "Repair of Concrete Blowups in Delaware," *Engineering News-Record*, September 10, 1925, p. 432.

[17] "Mapcracking in Concrete Pavements As Influenced by Soil Textures," by H. S. Sweet and K. B. Woods, *Proceedings*, Highway Research Board, National Research Council, December, 1946, pp. 286–301.

Since widely different materials and soil types are to be found throughout the United States, and since the nation is subjected to widely varying weather conditions (moisture and temperature), it is questionable as to whether or not such standardization could be accomplished with any degree of satisfaction for a unit as large as the entire country. A more logical engineering approach in regard to this problem might be that of standardizing designs by regions in which reasonably consistent types of materials, soils, climatic conditions, and even traffic conditions would prevail.

In addition to evaluating the moisture-temperature factors in studying blowups, a great quantity of data has been collected[15] on nonjointed pavements built before 1934 in Indiana. With similar conditions of age, soils, climate, and even traffic—within at least restricted locations—it still was found that some roads blew up while others did not. Detailed field checks were made on both types of pavement and information was collected in regard to construction materials and other factors that might have a bearing on the problem. These data were analyzed statistically and it was found that a significant correlation existed between the source of the coarse aggregate used in the concrete and pavement blowups. A total of 2,623 miles of pavements without joints was studied in this survey—2,404 blowups were recorded on 1,872 miles and 1,188 of these blowups occurred on only 284 miles of pavements constructed with coarse aggregates from only five different sources. Furthermore, 707 of the 1,188 blowups occurred on pavements built with one source of supply. In contrast, 851 miles of these nonjointed and old pavements contained no blowups. These data show that blowups occur in Indiana only on pavements constructed with certain types of aggregates, and that blowups do not occur on pavements constructed with certain other aggregates. To solve the problem, it would appear then that a determination of the characteristics of the materials which cause excessive expansion is required, rather than the adoption of designs which include such features as expansion joints. It is of interest to note that the State Highway Commission of Indiana has largely eliminated the use of expansion joints, as a concrete design feature, as a result of the data submitted in the research report on blowups.

As a result of the data obtained from the large-scale field performance surveys, a number of laboratory studies was initiated to determine the characteristics of the materials that were used in pavements susceptible to blowups as well as those which performed satisfactorily. In one laboratory study, Nung Yuen Lu[18] reported a number of interesting results including two which are pertinent to the discussion at hand: (*a*) The coefficient of expansion was different for different concretes and concrete materials when subjected to saturated, room-dry, and oven-dry conditions, and (*b*) concrete made with aggregates having good service performance records had a low coefficient of expansion whereas concrete made with aggregates having poor service performance records had a relatively high coefficient of expansion. However, it should be noted that, in this series of tests, the materials used had either extremely good or extremely bad service records. Thus, few data are available on the expansion characteristics of the materials with only average to moderately good service records. Furthermore, it is probable that this class of material is included in Mr. Anderson's materials with "normal expansion characteristics."

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[18] "Comparison of the Thermal and Moisture Expansion of Concrete and Stone," by Nung Yuen Lu, a thesis submitted to the faculty of Purdue Univ. at Lafayette, Ind., in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering, in February, 1948.

At the same time, H. S. Sweet,[19] Jun. ASCE, was able to produce elongation, mapcracking, and ultimately complete failures, in beams made with coarse aggregates similar to those used in pavements that had an excessive number of blowups, by saturating the aggregate before incorporation in the concrete and then by freezing and thawing the beams. Similar procedures used on beams made with aggregates that had good service records produced no appreciable distress in concrete beams, even after several hundred cycles of freezing and thawing. Furthermore, Mr. Sweet was able to develop a method of identifying inferior aggregates which, when used in concrete, produced mapcracking and associated blowups.[20]

Messrs. Lu and Sweet were each able to produce deterioration in concrete beams made with inferior aggregates by subjecting beams, made by saturating the aggregate before incorporation in the concrete, to alternate cycles of both wetting and drying and heating and cooling.

In summary, the reports on Indiana field and laboratory data are in accord, in general, with Mr. Anderson's conclusion that expansion joints are not needed in modern highway pavements. However, interpretation of these data leads to the conclusion that blowups in these pavements are correlated, primarily, with the type of coarse aggregate and that certain combinations of temperature, moisture, loss of strength due to freezing and thawing, and types of subgrade soil contribute to this action. Although additional research is being conducted in connection with expansion and durability characteristics of various combinations of aggregates and cements, the data to date (January, 1949), including many field observations, indicate the need for standardization of design on a regional rather than on a national basis.

J. K. Knoerle,[21] M. ASCE.—A great contribution has been made by the author to the establishment of a uniform practice for expansion joint spacing in concrete pavement construction. The writer agrees with Mr. Anderson's recommendation for the elimination of expansion joints. Observations made on concrete pavements in many states indicate that such an approach coincides with actual findings. With respect to contraction joints, however, several additional factors have been encountered in practice which are not mentioned by Mr. Anderson.

The paper is based upon the use of "dummy contraction joints" spaced on 20-ft centers. Where mesh reinforcement is not used, such a practice appears to be the only alternative; however, the use of 264 man-made cracks per mile of pavement may prove to be undesirable in the long run. The pavement acts as a roof to shed water, and excess openings in this roof can only lead to future troubles for the following reasons:

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[19] "Concrete Durability As Affected by Coarse Aggregate," by H. S. Sweet, a thesis submitted to the faculty of Purdue Univ. at Lafayette, Ind., in partial fulfilment of the requirements for the degree of Doctor of Philosophy in June, 1948.

[20] "Research on Concrete Durability As Affected by Coarse Aggregate," by H. S. Sweet, paper presented at the Annual Meeting, A.S.T.M., Detroit, Mich., June, 1948 (publication pending).

[21] Senior Associate, J. E. Greiner Co., Cons. Engrs., Baltimore, Md.

1. Infiltration at the edges and bottom of the pavement slab may occur unless costly remedial waterproofing measures are provided. An accumulation of foreign matter may result in differential movements of individual slabs causing wider openings at some joints and possibly blowups at others.
2. In cutting the pavement for the placement of dummy joints, it is necessary to disturb the smooth concrete surface which has been properly consolidated and screened by the finishing machine. The concrete aggregate becomes displaced or rearranged and excess mortar is floated to the surface.
3. Closely spaced joints require extra maintenance per mile of highway with the possibility of leakage and pumping under certain conditions.
4. Each joint increases pavement roughness because of the difficulty in "straightedging" a smooth surface after cutting the joint.
5. Where doweled contraction joints are used, there is always the danger of misalinement which may cause eccentric loading and cracks in the pavement. The more joints the greater will be the danger of uncontrolled cracks due to this hazard.
6. The assurance that the pavement will carry heavy loads over a long period of years with a minimum of interference to traffic is reduced by increasing the number of joints.

Where reinforcement is used in paving slabs, it has been found that contraction joints spaced from 46.5 ft to 61.5 ft on centers produce the most economical combination on the basis of the in-place cost of the joint material and the pavement reinforcement. A reinforced pavement with contraction joints spaced up to 61.5 ft on centers should be three times as sound as a nonreinforced pavement with joints spaced 20 ft on centers. The danger of joint failures and pumping, with resulting high maintenance costs, is reduced to one third. The new rubber type of joint filling compounds possess satisfactory waterproofing characteristics; therefore, joints sealed with this material and spaced 61.5 ft on centers should perform equally as well as joints spaced 20 ft on centers.

The main difficulty with contraction joints today is the need for a satisfactory load-transfer device that can be set accurately and that will function properly during the life of the pavement.

The trend in recent years is toward the elimination of joints, which have proved to be the chief source of pavement troubles. The states of New Jersey and Illinois are experimenting with heavily reinforced pavements poured continuously without either expansion or contraction joints. Such a design appears to be a step in the right direction and the results of those tests will be watched with interest by all highway engineers.

Mr. Anderson's paper has brought forth some interesting data and it is hoped that within the next few years highway engineers will agree upon a standard joint practice to simplify design, construction, and maintenance, thereby prolonging the life and improving the riding quality of concrete pavements.

William A. Conwell,[22] M. ASCE.—The profession is indebted to the author for compiling and presenting field test data on this highly important subject. Likewise the Public Roads Administration and the participating state highway departments are to be commended for the well conducted and successful series of tests which have done so much to remove the behavior of concrete pavements from the realm of speculation to the field of knowledge.

The writer would like to enter a minor objection to the use of the term "incompressible" in connection with foreign material that enters joints and prevents the closures required to take up the pavement expansion. Since all known materials are compressible to some degree, the substitution of "material of low compressibility" is recommended for "incompressible material."

The tests show that, after the initial expansion and contraction, the so-called contraction joints behave as both expansion and contraction joints. Thus, the expression "contraction joints" is a misnomer. Despite considerable thought, the writer has been unable to produce a term that would be more descriptive of the action. This point may be one of challenge to the profession, particularly to those members engaged in highway construction who can, undoubtedly, develop an excellent terminology from their picturesque vocabulary.

A. A. Anderson,[23] Assoc. M. ASCE —The lively interest in the subject of joints in concrete pavement continues after 60 years of experimentation, invention, and use under service conditions. The comments made by those who discussed this paper have added to the store of useful knowledge.

Mr. Gardner suggests an extension of the discussion into phases of the subject that were beyond the original intent of the writer. It is true that all the factors mentioned by Mr. Gardner do have a bearing on the subject, but a more comprehensive coverage would not have changed the conclusions.

Regarding Mr. Gardner's six predictions, the writer is in only partial agreement. Experience to date indicates that grooves cut to form contraction joints after the concrete has set do not perform their intended function. The tensile stresses that result from shrinkage will have come into existence before the concrete is sufficiently hard to permit the operation of the cutting machines. At this early age the concrete has little strength to resist tensile stresses, and cracks are likely to have formed before the grooves are cut. Unless the grooves can be cut much sooner than now appears possible, this method of forming contraction joints is not practical.

The installation of tie bars across contraction joints makes it impossible for the joints to relieve tensile stresses in the concrete. Tie bars thus placed will still relieve warping stresses; but, unless distributed steel is used in the pavement, open cracks may be expected to form in the slabs adjacent to the tied joints. If distributed steel is used, contraction joints are tied, and expansion joints are omitted, the result will be a continuously reinforced slab. The percentage of distributed steel that is required will be increased materially. Continuously reinforced pavements are currently under investigation, but tests have not progressed far enough to permit definite conclusions to be drawn.

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[22] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

[23] Mgr.. Highways & Municipal Bureau, Portland Cement Assn., Chicago, Ill.

It is agreed that, if tie bars are used across transverse joints (which cannot then be called contraction joints), load transfer devices may be omitted if the joint is the dummy groove type so that aggregate interlock is present. This procedure could not be followed if the joint were formed by a full-depth, nondeformed metal plate.

Certainly there is opportunity for development of joint sealing materials which will be better than most bituminous materials commonly used. Asphalts that are not too soft at summer temperatures are too hard and brittle at winter temperatures when watertight joints are of greatest importance. The thermoplastic rubber compounds recently developed give promise of much superior performance in this respect.

The need or effectiveness of subsealing in areas adjacent to joints on new construction is questioned. Presumably, in predicting this development, Mr. Gardner has in mind the problem of slab end pumping. Although subsealing has proved effective in correcting pumping on old pavements where steps were not taken at the time of construction to prevent it, other inexpensive and effective methods are available which will insure freedom from this difficulty if applied at the time of construction.

Use of distributed reinforcement will permit slab lengths to be increased as predicted by Mr. Gardner. The longer the slabs are made, the greater the weight of steel that will be required, and the greater the costs will become. Furthermore, the longer slabs will result in more movement at slab ends, increasing difficulties of joint construction and maintenance. For greatest economy and best performance, the joint spacings suggested in the paper are believed to be correct.

No conflict is found between the statements in the paper and conditions reported by Professor Woods or those in the paper[15] to which he refers. The Indiana roads covered in the latter paper[15] included 2,623 miles built without expansion joints. All were 10 years old or more at the time the report was made; thus, they were of ample age to have developed blowups if the blowups were going to appear in more than normal volume. Actually there were 851 miles with no blowups and another 864 miles with only 203 blowups. In contrast, on 284 miles of pavement with five coarse aggregates, there were 1,188 blowups. Nearly 30% of all blowups in the state were on 97 miles of pavement in which coarse aggregate from one source had been used.

It seems fairly evident that the Indiana pavements having excessive blowups were not built of materials with normal expansion characteristics. Thus, the facts agree with the writer's conclusion that little or no expansion space is needed if, among other qualifications, the pavement is constructed of materials having normal expansion characteristics. The abnormal behavior of the Indiana pavements may have been accentuated by capillary moisture supplied by some subgrades, but the difficulty was definitely connected with certain aggregates. It is understood that these unsatisfactory aggregates are no longer used in Indiana concrete pavements.

On other pavements covered in the Indiana survey—pavements not equipped with contraction joints designed for effective sealing—the number of blowups was small. With effective sealing of transverse contraction joints, the infiltration of inert, noncompressible material into open transverse cracks would be so limited that there would have been even fewer blowups than the small number reported.

At the same time, experience indicates that the use of expansion joints in the pavement having abnormal expansion properties would not have prevented the "mapcracking" or the blowups. The expansion likely to occur with abnormal aggregates cannot be predicted accurately but it may be too great to handle in this manner. Furthermore, internal stresses resulting from the abnormal expansion would have caused deterioration of the concrete even if it had been possible to place joints that produced complete freedom from restraint. Thus, even in this exceptional concrete, expansion joints probably would not have given permanent relief.

Mr. Knoerle suggests the use of distributed steel in proportions that will permit the spacing of contraction joints at 46.5 ft to 61.5 ft, stating that this method results in the lowest over-all cost of joint material and distributed steel. The writer's studies have led to the conclusion that the lowest over-all cost of joints and distributed steel results from a spacing of 28 ft to 35 ft, depending on the cost of joints. Moreover, observations in the field (and particularly on one of the test roads referred to in the paper) lead to the conclusion that movement at slab ends makes it difficult to prevent infiltration into joints where slab lengths are much in excess of 30 ft, so that it may be unsafe to omit expansion joints with the longer contraction joint spacings.

The difficulties of constructing and maintaining contraction joints do not seem as great as implied by Mr. Knoerle; and, as far as maintenance is concerned, the difficulties should be increased by the longer spacings. It is agreed that skilled workmen, good inspection, and supervision are necessary to insure proper construction, but these requirements would hold even if the number of joints were reduced as Mr. Knoerle suggests.

The writer recognizes the advantages accruing from the complete elimination of all transverse joints, requiring the use of fairly heavy continuous reinforcement if tests now (1949) under way demonstrate that design to be practical. Although these tests have not progressed far enough to permit definite conclusions, there are preliminary indications that continuously reinforced pavement is not a panacea for all joint problems.

Mr. Conwell is correct, of course, in pointing out that no material is "incompressible." The term is used to describe material that is compressible to approximately the same degree as concrete; therefore it offers no relief to the concrete. The same objection is applicable to the words "insoluble," "nonconductor," and numerous other terms widely used in technical literature. As long as there is no doubt as to meaning, it is questioned whether the advantages of greater accuracy are sufficient to justify the awkward wordiness required for technical exactitude.

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Paper No. 2385

DYNAMIC INSTABILITY OF TRUSS-STIFFENED SUSPENSION BRIDGES UNDER WIND ACTION

By Friedrich Bleich,[1] M. ASCE

With Discussion by Messrs. A. G. Pugsley; S. K. Ghaswala; F. B. Farquharson, George S. Vincent, and Robert E. McHugh, Jr.; and Friedrich Bleich.

Synopsis

An attempt is made in this paper to approach by rational analysis the problem of self-excitation of vibrations in suspension bridges. A mathematical solution of the problem of aerodynamic instability of truss-stiffened suspension bridges under wind action, which likewise applies to suspension bridges with extremely shallow stiffening girders, is presented herein. The theory leads to the explanation of the mechanism of catastrophic self-excitation in truss-type bridges, and ultimately to a rational method of bridge design against wind in order to assume aerodynamic stability.

1. Introduction

The tragic failure of the Tacoma Narrows Bridge in the State of Washington in the fall of 1940 directed the attention of the engineering profession to a problem only recently recognized as one of vital importance in suspension bridge design. The disaster brought the question of vibrations of suspension bridges under wind action into the focus of interest of bridge designers and investigators in the field of aerodynamics.

In structural engineering, wind forces customarily have been considered static forces—that is, steady forces acting on a structure at rest, like the horizontal wind forces usually taken into account in structural design. The heavy undulations of the Tacoma Narrows Bridge, which finally led to its collapse, demonstrated, however, that, even under action of a steady wind, unsteady forces (actually, periodic forces) may operate to excite oscillations with progressively augmented amplitudes, which may reach dangerous or even destructive proportions. These oscillations have been recognized as so-called self-induced or self-excited vibrations, closely connected with a peculiar state of dynamic instability of the vibrating structure.

Note.—Published in October, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] With Frankland and Lienhard, Cons. Engrs., New York, N. Y.; formerly Consultant to Advisory Board on the Investigation of Suspension Bridges, Public Roads Administration, Washington, D. C.

Elaborate research at the Daniel Guggenheim Aeronautical Laboratories at the California Institute of Technology in Pasadena, and at the University of Washington Laboratory in Seattle, instituted and directed by engineers of the Washington Toll Bridge Authority and Public Roads Administration and sponsored by these two organizations, has brought to light many significant facts and afforded much insight into the somewhat obscure mechanism of self-excitation in suspension bridges under wind action. The well-interpreted results of the wind tunnel tests on various models now provide the necessary fundamentals for an attempt to develop, with a prospect of success, a mathematical theory of dynamic instability under wind action, at least for a certain group of suspension bridges.

The purpose of a mathematical investigation into the problem of dynamic instability is a twofold one. The primary aim is the development of a theory with which observed facts can be correlated, which may be useful as a guide in conducting further laboratory tests, and which in turn may confirm the theory or indicate how the theory can be improved. The other purpose is to establish the theoretical and experimental foundation for a rational method of design permitting the specification of the condition of instability in any particular case.

The problem of aerodynamically excited oscillations in any suspension bridge structure under action of a steady uniform wind flow is one having a dual aspect. The problem proper is one of the mechanics of fluid flow involving the mutual relations between the shape of the suspended structure and the character and magnitude of the air forces acting on it. These relations and the interrelations between the air forces and the inherent dynamic properties of the structure form the basic feature of this phase of the research. The second aspect of the problem involves the detailed investigation of the response of the structure to the imposed aerodynamic forces—that is, the investigation of those conditions which operate to induce dynamic instability in any suspension bridge subjected to a steady wind stream.

The first-mentioned phase of the problem—the determination of the aerodynamic forces which act upon the oscillating structure—appears to be, in the first instance, a subject of laboratory research in view of the complexity of the geometric form and the variety of the structural make-up of the suspended floor. However, the possibility must not be overlooked that, at least in certain cases, perhaps an idealized enveloping surface of the floor structure may actually control the aerodynamic behavior of the structure, and that the influence that details of the geometric form may have on the aerodynamic behavior may be of the second order. Thus, by a process of abstraction of the type so often practiced in physics, it may be possible to arrive at a simplified form of the cross section for which theoretical research in the field of fluid flow may supply the fundamental data as to the magnitude and distribution of the acting air forces. Once in possession of a mathematically derived expression for the controlling air forces it is possible, as will be shown in this paper, to take into account the corrective effect of the particular geometry of the cross section. With the aerodynamic forces known, the solution of the aerodynamic problem reduces to its second phase, the investigation of the response of the structure to the action of the known resultant aerodynamic forces.

It may be in order to review the most important facts disclosed by an elaborate series of experiments on the behavior of suspension bridges under action of wind flow. These facts are:

Girder-stiffened sections develop maximum excitation when the angle of attack approaches zero (horizontal wind). The excited motion is purely vertical or purely torsional, and is identical in vibration form and frequency with one of the natural modes of the system. Vertical motions are noncatastrophic unless the ratio $\frac{d}{2b}$ (in which d is the girder depth, and $2b$ is the width of the bridge) exceeds a certain value somewhat greater than 0.2.

The wind velocity v at which critical torsional excitation starts increases considerably when the $\frac{d}{2b}$-ratio decreases. Experiments on section models, all vibrating with the same frequency but having various values of $\frac{d}{2b}$-ratios from 0.29 to 0.025, showed a continuous increase in v from 12 miles per hr to 65 miles per hr, indicating the outstanding effect of the particular shape of the suspended structure on the aerodynamic behavior of girder-stiffened bridges under wind action. The shallowest sections $\left(\frac{d}{2b} < 0.06\right)$ disclosed a behavior distinguished from that of the deeper sections, in that they revealed the aerodynamic characteristics of a thin flat plate.

Truss-type sections of the conventional form (closed deck) develop an optimum of stability in a horizontal wind, showing a critical velocity at zero angle of attack comparable with the critical velocity of a very shallow girder-type section or a flat plate. No self-excited vertical motion so far has been isolated in any truss-type section. However, tests indicated various cases of coupling of vertical and torsional catastrophic motion in truss-type sections of orthodox form. The period of the excited motion is distinctly different from the period of any natural mode of the vibrating system. Tests with section models having a fixed center of rotation in certain cases showed catastrophic torsional motion with a frequency always somewhat below the frequency of the natural torsional mode.

The aerodynamic properties of the truss-type section are controlled by those parts of the structure which are located near the deck level. The behavior under wind can be considerably improved by an adequate design of the chord and the adjacent part of the deck structure. Members of the stiffening frame remote from the deck appear to have little or no effect on the aerodynamic stability of the section. In fact, this type of section shows the characteristic behavior of a flat plate under horizontal wind, influenced to some extent by the particular make-up of the cross section in the vicinity of the leading or windward edge. Under upward acting wind the critical velocity decreases as the vertical angle of attack is increased.[2]

The previously enumerated experimental facts may be summarized as follows:

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[2] "Lessons in Bridge Design Taught by Aerodynamic Studies," by F. B. Farquharson, *Civil Engineering*, August, 1946, p. 344.

(*a*) Girder-stiffened suspension bridges are aerodynamically far more vulnerable to wind action than truss-stiffened bridges. The degree of stability in a horizontal wind stream is controlled in the first instance by the over-all shape of the cross section—that is, by the $\frac{d}{2b}$-ratio. The excited motion is purely vertical or purely torsional and is identical in vibration form and frequency with one of the natural modes of the system.

(*b*) The stability of truss-stiffened bridges with continuous solid roadway under horizontal wind apparently is affected in but a moderate degree by the particular shape of the cross section. There are strong indications that the behavior under horizontal wind is that of a flat plate of finite thickness. Of primary importance is the observed fact that the excited motion is a coupled motion having a vertical and an angular component with a frequency distinctly different from the frequency of any of the natural modes of the system.

These facts may be considered strong evidence that there are two entirely different types of aerodynamic forces, which induce two different types of excitation under wind action.

The first type of instability may be described briefly as follows: A prismatic structure of H-shaped cross section acted on by a horizontal steady wind stream induces a periodic disturbance of the surrounding air. That disturbance originates at the windward girder (leading edge) presenting a blunt obstacle in the horizontal wind stream. The motion is a result of vortices discharged alternately from the upper and lower edges of the vertical windward faces of the structure. At a certain critical wind velocity resonance between the frequency of the disturbance and the natural frequency of the structure generates vertical or torsional oscillations with steadily increasing amplitudes—the self-excited motion. These vibrations may have restricted amplitudes or may reach catastrophic proportions. The potential dynamic instability of the structure at certain wind velocities appears here as a consequence of the particular blunt shape of the cross section.

The phenomenon observed on truss-type sections represents the second type of dynamic instability. It indicates clearly that a truss-stiffened suspension bridge with the conventional closed deck may become unstable when the structure oscillates with two degrees of freedom, vibrating vertically and in torsion simultaneously. As will be shown hereinafter, the possibility of a coupled motion constitutes the essential condition that accounts for the occurrence of catastrophic self-induced vibrations in truss-stiffened suspension bridges under action of a horizontal wind stream.

The phenomenon as characterized in the foregoing paragraph is typical of the flutter frequently observed on vibrating flat plates and airfoils. It is closely related to the specific nature of the air forces acting on a flat plate oscillating in a steady horizontal wind stream.

However, the simple concept of flutter in many cases does not suffice to explain completely the actual behavior of truss-type cross sections under wind action. The floor structure of a truss-stiffened suspension bridge, in general, cannot be considered as a completely streamlined flat plate because of the blunt shape of the leading edge, where an alternating air force originates as a result of vortex shedding at that point. This consideration suggests the concept of an idealized cross section built up as a thin flat plate with shallow chords on both edges (Fig. 1). The vibrating structure may be thought of as acted on by air forces distributed over the plate (referred to herein as the "flat plate air forces") and an alternating resultant lift force F_v, applied to the windward chord.

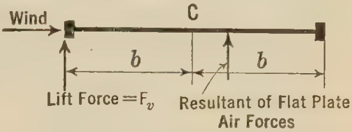

Fig. 1

The air forces acting on a horizontal flat plate freely oscillating vertically and torsionally and exposed to a horizontal wind stream are well known from theoretical investigations on the unsteady air flow around a thin flat plate. It is assumed, furthermore, that the magnitude and the nature of the lift force F_v, which depend on the particular shape of the truss chord and the adjacent part of the deck, can be determined experimentally in a rather simple manner. (It is shown hereinafter that the two unknown coefficients in the mathematical expression for F_v which determine magnitude and phase of F_v—a periodic time function—can be obtained by tests on a section model.) Theoretical solution of the problem of aerodynamic stability of the type of suspension bridges considered herein is therefore feasible, since the remaining research is confined to the investigation of the response of the structure to the given aerodynamic forces.

The following analytical study outlines a method of approach to the problem of aerodynamic instability in truss-stiffened suspension bridges and bridges with extremely shallow stiffening girders in order to obtain insight into the dynamic conditions which lead to self-induced vibrations. The analysis is based on the assumption that these two types of suspension bridge may be considered as having a suspended roadway cross section with the aerodynamic characteristics of the idealized section shown in Fig. 1. The subsequent theory supplies the rational explanation of the phenomena observed in various tests made on the full model of the preliminary design of the Tacoma Narrows Bridge with closed deck, and on a section model of the Golden Gate Bridge (San Francisco Bay, California). The preliminary design of the Tacoma Narrows Bridge had a continuous concrete slab deck. In the final design slots of open grating were introduced to improve the aerodynamic characteristics.

For the sake of simplicity, the analytical presentation of the somewhat involved theory has been divided into two parts. Sections 2 to 5 deal with the flutter theory of suspension bridges having a floor structure with flat plate characteristics. Utilizing the results of the theory of flutter outlined in these sections, the extended theory involving the flat plate having chords is developed in Section 6.

The subsequent investigation is limited to the case where the wind acts at zero angle of attack.

Notation.—The letter symbols used in this paper are defined where they first appear, in the text or by illustrations, and are assembled for convenience of reference in the Appendix. Discussers are requested to conform to this notation.

2. Free Vibrations and Self-Induced Oscillations of Systems of Two Degrees of Freedom

Free Vibrations.—A rigid bar suspended on two equal springs (Fig. 2(*a*)) and vibrating in its vertical plane represents a mechanical system of two degrees of freedom. The center of gravity is located at point C. The two principal modes of vibration having the periods p_1 and p_2 are shown in Fig. 2(*b*). The corresponding motions may be considered as rotations about points P and Q, respectively. The distances l_3 and l_4 from the center of gravity are characteristics of the system.[3] Any disturbance will cause oscillations whereby the motion of any point of the body consists of two harmonic motions of different periods. The resultant oscillations, in general, are not harmonic.

For the particular case when $l_1 = l_2$, $l_3 = \infty$ and $l_4 = 0$, the two principal modes of oscillation reduce to a vertical oscillation η and to a rotation about the center of gravity, the angular oscillation ϕ. The implication from this reasoning is that the principal modes of oscillation of a suspension bridge with a symmetrical cross section, considered a system of two degrees of freedom (bending and twisting), are identical with the principal modes of vertical vibration and the principal modes of torsional oscillation of the system. (In a strict sense, since the configurations possible to an elastic structure are infinitely various, the degree of freedom of a suspension bridge actually is infinite. For the sake of simplicity it is considered that the two spectra of possible displacements (vertical and torsional) are two degrees of freedom.) Any configuration of this system may be thought of as a linear combination of a finite or infinite number of vertical and torsional modes. This implication is of primary importance in the mathematical treatment of the problem under consideration, since it may be concluded that the principal vertical and torsional modes are, in a major degree, suitable for depicting any forced or self-induced vibrations of the structure. Frequencies and displacement form of these principal modes can be conveniently determined by an energy method, outlined in its essential features in Section 3.

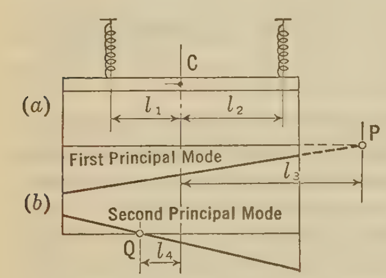

Fig. 2

Self-Induced Vibrations.—In Section 3 it is demonstrated that the air forces acting on a vibrating thin flat plate are functions of the amplitudes and their derivatives with respect to time. They are, therefore, periodic forces which cause a principal change of the form of the free vibrations of the plate.

It is assumed that a periodic force *F*, of suitable frequency, is applied to a spring-suspended elastic flat plate vibrating under action of wind. Both ends of the long plate have hinged supports (Fig. 3). The force *F* is distributed along the span and is assumed of such magnitude and location that, in spite of the damping effect of the air forces at moderate wind speed, a steady-state oscillation is maintained. This stable motion is characterized by a vertical and torsional component different from the corresponding modes of the free vibration, the frequencies and displacement forms depending on the wind velocity v. The two components have different frequencies and, as a result of the particular nature of the air forces, show a difference of phase; the motion is not harmonic.

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[3] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1937, p. 131.

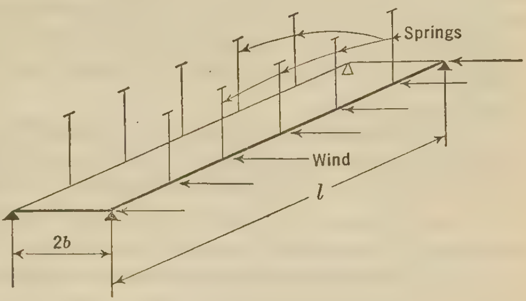

FIG. 3

With steadily increasing wind velocity the external force F, necessary to maintain the motion, at first increases and then decreases until a point is reached where the air forces alone sustain a constant amplitude of the oscillation. The corresponding velocity v is called the critical velocity or the flutter velocity. In Fig. 4, F is plotted against v. At critical wind speed, v_c, the two components of the coupled motion have the same frequency, but show a difference of their phase angles. This difference of the phase angles accounts for the fact that above the critical velocity the structure absorbs energy from the air, with the effect that the amplitudes of small initial oscillations increase steadily until they reach dangerous proportions.

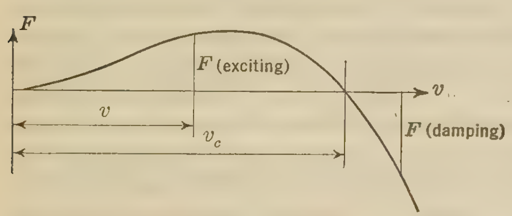

FIG. 4

Below the critical velocity an exciting force is necessary to maintain a steady-state motion; above the critical velocity the direction of the force must be reversed (damping force) to maintain this steady-state motion (Fig. 4). In the absence of such a damping force the slightest increase of v above v_c causes an augmentation of the amplitude.

The motion, therefore, is stable below the critical velocity and becomes unstable above this velocity. An increase of v beyond that limit leads to a catastrophic response of the vibrating structure. The specification of the condition of instability (that is, the determination of the critical velocity and the flutter frequency) is the problem with which this paper deals.

3. AIR FORCES ACTING ON A VIBRATING FLAT PLATE

The determination of the dynamic air forces acting on a flat plate or on an airfoil which oscillates vertically and torsionally in a steady air stream has been treated by several authors in the last two decades. Expressions for the air forces suitable for the following investigation were derived by T. Theodorsen.[4]

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[4] "General Theory of Aerodynamic Instability and the Mechanism of Flutter," by T. Theodorsen, *Technical Report No. 496*, National Advisory Committee for Aeronautics, Washington, D. C., 1935.

Equivalent equations have been developed independently by R. Kassner and H. Fingado[5] and by Theodor von Kármán, M. ASCE, and W. R. Sears.[6] The Theodorsen solution is based on the assumption that the vibrating flat plate is infinitely thin and that the air flow is two dimensional. Under the latter assumption the equations apply only to cases where the object under wind action is sufficiently extended perpendicularly to the air flow so that the altered conditions at the ends influence the total effect of the air forces only to a negligible degree. This condition is satisfied to a large extent in the case of suspension bridges.

The Theodorsen equations were derived for small harmonic oscillations about the position of equilibrium, composed of simultaneous vertical and torsional vibrations of constant amplitudes. Consequently, they apply solely to the narrow zone of transition from stable to unstable motion, which may be considered a sustained harmonic oscillation of constant amplitude. Therefore, the Theodorsen expressions for the dynamic air forces suffice to solve the problem of self-excitation from its practical viewpoint—that is, the specification of the conditions of instability or, more exactly, the determination of the critical wind velocity and the characteristics of the motion at that wind speed.

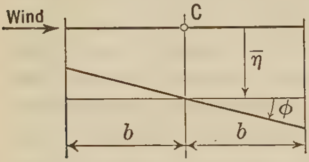

FIG. 5

Consider a long flat plate having the width $2\,b$, and vibrating vertically and torsionally about its position of equilibrium; $\bar{\eta}$ is the vertical or bending amplitude of the center point C, downward displacements being positive; ϕ is the angle of rotation, the torsional amplitude, counted positive when the plate rotates clockwise (Fig. 5). The air forces are expressed as functions of the coordinates $\bar{\eta}$ and ϕ and their derivatives with respect to time. According to Mr. Theodorsen, a unit length of the plate is acted on by the resultant lift force—

$$F_L = -\pi\,\rho\,b^2\,(v\,\dot{\phi} + \ddot{\bar{\eta}}) - 2\,\pi\,\rho\,b\,v\,C(k)\left(v\,\phi + \dot{\bar{\eta}} + \frac{b}{2}\,\dot{\phi}\right) \ldots\ldots (1)$$

—and the resultant lift moment—

$$M_L = -\pi\,\rho\,b^2\left(v\,\frac{b}{2}\,\dot{\phi} + \frac{b^2}{8}\,\ddot{\phi}\right) + \pi\,\rho\,b^2\,v\,C(k)\left(v\,\phi + \dot{\bar{\eta}} + \frac{b}{2}\,\dot{\phi}\right) \ldots (2)$$

—in which ρ is the mass density of air (assumed 0.00238 slugs); $\dot{\bar{\eta}}$ and $\dot{\phi}$ are the first derivatives with respect to time of the coordinates $\bar{\eta}$ and ϕ, respectively (velocities); $\ddot{\bar{\eta}}$ and $\ddot{\phi}$ are the second derivitives with respect to time of the coordinates $\bar{\eta}$ and ϕ, respectively (accelerations); and $C(k)$ is a complex function of the dimensionless ratio $k = \frac{\omega\,b}{v}$, ω being the flutter frequency. Positive F_L denotes a downward acting force; positive M_L, a clockwise acting moment.

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[5] "The Two-Dimensional Problem of Wind Vibration," by R. Kassner and H. Fingado, *Journal*, Royal Aeronautical Soc., Vol. 41, 1937, p. 921.

[6] "Airfoil Theory for Non-Uniform Motion," by Theodor von Kármán and W. R. Sears, *Journal of the Aeronautical Sciences*, August, 1938, p. 379.

Mr. Theodorsen developed his solution for a system of three degrees of freedom considering the air flow over an airfoil with an aileron. Eqs. 1 and 2 are derived from the Theodorsen equations[7] by omitting all terms pertaining to the aileron and setting $a = 0$ and $h = \bar{\eta}$.

The terms in Eqs. 1 and 2 containing $\ddot{\bar{\eta}}$ and $\ddot{\phi}$ represent the inertia effect of a cylinder of air having a diameter equal to the width $2\,b$. These terms could be easily combined with the terms expressing the inertia force of the vibrating body in the equations of motion. It is apparent, however, that, in the case of a vibrating suspension bridge, the inertia effect $\pi\,\rho\,b^2\,\ddot{\bar{\eta}}$ in Eq. 1 is very small in comparison with the inertia force $m\,\ddot{\bar{\eta}}$ of the vibrating structure, m being the mass of the bridge per unit length. The same holds true for the inertia term in Eq. 2. Accordingly, these terms may be disregarded without substantial error. Rearranging, Eqs. 1 and 2 may be written

$$F_L = -\,2\,\pi\,\rho\,b\,v^2 \left\{ C(k) \left(\phi + \frac{\dot{\eta}\,b}{v} \right) + [1 + C(k)]\,\frac{b}{2}\,\frac{\dot{\phi}}{v} \right\} \ldots\ldots (3)$$

$$M_L = \pi\,\rho\,b^2\,v^2 \left\{ C(k) \left(\phi + \frac{\dot{\eta}\,b}{v} \right) + [1 - C(k)]\,\frac{b}{2}\,\frac{\dot{\phi}}{v} \right\} \ldots\ldots\ldots (4)$$

in which the dimensionless quantity $\eta = \frac{\bar{\eta}}{b}$ has been introduced. Both ϕ and η are now dimensionless coordinates.

The function $C(k)$ is a complex function having the form:

$$C(k) = F(k) - i\,G(k) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which $F(k)$ and $G(k)$ represent transcendental functions of k and $i = \sqrt{-1}$;

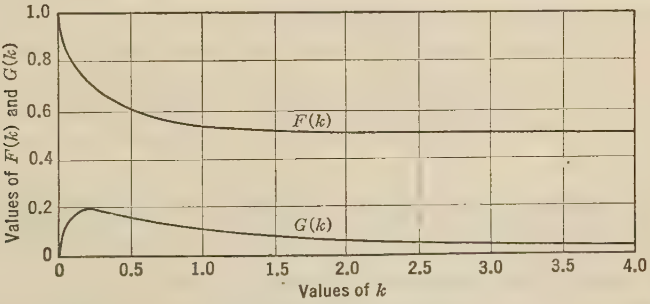

FIG. 6

$F(k)$ and $G(k)$ are plotted against k in Fig. 6. It may be noted that $F(k)$ approaches the value 0.5 and $G(k)$ approaches 0 when k approaches ∞. The critical value of k corresponding with the critical velocity v_c, as far as suspension bridges are concerned, lies below $k = 1$. For that reason, Table 1, which shows the numerical values of $F(k)$ and $G(k)$ at appropriate intervals, is restricted to values of k between 0 and 1.[8]

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[7] "General Theory of Aerodynamic Instability and the Mechansim of Flutter," by T. Theodorsen, *Technical Report No. 496*, National Advisory Committee for Aeronautics, Washington, D. C., 1935, p. 419, equations XVIII and XX.

It may be proper to discuss briefly the complex form of the function $C(k)$ which simply signifies that $C(k)$ may be thought of as composed of two components, different as to their magnitude, one component a quarter of the period ahead in time with respect to the other. The functions $C(k)$ affect the time factor in η and ϕ in a different degree, thus producing a difference in the phase angles of the vertical and torsional oscillations, as pointed out in Section 2.

TABLE 1.—VALUES OF FUNCTIONS $F(k)$ AND $G(k)$ FROM $k = 0$ TO $k = 1$

k	$F(k)$	$G(k)$	k	$F(k)$	$G(k)$	k	$F(k)$	$G(k)$	k	$F(k)$	$G(k)$
(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
0	1.0000	0.0000	0.18	0.7443	0.1887	0.36	0.6390	0.1709	0.60	0.5788	0.1378
0.02	0.9637	0.0752	0.20	0.7276	0.1886	0.38	0.6317	0.1679	0.65	0.5713	0.1319
0.04	0.9267	0.1160	0.22	0.7125	0.1877	0.40	0.6250	0.1650	0.70	0.5648	0.1264
0.06	0.8920	0.1426	0.24	0.6989	0.1862	0.42	0.6187	0.1621	0.75	0.5591	0.1213
0.08	0.8604	0.1604	0.26	0.6865	0.1842	0.44	0.6130	0.1592	0.80	0.5541	0.1165
0.10	0.8319	0.1723	0.28	0.6753	0.1819	0.46	0.6076	0.1563	0.85	0.5498	0.1121
0.12	0.8063	0.1801	0.30	0.6650	0.1793	0.48	0.6026	0.1535	0.90	0.5459	0.1078
0.14	0.7834	0.1849	0.32	0.6556	0.1766	0.50	0.5979	0.1507	0.95	0.5425	0.1039
0.16	0.7628	0.1876	0.34	0.6469	0.1738	0.55	0.5866	0.1444	1.00	0.5394	0.1003

Substituting, finally,

$$C(k) = f_1 \tag{6a}$$

$$1 + C(k) = f_2 \tag{6b}$$

$$1 - C(k) = f_3 \tag{6c}$$

and

$$2 \pi \rho b = s \tag{6d}$$

Eqs. 3 and 4 are preferably written in the form:

$$F_L = - s v^2 \left[f_1 \left(\phi + \frac{b}{v} \dot{\eta} \right) + f_2 \frac{b}{2 v} \dot{\phi} \right] \tag{7}$$

$$M_L = \frac{s b v^2}{2} \left[f_1 \left(\phi + \frac{b}{v} \dot{\eta} \right) - f_3 \frac{b}{2 v} \dot{\phi} \right] \tag{8}$$

4. THE DIFFERENTIAL EQUATIONS OF SELF-EXCITED VIBRATIONS IN SUSPENSION BRIDGES DUE TO THE FLAT PLATE AIR FORCES

Establishment of the differential equations of motion is the aim of this section, the final solution of the flutter problem being treated in Section 5.

Some introductory explanations may facilitate understanding of the peculiarity of the problem of self-excitation under wind action.

A self-induced vibration may be defined as a phenomenon in which the alternating forces furnishing the energy to the oscillations are controlled by the motion itself. When motion ceases, the alternating forces disappear. This type of periodic motion is distinctly different from a forced vibration where the periodic forces exist independently of the motion and persist when the motion is stopped.

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[8] "The Two-Dimensional Problem of Wind Vibrations," by R. Kassner and H. Fingado, *Journal*, Royal Aeronautical Soc., Vol. 41, 1937, p. 945.

A self-induced oscillation is described by a system of homogeneous linear differential equations, because the acting air forces are functions of the amplitudes and their derivatives with respect to time. These equations may be interpreted as the differential equations of a free, but damped, oscillation of a mechanical system. Because of the peculiarity of the dynamic air forces, resulting from the nonstationary flow around the oscillating structure, the damping effect of these forces may be positive, so that any initial oscillation decreases and finally disappears; or the forces may be negative, in which case the initial amplitude increases steadily. In the latter case they are referred to as self-induced vibrations. The point at which damping changes from positive to negative determines the incipient state of dynamic instability. Above this critical point—associated with a certain wind velocity—the motion shows, in general, disproportionately large increases of the amplitudes at slight increases of the wind velocity.

The solution of this problem—namely, the determination of the border line between stable and unstable motion—is facilitated by the following facts: The motion at critical wind velocity is a steady-state motion. The vertical and torsional vibrations composing the coupled motion have the same frequency ω as was outlined in Section 2, so that the motion is harmonic. Applying the Theodorsen expressions for the air forces, the vibration problem can be solved, at least to the point that the condition of instability can be specified. It is worthy of note that the homogeneous linear differential equations which define the motion at the incipient state of instability are not suitable for determining the character of the motion when the critical velocity is exceeded.

Principal Modes of Free Vibrations of Suspension Bridges.—Because investigation of the condition of flutter basically is tied to the natural modes of vibration, it appears necessary to outline briefly the essential features of an energy method that permits specification of frequencies and displacement forms of the natural modes of suspension bridges without the use of a complex mathematical framework and without cumbersome numerical calculations. The method affords an easy and a quick solution of the problem, even in cases where the effect of additional structural devices, such as cable ties, diagonal stays, double system of lateral bracing, and the like, is involved.

The method discussed hereinafter is an approximation method and is based on a minimum principle of dynamics. The application of such a minimum condition practically reduces the determination of the frequencies to the solution of an algebraic equation of second or third degree.

A freely oscillating and undamped suspension bridge represents a conservative mechanical system. No energy is imparted to the structure from any external source and no energy is dissipated by any kind of damping. The frequency ω and the configuration of any natural mode of vibration of such a conservative system are determined by the minimum condition:

$$T - V = \text{minimum} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which T denotes the maximum value of the kinetic energy of the entire structure, a condition occurring when the structure passes its equilibrium position; and V denotes the maximum potential energy (elastic and gravitational) accumulated in the structure when it occupies its extreme position—T and V are homogeneous quadratic functions of the amplitude η and its derivatives with respect to x. (Since η varies along the span it is a function of the abscissa x.) Assuming small oscillations about the position of equilibrium for the two types of motion, T and V can be expressed as follows:

Vertical Vibrations.—

$$T = \frac{m\,\omega^2}{2} \int_L \eta^2\,dx \tag{10}$$

and

$$V = \frac{E_f\,I_f}{2} \int_L (\eta'')^2\,dx + \frac{H_w}{2} \int_L (\eta')^2\,dx + \frac{4\,h}{L^2_m} H_i \int_L \eta\,dx \tag{11}$$

in which L_m represents the length of the main span; h is the cable sag of the main span; m is the mass of the bridge per unit length; E_f is the modulus of elasticity of the stiffening frames; I_f is the total moment of inertia of the stiffening frames; H_w is the cable tension (all cables) due to dead weight w; and H_i is the cable tension (all cables) due to the inertia forces. The integrals extend over the total length of the bridge L.

The cable tension H_i is connected with η by the linearized cable equation:

$$\frac{L_c}{E_c\,A_c} H_i - \frac{8\,h}{L^2_m} \int_L \eta\,dx = 0 \tag{12}$$

in which A_c and E_c denote the cross-sectional area and the modulus of elasticity of the cables; and the length L_c is defined by

$$L_c = \int \left(\frac{ds}{dx}\right)^2 ds \tag{13}$$

—the integral extending from anchorage to anchorage of the cables.

Eliminating H_i by use of Eq. 12 gives the following expression for the energy difference,

$$T - V = \frac{m\,\omega^2}{2} \int_L \eta^2\,dx - \frac{E_f\,I_f}{2} \int_L (\eta'')^2\,dx - \frac{H_w}{2} \int_L (\eta')^2\,dx - \frac{32\,h^2}{L^4_m} \frac{E_c\,A_c}{L_c} \left(\int_L \eta\,dx\right)^2 \tag{14}$$

Torsional Vibrations.—If ϕ is the angle of rotation and r is the mass radius of gyration of the cross section of the bridge, immediately $T - V$ may be derived from Eq. 14 by substituting

$$\eta^2 = r^2\,\phi \tag{15a}$$

in the first term and substituting

$$\eta = b\,\phi \tag{15b}$$

in the other terms. Thus,

$$T - V = \frac{m\,r^2\,\omega^2}{2}\int_L \phi^2\,dx - \frac{E_f\,I_f\,b^2}{2}\int_L (\phi'')^2\,dx - \frac{H_w\,b^2}{2}\int_L (\phi')^2\,dx - \frac{32\,h^2\,b^2}{L^4_m}\frac{E_c\,A_c}{L_c}\left(\int_L \phi\,dx\right)^2 \qquad (16)$$

Eqs. 14 and 16 apply to suspension bridges of the conventional type. When structural devices are installed to augment vertical or torsional rigidity, Eqs. 14 and 16 must be extended by terms indicating the amount of energy—expressed as a function of η or ϕ—accumulated in the added structural parts.

The condition expressed by Eq. 9 requires that under all possible configurations of the vibrating structure that configuration η or ϕ must be found which makes Eq. 14 or Eq. 16 have a minimum value. A general method for solving such minimum problems by approximation has been devised by W. Ritz.[9] This method is based on the following reasoning:

If η could be represented by a linear combination of a sequence of suitable functions $\Phi_1, \Phi_2, \cdots, \Phi_n$ satisfying certain conditions which will be specified subsequently, and $a_1, a_2, \cdots, a_n$ represent a corresponding set of parameters, then η may be assumed as having the form:

$$\eta = a_1\,\Phi_1 + a_2\,\Phi_2 + \cdots + a_n\,\Phi_n \qquad (17)$$

The nature of the problem dealt with imposes an important restriction on the form of the functions Φ. The displacements defined by Φ must be possible displacements of the elastic system under consideration and, as a logical consequence, must satisfy the conditions of restraint—that is, the end conditions. Thereafter the form of the functions Φ, with certain limitations, remains arbitrary. These limitations are governed only by the requirement that the values of η are to be determined with sufficient accuracy by as small a number of parameters as possible.

Introducing Eq. 17 into the energy equation (Eq. 14) and performing the integrations indicated leads to an expression giving $T - V$ as a quadratic junction of the n parameters a. Inasmuch as η must satisfy Eq. 9, the parameters $a_1, a_2, \ldots, a_n$ are determined by the condition that $T - V$ becomes a minimum. Thus,

$$\frac{\partial(T - V)}{\partial a_1} = 0,\ \frac{\partial(T - V)}{\partial a_2} = 0,\ \cdots,\ \frac{\partial(T - V)}{\partial a_n} = 0 \qquad (18)$$

The first derivative of a quadratic function is a linear function. Eqs. 18 therefore represent a system of n homogeneous linear equations from which the parameters a can be determined. This system of equations does not have any solution unless the determinant Δ of the coefficients is equal to zero. Thus,

$$\Delta = 0 \qquad (19)$$

which is an equation of the nth degree of the unknown ω^2, represents the frequency equation that determines ω. Eq. 19 yields n different roots ω^2 associated with the first, second, $\cdots$, nth mode of vibration. In general, the degree of accuracy decreases somewhat for the higher modes. By substituting the numerical value of any of the roots of Eq. 19 into the linear equations (Eqs. 18) the relative values of the parameters a, expressed by one arbitrary parameter, can be calculated and the configuration of the modes of vibration associated with that root thereby specified.

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[9] "Über eine neue Methode zur Lösung gewisser Varitionsprobleme der mathematischen Physik", by W. Ritz, *Journal für die reine und angewandte Mathematik*, Vol. 135, 1909, p. 1.

The Ritz method, when based upon a proper set of functions Φ, furnishes a sequence of parameters a, rapidly converging to the limit zero, so that a few terms of the series (Eq. 17) suffice to determine ω and η with the necessary degree of accuracy. Success or failure in applying the Ritz method depends largely on the proper choice of the functions Φ. Such an appropriate set of functions can be found in the sequence of displacement forms representing the solution of a related vibration problem having the same boundary conditions. For instance, the functions $\sin \frac{\pi x}{L_m}$, $\sin \frac{2\pi x}{L_m}$, $\cdots$, $\sin \frac{n\pi x}{L_m}$ represent the principal modes of vibration of a hinged beam (the stiffening frame without cable) It may be expected, therefore, that these functions will prove to be that sequence of suitable Φ-functions on which the approximate solution of the problem of free vibration of suspension bridges with hinged stiffening frame can be based.

Equation of Motion of Self-Induced Vibrations.—The complexity of the vibrating system and the involved form of the mathematical expressions for the air forces render it difficult to devise a direct solution of the dynamic problem of self-excitation by starting from the differential equation of small vibrations. However, another method is available to deal successfully with the somewhat involved problem.

The solution of the problem of the free vibrations of suspension bridges as discussed hereinbefore has been greatly simplified by using the Ritz method, in which the parameters a represent a particular type of the so-called "generalized coordinates," a concept which has proved to be an important tool in connection with the Lagrangian equations of motion for dynamic problems.

Following the pattern established by the energy method the assumption can be made that the configuration of the elastic system at flutter speed may be depicted by an appropriately chosen set of functions Φ. It may be assumed, therefore, that the two vibration coordinates, η and ϕ, the space-time functions which define the configuration of the vibrating system at any instant, are satisfactorily expressed by two finite series of the form:

$$\eta = q_{\eta 1}\Phi_{\eta 1} + q_{\eta 2}\Phi_{\eta 2} + \cdots + q_{\eta n}\Phi_{\eta n} \qquad (20a)$$

and

$$\phi = q_{\phi 1}\Phi_{\phi 1} + q_{\phi 2}\Phi_{\phi 2} + \cdots + q_{\phi n}\Phi_{\phi n} \qquad (20b)$$

in which the coefficients q are products $(u\,\psi)$, u being a parameter having the dimension of η or ϕ, respectively, and ψ representing a time function. The quantities q are the generalized coordinates. In combination with the chosen space functions Φ they determine the configuration of the vibrating system at any moment.

An elastic structure representing a continuous mechanical system has an infinite number of degrees of freedom; that is, an infinite number of coordinates is necessary to describe the configuration of the system. Introduction of generalized coordinates has the effect of expressing the configuration of such a system by a small number of appropriate coordinates. Thus, the degree of freedom of the system, at least as far as the mathematical treatment is concerned, is reduced to the number of generalized coordinates.

There is no difficulty concerning the choice of the Φ-functions that are to be introduced in the subsequent analysis of the binary motion of suspension bridges. The principal modes of the vertical and the torsional vibrations of that suspension bridge whose stability conditions are to be specified offer themselves as a suitable set of Φ-functions, as was indicated in Section 2 in discussing the free vibration of a suspension bridge considered as a system of two degrees of freedom. Since the first occurrence of dynamic instability logically is associated with the lowest frequencies, corresponding to a symmetric or an asymmetric vibration, the set of Φ-functions always must include the first symmetric or the first asymmetric principal modes of vertical and torsional vibrations.

Lagrange's Equations of Motion.—In the previous discussion of the free vibrations a minimum principle of mechanics was utilized in order to obtain the equations from which the values of the parameters a (generalized coordinates) determining the various modes of free vibration could be derived. In dealing with the problem of self-excitation resort must be had to a more general form of the equation of motion—the Lagrangian equations for small motion. The condition of Eq. 9, on which the theory of free vibration could be based, applies only to a conservative mechanical system represented, for instance, by an undamped, freely vibrating structure. However, despite the fact that the motion under wind action at flutter speed may be considered a sustained motion, the oscillating system, in reality, is a nonconservative one. Energy input from the surrounding air takes place during one part of the cycle, whereas an equal amount of energy is dissipated during another part of the cycle.

The subsequent theory of self-excitation in suspension bridges utilizes the fundamentals of a concept which already has been applied to the flutter problem in aircraft design.[10]

The Lagrangian equations for small motion of a mechanical system about a position of equilibrium are:[11]

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} - Q_j = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which $j = 1, 2, \cdots, n$; q_j are the n generalized coordinates; $\dot{q}_j$ is the first derivative of q_j with respect to time (velocity); T is the kinetic energy of the oscillating system and is a quadratic function of the generalized coordinates; V is the total potential energy of the system and is a quadratic function of the generalized coordinates; and Q_j is the generalized force corresponding to the generalized coordinate q_j. The generalized coordinates q_j have, as previously stated, the form of products—

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[10] "General Approach to the Flutter Problem," by S. J. Loring, *Journal*, S.A.E., August, 1941, p. 345.

[11] "Mathematical Methods in Engineering," by Theodor von Kármán and Maurice A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940.

$$q_j = u_j \psi_j \text{.........................} (22)$$

The parameters u_j may be considered in the following dimensionless quantities since η and ϕ have been defined as magnitudes of zero dimension (see Section 3). The symbol ψ_j is the time function $e^{\omega_j t}$, in which e is the base of the Naperian logarithms; ω_j is a complex number; and t represents time. For the system under consideration the vertical amplitudes η and the angle of rotation ϕ are expressed by the two series (Eqs. 20) involving, together, $2n$ generalized coordinates q_j. There are $2n$ equations of the type of Eq. 21. Accordingly, a set of $2n$ values of q_j is determined by $2n$ equations.

Application of Lagrange's Equations to the Flutter Problem of Suspension Bridges.—The derivation of the equations of motion from the basic Eq. 21 will be shown conveniently in applying the Lagrangian equations to the most simplified case, in which it is assumed that a set of only two generalized coordinates leads to a sufficiently accurate solution. It is easily recognized that the coupled motion associated with a flutter frequency, which always lies between the two frequencies of the first vertical and the first torsional principal modes of free vibration (either symmetric or asymmetric), is controlled by these two frequencies. Therefore, the displacement form of the two components of the coupled motion cannot be too far different from the configuration of the first vertical and first torsional principal mode, respectively. The next higher modes may have only a corrective effect on the computed critical velocity and flutter frequency.

The following analysis reveals that, in the case of the conventional type of suspension bridges, in which the torsional rigidity of the bridge structure is determined only by the vertical rigidity of the compound system composed of cables and trusses, a set of two generalized coordinates suffices to arrive at an exact solution of the problem. The particular reason for that behavior, as will be shown subsequently, is that the displacement forms Φ_η and Φ_ϕ associated with the chosen generalized coordinates are identical ($\Phi_\eta = \Phi_\phi$) for each principal mode. This simple relationship does not hold true for bridges whose torsional rigidity is materially affected by the torsional resistance of the towers or by installation of a double lateral system or other devices. However, even in these cases, restriction to two coordinates leads to an approximate solution of the flutter problem which is sufficiently accurate.

If a three-span suspension bridge oscillating in a symmetric configuration under action of wind is considered and it is assumed, in accordance with the foregoing explanation, that

$$\eta = q_1 \Phi_1 \text{.........................} (23a)$$

and

$$\phi = q_2 \Phi_2 \text{.........................} (23b)$$

in which Φ_1 represents the first symmetric principal mode of vertical oscillation and Φ_2 represents the corresponding mode of angular vibration, the kinetic energy of the oscillating system is given by

$$T = \frac{m\,b^2}{2} \int_L \dot{\eta}^2\,dx + \frac{m\,r^2}{2} \int_L \dot{\phi}^2\,dx \text{................} (24)$$

In Eq. 24, the integrals extend over the three spans with a total length L. Substituting Eqs. 23 in Eq. 24 gives

$$T = \frac{m\,b^2}{2}\,\dot{q}^2_1 \int_L \Phi^2_1\,dx + \frac{m\,r^2}{2}\,\dot{q}^2_2 \int_L \Phi^2_2\,dx \qquad (25)$$

Differentiation of Eq. 25 with respect to $\dot{q}_1$ and $\dot{q}_2$, respectively, and again, differentiation with respect to t, gives

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_1} = m\,b^2\,\ddot{q}_1 \int_L \Phi^2_1\,dx \qquad (26a)$$

and

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_2} = m\,r^2\,\ddot{q}_2 \int_L \Phi^2_2\,dx \qquad (26b)$$

These are the first terms in each of the two Lagrangian equations corresponding with the generalized coordinates q_1 and q_2; $\ddot{q}$ denotes the second derivative with respect to time, or the acceleration.

The second term in each of the two Lagrangian equations is dependent on the potential energy. Thus,

$$V = \frac{E\,I\,b^2}{2}\left[\int_L (\eta'')^2\,dx + \int_L (\phi'')^2\,dx\right] - \frac{H_w\,b^2}{2}\left[\int_L \eta\,\eta''\,dx + \int_L \phi\,\phi''\,dx\right]$$
$$+ \frac{K}{2}\left[\left(\int_L \eta\,dx\right)^2 + \left(\int_L \phi\,dx\right)^2\right] \qquad (27)$$

in which

$$K = \frac{64\,h^2\,b^2}{L^4_m}\,\frac{E_c\,A_c}{L_c} \qquad (28)$$

Substitution of Eqs. 23 in Eq. 27 gives

$$V = \frac{E\,I\,b^2}{2}\left[q^2_1 \int (\Phi''_1)^2\,dx + q^2_2 \int_L (\Phi''_2)^2\,dx\right]$$
$$- \frac{H_w\,b^2}{2}\left[q^2_1 \int_L \Phi_1\,\Phi''_1\,dx + q^2_2 \int_L \Phi_2\,\Phi''_2\,dx\right]$$
$$+ \frac{K}{2}\left[q^2_1\left(\int_L \Phi_1\,dx\right)^2 + q^2_2\left(\int_L \Phi_2\,dx\right)^2\right] \qquad (29)$$

from which

$$\frac{\partial V}{\partial q_1} = q_1\left[E\,I\,b^2 \int (\Phi''_1)^2\,dx - H_w\,b^2 \int_L \Phi_1\,\Phi''_1\,dx + K\left(\int_L \Phi_1\,dx\right)^2\right] \qquad (30a)$$

and

$$\frac{\partial V}{\partial q_2} = q_2\left[E\,I\,b^2 \int_L (\Phi''_2)^2\,dx - H_w\,b^2 \int_L \Phi_2\,\Phi''_2\,dx + K\left(\int_L \Phi_2\,dx\right)^2\right] \qquad (30b)$$

Eq. 21, therefore, takes the form:

$$m\, b^2\, \ddot{q}_1 \int_L \Phi^2{}_1\, dx + q_1 \Big[E\, I\, b^2 \int_L (\Phi''{}_1)^2\, dx$$
$$- H_w\, b^2 \int_L \Phi_1\, \Phi''{}_1\, dx + K \left(\int_L \Phi_1\, dx \right)^2 \Big] = Q_1 \quad \text{(31a)}$$

and

$$m\, r^2\, \ddot{q}_2 \int_L \Phi^2{}_2\, dx + q_2 \Big[E\, I\, b^2 \int_L (\Phi''{}_2)^2\, dx$$
$$- H_w\, b^2 \int_L \Phi_2\, \Phi''{}_2\, dx + K \left(\int_L \Phi_2\, dx \right)^2 \Big] = Q_2 \quad \text{(31b)}$$

Before determining the generalized forces Q_1 and Q_2, Eqs. 31 may be simfied by introducing the frequencies ω_1 and ω_2 associated with the two principal modes Φ_1 and Φ_2. Assuming $Q_1 = 0$ and $Q_2 = 0$ in Eqs. 31, two homogeneous equations, independent of each other, are derived which define the vertical and the torsional free vibrations of the system. Consequently, the solutions

$$q_1 = e^{i\omega_1 t} \quad \text{(32a)}$$

and

$$q_2 = e^{i\omega_2 t} \quad \text{(32b)}$$

must satisfy these equations.

By introducing Eqs. 32 and substituting,

$$\ddot{q}_1 = -\, \omega^2{}_1\, e^{i\omega_1 t} \quad \text{(33a)}$$

and

$$\ddot{q}_2 = -\, \omega^2{}_2\, e^{i\omega_2 t} \quad \text{(33b)}$$

in the two homogeneous equations, gives

$$-\, m\, b^2\, \omega^2{}_1 \int_L \Phi^2{}_1\, dx + E\, I\, b^2 \int_L (\Phi''{}_1)^2\, dx$$
$$- H_w\, b^2 \int_L \Phi_1\, \Phi''{}_1\, dx + K \left(\int_L \Phi_1\, dx \right)^2 = 0 \quad \text{(34a)}$$

and

$$-\, m\, r^2\, \omega^2{}_2 \int_L \Phi^2{}_2\, dx + E\, I\, b^2 \int_L (\Phi''{}_2)^2\, dx$$
$$- H_w\, b^2 \int_L \Phi_2\, \Phi''{}_2\, dx + K \left(\int_L \Phi_2\, dx \right)^2 = 0 \quad \text{(34b)}$$

When using these relations Eqs. 31 may be written in the more condensed form:

$$(\ddot{q}_1 + \omega^2{}_1\, q_1) \int_L \Phi^2{}_1\, dx = \frac{Q_1}{m\, b^2} \quad \text{(35a)}$$

and

$$(\ddot{q}_2 + \omega^2{}_2\, q_2) \int_L \Phi^2{}_2\, dx = \frac{Q_2}{m\, r^2} \qquad \text{(35}b\text{)}$$

So far, it has been tacitly assumed that the rigidity of the structure is due only to the combined effect of cables and stiffening frames. Suppose now that Eqs. 31 have been extended by additional terms related to that part of the elastic energy which is accumulated in reinforcing structural elements, such as tower stays, center stays, and the like. The same additional terms also appear in Eqs. 34, so that the end results are equations of motion of the form of Eqs. 35. Eqs. 35, therefore, apply to any type of suspension bridge.

In determining the generalized forces Q_1 and Q_2, the external forces F_L and M_L are defined by Eqs. 7 and 8 as functions of the coordinates η and ϕ and their derivatives with respect to t. The work done by these forces during a virtual change of the configuration of the system defined by the virtual displacements $b\,\delta\eta$ and $\delta\phi$, may be expressed as

$$\delta W = \int_L (F_L\, b\, \delta\eta + M_L\, \delta\phi)\, dx \qquad (36)$$

On the other hand, δW can also be determined as the work done by the generalized forces Q_1 and Q_2 due to the virtual displacements δq_1 and δq_2. Thus,

$$\delta W = Q_1\, \delta q_1 + Q_2\, \delta q_2 \qquad (37)$$

The variations δq_1 and δq_2 are thought of as due to a variation of the dimensionless parameters u_1 and u_2 connected with the coordinates q_1 and q_2.

Expressing F_L, M_L, $\delta\eta$, and $\delta\phi$ in Eq. 36 by the generalized coordinates q_1 and q_2,

$$\delta W = \delta q_1 \int_L R_1\, dx + \delta q_2 \int_L R_2\, dx \qquad (38)$$

in which R_1 and R_2 are expressions depending on the air forces F_L and M_L. Equating the coefficients of δq_1 and δq_2 in Eqs. 37 and 38 furnishes the expressions for the generalized forces. Accordingly,

$$Q_1 = \int_L R_1\, dx \qquad \text{(39}a\text{)}$$

and

$$Q_2 = \int_L R_2\, dx \qquad \text{(39}b\text{)}$$

Using Eqs. 7 and 8 and substituting

$$\eta = q_1\, \Phi_1 \qquad \text{(40}a\text{)}$$

and

$$\phi = q_2\, \Phi_2 \qquad \text{(40}b\text{)}$$

The terms in Eq. 36 become

$$\int_L F_L\, b\, \delta\eta\, dx = -\, s\, b\, v^2 \int_L \Big[f_1 \Big(q_2\, \Phi_2 + \frac{b}{v}\, \dot{q}_1\, \Phi_1 \Big) + f_2\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi_2 \Big]\, \Phi_1\, \delta q_1\, dx \tag{41a}$$

and

$$\int_L M_L\, \delta\phi\, dx = \frac{s\, b\, v^2}{2} \int_L \Big[f_1 \Big(q_2\, \Phi_2 + \frac{b}{v}\, \dot{q}_1\, \Phi_1 \Big) - f_3\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi_2 \Big]\, \Phi_2\, \delta q_2\, dx \tag{41b}$$

and, finally,

$$\delta W = -\, s\, b\, v^2\, \delta q_1 \int_L \Big[f_1 \Big(q_2\, \Phi_1\, \Phi_2 + \frac{b}{v}\, \dot{q}_1\, \Phi^2{}_1 \Big) + f_2\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi_1\, \Phi_2 \Big]\, dx + \frac{s\, b\, v^2}{2}\, \delta q_2 \int_L \Big[f_1 \Big(q_2\, \Phi^2{}_2 + \frac{b}{v}\, \dot{q}_1\, \Phi_1\, \Phi_2 \Big) - f_3\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi^2{}_2 \Big]\, dx \tag{42}$$

In Eq. 42 the terms within the brackets when multiplied by $-\, s\, b\, v^2$ and $\frac{s\, b\, v^2}{2}$, respectively, represent the magnitudes R_1 and R_2 of Eq. 38 and define the generalized forces Q_1 and Q_2 according to Eqs. 39.

The differential equations of motion (Eqs. 35) therefore assume the form:

$$(\ddot{q}_1 + \omega^2{}_1\, q_1) \int_L \Phi^2{}_1\, dx + \frac{s\, b\, v^2}{m\, b^2} \Big(f_1\, q_2 \int_L \Phi_1\, \Phi_2\, dx + f_1\, \frac{b}{v}\, \dot{q}_1 \int_L \Phi^2{}_1\, dx + f_2\, \frac{b}{2\, v}\, \dot{q}_2 \int_L \Phi_1\, \Phi_2\, dx \Big) = 0 \tag{43a}$$

and

$$(\ddot{q}_2 + \omega^2{}_2\, q_2) \int_L \Phi^2{}_2\, dx - \frac{s\, b\, v^2}{2\, m\, r^2} \Big(f_1\, q_2 \int_L \Phi^2{}_2\, dx + f_1\, \frac{b}{v}\, \dot{q}_1 \int_L \Phi_1\, \Phi_2\, dx - f_3\, \frac{b}{2\, v}\, \dot{q}_2 \int_L \Phi^2{}_2\, dx \Big) = 0 \tag{43b}$$

The definite integrals represent constants which can be easily computed from the assumed displacement forms Φ_1 and Φ_2. They reflect the geometric and dynamic properties of the whole structure under consideration. The unknown functions in these two differential equations of second order are q_1 and q_2.

The principal modes Φ_1 and Φ_2 contain arbitrary factors N_1 and N_2 which may be chosen in such a way that

$$\int_L \Phi^2{}_1\, dx = 1 \tag{44a}$$

and

$$\int_L \Phi^2{}_2\, dx = 1 \tag{44b}$$

The functions Φ are normalized. In using the normalized form of Φ_1 and Φ_2, the substitution factor, D, defined by

$$\int_L \Phi_1 \Phi_2 \, dx = D \qquad (45)$$

may be introduced. Eqs. 26, therefore, assumed the more condensed form:

$$\ddot{q}_1 + \omega^2{}_1 q_1 + \frac{s\,b\,v^2}{m\,b^2}\left[f_1\left(q_2 D + \frac{b}{v}\dot{q}_1\right) + f_2 \frac{b}{2\,v}\dot{q}_2 D\right] = 0 \qquad (46a)$$

and

$$\ddot{q}_2 + \omega^2{}_2 q_2 - \frac{s\,b\,v^2}{2\,m\,r^2}\left[f_1\left(q_2 + \frac{b}{v} D \dot{q}_1\right) - f_3 \frac{b}{2\,v}\dot{q}_2\right] = 0 \qquad (46b)$$

Eqs. 46 have been derived under the assumption that the structure vibrates in a symmetric configuration. Proceeding in the same manner as previously shown, but presuming an asymmetric oscillation, again leads to the differential equations (Eqs. 46) in which Φ_1, Φ_2, ω_1, and ω_2 now refer to two corresponding asymmetric normal modes. Therefore, Eqs. 46 may be considered the basic system of differential equations on which at least an approximate solution of the flutter problem of suspension bridges can be based.

In the particular case of suspension bridges of orthodox design, Eqs. 46 can be somewhat simplified. It was pointed out previously that in that case the vertical and torsional mode are identical—that is, $\Phi_1 = \Phi_2$. Thus, it follows that the integrals in Eqs. 43 drop out and that these equations reduce to

$$\ddot{q}_1 + \omega^2{}_1 q_1 + \frac{s\,b\,v^2}{m\,b^2}\left[f_1\left(q_2 + \frac{b}{v}\dot{q}_1\right) + f_2 \frac{b}{2\,v}\dot{q}_2\right] = 0 \qquad (47a)$$

and

$$\ddot{q}_2 + \omega^2{}_2 q_2 - \frac{s\,b\,v^2}{2\,m\,r^2}\left[f_1\left(q_2 + \frac{b}{v}\dot{q}_1\right) - f_3 \frac{b}{2\,v}\dot{q}_2\right] = 0 \qquad (47b)$$

Eqs. 47 are independent of Φ_1 and Φ_2—that is, of the particular displacement form of the two modes associated with the frequencies ω_1 and ω_2. Eqs. 47 become identical with the equation of motion of a small independent section of the bridge of unit length. Thus, the flutter characteristics—critical velocity and flutter frequency of the whole suspension bridge—are the same as the flutter characteristics of an independent section of the bridge. Therefore, it may be concluded that self-excitation of suspension bridges of that type, where vertical and angular vibrations show the same vibration form, can be studied on section models free to vibrate both vertically and in torsion. The test results, if corrected for the difference in aspect ratio, must coincide with the results of tests on full models.

As the outset of the foregoing discussion concerning the equations of motion, the statement was made that two generalized coordinates suffice to arrive at an exact solution of the flutter problem in the case of suspension bridges of orthodox type (Eqs. 47). To prove this, it may be assumed that

$$\eta = q_1 \Phi + q_3 \Phi' \qquad (48a)$$

and

$$\phi = q_2 \Phi + q_4 \Phi' \qquad (48b)$$

in which Φ' is a principal mode different from Φ. The four generalized coordinates q lead to four differential equations with four unknowns. However, as a consequence of the orthogonality relation,

$$\int_L \Phi \, \Phi' \, dx = 0 \qquad (49)$$

which exists between two different principal modes of oscillation of a vibrating mechanical system, it will be found that the first two equations involve only the unknowns q_1 and q_2; the third and fourth equations, the unknowns q_3 and q_4. Accordingly, the four equations split up into two independent systems of two equations having the form of Eqs. 47. The group controlled by the lowest frequencies, ω_1 and ω_2, furnishes the lowest critical velocity. The second group leads to a higher value of the critical velocity and is therefore meaningless. From this it is concluded that Eqs. 47 actually constitute the complete system of differential equations involved in the solution of the flutter problem of the orthodox system and the solution, therefore, must be exact.

Effect of Structural Damping.—Thus far the vibration has been considered as not being influenced by any kind of structural damping. It is not difficult, however, to insert in Eqs. 46 or Eqs. 47 an adequate damping term to account for the dissipation of energy due to internal and frictional damping. It is convenient to introduce the internal damping forces as being in phase with the velocity and of a magnitude proportional to the restoring forces. This is tantamount to the assumption that the damping forces are proportional to the amplitude and are independent of the frequency. The restoring forces are represented by the terms $\omega^2{}_1 \, q_1$ and $\omega^2{}_2 \, q_2$, which may be replaced by

$$\omega^2{}_1 \, q_1 \, (1 + i \, g_1) \qquad (50a)$$

and

$$\omega^2{}_2 \, q_2 \, (1 + i \, g_2) \qquad (50b)$$

respectively, in which g_1 and g_2 denote the damping coefficients for vertical and torsional vibrations, respectively,[12] and are related to δ_s, the respective logarithmic decrements of internal damping, by

$$g = \frac{\delta_s}{\pi} \qquad (50c)$$

In so far as the over-all structural damping due to internal and frictional damping forces can be replaced by some equivalent internal damping, characterized by a constant logarithmic decrement δ_s, the coefficients g_1 and g_2 may be understood as related to the over-all damping capacity of the bridge. The effect of structural damping on the flutter characteristics may then be traced by Eqs. 46 or 47 in which $\omega^2{}_1 \, q_1$ and $\omega^2{}_2 \, q_2$ have been replaced by Eqs. 50*a* and

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[12] "Mechanism of Flutter, a Theoretical and Experimental Investigation of the Flutter Problem," by T. Theodorsen and I. E. Garrick, *Technical Report No. 685*, National Advisory Committee for Aeronautics, Washington, D. C., 1940.

50*b*, respectively. It should be emphasized that the mathematical expressions for the air forces represent the totality of the damping and exciting air forces acting on the vibrating structure. It would be erroneous to include in the damping decrement the effect of atmospheric damping in still air.

5. DETERMINATION OF THE FLUTTER CHARACTERISTICS

In the following mathematical analysis reference is made to the general form of the differential equations of motion (Eqs. 46). The incipient state of flutter has been characterized in Section 2 as a sinusoidal motion of constant amplitude where both components of the coupled motion have the same frequency. Such an oscillation can be represented by the following set of solutions of Eqs. 46:

$$q_1 = u_1 e^{i\omega t} \ldots\ldots (51a)$$

and

$$q_2 = u_2 e^{i\omega t} \ldots\ldots (51b)$$

in which u_1 and u_2 are dimensionless parameters determining the contributions of the vertical and torsional vibrations to the resultant motion. Substitution of Eqs. 51 in Eqs. 46 gives

$$b\left(-\omega^2 + \omega^2_1 + \frac{s\,v}{m} f_1\, i\,\omega\right) u_1 + \left(\frac{s\,v^2}{m} f_1 + \frac{s\,b\,v}{2\,m} f_2\, i\,\omega\right) D\,u_2 = 0 \ldots (52a)$$

and

$$b\left(-\frac{s\,b\,v}{2\,m\,r^2} f_1\, i\,\omega\right) D\,u_1 + \left(-\omega^2 + \omega^2_2 - \frac{s\,b\,v^2}{2\,m\,r^2} f_1 + \frac{s\,b^2\,v}{4\,m\,r^2} f_3\, i\,\omega\right) u_2 = 0 \ldots (52b)$$

Since f_1, f_2, and f_3 are functions of $k = \frac{\omega\,b}{v}$, it is convenient to eliminate the wind velocity v by replacing it by $\frac{\omega\,b}{k}$. By doing this and dividing both equations by ω^2, Eqs. 52 become

$$b\left(-1 + \frac{\omega^2_1}{\omega^2} + \frac{s\,b}{m}\frac{f_1}{k}\, i\right) u_1 + \left(\frac{s\,b^2}{m}\frac{f_1}{k^2} + \frac{s\,b^2}{m}\frac{f_2}{2\,k}\, i\right) D\,u_2 = 0 \ldots (53a)$$

and

$$\left(-\frac{s\,b}{m}\frac{b^2}{2\,r^2}\frac{f_1}{k}\, i\right) D\,u_1 + \left(-1 + \frac{\omega^2_2}{\omega^2_1}\frac{\omega^2_1}{\omega^2} - \frac{s\,b}{m}\frac{b^2}{2\,r^2}\frac{f_1}{k^2} + \frac{s\,b}{m}\frac{b^2}{4\,r^2}\frac{f_3}{k}\, i\right) u_2 = 0 \ldots (53b)$$

Substituting,

$$\mu = \frac{s\,b}{m} \ldots\ldots (54a)$$

$$\kappa = \frac{2\,r^2}{b^2} \ldots\ldots (54b)$$

and

$$Z = \frac{\omega^2{}_1}{\omega^2} \qquad (54c)$$

Eqs. 53 may finally be written

$$\left(-1 + Z + \mu \frac{f_1}{k} i\right) u_1 + \mu \left(\frac{f_1}{k^2} + \frac{f_2}{2\,k} i\right) D\, u_2 = 0 \qquad (55a)$$

and

$$-\mu \frac{f_1}{k} i\, D\, u_1 + \left[-\kappa + \kappa \frac{\omega^2{}_2}{\omega^2{}_1} Z - \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right)\right] u_2 = 0 \qquad (55b)$$

which represent two homogeneous linear equations with the variables u_1 and u_2. Finite values of u_1 and u_2 exist only when the coefficient determinant Δ of the system becomes zero. Therefore, $\Delta = 0$ represents the frequency equation from which ω (included in Z) and also—as explained immediately hereafter—the critical velocity v_c, associated with ω, may be determined.

The coeficients of u_1 and u_2 in Eqs. 55 contain imaginary terms. Moreover, the functions f_1, f_2, and f_3 are complex functions of k. The determinant Δ, therefore, is a determinant of second order of complex numbers. Accordingly, the frequency equation or, as it may now be called, the stability condition, takes the form:

$$\Delta = \Delta_1 + i\,\Delta_2 = 0 \qquad (56a)$$

and this condition is satisfied when

$$\Delta_1 = 0 \qquad (56b)$$

and

$$\Delta_2 = 0 \qquad (56c)$$

Thus two equations are obtained from which the two remaining unknowns k and Z, which finally define ω and v_c, may be computed. By substitution of the numerical values of k and Z in either Eq. 55a or Eq. 55b, the ratio $\frac{u_1}{u_2}$ can be determined. In this way flutter speed and flutter mode are found. The ratio $\frac{u_1}{u_2}$ is a complex number—

$$a + b\,i = e^{i\theta} \qquad (57)$$

—and the solutions (Eqs. 51), therefore, appear in the form:

$$q_2 = u_2\, e^{i\omega t} \qquad (58a)$$

and

$$q_1 = u_2\, e^{i\theta}\, e^{i\omega t} = u_2\, e^{i(\omega t + \theta)} \qquad (58b)$$

The symbol θ indicates the difference between the phase angles of the vertical and torsional component of the motion; u_2 is an arbitrary parameter.

From Eqs. 55 the frequency equation (Eq. 56a) can be derived by performing the multiplications. Replacing $\frac{\omega^2{}_2}{\omega^2{}_1}$ by X the stability condition—

$$\kappa\, X\, Z^2 - \left[\kappa + \kappa\, X - \mu\,\kappa\, X \frac{f_1}{k} i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right)\right] Z + \left[\kappa - \kappa\,\mu \frac{f_1}{k} i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right) - \mu^2 \frac{f^2{}_1}{k^3} (1 - D^2)\, i - \mu^2 \frac{f_1}{2\,k^2} (f_2\, D^2 + f_3)\right] = 0 \qquad (59)$$

—is obtained. When the frequencies ω_1 and ω_2 of a suspension bridge are associated with the same displacement form, or in the case of vibration of a section model, Eq. 59 may be simplified by assuming $D = 1$.

For any given design, ω_1, ω_2, κ, and μ are known magnitudes. The two variables Z and k, representing functions of the flutter frequency ω and the critical velocity v_c, remain as unknown quantities. After substituting the numerical values for ω_1, ω_2, κ, and μ in Eq. 59, the real terms can be separated from the imaginary terms. Equating the real part Δ_1 of Eq. 59 and the imaginary part Δ_2, respectively, to zero, two equations with real coefficients are obtained, which finally define Z and k.

The coefficients f_1, f_2, and f_3 are complicated complex functions of k and depend, therefore, on v_c and ω, both unknown quantities at the start of the calculation. It is not possible to solve Eqs. 56*b* and 56*c* explicitely for Z and k, and even solution by a process of trial and error would be rather laborious. It is far more convenient to consider k as a parameter and to solve the two equations by treating Z and one of the design factors (for instance, X) as variables. This leads to the following procedure:

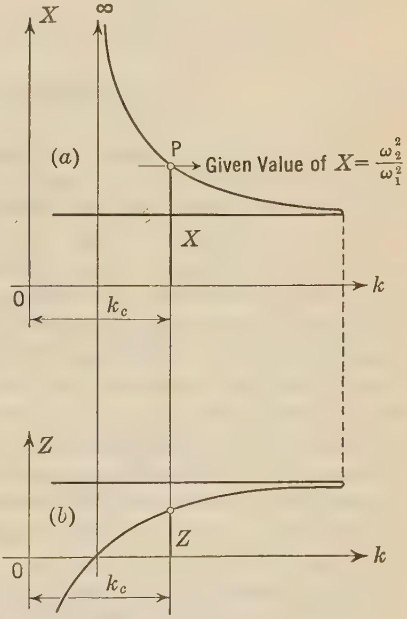

FIG. 7

A set of consecutive values of k in the neighborhood of the expected value of this magnitude may be chosen and for each of them the terms f_1 and f_2, and f_3 may be calculated using the values of $F(k)$ and $G(k)$ from Table 1. Each value of k and the corresponding value of f_1, f_2, and f_3 are introduced in Eq. 59, which can be split in two equations of the form:

$$X Z^2 + a X Z + b Z + c = 0 \quad (60a)$$

and

$$X Z + dZ + e = 0 \quad (60b)$$

In the case of a damped vibration Eq. 60*b* takes the form:

$$X Z^2 + dZ^2 + e Z + f = 0 \quad (60c)$$

In Eqs. 60 the coefficients a, b, c, d, e, and f are real numbers. Elimination of $X Z$ yields a quadratic in Z, from which two Z-roots and therefore two X-roots result for each assumed value of k. Plotting Z and X against k gives two curves of the type shown in Fig. 7. That point P of the X-diagram which coincides with the given value of $X = \frac{\omega^2_2}{\omega^2_1}$ determines the abscissa k_c, the critical value of k, and, simultaneously, the corresponding ordinate in the Z-diagram. Both the values of k_c and Z ultimately define ω and v_c inasmuch as, by definition,

$$\omega = \frac{\omega_1}{\sqrt{Z}} \qquad (61)$$

and

$$v_e = \frac{\omega\, b}{k_e} \qquad (62)$$

in which ω_1 is the known frequency of the first vertical principal mode of vibration of the system.

Damped Vibration.—In substituting the expressions (Eq. 50) for the terms $\omega^2{}_1\, q_1$ and $\omega^2{}_2\, q_2$ in Eqs. 46, the equations of the damped motion are obtained. For the sake of simplicity, it may be assumed that $g_1 = g_2 = g$. Again proceeding in the same manner as before, it can be easily verified that the magnitude Z in Eqs. 55 transforms to $Z(1 + i\, g)$. Accordingly, Eq. 59 assumes the form:

$$\begin{aligned} &\kappa\, X\, Z^2\,(1 + i\, g)^2 - \left[\kappa\,(1 + X) - \kappa\,\mu\, X \frac{f_1}{k} i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right)\right] Z\,(1 + i\, g) \\ &\quad + \left[\kappa - \kappa\,\mu \frac{f_1}{k} i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right) - \mu^2 \frac{f^2{}_1}{k^3}(1 - D^2)\, i \right. \\ &\qquad \left. - \mu^2 \frac{f_1}{2\,k^2}(f_2\, D^2 + f_3)\right] = 0 \qquad (63) \end{aligned}$$

Since g is small compared with unity, the term $(1 + i\, g)^2$ conveniently may be replaced by $(1 + 2\, i\, g)$. As for the remainder, the procedure of solving Eq. 63 is the same as discussed previously.

Illustrative Problem.—The values underlying the following calculation coincide with the design data of the prototype of one of the full models of the Tacoma Narrows Bridge (preliminary design with stiffening trusses and closed deck) which have been investigated in the wind tunnel of the Structural Research Laboratory of the University of Washington. The behavior of that model under wind action so clearly indicated a manifestation of flutter that the design of the prototype may be chosen to demonstrate the application of the method outlined in Section 5 as an example of self-excitation by pure flutter. Because of the favorable shape of the leading edge, the dynamic behavior of the model apparently was influenced only to a minor degree by vortex shedding at that point. Consequently, the suspended floor structure may be considered as having flat plate characteristics.

The frequencies of the first asymmetric vertical and torsional modes of the bridge system under consideration are considerably higher than the frequencies of the corresponding symmetric natural modes. Accordingly, it must be expected that self-excitation under wind action is associated with a symmetric configuration of the vibrating system. The investigation, therefore, must start from frequencies and displacement forms of the symmetric modes of vibration.

As a consequence of the considerable torsional stiffness of the towers of the model, the shape of the first torsional mode deviates appreciably from the shape of the corresponding vertical mode. The two configurations are shown in Fig. 8.

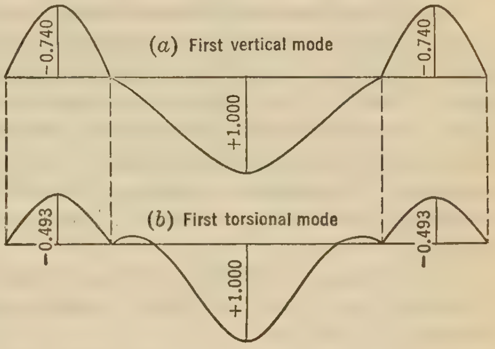

FIG. 8

Design data for the prototype include: Length of main span, $L_m = 2{,}800$ ft; length of side span, $L_s = 1{,}100$ ft; width of bridge, $2\,b = 60$ ft; mass of bridge, $m = 269$ slugs per lin ft; and mass radius of gyration, $r = 23.4$ ft. Frequencies and displacement forms of the first vertical and the first torsional principal modes have been calculated by the Ritz method, discussed in Section 4, using a two-term sine expansion for the main span and a single sine function for the side spans. The calculated and observed frequencies show close agreement. For the first vertical mode the frequency is

$$\omega^2{}_1 = 0.755 \qquad (64a)$$

and the displacement forms for the main span and the side span, respectively, are

$$\Phi_1 = 0.792\left(\sin\frac{\pi x}{L_m} - 0.113\sin\frac{3\,\pi x}{L_m}\right) \qquad (64b)$$

and

$$\Phi_1 = 0.792\left(0.740\sin\frac{\pi x}{L_s}\right) \qquad (64c)$$

For the first torsional mode the frequency is

$$\omega^2{}_2 = 2.41 \qquad (65a)$$

and the displacement forms for the main span and the side span, respectively, are

$$\Phi_2 = 0.835\left(\sin\frac{\pi x}{L_m} - 0.493\sin\frac{3\,\pi x}{L_m}\right) \qquad (65b)$$

and

$$\Phi_2 = 0.835\left(0.493\sin\frac{\pi x}{L_s}\right) \qquad (65c)$$

The functions Φ_1 and Φ_2 are given in normalized form, the factors of normalization being $N_1 = 0.792$ and $N_2 = 0.835$. According to Eq. 45, therefore,

$$D = \int_L \Phi_1\,\Phi_2\,dx = 0.792 \times 0.835\left[\int_0^{L_m}\left(\sin\frac{\pi x}{L_m} - 0.113\sin\frac{3\,\pi x}{L_m}\right)\right.$$
$$\left(\sin\frac{\pi x}{L_m} - 0.493\sin\frac{3\,\pi x}{L_m}\right)dx + 2\int_0^{L_s}\left(0.740\sin\frac{\pi x}{L_s}\right.$$
$$\left.\left.\times\,0.493\sin\frac{\pi x}{L_s}\right)dx\right] \qquad (66)$$

from which $D = 0.888$ and $D^2 = 0.788$. Using $s = 2\,\rho\,\pi\,b = 6.28 \times 0.00238 \times 30 = 0.448$, Eq. 54*a* gives $\mu = \frac{0.448 \times 30}{269} = 0.05$. From Eq. 54*b* it follows that $\kappa = \frac{2 \times 560}{30^2} = 1.25$; $X = \frac{\omega^2_2}{\omega^2_1} = \frac{2.41}{0.755} = 3.19$.

After these preparatory calculations the stability condition (Eq. 59) may be applied to a set of assumed values of k to obtain the two diagrams shown in Fig. 7 or, at least, those parts of the two graphs involving the region where the roots X and Z of Eq. 59 may be expected to be found. Assuming that the value of k lies between 0.20 and 0.25, the determination of three values of Z and the corresponding three values of X suffices to determine k_c with sufficient accuracy. The stability condition (Eq. 59) has been solved, therefore, for k-values of 0.20, 0.22, and 0.25. The computation based on $k = 0.20$ is as follows:

From Table 1, $F(k) = 0.728$ and $G(k) = 0.189$. Therefore, $f_1 = 0.728 - 0.189\,i$; $f_2 = 1 + f_1 = 1.728 - 0.189\,i$; and $f_3 = 1 - f_1 = 0.272 + 0.189\,i$. According to Eq. 59,

$$\kappa\, X\, Z^2 = 1.25\, X\, Z^2 \qquad \text{(67}a\text{)}$$

$$-\left[\kappa\,(1 + X) - \kappa\,\mu\,X\frac{f_1}{k}\,i + \mu\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right)\right] Z$$

$$= -\,1.1909\,X\,Z - 2.1842\,Z + 0.2275\,i\,X\,Z + 0.2703\,i\,Z \qquad \text{(67}b\text{)}$$

and

$$\kappa + \mu\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right) - \kappa\,\mu\frac{f_1}{k}\,i - \mu^2\left[\frac{f^2_1}{k^3}\,i\,(1 - D^2) + \frac{f_1}{2\,k^2}\,(f_2\,D^2 + f_3)\right] = 2.0696 - 0.5215\,i \qquad \text{(67}c\text{)}$$

The sum of Eqs. 67 represents Eq. 59; the real terms furnishing the equation,

$$1.25\,X\,Z^2 - 1.1909\,X\,Z - 2.1842\,Z + 2.0696 = 0 \qquad \text{(68}a\text{)}$$

and the imaginary terms furnishing the equation,

$$0.2275\,X\,Z + 0.2703\,Z - 0.5215 = 0 \qquad \text{(68}b\text{)}$$

From Eq. 68*b*,

$$X\,Z = -\,1.1881\,Z + 2.2923 \qquad \text{(68}c\text{)}$$

which, when substituted in Eq. 68*a*, gives the quadratic equation,

$$Z^2 - 1.4114\,Z - 0.6603 = 0 \qquad \text{(68}d\text{)}$$

having the roots: $Z_1 = 0.475$ and $Z_2 = 0.937$. Substituting these values in Eq. 68*c* gives $X_1 = -\,1.1881 + \frac{2.292}{0.475} = 3.64$ and $X_2 = -\,1.1881 + \frac{2.292}{0.937} = 1.26$.

In the same manner, two other pairs of the roots X and Z have been determined for $k = 0.22$ and $k = 0.25$. The results are compiled in Table 2.

Since the values of the roots X_2, representing the lower branch of the X-curve, are far below the given value of $X = \frac{\omega^2_2}{\omega^2_1} = 3.19$, these values and

TABLE 2.—VALUES OF X AND Z IN EQ. 59 CORRESPONDING TO ASSUMED VALUES OF k IN ILLUSTRATIVE PROBLEM

k (assumed) (1)	X_1 (2)	X_2 (3)	Z_1 (4)	Z_2 (5)
0.200	3.64	1.26	0.475	0.937
0.220	2.87	1.24	0.552	0.938
0.250	2.31	1.18	0.620	0.939

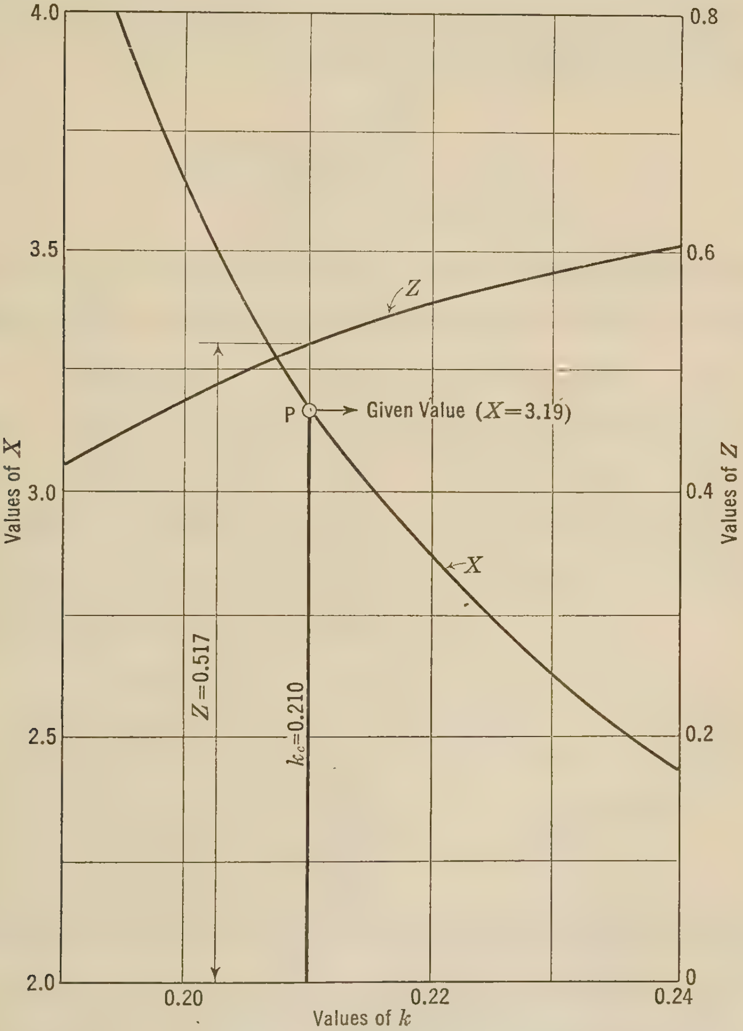

FIG. 9

therefore the corresponding values of Z_2 are of no significance for the solution. The curves X_1 and Z_1 are shown in Fig. 9. A parallel to the k-axis at the distance $X = 3.19$ from it intersects the X-diagram at P. The corresponding abscissa, $k_c = 0.210$, represents the critical value of k. From the Z-diagram the value of $Z = 0.517$, which corresponds to the abscissa $k = 0.210$, is inferred. These values, when used in Eqs. 61 and 62, give the flutter frequency $\omega = \frac{0.869}{\sqrt{0.517}} = 1.21$ or 11.6 cycles per min; and the critical velocity $v_c = \frac{1.21 \times 30}{0.210} = 173$ ft per sec, or 118 miles per hr. The flutter frequency lies between $\omega_1 = 0.869$ and $\omega_2 = 1.553$, the frequencies of the principal modes.

In order to obtain information about the particular character of the excited motion reference is made to Eq. 55a, from which

$$\frac{u_1}{u_2} = -\frac{\mu\left(\frac{f_1}{k^2} + \frac{f_2}{2\,k}\,i\right)D}{-1 + Z + \mu\frac{f_1}{k}\,i} \qquad \text{(69)}$$

Substituting the numerical values of f_1 and f_2 from Table 1 for the value of $k = 0.210$ ($f_1 = 0.720 - 0.189\,i$ and $f_2 = 1.720 - 0.189\,i$), $-\mu\left(\frac{f_1}{k^2} + \frac{f_2}{2\,k}\,i\right)D = -0.747 + 0.0085\,i$ and $-1 + Z + \mu\frac{f_1}{k}\,i = -0.438 + 0.1714\,i$. Using vector representation these values may be written: $-0.747 + 0.0085\,i = 0.747\,e^{i\theta_1}$ and $-0.438 + 0.1714\,i = 0.470\,e^{i\theta_2}$, from which $\tan\theta_1 = -0.011$ and $\tan\theta_2 = -0.391$, so that $\theta_1 = -0.011$ and $\theta_2 = -0.372$. Therefore, $\frac{u_1}{u_2} = \frac{0.747\,e^{-0.011i}}{0.470\,e^{-0.372i}} = 1.59\,e^{0.36i}$, which means that the ratio of the vertical and the torsional amplitudes becomes 1.59 and the difference of the phase angles is 0.36 radian, or 20.5°. The motion, therefore, is described by the equations:

$$\phi = C \sin \omega t \qquad \text{(70a)}$$

and

$$\eta = 1.59\,C \sin(\omega t + 0.36) \qquad \text{(70b)}$$

Fig. 10 shows successive positions of the bridge floor during one cycle. The extreme position of the apparent center of rotation, indicated by point C, the intersection of the two lines 1 and 3 (extreme positions) lies upstream ahead of the leading edge. Lines 2 and 4 indicate the positions when η approaches zero.

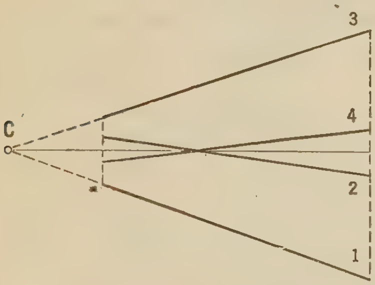

FIG. 10

To check the effect of damping on the flutter characteristic the procedure was repeated by using Eq. 63, assuming $g = 0.016$, which value corresponds with a logarithmic decrement of structural damping, $\delta = 0.05$, which in turn agrees fairly well with the observed damping rate of the model of the Tacoma Narrows Bridge.

The computed X-curve and the Z-curve are shown in Fig. 11. Point P, having the ordinate $X = 3.19$, defines the abscissa $k_c = 0.195$ and the corresponding ordinate $Z = 0.545$. Using these values, $\omega = \dfrac{0.869}{\sqrt{0.545}} = 1.18$, or 11.3 cycles per min; and $v_c = \dfrac{1.18 \times 30}{0.195} = 182$ ft per sec, or 124 miles per hr. The flutter frequency of the damped motion is slightly less than that of the undamped motion. The damping effect increases the critical velocity about 6%.

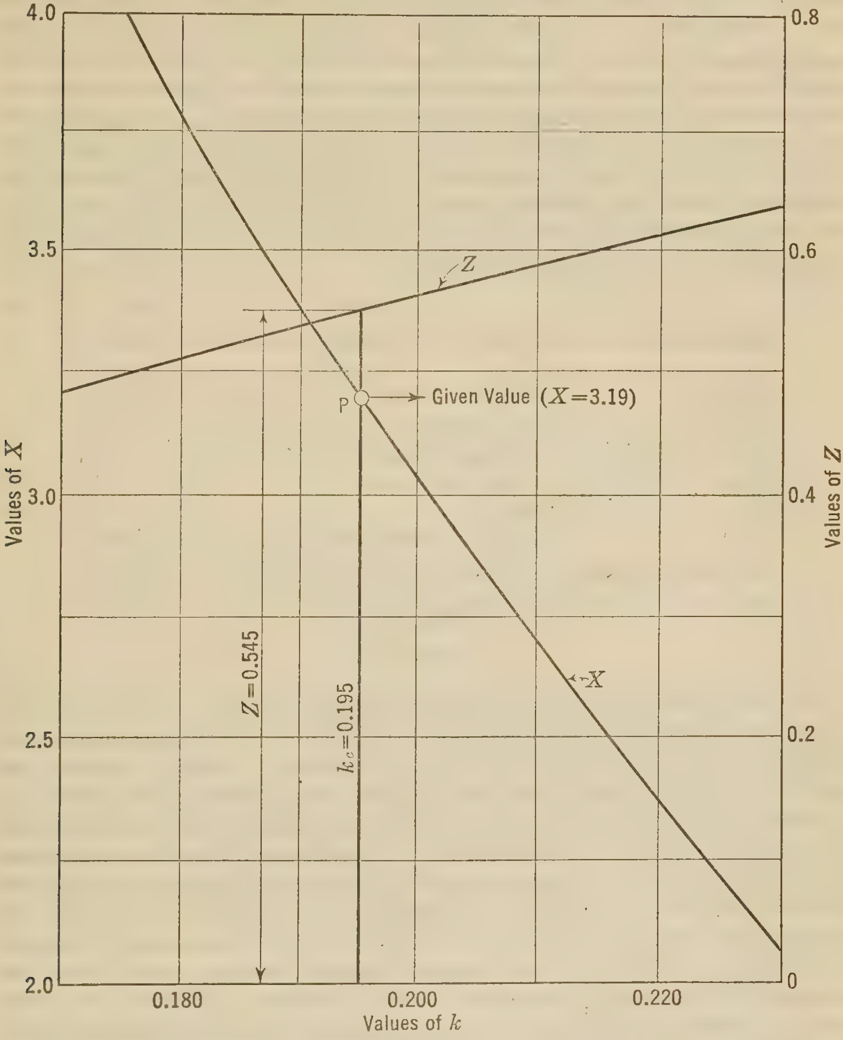

FIG. 11

No precise data concerning observations on the coupled motion of the full model of the Tacoma Narrows Bridge are available. The purpose of the test was achieved after studying the behavior of the model under various angles of attack. Within the limits of wind velocity available in the wind tunnel (118 miles per hr), no self-excited vibration could be observed at zero angle of attack. However, by a process of extrapolation a probable value of v_c at zero angle of attack could be disclosed. The curve in Fig. 12 is taken from a report of the Structural Research Laboratory of the University of Washington showing the critical wind velocities plotted against angle of attack. In projecting the curve toward $\alpha = 0°$ as indicated by the dashed part of the curve, a probable value of v_c at $\alpha = 0°$ can be estimated, inasmuch as the curve intersects the vertical axis somewhere between values of 120 miles per hr and 130 miles per hr. The previously derived theoretical value of 124 miles per hr is in fair agreement with any value between these two limits.

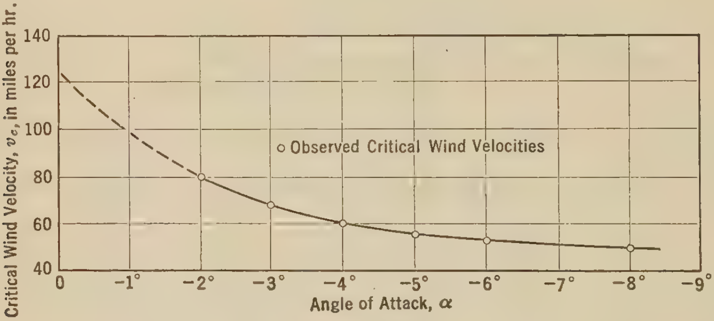

Fig. 12

The self-excited motion observed (under angles of attack of −2° and more) showed the characteristic feature of the flutter phenomenon—a torsional motion superimposed on a vertical oscillation. The apparent center of rotation was offset toward the windward truss as predicted by the theory. The distance of the center of rotation from the longitudinal axis of the structure changed with the changing angle of the acting wind. A preliminary theoretical investigation of the effect of varying angles of attack on the critical wind velocity and the flutter frequency shows that the torsional component of the coupled motion becomes more and more dominant with steadily increasing angle of attack. Accordingly, the distance of the center of rotation from the center line of the structure diminishes when the angle of attack increases. Likewise, the flutter frequency increases and approaches the frequency ω_2 of the torsional mode when α steadily increases.

It is interesting to check the behavior under wind action of a section model having the same dynamic properties as the full model. The fact that the two natural modes composing the excited motion are distinctly different in their displacement form renders it probable that ω and v_c might have somewhat different values for the two kinds of models—which is actually the case. Application of Eq. 59 (using $D = 1$) yields, when the values are adjusted to prototype scale, $\omega = 1.23$ and $v_c = 114$ miles per hr, as compared with $\omega = 1.18$ and $v_c = 124$ miles per hr for the complete bridge, indicating a noticeable difference between the critical velocities of the full model and the section model. The same structural damping rate as introduced in the calculations for the full model has been taken into account.

6. GENERALIZED THEORY OF DYNAMIC STABILITY OF TRUSS-STIFFENED SUSPENSION BRIDGES

Extension of the Theory of Flutter.—In the preceding sections concern was centered on the specification of the stability condition of suspension bridges having an ideal flat plate cross section. The stability condition has been derived under the assumption that the dynamic air forces responsible for the occurrence of self-excitation are defined by the Theodorsen equation for a perfectly streamlined thin flat plate. It is easily noted that this greatly simplified concept—presumably in the majority of cases—might be not quite consistent with the actual conditions regarding the magnitude and distribution of the air forces.

It is felt, therefore, that the theory developed in the preceding sections needs further refinement, taking into account the more or less marked effect of the particular shape of the cross section in the vicinity of the leading edge. The blunt form of the leading edge gives rise to a periodic lift force F_v, as pointed out in Section 1, which may be considered as simply superimposed upon the air forces acting on the flat plate (Fig. 1).

The subsequent theory is based on three assumptions, the first of which is that the air forces acting on the flat plate are defined by the Theodorsen equations (Eqs. 7 and 8). It is readily realized that the lift force F_v may have some influence on the air flow around the structure; but it can be assumed that, when F_v is of magnitude comparable to that of the flat plate forces, magnitude and distribution of those forces will not be influenced substantially. Comparison of the prediction of the theory with test results indicates that the assumption is fairly correct. The second assumption is that the frequency of the vortex shedding at the windward chord—the vortices assumed to be responsible for the alternating force F_v—is controlled by the frequency of the oscillating structure; the third, that the lift force F_v is assumed to be a function of the wind velocity v, the torsional amplitude ϕ, and the first derivative $\dot{\eta}$ of the vertical amplitude and that this function has the form:

$$F_v = 2\,\pi\,\rho\,b\,v^2\,(A_v + B_v\,i)\left(\phi + \frac{b\,\dot{\eta}}{v}\right)\ldots\ldots\ldots\ldots(71)$$

in which A_v and B_v are parameters depending on the shape of the cross section, are supposed to be determined by tests on a section model of the design under consideration, and are functions of k presumably slightly decreasing with increasing k. Eq. 71 may be regarded as a hypothesis whose validity and limitation can be proved only by a number of laboratory tests. No attempt is made to establish in a rational way the mathematical form of this equation.

The following analysis is confined to cases—apparently including suspension bridges of the truss type—where the vortex shedding effect is comparatively small, about of the order of magnitude of the damping effect of the Theodorsen air forces. The theory does not apply to girder-stiffened bridges, where F_v becomes dominant and the air forces acting on the plate presumably deviate considerably in magnitude and distribution from those given by the Theodorsen equations.

By introducing the substitution factors, s, and

$$f_4 = A_v + B_v\, i \quad\text{.......................(72)}$$

Eq. 71 becomes

$$F_v = s\, v^2 f_4 \left(\phi + \frac{b\, \dot{\eta}}{v}\right) \quad\text{....................(73)}$$

which is a more suitable form for the analysis. The lift force F_v produces a moment $b\, F_v$ about the longitudinal center line of the structure. Adding F_v and $b\, F_v$ to the flat plate forces as represented by Eqs. 7 and 8, the expressions for the resultant lift force F_L and the resultant moment M_L which operate to produce self-excitation now assume the form:

$$F_L = -\, s\, v^2 \left[f_1 \left(\phi + \frac{b}{v}\, \dot{\eta}\right) + f_2 \frac{b}{2\, v}\, \dot{\phi} + f_4 \left(\phi + \frac{b}{v}\, \dot{\eta}\right)\right] \quad\text{....(74)}$$

$$M_L = \frac{s\, b\, v^2}{2} \left[f_1 \left(\phi + \frac{b}{v}\, \dot{\eta}\right) - f_3 \frac{b}{2\, v}\, \dot{\phi} + 2\, f_4 \left(\phi + \frac{b}{v}\, \dot{\eta}\right)\right] \quad\text{...(75)}$$

Proceeding in the same manner as in Section 4, under the heading, "Application of Lagrange's Equations to the Flutter Problem of Suspension Bridges," Eqs. 46 become

$$\ddot{q}_1 + \omega^2{}_1\, q_1 + \frac{s\, b\, v^2}{m\, b^2} \left[f_1 \left(q_2\, D + \frac{b}{v}\, \dot{q}_1 \right) \right.$$
$$\left. + \frac{b\, f_2}{2\, v}\, \dot{q}_2\, D + f_4 \left(q_2\, D + \frac{b}{v}\, \dot{q}_1 \right) \right] = 0 \quad\text{.............(76a)}$$

and

$$\ddot{q}_2 + \omega^2{}_2\, q_2 - \frac{s\, b\, v^2}{2\, m\, r^2} \left[f_1 \left(q_2 + \frac{b}{v}\, D\, \dot{q}_1 \right) \right.$$
$$\left. - \frac{b\, f_3}{2\, v}\, \dot{q}_2 + 2\, f_4 \left(q_2 + \frac{b}{v}\, D\, \dot{q}_1 \right) \right] = 0 \quad\text{.............(76b)}$$

Since the first and third terms in Eqs. 74 and 75 show the same factor, $\phi + \frac{b\, \dot{\eta}}{v}$, it is easily noted that the third terms within the brackets in Eqs. 76 are derived from the first terms simply by replacing f_1 with f_4 and $2\, f_4$, respectively. Eqs. 55, therefore, become

$$\left(-\, 1 + Z + \mu \frac{f_1}{k}\, i + \mu \frac{f_4}{k}\, i \right) u_1 + \mu \left(\frac{f_1}{k^2} + \frac{f_2}{2\, k}\, i + \frac{f_4}{k^2} \right) D\, u_2 = 0 \quad\text{..(77a)}$$

and

$$-\left(\mu \frac{f_1}{k}\, i + 2\, \mu \frac{f_4}{k}\, i \right) D\, u_1 + \left[-\, \kappa + \kappa \frac{\omega^2{}_2}{\omega^2{}_1}\, Z \right.$$
$$\left. -\, \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\, k}\, i \right) - 2\, \mu \frac{f_4}{k^2} \right] u_2 = 0 \quad\text{..............(77b)}$$

If the determinant of Eqs. 55 is written in condensed form—

$$\Delta_o = \begin{vmatrix} A & B \\ C & E \end{vmatrix} \qquad (78)$$

—in which A, B, C, and E are the coefficients of the variables u_1 and u_2 in Eqs. 55—it is readily noted that the determinant of Eqs. 77 can be written

$$\Delta = \begin{vmatrix} A + \mu \frac{f_4}{k} i & B + \mu \frac{f_4}{k^2} D \\ -\left(C + 2\,\mu \frac{f_4}{k} i\, D\right) & E - 2\,\mu \frac{f_4}{k^2} \end{vmatrix} \qquad (79)$$

Therefore, the stability condition, $\Delta = 0$, assumes the form:

$$\Delta = \Delta_o + \mu f_4 \left(- \frac{2\,A}{k^2} + \frac{E}{k} i + \frac{2\,B}{k} i\, D + \frac{C}{k^2} D \right) \qquad (80)$$

in which the terms containing f^2_4 have been disregarded as small magnitudes of second order. Their sum vanished when $D = 1$. The symbol Δ_o is the determinant $A\,E - B\,C$ and is therefore identical with the left side of Eq. 59. Without introducing appreciable error Eq. 80 may be further simplified by setting

$$A = -1 + Z \qquad (81a)$$

$$B = 0 \qquad (81b)$$

$$C = 0 \qquad (81c)$$

and

$$E = -\kappa + \kappa\, Z\, X \qquad (81d)$$

and by neglecting also all terms containing the product of f_4 and one of the functions f_1, f_2, and f_3. Thus, the expression is finally obtained for the stability condition:

$$\Delta = \Delta_o + \mu f_4 \left[\frac{2}{k^2} (1 - Z) + \frac{\kappa}{k} (-1 + Z\, X)\, i \right] \qquad (82)$$

When structural damping is to be taken into account, Δ_o becomes identical with the left side of Eq. 63.

Eq. 82 departs from the stability condition (Eq. 59) by the two additional terms which depend on f_4 and which represent the corrective effect of the particular shape of the cross section. To specify the condition of instability the same procedure may be used as in Section 5. Eq. 82 can be split into a real part and an imaginary part, thus yielding two equations that ultimately determine critical velocity and flutter frequency.

Dynamic instability of the type of structure herein considered under action of a horizontal wind flow is the combined effect of the flat plate air forces and the lift force F_v. It remains in its essential features a flutter phenomenon—that is, an excited motion composed of a vertical and a torsional vibration. However, the critical velocity and the flutter frequency now are controlled by both the dynamic properties of the system and the lift force F_v, the latter depending on the particular shape of the cross section. With increasing magnitude of F_v, the critical velocity v_c decreases, whereas the flutter frequency ω increases, tending to approach the frequency of the natural torsional mode which forms one of the two components of the excited motion.

Instability of an Oscillating System with One Degree of Freedom.—The combined effect of the flat plate air forces and the lift force F_v changes the most characteristic feature of the flutter phenomenon. As long as the wind flow produces only the aerodynamic forces as described by the Theodorsen equations, dynamic instability can develop only in a vibrating system which has at least two degrees of freedom. Thus, in the absence of air forces of the type represented by the lift force F_v, the flat plate air forces alone cannot cause any kind of dynamic instability in those cases where the system, due to its particular restraint, oscillates purely vertically or purely torsionally.

This fact can readily be shown: Eq. 47*a* represents, on dropping the terms which depend on the torsional coordinate q_2, the equation of pure vertical motion—

$$m\,\ddot{q}_1 + m\,\omega^2_1\,q_1 + s\,v\,f_1\,\dot{q}_1 = 0 \quad\ldots\ldots\ldots\ldots\ldots\;(83a)$$

Inasmuch as

$$f_1 = F - G\,i \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(83b)$$

Eq. 83*a* becomes

$$m\,\ddot{q}_1 + m\,\omega^2_1\,q_1 + F\,s\,v\,\dot{q}_1 - G\,s\,v\,\dot{q}_1\,i = 0 \quad\ldots\ldots\ldots\;(83c)$$

The last term in Eq. 83*c* represents a force in phase with the amplitude q_1. Its sole effect is a slight change of the spring constant $m\,\omega^2_1$, it has no bearing at all upon the energy balance of the vibrating system. The preceding term is in phase with the velocity q_1 and represents, because of its plus sign, a damping force which dissipates the energy of the oscillating system. Eq. 83*c* obviously defines a damped vibration.

From Eq. 47*b*, by omitting the terms having the factor q_1, the equation of pure angular vibration—

$$m\,r^2\,\ddot{q}_2 + m\,r^2\,\omega^2_2\,q_2 - \frac{s\,b\,v^2}{2} f_1\,q_2 + \frac{s\,b^2\,v}{4} f_3\,\dot{q}_2 = 0 \quad\ldots\ldots\;(84)$$

—is derived. Substituting Eq. 83*b* and

$$f_3 = 1 - F + G\,i \quad\ldots\ldots\ldots\ldots\ldots\ldots\;(85)$$

in Eq. 84 gives

$$m\,r^2\,\ddot{q}_2 + m\,r^2\,\omega^2_2\,q_2 - \frac{s\,b\,v}{2}\left(F\,v\,q_2 - \frac{G\,b}{2}\,i\,\dot{q}_2\right) + \frac{s\,b\,v}{2}\left[G\,v\,i\,q_2 + (1 - F)\,\frac{b}{2}\,\dot{q}_2\right] = 0 \quad\ldots\ldots\ldots\;(86)$$

The third term in Eq. 86 represents a force in phase with the amplitude q_2 which affects only the spring constant. The fourth term always is positive, as G and $(1 - F)$ are real positive numbers. The product $i\,q_2$ is in phase with the velocity $\dot{q}_2$, accordingly, the last term is a dissipative force and Eq. 86 defines a damped vibration, regardless of the value of the wind velocity v. The foregoing discussion demonstrates that no dynamic instability occurs when an ideal flat plate vibrates in a purely torsional or purely vertical mode.

Now let it be assumed that the complex factor f_4 in the expression for the lift force F_v be so defined that F_v represents an exciting force increasing with increasing wind velocity. Acting on a section model with fixed center of rotation, for instance, the force F_v, at a certain wind velocity at which structural damping is overcome, would generate excited vibrations. However, the flat plate air forces, which are thought of as acting simultaneously on the model, are damping forces, as demonstrated previously, and delay the excitation until a wind velocity is reached at which equilibrium exists between the exciting and damping forces. Any further increase of v causes vibrations of steadily increasing amplitude, and the system becomes dynamically unstable. The wind velocity at which steady state harmonic motion can be maintained defines the critical velocity of the section model under wind action.

Thus, another type of self-excitation is disclosed whereby periodic vortex discharge at the leading edge is the real source of instability. The air forces that act on the flat plate play the role of damping forces with the effect of delaying the start of the self-excited vibration. The frequency of the excited vibration is reduced as compared with the frequency of the motion in still air.

A particular question arises as to the correlation of the behavior under wind action of a system of one degree of freedom, previously discussed, and the behavior of a system of two degrees of freedom treated in this section (see "Extension of the Theory of Flutter"). In a system having two degrees of freedom, the flutter phenomenon with its peculiar mechanism and the vortex shedding effect operate together to produce the dynamic instability of the structure, whereas in a system whose motion is restricted to torsional oscillations, only F_v is responsible for the occurrence of excited vibrations. Thus, it follows that two identical section models, one of them spring suspended and the other with fixed center of rotation, will show different values of v_c.

The detailed discussion of the particular behavior of an oscillating system with fixed center of rotation was deemed necessary because of the importance of that class of mechanical system for the determination of those experimental data from which magnitude and angle of phase of F_v can be derived. As only one free coordinate is involved, the theory of a section model whose motion under wind action is restricted to angular vibration becomes very simple.

The equation of motion of a torsionally vibrating section model acted on by the flat plate air forces and by the lift force F_v can be easily derived from Eq. 76*b* by omitting the term containing the coordinate q_1. Thus,

$$\ddot{q}_2 + \omega^2{}_2\, q_2 - \frac{s\, b\, v^2}{2\, m\, r^2}\left(f_1\, q_2 - \frac{b\, f_3}{2\, v}\, \dot{q}_2 + 2\, f_4\, q_2\right) = 0 \;........ (87)$$

Substituting the solution $q_2 = e^{i\omega t}$, replacing v with $\frac{\omega\, b}{k}$, Eq. 87 becomes

$$-\,\omega^2 + \omega^2{}_2 - \frac{s\, b}{m}\left(\frac{b^2}{2\, r^2}\frac{f_1}{k^2}\,\omega^2 - \frac{b^2}{4\, r^2}\frac{f_3}{k}\, i\, \omega^2 + 2\,\frac{b^2}{2\, r^2}\frac{f_4}{k^2}\,\omega^2\right) = 0 \;... (88)$$

Dividing by ω^2 and substituting Eqs. 54 gives

$$-1 + \frac{\omega^2_2}{\omega^2} - \frac{\mu}{\kappa}\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right) - 2\,\frac{\mu}{\kappa}\,\frac{f_4}{k^2} = 0 \quad \text{(89)}$$

which represents the stability condition. In order to include structural damping also, the term $\frac{\omega^2_2}{\omega^2}$ is extended by the factor $(1 + i\,g)$, so that

$$-1 + \frac{\omega^2_2}{\omega^2}(1 + i\,g) - \frac{\mu}{\kappa}\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right) - 2\,\frac{\mu}{\kappa}\,\frac{f_4}{k^2} = 0 \quad \text{(90)}$$

Eq. 90 represents a relationship between the frequency ω of the induced vibration and the magnitude $k = \frac{\omega\,b}{v}$. Because f_1, f_3, and f_4 are complex functions, Eq. 90, on equating the real and the imaginary parts to zero, yields two equations with real coefficients. These equations define ω and k if A_v and B_v are considered known properties of the model setup. In turn, Eq. 90 determines the characteristics A_v and B_v when ω and k would be known through laboratory tests on a section model by observing the frequency ω of the excited motion and the critical velocity v_c.

Eq. 90 forms the basic relation for the interpretation of a group of tests on a section model with fixed center of rotation, which may furnish A_v and B_v, determining the magnitude and the angle of phase of F_v as a function of k for the cross section under consideration. Introduction of these two experimental values into the stability conditions (Eq. 82) permits prediction of flutter frequency and critical wind velocity for any suspension bridge having a cross section of the same geometric form as the section model.

If A_v and B_v were constants—that is, were independent of k—a single test would yield the values of A_v and B_v, because two relations exist between A_v and B_v on the one hand and the observed values of ω and k on the other hand. To find any variation of A_v and B_v at least two or, if necessary, three experiments with different spring constants must be carried out. In this way, two or three values of A_v and B_v can be found which determine sufficiently accurately the A_v-diagram and the B_v-diagram indicating the variation of these two magnitudes with k (Fig. 13).

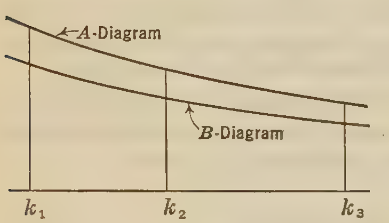

FIG. 13

It is of interest to note that in the case of purely torsional vibration the frequency ω of the excited oscillation lies, according to the theory, definitely below the frequency ω_2 of the free torsional vibration (motion in still air). This shift in frequency actually has been observed in purely torsional motion of section models of truss type.

Illustrative Problem.—Experiments at the University of Washington with section models of the Golden Gate Bridge afford a welcome opportunity to explain the previously outlined method by an instructive example and to show that the observed data comply fairly well with the predictions of the theory. It will be indicated subsequently: (1) How the air force function f_4 can be found from the observed frequencies and critical velocities of a section model with fixed center of rotation (model A); and (2) how to determine the condition of stability of another section model with free spring mount (coupled motion) having the same geometric form as model A (model B). Using the function f_4 derived from the tests on model A, the stability condition (Eq. 82) applied to model B must yield results which are in accordance with the measurements made on that model.

Section Model with Fixed Center of Rotation (Model A).—The design characteristics of model A, after adjustment to prototype scale are: $2\,b = 90$ ft; $m = 555$ slugs; and $r = 27.8$ ft. In the first test, observed values were as follows: Natural torsional frequency (wind off), $\omega_2 = 0.929$; frequency of the excited torsional motion, $\omega = 0.856$; and critical wind velocity, $v_c = 94$ ft per sec. The frequencies $\omega_2 = 0.929$ and $\omega = 0.856$ correspond, respectively, to 76.8 cycles per min and 70.8 cycles per min observed on the 75-scale model. The logarithmic decrement of structural damping of the model was $\delta_s = 0.073$, which value was determined by subtracting the observed logarithmic decrement of aerial damping, $\delta_a = 0.010$, from the total damping, $\delta_s + \delta_a = 0.083$.

The following values are computed first: $s = 2\,\pi\,\rho\,b = 6.28 \times 0.00238 \times 45 = 0.673$; $\mu = \frac{s\,b}{m} = \frac{0.673 \times 45}{555} = 0.0545$; $\kappa = \frac{2\,r^2}{b^2} = \frac{2 \times 27.8^2}{45^2} = 0.763$; $\frac{\mu}{\kappa} = 0.0715$; $\frac{\omega^2{}_2}{\omega^2} = \frac{0.929^2}{0.856^2} = 1.178$; $k = \frac{\omega\,b}{v_c} = \frac{0.856 \times 45}{94} = 0.410$; and $g = \frac{\delta_s}{\pi} = 0.0232$. Entering Table 1 with the argument $k = 0.41$, it is found that $f_1 = 0.622 - 0.163\,i$ and $f_3 = 0.378 + 0.163\,i$.

Introducing the numerical values of $\frac{\mu}{\kappa}$, $\frac{\omega^2{}_2}{\omega^2}$, k, and g in the stability conditions (Eq. 90) gives

$$- 1 + 1.178\,(1 + 0.0232\,i) - 0.0715\left(\frac{0.622 - 0.163\,i}{0.410^2} - \frac{0.378\,i - 0.163}{0.820}\right) - \frac{2 \times 0.0715}{0.410^2}\,(A_v + B_v\,i) = 0 \qquad (91a)$$

from which

$$0.1780 + 0.0273\,i - 0.2788 + 0.1023\,i - 0.8507\,A_v - 0.8507\,i\,B_v = 0 \qquad (91b)$$

Separating the real and imaginary terms leads to

$$\text{Real terms: } -0.1008 - 0.8507\,A_v = 0 \qquad (92a)$$

$$\text{Imaginary terms: } 0.1296 - 0.8507\,B_v = 0 \qquad (92b)$$

—from which $A_v = -0.1185$ and $B_v = 0.1524$. Thus, the complex term, $f_4 = -0.1185 + 0.1524\,i$, determines (for $k = 0.41$ according to Eq. 73) the lift force F_v acting on the windward chord.

In the second test, observed values included: Natural frequency (wind off), $\omega_2 = 1.740$; frequency of the excited torsional motion, $\omega = 1.670$; critical velocity, $v_c = 127$ ft per sec; and logarithmic decrement of structural damping, $\delta_s = 0.009$. The corresponding value of k is $k = \frac{\omega b}{v_c} = \frac{1.67 \times 45}{127} = 0.591$. Proceeding in the same manner as before, it was found that $f_4 = -0.1006 + 0.1392\,i$.

Inspection of the values of f and f_4 shows that the parameters A_v and B_v diminish slightly with increasing k. For the purpose of the following investigation of model B it is satisfactory to assume that, within the range of k-values associated with the stability problems of models A and B, the parameters A_v and B_v vary proportionately with k.

Section Model Mounted on Free Springs (Model B).—The design characteristics for model B are the same as those for model A. Observed test values included: Natural vertical frequency, $\omega_1 = 0.814$; natural torsional frequency, $\omega_2 = 1.320$; and logarithmic decrement of structural damping, $\delta_s = 0.003$. The values of s, μ, and κ are the same as those for model A. For model B, however, $\frac{\omega^2{}_2}{\omega^2{}_1} = X = 2.62$ and $g = \frac{\delta_s}{\pi} = 0.001$.

The procedure in determining the natural frequency and the critical velocity at which excited motion begins is essentially the same as shown in Section 5, "Illustrative Problem," whereby the extended stability condition (Eq. 82) must be applied. A sequence of k-values is chosen, for each of which the expression Δ_o is computed by Eq. 63, assuming $D = 1$ for section model investigations. In adding to each Δ_o the additional term depending on f_4, the complete Eq. 82 is obtained, which finally yields the two equations from which the parameters X and Z corresponding to the assumed value of k can be calculated. On drawing the X-diagram and the Z-diagram as demonstrated in the previous section, the value of k_c and the corresponding value of Z can be found. The computation, using $k = 0.73$ is shown subsequently.

Determination of Δ_o from Eq. 63.—Introducing the numerical values for s, μ, κ, and g in Eq. 63 and using $f_1 = 0.5614 - 0.1233\,i$, $f_3 = 0.4386 - 0.1233\,i$, and $D = 1$, and separation of the real from the imaginary terms gives

$$0.763\,X\,Z^2 - 0.7560\,X\,Z - 0.8251\,Z + 0.8148 = 0 \quad \ldots\ldots (93a)$$

and

$$0.00153\,X\,Z^2 + 0.03121\,X\,Z + 0.02816\,Z - 0.06026 = 0 \quad \ldots\ldots (93b)$$

The next step is to compute the terms in Eq. 82 which depend on f_4. Thus,

$$\frac{2\,\mu}{k^2}(1 - Z) = \frac{0.109}{0.5329}(1 - Z) = 0.2045\,(1 - Z) \quad \ldots\ldots (94a)$$

and

$$\frac{\mu\,\kappa}{k}(-1 + X\,Z)\,i = \frac{0.763 \times 0.545}{0.73}(-1 + X\,Z)\,i$$
$$= 0.0570\,i\,(-1 + X\,Z) \quad \ldots\ldots (94b)$$

From the two values for f_4 computed previously the expression f_4 for $k = 0.73$ is derived—namely, $f_4 = -0.087 + 0.129\,i$. Multiplying Eqs. 94 by this value of f_4 gives

$$-0.01779 + 0.01779\,Z + 0.02638\,i - 0.02638\,i\,Z \qquad (95a)$$

and

$$0.00496\,i - 0.00496\,i\,X\,Z - 0.00735\,X\,Z + 0.00735 \qquad (95b)$$

Adding the real terms of Eqs. 95 to Eq. 93*a*, and the imaginary terms to Eq. 93*b*, finally leads to

$$0.763\,X\,Z^2 - 0.7633\,X\,Z - 0.8073\,Z + 0.8044 = 0 \qquad (96a)$$

and

$$0.00153\,X\,Z^2 + 0.02625\,X\,Z - 0.00178\,Z - 0.02892 = 0 \qquad (96b)$$

From Eq. 96*b*,

$$X\,Z = \frac{0.00178\,Z + 0.02892}{0.00153\,Z + 0.02625} \qquad (97)$$

Substituting Eq. 97 in Eq. 96*a* yields the quadratic equation,

$$Z^2 - 1.3360\,X + 0.3464 = 0 \qquad (98)$$

having the roots $Z_1 = 0.943$ and $Z_2 = 0.392$. From Eq. 97 the associated values of X are computed—that is, $X_1 = 1.04$ and $X_2 = 2.68$.

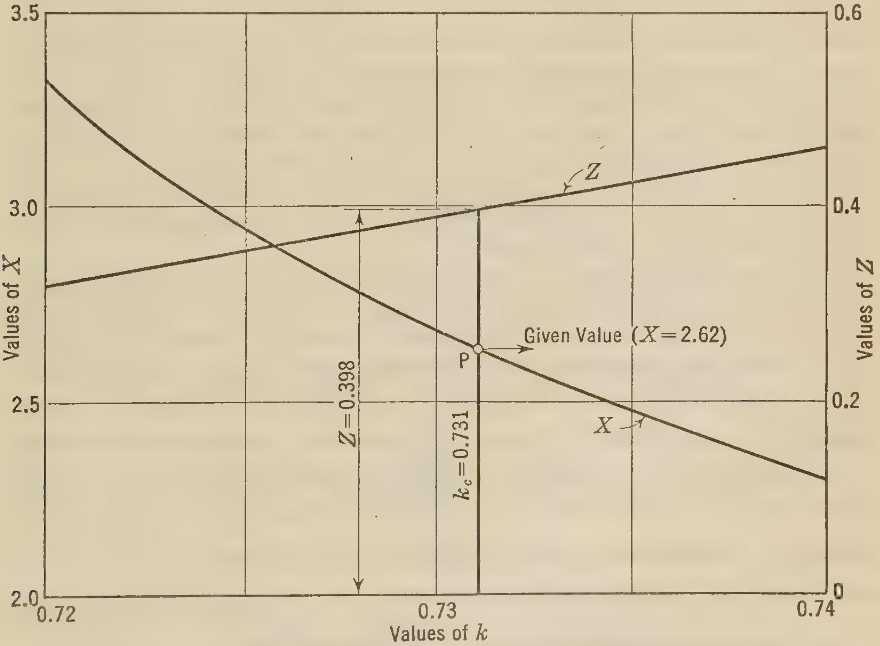

FIG. 14

In the same manner, the roots Z and X were computed for $k = 0.72$ and $k = 0.74$. The X-diagram and the Z-diagram built up from the X_2-values and the Z_2-values are shown in Fig. 14. The ratio $X = \frac{\omega^2{}_2}{\omega^2{}_1} = 2.62$ determines point P on the X-diagram which finally defines the critical values $k_c = 0.731$ and $Z = 0.398$—from which the flutter frequency, $\omega = \frac{\omega_1}{\sqrt{Z}} = \frac{0.814}{\sqrt{0.398}} = 1.29$, and the critical wind velocity, $v_c = \frac{\omega b}{k} = \frac{1.29 \times 45}{0.731} = 79.5$ ft per sec, are obtained. Reduced to model scale, therefore, the computed and observed values compare as follows:

Value	Computed	Observed
ω	11.17 (107 cycles per min)	109 cycles per min
v_c	9.2 ft per sec	9.4 ft per sec

—showing that the theoretical results are in good agreement with the observed values of ω and v_c.

Observing that the frequency ω of the excitation lies very close to the natural frequency ω_2, it is concluded that the torsional component of the coupled vibration dominates the motion. The determination of the ratio $\frac{u_1}{u_2}$ yields $\frac{\eta}{\phi} = 0.134$—that is, the angular amplitude is about seven times as great as the vertical amplitude. Accordingly, the lateral shift of the center of rotation is small, its theoretical maximum value amounting to 50 in., or about 2/3 in. when reduced to model scale. The observed windward shift of the center of rotation was 3/4 in. The close accord between the theoretical and the measured values of the flutter characteristics indicates that the premises on which the theory is based—namely, the supposed mathematical form of F_v and the independence from F_v of the flat plate forces that act simultaneously on the structure—are approximately correct.

The illustrative example treated in this section offers the opportunity to demonstrate the marked effect of the configuration of the leading edge upon the dynamic stability of the bridge structure. It may be recalled that the lift force F_v associated with the vortex discharge at the windward chord always effects reduction of the critical velocity v_{c0} which would occur when F_v approaches zero (pure flutter). The ratio $\frac{v_c}{v_{c0}}$, therefore, offers an excellent measure for classifying the degree of the inherent aerodynamic stability of any cross section, and will be referred to as the "stability factor of the cross section." Actually $\frac{v_c}{v_{c0}}$ varies somewhat with the other dynamic properties, such as stiffness, mass, and the like, but these variations are of such minor degree that $\frac{v_c}{v_{c0}}$ may be considered a characteristic of the cross section.

As an illustration, calculations have been made to establish the stability factor of model B. Applying Eq. 59 to model B, the flutter speed is found to be $v_{c0} = 18.6$ ft per sec (model scale). Hence, the stability factor becomes $\frac{v_c}{v_{c0}} = \frac{9.2}{18.6} = 0.49$. Therefore, the particular shape of the chord and the adjacent part of the deck structure of model B accounts for the reduction of v_c to one half of its possible optimum, v_{c0}. The behavior of the model of the Tacoma Narrows Bridge discussed in Section 5 and having the stability factor $\frac{v_c}{v_{c0}} \doteq 1$ proves that the efficiency of the cross section can be materially improved by an adequate design of certain structural details based on the information obtained from comparative tests in the wind tunnel.

The marked effect of the particular shape of the leading edge on the dynamic stability of a truss-type suspension bridge has already been disclosed in the tests made by F. B. Farquharson, M. ASCE, at the University of Washington Laboratory. The importance of an adequate shape of chord and adjoining deck structure has been repeatedly stressed in his reports on these tests. The theory developed hereinbefore provides a procedure of judging quantitatively the efficiency of any configuration of the leading edge.

7. Summary and Conclusion

The primary objective of this paper is to outline a rational method of approach for solving the problem of self-excited motion due to wind action of truss-stiffened suspension bridges and bridges with extremely shallow stiffening girders. The analysis developed in this paper is confined to the investigation of the effect of horizontal wind.

The method is based on the conception that these types of bridges behave aerodynamically like a flat plate acted on by air forces distributed over the width of the plate, and by additional air forces represented by a lift force F_v at the windward chord. The flat plate air forces are assumed to be identical with the aerodynamic air forces that originate on an oscillating streamlined thin flat plate under horizontal wind action, and for which the mathematical expressions are well established. The lift force F_v depending in magnitude and phase angle on the particular shape of the leading edge must be determined experimentally on a section model of the type of cross section under consideration.

The theory suggested herein explains all the characteristic phenomena which have been observed on section models and full-scale models of truss-type cross sections with closed deck acted on by a horizontal wind stream. Good quantitative agreement between theory and experiment is demonstrated in two cases. Other test results useful for checking the theory have not been available to the author.

In closing, the following remarks may be of value in properly judging the theory outlined and for its significance within the scope of the whole problem of self-excitation in suspension bridges.

The mathematical solution of the problem of dynamic instability of suspension bridges, as herein developed, is based on well-established principles of dynamics. The solution is sufficiently general so that the outlined method may serve as a pattern in solving all those problems of dynamic instability of suspension bridges which, mathematically speaking, reduce to a system of homogeneous linear equations. Assuming that the coefficients which determine magnitude, distribution, and timing of the air forces are known, the response of any type of suspension bridge structure (including also bridges with double lateral systems, tower stays, and other stiffening elements) can be determined theoretically and an exact correlation between the behavior of a section model and the prototype readily can be established.

The special theory discussed herein covers those types of suspension bridges which may be considered of importance for future suspension bridge design—namely, truss-stiffened bridges and bridges with extremely shallow stiffening girders. Girder cross sections of any considerable depth-to-width ratio exhibit such high vulnerability to aerodynamic forces that, in the author's opinion, they cannot compete successfully with these other types.

The introduction of the lift force F_v in the analysis of the problem is an attempt to explain the phenomena of self-excitation by the combined action of the known flat plate air forces and of a hypothetical dynamic force F_v without referring to the actual mechanism which generates that force. The concept of hypothetical forces has been used repeatedly in solving even basic problems in physics. Success or failure depends only on the proper choice and the adaptability of the mathematical expressions for such forces introduced in the analysis. Further tests may justify the suggested mathematical form or may indicate some necessary refinement. Close collaboration between theoretical and laboratory research may lead to a satisfying solution of the particular problem discussed, thus establishing a mathematical method for the solution of related problems of self-excitation in suspension bridges.

8. Acknowledgments

The theoretical studies reported herein are part of an extensive investigation of the problem of vibration in suspension bridges sponsored by the Advisory Board on Investigation of Suspension Bridges of the Public Roads Administration. They represent the essentials of two reports submitted by the author to the members of the Advisory Board, dated November 5, 1945, and February 27, 1947. The author is especially indebted to Professor Farquharson, director of the Engineering Experiment Station, University of Washington, and to George S. Vincent, M. ASCE, highway bridge engineer, Public Roads Administration, for their cooperation and constructive criticism of his studies.

APPENDIX. NOTATION

The following letter symbols, adopted for use in the paper, and in its discussion, conform essentially to American Standard Letter Symbols for Science and Technology (ASA—Z10) prepared by a Committee of the American Standards Association, with Society participation:

A = cross-sectional area; A_c = cross-sectional area of cables;
A_v = a coefficient in the expression (Eq. 71) for the lift force F_v;
$a_1, a_2, \cdots, a_n$ = coefficients in Φ-expansion for η;

B_v = coefficient in Eq. 71 for the lift force F_v;
b = half bridge width;
$C(k)$ = a complex function of k (defined by Eq. 5);
D = defined by Eq. 45;
d = depth of girder;
E = modulus of elasticity:
  E_c = modulus of elasticity of the cables;
  E_f = modulus of elasticity of the stiffening frames;
e = base of natural logarithms;
F = a periodic force:
  F_L = a resultant lift force acting on a vibrating flat plate;
  F_v = a periodic lift force acting on the leading edge;
$F(k)$ = a transcendent function of k (Table 1);
f = air force function:
  f_1, f_2, f_3 = air force functions defined by Eqs. 6;
  f_4 = air force function defined by Eq. 72;
$G(k)$ = a transcendent function of k (Table 1);
g = damping coefficient:
  g_1 = vertical vibrations;
  g_2 = torsional vibrations;
H = cable tension:
  H_i = cable tension due to inertia forces;
  H_w = cable tension due to dead weight w;
h = sag of cable in main span;
I = moment of inertia; I_f = moment of inertia of total stiffening frames;
i = imaginary unit ($= \sqrt{-1}$);
K = a parameter defined by Eq. 28;
k = a dimensionless ratio $\left(= \frac{\omega b}{v}\right)$; k_c = critical value of k;
L = length of total bridge:
  L_c = length defined by Eq. 13;
  L_m = length of the main span;
  L_s = length of the side span;
l = a distance;
M = a moment; M_L = resultant lift moment acting on a vibrating flat plate;
m = mass of bridge per unit length;
N = factors of normalization of the functions Φ;
n = an integer;
p = period of oscillation;
Q_j = the generalized force corresponding to the generalized coordinate q_j;
q_j = generalized coordinates:
  $\dot{q}_j$ = first derivative with respect to time;
  $\ddot{q}_j$ = second derivative with respect to time;
R_1 and R_2 = magnitudes in Eq. 38 depending on the air forces F_L and M_L;
r = mass radius of gyration;
s = a substitution factor ($= 2 \pi \rho b$);

T = kinetic energy;
t = time;
u_1, u_2 = dimensionless parameters determining the contributions of the vertical and torsional vibrations to the resultant motion;
V = potential energy;
v = wind velocity; v_c = critical wind velocity;
W = work done by the external forces;
w = dead weight;
X = a substitution factor $\left(= \frac{\omega^2{}_2}{\omega^2{}_1}\right)$;
x = a rectangular coordinate;
Z = a substitution factor $\left(= \frac{\omega^2{}_1}{\omega^2}\right)$;
α = angle of attack;
Δ = a determinant;
δ = logarithmic decrement of damping:
  δ_a = logarithmic decrement of aerial damping;
  δ_s = logarithmic decrement of structural damping;
η = dimensionless vertical amplitude $\left(= \frac{\bar{\eta}}{b}\right)$:
  $\dot{\eta}$, $\ddot{\eta}$ = first and second derivatives with respect to time;
  $\bar{\eta}$ = vertical or bending amplitude;
  $\dot{\bar{\eta}}$, $\ddot{\bar{\eta}}$ = first and second derivatives with respect to time;
θ = difference between phase angles of the vertical and torsional components of the motion;
κ = a substitution factor $\left(= \frac{2\,r^2}{b^2}\right)$;
μ = a substitution factor $\left(= \frac{s\,b}{m}\right)$;
ρ = mass density of air (assumed 0.00238 slug);
Φ = a function of x;
ϕ = torsional amplitude; $\dot{\phi}$, $\ddot{\phi}$ = first and second derivatives with respect to time;
ψ = a time function; and
ω = flutter frequency:
  ω_1 = frequency of the vertical component of motion;
  ω_2 = frequency of the torsional component of motion.

DISCUSSION

A. G. Pugsley,[13] Assoc. M. ASCE.—This interesting attempt to link the suspension bridge flutter problem with the extensive knowledge of flutter already developed by aeronautical workers is very welcome. However, it is regrettable that the author has so wholeheartedly adopted the mathematical presentation customary among analysts of airplane flutter in the United States, rather than taken advantage of the simpler modes of presentation that have always been usual in England. Any sound simplification at this stage seems especially advantageous, as the aim presumably is to interest and guide practicing civil and structural engineers who have none of the flutter background that is relatively common among aeronautical engineers.

Eqs. 1 and 2 express the lift and moment on a plate in terms of the torsional and "flexural" coordinates, ϕ and $\bar{\eta}$, as given by a particular vortex theory commonly associated with T. Theodorsen in the United States, and with H. Glauert[14] and W. J. Duncan in England. Regardless of the theory (or experimental background) used, however, the lift and moment can still be expressed simply in terms of the coordinates ϕ and $\bar{\eta}$, thus:

$$F_L = a_1 \phi + b_1 \phi + c_1 \ddot{\phi} + d_1 \bar{\eta} + e_1 \dot{\bar{\eta}} + f_1 \ddot{\bar{\eta}} \dots\dots\dots\dots (99)$$

and

$$M_L = a_2 \phi + b_2 \ddot{\phi} + c_2 \dot{\phi} + d_2 \bar{\eta} + e_2 \dot{\bar{\eta}} + f_2 \ddot{\bar{\eta}} \dots\dots\dots\dots (100)$$

in which the coefficients a_1 to f_1 and a_2 to f_2 (some may be zero) are approximately constant and may be based on the best available theory or experiment. This type of presentation would reduce the length of algebra without detracting from the theory of flutter, and would be independent of a particular derivation of such coefficients as a_1, b_1, etc. This is an English criticism of much American work on flutter in general; the custom in the United States appears to have been started by Mr. Theodorsen at a time when he was very interested in vortex theory and was practically the only worker on the subject of flutter in the United States.

The writer is particularly anxious that this suspension bridge problem should not be obscured and made unnecessarily difficult at this stage to civil engineers generally. In place of too much theory, therefore, he would advocate even an oversimplification, and to illustrate this offers the following comments.

The writer agrees with the author's main thesis—that in the suspension bridge problem there are probably two main types of flutter: One is the classical flutter which is typical of airplane wing flutter and involves essentially the coupling of at least two degrees of freedom—in the bridge case, vertical and torsional motions—and the other arises from forces caused by the eddies shed by the bridge—primarily from its bluff "leading edge." The first type of flutter has a critical speed, as indicated by the author, which is essentially a function of the elastic, inertial, and aerodynamic characteristics of the bridge, whereas the second is of the nature of the "galloping" of telegraph wires and is comparable with what English workers in the field (having in mind the buffets from the shed eddies) have called "auto-buffeting." In the first case the amplitude of motion may rapidly become excessive at speeds greater than the critical. The amplitude of motion in the second case becomes appreciable only when the eddies are fairly large and shed at regular intervals at a frequency approximating a natural frequency of the bridge structure. It is thus understandable that such oscillations occur at the natural frequencies of the bridge and are most pronounced for decks with side girders having continuous plate webs.[15]

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[13] Prof., Civ. Eng., Univ. of Bristol, Bristol, England.

[14] "The Force and Moment on an Oscillating Aerofoil," by H. Glauert, *Reports and Memoranda No. 1242*, Aeronautical Research Committee, H.M. Stationery Office, London, 1929.

The possible remedies for the two types of flutter are quite different. The occurrence of classical flutter can be postponed to higher and higher speeds by increasing the torsional stiffness, and by what aeronautical workers call "mass balancing," which aims at removing any coupling between the torsional and vertical motions of the deck. These steps have little effect on any resonance between the shed eddy frequency and the natural frequency (for vertical or torsional motion of the deck). The remedy for the resonance is to break up, both in space and in regularity, the eddies shed by the "leading" deck girder—for example, by changing from a plate web to an irregular lattice web.

From what the writer has seen of suspension bridge flutter in wind tunnel model work, it seems that most of the "critical speeds" observed may be attributed primarily to eddy resonance, the particular speed itself arising from the fact that in a given case eddy frequency varies with wind speed and thus "picks up" in succession the natural frequencies of the structure. (The latter are mostly flexural in the lower ranges.) Only occasionally will one of the critical speeds derive from coupled conditions associated with classical flutter.

It is with the foregoing thoughts in mind—no doubt an oversimplification of the physics of the problem, yet a useful one—that the writer expresses regret for the detailed mathematics which the author chose as his vehicle. Such treatment also may inadvertently present an appearance of greater accuracy than is possible. There is little doubt, for example, that the vortex theory behind Eqs. 1 and 2 (which may be appreciably in error even for simple airfoils) can be only a rough guide for the complex shapes presented by bridge decks. Nevertheless, the application of aeronautical flutter theory that this paper presents is very welcome.

S. K. Ghaswala,[16] Jun. ASCE.—An interesting mathematical analysis of the stability of suspension bridges under wind action is given in this paper. Considerations for a comprehensive study of these problems will necessarily include (1) the question of the damping capacity of the bridge; (2) a study of the winds as they exist in nature, especially the meteorological aspects; and (3) model tests in wind tunnels involving the coordination of the model and prototype and scale effect on the Reynolds number when turbulent flow is involved.[17] No discussion of this subject is complete without reference to the pioneering work of D. B. Steinman,[18] M. ASCE. Dr. Steinman appears to be the first to have developed simple formulas for determining the aerodynamic stability of suspension bridges. In fact his equation,

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[15] "An Experimental Investigation of the Aerodynamic Stability of Suspension Bridges," by C. Scruton, 3d Cong. of the International Assn. for Bridge and Structural Eng., Zurich, 1948.

[16] Designing Engr., Concrete Constr. and Eng. Co., Ltd., Bombay, India.

$$E_f I'_f > \frac{b L^4}{60 \sqrt{h}} \qquad (101)$$

represents a simple criterion for determining the dividing line between aerodynamically stable and unstable sections.[19] In the relation, Eq. 101, E_f is the modulus of elasticity of the stiffening girder; I'_f is the moment of inertia of the half section of one stiffening girder; b is the half width of the section between center lines of the cables; L is the length of span; and h is the vertical cable sag.

In the normally required range of wind velocities, any suspension bridge with a stiffening system, satisfying Eq. 101, will be aerodynamically stable. When proportioned below this requirement it becomes unstable.

It is true that in the light of present knowledge the development of a design that would be structurally and aerodynamically stable, and at the same time the most economical, would be most difficult. The problem now facing engineers (after the collapse of the Tacoma Narrows Bridge) is without precedent in engineering history for the major part of preliminary designing falls into the field of aeronautics and aeronautical meteorology as stressed by the writer elsewhere.[20]

The Tacoma Narrows Bridge was by far the most flexible and most economical bridge of comparable span. Had it been aerodynamically stable it would have represented great progress in bridge design and construction. In fact, it is the writer's belief that, once the aerodynamic theory of suspension bridges is thoroughly understood and verified by model tests, it may be possible to build bridges even more flexible and economical than this ill-fated structure. This paper appears to go a long way toward the realization of this goal and toward a deeper understanding of the laws of dynamics and their application to bridge design.

F. B. FARQUHARSON[21] AND GEORGE S. VINCENT,[22] MEMBERS, ASCE, AND ROBERT E. McHUGH, JR.,[23] JUN. ASCE.—At the request of the Advisory Board on the Investigation of Suspension Bridges, tests have been made at the University of Washington in Seattle for the purpose of verifying, experimentally, the flutter analysis presented by Mr. Bleich. The purpose of the tests was:

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[17] "Wind Tunnels," by S. K. Ghaswala, *Science and Culture*, December, 1943, p. 224.

[18] "Rigidity and Aerodynamic Stability of Suspension Bridges," by D. B. Steinman, *Transactions*, ASCE, Vol. 110, 1945, p. 439.

[19] *Ibid.*, p. 575.

[20] "The Role of Meteorology in Civil Engineering," by S. K. Ghaswala, *Civil Engineering and Public Works Review*, London (publication pending).

[21] Prof., Civ. Eng., and Director of Eng. Experiment Station, Univ. of Washington, Seattle, Wash.

[22] Prin. Highway Bridge Engr., Public Roads Administration, Structural Research Laboratory, Univ. of Washington, Seattle, Wash.

[23] Engr. Assistant, Univ. of Washington, Suspension Bridge Investigation, Seattle, Wash.

(1) To verify the pure flutter analysis as applied to a thin flat plate in a horizontal wind;
(2) To verify the combined flutter and vortex force analysis as applied to a flat plate with blunt edges or rudimentary girders; and
(3) To determine the degree to which the analysis, developed for a horizontal wind, will indicate the behavior of the section in an inclined wind.

The basic model was a built-up section 12 in. wide, 60.75 in. long, and $\frac{1}{4}$ in. thick. For models B and C (Table 3), the edges were semicircular and sharp, respectively, to minimize vortex discharge in a horizontal wind and thus approach the theoretical conditions of an infinitely thin flat plate. For model A, the edges were square, giving a depth-to-width ratio of 0.0208 and bringing into play small vortex forces. For models G-1 and G-2, model A was used with rudimentary girder webs of fiberboard 0.012 in. thick, attached to the edges to give depth-to-width ratios of 0.05 and 0.067, respectively.

The plates were built up of magnesium with copper edges to give values of the ratio of mass radius of gyration to width within the range generally found for suspension bridges. A wood model H, 13.21 in. by 1.58 in., with semicircular edges, was available and was tested for the purpose of comparison, although its depth-to-width ratio was 0.12.

The mass properties of the various models (including attachments and one third of the spring) are shown in Table 3. They differed little and all models were tested on the same set of coil springs, with freedom to move vertically as well as to rotate about a longitudinal axis. However, models G-1 and G-2 were also tested with added mass to alter the vertical and torsional frequencies, and again with torsional spring action of three different strengths acting in parallel with the vertical springs, in order to modify further the relative vertical and torsional frequencies.

The motion of the model was recorded on a Brush oscillograph, receiving the amplified output of a Statham gage incorporated in the support of one of the springs.

The values of ω_1 and ω_2, the circular frequencies (radians per second) of the vertical and torsional oscillations of the models, were determined by manual excitation of these modes in still air. When the model was manually excited in torsion in a wind stream, the frequency was less than in still air and the axis of rotation was no longer on the longitudinal axis of the plate but was shifted upwind several inches. The frequency, ω, at the critical velocity, v_c (feet per second), is recorded in Table 3 for each model, together with the observed value of $k_c = \frac{\omega b}{v_c}$.

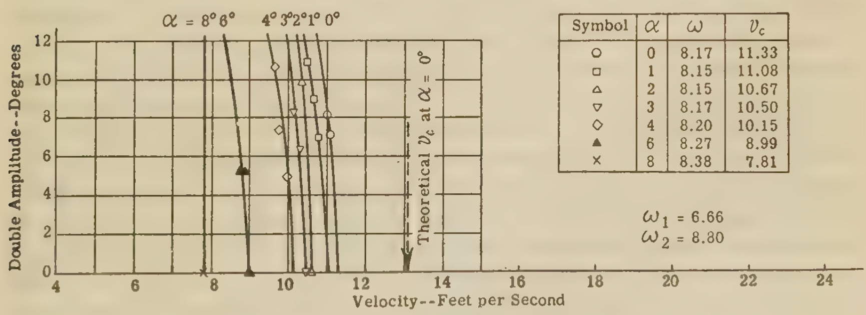
The logarithmic decrements, δ_o, were determined in still air. These averaged about 0.015 for the vertical mode and 0.005 for the torsional mode. These values of δ_o result mostly from air resistance. Previous tests of the same springs with streamlined weights of about the same mass as these models showed δ_s, the logarithmic decrement created by structural damping, to be about 0.002. The tests demonstrated that the aerodynamic damping as measured by the logarithmic decrement increases with amplitude—that is, with the velocity of vibration. This increase is an indication that the air forces opposing the oscillation vary as the square of the velocity in accordance with Froude's law. Tests of bridge models have indicated that only a small part of the air damping has a constant logarithmic decrement which is characteristic of viscous damping, on which Reynolds' law is based.

TABLE 3.—COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Model[a]	Weight (lb)	r (in.)	$\left(\frac{\omega_2}{\omega_1}\right)^2$	FLUTTER FREQUENCIES ω (RADIANS PER SEC)		CRITICAL VELOCITY v_c (FT PER SEC)		k_c (CRITICAL VALUE OF k)		THEORETICAL[b] AIR FORCE FUNCTION, f_4	
				Observed	Theoretical	Observed	Theoretical	Observed	Theoretical	A	B
A	15.68	4.784	1.744	8.168	7.900[c]	11.33	13.58[c]	0.360	0.291[c]	0.0271	0.0360
B	15.37	4.770	1.786	8.273	8.032[c]	12.15	13.94[c]	0.340	0.288[c]	0.0164	0.0314
C	14.32	4.639	1.848	8.650	8.357[c]	12.32	14.26[c]	0.351	0.293[c]	0.0112	0.0354
H	14.54	4.314	1.971	8.671	....	11.67	....	0.409	....	0.0588	0.0251
G-1	15.68	4.784	1.744	8.273	....	8.83	....	0.468	....	0.1680	0.0297
			2.849	10.263	....	12.29	....	0.418	....	0.1707	0.0552
			3.129	10.891	....	13.16	....	0.414	....	0.1221	0.0675
			3.327	11.331	....	14.32	....	0.396	....	0.0667	0.0751
G-1	17.31	4.569	1.877	8.168	....	9.15	....	0.446	....	0.1552	0.0445
G-2	15.68	4.784	1.744	8.482	8.584[d]	6.84	6.66[d]	0.620	0.644[d]	0.1900	0.0672
			2.856	10.660	10.650[d]	8.66	8.63[d]	0.616	0.617[d]	0.2037	0.0896
			3.149	11.205	11.236[d]	9.32	9.15[d]	0.601	0.614[d]	0.2472	0.0838
			3.357	11.624	11.624[d]	9.42	9.49[d]	0.617	0.612[d]	0.2557	0.0867
G-2	17.11	5.228	1.477	7.540	7.605[d]	6.00	5.78[d]	0.628	0.658[d]	0.1712	0.0594

[a] Descriptions: Model A, square edges; model B, rounded edges; model C, sharp edges; model H, thick plate; model G-1, 0.6-in. girder; and model G-2, 0.8-in. girder. [b] Theoretical value of f_4 based on the corresponding observed value of k_c. [c] Theoretical values, assuming pure flutter—that is, $f_4 = 0$. [d] Theoretical values based on values of f_4 taken from Fig. 18.

For the flat plate models A, B, and C, the theoretical flutter velocity, v_c, and flutter frequency, ω, were computed using Eq. 59 with $D = 1$. (Since the section model is not distorted but moves as a unit, the vertical and torsional motions have the same displacement form.) The logarithmic decrement of structural damping, $\delta_s = 0.002$, was only one twenty fifth of that damping, which accounted for 6% increase in critical velocity in the author's illustrative example (see text discussion of Figs. 10 and 11). It was therefore neglected in these calculations.

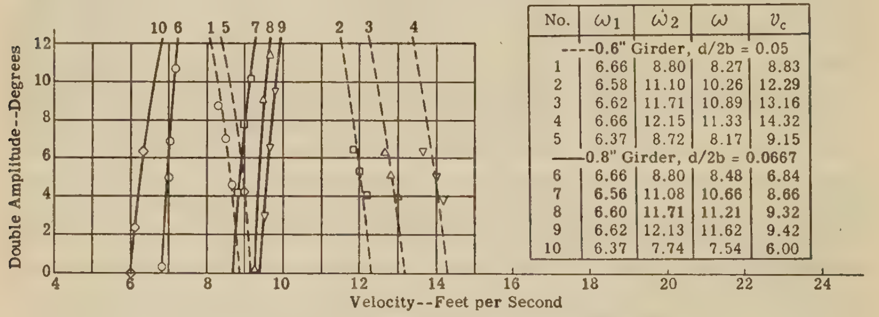


Symbol	α	ω	v_c
○	0	8.17	11.33
□	1	8.15	11.08
△	2	8.15	10.67
▽	3	8.17	10.50
◇	4	8.20	10.15
▲	6	8.27	8.99
×	8	8.38	7.81

FIG. 15.—FLAT PLATE, SQUARE EDGES

The curves in Figs. 15 and 16 show the amplitude plotted against the wind velocity for typical tests. The curves that are vertical or slope backward indicate that as the velocity is slowly increased no oscillation occurs until the critical velocity is reached, whereupon motion begins spontaneously and the amplitude increases indefinitely. The condition is one of instability—that is, there is no steady-state amplitude. This characterizes the flutter phenomenon. This is the sense in which the author uses the term "instability" as, for example, in the paragraphs following Eq. 86. He does not mean that a "stable" section remains stationary but, rather, that it maintains a stable condition of either rest or steady-state vibration.

A backward sloping curve also indicates that if, at velocities slightly below the critical value, the model is disturbed in any way so that its displacement reaches the amplitude indicated by the curve for that velocity, then the motion will increase indefinitely. In other words, an increase in either velocity or displacement which plots to the right or above the curve precipitates instability.



No.	ω_1	ω_2	ω	v_c
----0.6" Girder, d/2b = 0.05				
1	6.66	8.80	8.27	8.83
2	6.58	11.10	10.26	12.29
3	6.62	11.71	10.89	13.16
4	6.66	12.15	11.33	14.32
5	6.37	8.72	8.17	9.15
——0.8" Girder, d/2b = 0.0667				
6	6.66	8.80	8.48	6.84
7	6.56	11.08	10.66	8.66
8	6.60	11.71	11.21	9.32
9	6.62	12.13	11.62	9.42
10	6.37	7.74	7.54	6.00

Fig. 16.—Girder Sections ($\alpha = 0°$)

A response curve that slopes forward has an entirely different meaning. At any velocity above the critical, it shows the steady-state amplitude that will be built up and maintained until the velocity is increased or decreased. Although this type of response often becomes catastrophic when the velocity increases, its stability or steadiness of amplitude at a given velocity is in contrast to the instability of pure flutter. It is attributed to the combined action of the flutter forces and other forces induced by vortex action discharged from the leading edge. (For different shapes, the relative influence of flutter and vortex forces can vary over a wide range, including many shapes for which flutter is essentially nonexistent.) These vortices, readily visualized by smoke stream tests on stationary and oscillating models, may at first be discharged at some multiple of the natural frequency of the model, but when vibration begins they quickly take on the frequency of the vibration, giving rise to zones of modified pressure moving periodically across the section.

This second type of response curve (with a positive slope) was found in a horizontal wind on only model G-2 (0.8-in. girder, Fig. 16), having a $\frac{d}{2b}$-ratio of 0.0667. When the angle of attack was increased to 8°, it appeared also on model B, the round-edged plate. The vortex forces influence all the other response curves also, even though they do not reverse the slopes. A measure of their influence is indicated by the difference between the observed critical velocity and the theoretical value that was computed by solving Eq. 59 (as in the author's example under the heading, "Illustrative Problem," in Section 5) and is shown in Fig. 15.

In Fig. 17 the amplitude response curves for the various sections in a horizontal wind are plotted against k, which places them on a common basis, except for the small effect of minor differences in mass distribution. The presence of vortex forces tends to reduce the critical velocity and increase k_c, this effect naturally being greatest for the blunt sections and least for the round-edged and sharp-edged ones. The marked influence of streamlining is shown by the relative position of the curve for model H, which has a $\frac{d}{2\,b}$-ratio nearly twice that of the deeper girder section, yet shows a vortex effect only slightly greater than does the thin, square-edged model A. It should not be concluded, however, that streamlining affords the solution to objectionable oscillation. Although it does reduce the vortex forces and therefore increases the critical velocity, it does not insure a safe section if a violent flutter, which it cannot eliminate, falls within the range of possible wind velocity.

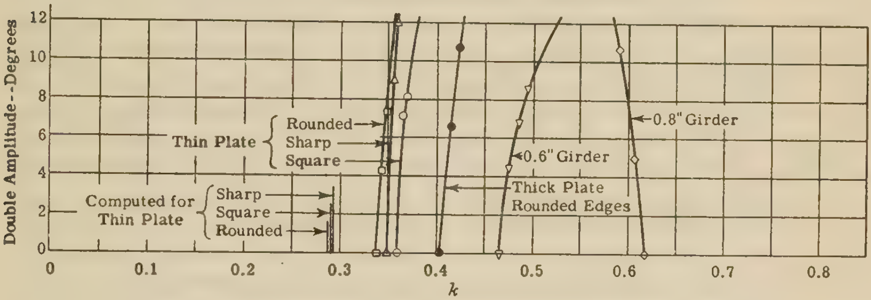


FIG. 17.—FLAT PLATE AND SHALLOW GIRDER SECTIONS ($\alpha = 0°$)

The magnitude and phase of the vortex force as represented by the coefficients, A and B, in Eqs. 71 and 72 were computed using the experimental values of ω, ω_1, ω_2, and k_c and solving Eq. 82, in which Eq. 59 was substituted for Δ_o. These computed values for model G-2 are plotted in Fig. 18. They show a somewhat random variation with k_c over the limited range of k_c-values covered by the different spring conditions. It was determined that the probable variations in test values (of the order of 0.2 ft per sec for v_c and 0.2 cycles per min for frequency) could account for only a small part of the variations of A and B. Both the test values and the calculations are sensitive to changes in mass distribution. Perhaps the simplifications of Eq. 81 account for some discrepancy. In an attempt at closer refinement, model G-2 was tested with pivots placed on the longitudinal axis of symmetry to prevent vertical motion so that the values of A and B could be computed by Eq. 89 (earlier tests had shown that the pivots increase the damping by only a small amount). Fig. 18

shows these computed values also plotted against k_c. For coefficient A the values determined for the free model by Eq. 82 show considerable consistency and were taken into account in drawing the curve. For B the values from the free model calculations were more erratic and the curve was drawn through the points obtained for the pivoted model. The theoretical values of k_c, ω, and v_c for the unpivoted tests on model G-2 were computed using values of A and B taken from the curves of Fig. 18. These theoretical values, shown in Table 3, agree very well with the observed values.

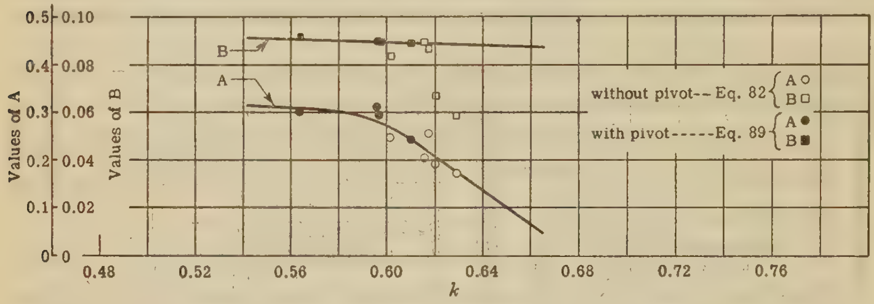


Fig. 18.—Computed Vortex Force $f_4 = A + B\,i$ (0.8-In. Girder; $\alpha = 0°$)

For angles of attack up to 2° the response differs little from that in a horizontal wind but for greater angles the difference becomes considerable. From the few observations available, it is believed the wind at some bridge sites may have an upward angle up to 6° or 8°. (Values of this order were measured on the Golden Gate Bridge, San Francisco Bay, California, during August, 1944.)

In all the tests reported in this discussion, "end plates" were used to prevent air circulation around the ends of the test specimens. Parallel tests were also made without the end plates, and it was found that the differences were small and principally accounted for by the change in mass which they caused. This agrees with section model tests made at the laboratory, indicating that with an aspect ratio (length to width) of as much as 4.5 to 5.0 the end effect is negligible.

It was expected that the models with sharp, thin edges or with rounded edges would show results much nearer the theoretical values and they did, and considerable effort was made to perfect the technique to reduce any chance vortex excitation. Knowing that angularity of wind causes vortex discharge, small, unmeasurable angle changes in the vicinity of 0° were tried with the thought that a perfectly horizontal wind stream would show the highest critical velocity. This was unfruitful, as might be anticipated, because the angle of attack can be measured to an accuracy of about $\frac{1}{4}$°.

The conclusions seem inescapable that any suspension bridge will be subject to vortex forces in addition to the flutter forces and that it will be necessary to run wind tunnel tests on an oscillating model to determine the values of A and B in the expression, $f_4 = A + B\,i$.

The author's mathematical treatment of the problem is especially valuable because it takes into account both the flutter and vortex forces in the varying proportions in which they may act on a given structure, particularly when trusses or shallow girders are used. It is a distinct advantage that his method can make use of data from aerodynamic section model tests; it does not rely on purely theoretical considerations for determining the important and sensitive parameters representing the vortex influence. The analysis should be extended to cover vertical angularity of the wind, which will be needed to assure the stability of bridges at some sites.

One limitation of the analysis is that, although it determines the wind velocity at which motion will occur, it gives no indication of the seriousness of the motion. In many cases, particularly where flutter is dominant, this information is sufficient since the motion is so violent that the possibility of its occurrence must be avoided. However, there are many cases in which a vortex-excited motion, revealed at a moderate velocity, may be difficult to avoid but may be noncastastrophic, causing limited amplitude and being subject to effective control, if desirable, through damping or other methods. A major objective of the current program of the Cooperative Research Project at the University of Washington is the development of a combined mathematical and experimental method in which the results of aerodynamic tests on section models are used to predict the steady-state amplitude that will develop on the bridge in the corresponding wind stream against a given damping. However, little is known concerning the structural damping of suspension bridges since the mass of data collected on other types of bridges is inapplicable. The development of technique for determining the damping of different types of suspension bridges is another major objective of the project.

[The corroborative tests reported in this discussion were made as a cooperative research project of the University of Washington and the U. S. Public Roads Administration, under the direction of Professor Farquharson and with the supervision and active participation of Mr. Vincent. Mr. McHugh made the tests and plotted and tabulated the data. Graduate students assisted in the computations involved.—Ed.]

Friedrich Bleich,[24] M. ASCE.—Professor Pugsley's criticism is concerned with some general aspects of the solution of the problem of self-excitation in suspension bridges presented in the paper. The writer does not agree with Professor Pugsley's statement that most of the critical speeds observed may be attributed primarily to eddy resonance. In so far as truss-striffened bridges or bridges with very shallow stiffening girders are concerned, all wind tunnel experiments strongly indicate that flutter always has an important bearing on the phenomenon of catastrophic instability but that, to a greater or less extent, the critical velocity also is influenced by a vortex shedding effect at the leading edge. This general concept cannot be simplified further, as Professor Pugsley suggests, without impairing any theory of self-excitation in suspension bridges. Any simplified concept would deviate from the real facts, as observed in wind tunnel tests, and would lead to an unrealistic theory incapable of explaining the actual behavior of a suspension bridge under wind action.

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[24] With Frankland and Lienhard, Cons. Engrs., New York, N. Y.

The discussion by Messrs. Farquharson, Vincent, and McHugh is especially welcome since the experimental verifications which they report are as satisfactory as can be expected with a subject of such complexity as the problem under consideration. The five tests on model G-2 with substantial variations of the spring constant prove, without any doubt, that dependable results can be obtained by the combined theoretical and experimental method as suggested in the paper. It is regrettable that, within the limits of wind velocity available in the wind tunnel, no excitations could be observed on paprallel tests with a pivoted model G-1 in order to determine, with sufficient accuracy, the values of the parameters A and B which define the aerodynamic force F_v at the leading edge.

The attempt made by Messrs. Farquharson, Vincent, and McHugh to derive the parameters A and B, from observations of flutter frequency and critical wind speed on the unpivoted models, led to more or less random values of the characteristics A and B. This can be seen from an inspection of the last column under the heading f_4 in which the A-values and B-values for the models G-1 and G-2 in each group are widely scattered. The failure to arrive at a series of consistent values A and B in each of these two series of tests is explained as follows: The stability equation (Eq. 82) from which A and B have been computed is completely inappropriate for the determination of these two characteristics. The two equations that follow from Eq. 82 are so sensitive, when solved for A and B (for given values of the flutter frequency ω and critical wind velocity v_c), that a slight variation of less than 1% in ω or v_c suffices to produce very substantial changes of the values of the computed A and B. The investigators admit that the probable error in the observed values of v_c is about of the order of 0.2 ft per sec—that is, about 1.5% to 2% of the test values. This fact, and only this fact, explains the considerable scatter of the values in the last two columns.

Therefore, no conclusion as to the actual values of the air force characteristics A and B can be drawn from these data. There is no method of obtaining reliable values of these parameters other than computing them from observations on a model with a fixed center of rotation (pivoted model) by means of Eq. 89. This equation was especially devised to provide a dependable derivation of the characteristics A and B and its application in the case of model G-2 led to satisfactory results which showed fairly good agreement between the observed and computed values of critical velocity and flutter frequency.

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FLEXURAL CONSTANTS OF HAUNCHED BEAMS BY AREA COMPUTATION

By Fang-Yin Tsai,[1] M. ASCE

With Discussion by Messrs. J. Edmund Fitzgerald, Harris Solman, Robert V. Hauer, George Epps, and William A. Conwell.

Synopsis

A simple and accurate method for computing the flexural constants (shape and load constants) of haunched beams is presented in this paper. For a haunched beam considered as simply supported, the various end "angle changes" are used as the flexural constants, whose values are computed from the various "momental" (moment of area) diagrams of the absolute or relative reciprocal I-diagram, all about one end of the beam. The method differs from existing methods mainly in the following two respects: (1) The various angle changes are computed from the reciprocal I-diagram instead of from the M/I-diagram as in the usual moment-area method for computing beam deflections; and (2) the values of the various moments of the reciprocal I-diagram are computed from the areas of its respective momental diagrams, obtained one from another, simply by multiplying the ordinates of the preceding diagram by their respective moment arms, instead of summing up, always inaccurately, the moments of its many elementary areas of triangular, trapezoidal, or even rectangular shape, as in the usual methods.

The advantages of the method over existing methods are obvious, because, for the various common types of loading, the relations between the angle changes and the momental areas of the reciprocal I-diagram may be expressed by comparatively simple exact formulas, and the area of any diagram, no matter how irregular in character, may be computed with satisfactory accuracy by simple methods.

For those who prefer it, a simple graphical method of constructing the various momental diagrams from the reciprocal I-diagram is presented. Also presented are five theorems of momental areas for further simplifying the

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Note.—Published in September, 1948, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof. of Structural Eng. and formerly Dean of College of Eng., National Chung Cheng Univ., Nanchang, Kiangsi, China; Prof. of Structural Eng., The Kiangsi Provincial Polytechnic Inst., Nanchang, Kiangsi, China.

computations of the method. Three typical numerical examples illustrate the application of the method to the computation of the various angle changes, and also the conversion of the angle changes into the various constants used in the method of moment distribution introduced by Hardy Cross, Hon. M. ASCE.

INTRODUCTION

Analysis or design of continuous frames having haunched beams by any method necessitates the use of certain flexural constants of the beams. For haunched beams of several common types, there are available many comprehensive tables and charts showing these flexural constants variously expressed, but for haunched beams of uncommon types the predetermination of their flexural constants is always imperative. As the exact determination of such flexural constants by direct integration is usually difficult if not impossible, approximation by either numerical or graphical summation is always necessary. Existing methods for such approximation, under whatever disguise, fall mainly into the categories: (*a*) Replacement of the actual M/I-diagram or the $1/I$-diagram by a number of trapezoidal or triangular elementary areas as represented by the method[2] of G. E. Large and Clyde Morris, Members, ASCE; and (*b*) substitution for the actual reciprocal I-curve of one or two arbitrary curves through only a few points as represented by Walter H. Weiskopf and John W. Pickworth,[3] Members, ASCE. The first method is equivalent to approximating the area under any curve by the so-called "trapezoidal rule," the accuracy of which can scarcely be judged satisfactory unless a fairly large number of segments is used. The second method can scarcely be considered simple, since fitting of the substitute reciprocal I-curve usually requires a few trials. Its accuracy is also questionable, inasmuch as the substitute curve rarely fits the actual one exactly, especially when the actual reciprocal I-curve is irregular or changes direction abruptly.

The object of this paper is to present a simple and accurate method for computing the flexural constants of haunched beams, with all the aforementioned objections of existing methods removed. In this method, all the flexural constants are expressed first in terms of the moments (first, second, third, and fourth as any case requires), about one end of the beam, of the I_o/I-diagram which may be called the "relative reciprocal" I-diagram. In this concept I is the moment of inertia of the cross section at any section of a beam, and I_o is some arbitrary value of I used as a standard in the computation, for which the maximum, the minimum, or some other value may be employed according to the user's preference. The use of the $1/I$-diagram suffices to serve the purpose as well, but the use of the I_o/I-diagram considerably facilitates the numerical work of computation. Values of these moments are then expressed in terms of the areas of the various momental diagrams obtained by multiplying the ordinates of the relative reciprocal I-diagram by the various powers of their respective moment arms. Therefore, determination of the flexural constants of haunched beams is reduced to area computations. The advantages of the method presented herein over existing methods are obvious when it is considered that: (1) The relations between the flexural constants and the areas of the various momental diagrams are theoretically as exact as any principle of applied mechanics; (2) the various momental diagrams can be easily obtained, one after another, from the relative reciprocal I-diagram by simple processes of numerical or graphical multiplication with the integers 1, 2, 3, 4, $\cdots$, n as multipliers; and (3) the computation of the area of any diagram, no matter how irregular its character, is simple and may be easily accomplished with satisfactory accuracy by any simple method such as the planimeter or Simpson's rule.

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[2] "The Moment Distribution Method of Structural Analysis Extended to Lateral Loads and Members of Variable Section," by G. E. Large and Clyde Morris, *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio, 1932, p. 14.

[3] "Tapered Structural Members: An Analytical Treatment," by Walter H. Weiskopf and John W Pickworth, *Transactions*, ASCE, Vol. 102, 1937, p. 1.

Notation.—The letter symbols used in this paper are defined where they first appear, in text or in illustrations, and are assembled for convenience of reference in the Appendix.

DEFINITION OF FLEXURAL CONSTANTS

For a beam having asymmetrical haunches and subjected to any loading, there are five and only five flexural constants of independent character. Determination of these constants must precede analysis or design of the beam as a member in a continuous frame. The values of three of such flexural constants, known as "shape constants," depend only on the shape of beam, whereas those of the remaining two, known as "load constants," depend on both the shape of the beam and the type of loading. If the shape of beam is symmetrical with respect to the center line of its span, the number of shape constants is reduced to two; if the loading is also symmetrical, there is only one load constant. Certain methods of analysis for continuous frames, such as the moment distribution method,[4] necessitate use of four shape constants, of which three are of independent character, the fourth being easily derived when any three are determined.

The shape and load constants of haunched beams may be expressed in various systems to suit any particular method of analysis. Elsewhere,[5] the author has already explained those constants in the various prevailing systems with regard to their characteristics and adaptability to certain methods in conjunction with a comprehensive conversion table showing their equivalents.

Among the various systems of expressing the shape and load constants for haunched beams, that of using the end "angle changes" with the beam considered as simply supported, apparently first introduced by German authors and used by Ernst Suter[6] and A. Strassner[7] in developing their methods of analysis, is perhaps the most fundamental and also the most elegant because the end angle changes: (a) Possess a clear and definite physical meaning; (b) represent, directly, the predominant deformations of continuous frames, on the basis of which most of the methods of analysis are developed; (*c*) are computed from the condition of a simply-supported beam, the simplest basic determinate structure, to which continuous structures are usually reduced before proceeding with their analysis; (*d*) always first appear, either explicitly or implicitly, in the computation of the flexural constants in any other system and by any method; and (*e*) give exceedingly simple results in certain methods of analysis, as in the case of the theorem of three moments presented previously by the author.[8] In this paper, therefore, the end angle changes are used as the flexural constants for haunched beams.

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[4] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

[5] *Ibid.*, Vol. 102, 1937, pp. 41–53.

[6] "Die Methode der Festpunkte," by Ernst Suter, Julius Springer, Berlin, 1932.

[7] "Neuere Methoden zur Statik der Rahmenträgwerke und der elastischen Bogenträger," by A. Strassner, Wilhelm Ernst und Sohn, Berlin, Band I, 1925.

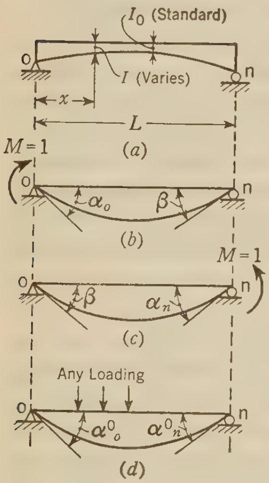
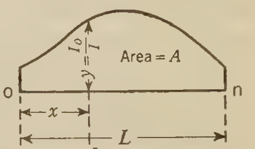
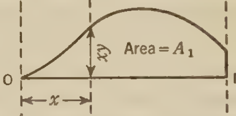


FIG. 1

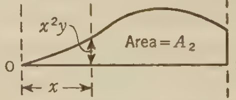
Considering a haunched beam as simply supported at its two ends (Fig. 1(*a*)) and subjected successively to the loadings shown in Figs. 1(*b*), 1(*c*), and 1(*d*), the following end angle changes occur: α_o or α_n represents the angle change at end o or end n due to a unit moment applied, respectively, at the same end; β represents the angle change at either end due to a unit moment applied at the other end; and α^o_o or α^o_n represents the angle change at the ends o or n, respectively, due to any given loading. It is evident that α_o, α_n, and β are shape constants, whereas α^o_o and α^o_n are load constants. Of course, the flexural constants in terms of those angle changes cannot be used directly in all methods of analysis, but, with their values determined, the constants in any other system needed in any method of analysis may be easily derived according to the previously mentioned conversion table.[5]



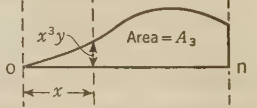
(*a*) $\frac{I_o}{I}$-DIAGRAM



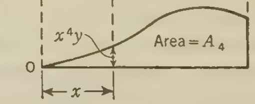
(*b*) 1st MOMENTAL DIAGRAM



(*c*) 2nd MOMENTAL DIAGRAM



(*d*) 3rd MOMENTAL DIAGRAM



(*e*) 4th MOMENTAL DIAGRAM

FIG. 2

MOMENTS AND MOMENTAL AREAS OF I_o/I-DIAGRAMS

Fig. 2(*a*) shows the I_o/I-diagram for a haunched beam of length L, the ordinate at any distance x from end o being designated, for the sake of brevity, by y, so that

$$y = \frac{I_o}{I} \qquad (1)$$

---

[8] *Transactions*, ASCE, Vol. 102, 1937, p. 45.

The area of this relative reciprocal I-diagram is

$$A = \int_o^n y\,dx \tag{2}$$

and its first moment about point ● will be

$$Q = \int_o^n x\,y\,dx \tag{3}$$

Multiplication of all the ordinates of the relative reciprocal I-diagram by their respective values of x gives the first momental diagram (Q-diagram) about point o (Fig. 2(b)). The area A_1 of this diagram is evidently equal to Q. Thus,

$$A_1 = \int_o^n y\,x\,dx \tag{4}$$

Similarly, if all the ordinates of the first momental diagram be multiplied, successively and thrice, by their respective values of x, the second, third, and fourth momental diagrams, all about end o, of the relative reciprocal I-diagram will be obtained, one after another, as shown, respectively, in Figs. 2(c), 2(d), and 2(e). Letting the areas of these momental diagrams be called "momental areas" and designated by A_2, A_3, and A_4, respectively,

$$A_2 = \int_o^n y\,x^2\,dx \tag{5}$$

$$A_3 = \int_o^n y\,x^3\,dx \tag{6}$$

and

$$A_4 = \int_o^n y\,x^4\,dx \tag{7}$$

For beams with symmetrical haunches, the various momental areas about end o are equal, respectively, to those about end n, from which, by the parallel axis theorem, may be obtained

$$L\,A_1 = \tfrac{1}{2}\,L^2\,A \tag{8}$$

and

$$\frac{A_3}{L} = -\tfrac{1}{2}\,L\,A_1 + \tfrac{3}{2}\,A_2 = -\tfrac{1}{4}\,L^2\,A + \tfrac{3}{2}\,A_2 \tag{9}$$

Eqs. 8 and 9 are useful in simplifying the results for symmetrical haunched beams.

If preferred, the various momental diagrams may be obtained from the relative reciprocal I-diagram by simple graphical constructions. For example, in Fig. 3, project the ordinate y at any point m of the relative reciprocal I-diagram to the vertical line through point n. The line joining points a and o intersects y at point b. It is evident that

$$\frac{b\,m}{x} = \frac{y}{L} \qquad \text{(10}a\text{)}$$

or

$$b\,m = \frac{x\,y}{L} \qquad \text{(10}b\text{)}$$

It is obvious that $b\,m$ is the ordinate, divided by the constant L, at point m for the desired first momental diagram about point o. Similarly, projection of $b\,m$ to the vertical line through n gives point f, and the line joining this point with o intersects the ordinate at c. Then,

$$c\,m = \frac{x}{L}\,b\,m = \frac{x^2\,y}{L^2} \qquad \text{(11)}$$

which is the ordinate, divided by the constant L^2, at point m, for the desired second momental diagram about point o. Proceeding in the same manner, the ordinates $d\,m$ and $e\,m$, divided, respectively, by the constants L^3 and L^4, at the same point for the desired third and fourth momental diagrams, may be obtained as follows:

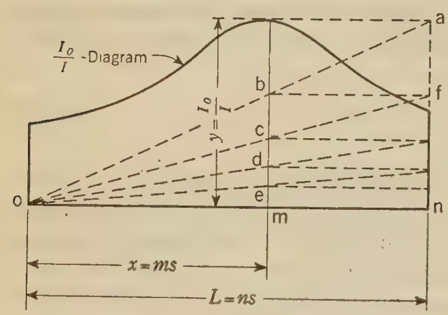


FIG. 3

$$d\,m = \frac{x}{L}\,c\,m = \frac{x^3\,y}{L^3} \qquad \text{(12)}$$

and

$$e\,m = \frac{x}{L}\,d\,m = \frac{x^4\,y}{L^4} \qquad \text{(13)}$$

Hereinafter it will be shown that the three shape constants of a haunched beam, α_o, α_n, and β, may be expressed entirely in terms of the momental diagram areas A, A_1, and A_2, and the two load constants, α°_o and α°_n, also may be expressed entirely in terms of the areas. The factors needed for each of the common types of loading are as follows: A, A_1, and A_2 for concentrated loads; A, A_1, A_2, and A_3 for uniform load distributed in any manner along the length of beam; and A_1, A_2, A_3, and A_4 for uniformly varying load. For symmetrical beams or loadings, the items of the areas needed for both shape and load constants may be somewhat modified.

SHAPE CONSTANTS

Referring to Figs. 1(b) and 1(c) and applying the method of moment area or of virtual work, the three shape constants, α_o, α_n, and β, may be determined as follows:

$$\alpha_n = \frac{1}{E\,I_o\,L^2}\int_o^n y\,x^2\,dx \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$\alpha_o = \frac{1}{E\,I_o\,L^2}\int_o^n y\,(L - x)^2\,dx$$

$$= \frac{1}{E\,I_o\,L^2}\left(L^2\int_o^n y\,dx - 2\,L\int_o^n y\,x\,dx + \int_o^n y\,x^2\,dx\right) \ldots (15)$$

$$\beta = \frac{1}{E\,I_o\,L^2}\int_o^n y\,x\,(L - x)\,dx = \frac{1}{E\,I_o\,L^2}\left(L\int_o^n y\,x\,dx - \int_o^n y\,x^2\,dx\right) \ldots (16)$$

in which E is the modulus of elasticity of the beam material. Eqs. 14, 15, and 16 may be rewritten in terms of the various momental areas as follows:

$$E\,I_o\,\alpha_n = \frac{1}{L^2}\,A_2 \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

$$E\,I_o\,\alpha_o = A - \frac{2}{L}\,A_1 + \frac{1}{L^2}\,A_2 \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

and

$$E\,I_o\,\beta = \frac{1}{L}\,A_1 - \frac{1}{L^2}\,A_2 \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

For beams with symmetrical haunches, it is obvious, from the relations in Eqs. 8 and 9, that

$$E\,I_o\,\alpha_n = E\,I_o\,\alpha_o = E\,I_o\,\alpha = \frac{1}{L^2}\,A_2 \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and

$$E\,I_o\,\beta = \tfrac{1}{2}\,A - \frac{1}{L^2}\,A_2 \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

For beams with constant I ($y = 1$), it is evident from Eqs. 14 and 16 that

$$E\,I\,\alpha = \frac{L}{3} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

and

$$E\,I\,\beta = \frac{L}{6} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

LOAD CONSTANTS

The load constants, $\alpha^o{}_o$ and $\alpha^o{}_n$, for a haunched beam subjected to any of the common types of loading may be expressed entirely in terms of the various momental areas in the same manner as are their shape constants. They will be determined by the two general equations—

$$\alpha^o{}_o + \alpha^o{}_n = \int_o^n \frac{M}{E\,I}\,dx \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

and

$$\alpha^o{}_n = \frac{1}{L}\int_o^n \frac{M\,x}{E\,I}\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

—based on the principle of moment area or virtual work.

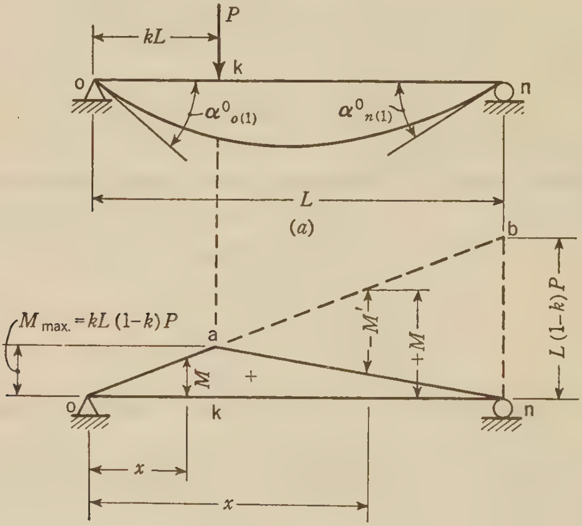
For the sake of brevity, Eqs. 24 and 25 may be rewritten

$$E\,I_o\,(\alpha^o{}_o + \alpha^o{}_n) = \int_o^n M\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

and

$$E\,I_o\,\alpha^o{}_n = \frac{1}{L}\int_o^n M\,y\,x\,dx \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The difference between Eqs. 26 and 27 obviously will give the value of $\alpha^o{}_o$. Formulas for load constants will be deduced for six common cases of basic loading. Those for several other cases of loading can be easily derived therefrom by superposition or subtraction.



(a)

(b)

Fig. 4

Case 1. Concentrated Load at Any Point.—Fig. 4(*a*) shows a simply-supported haunched beam subjected to a concentrated load P at any point k at a distance $k\,L$ from end o. As shown in Fig. 4(*b*), the bending moment diagram is the triangle oan, which may be considered as the sum of the positive triangle obn and the negative triangle abn. It may be easily shown that the positive bending moment M, at any section at a distance x from the left end, for the triangle obn covering the length on, will be

$$M = x\,(1 - k)\,P \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The negative bending moment M' for the triangle abn covering the length kn will be

$$-M' = -(x - k\,L)\,P \tag{29}$$

Substituting the values of M and $-M'$ from Eqs. 28 and 29 in Eqs. 26 and 27, and designating, for the sake of clearness, the two load constants in this case of loading $\alpha^o{}_{o(1)}$ and $\alpha^o{}_{n(1)}$,

$$E\,I_o\,(\alpha^o{}_{o(1)} + \alpha^o{}_{n(1)} = \left[(1-k)\int_o^n y\,x\,dx - \int_k^n y\,x\,dx + k\,L\int_k^n y\,dx\right]P = [k\,L\,A_{(kn)} - A_{1(kn)} + (1-k)\,A_1]\,P \tag{30}$$

and

$$E\,I_o\,\alpha^o{}_{n(1)} = \frac{1}{L}\left[\int_o^n y\,x^2\,dx - k\int_o^n y\,x^2\,dx - \int_k^n y\,x^2\,dx + k\,L\int_k^n y\,x\,dx\right]P \tag{31}$$

Therefore, noting that

$$A_2 - A_2\,(kn) = A_{2(ok)} \tag{32}$$

and, simplifying,

$$E\,I_o\,\alpha^o{}_{n(1)} = \left[k\,A_{1(kn)} + \frac{1}{L}\,A_{2(ok)} - \frac{k}{L}\,A_2\right]P \tag{33a}$$

and

$$E\,I_o\,\alpha^o{}_{o(1)} = \left[k\,L\,A_{kn} + (1+k)\,A_{1(ok)} - 2\,k\,A_1 - \frac{1}{L}\,A_{2(ok)} + \frac{k}{L}\,A_2\right]P \tag{33b}$$

In Eqs. 33, the various areas with the subscript (ok) indicate the areas to be computed from their respective diagrams between the ordinates at points o and k only. A similar situation exists for areas with the subscript (kn). Eqs. 33 may be used conveniently for computing the influence diagrams for the load constants, since any change in the position of the single concentrated load affects only the value of k and the position of point k in those equations.

For a symmetrical haunched beam with a concentrated load at its center,

$$\alpha^o{}_{o(1)} = \alpha^o{}_{n(1)} = \alpha^o{}_{(1)} \tag{34}$$

Noting that $k = 0.5$; $A_{(kn)} = 0.5\,A$; $A_{1(ok)} = A_1 - A_{1(kn)}$ and the relation in Eq. 8, it is obvious from Eq. 30 that

$$E\,I_o\,\alpha^o{}_{(1)} = \tfrac{1}{2}\,A_{1(ok)}\,P \tag{35}$$

in which $A_{1(ok)}$ is the area of the first momental diagram between points o and k.

Case 2. Uniform Load Covering Whole Length of Beam.—The two load constants for this case of loading will be designated $\alpha^o{}_{n(2)}$ and $\alpha^o{}_{o(2)}$. The bending moment at any section at a distance m from end o is

$$M = \tfrac{1}{2}(L\,x - x^2)\,w \quad \text{.....................(36)}$$

Substituting Eq. 36 in Eqs. 26 and 27,

$$E\,I_o\,[\alpha^o_{o(2)} + \alpha^o_{n(2)}] = \frac{1}{2}\left(L\int_o^n y\,x\,dx - \int_o^n y\,x^2\,dx\right)w = \tfrac{1}{2}(L\,A_1 - A_2)w \quad \text{..(37)}$$

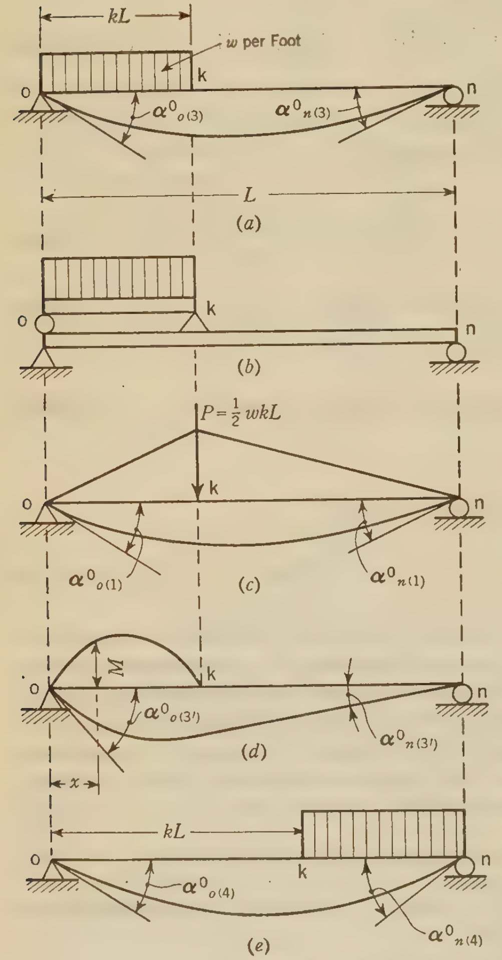

kL
w per Foot
k
o
n
$\alpha^0_{o(3)}$
$\alpha^0_{n(3)}$
L
(a)
o
k
n
(b)
$P=\frac{1}{2}wkL$
k
o
n
$\alpha^0_{o(1)}$
(c)
$\alpha^0_{n(1)}$
M
o
k
n
$\alpha^0_{o(3')}$
(d)
$\alpha^0_{n(3')}$
x
kL
o
k
n
$\alpha^0_{o(4)}$
$\alpha^0_{n(4)}$
(e)

FIG. 5

and

$$E\,I_o\,\alpha^o_{n(2)} = \frac{1}{2}\left(\int_o^n y\,x^2\,dx - \frac{1}{L}\int_o^n y\,x^3\,dx\right)w \quad \text{..(38)}$$

Therefore,

$$E\,I_o\,\alpha^o_{n(2)} = \frac{1}{2}\left(A_2 - \frac{1}{L}A_3\right)w \quad \text{..(39)}$$

and

$$E\,I_o\,\alpha^o_{o(2)} = \left[\frac{1}{2}\left(L\,A_1 + \frac{1}{L}A_3\right) - A_2\right]w \quad \text{..(40)}$$

For a symmetrical haunched beam,

$$\alpha^o_{o(2)} = \alpha^o_{n(2)} = \alpha^o_{(2)} \quad \text{..(41)}$$

From Eqs. 37 and 38,

$$E\,I_o\,\alpha^o_{(2)} = \tfrac{1}{4}(\tfrac{1}{2}L^2 A - A_2)w \quad \text{..(42)}$$

Comparing Eq. 42 with Eq. 21,

$$\alpha^o_{(2)} = \tfrac{1}{4}\,\beta\,w\,L^2 \quad \text{....(43)}$$

Case 3. Uniform Load Covering Partial Length at Left End.—The loading is shown in Fig. 5(a). If the load is considered to be applied to the beam on through the stringer ok as shown in Fig. 5(b), the bending moment diagram is then resolved into two component parts: A triangle with length on as base, due to a concentrated load of $\frac{1}{2}\,w\,k\,L$ at point k as shown in Fig. 5(c); and a parabola with length ok as base, due to a uniform load on the stringer ok as shown in Fig. 5(*d*). Evidently, the sum of the load constants for the two component parts at either end will be equal to that for the given loading at the same end. The load constants for Fig. 5(*c*) will be the same as those for case 1 when $P = \frac{1}{2} w k L$. Those for Fig. 5(*d*), designated by $\alpha^o_{n(3')}$ and $\alpha^o_{o(3')}$, may be computed as before by Eqs. 26 and 27:

$$M = (\tfrac{1}{2} k L x - \tfrac{1}{2} x^2) w \ldots\ldots (44)$$

$$E I_o [\alpha^o_{n(3')} + \alpha^o_{o(3')}] = \left(\tfrac{1}{2} k L \int_o^k y x \, dx - \frac{1}{2} \int_o^k y x^2 \, dx \right) w$$
$$= \frac{w}{2} [k L A_{1(ok)} - A_{2(ok)}] \ldots (45)$$

$$E I_o \alpha^o_{n(3')} = \left(\frac{k}{2} \int_o^k y x^2 \, dx - \frac{1}{2L} \int_o^k y x^3 \, dx \right) w$$
$$= \frac{1}{2} \left[k A_{2(ok)} - \frac{1}{L} A_{3(ok)} \right] w \ldots (46)$$

$$E I_o \alpha^o_{o(3')} = \frac{1}{2} \left[k L A_{1(ok)} - (1 + k) A_{2(ok)} + \frac{1}{L} A_{3(ok)} \right] w \ldots (47)$$

Adding Eqs. 46 and 47 to the results obtained (with $P = \frac{1}{2} w k L$) from Eqs. 32 and 33, respectively, and simplifying,

$$E I_o \alpha^o_{n(3)} = \frac{1}{2} \left[k^2 L A_{1(kn)} + 2 k A_{2(ok)} - k^2 A_2 - \frac{1}{L} A_{3(ok)} \right] w \ldots (48)$$

and

$$E I_o \alpha^o_{o(3)} = \frac{1}{2} \Big[k^2 L^2 A_{(kn)} + (2 + k) k L A_{1(ok)} - 2 k^2 L A_1$$
$$- (1 + 2k) A_{2(ok)} + k^2 A_2 + \frac{1}{L} A_{3(ok)} \Big] w \ldots (49)$$

When $k = 1$, $A_{(kn)} = A_{1(kn)} = 0$, and all subscripts (ok) are deleted, then Eqs. 48 and 49 become, respectively, identical with Eqs. 39 and 40, as they should be.

Case 4. Uniform Load Covering Partial Length at Right End.—The loading is shown in Fig. 5(*e*), in which the left end of the loading is at a distance $k L$ from the left end of the beam. The two load constants, $\alpha^o_{n(4)}$ and $\alpha^o_{o(4)}$, may be obtained by taking the differences between those in cases 2 and 3.

$$\alpha^o_{(4)} = \alpha^o_{(2)} - \alpha^o_{(3)} \ldots\ldots (50)$$

Substituting the proper load constants from cases 2 and 3 in Eq. 50 and simplifying,

$$E I_o \alpha^o_{n(4)} = \frac{1}{2} \left[- k^2 L A_{1(kn)} + (1 + k^2) A_2 - 2 k A_{2(ok)} - \frac{I}{L} A_{3(kn)} \right] w \ldots (51)$$

$$E I_o \alpha^o_{o(4)} = \frac{1}{2} \Big[- k^2 L^2 A_{(kn)} - (2 + k) k L A_{1(ok)}$$
$$+ (1 + 2 k^2) L A_1 + (1 + 2k) A_{2(ok)} - (2 + k^2) A_2 + \frac{I}{L} A_{3(kn)} \Big] w \ldots (52)$$

When $k = 0$, Eqs. 51 and 52 will be reduced, respectively, to Eqs. 39 and 40, as they should be.

Case 5. Triangular Load with Maximum Intensity at Right End.—The bending moment at any section a distance x from the left end is

$$M = \frac{1}{6}\left(L\,x - \frac{1}{L}\,x^3 \right) w \ldots\ldots (53)$$

Letting the load constants for this loading be designated by $\alpha^o{}_{n(5)}$ and $\alpha^o{}_{o(5)}$, and proceeding as before by Eqs. 26 and 27,

$$E\,I_o\,[\alpha^o{}_{o(5)} + \alpha^o{}_{n(5)}] = \frac{1}{6}\left(L\,A_1 - \frac{1}{L}\,A_3\right) w \ldots\ldots (54a)$$

$$E\,I_o\,\alpha^o{}_{n(5)} = \frac{1}{6}\left(A_2 - \frac{1}{L^2}\,A_4\right) w \ldots\ldots (54b)$$

and

$$E\,I_o\,\alpha^o{}_{o(5)} = \frac{1}{6}\left(L\,A_1 - A_2 - \frac{1}{L}\,A_3 + \frac{1}{L^2}\,A_4\right) w \ldots\ldots (55)$$

For a symmetrical haunched beam, Eq. 55 may be simplified by the relation of Eq. 9 so that

$$E\,I_o\,\alpha^o{}_{o(5')} = \frac{1}{6}\left(\frac{3}{4}\,L^2\,A - \frac{5}{2}\,A_2 + \frac{1}{L^2}\,A_4\right) w \ldots\ldots (56)$$

Eq. 54*b* admits no simplification, hence

$$\alpha^o{}_{n(5')} = \alpha^o{}_{n(5)} \ldots\ldots (57)$$

Case 6. Triangular Load with Maximum Intensity at the Left End.—The bending moment at any section a distance x from the left end is

$$M = \frac{1}{6}\left(2\,L\,x - 3\,x^2 + \frac{1}{L}\,x^3\right) w \ldots\ldots (58)$$

Letting the load constants for this loading be designated by $\alpha^o{}_{n(6)}$ and $\alpha^o{}_{o(6)}$ and proceeding as before by Eqs. 26 and 27,

$$E\,I_o\,[\alpha^o{}_{o(6)} + \alpha^o{}_{n(6)}] = \frac{1}{6}\left(2\,L\,A_1 - 3\,A_2 + \frac{1}{L}\,A_3\right) w \ldots\ldots (59)$$

$$E\,I_o\,\alpha^o{}_{n(6)} = \frac{1}{6}\left(2\,A_2 - \frac{3}{L}\,A_3 + \frac{1}{L^2}\,A_4\right) w \ldots\ldots (60)$$

and

$$E\,I_o\,\alpha^o{}_{o(6)} = \frac{1}{6}\left(2\,L\,A_1 - 5\,A_2 + \frac{4}{L}\,A_3 - \frac{1}{L^2}\,A_4\right) w \ldots\ldots (61)$$

For a symmetrical haunched beam, Eqs. 60 and 61 may be simplified by the relations of Eqs. 8 and 9 so that

$$E\,I_o\,\alpha^o{}_{n(6')} = \frac{1}{6}\left(\frac{3}{4}\,L^2\,A - \frac{5}{2}\,A_2 + \frac{1}{L^2}\,A_4\right) w \ldots\ldots (62)$$

and

$$E\,I_o\,\alpha^o_{o(6')} = \frac{1}{6}\left(A_2 - \frac{1}{L^2}A_4\right)w \qquad (63)$$

It may be noted that the following relations are always valid and may serve as checks:

$$\alpha^o_{n(5)} + \alpha^o_{n(6)} = \alpha^o_{n(2)} \qquad (64)$$

$$\alpha^o_{o(5)} + \alpha^o_{o(6)} = \alpha^o_{o(2)} \qquad (65)$$

$$\alpha^o_{n(5')} = \alpha^o_{o(6')} \qquad (66)$$

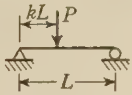
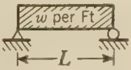
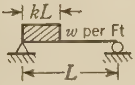
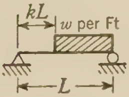
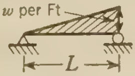
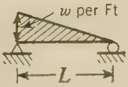
$$\alpha^o_{o(5')} = \alpha^o_{n(6')} \qquad (67)$$

and

$$\alpha^o_{n(5')} + \alpha^o_{o(5')} = \alpha^o_{n(6')} + \alpha^o_{o(6')} = \alpha^o_{(2)} \qquad (68)$$

The foregoing six cases cover the common types of loading usually encountered in practice. For convenience of reference and application, the

TABLE 1.—SUMMARY OF FORMULAS FOR LOAD CONSTANTS

Case	Loading	Beam	LOAD CONSTANTS	
			$E\,I_o\,(\alpha^o_o + \alpha^o_n)$	$E\,I_o\,\alpha^o_n$
(1)	(2)	(3)	(4)	(5)
1		Asymmetrical	$[k\,L\,A_{(kn)} - A_{1(kn)} + (1-k)\,A_1]\,P$	$\left[k\,A_{1(kn)} + \frac{1}{L}A_{2(ok)} - \frac{k}{L}A_2\right]P$
		Symmetrical	Also $k = \frac{1}{2}$: $P\,A_{1(ok)}$	$\frac{1}{2}\,P\,A_{1(ok)}$
2		Asymmetrical	$\frac{1}{2}\,[L\,A_1 - A_2]\,w$	$\frac{1}{2}\left[A_2 - \frac{1}{L}A_3\right]w$
		Symmetrical	$\frac{1}{2}\,[\frac{1}{2}L^2A - A_2]\,w$ or $\frac{1}{2}\,E\,I_o\,\beta\,w\,L^2$	$\frac{1}{4}\,[\frac{1}{2}L^2A - A_2]\,w$ or $\frac{1}{4}\,E\,I_o\,\beta\,w\,L^2$
3		Asymmetrical	$\frac{1}{2}\,[k^2L^2A_{(kn)} + 2\,k\,L\,A_{1(ok)} - k^2\,L\,A_1 - A_{2(ok)}]\,w$	$\frac{1}{2}\left[k^2\,L\,A_{1(kn)} + 2\,k\,A_{2(ok)} - k^2A_2 - \frac{1}{L}A_{3(ok)}\right]w$
4		Asymmetrical	$\frac{1}{2}\,[-k^2L^2A_{(kn)} - 2\,k\,L\,A_{1(ok)} + (1+k^2)\,L\,A_1 - A_{2(kn)}]\,w$	$\frac{1}{2}\left[-k^2\,L\,A_{1(kn)} - 2\,k\,A_{2(ok)} + (1+k^2)\,A_2 - \frac{1}{L}A_{3(kn)}\right]w$
5		Asymmetrical	$\frac{1}{6}\left[L\,A_1 - \frac{1}{L}A_3\right]w$	$\frac{1}{6}\left[A_2 - \frac{1}{L^2}A_4\right]w$
		Symmetrical	$\frac{1}{4}\,[\frac{1}{2}L^2A - A_2]\,w$	
6		Asymmetrical	$\frac{1}{6}\left[2\,L\,A_1 - 3\,A_2 + \frac{1}{L}A_3\right]w$	$\frac{1}{6}\left[A_2 - \frac{3}{L}A_3 + \frac{1}{L^2}A_4\right]w$
		Symmetrical	$\frac{1}{4}\,[\frac{1}{2}L^2A - A_2]\,w$	$\frac{1}{6}\left[\frac{3}{4}L^2A - \frac{5}{2}A_2 + \frac{1}{L^2}A_4\right]w$

formulas for the load constants for these six cases of loading are tabulated in Table 1, in which Col. 4 gives the formulas for the sum $E\,I_o\,(\alpha^o{}_o + \alpha^o{}_n)$ instead of that for $E\,I_o\,\alpha^o{}_o$ alone, because the former are much shorter and therefore more convenient to apply than the latter. From Table 1 formulas for load constants for several other types of loading not given may be derived. Thus, for a uniform load covering partial length near the middle of a beam (Fig. 6(*a*)), the load constants may be considered as equal to the difference between those for the loadings shown in Figs. 6(*b*) and 6(*c*), and, therefore, may be computed by the formulas given for case 3. For the loading shown in Figs. 6(*d*) and 6(*e*), load constants may be obtained by adding those for case 2 to those for cases 5 and 6, respectively.

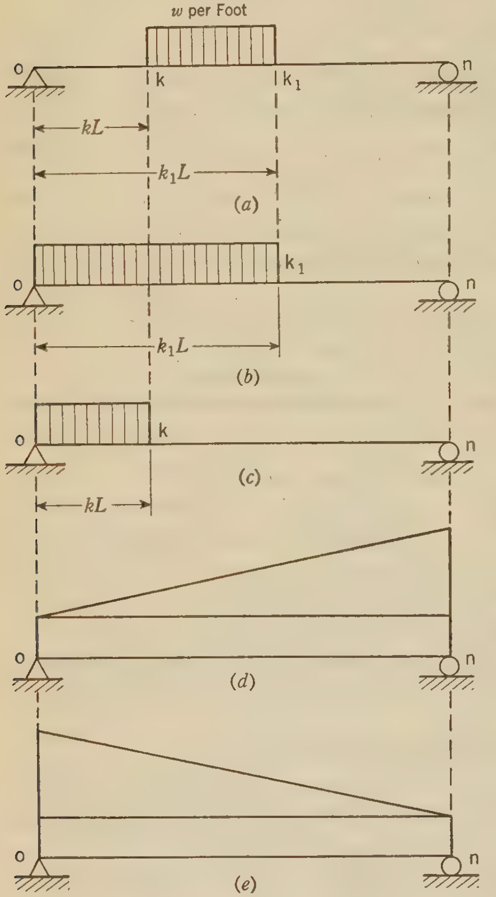


Fig. 6

For nonuniform load distributed in any irregular manner, the M/I-diagram or $M\,I_o/I$-diagram must be used. For computing the ordinates of its bending moment diagram, the simplified numerical procedure presented by N. M. Newmark,[9,10,11] M. ASCE, is recommended. As soon as the $M\,I_o/I$-diagram has been obtained, the load constants may be computed by the following equations, which are evident from Eqs. 26 and 27:

$$E\,I_o\,(\alpha^o{}_o + \alpha^o{}_n) = A_{(m)} \quad \text{..(69)}$$

$$E\,I_o\,\alpha^o{}_n = \frac{1}{L}\,A_{1(m)} \quad \text{.......................(70)}$$

and

$$E\,I_o\,\alpha^o{}_o = A_{(m)} - \frac{1}{L}\,A_{1(m)} \quad \text{...................(71)}$$

in which $A_{(m)}$ is the area of the $M\,I_o/I$-diagram, and $A_{1(m)}$ is the area of its first momental diagram about the left end.

---

[9] "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads," by N. M. Newmark, *Transactions*, ASCE, Vol. 108, 1943, p. 1161.

[10] *Ibid.*, p. 1169, Fig. 7.

[11] *Ibid.*, p. 1171, Fig. 10.

AREA COMPUTATION

The total or partial areas of the various momental diagrams may be easily computed with satisfactory accuracy by Simpson's rule:

$$A = \tfrac{1}{3}s\left[(y_o + y_n) + 4(y_1 + y_3 + \cdots + y_{n-i}) \right.$$
$$\left. + 2(y_2 + y_4 + \cdots + y_{n-2})\right] \quad \text{..(72)}$$

In applying Simpson's rule, the beam length is divided into n number of equal segments of length s. The ordinates of the various momental diagrams at the points of division o, 1, 2, $\cdots$, n are, respectively, $y_o, y_1, y_2, \cdots, y_n$. Simpson's rule is obtained by assuming that the curve connecting three consecutive ordinates is a quadratic parabola, and therefore is applicable only when n is an even number. When n is an odd number, the area between y_o and y_1, designated by A_{o-1}, may be computed separately and the remaining area between y_1 and y_n, designated by A_{1-n}, may be computed by a modification of Simpson's rule—

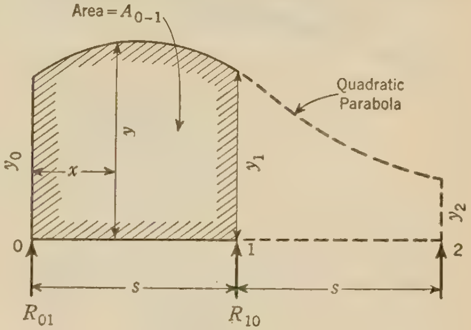


FIG. 7

$$A_{1-n} = \tfrac{1}{3}s\left[(y_1 + y_n) + 4(y_2 + y_4 + \cdots + y_{n-1}) \right.$$
$$\left. + 2(y_3 + y_5 + \cdots + y_{n-2})\right] \quad \text{..(73)}$$

Fig. 7 shows a quadratic parabola passing the three ordinates y_o, y_1, and y_2. The ordinate at any point will be

$$y = \frac{1}{2}\left(\frac{x}{s} - 1\right)\left(\frac{x}{s} - 2\right)y_0 - \frac{x}{s}\left(\frac{x}{s} - 2\right)y_1 + \frac{x}{2s}\left(\frac{x}{s} - 1\right)y_2 \quad \text{..(74)}$$

Therefore,

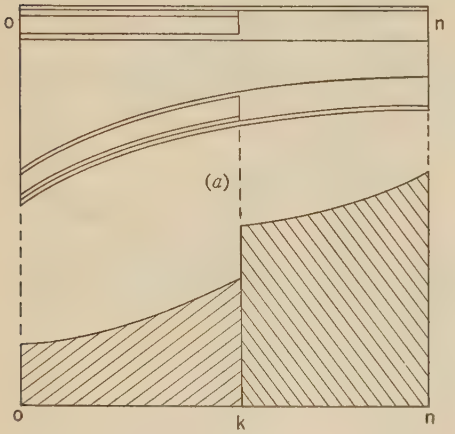
$$A_{o-1} = \int_o^s y\,dx = \frac{s}{3}\left(\frac{5}{4}y_o + 2y_1 - \tfrac{1}{4}y_2\right) \quad \text{..............(75)}$$

Combining Eqs. 73 and 75 and simplifying, Simpson's rule may be obtained for an odd value of n,

$$A = \frac{s}{3}\left[\frac{5}{4}y_o + 3y_1 + \frac{15}{4}y_2 + y_n + 4(y_4 + y_6 + \cdots + y_{n-1}) \right.$$
$$\left. + 2(y_3 + y_5 + \cdots + y_{n-2})\right] \quad \text{..(76)}$$

Eq. 76 is not so accurate as Eq. 72, and therefore should not be used unless unavoidable.

Application of Eqs. 72 to 76 is limited to the case where the relative reciprocal I-diagram is a smooth curve without any change throughout the length of the beam. For the type of haunched beam shown in Fig. 8(*a*), in which the relative reciprocal I-diagram has an abrupt change at point k (Fig. 8(*b*)), the diagram should be divided into two sections by the ordinate at point k. The areas of these sections should be computed separately and then added.



(*b*) $\frac{I_o}{I}$ – DIAGRAM

FIG. 8

Let y_m be the ordinate of the relative reciprocal I-diagram at any point m which is at a distance $x = m\,s$ from end o. The ordinates of the various momental diagrams A_1, A_2, A_3, and A_4 are, respectively,

$$y'_m = s\,(m\,y_m) \ldots . (77)$$

$$y''_m = s\,(m\,y'_m) = s^2\,(m^2\,y_m) \ldots (78)$$

$$y'''_m = s\,(m\,y''_m) = s^3\,(m^3\,y_m) \ldots (79)$$

and

$$y''''_m = s\,(m\,y'''_m) = s^4\,(m^4\,y_m) \ldots (80)$$

Thus the various momental diagrams may be obtained, one after another, from the relative reciprocal I-diagram by multiplying the ordinates of the preceding diagram successively by the integers 0, 1, 2, $\cdots$, n. Of course, the areas computed from these diagrams must be multiplied, respectively, by s, s^2, s^3, and s^4 to obtain their correct values. If the values of y'_m, y''_m, $\cdots$, y^n_m in terms of y_m from Eqs. 77 to 80 are substituted in Eq. 72 or Eq. 76, the areas A_2, A_3, $\cdots$, A_n are obtained directly without constructing their diagrams.

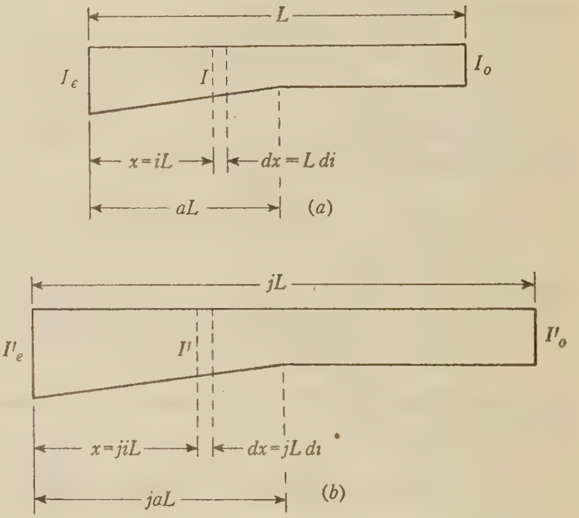


FIG. 9

THEOREMS OF MOMENTAL AREAS

Another distinct advantage of the method is that the computations may be further simplified by application of the theorems of momental areas, which will reduce to a minimum the work of plotting the various momental diagrams and of computing their areas.

Theorem of Similitude.—When two haunches or haunched beams of the same type (straight, parabolic, or sharply curved) are proportional in length and have the same ratios between their moments of inertia at all the corresponding sections ($I'/I = I'_o/I_o = I'_e/I_e$) as shown in Fig. 9, the areas of their I_o/I-diagrams and their various momental areas about the same end have definite relations of proportionality. Let A, A_1, and A_2 represent elements for the beam shown in Fig. 9(a), and let A_j, A_{1j}, and A_{2j} represent elements for the beam shown in Fig. 9(b). Also, if $x = i\,L$, and $dx = L\,di$, by Eq. 2,

$$A = \int_0^L \frac{I_o}{I}\,dx = L\int_0^1 y\,di \qquad (81)$$

and

$$A_j = \int_0^{jL} \frac{I'_o}{I'}\,dx = j\,L\int_0^1 y\,di \qquad (82)$$

Therefore,

$$\frac{A_j}{A} = j \qquad (83)$$

Similarly,

$$\frac{A_{1j}}{A_1} = j^2 \qquad (84)$$

$$\frac{A_{2j}}{A_2} = j^3 \qquad (85)$$

$$\frac{A_{3j}}{A_3} = j^4 \qquad (86)$$

and

$$\frac{A_{4j}}{A_4} = j^5 \qquad (87)$$

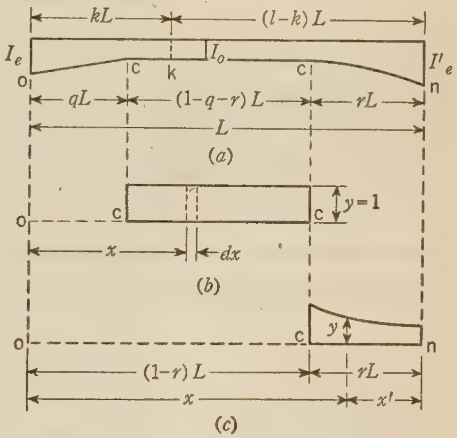


FIG. 10

Theorem of Constant Section.—When a haunched beam contains a section with constant moment of inertia (section cc, Fig. 10(a)), the area of the I_o/I-diagram of that section and its various momental areas about the left end, designated by A_c, A_{1c}, A_{2c}, $\cdots$, A_{nc}, may be computed separately without plotting its various momental diagrams. Referring to Fig. 10(b),

$$A_c = (1 - q - r)\,L \qquad (88)$$

$$A_{1c} = \int_{qL}^{(1-r)L} x\,dx = \tfrac{1}{2}\,L^2\,(1 - q - r)\,(1 + q - r)$$
$$= \tfrac{1}{2}\,L\,A_c\,(1 + q - r) \qquad (89)$$

$$A_{2c} = \int_{qL}^{(1-r)L} x^2\,dx = \tfrac{1}{3}\,L^3\,(1 - q - r)\,[(1 - r)^2 + q\,(1 + q - r)]$$
$$= \tfrac{2}{3}\,q\,L\,A_{1c} + \tfrac{1}{3}\,(1 - r)^2\,L^2\,A_c \qquad (90)$$

$$A_{3c} = \int_{qL}^{(1-r)L} x^3\,dx = \tfrac{1}{4} L^4 (1-q-r)(1+q-r)\left[(1-r)^2 + q^2\right]$$
$$= \tfrac{1}{2} L^2 A_{1c} \left[(1-r)^2 + q^2\right] \quad (91)$$

$$A_{4c} = \int_{qL}^{(1-r)L} x^4\,dx = \tfrac{1}{5} L^5 (1-r)(1-q-r)(1+q-r)$$
$$\times \left[(1-r)^2 + q^2\right] + \tfrac{1}{5} L^5 q^4 (1-q-r)$$
$$= \tfrac{4}{5} (1-r) L A_{3c} + \tfrac{1}{5} q^4 L^4 A_c \quad (92)$$

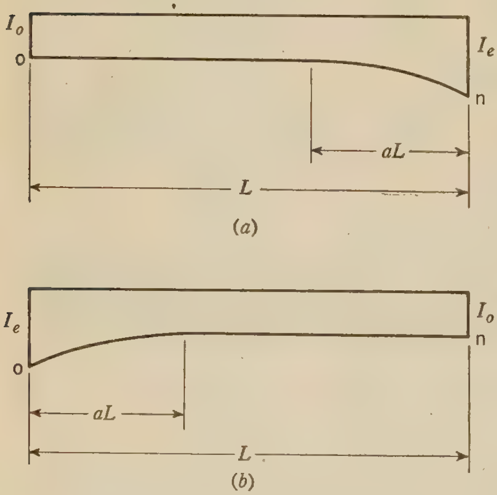
Theorem of Parallel Axes.—When the area of the I_o/I-diagram and its various momental areas of the haunch (or haunched beam) cn (Fig. 10(*c*)) about its right end, designated by A'_r, A'_{1r}, A'_{2r}, $\cdots$, A'_{4r}, are known, those of the same haunch (or haunched beam) about the left end, designated by A_r, A_{1r}, A_{2r}, $\cdots$, A_{4r}, may be computed by the following:

$$A_r = A'_r \quad (93)$$

$$A_{1r} = \int_{(1-r)L}^{L} x\,y\,dx = \int_0^{rL} (L - x')\,y\,dx$$
$$= L \int_0^{rL} y\,dx - \int_0^{rL} x'\,y\,dx = L\,A'_r - A'_{1r} \quad (94)$$

and

$$A_{2r} = \int_{(1-r)L}^{L} x^2\,y\,dx = \int_0^{rL} (L - x')^2\,y\,dx$$
$$= L^2 \int_0^{rL} y\,dx - 2\,L \int_0^{rL} x'\,y\,dx + \int_0^{rL} (x')^2\,y\,dx$$
$$= L^2 A'_r - 2\,L\,A'_{1r} + A'_{2r} \quad (95)$$



(*a*)

(*b*)

FIG. 11

Similarly,

$$A_{3r} = L^3 A'_r - 3\,L^2 A'_{1r} + 3\,L\,A'_{2r} - A'_{3r} \quad (96)$$

and

$$A_{4r} = L^4 A'_r - 4\,L^3 A'_{1r} + 6\,L^2 A'_{2r} - 4\,L\,A'_{3r} + A'_{4r} \quad (97)$$

It may be noted that Eqs. 93 to 97 are independent of the value of r and are equally applicable when $r = 1$. Therefore, when the area of the I_o/I-diagram and its various momental areas for the asymmetrical haunched beam (Fig. 11(*a*)) about either end are known and are taken as A'_r, A'_{1r}, A'_{2r}, $\cdots$, A'_{4r}, those about the same end for the same beam placed in the reversed position (Fig. 11(*b*)) will obviously be A_r, A_{1r}, A_{2r}, $\cdots$, A_{4r} which may be computed by Eqs. 93 to 97.

When the right haunch cn of the beam in Fig. 10(*a*) is of the same type (straight, parabolic, or sharply curved) as the left haunch oc, and also $I'_e = I_e$, the values of areas for the right haunch about its right end may be obtained from those for the left haunch about its left end by Eqs. 83 to 87.

Theorem of Combination.—When a haunched beam is composed of several different sections (such as the beam in Fig. 10(*a*)), the area of the I_o/I-diagram and its various momental areas about the left end will be equal to the sum of those of its component sections oc, cc, and cn about the same end. Thus, the value of the momental diagram areas for the whole beam will be

$$A = A_q + A_c + A_r \tag{98}$$

$$A_1 = A_{1q} + A_{1c} + A_{1r} \tag{99}$$

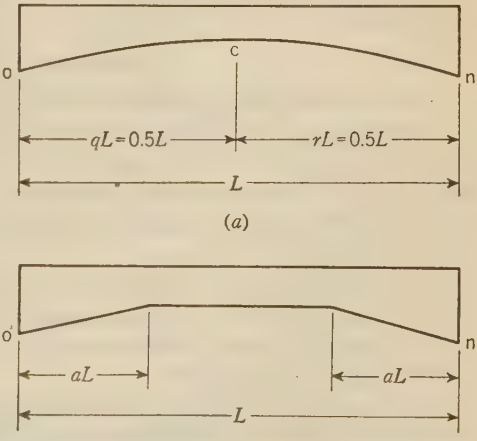
$$A_2 = A_{2q} + A_{2c} + A_{2r} \tag{100}$$

$$A_3 = A_{3q} + A_{3c} + A_{3r} \tag{101}$$

and

$$A_4 = A_{4q} + A_{4c} + A_{4r} \tag{102}$$

The various partial areas with subscript (*ok*) covering the partial length $k\,L$ from the left end of the beam, which are needed for determining the load constants in cases 1, 3, and 4, may be computed as usual when $k < q$. When $q < k < (1 - r)$ (see Fig. 10(*a*)), they will be equal to the sum of those for the sections oc and ck, the latter being obtained from Eqs. 88 to 92 by substituting $1 - k$ for r. When $(1 - r) < k < 1$, the various partial areas with subscript (*kn*) will be first computed as usual and then subtracted from the various total areas for the entire length of beam to obtain those partial areas with subscript (*ok*).



(*a*)

(*b*)

FIG. 12

Theorem of Symmetry.—For symmetrical haunched beams such as those shown in Fig. 12, the areas of the I_o/I-diagram and its various momental areas about the left end may be obtained from those of the left half also about the same end. Since the beams are symmetrical, the area of the I_o/I-diagram and its various momental areas of the right half about the right end are evidently equal to those of the left half about the left end. Thus, when Eqs. 93 to 97 are substituted in Eqs. 98 to 102 with $A_c = A_{1c} = \cdots = 0$, and, with subscripts r transformed to subscripts q, equations are obtained for computing the values of the various momental areas of symmetrical haunched beams—

$$A = 2\,A_q \quad \text{(103)}$$

$$A_1 = L\,A_q \quad \text{(104)}$$

$$A_2 = L^2 A_q - 2\,L\,A_{1q} + 2\,A_{2q} \quad \text{(105)}$$

$$A_3 = L^3 A_q - 3\,L^2 A_{1q} + 3\,L\,A_{2q} \quad \text{(106)}$$

and

$$A_4 = L^4 A_q - 4\,L^3 A_{1q} + 6\,L^2 A_{2q} - 4\,L\,A_{3q} + 2\,A_{4q} \quad \text{(107)}$$

The various partial areas with subscript (ok) covering the partial length $k\,L$ at the left half of the beam, needed for computing the load constants in cases 1, 3, and 4, may be computed as usual when $k < 0.5$. When $k > 0.5$, the symmetrical haunched beam may be reversed in position so that $k < 0.5$.

The previously given five theorems obviously offer simplified procedures for computation. When the area of the I_o/I-diagram and its various momental areas of certain haunch about either end are known, the corresponding elements easily may be computed without plotting the various momental diagrams for:

(*a*) Any other haunch of the same type with the same ratio between the moments of inertia at both ends, but with different lengths; and

(*b*) Any haunched beam, symmetrical or asymmetrical, with or without a section of constant moment of inertia, containing either the same haunch or any other haunch as defined in (*a*).

It is shown subsequently by numerical examples that the application of the foregoing five theorems in many cases will simplify greatly the work of computation.

NUMERICAL EXAMPLES

To illustrate the application of the method as well as the five theorems of momental areas, three typical numerical examples are given. In order that the results computed by this method may be checked, the shape of a haunched beam is taken from previously published[12,13] tables. With the I_o/I-diagram and its various momental diagrams plotted, the computations are divided into the following steps:

(1) Computation of the total areas of the various momental diagrams;

(2) Computation of the shape constants in terms of the end angle changes;

(3) Computation of the load constants in terms of the end changes for the desired type of loading; and

(4) Conversion of the shape and load constants in terms of the end angle changes into other systems to suit any method of continuous frame analysis, such as the Cross method of moment distribution.[4]

Example I.—The relative depth and moment of inertia and the I_o/I-diagram of the asymmetrical haunched beam are shown in Figs. 13(*a*), 13(*b*), and 13(*c*).

---

[12] "Neuere Methoden zur Statik der Rahmenträgwerke und der elastischen Bogenträger," by A. Strassner, Wilhelm Ernst und Sohn, Berlin, Band I, 1925, pp. 101–112.

[13] "Analysis of Continuous Frames," by E. B. Russell, San Francisco, Calif., 1934 (lithographed), pp. 24–29.

Note that, according to the notation used by E. B. Russell,[13] the length of haunch is $q\,L = 0.5\,L$; b, the ratio of the moment of inertia at the small end to that at the large end of the haunch, is equal to 0.2; and L is the length of the beam. The theorems of constant section and of combination will be applied by first computing the various momental areas for the haunch only, and then adding them to those of the constant section to obtain the values for the whole beam. The momental diagrams for the haunch only are obtained as shown in Figs. 13(d), 13(e), 13(f), and 13(g), from which the areas are

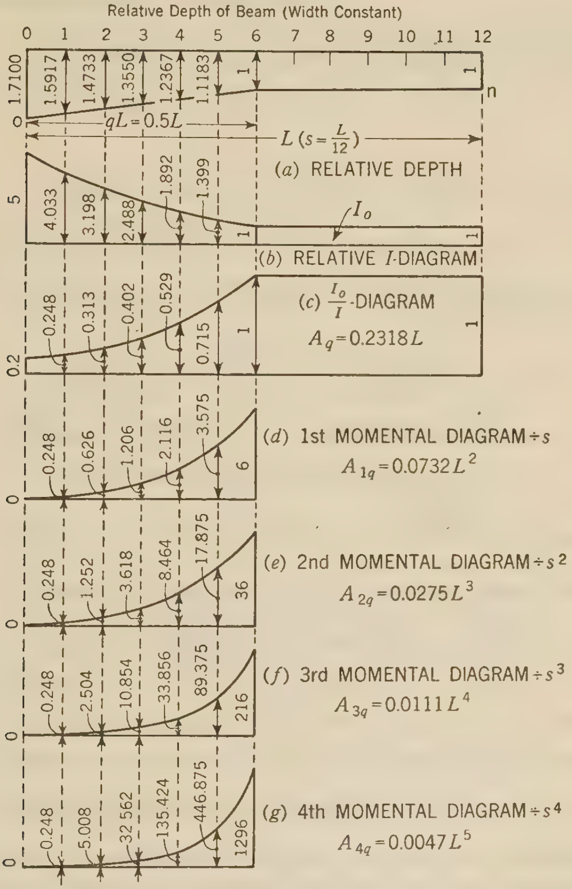

Relative Depth of Beam (Width Constant)
0 1 2 3 4 5 6 7 8 9 10 11 12
1.7100
1.5917
1.4733
1.3550
1.2367
1.1183
1
1
n
0
qL=0.5L
L (s=L/12)
(a) RELATIVE DEPTH
5
4.033
3.198
2.488
1.892
1.399
1
I_o
1
(b) RELATIVE I-DIAGRAM
0.2
0.248
0.313
0.402
0.529
0.715
1
(c) I_o/I-DIAGRAM
A_q=0.2318L
1
0
0.248
0.626
1.206
2.116
3.575
6
(d) 1st MOMENTAL DIAGRAM÷s
A_1q=0.0732L²
0
0.248
1.252
3.618
8.464
17.875
36
(e) 2nd MOMENTAL DIAGRAM÷s²
A_2q=0.0275L³
0
0.248
2.504
10.854
33.856
89.375
216
(f) 3rd MOMENTAL DIAGRAM÷s³
A_3q=0.0111L⁴
0
0.248
5.008
32.562
135.424
446.875
1296
(g) 4th MOMENTAL DIAGRAM÷s⁴
A_4q=0.0047L⁵

FIG. 13

$$A_q = \frac{1}{3}\left(\frac{L}{12}\right)[(0.2 + 1) + 4(0.248 + 0.402 + 0.715) + 2(0.313 + 0.529)] = 0.2318\,L \quad \text{..(108}a\text{)}$$

$$A_{1q} = \frac{1}{3}\left(\frac{L}{12}\right)^2 [(0 + 6) + 4(0.248 + 1.206 + 3.575) + 2(0.626 + 2.116)] = 0.0732\ L^2 \ldots (108b)$$

$$A_{2q} = \frac{1}{3}\left(\frac{L}{12}\right)^3 [(0 + 36) + 4(0.248 + 3.618 + 17.875) + 2(1.252 + 8.464)] = 0.0275\ L^3 \ldots (108c)$$

$$A_{3q} = \frac{1}{3}\left(\frac{L}{12}\right)^4 [(0 + 216) + 4(0.248 + 10.854 + 89.375) + 2(2.504 + 33.856)] = 0.0111\ L^4 \ldots (108d)$$

and

$$A_{4q} = \frac{1}{3}\left(\frac{L}{12}\right)^5 [(0 + 1{,}296) + 4(0.248 + 32.562 + 446.875) + 2(5.008 + 135.424)] = 0.0047\ L^5 \ldots (108e)$$

The momental areas of the constant section are computed by Eqs. 88 to 92 with $q = 0.5$ and $r = 0$:

$$A_c = 0.5000\ L \ldots\ldots (109a)$$

$$A_{1c} = \tfrac{1}{2}\ (0.5)\ (1 + 0.5)\ L^2 = 0.3750\ L^2 \ldots\ldots (109b)$$

$$A_{2c} = \tfrac{2}{3}\ (0.5)\ (0.3750)\ L^3 + \tfrac{1}{3}\ (0.5)\ L^3 = 0.2917\ L^3 \ldots\ldots (109c)$$

$$A_{3c} = \tfrac{1}{2}\ (0.3750)\ (1 + \overline{0.5^2})\ L^4 = 0.2344\ L^4 \ldots\ldots (109d)$$

and

$$A_{4c} = \tfrac{4}{5}\ (0.2344)\ L^5 + \tfrac{1}{5}\ (0.5)^4\ (0.5000)\ L^5 = 0.1938\ L^5 \ldots\ldots (109e)$$

The various total momental areas of the beam are, therefore,

$$A = A_q + A_c = 0.2318\ L + 0.5000\ L = 0.7318\ L \ldots\ldots (110a)$$

$$A_1 = A_{1q} + A_{1c} = 0.0732\ L^2 + 0.3750\ L^2 = 0.4482\ L^2 \ldots\ldots (110b)$$

$$A_2 = A_{2q} + A_{2c} = 0.0275\ L^3 + 0.2917\ L^3 = 0.3192\ L^3 \ldots\ldots (110c)$$

$$A_3 = A_{3q} + A_{3c} = 0.0111\ L^4 + 0.2344\ L^4 = 0.2455\ L^4 \ldots\ldots (110d)$$

and

$$A_4 = A_{4q} + A_{4c} = 0.0047\ L^5 + 0.1938\ L^5 = 0.1985\ L^5 \ldots\ldots (110e)$$

The coefficients for the angle changes computed from Strassner's tables[12] are given in parentheses immediately following the results obtained by the method herein described:

$$E\ I_o\ \alpha_n = \frac{1}{L^2} A_2 = 0.3192\ L\ \ (0.3190) \ldots\ldots (111a)$$

$$E\ I_o\ \alpha_o = A - \frac{2}{L} A_1 + \frac{1}{L^2} A_2 = (0.7318 - 2 \times 0.4482 + 0.3192)\ L = 0.1546\ L\ \ (0.1547) \ldots (111b)$$

and

$$E\ I_o\ \beta = \frac{1}{L} A_1 - \frac{1}{L^2} A_2 = (0.4482 - 0.3192)\ L = 0.1290\ L\ (0.1290) \ldots (111c)$$

Load constants in terms of angle changes are computed for the following four cases of loading. Their coefficients computed from Strassner's tables[12] for cases 1 and 2 of loading (Strassner's tables covering only these two cases) are given in parentheses immediately following the results obtained by the method of this paper.

Case 1. Concentrated Load at Point 8 ($k = \frac{2}{3}$).—The various partial areas with subscript (kn) needed in this case are computed by Eqs. 88 to 90 with $q = \frac{2}{3}$ and $r = 0$, since between point k and point n the beam section is constant:

$$A_{(kn)} = (1 - \tfrac{2}{3})\,L = \tfrac{1}{3}\,L = 0.3333\,L \ldots\ldots\ldots\ldots (112a)$$

$$A_{1(kn)} = \tfrac{1}{2} \times \tfrac{1}{3}\,(1 + \tfrac{2}{3})\,L^2 = \frac{5}{18}\,L^2 = 0.2778\,L^2 \ldots\ldots\ldots (112b)$$

and

$$A_{2(kn)} = (\tfrac{2}{3} \times \tfrac{2}{3} \times \frac{5}{18} + \tfrac{1}{3} \times \tfrac{1}{3})\,L^3 = \frac{19}{81}\,L^3 = 0.2346\,L^3 \ldots (112c)$$

$$A_{1(ok)} = A_1 - A_{1(kn)} = (0.4482 - 0.2778)\,L^2 = 0.1704\,L^2 \ldots (113a)$$

and

$$A_{2(ok)} = A_2 - A_{2(kn)} = (0.3192 - 0.2346)\,L^3 = 0.0846\,L^3 \ldots (113b)$$

$$E\,I_o\,[(\alpha^o_{o(1)} + \alpha^o_{n(1)}]$$
$$= [\tfrac{2}{3} \times 0.3333 - 0.2778 + (1 - \tfrac{2}{3})\,0.4482]\,P\,L^2 = 0.0938\,P\,L^2 \ldots (114a)$$

$$E\,I_o\,\alpha^o_{n(1)} = (\tfrac{2}{3} \times 0.2778 + 0.0846 - \tfrac{2}{3} \times 0.3192)\,P\,L^2$$
$$= 0.0570\,P\,L^2 \;\; (0.0570) \ldots (114b)$$

and

$$E\,I_o\,\alpha^o_{o(1)} = (0.0938 - 0.0570)\,P\,L^2 = 0.0368\,P\,L^2 \;\; (0.0369) \ldots (114c)$$

Case 2. Uniform Load Covering Whole Length of Beam.—

$$E\,I_o\,\alpha^o_{n(2)} = \tfrac{1}{2}\,(0.3192 - 0.2455)\,w\,L^3 = 0.0368\,w\,L^3 \;\; (0.0368) \ldots (115a)$$

and

$$E\,I_o\,\alpha^o_{o(2)} = [\tfrac{1}{2}\,(0.4482 + 0.2455) - 0.3192]\,w\,L^3$$
$$= 0.0277\,w\,L^3 \;\; (0.0277) \ldots (115b)$$

Case 5. Triangular Load with Maximum Intensity of w per Foot at Right End.—

$$E\,I_o\,[\alpha^o_{o(5)} + \alpha^o_{n(5)}] = \tfrac{1}{6}\,(0.4482 - 0.2455)\,w\,L^3 = 0.0338\,w\,L^3 \ldots (116a)$$

$$E\,I_o\,\alpha^o_{n(5)} = \tfrac{1}{6}\,(0.3192 - 0.1985)\,w\,L^3 = 0.0201\,w\,L^3 \ldots\ldots (116b)$$

and

$$E\,I_o\,\alpha^o_{o(5)} = (0.0338 - 0.0201)\,w\,L^3 = 0.0137\,w\,L^3 \ldots\ldots (116c)$$

Case 6. Triangular Load with Maximum Intensity of w per Foot at the Left End.—

$$E\,I_o\,[\alpha^o_{o(6)} + \alpha^o_{n(6)}] = \tfrac{1}{6}\,(2 \times 0.4482 - 3 \times 0.3192 + 0.2455)\,w\,L^3$$
$$= 0.0307\,w\,L^3 \ldots (117a)$$

$$E\,I_o\,\alpha^o_{n(6)} = \tfrac{1}{6}\,(2 \times 0.3192 - 3 \times 0.2455 + 0.1985)\,w\,L^3$$
$$= 0.0167\,w\,L^3 \ldots (117b)$$

and

$$E\,I_o\,\alpha^o{}_{o(6)} = (0.0307 - 0.0167)\,w\,L^3 = 0.0140\,w\,L^3 \ldots\ldots (117c)$$

Note that these results satisfy Eqs. 64 and 65.

Formulas for conversion of angle changes into the constants used in the Cross method are taken from the author's table, previously[5] mentioned with the one exception that the relative stiffness factors (which are equal to the absolute stiffness factors divided by 4 E and correspond to I/L for a beam with constant moment of inertia) are used to check Russell's coefficients, which are given in parentheses immediately following the results obtained by the method of this paper. The carry-over factors are

$$C_{on} = \frac{\beta}{\alpha_n} = \frac{0.1290}{0.3192} = 0.404 \quad (0.405) \ldots\ldots\ldots\ldots (118a)$$

and

$$C_{no} = \frac{\beta}{\alpha_o} = \frac{0.1290}{0.1546} = 0.834 \quad (0.835) \ldots\ldots\ldots\ldots (118b)$$

The relative stiffness factors are

$$S'_o = \frac{S_o}{4\,E} = \frac{\alpha_n}{4\,E\,(\alpha_o\,\alpha_n - \beta^2)} = \frac{0.3192}{4\,(0.1546 \times 0.3192 - 0.1290^2)} \times \frac{I_o}{L} = \frac{0.3192}{4 \times 0.0327} \times \frac{I_o}{L} = 2.44\,\frac{I_o}{L} \quad (2.44) \ldots (119a)$$

and

$$S'_n = \frac{S_n}{4\,E} = \frac{\alpha_o}{4\,E\,(\alpha_o\,\alpha_n - \beta^2)} = \frac{0.1546}{4 \times 0.0327} \times \frac{I_o}{L} = 1.18\,\frac{I_o}{L} \quad (1.18) \ldots (119b)$$

Fixed-end moments are computed by

$$M_{fn} = \frac{\alpha_o\,\alpha^o{}_n - \beta\,\alpha^o{}_o}{\alpha_o\,\alpha_n - \beta^2} \ldots\ldots\ldots\ldots (120a)$$

and

$$M_{fo} = \frac{\alpha_n\,\alpha^o{}_o - \beta\,\alpha^o{}_n}{\alpha_o\,\alpha_n - \beta^2} \ldots\ldots\ldots\ldots (120b)$$

For cases 1, 2, 5, and 6, Eqs. 120 become, respectively,

$$M_{fn(1)} = \frac{0.1546 \times 0.0570 - 0.1290 \times 0.0368}{0.0327}\,P\,L = \frac{0.004065}{0.0327}\,P\,L = 0.1243\,P\,L \quad (0.124) \ldots (121a)$$

and

$$M_{fo(1)} = \frac{0.3192 \times 0.0368 - 0.1290 \times 0.0570}{0.0327}\,P\,L = \frac{0.004394}{0.0327}\,P\,L = 0.1343\,P\,L \quad (0.135) \ldots (121b)$$

$$M_{fn(2)} = \frac{0.1546 \times 0.0368 - 0.1290 \times 0.0277}{0.0327}\,w\,L^2 = \frac{0.002116}{0.0327}\,w\,L^2 = 0.0647\,w\,L^2 \quad (0.065) \ldots (122a)$$

and

$$M_{fo(2)} = \frac{0.3192 \times 0.0277 - 0.1290 \times 0.0368}{0.0327} \, w\,L^2$$

$$= \frac{0.004095}{0.0327} \, w\,L^2 = 0.1251 \, w\,L^2 \quad (0.125) \,..\, (122b)$$

$$M_{fn(5)} = \frac{0.1546 \times 0.0201 - 0.1290 \times 0.0137}{0.0327} \, w\,L^2$$

$$= \frac{0.001340}{0.0327} \, w\,L^2 = 0.0820 \, \frac{w\,L^2}{2} \quad (0.082) \,..\, (123a)$$

and

$$M_{fo(5)} = \frac{0.3192 \times 0.0137 - 0.1290 \times 0.0201}{0.0327} \, w\,L^2$$

$$= \frac{0.001780}{0.0327} \, w\,L^2 = 0.1088 \, \frac{w\,L^2}{2} \quad (0.108) \,..\, (123b)$$

$$M_{fn(6)} = \frac{0.1546 \times 0.0167 - 0.1290 \times 0.0140}{0.0327} \, w\,L^2$$

$$= \frac{0.000773}{0.0327} \, w\,L^2 = 0.0472 \, \frac{w\,L^2}{2} \quad (0.047) \,..\, (124a)$$

and

$$M_{fo(6)} = \frac{0.3192 \times 0.0140 - 0.1290 \times 0.0167}{0.0327} \, w\,L^2$$

$$= \frac{0.002315}{0.0327} \, w\,L^2 = 0.1416 \, \frac{w\,L^2}{2} \quad (0.142) \,..\, (124b)$$

Example II.—The symmetrical haunched beam shown in Fig. 14 is composed of two asymmetrical beams, each of which has the same shape as that in Example I, except that the length is 0.5 L, hence the values of the momental areas of the beam may be computed by the application of the foregoing theorems from the results obtained in Example I without plotting the various momental diagrams.

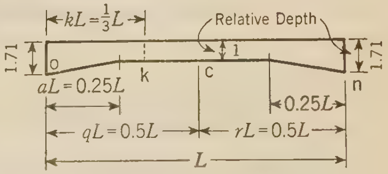


Fig. 14

The load constants are computed for cases 1 and 2 only, hence the values of A_3 and A_4 are not needed. The values of A_q, A_{1q}, and A_{2q} for the left half, oc of the beam may be computed from the values of A, A_1, and A_2 obtained in Example I by Eqs. 83 to 85 with $j = 0.5$:

$$A_q = 0.5 \times 0.7318\, L = 0.3659\, L \,.............. (125a)$$

$$A_{1q} = (0.5)^2\, 0.4482\, L^2 = 0.1121\, L^2 \,.............. (125b)$$

and

$$A_{2q} = (0.5)^3\, 0.3192\, L^3 = 0.0399\, L^3 \,.............. (125c)$$

The values of A, A_1, and A_2 for the whole beam may be computed by Eqs. 103 to 105:

$$A = 2\,A_q = 2 \times 0.3659\,L = 0.7318\,L \ldots\ldots\ldots\ldots (126a)$$

$$A_1 = L\,A_q = 0.3659\,L^2 \ldots\ldots\ldots\ldots (126b)$$

and

$$\begin{aligned} A_2 &= L^2 A_q - 2\,L\,A_{1q} + 2\,A_{2q} \\ &= (0.3659 - 2 \times 0.1121 + 2 \times 0.0399)\,L^3 = 0.2216\,L^3 \ldots (126c) \end{aligned}$$

Shape constants, in terms of angle changes, are

$$E\,I_o\,\alpha = \frac{1}{L^2} A_2 = 0.2216\,L \quad (0.2217) \ldots\ldots\ldots\ldots (127a)$$

and

$$\begin{aligned} E\,I_o\,\beta &= \tfrac{1}{2} A - \frac{1}{L^2} A_2 = (0.5 \times 0.7318 - 0.2216)\,L \\ &= 0.1443\,L \quad (0.1443) \ldots (127b) \end{aligned}$$

Load constants in terms of angle changes are as follows:

Case 1. Concentrated Load at $k\,L = L/3$.—The various partial areas needed also are computed from those obtained in Example I. In this example $A_{(kc)}$ corresponds to $A_{(kn)}$ in Example I, and the same is true for $A_{1(ok)}$ and $A_{2(ok)}$ in the two examples:

$$A_{(kc)} = 0.5 \times 0.3333\,L = 0.1667\,L \ldots\ldots\ldots\ldots (128a)$$

$$A_{(kn)} = A_{(kc)} + A_{(cn)} = A_{(kc)} + A_q = (0.1667 + 0.3659)\,L = 0.5326\,L \ldots (128b)$$

$$A_{1(ok)} = (0.5)^2\,0.1704\,L^2 = 0.0426\,L^2 \ldots\ldots\ldots\ldots (128c)$$

$$A_{1(kn)} = A_1 - A_{1(ok)} = (0.3659 - 0.0426)L^2 = 0.3233\,L^2 \ldots (128d)$$

and

$$A_{2(ok)} = (0.5)^3\,0.0846\,L^3 = 0.0106\,L^3 \ldots\ldots\ldots\ldots (128e)$$

Also,

$$\begin{aligned} &E\,I_o\,[\alpha^o{}_{o(1)} + \alpha^o{}_{n(1)}] \\ &\quad = [\tfrac{1}{3} \times 0.5326 - 0.3233 + (1 - \tfrac{1}{3})\,0.3659]\,P\,L^2 = 0.0982\,P\,L^2 \ldots (129a) \end{aligned}$$

$$\begin{aligned} E\,I_o\,\alpha^o{}_{n(1)} &= [\tfrac{1}{3} \times 0.3233 + 0.0106 - \tfrac{1}{3} \times 0.2216]\,P\,L^2 \\ &= 0.0445\,P\,L^2 \quad (0.0445) \ldots (129b) \end{aligned}$$

and

$$E\,I_o\,\alpha^o{}_{o(1)} = (0.0982 - 0.0445)P\,L^2 = 0.0537\,P\,L^2 \quad (0.0537) \ldots (129c)$$

Case 2. Uniform Load Covering Whole Length of Beam.

$$E\,I_o\,\alpha^o{}_{(2)} = \tfrac{1}{4}\,E\,I_o\,\beta\,w\,L^2 = 0.25 \times 0.1443\,w\,L^3 = 0.0361\,w\,L^3 \quad (0.0361) \ldots (130)$$

Conversion of angle changes into the constants used in the Cross method[4] is as shown hereafter. The beam is that in Russell's tables[13] with $a = 0.25$

and $b = 0.2$. Russell's coefficients are again given in parentheses immediately following those obtained by this method to serve as a check. The carry-over factor is

$$C_{on} = C_{no} = C = \frac{0.1443}{0.2216} = 0.651\ (0.650)\ldots\ldots\ldots\ldots(131)$$

The relative stiffness factor is

$$S'_o = S'_n = S' = \frac{0.2216}{4\,[(0.2216)^2 - (0.1443)^2]} \times \frac{I_o}{L} = \frac{0.0554}{0.0283} \times \frac{I_o}{L} = 1.96\,\frac{I_o}{L}\ (1.95)\ldots(132)$$

Fixed-end moments for case 1 are

$$M_{fn(1)} = \frac{0.2216 \times 0.0445 - 0.1443 \times 0.0537}{0.0283}\,P\,L = \frac{0.00210}{0.0283}\,P\,L = 0.0746\,P\,L\ \ (0.075)\ldots(133a)$$

and

$$M_{fo(1)} = \frac{0.2216 \times 0.0537 - 0.1443 \times 0.0445}{0.283}\,P\,L = \frac{0.00548}{0.0283}\,P\,L = 0.1938\,P\,L\ \ (0.194)\ldots(133b)$$

For case 2, the fixed-end moments are

$$M_{f(2)} = \frac{\alpha^o\,(\alpha - \beta)}{\alpha^2 - \beta^2} = \frac{\beta}{4\,(\alpha + \beta)}\,w\,L^2 = \frac{0.1443}{4\,(0.2216 + 0.1443)}\,w\,L^2 = 0.0987\,w\,L^2\ \ (0.099)\ldots(133c)$$

Example III.—To demonstrate further the application of the five theorems of momental areas in simplifying the computations of this method, the values of A, A_1, and A_2 for the asymmetrical beam with haunch at both ends (Fig. 15) will be computed from the results obtained in Example I without plotting the various momental diagrams. Two different methods may be used.

(*a*) The beam is considered as composed of the three sections oq, qr, and rn, the haunches oq and rn being similar to haunch oc of the beam in Example I.

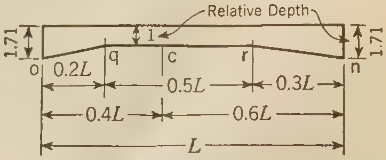


FIG. 15

For haunch oq, $j = \dfrac{0.2}{0.5} = 0.4$,

$$A_q = 0.4 \times 0.2318\,L = 0.092720\,L\ldots\ldots\ldots\ldots(134a)$$

$$A_{1q} = (0.4)^2\,0.0732\,L^2 = 0.011712\,L^2\ldots\ldots\ldots\ldots(134b)$$

and

$$A_{2q} = (0.4)^3\,0.0275\,L^3 = 0.001760\,L^3 \quad\text{(134c)}$$

For haunch rn, $j = \dfrac{0.3}{0.5} = 0.6$,

$$A'_r = 0.6 \times 0.2318\,L = 0.139080\,L \quad\text{(135a)}$$

$$A'_{1r} = (0.6)^2\,0.0732\,L^2 = 0.026352\,L^2 \quad\text{(135b)}$$

$$A'_{2r} = (0.6)^3\,0.275\,L^3 = 0.005940\,L^3 \quad\text{(135c)}$$

$$A_r = 0.139080\,L \quad\text{(135d)}$$

$$A_{1r} = (0.139080 - 0.026352)L^2 = 0.112728\,L^2 \quad\text{(135e)}$$

and

$$A_{2r} = (0.139080 - 2 \times 0.026352 + 0.005940)L^3 = 0.092316\,L^3 \quad\text{(135f)}$$

For the constant-depth section, qr, $q = 0.2$, $r = 0.3$,

$$A_c = (1 - 0.2 - 0.3)\,L = 0.500000\,L \quad\text{(136a)}$$

$$A_{1c} = (0.5)\,(0.500000)\,(1 + 0.2 - 0.3)L^2 = 0.225000\,L^2 \quad\text{(136b)}$$

and

$$A_{2c} = \tfrac{2}{3}\,(0.2)\,(0.225000)L^3 + \tfrac{1}{3}\,(0.7)^2\,(0.500000)L^3 = 0.111667\,L^3 \quad\text{(136c)}$$

For the whole beam,

$$A = (0.092720 + 0.500000 + 0.139080)L = 0.731800\,L \quad\text{(137a)}$$

$$A_1 = (0.011712 + 0.225000 + 0.112728)L^2 = 0.349440\,L^2 \quad\text{(137b)}$$

and

$$A_2 = (0.001760 + 0.11667 + 0.092316)L^3 = 0.205743\,L^3 \quad\text{(137c)}$$

(*b*) The beam is considered as composed of the two asymmetrical beams oc and cn, both being similar to the beam in Example I. For asymmetrical beam oc, $j = 0.4$,

$$A_q = 0.4 \times 0.7318\,L = 0.292720\,L \quad\text{(138a)}$$

$$A_{1q} = (0.4)^2\,0.4482\,L^2 = 0.071712\,L^2 \quad\text{(138b)}$$

and

$$A_{2q} = (0.4)^3\,0.3192\,L^3 = 0.20429\,L^3 \quad\text{(138c)}$$

For asymmetrical beam cn, $j = 0.6$,

$$A'_r = 0.6 \times 0.7318\,L = 0.439080\,L \quad\text{(139a)}$$

$$A'_{1r} = (0.6)^2\,0.4482\,L^2 = 0.161352\,L^2 \quad\text{(139b)}$$

$$A'_{2r} = (0.6)^3\,0.3192\,L^3 = 0.068947\,L^3 \quad\text{(139c)}$$

$$A_r = 0.439080\,L \quad\text{(139d)}$$

$$A_{1r} = (0.439080 - 0.161352)\,L^2 = 0.277728\,L^2 \quad\text{(139e)}$$

and

$$A_{2r} \doteq (0.439080 - 2 \times 0.161352 + 0.068947)L^3 = 0.185323\,L^3 \quad\text{..(139}f\text{)}$$

For the whole beam,

$$A = (0.292720 + 0.439080)L = 0.731800\,L \quad\text{........(140}a\text{)}$$

$$A_1 = (0.071712 + 0.277728)L^2 = 0.349440\,L^2 \quad\text{........(140}b\text{)}$$

and

$$A_2 = (0.020429 + 0.185323)L^3 = 0.205752\,L^3 \quad\text{........(140}c\text{)}$$

These two methods practically check. The values of A_3 and A_4, if desired, may be computed in the same manner.

CONCLUSION

The foregoing three numerical examples suffice to demonstrate the simplicity as well as the accuracy of the method presented in this paper. With the various momental diagrams for a given haunched beam once constructed, the shape and load constants for any of the common types of loading all can be computed from the areas of those diagrams. No separate M-diagram or M/I-diagram is needed for each loading, as is necessary in any of the existing methods. Since the computation of the flexural constants is reduced at last to that of the area of a plane figure under a curve, the accuracy of the method lies entirely in the area computation. Hence, the method can yield results with any desired accuracy if the length of beam is divided into a sufficient number of equal segments. Ordinarily, the division of the length of beam into ten or twelve equal segments will give satisfactory accuracy for practical purposes.

When the flexural constants of a number of haunched beams containing the same or similar haunches are to be computed, the five theorems of momental areas may be applied in simplifying the necessary computations. These five theorems represent the unique feature of the method, which is unparalleled by other methods within the author's knowledge.

APPENDIX. NOTATION

The following letter symbols, adopted for use in the paper and in its discussions, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

The dimensions of the letter symbols are indicated in parentheses following each symbol with f representing a force, l representing a length, and p representing a pure number or ratio.

A (l) = area of the I_o/I-diagram for entire length of beam, with the following subscripts for discrimination: (ok) and (kn) for the partial lengths ok and kn, respectively; c for the section of the beam with a constant moment of inertia; q for the left haunch or part of the beam; r for the right haunch or part of the beam (these rules applying also to the following symbols: A_1, A_2, A_3, and A_4);

A_1 (l^2) = area of first momental diagram of the I_o/I-diagram for the entire length about the left end of the beam;

A'_{1r} = area of first momental diagram of I_o/I-diagram for the right haunch or the part of the beam about the right end of the beam (the same rule applying also to A_2, A_3, and A_4);

A_2 (l^3) = area of the second momental diagram of the I_o/I-diagram for the entire length about the left end of the beam;

A_3 (l^4) = area of the third momental diagram of the I_o/I-diagram for the entire length about the left end of the beam;

A_4 (l^5) = area of the fourth momental diagram of the I_o/I-diagram for the entire length about the left end of the beam;

$A_{(m)}$ ($f\,l^2$) = area of the $M\,I_o/I$-diagram for the entire length of the beam;

$A_{1(m)}$ ($f\,l^3$) = area of the first momental diagram of the $M\,I_o/I$-diagram about the left end of the beam;

a (p) = proportional length of the haunch;

b (p) = ratio of the moment of inertia at the small end to that at the large end of haunch;

C (p) = carry-over factor;

C_{on} = carry-over factor from end o to end n;

C_{no} = carry-over factor from end n to end o;

E ($f\,l^{-2}$) = modulus of elasticity of the material;

I (l^4) = moment of inertia of the cross section at any section of the beam;

I' = moment of inertia of the cross section for another beam;

I_e (l^4) = moment of inertia of cross section at end of beam; I'_e = moment of inertia of cross section for another end or another beam;

I_o (l^4) = an arbitrary value of I used as a standard in computation;

I'_o = an arbitrary value of I used as a standard in computation for another beam;

i (p) = a ratio, x/L;

j (p) = ratio between length of two similar haunches, beams, or parts of beams;

k (p) = ratio of distance of point k measured from left end of the beam to the entire beam length;

L (l) = length of the beam;

M ($f\,l$) = bending moment at any section of the beam;

M_f ($f\,l$) = fixed-end moment due to applied loads, with subscript o or n indicating, respectively, at left end or right end of beam;

m (p) = ratio of distance of point m measured from the left end of the beam to the entire beam length;

n (p) = number of equal segments into which the length of the beam is divided;

P = a concentrated load;

Q (l^2) = first moment of A about left end of beam;

$q\ (p)$ = proportional length of the left haunch or part of the beam;
$r\ (p)$ = proportional length of the right haunch or part of the beam;
$S\ (f\,l)$ = absolute stiffness factor, with subscript o or n indicating left or right end of beam, respectively;
$S'\ (l^3)$ = relative stiffness factor $= \frac{S}{4\,E}$, with subscript o or n indicating left or right end of beam, respectively;
$s\ (l)$ = length of equal segments into which the length of the beam is divided;
w = uniform load per unit length;
$x\ (l)$ = abscissa of any point measured from the left end of beam;
$x'\ (l) = L - x$;
$y\ (p)$ = the ordinate of the I_o/I-diagram $\left(= \frac{I_o}{I}\right)$, with subscripts $o, 1, 2, \cdots, m, \cdots, n$ indicating its value at points at a distance of $os, 1s, 2s, \cdots, ms, \cdots, ns$ from the left end of the beam;
$y'_m\ (l)$ = ordinate at any point m of the first momental diagram of the I_o/I-diagram about the left end of the beam;
$y''_m\ (l^2)$ = ordinate at any point m of the second momental diagram of the I_o/I-diagram about the left end of the beam;
$y'''_m\ (l^3)$ = ordinate at any point m of the third momental diagram of the I_o/I-diagram about the left end of the beam;
$y''''_m\ (l^4)$ = ordinate at any point m of the fourth momental diagram of the I_o/I-diagram about the left end of the beam;
$\alpha\ (f^{-1}\,l^{-1})$ = angle change at either end of a simply-supported beam due to a unit moment applied at the same end, with subscript o or n indicating, respectively, at left or right end;
$\alpha^o\ (p)$ = angle change at either end of a simply-supported beam due to any applied loads, with subscript o or n indicating, respectively, at left or right end; and
$\beta\ (f^{-1}\,l^{-1})$ = angle change at either end of a simply-supported beam due to a unit moment applied at the other end.

DISCUSSION

J. Edmund Fitzgerald,[14] Jun. ASCE.—The author has shown true originality by avoiding the generally accepted methods of approach based on the $\frac{M}{E\,I}$-diagrams and launching his investigation on the basis of the relative reciprocal I-diagram. This method enables one to construct tables for haunched beams with no-load conditions. By including the values of the momental areas A for various haunches it is then possible to derive expressions for end moments quite directly for any load conditions.

The graphical method of determining A_1, A_2, A_3, etc., suggested by the author proves too involved with respect to the number of lines required and the necessity of multiplying all values by L, to compete favorably with the analytical method. The writer solved a number of haunches by both methods and is decidedly in favor of the analytical method.

The most encouraging aspect of this paper is the fact that one may now approach the problem of haunched beams with far more freedom than has ever before been possible. Beam tables, at best, have been only an expedient made necessary by the formidable amount of calculations heretofore required in the solutions of haunched beams. The author's method is rapid and adaptable to "mass production" methods of calculation. The use of the Simpson rule to calculate areas is no more accurate than when used to calculate the areas of $\frac{M}{E\,I}$-diagrams, but the calculations of ordinates have been changed from a second source of error to an exact method in so far as each ordinate is concerned.

The theorem of similtude is by far one of the most valuable contributions made by the author.

In Eqs. 44 to 49, inclusive, the author avoids the direct approach for no apparent reason. Actually, his analysis mars the structural concept. The more generally used and understood approach is as follows:

Referring to Fig. 5,

$$M_{ok} = \left[\tfrac{1}{2}\,k\,L\,x\,(2-k) - \tfrac{1}{2}\,x^2\right] w \ldots\ldots\ldots\ldots\ldots (141a)$$

and

$$M_{kn} = \frac{k^2\,L}{2}\,(L-x)\,w \ldots\ldots\ldots\ldots\ldots\ldots\ldots (141b)$$

From Eq. 26,

$$E\,I_o\,(\alpha^\circ{}_{n(3)} + \alpha^\circ{}_{o(3)}) = \int_o^n M\,y\,dx = \left[\tfrac{1}{2}\,k\,L\,(2-k)\int_o^k y\,x\,dx - \tfrac{1}{2}\int_o^k y\,x^2\,dx + \frac{k^2\,L^2}{2}\int_k^n y\,dx - \frac{k^2\,L}{2}\int_k^n y\,x\,dx\right] w$$

$$= \frac{w}{2}\left[k^2\,L^2\,A_{(kn)} + k\,L\,(2-k)\,A_{1(ok)} - k^2\,L\,A_{1(nk)} - A_{2(ok)}\right] \ldots (142)$$

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[14] Asst. Prof. of Civ. Eng., School of Eng., North Dakota Agri. College, Fargo, N. Dak.

From Eq. 27,

$$E\,I_o\,\alpha^{o}{}_{n(3)} = w\left[\tfrac{1}{2}\,k\,(2-k)\int_o^k y\,x^2\,dx - \frac{1}{2\,L}\int_o^k y\,x^3\,dx\right.$$
$$\left.+\frac{k^2\,L^2}{2}\int_k^n y\,x\,dx - \frac{k^2\,L}{2}\int_k^n y\,x\,dx\right]$$
$$= \frac{w}{2}\left[k^2\,L\,A_{1(kn)} + k\,(2-k)\,A_{2(ok)} - k^2\,A_{2(kn)} - \frac{1}{L}\,A_{3(ok)}\right]\ldots(143)$$

Subtracting Eq. 143 from Eq. 142,

$$E\,I\,\alpha^{o}{}_{o(3)} = \frac{w}{2}\left[k^2\,L^2\,A_{(kn)} + (2+k)\,k\,L\,A_{1(ok)} - 2\,k^2\,L\,A_1\right.$$
$$\left.-\,(1+2\,k)\,A_{2(ok)} + k^2\,A_2 + \frac{1}{L}\,A_{3(ok)}\right]\ldots\ldots\ldots\ldots(144)$$

Eqs. 143 and 144 agree with the author's results, but are found in a more direct manner.

HARRIS SOLMAN,[15] M. ASCE.—The avowed purpose of the author, as stated in the "Synopsis," is to present a method of expressing the flexural constants of a beam in terms of the characteristics of the beam as represented by the momental areas of the reciprocal *I*-diagram and in the form of exact, easily evaluated formulas. The use of this method is not limited to any particular type of beam and may be applied with equal advantage to cases of beams of any desired sectional variation and even to prismatic beams. It is difficult to understand, therefore, why the author chose to present his discussion under the title and within the framework of "haunched beams," which may be mistakenly assumed to imply a restriction of the discussion to one particular type of beam.

Of especial interest to the writer is the type of beam in which the section undergoes abrupt changes at a number of points in the span—the points dividing the beam into a number of segments of cross sections different from each other but constant for the length of each segment. This is the beam of prismatic segments and it represents the type to which belong the most steel beams where the sections are built up by the addition of cover plates. In spite of the fact that this type is commonly used, little has been written about its characteristics. The writer's interest in this type is further augmented by the extended application of its characteristics, even beyond the beams that actually belong to this category. Any type of sectional variation may be viewed as a limiting condition of a formation of a large number of prismatic segments and therefore may be treated as a beam of this type. Indeed, it is a matter of common practice, in an approximate analysis of a beam of any type of sectional variation, to divide the beam into a number of parts and to treat each part as a prismatic segment, the degree of accuracy of such an approximation depending on the number of segments assumed.

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[15] Highway Bridge Engr., U. S. Public Roads Administration, Div. 1, Albany, N. Y.

The method under discussion is not only applicable to this type of beam, but its application here is particularly easy and adaptable. The use of this method for beams of prismatic segments was advocated[16] by the writer in 1947. The writer designated the momental areas A, A_1, and A_2, respectively, by a, m, and i, since these momental areas can be readily recognized as the area, moment, and moment of inertia of the analogous column. Subscripts A and B were used to indicate the particular point of reference, A and B being the ends of the span under consideration. The writer also used the symbol j to denote the momental area A_3.

In an approximate analysis, the loading, too, may be approximated with a sufficient degree of accuracy as a series of concentrated loads. Under such a treatment only the commonly familiar momental areas A, A_1, and A_2 are involved. However, since the determination of A_3 is so easy in the case of prismatic segments,[16] the writer could not resist the temptation of introducing it as j in a direct treatment of uniform loads. Incidentally, the direct treatment of uniform loads afforded the writer the opportunity of introducing the case of a uniform load on a symmetrical beam with its exceeding simplicity.

In his generalized discussion of the subject, the author used six cases of loading. Case 1 is a point loading, cases 2, 3, and 4 are area loadings of constant intensity, and cases 5 and 6 are area loadings of which the intensity is a linear function of the distance along the span. For a more complete generalization another area loading could have been introduced, the intensity of which is an nth degree function of the distance along the span. In addition to its theoretical generalization, such a loading would represent the weight of the beam itself. This loading, however, would require the use of momental areas of a degree up to and including $n + 3$. It is agreed that the additional complexity would not be warranted by the advantage gained by this generalization.

It may be noted that the author recognizes the existence of three "shape constants" for the general condition, whereas the writer[16] derived formulas for two constants at each end of the beam (carry-over and stiffness factors), making a total of four constants for the beam. This apparent discrepancy is not to be interpreted as a disagreement, as no attempt has been made in the writer's paper to eliminate redundancy. Indeed, the well-known relationship between the constants, $r_A K_A = r_B K_B$, reduces the number of independent constants to three, thus bearing out the author's statement.

ROBERT V. HAUER,[17] ESQ.—The method presented by Professor Tsai for analyzing haunched beams (in fact, beams of any shape) is an interesting contribution to the theory of flexure in so far as it shows that it is possible to derive the shape constants and the load constants for all common loading conditions from the reciprocal I-diagram and its successive momental areas alone. As the same problem can be solved in several other ways, the relative merits of this new method must be judged by its simplicity and its accuracy.

The title of the paper does not quite reveal the characteristic feature of the method described; for, as every end angle change of a beam is represented by a definite integral, the evaluation of such an integral, if not done directly, is always (in more or less concealed form) the computation of an area. It is only the special arrangement of the various area computations that characterizes the author's method. Professor Tsai's procedure is based on the fact that the moments for all common loading conditions (including those connected with the shape constants) are whole algebraic functions of the distance x from one end of the beam. This makes it possible to compute first the component areas defined by the successive powers of x in the integrand, instead of directly evaluating the areas representing the various end angle changes. By combining these components, or "momental," areas properly, the required angle changes are found.

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[16] "Design Constants Facilitate Analysis of Beams Composed of Prismatic Segments," by Harris Solman, *Civil Engineering*, May, 1947, p. 43.

[17] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

This procedure would result in a simplification only in the exceptional case when the number of component areas was smaller than the number of required angle changes. For the three angle changes representing the shape constants three momental areas must be evaluated (A, A_1, and A_2). One more (A_3) is required for the two angle changes that occur under a uniformly distributed load; but this case can also be solved directly with only one additional operation, because it follows from Eqs. 37 and 19 that

$$\alpha^o_{o(2)} + \alpha^o_{n(2)} = \tfrac{1}{2}\,\beta\, w\, L^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (145)$$

—a relation which is a generalization of Eq. 43. If a concentrated load acts on the beam, three new areas have to be evaluated—namely, $A_{(kn)}$, $A_{1(kn)}$, and $A_{2(ok)}$—and a uniform load covering part of the beam requires the computation of even five additional areas, $A_{(kn)}$, $A_{1(ok)}$, $A_{2(ok)}$, $A_{1(kn)}$, and $A_{3(ok)}$, whereas for each of these loading conditions the two load constants can be found directly in two operations. Only for a triangular load can a saving be achieved by the author's method, as only one additional area (A_4) is required in this case.

It must be added that, when partial momental areas are to be evaluated, it will often be necessary to divide the beam into a greater number of sections. For instance, in Example I, if a concentrated load were assumed to act at $x = 0.1\ L$, the division of the haunched part of the beam into six sections would evidently not be sufficient.

Finally, considering that, after the various areas are found, the task remains to combine them properly, the writer doubts if the method suggested by the author really excels in simplicity.

The process of plotting and evaluating the I_o/I-diagram and its momental areas can be considered a preparatory work intended to facilitate the computation of the angle changes. That this does not result in a greater simplification is explained by the fact that the preparatory work must be repeated for each beam of individual shape. The writer believes, therefore, that Professor Tsai's method cannot compete in convenience with another method of computing angle changes, a procedure which, like the author's method, is based on some preparatory work eliminating the necessity of plotting the various M/I-diagrams, but has the great advantage that the preparatory work needs to be done only once.

Briefly outlined, this scheme consists of dividing the beam into a number of equal parts (say, ten) and attributing to each section a constant reciprocal moment of inertia equal to its average value in this part. If this is done, the integral representing an angle change for a certain loading condition can evidently be expressed as a sum of ten terms with numerical coefficients that can be evaluated and tabulated once and for all and used for a beam of any shape. The actual computation of an angle change then simply consists of multiplying a set of tabulated figures with the respective I_o/I-values of the beam in consideration and summing up, which is a very convenient and relatively time saving operation.

If reasonably applied, all methods of computing the end angle changes of a haunched beam are sufficiently accurate from a practical standpoint. It must be borne in mind, however, that the basic integral formula for the angle changes is itself strictly correct only for a prismatic beam. When applied to a beam with variable cross section, even if the integration is performed with mathematical exactness, it leads to a result that is only a physical approximation.

In order to investigate the sensitivity of the angle changes with respect to the manner of computation, the writer used the foregoing procedure for recomputing all cases of Example I. A maximum difference of 0.7% and an average difference for the eleven angle changes of only 0.37% was found. This comparison confirms the writer's opinion that the accuracy of all methods for computing angle changes is satisfactory for all practical purposes.

In conclusion, it would be valuable if Table 1 were extended to include the formulas for the angle changes corresponding to an external moment acting at any section of the beam, as such a loading condition is often important in frames carrying cranes.

George Epps,[18] Assoc. M. ASCE.—The subject of beam constants, as used in various methods of analysis for statically indeterminate structures, has been discussed and presented from several different "angles." In fact, it seems that nearly everyone who has extensive use for these constants develops his own method for calculating them. Professor Tsai has presented an approach which, at first, seems to be rather complex; but after a little study it appears that it might be useful, particularly in problems involving fixed loadings.

In the design of bridges it is necessary to design for moving loads. Thus, fixed-end moments should be available for concentrated loads at several points. Most methods for deriving beam constants, including the author's, involve the use of partial moment areas for each position of the load. For this reason, the writer developed a method for determining such fixed-end moments which was presented in 1938.[19]

In the "Introduction," the author questions the accuracy of the trapezoidal rule of summation for use in the determination of beam constants. The writer has always used the rectangular rule, which is generally considered to be even less accurate, but he feels that the results obtained are sufficiently accurate for practical purposes. Many factors, such as cracking of the concrete, the effect of reinforcing steel, or the composition and connections of steel beams, all influence the beam constants. Various assumptions regarding these effects are made, which probably introduce errors much greater than any caused by the method of summation. Also, the actual loading on a beam seldom corresponds to the assumed or design load. This being so, it does not appear to be worthwhile to strive for great refinement in the calculation of beam constants. An indication of the comparative accuracy of various methods of summation is afforded by Table 2, which includes the momental areas for Example I as computed by the methods indicated. The difference between the correct values, as determined by integration (Col. 2, Table 2) and any of the others, is quite small.

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[18] Bridge Engr., Secondary Roads Dept., Kansas State Highway Comm., Topeka, Kans.

[19] "Beam Constants for Continuous Trusses and Beams," by George Epps, *Transactions*, ASCE, Vol. 104, 1939, p. 1522.

The values of the momental areas in the paper are calculated for one end of the beam and those for the other end are found by differences in deriving the constants for the other end of the beam (as in Eq. 111*b*). The writer prefers separate computations for each end of the beam, thus making the differences noted previously available for checking purposes.

Professor Tsai develops beam constants for use in the distribution of angle changes and then converts them to the beam constants required for moment distribution. All the designers with whom the writer is acquainted use the moment distribution or the slope deflection methods. For this reason, the author's method would be more useful if it were set up to give the required constants for these latter methods directly from the momental areas.

TABLE 2.—COMPARATIVE ACCURACY OF METHODS OF COMPUTING MOMENTAL AREAS

Momental areas (1)	Integration (2)	Tsai (3)	Rectangular rule (4)	Trapezoidal rule (5)
A	0.7317	0.7318	0.7302	0.7339
A_1	0.4481	0.4482	0.4472	0.4498
A_2	0.3191	0.3192	0.3182	0.3203
A_3	0.2455	0.2455	0.2444	0.2452
A_4	0.1986	0.1985	0.1972	0.1989

In the "Conclusion," Professor Tsai states that: "No separate M-diagram or M/I-diagram is needed for each loading, as is necessary in any of the existing methods." It appears to the writer that the momental areas in Eqs. 112 and 113 are areas of some M/I-diagram even though known by another name.

In general, it seems that the author's method is an attempt to produce a number of short cuts in the computation of beam constants through the use of the theorems of momental areas. This procedure should prove quite useful to one who is required to compute constants for a great many beams. For one who has use for such a calculation only occasionally it is believed that a simpler procedure might be more useful—such as, for example, an adaptation of the writer's procedure.[19]

William A. Conwell,[20] M. ASCE.—A comparison of this paper with a current paper entitled "Beam Deflections by Second and Third Moments" by Hsu Shih-Chang[21] leads to the conclusion that, although the papers treat different subjects, their underlying philosophy is the same. In each instance there is a preference for what might be termed the mechanical method of determining the moment of a load or an area—that is, multiplying the load or the area by its distance from the point about which moments are being taken. In view of the prevalence of the mathematical viewpoint of considering moments as the integral of shears and, in most instances, the convenience of so handling the problem, the use of fundamentals only by these authors is most interesting. Perhaps in his closing discussion, Professor Tsai would like to comment on this point. It would be of interest to know whether this approach represents a return to fundamentals by engineers in China or whether they never left them.

Even though novel methods, such as the method suggested by the author may not immediately replace others now in vogue, it is important that they receive consideration and be published. Only by charting these new paths is it possible to stimulate thought, strike a sympathetic chord in the minds of others thinking of the subject, and perhaps, lead the way in the development of new and powerful methods.

The writer believes that the author, unduly and modestly, limited the scope of this paper in his choice of title. The implications and the content of the text carry far beyond the field of haunched beams. Likewise, the reasons Professor Tsai gives for advancing his theses are stated only in terms of its advantages over existing procedures and do not touch the tremendous imaginative appeal inherent in these new thoughts.

In his review of existing methods in the "Introduction," the author should mention a possible extension to the trapezoidal method which almost completely overcomes his objection to it. By making the basic assumption used in the development of Simpson's rule (which the author recommends in his method), it is possible to obtain corrections to the trapezoidal diagrams which bring them very close to the actual diagrams. This assumption (that the curve is a second degree parabola between any three consecutive points) yields a substitute diagram of trapezoids and corrective parabolic segments. Since such a substitute would closely follow the actual curve and since the properties of the geometric figures involved may be quickly determined, it would appear that a much higher degree of accuracy than the author implied can be obtained with little effort.

In his "Introduction," the author mentions the "many comprehensive tables and charts" of flexural constants which are available for common types of beams. It is now generally accepted that advances in knowledge in this field will be marked by a gradual accumulation of data in chart and tabular form until common types are completely covered. The "Handbook of Frame Constants, Beam Factors, and Moment Coefficients for Members of Variable

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[20] Gen Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

[21] "Beam Deflections by Second and Third Moments," by Hsu Shih-Chang, *Transactions*, ASCE, Vol. 113, 1948, p. 445.

Section,"[22] prepared by the Portland Cement Association, with its mass of data for straight and parabolic haunches of varying lengths, symmetrically and unsymmetrically placed would seem to be close to the ultimate authority in this regard. Although the author seems to think of his work as being applicable to the more uncommon type of beam, it is apparent that it could be valuable in the extension of tables and charts as more types of beams become sufficiently common to warrant the expression of beam characteristics in this form.

The very fact that certain types of beams may be designated as uncommon leads to the conclusion that they occur so infrequently that the development of of special methods for their analysis would scarcely seem justified. Should some of the uncommon types be of sufficient importance, of course, tabular data should be developed for them.

Although the author stresses the application of this method to unusual types, he chooses for his numerical examples, types which are covered by tables. He quotes results from the Russell tables[13] which check the computations by momental areas. It would be of value to know if the data from those tables were read directly or if interpolation was required. The writer had to use interpolation to check the results with the Handbook. A comparison of the author's coefficients with those obtained by interpolation from the Handbook are shown in Table 3 for Examples I and II. This set of coefficients shows that, in general, linear interpolation of Table 3 yields results within the accuracy normally required for engineering design. The maximum error occurred in Example II where a 5.3% variation in the coefficient for $M_{fo(1)}$ was noted.

TABLE 3.—COMPARISON OF COEFFICIENTS AS COMPUTED IN THE TSAI METHOD AND AS GIVEN IN THE "HANDBOOK OF FRAME CONSTANTS, BEAM FACTORS, AND MOMENT COEFFICIENTS FOR MEMBERS OF VARIABLE SECTION"[a]

(a) EXAMPLE I						(b) EXAMPLE II		
Item	Tsai	Handbook	Item	Tsai	Handbook	Item	Tsai	Handbook
C_{on}	0.404	0.405	$M_{fn(2)}$	0.0647	0.0652	C	0.651	0.646
C_{no}	0.834	0.832	$M_{fo(2)}$	0.1251	0.1246	S'	1.96	1.96
S'_o	2.44	2.44	$M_{fn(5)}$	0.0820	0.0858	$M_{fn(1)}$	0.0746	0.0754
S'_n	1.18	1.18	$M_{fo(5)}$	0.1088	0.1078	$M_{fo(1)}$	0.1938	0.1835
$M_{fn(1)}$	0.1243	0.1203	$M_{fn(6)}$	0.0472	0.0478	$M_{f(2)}$	0.0987	0.0991
$M_{fo(1)}$	0.1343	0.1309	$M_{fo(6)}$	0.1416	0.1447			

[a] "Handbook of Frame Constants, Beam Factors, and Moment Coefficients for Members of Variable Section," Portland Cement Assn., Chicago, Ill., 1947.

When parabolic interpolation was employed, the Handbook coefficient of $M_{fo(1)} = 0.1835$ in Table 3(*b*) became 0.1930; and the error, 0.4%. The parabolic interpolation was based on values for the three points nearest the point for which a value is desired. A similar improvement in accuracy can be obtained at any time when there is doubt as to the efficacy of straight-line or trapezoidal methods.

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[22] "Handbook of Frame Constants, Beam Factors, and Moment Coefficients for Members of Variable Section," Portland Cement Assn., Chicago, Ill., 1947.

As long as engineers analyze structures; there will probably be debate as to the types of functions that should be taken as flexural constants. The author makes a strong case for the use of end angle changes on the basis that they assist in visualizing the action of the structure. There is no questioning the desirability of visualizing structural action. Hardy Cross, Hon. M. ASCE (who, because of his work in moment distribution, may be said to be a proponent of the use of end moments for flexural constants), took cognizance of the importance of such visualization when he wrote:[23]

> "The ability of a designer of continuous structures is measured chiefly by his ability to visualize the deformation of the structure under load. If he cannot form a rough picture of these deformations when he begins the analysis he will probably analyze the structure in some very awkward and difficult way; if he cannot picture these deformations after he has made the analysis, he doesn't know what he is talking about."

This statement implies that a knowledge of moments should enable a designer to visualize the deformations of the structure and such is the case. Since moments are required for design, they must always be computed and there is thus a good argument for making use of them as flexural constants and later determining such actual deflections as may be required. To the writer and many others this procedure seems more logical than first determining deformations that may or may not be required and then finding the moments that will, of course, be required for design.

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[23] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932.

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MISSISSIPPI RIVER BRIDGE AT DUBUQUE, IOWA

By R. N. Bergendoff[1] and Josef Sorkin,[2] Members, ASCE

With Discussion by Messrs. Jonathan Jones, A. Floris, T. H. Rust, Charles W. Dohn, Louis Balog, and R. N. Bergendoff and Josef Sorkin.

Synopsis

A number of unique and interesting features have been incorporated in the Julien Dubuque Bridge, opened on August 31, 1943, for highway traffic across the Mississippi River between Dubuque, Iowa, and East Dubuque, Ill. Not only is the 845-ft main channel span the second longest clear span across the Mississippi River, but the 1,537-ft three-span continuous truss of which it forms the central span is the longest of its type in the world. There are only two similar structures in North America—the Saint Lawrence River Bridge west of Montreal, Que., Canada, and the Meramec River Bridge south of St. Louis, Mo., both of which have much shorter arch spans (400 ft and 264 ft, respectively).

The 1,537-ft three-span continuous structure is supported on eight shoes, all expansion type. End bearings are 8-ft-long rocker columns; intermediate bearings have shoes riding on nested rollers. At the east end of the central arch span a large pin embedded on the center line of a supporting pier engages a floor beam of the superstructure to transfer longitudinal and transverse forces from the three-span superstructure unit to the substructure. Torque in piers due to transverse forces is thus largely eliminated. The dead load weight of pier 18, at the west end of the central arch span, was materially reduced by using sand fill in the central section of the pier base, thus minimizing the size of the pier and the number of supporting piles required.

The three-span continuous truss structure comprising the 845-ft tied arch center span, flanked by two 346-ft end spans, presents a pleasing appearance, affords to traffic a view unobstructed by diagonal truss members across the 845-ft main channel span, and has inherent advantages for erection. In economy it compared favorably with cantilever, tied arch, and suspension span layouts studied for the crossing.

Note.—Published in June, 1947, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., Howard, Needles, Tammen & Bergendoff, Kansas City, Mo., and New York, N. Y.

[2] Engr. of Design, Howard, Needles, Tammen & Bergendoff, Kansas City, Mo.

INTRODUCTION

The bridge serves local traffic between Dubuque, Iowa and East Dubuque, Ill., as well as heavy through traffic on United States Highway No. 20 and the north-south traffic of United States highways Nos. 61 and 51. The new structure replaced a privately-owned toll bridge erected in 1887 some distance upstream from the new location.

Various sites were studied. Navigation requirements at some of the alternate sites permitted shorter channel spans, but none provided as satisfactory alinement or as satisfactory connections to principal routes of highway traffic. Viaduct approaches to the river spans pass above all railroad tracks at both ends of the structure. Both the length of the channel span and the locations of the piers supporting it were dictated by the War Department to satisfy navigation interests and to permit maneuverability of boats approaching the barge terminal on the west bank of the river directly north of the bridge. A vertical clearance of 64 ft above normal pool elevation over a horizontal distance of 800 ft is provided by the main channel span.

DESCRIPTION OF STRUCTURE

At Dubuque the Mississippi River is about 1,700 ft wide at pool level, at which stage it has a maximum depth of 35 ft. High water at times is more than 15 ft above pool level, and inundates the entire valley from abutment to abutment of the bridge—a distance of approximately 6,000 ft.

Outcrops of limestone are in evidence at the bluffs on both sides of the valley. Within the limits of the bridge site, however, bedrock is 400 ft below pool level, and is overlain with a deep deposit of coarse sand and gravel.

The 7,082-ft total length of the bridge project is divided into 1,320 ft of paved embankment approaches and 5,762 ft of open structure. The 1,537-ft main river channel span unit is a three-span continuous through truss structure, consisting of an 845-ft tied arch center span flanked by 346-ft truss side spans as shown in Fig. 1. Approach viaducts are steel continuous deck girder units, with spans up to 187 ft long. Long girder spans adjacent to the river structure have curved bottom flanges; all other spans are of parallel flange girder type. The Iowa approach, made up of seventeen spans from 44 ft to 187 ft long, has a total length of 1,448 ft; the Illinois approach contains twenty-three spans from 85 ft to 184 ft long and has a total length of 2,757 ft. The bridge, which crosses

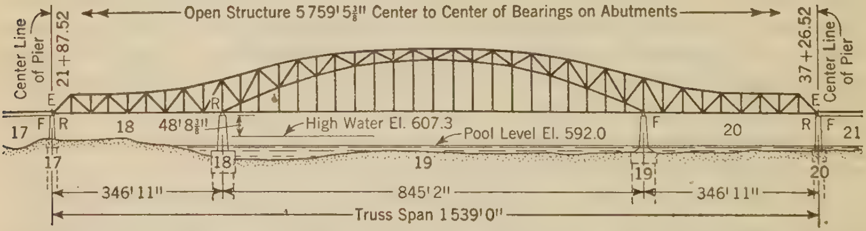

FIG. 1.—ELEVATION OF THREE-SPAN MAIN RIVER CHANNEL UNIT OF MISSISSIPPI RIVER BRIDGE AT DUBUQUE, IOWA

above twenty-nine tracks of four different railroads, provides one 5-ft sidewalk and a two-lane roadway 24 ft wide. The roadway is designed for H-20 loading as given by the 1935 specifications of the American Association of State Highway Officials (AASHO), modified to assume 12-ft lanes in the design of main trusses and girders, but using 9-ft lanes in the design of the floor system. Fig. 2 shows the roadway cross section of the truss spans.

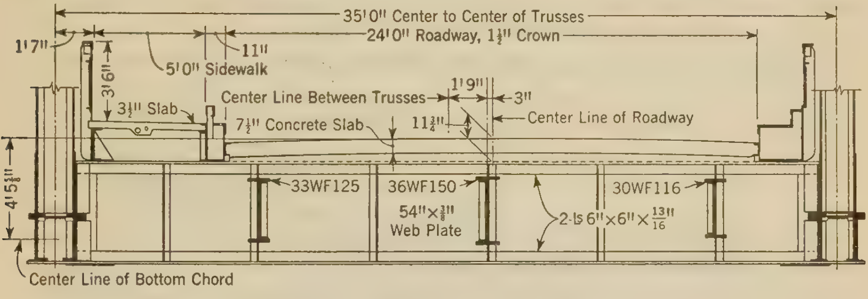

FIG. 2.—CROSS SECTION OF TRUSS SPANS SHOWING ROADWAY LAYOUT

The entire substructure is supported on timber piles. Untreated timber piles were used wherever fully embedded below permanent ground-water elevation; elsewhere creosoted timber piles were used. Except for pier 18, the entire substructure and the viaduct approaches are of conventional design.

The principal purpose of this paper is to present the development of the design and the expedient erection of the main three-span continuous unit.

THREE-SPAN THROUGH TRUSS CONTINUOUS UNIT

With the length of river channel span determined by navigation requirements, the west span of the continuous unit was made sufficiently long to clear the railroad tracks on that bank of the river and to provide a nominal positive dead load reaction at the end bearing. Symmetry was obtained without sacrificing economy by making both end spans of the continuous unit of equal length. By this arrangement a maximum negative moment was introduced at the intermediate supports through utilization of the full dead weight of the end span.

After studies had been made of appearance and section requirements for various alternative heights, the rise of the 845-ft 2-in. central arch span was selected as 106 ft from center of tie to center of bottom chord of the truss. For an 845-ft tied arch span of conventional design and a 1:6 rise ratio, a corresponding height of approximately 130 ft would have been required. Truss depths were selected as 40 ft from center to center of chords at the end portal, 70 ft at the intermediate supports, and 24 ft at the center of the arch. Transverse spacing of trusses is 35 ft on centers. The center line of roadway is offset 2 ft to balance the sidewalk and roadway loads between the two trusses. With this arrangement the maximum stresses in the two trusses are approximately the same, so the trusses were made identical. A saving of floor beam metal would have resulted from a closer spacing of trusses, with sidewalk outside; but truss loadings would have so differed that the trusses would not have been identical and duplication of fabrication would have been sacrificed. Also, closer truss spacing would have reduced what was regarded as a desirable width-length ratio for lateral stability.

Study of various truss systems resulted in adoption of the Warren system. Compared to the Pratt type, the Warren truss has the advantage of single-panel loading on vertical members and constant chord stresses in double-panel lengths. Subdivided truss systems with shorter floor panels would have secured some saving in the floor system metal, but this saving would be offset by a greater number of members and joints. In appearance the Warren system seemed to harmonize most satisfactorily with the general outline of the structure.

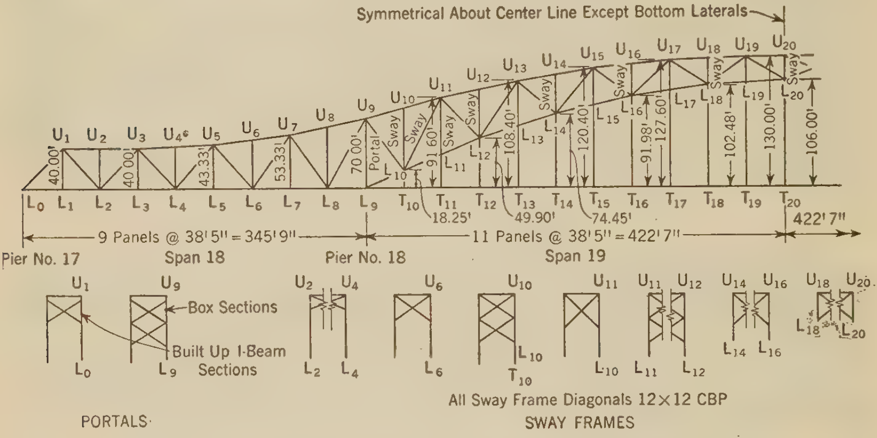
Symmetrical About Center Line Except Bottom Laterals
U1 U2 U3 U4 U5 U6 U7 U8 U9 U10 U11 U12 U13 U14 U15 U16 U17 U18 U19 U20
L0 L1 L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14 L15 L16 L17 L18 L19 L20
T10 T11 T12 T13 T14 T15 T16 T17 T18 T19 T20
40.00'
40.00'
43.33'
53.33'
70.00'
Portal
Sway
91.60'
108.40'
120.40'
91.98'
127.60'
102.48'
130.00'
106.00'
18.25'
49.90'
74.45'
422' 7"
9 Panels @ 38' 5" = 345' 9"
11 Panels @ 38' 5" = 422' 7"
Pier No. 17
Span 18
Pier No. 18
Span 19
Box Sections
Built Up I-Beam Sections
All Sway Frame Diagonals 12×12 CBP
PORTALS
SWAY FRAMES

FIG. 3.—GENERAL DIMENSIONS OF TRUSS SPANS, LATERAL BRACING, SWAY FRAMES, AND PORTALS

The general arrangement of lateral and sway bracing is shown in Fig. 3. The bottom lateral system in the plane of the roadway deck is a conventional rigid single-plane system of double-angle members. The top lateral system in the planes of the top and bottom chords of the arch span is a rigid double-plane system of H-members. Sway frames are provided in the plane of all vertical posts except U8–L8. They are also provided in the planes of hangers L11–U11 and in the planes of diagonals L10–U11. The end posts and the posts at the intermediate piers are braced with massive portals. Horizontal members were omitted from sway frames and portals in end spans to improve appearance.

All panels are 38 ft 5 in. long, there being nine panels in the end spans and twenty-two panels in the center span. Floor beams rigidly framed to the trusses and hangers support three lines of roadway stringers on 10-ft, 3-in. centers, together with a sidewalk fascia beam. Transverse crossbeams, 4 ft $9\frac{5}{8}$ in. on centers, rest on the top flanges of the stringers. The roadway deck is a $7\frac{1}{2}$-in. concrete slab, reinforced transversely with rail steel bars. A steel safety curb separates the 24-ft roadway, from the 5-ft sidewalk on the north. Along the south side of the roadway a similar curb is provided, also steel. The south handrail is 24 in. outside the edge of the roadway, and the intervening space is floored with steel plate to provide an auxiliary sidewalk. Handrails 3 ft 6 in. high above the sidewalks are steel, with steel posts. Roadway curbs are 10 in. high. Above streets and railroad tracks the curbs are closed; elsewhere they are left open to provide full drainage and to facilitate cleaning the roadway and removing snow. To eliminate undesirable chord stress participation and to relieve possible temperature stresses due to unequal thermal movement of the floor system, floor contraction joints and stringer expansion bearings were provided at L4, L7, T11, T15, and T18.

DESIGN PROCEDURE

Because of the magnitude and statical indeterminacy of the structure, erection methods had to be predetermined and sections provided for stresses encountered during the various stages of erection. The design finally adopted contemplated a three-stage erection program comprising: (1) Cantilever erection of the trusses and bracing of the central span; (2) erection of hangers, floor beams, and arch span ties after the trusses had been connected to act as a three-span continuous unit; and (3) installation of the floor system and deck after making the tie connection of the arch span. Although a 50% increase of normal allowable stresses was permitted for erection purposes, a number of members nevertheless were governed by erection stresses.

The form of the trusses of the central span lends itself to an economical arrangement for cantilever erection with maximum depth over intermediate supports tapering to a minimum depth at the center of the span. To minimize the added material required for erection purposes, erection of the arch by cantilever methods was limited to placement of trusses and bracing. After closure of the trusses at the center of the arch span, added carrying capacity of the trusses became available through conversion to a three-span continuous system by vertical adjustment of the end reactions. Utilizing this increased capacity, the hangers, floor beams, and ties of the center span were added. By adjustment of the horizontal distance between L9 and L20, the tie closure could then be made. Stringers, crossbeams, curbs, railings, and concrete floor slabs were then added to complete the span. The design drawings were complete in every detail, including the make-up of the members and details of splices and joints. It may be noted that the superstructure contractor elected to carry out erection generally in full accordance with the original design scheme.

Floor System.—For design of the concrete roadway slab the so-called Westergaard theory[3,4] of wheel load distribution was used. Welded bar trusses were planned originally for the slab reinforcement; subsequently, emergencies of World War II made it necessary to use bent bars of equivalent area. Crossbeams, designed as continuous beams over the stringer supports, are 12-in. I at 31.8 lb in intermediate panels and 12-in. I at 40.8 lb in end panels. Stringers were designed as simple beams spanning between floor beams with maximum load reactions transmitted by the crossbeams, and are rolled beams. Generally, all floor beams are plate girders 4 ft 6 in. deep. At intermediate piers, special double-web floor beams, 6 ft 2 in. deep, are provided as an integral part of the portal frames.

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[3] "Distribution of Wheel Loads and Design of Reinforced Concrete Bridge Floor Slabs," by H. R. Erps, A. L. Googins, and J. L. Parker, *Public Roads*, October, 1937, p. 149.

[4] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *ibid.*, March, 1930, p. 1.

Lateral and Sway Bracing.—Three lateral systems are provided as follows: (*a*) a lower lateral system in the plane of the bottom chords of the end spans and the arch tie of the center span; (*b*) a lateral system in the plane of the truss bottom chord of the arch span; and (*c*) a top lateral system in the plane of the top chord of the three spans.

The bottom lateral system was designed as a three-span continuous truss, with the bottom chords of the end spans and the arch ties of the center span acting as chords for the lateral truss. This system comprises a rigid single plane with diagonals of double-angle section proportioned to take either tension or compression. Except for the six central panels, where carbon steel was used, all diagonals of this lateral system are silicon steel. In panels L8′–L9′ and L9′–L10′, adjacent to pier 19, special diagonals were provided to transmit all longitudinal forces of the structure to the vertical pin anchor.

The lateral system in the plane of the bottom chord of the arch truss was provided primarily for the purpose of bracing the chord. This system extends only from L11 to L11′, because of clearance requirements. Chord bracing in other panels is provided by the sway frames in the planes of L10–U10, L11–U11, and L10–U11.

The top lateral system was designed as a three-span continuous truss with the spans equivalent to the developed lengths of the top chord plane. A rigid, double-plane, double-web system was used with H-section diagonals proportioned to resist either tension or compression.

Nominal sway bracing is provided in the plane of all vertical posts and at hanger U11–L11. The H-sections were used for all sway bracing diagonals. Portal frames in the plane of the end posts and in the plane of the vertical posts at intermediate piers, L9–U9, were designed to transmit computed lateral loads to the foundation. To obtain the desired rigidity, and, because of esthetic considerations, the struts and diagonals of these frames have fairly massive sections. Deep plate girder sections were used for end portal framing. Box girder sections were used for the intermediate portals.

Main Trusses.—Truss chords, arch ties, and some of the heavier diagonals are made up of box girder sections consisting of two I-beams, top and bottom cover plates, and additional side plates as required. Handholes, 9 in. by 18 in. spaced at about 3-ft 6-in. centers, were provided to permit riveting and painting. All other diagonals are also box sections consisting of channels and plates, with handholes 6 in. by 12 in., at 3-ft centers. Box sections were used throughout, to the complete exclusion of laced members. Nearly all truss members are silicon steel. Carbon steel was used for such details, as diaphragms, fills, connection angles, etc., and for members of minimum section, where size of material rather than stress governed. In proportioning the members, the material in line of the handholes was considered ineffective in carrying the primary stresses. All posts and hangers are made up of single-web girders, consisting of four bulb angles and a web plate.

Bearings.—Substantially to eliminate pier torsion, which would have resulted with fixed shoes, because of lateral deformation of the lower lateral system under transverse wind loads, expansion-type bearings are provided at all supports. Longitudinal forces of the entire three-span structure are transmitted to pier 19 through a large pintle pin anchored in an encased structural frame embedded in the pier. The forged steel pin has a 5-in.-diameter hollow center, is 4 ft 4½ in. long, and varies in outside diameter from 12 in. at the top to 8 in. at the bottom. The upper end, 13 in. in diameter, extends 9 in. into the double floor beam of the superstructure to engage a 5-in.-thick bearing plate on the floor beam.

Expansion shoes at both ends of the central arch span are each supported on a nest of five 12-in.-diameter alloy steel rollers 5 ft 8 in. long. Rotation resulting from span deflection is provided through a line bearing detail connecting upper and lower sections of the shoe rather than by the conventional pin type arrangement. The shoes are 6 ft 8 in. square and 5 ft 10 in. high.

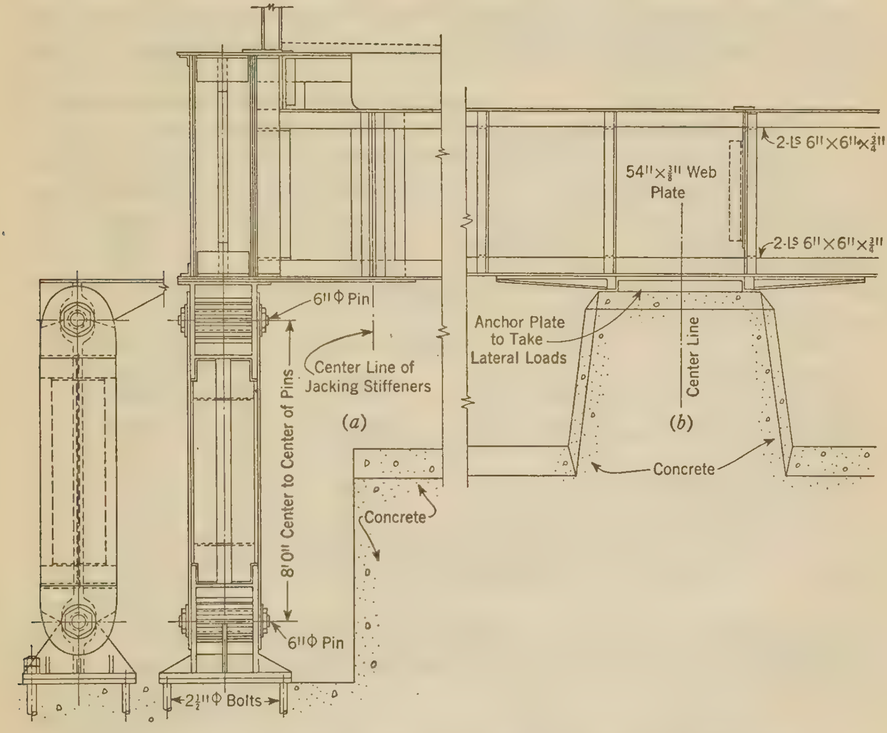

6″Φ Pin
Center Line of Jacking Stiffeners
8′0″ Center to Center of Pins
(a)
Concrete
6″Φ Pin
2½″Φ Bolts
2-Ls 6″×6″×¾″
54″×⅜″ Web Plate
2-Ls 6″×6″×¾″
Anchor Plate to Take Lateral Loads
Center Line
(b)
Concrete

FIG. 4.—EXPANSION ROCKER COLUMNS AND SPECIAL GUIDES FOR TRANSMISSION OF TRANSVERSE WIND SHEARS FROM END FLOOR BEAMS TO PIERS 17 AND 20

A number of factors governed the design of bearings for ends of the three-span unit. Provision had to be made for: (1) The lowering of panel point L0, during erection operations, 3.56 ft below its final position; (2) a tie-down anchorage to resist a 125.5-kip uplift during cantilevering operations; (3) a 403-kip positive reaction for final dead plus live load; (4) an expansion of 1,190 ft of structure; and (5) the transmission of a transverse horizontal wind shear of 159 kips from the superstructure to the pier.

During erection the ends of the trusses were supported and tied down by two 4-in.-diameter anchor bolts embedded in the pier. The raising and lowering of the end of the span was controlled by a 250-ton hydraulic jack placed between pairs of beams engaging these bolts. After erection was completed, rocker columns 8 ft high (see Fig. 4(*a*)) were inserted to act as bearings for the ends of the span. These care for expansion movements of the span with only a nominal change in roadway elevation, and were designed to take both positive and negative reactions. Transverse wind shears are transmitted to a built-up section of the pier through special guides in the plane of the lower flanges of the end floor beams.

Deflection and Camber.—The maximum dead load deflection of 15⅛ in. occurs at the center line of the arch span.

Materials and Stresses.—Properties of the principal materials used in the superstructure, together with the design stresses used for normal dead, live, and impact loadings, are given in Table 1. For stresses caused by gravity loads in combination with lateral or longitudinal loads, and for erection stresses, overstress was permitted in accordance with the following schedule:

TABLE 1.—PROPERTIES AND DESIGN STRESSES OF PRINCIPAL SUPERSTRUCTURE MATERIALS, IN POUNDS PER SQUARE INCH

Property or design stress	Carbon steel	Silicon Steel
ASTM[a] designation	A7–39	A94–39
Minimum tensile strength	60,000	80,000
Minimum yield point	30,000	45,000
Axial tension, net section	18,000	24,000
Axial compression, gross section	$15{,}000 - 0.25\left(\frac{L}{r}\right)^2$	$20{,}000 - 0.46\left(\frac{L}{r}\right)^2$
Bending on Extreme Fiber:		
Compression	$18{,}000 - 5.0\left(\frac{L}{b}\right)^2$	$24{,}000 - 6.67\left(\frac{L}{b}\right)^2$
Tension, net section	18,000	24,000
Pins	27,000	36,000
Shear:		
Girder webs, gross section	11,000	14,000
Pins and shop-driven rivets	13,500	18,000
Power-driven rivets	11,000	....
Bearing:		
Pins and shop-driven rivets	27,000	....
Power-driven rivets	22,500	....

[a] American Society for Testing Materials.

Loads	Permissible overstress, % of normal
Dead load + 30-lb wind	0
Dead load + live load + impact + 30-lb wind	25
Dead load + 50-lb wind	50
Erection stresses	50

Compression Members.—In compression members consisting of segments connected by cover plates or of segments connected by webs, the thickness of the segment webs was limited to a minimum of 1/28 of the unsupported distance of plate for silicon steel, and 1/32 of that for carbon steel. Cover plates connecting the segments were limited in thickness to a minimum of 1/36 of the unsupported distance of the plate for silicon steel, and 1/40 of that for carbon steel.

DESIGN LOADS

Dead Load.—The dead load per linear foot of bridge is 8,545 lb for the arch span and 6,745 lb for the end spans, and is made up as follows:

Load	Arch span (lb)	End spans (lb)
Concrete roadway and sidewalk	2,540	2,540
Curbs and railing	210	210
Floor system	845	845
Truss steelwork (average)	4,950	3,150
Total	8,545	6,745

For final stress calculations, the actual estimated weights of truss members, lateral system, and other component parts were properly distributed among the various panel points.

Live Load.—The floor system was designed for an H-20 loading on 9-ft lanes in accordance with the 1935 AASHO specifications. This loading consists of a 20-ton truck followed and preceded by 15-ton trucks spaced 30 ft apart. The main trusses were designed for the same loading, except that the lanes were assumed to be 12 ft wide. Such a modification of the design loading was deemed justifiable since a concentration of a great number of exceptionally heavy trucks, required to produce maximum stresses in the various truss members of the long spans, is highly improbable. Equivalent uniform loads were used for the truss design.

Impact.—For dynamic vibratory and impact effects, stresses produced by live load were increased in accordance with the formula: $I = \frac{50}{L + 125}$, in which I is the impact fraction, and L is the length, in feet, of the section of the span which is loaded to produce the maximum stress in the member.

Sidewalk Load.—The sidewalk slab and the immediate supports were designed to carry 100 lb per sq ft. For design of the trusses, 50 lb per sq ft was used.

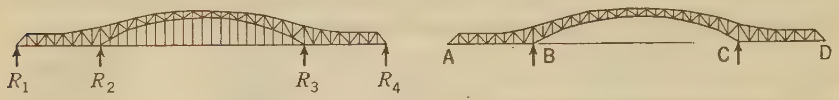
Wind Load.—Wind forces on the structure were assumed as a moving horizontal load equal to 30 lb per sq ft on one and one half times the area of the structure as seen in elevation. The lateral force due to the moving live load and wind force against this load were considered as 200 lb per lin ft of bridge, acting 6 ft above the roadway. Stability of the structure and of the stresses in the members was also checked for a 50-lb wind pressure.

Longitudinal Forces.—Provision was made for the effect of a longitudinal force equivalent to 10% of the live load on the structure, acting 4 ft above the floor. Appropriate provisions were also made for longitudinal wind loads.

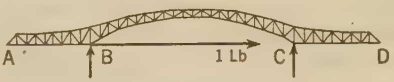
CALCULATION OF STRESSES

The continuous tied arch unit is statically indeterminate in the third degree, requiring for its solution three elastic equations. For the analysis of the structure the theory of redundant members was used.

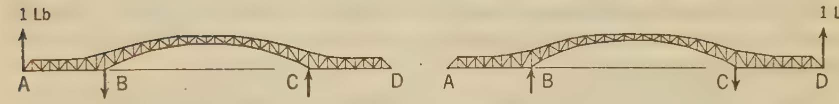
As shown in Fig. 5(*b*), the end reactions and the arch tie were assumed to be the redundant forces. With these forces removed, the truss is a simple span with two end cantilevers, in which the stresses can be calculated readily for any given loading. Referring to Fig. 5(*b*), and with the redundants removed as shown, let S' be the stress in each member due to a given loading; u be the stress due to a horizontal unit load applied as shown in Fig. 5(*c*); v be the stress due to a vertical unit load applied as shown in Fig. 5(*d*); w be the stress due to a vertical unit load applied as shown in Fig. 5(*e*); L be the length of each member, in inches; E be the modulus of elasticity of the material, in pounds per square inch; and A be the average gross area of each member, in square inches. The deflections and horizontal movements due to unit loads can be expressed as in Table 2, in which the first line, for example, denotes vertical deflection at point A due to a horizontal unit load applied as shown in Fig. (5*c*), and is represented by $\sigma\, u\, a = \Sigma\, u\, v\, L/(E\, A)$.



(*a*) BRIDGE FRAME (*b*) REDUNDANTS REMOVED



(*c*) UNIT TENSION IN TIE



(*d*) UNIT LOAD AT LEFT END (*e*) UNIT LOAD AT RIGHT END

Fig. 5.—Methods of Loading to Determine Unit Deflections at Various Points

TABLE 2.—Deflections and Movements Due to the Unit Loadings Shown in Fig. 5

Direction of movement (1)	Point (2)	Method of loading (3)	Designation (4)	Eq. No. (5)
Vertical . . .	A	(See Fig. 5(*c*))	$\sigma\, u\, a = \Sigma\, u\, v\, L/(E\, A)$	1
Horizontal.	B	(See Fig. 5(*c*))	$\sigma\, u\, b = \Sigma\, u^2\, L/(E\, A)$	2
Vertical . . .	D	(See Fig. 5(*c*))	$\sigma\, u\, d = \Sigma\, u\, w\, L/(E\, A)$	3
Vertical . . .	A	(See Fig. 5(*d*))	$\sigma\, v\, a = \Sigma\, v^2\, L/(E\, A)$	4
Horizontal.	B	(See Fig. 5(*d*))	$\sigma\, v\, b = \Sigma\, u\, v\, L/(E\, A)$	5
Vertical . . .	D	(See Fig. 5(*d*))	$\sigma\, v\, d = \Sigma\, v\, w\, L/(E\, A)$	6
Vertical . . .	A	(See Fig. 5(*e*))	$\sigma\, w\, a = \Sigma\, v\, w\, L/(E\, A)$	7
Horizontal.	B	(See Fig. 5(*e*))	$\sigma\, w\, b = \Sigma\, u\, w\, L/(E\, A)$	8
Vertical . . .	D	(See Fig. 5(*e*))	$\sigma\, w\, d = \Sigma\, w^2\, L/(E\, A)$	9
Vertical . . .	A	Any given load	$\sigma\, a = \Sigma S'\, v\, L/(E\, A)$	10
Horizontal.	B	Any given load	$\sigma\, b = \Sigma S'\, u\, L/(E\, A)$	11
Vertical . . .	D	Any given load	$\sigma\, d = \Sigma S'\, w\, L/(E\, A)$	12

In accordance with the law of reciprocal deflections, however,

$$\sigma\, u\, a = \sigma\, v\, b \qquad (13a)$$

$$\sigma\, u\, d = \sigma\, w\, b \qquad (13b)$$

$$\sigma\, v\, d = \sigma\, w\, a \qquad (13c)$$

Furthermore, because of symmetry of the structure,

$$\sigma\, u\, a = \sigma\, u\, d \tag{14a}$$

$$\sigma\, v\, a = \sigma\, w\, d \tag{14b}$$

To satisfy the condition of zero deflection at point A and point D and no movement at point B, the following expressions are developed:

$$R_1\, \sigma\, v\, a + H\, \sigma\, u\, a + R_4\, \sigma\, w\, a + \sigma\, a = 0 \tag{15a}$$

$$R_1\, \sigma\, v\, b + H\, \sigma\, u\, b + R_4\, \sigma\, w\, b + \sigma\, b = 0 \tag{15b}$$

$$R_1\, \sigma\, v\, d + H\, \sigma\, u\, d + R_4\, \sigma\, w\, d + \sigma\, d = 0 \tag{15c}$$

in which R_1 and R_4 are the final end reactions and H is the final stress in the arch tie, for any given loading.

Substituting in these equations the values of Eqs. 13 and 14, Eqs. 15 can be rewritten as:

$$R_1\, \sigma\, v\, a + H\, \sigma\, u\, a + R_4\, \sigma\, v\, d + \sigma\, a = 0 \tag{16a}$$

$$R_1\, \sigma\, u\, a + H\, \sigma\, u\, b + R_4\, \sigma\, u\, a + \sigma\, b = 0 \tag{16b}$$

$$R_1\, \sigma\, v\, d + H\, \sigma\, u\, a + R_4\, \sigma\, v\, a + \sigma\, d = 0 \tag{16c}$$

in which there are three unknowns—R_1, the final left end reaction for a given loading; R_4, the final right end reaction; and H, the final stress in the arch tie. The other components of the equations are constants. Thus, $\sigma\, v\, a$, $\sigma\, u\, a$, $\sigma\, v\, d$, and $\sigma\, u\, b$ are constants based on the geometry of the statically determinate structure (Fig. 5(*b*)) and the areas of members, and $\sigma\, a$, $\sigma\, b$, and $\sigma\, d$ are constants based on the position and magnitude of a given loading on the statically determinate structure.

Direct general expressions for the redundants can be developed from Eqs. 16. Such expressions, however, would be quite involved, and would serve no practical purpose. It is simpler and more expedient first to determine the numerical values of $\sigma\, v\, a$, $\sigma\, u\, b$, $\sigma\, u\, a$, and $\sigma\, v\, d$, and then to solve the simultaneous equations, thus obtaining general expressions for the redundants in terms of σa, $\sigma\, b$, and $\sigma\, d$.

Computations leading to determination of values of $\sigma\, v\, a$, $\sigma\, u\, b$, $\sigma\, u\, a$, and $\sigma\, v\, d$ are shown in Table 3. Dividing the totals of Cols 5, 7, 8, and 10 by E (equal to 30,000,000 lb per sq in.) gives the following:

$$\sigma\, v\, a = 0.0002538867 \text{ in.} \qquad \sigma\, u\, a = -0.000106734 \text{ in.}$$
$$\sigma\, u\, b = 0.0000730703 \text{ in.} \qquad \sigma\, v\, d = 0.000143508 \text{ in.}$$

Using these numerical values, Eqs. 16 become:

$$0.0002538867\, R_1 - 0.000106734\, H + 0.000143508\, R_4 + \sigma\, a = 0 \tag{17a}$$

$$-0.000106734\, R_1 + 0.0000730703\, H - 0.000106734\, R_4 + \sigma\, b = 0 \tag{17b}$$

$$0.000143508\, R_1 - 0.000106734\, H + 0.0002538867\, R_4 + \sigma\, d = 0 \tag{17c}$$

from which:

$$R_1 = -\,10{,}372.1\,\sigma\,a - 17{,}067.78\,\sigma\,b - 1{,}312.5\,\sigma\,d \dots\dots\dots (18)$$

$$H = -\,17{,}067.7\,\sigma\,a - 63{,}545.5\,\sigma\,b - 17{,}067.7\,\sigma\,d \dots\dots\dots (19)$$

$$R_4 = -\,1{,}312.5\,\sigma\,a - 17{,}067.7\,\sigma\,b - 10{,}372.1\,\sigma\,d \dots\dots\dots (20)$$

For symmetrical loading, such as for dead load, $\sigma\,a = \sigma\,d$, and, consequently:

$$R_1 = R_4 = -\,11{,}684.6\,\sigma\,a - 17{,}067.7\,\sigma\,b \dots\dots\dots\dots (21)$$

and

$$H = -\,34{,}035.4\,\sigma\,a - 63{,}545\,\sigma\,b \dots\dots\dots\dots\dots (22)$$

With these reactions determined, R_2 and R_3 were obtained by statics, and influence lines were developed for the reactions, as well as for the stresses in individual truss members. From Eqs. 18, 19, and 20, other pertinent proper-

TABLE 3.—COMPUTATIONS FOR VALUES OF $\sigma\,v\,a$, $\sigma\,u\,b$, $\sigma\,u\,a$, AND $\sigma\,v\,d$

Member	Length, L (in.)	Area, A (sq in.)	$\sigma\,v\,a\,E$		$\sigma\,u\,b\,E$		$\sigma\,u\,a\,E$	$\sigma\,v\,d\,E$	
			v	$\frac{v^2 L}{A}$	u	$\frac{u^2 L}{A}$	$\frac{u\,v\,L}{A}$	w	$\frac{w\,v\,L}{A}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
L0-U1......	665	86.5	−1.39	14.85	....	....	....	....	....
U1-U3......	922	67.0	−1.92	50.70	....	....	....	....	....
U3-U5......	923	79.5	−3.69	158.00	....	....	....	....	....
U5-U7......	927	89.0	−4.82	242.0	....	....	....	....	....
U7-U9......	944	108.0	−5.08	225.0	....	....	....	....	....
L0-L2......	922	67.0	+0.96	12.67	....	....	....	....	....
L2-L4......	922	71.0	+2.88	107.50	....	....	....	....	....
L4-L6......	922	80.7	+4.44	225.00	....	....	....	....	....
L6-L8......	922	93.5	+5.05	251.00	....	....	....	....	....
L8-L9......	461	111.5	+4.94	101.00	....	....	....	....	....
U1-L2......	665	42.4	+1.39	30.30	....	....	....	....	....
L2-U3......	665	42.4	−1.39	30.30	....	....	....	....	....
U3-L4......	665	42.4	+1.16	21.10	....	....	....	....	....
L4-U5......	695	46.5	−1.13	19.10	....	....	....	....	....
U5-L6......	695	52.0	+0.506	3.42	....	....	....	....	....
L6-U7......	788	47.2	−0.466	3.62	....	....	....	....	....
U7-L8......	788	57.3	−0.098	0.13	....	....	....	....	....
L8-U9......	958	50.7	+0.086	0.16	....	....	....	....	....
U9-L9......	840	160	−0.93	4.54	+0.48	1.21	− 2.34	−0.41	+ 2.00
U9-U11.....	960	100.0	−5.47	287.00	+0.302	0.876	− 15.85	−0.288	+ 15.10
U11-U13....	944	78.0	−6.10	450.00	+1.02	12.60	− 75.20	−0.963	+ 71.15
U13-U15....	934	78.0	−6.76	547.00	+1.88	42.30	−152.00	−1.99	+161.00
U15-U17....	925	93.2	−7.36	537.0	+2.86	81.30	−208.70	−3.44	+251.00
U17-U19....	922	144.5	−7.75	383.0	+3.90	97.20	−192.50	−5.37	+265.00
U19-U′19...	922	169.6	−7.20	282.5	+4.42	106.20	−173.40	−7.20	+282.50
U′19-U′17...	922	144.5	−5.37	184.0	+3.90	97.20	−133.50	−7.75	+265.00
U′17-U′15...	925	93.2	−3.44	117.6	+2.86	81.30	− 97.80	−7.36	+252.00
U′15-U′13...	934	78.0	−1.99	47.4	+1.88	42.30	− 44.80	−6.76	+161.00
U′13-U′11...	944	78.0	−0.963	11.2	+1.02	12.60	− 11.88	−6.10	+ 71.15
U′11-U′9....	960	100.0	−0.288	0.8	+0.302	0.876	− 0.83	−5.47	+ 15.10
U′9-L′9.....	840	160.0	−0.41	0.89	+0.48	1.21	− 1.03	−0.93	+ 2.00
L9-L10.....	511	254.0	+5.46	60.0	−1.11	2.485	− 12.20	0	0
L10-L12....	997	199.0	+5.89	173.7	−1.71	14.63	− 50.40	+0.59	+ 17.40
L12-L14....	970	150.0	+6.42	266.5	−2.46	39.20	−102.00	+1.42	+ 58.85
L14-L16....	946	92.0	+6.90	489.0	−3.32	113.2	−235.50	+2.62	+186.00
L16-L18....	931	77.2	+7.30	643.0	−4.24	217.0	−373.50	+4.15	+365.50
L18-L20....	924	71.2	+7.30	694.0	−5.05	331.0	−478.00	+6.11	+580.00
L20-L′18 ...	924	71.2	+6.11	484.0	−5.05	331.0	−400.00	+7.30	+580.00
L′18-L′16...	931	77.2	+4.15	208.0	−4.24	217.0	−212.50	+7.30	+365.50
L′16-L′14...	946	92.0	+2.62	70.6	−3.32	113.2	− 89.50	+6.90	+186.00

TABLE 3.—(*Continued*)

Member	Length, L (in.)	Area, A (sq in.)	$\sigma v a E$		$\sigma u b E$		$\sigma u a E$	$\sigma v d E$	
			v	$\frac{v^2 L}{A}$	u	$\frac{u^2 L}{A}$	$\frac{u v L}{A}$	w	$\frac{w v L}{A}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
L′14–L′12...	970	150.0	+1.42	13.0	−2.46	39.2	− 22.50	+6.42	+ 58.85
L′12–L′10...	997	199.0	+0.59	1.7	−1.71	14.63	− 5.04	+5.89	+ 17.40
L′10–L′9....	511	254.0	0	0	−1.11	2.48	0	+5.46	0
U9–L10.....	773	46.5	+0.561	5.23	−0.487	3.94	− 4.54	+0.40	+ 3.73
L10–U11....	995	91.6	−0.396	1.71	+0.643	4.60	− 2.81	−0.629	+ 2.71
U11–L12....	681	42.4	+0.707	8.03	−0.597	5.73	− 6.78	+0.582	+ 6.61
L12–L13....	840	79.3	−0.275	0.80	+0.646	4.43	− 1.88	−0.760	+ 2.21
U13–L14....	615	42.4	+0.755	8.28	−0.698	7.08	− 7.65	+0.806	+ 8.84
L14–U15....	719	70.1	−0.104	0.11	+0.586	3.52	− 0.62	−0.895	+ 0.95
U15–L16....	574	42.4	+0.726	7.14	−0.791	8.48	− 7.78	+1.113	+ 10.96
L16–U17....	630	61.2	+0.164	0.28	+0.451	2.09	+ 0.76	−0.952	− 1.61
U17–L18....	551	42.4	+0.637	5.28	−0.842	9.21	− 6.97	+1.468	+ 12.17
L18–U19....	567	45.2	+0.550	3.77	+0.187	0.44	+ 1.28	−0.894	− 6.15
U19–L20....	544	39.7	−0.141	0.27	−0.437	2.62	+ 0.86	+1.30	− 2.55
L20–U′19....	544	39.7	+1.30	23.15	−0.437	2.62	− 8.15	−0.141	− 2.55
U′19–L′18...	567	45.2	−0.894	10.00	+0.187	0.44	− 2.31	+0.550	− 6.15
L′18–U′17...	551	42.4	+1.468	28.00	−0.842	9.21	− 16.10	+0.637	+ 12.17
U′17–L′16...	630	61.2	−0.952	9.32	+0.451	2.09	− 4.42	+0.164	− 1.61
L′16–U′15...	574	42.4	+1.113	16.80	−0.791	8.48	− 11.95	+0.726	+ 10.96
U′15–L′14...	719	70.1	−0.895	8.20	+0.586	3.52	− 5.35	−0.104	+ 0.95
L′14–U′13...	615	42.4	+0.806	9.44	−0.698	7.08	− 8.17	+0.755	+ 8.84
U′13–L′12...	840	79.3	−0.760	6.12	+0.646	4.43	− 5.20	−0.275	+ 2.21
L′12–U′11...	681	42.4	+0.582	5.43	−0.597	5.73	− 5.58	+0.707	+ 6.61
U′11–L′10...	995	91.6	−0.629	4.30	+0.643	4.60	− 4.45	−0.396	+ 2.71
L′10–U′9....	773	46.5	+0.400	2.66	−0.487	3.94	− 3.24	+0.561	+ 3.73
L9–T10.....	461	142.5	....	....	1.0	3.24	....	....	....
T10–T11....	461	137.5	....	....	1.0	3.35	....	....	....
T11–T16....	2,305	127.5	....	....	1.0	18.10	....	....	....
T16–T18....	922	137.5	....	....	1.0	6.70	....	....	....
T18–T′18....	1,844	142.5	....	....	1.0	12.84	....	....	....
T′18–T′16...	922	137.5	....	....	1.0	6.70	....	....	....
T′16–T′11...	2,305	127.5	....	....	1.0	18.10	....	....	....
T′11–T′10...	461	137.5	....	....	1.0	3.35	....	....	....
T′10–L′9....	461	142.5	....	....	1.0	3.24	....	....	....
Total.....	....	....	....	7,616.60	....	2,192.10	−3,202.02	....	4,305.24

ties of the structure are readily obtainable. Thus, for a 1-in. settlement at point A, with $\sigma b = 0$ and $\sigma d = 0$, $R_1 = -10{,}372.1$ lb; $H = -17{,}067.7$ lb; and $R_4 = -1{,}312.5$ lb. Similarly, for a 1-in. horizontal displacement at point B, with $\sigma a = 0$ and $\sigma d = 0$, $R_1 = -17{,}067.7$ lb; $H = -63{,}545.5$ lb; and $R_4 = -17{,}067.7$ lb. Also, with $\sigma a = 0$ and $\sigma d = 0$, for $H = 1$ lb, $R_1 = R_4 = \frac{17{,}067.7}{63{,}545.5} = 0.269$ lb. Other reactions, as well as stresses, can be obtained by statics. These relations were important for studying the effect of settlement of supports, as well as for determination of erection stresses through the various stages and operations.

As is necessary for statically indeterminate structures, the areas of truss members had to be assumed or estimated before a stress analysis could be made. Furthermore, any effect the method of erection might have on the size of truss members had to be predetermined, since any subsequent change of member area for erection conditions would have unbalanced the system. Since a cantilever erection method was indicated, it was apparent that some of the members

TABLE 4.—STRESSES,[a] IN KIPS, AND GENERAL

Member	STATIC LOADS				Erection	Erection wind	Col. 6 plus Col. 7	DESIGN FOR		EFFECTIVE AREA		$\frac{L}{r}$
	Dead	Live	Impact	Total				Col.	Stress	Gross	Net	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
U1–U3...	242 *C*	200 *C*	19 *C*	461 *C*	500 *T*	390 *T*	890 *T*	5	461 *C*	64.13	52.7	45
U3–U5...	44 *C*	250 *C*	28 *C*	322 *C*	1,260 *T*	444 *T*	1,704 *T*	8	1,704 *T*	77.04	63.79	...
U11–U13.	428 *T*	191 *T*	18 *T*	637 *T*	1,730 *T*	422 *T*	2,152 *T*	8	2,152 *T*	74.79	61.54	...
U13–U15.	430 *C*	218 *C*	22 *C*	670 *C*	1,240 *T*	385 *T*	1,625 *T*	8	1,625 *T*	74.79	61.54	...
L0–U1...	256 *C*	172 *C*	17 *C*	445 *C*	....	....	....	5	445 *C*	74.79	60.29	37
L2–L4...	197 *T*	240 *T*	27 *T*	464 *T*	945 *C*	208 *C*	1,153 *C*	8	1,153 *C*	68.97	55.4	45
L18–L20.	465 *T*	237 *T*	23 *T*	725 *T*	....	....	....	5	725 *T*	68.97	55.40	...
L4–L6...	203 *C*	205 *C*	20 *C*	428 *C*	1,613 *C*	287 *C*	1,900 *C*	8	1,900 *C*	78.10	....	45
L6–L8...	820 *C*	245 *C*	42 *C*	1,107 *C*	2,100 *C*	487 *C*	2,587 *C*	8	2,587 *C*	89.85	....	45
L16–L18.	350 *C*	240 *C*	23 *C*	613 *C*	393 *C*	....	....	5	613 *C*	74.91	....	45
U5–U7...	496 *T*	221 *T*	25 *T*	742 *T*	1,870 *T*	637 *T*	2,507 *T*	8	2,507 *T*	85.96	71.6	...
U15–U17.	1,335 *C*	359 *C*	36 *C*	1,730 *C*	692 *T*	....	....	5	1,730 *C*	90.00	81.00	44
L14–L16.	1,238 *C*	376 *C*	36 *C*	1,650 *C*	951 *C*	....	....	5	1,650 *C*	91.21	....	46
U7–U9...	1,137 *T*	298 *T*	37 *T*	1,472 *T*	2,260 *T*	776 *T*	3,036 *T*	8	3,036 *T*	104.59	87.09	...
U9–U11..	1,126 *T*	281 *T*	27 *T*	1,434 *T*	2,210 *T*	910 *T*	3,120 *T*	8	3,120 *T*	97.72	80.97	...
L8–L9...	1,380 *C*	353 *C*	34 *C*	1,767 *C*	2,360 *C*	440 *C*	2,800 *C*	8	2,800 *C*	107.51	....	45
U17–U19.	2,160 *C*	468 *C*	47 *C*	2,675 *C*	129 *T*	....	....	5	2,675 *C*	140.26	....	43
L9–T10..	2,005 *T*	439 *T*	42 *T*	2,486 *T*	928 *T*[e]	2,933 *T*[f]	....	7	2,933 *T*	....	119.37	...
L12–L14.	2,105 *C*	520 *C*	50 *C*	2,675 *C*	1,490 *C*	....	....	5	2,675 *C*	145.26	....	48
T10–T11.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	720 *T*[e]	2,725 *T*[f]	....	7	2,725 *T*	....	115.14	...
T11–T14.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	544 *T*[e]	2,549 *T*[f]	....	7	2,549 *T*	....	106.64	...
T14–T16.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	466 *T*[e]	2,471 *T*[f]	....	7	2,471 *T*	....	106.64	...
T16–T18.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	617 *T*[e]	2,622 *T*[f]	....	7	2,622 *T*	....	115.14	...
T18–T20.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	683 *T*[e]	2,688 *T*[f]	....	7	2,688 *T*	....	119.39	...
L0–L2...	177 *T*	119 *T*	13 *T*	309 *T*	....	....	....	5	340 *T*[g]	64.13	52.7	...
U19–U'19	2,565 *C*	539 *C*	52 *C*	3,156 *C*	....	....	....	5	3,156 *C*	165.26	....	43
L9–L10..	3,788 *C*	762 *C*	73 *C*	4,623 *C*	2,630 *C*	....	...	5	4,643 *C*	248.72	....	46
L10–L12.	2,920 *C*	628 *C*	60 *C*	3,608 *C*	2,000*C*	....	....	5	3,608 *C*	194.91	....	46
U1–L2...	94 *T*	131 *T*	15 *T*	240 *T*	365 *C*	85 *C*	450 *C*	8	450 *C*	38.81	....	94
L2–U3...	64 *T*	94 *T*	23 *T*	181 *T*	450 *T*	85 *T*	535 *T*	8	535 *T*	....	32.81	...
U3–L4...	223 *C*	113 *C*	23 *C*	359 *C*	462 *C*	82 *C*	544 *C*	8	544 *C*	38.81	....	94
L6–U7...	572 *T*	171 *T*	23 *T*	766 *T*	....	....	....	5	766 *T*	....	36.06	...
U9–L10..	521 *T*	141 *T*	14 *T*	682 *T*	....	....	....	5	682 *T*	....	36.06	...
U11–L12.	437 *T*	150 *T*	17 *T*	604 *T*	....	....	....	5	604 *T*	....	32.81	...
U13–L14.	494 *T*	144 *T*	20 *T*	658 *T*	....	....	....	5	658 *T*	....	32.81	...
U15–L16.	520 *T*	156 *T*	21 *T*	697 *T*	....	....	....	5	697 *T*	....	32.81	...
U17–L18.	478 *T*	172 *T*	17 *T*	667 *T*	....	....	....	5	667 *T*	....	32.81	...
L18–U19.	301 *C*	190 *C*	18 *C*	509 *C*	....	....	....	5	509 *C*	38.81	....	81
U19–L20.	154 *T*	174 *T*	17 *T*	345 *T*	....	....	....	5	366 *T*[d]	....	30.56	...
L4–U5...	374 *T*	145 *T*	25 *T*	544 *T*	....	....	....	5	544 *T*	....	36.00	...
U5–L6...	443 *C*	145 *C*	22 *C*	610 *C*	....	....	....	5	610 *C*	47.76	....	91
U7–L8...	472 *C*	153 *C*	17 *C*	642 *C*	....	....	....	5	642 *C*	53.26	....	94
L8–U9...	590 *T*	179 *T*	20 *T*	789 *T*	....	....	....	5	789 *T*	....	39.26	...
L14–U15.	684 *C*	198 *C*	27 *C*	909 *C*	....	....	....	5	909 *C*	64.56	....	86
L16–U17.	559 *C*	193 *C*	19 *C*	771 *C*	...	....	....	5	771 *C*	55.56	....	82
L12–U13.	734 *C*	200 *C*	20 *C*	954 *C*	....	....	....	5	954 *C*	73.65	....	97
L10–U11.	813 *C*	216 *C*	21 *C*	1,050 *C*	....	....	....	5	1,050 *C*	85.90	....	106
U9–L9...	963 *C*	263 *C*	29 *C*	1,255 *C*	....	....	....	5	1,228 *C*	132.38	....	...
U1–L1, U3–L3, U5–L5, U7–L7, U15–L15, U17–L17, U19–L19...............									167 *T*	20.34	17.02	224
U2–L2, U4–L4, U6–L6, U14–L14, U16–L16, U18–L18, U20–L20............									167 *T*	20.34	....	203
U8–L8, U12–L12..									167 *T*	23.94	....	210
U11–L11, U13–L13..									167 *T*	23.94	....	196
L11–T11, L12–T12, L13–T13, L14–T14, L15–T15....................									167 *T*	23.94	....	284
U10–L10..									167 *T*	27.06	....	178
L16–T16, L17–T17, L18–T18, L19–T19, L20–T20....................									167 *T*	27.06	....	302
L10–T10..									....[m]	69.39	....	52

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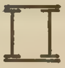
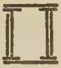
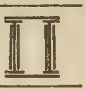
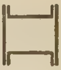
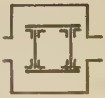
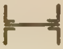
[a] *T* = tension, *C* = compression. Unit stresses as shown for members governed by static loads were re-
carbon steel, S = silicon steel. [d] Add two plates 20 × $\frac{5}{8}$ from L0 to 12 ft 6 in. up, and at lower portal connection
[g] L0–L2 section determined by Col. 5 + 15-lb wind at 1.25 normal stress. Bending at L0 = 466,000 ft-lb. [h] For
these members see Col. 1. [m] Designed for direct stress of 167 kips plus a moment of 1,250 ft-kips.

Make-Up of Continuous-Span Members

Unit Stress[b]		Material[c]	Make-Up of Section				Shape	Member
f_c	f_t		Top plate	Bottom plate	Side plates	Is, [s, or ∠s		
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
14,500	27,000	C	$30\times\frac{3}{8}$	$30\times\frac{3}{8}$	....	2-24 I 79.9		U1–U3
....	27,000	C	$32\times\frac{1}{2}$	$32\times\frac{5}{8}$	....			U3–U5
....	36,000	S	$30\times\frac{1}{2}$	$30\times\frac{5}{8}$	....			U11–U13
....	27,000	C	$30\times\frac{1}{2}$	$30\times\frac{5}{8}$	....			U13–U15
13,775	....	C	$30\times\frac{1}{2}$	$30\times\frac{5}{8}$	....[d]			L0–U1
21,750	....	C	$30\times\frac{7}{16}$	$30\times\frac{7}{16}$	....			L2–L4
....	24,000	S	$30\times\frac{7}{16}$	$30\times\frac{7}{16}$	....			L18–L20
28,600	....	S	$30\times\frac{1}{2}$	$30\times\frac{1}{2}$	....	2-24 I 90		L4–L6
28,600	....	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	....			L6–L8
14,500	....	C	$30\times\frac{7}{16}$	$30\times\frac{7}{16}$	....			L16–L18
....	36,000	S	$32\times\frac{1}{2}$	$32\times\frac{5}{8}$	....	2-24 I 95		U5–U7
19,110	....	S	$33\times\frac{1}{2}$	$33\times\frac{5}{8}$	....	2-24 I 100		U15–U17
19,000	....	S	$32\times\frac{1}{2}$	$32\times\frac{5}{8}$	....			L14–L16
....	36,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	....	2-24 I 115		U7–U9
....	36,000	S	$32\times\frac{1}{2}$	$32\times\frac{5}{8}$	....			U9–U11
28,600	....	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	....	2-24 I 120		L8–L9
19,100	....	S	$32\times\frac{3}{4}$	$33\times\frac{7}{8}$	$2\text{–}20\times\frac{5}{8}$			U17–U19
....	24,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	$4\text{–}20\times\frac{7}{16}$			L9–T10
18,900	....	S	$32\times\frac{3}{4}$	$33\times\frac{7}{8}$	$2\text{–}20\times\frac{3}{4}$			L12–L14
....	24,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	$2\text{–}20\times\frac{3}{4}$			T10–T11
....	24,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	$2\text{–}20\times\frac{1}{2}$			T11–T14
....	24,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	$2\text{–}20\times\frac{1}{2}$			T14–T16
....	24,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	$2\text{–}20\times\frac{3}{4}$			T16–T18
....	24,000	S	$32\times\frac{5}{8}$	$32\times\frac{3}{4}$	$4\text{–}20\times\frac{7}{16}$			T18–T20
....	18,000	C	$30\times\frac{3}{8}$	$30\times\frac{3}{8}$	$2\text{–}20\times\frac{3}{8}$[h]	2-24 I 79.9		L0–L2
19,100	....	S	$32\times\frac{3}{4}$	$33\times\frac{7}{8}$	$4\text{–}20\times\frac{5}{8}$	2-24 I 120		U19–U′19
18,330	....	S	$34\times\frac{3}{4}$	$34\times\frac{7}{8}$	$2\text{–}32\times\frac{3}{4}$ $2\text{–}32\times\frac{5}{8}$	2-36 I 194		L9–L10
19,000	....	S	$33\times\frac{3}{4}$	$33\times\frac{7}{8}$	$2\text{–}26\times\frac{3}{4}$ $2\text{–}26\times\frac{5}{8}$	2-30 I 132		L10–L12
19,200	....	C	....	....	$2\text{–}22\times\frac{7}{16}$	2-18 [42.7		U1–L2
....	27,000	C	....	....	$2\text{–}22\times\frac{7}{16}$			L2–U3
19,200	....	C	....	....	$2\text{–}22\times\frac{7}{16}$			U3–L4
....	21,200	S	....	....	$2\text{–}24\times\frac{1}{2}$			L6–U7
....	21,100	S	....	....	$2\text{–}24\times\frac{1}{2}$			U9–L10
....	21,200	S	....	....	$2\text{–}22\times\frac{7}{16}$			U11–L12
....	21,200	S	....	....	$2\text{–}22\times\frac{7}{16}$			U13–L14
....	21,700	S	....	....	$2\text{–}22\times\frac{7}{16}$			U15–L16
....	21,700	S	....	....	$2\text{–}22\times\frac{7}{16}$			U17–L18
14,600	....	S	....	....	$2\text{–}22\times\frac{1}{2}$			L18–U19
....	10,800	C	....	....	$2\text{–}22\times\frac{3}{8}$			U19–L20
....	18,000	C	....	....	$2\text{–}22\times\frac{1}{2}$	2-18 [45.8		L4–U5
13,400	....	S	....	....	$2\text{–}24\times\frac{9}{16}$			U5–L6
13,040	....	S	....	....	$2\text{–}26\times\frac{5}{8}$			U7–L8
....	20,500	S	....	....	$2\text{–}26\times\frac{1}{2}$	2-18 [58		L8–U9
14,000	....	S	....	....	$2\text{–}26\times\frac{3}{4}$	2-18 [58		L14–U15
14,100	....	S	....	....	$2\text{–}24\times\frac{5}{8}$			L16–U17
13,260	....	S	....	....	$2\text{–}30\times\frac{11}{16}$	2-18 I 70		L12–U13
12,400	....	S	....	....	$2\text{–}30\times\frac{5}{8}$ $2\text{–}16\times\frac{1}{2}$	2-18 I 70		L10–U11
19,480	....	S	$42\times\frac{3}{4}$	$42\times\frac{3}{4}$	$2\text{–}17\frac{3}{4}\times\frac{5}{8}$[j]	$8\text{–}5\times5\times\frac{5}{8}$ ∠		U9–L9
....	18,000	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}6\times3\frac{1}{2}$ ∠ 12.3[k]		U1–L1[l]
....	....	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}6\times3\frac{1}{2}$ ∠ 12.3[k]		U2–L2[l]
....	....	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}7\times3\frac{1}{2}$ ∠ 15.3[k]		U8–L8[l]
....	18,000	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}7\times3\frac{1}{2}$ ∠ 15.3[k]		U11–L11[l]
....	18,000	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}7\times3\frac{1}{2}$ ∠ 15.3[k]		L11–T11[l]
....	....	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}8\times3\frac{1}{2}$ ∠ 18[k]		U10–L10
....	18,000	C	$19\times\frac{5}{16}$[j]	....	....	$4\text{–}8\times3\frac{1}{2}$ ∠ 18[k]		L16–T16[l]
....	24,000	S	$17\frac{1}{2}\times\frac{3}{4}$[j]	....	$2\text{–}18\times\frac{3}{4}$	$4\text{–}8\times6\times\frac{3}{4}$ ∠		L10–T10

---

duced to compensate for bending stresses due to weight of member itself. [b] In pounds per square inch. [c] C = 8 ft below and 5 ft above portal brace. [e] 30-lb quartering wind. [f] Dead load plus 30-lb quartering wind. 10 ft from L0 only. [i] Determined by live load reversal. [j] Web plates. [k] Bulb angles. [l] For complete listing of would be governed by erection stresses. The following design procedure was, therefore, adopted:

1. Areas of members were approximated for the final structure, based on specified loading.
2. The areas were checked for erection stresses as determined by the conditions shown in Fig. 6, and increased as required.
3. The structure was analyzed for final loading stresses.

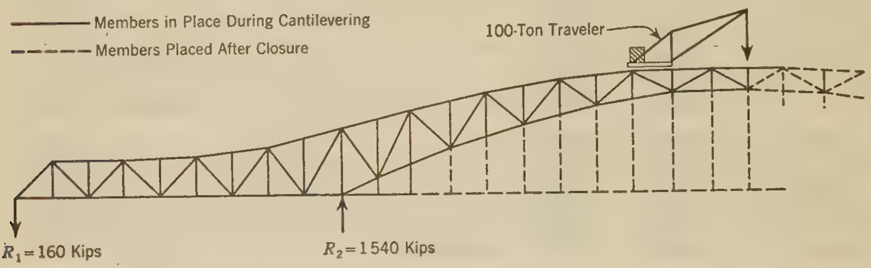

Members in Place During Cantilevering
Members Placed After Closure
100-Ton Traveler
R_1 = 160 Kips
R_2 = 1 540 Kips

Fig. 6.—Maximum Loading Assumed in Designing for Erection Stresses

A summary of the final design stresses and of the general make-up of members is given in Table 4, and a summary of dead load panel concentrations in Table 5.

TABLE 5.—Dead Load

Item	Weight, kips per lin ft	Panel								
		0	1	2	3	4	5	6	7	8
Roadway concrete	1.316	28.00	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60
Handrail and curbs	0.107	2.25	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10
Crossbeams	0.092	2.30	3.35	3.35	3.35	4.00	3.35	3.35	4.00	3.35
Stringers	0.223	4.85	8.70	8.70	8.70	9.00	8.70	8.70	9.00	12.70
Floor beam[a]	0.112	6.20	4.32	4.32	4.32	4.50	4.32	4.32	4.50	4.32
Bottom laterals[a]	0.090	1.25	2.25	2.00	2.00	2.50	3.00	3.15	3.25	6.35
Top laterals[a]	0.112	....	1.82	3.64	3.64	3.64	3.64	3.64	3.65	3.65
Sway frame[a]	0.032	....	....	4.50	0.93	4.70	0.93	5.00	0.93	0.93
Portals and arch chord laterals	0.058	2.50	8.70	....	....	....	....	....	....	....
Truss	....	15.75	32.75	32.80	34.20	36.10	39.00	42.40	46.90	53.10
Miscellaneous	....	1.20	....	....	....	0.26	....	....	....	....
Total	....	64.30	116.59	114.01	111.84	119.40	117.64	125.26	127.20	139.10

[a] One

Erection Procedure

Completion of the entire project in fifteen months was provided in the contract. Construction operations began in June, 1941. Following the attack on Pearl Harbor, Hawaii, on December 7, 1941, with the resulting emergency, construction progress was considerably slowed down, primarily because of shortages of materials and delayed deliveries. As a result, the project was finally completed in April, 1944, although the bridge was opened to traffic in August, 1943.

General Procedure.—Erection of the viaduct girders presented no particular problem. A locomotive crane, a crawler crane, and a traveling derrick were used to place steel. Detailed erection plans and a stress analysis covering all principal steps for the three-span unit were developed by the contractor and reviewed and compared with the engineers' analysis. The diagrams in Fig. 7 illustrate the general procedure.

Three-Span Continuous Truss Unit.—Erection of the trusses of the three-span unit was started at the west end span. Commencing at pier 17, the four west panels were erected on falsework bents and the balance of the truss of span 18 was then placed by cantilevering to pier 18. On completion of this work simultaneous erection operations were carried out from pier 18 eastward and from pier 19 westward and eastward.

The east half of the three-span unit was erected by balanced cantilever methods, with pier 19 acting as the sole supporting substructure unit. In addition to resting on the shoes placed on the pier, the span was temporarily supported during erection at the first panel point east of the pier on inclined steel columns framed into the pier base. Fig. 8 shows the first stage of the balanced construction operations. Working eastward from pier 19 a guy derrick mounted at floor level erected the steel of span 20 simultaneously with placement of the steel of the east half of span 19 by a 55-ton traveling derrick operating on the top chord. Prior to landing at pier 20, 699 ft of the structure with a weight of 1,520 tons, exclusive of 110 tons of erecting equipment, were balanced on pier 19.

PANEL CONCENTRATIONS

POINTS											
9	10	11	12	13	14	15	16	17	18	19	20
52.00	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60
4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10
3.35	3.35	4.00	3.35	3.35	3.35	4.00	3.35	3.35	4.00	3.35	3.35
16.70	12.70	9.00	8.70	8.70	8.70	9.00	8.70	8.70	9.00	8.70	8.70
26.00	4.32	4.50	4.32	4.32	4.32	4.50	4.32	4.32	4.50	4.32	4.32
12.40	7.00	4.15	3.50	3.25	3.15	2.75	2.50	2.50	2.50	2.25	2.00
3.70	3.70	3.65	3.65	3.65	3.64	3.64	3.64	3.64	3.64	3.64	3.64
....	6.60	6.60	8.15	1.86	5.56	1.86	5.23	1.86	5.00	1.86	5.00
33.50	....	18.20	3.64	3.64	3.64	3.64	3.64	3.64	3.64	3.64	3.64
111.90	110.90	94.10	84.00	79.70	72.90	69.80	68.80	71.50	73.10	74.30	75.90
....	....	0.28	....	....	....	0.26	....	....	0.26	....	....
263.65	203.27	182.80	174.01	163.17	159.96	154.15	154.88	154.21	160.34	156.76	161.25

half.

The west half of span 19 was placed by cantilevering out from pier 18 with a 55-ton traveling derrick mounted on the top chord. Fig. 7(*a*) shows the geometric position of the trusses at this stage. On completion of the erection of the trusses and bracing, closure was made at the lower chord panel point L20

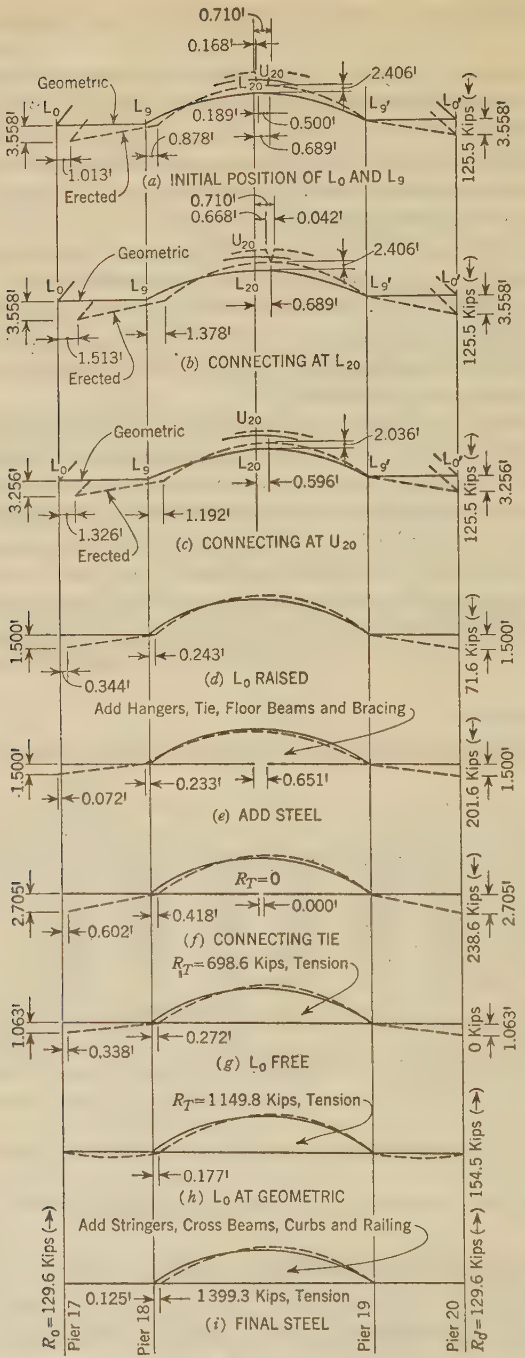

0.710'
0.168'
U20
L20
2.406'
Geometric
L0
L9
L9'
L0'
3.558'
0.189'
0.500'
0.878'
0.689'
1.013'
Erected
(a) INITIAL POSITION OF L0 AND L9
125.5 Kips (←)
3.558'
0.710'
0.668'
0.042'
U20
2.406'
Geometric
L0
L9
L20
L9'
L0'
3.558'
0.689'
1.378'
1.513'
Erected
(b) CONNECTING AT L20
125.5 Kips (←)
3.558'
U20
Geometric
2.036'
L0
L9
L20
L9'
L0'
3.256'
0.596'
1.192'
1.326'
Erected
(c) CONNECTING AT U20
125.5 Kips (←)
3.256'
1.500'
0.243'
0.344'
(d) L0 RAISED
71.6 Kips (←)
1.500'
Add Hangers, Tie, Floor Beams and Bracing
1.500'
0.072'
0.233'
0.651'
(e) ADD STEEL
201.6 Kips (←)
1.500'
RT=0
2.705'
0.602'
0.418'
0.000'
(f) CONNECTING TIE
238.6 Kips (←)
2.705'
RT=698.6 Kips, Tension
1.063'
0.338'
0.272'
(g) L0 FREE
0 Kips
1.063'
RT=1 149.8 Kips, Tension
0.177'
(h) L0 AT GEOMETRIC
154.5 Kips (→)
Add Stringers, Cross Beams, Curbs and Railing
R0=129.6 Kips (→)
0.125'
1 399.3 Kips, Tension
(i) FINAL STEEL
Pier 17
Pier 18
Pier 19
Pier 20
Rd=129.6 Kips (→)

Fig. 7.—Positions of the Trusses During Each Erection Step (Dotted Lines) Compared with Unstrained Position (Solid Lines)

followed by closure of the top chord at panel points U19 and U′19. Vertical adjustments at these points were controlled by regulating the jacks and tie downs at L0 and L′0, as shown in Figs. 7(*b*) and 7(*c*), respectively. Horizontal control was provided by jacking against the ends of the trusses at pier 17 with the shoes at pier 18 free to roll. Illustrative of the accuracy of the fabrication and erection is the fact that, upon completion of the truss erection, prior to closing, only ¼-in. discrepancy was found in the horizontal length of the trusses.

At this stage of erection the unit became a three-span continuous truss. To limit erection stresses to a predetermined economic maximum, the end reactions were raised as shown in Fig. 7(*d*), resulting in a decrease in the negative end reactions. A redistribution of stresses set up in step (*a*) was thus effected. After closure of the trusses, the hangers, floor beams, and ties were placed as the traveling derricks were backed down off the span.

During these operations all arch tie splices were riveted, except at the west end of the span at panel point T12, where final closure of the tie was to be effected. The geometric position of the trusses, and the end reaction at the end of this step are shown in Fig. 7(*e*). Fig. 9 shows the truss spans with all the hangers and ties erected just prior to closing the tie.

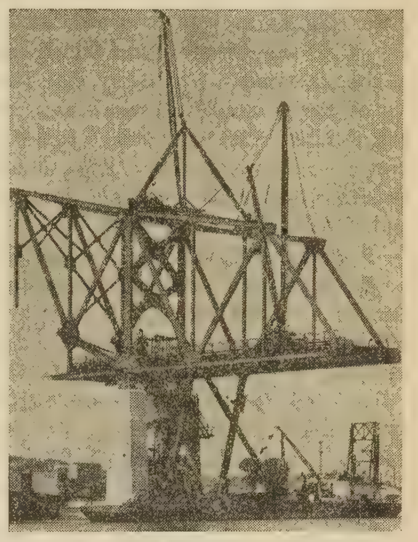


Fig. 8.—Early Stage of Balanced Erection on Pier 19, with Stability Provided Wholly by Diagonal Struts to Panel Point L′8

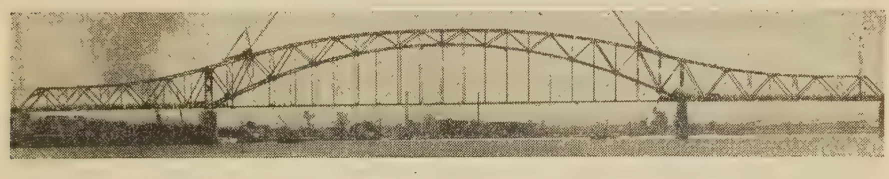


Fig. 9.—Truss Span Functioning as a Conventional Three-Span Continuous Structure Following Cantilever Closure and Prior to Final Closure of Arch Tie

The design contemplated lowering the ends of trusses, as shown in Fig. 7(*f*), a distance sufficient to shorten the arch span and to effect a closure of the tie in unstressed condition. To avoid the increased erection stresses in the top chord over the intermediate support, the contractor elected to effect a closure by a direct horizontal pull on the ties.

Closure of arch ties at this stage of erection required a 10-in. horizontal shortening of the arch span. Special jacking devices were used, incorporating two 350-ton hydraulic jacks per tie. Each pair of jacks was connected to a single pump, thus assuring a uniform pull on the component parts of each tie. The jacking device consisted of a temporary diaphragm bolted near the end across each length of unconnected tie, with connecting links extending from each diaphragm to the jacking beams. To control simultaneous closure of both trusses, the pump pressures throughout the operation were limited to a maximum difference of 50 lb per sq in. After thus shortening the arch span, the tie splices were riveted, and the structure was then ready to receive the balance of the steel and the roadway and sidewalk slab.

After the arch tie closure was made, the end truss anchorages were released, and the L0 points were jacked to final elevation. The rocker column end bearings were then installed. The amount of vertical movement and the resulting reactions, as checked by gage readings, corresponded closely to the computed end reactions.

A 16-ton floor traveler was used for the erection of the balance of steelwork on the arch span, including stringers, crossbeams, curbs, and railing. To prevent undue overstress of truss members due to unbalanced loading, symmetrical placing of concrete slab, in so far as practical, was required, conforming to a carefully devised pouring schedule.

Substructure.—With the exception of piers 18 and 19, at the ends of the arch span, the forty-two piers of the structure are of conventional type. Excavation for the construction of all piers was done in the open. Cofferdams were used in constructing bases of certain piers—timber crib cofferdams for piers 18 and 19, and steel sheet pile cofferdams for pier 17 and piers 20 to 25, inclusive.

Piers 26 to 31, inclusive, were constructed in open excavations kept essentially free of ground water by a system of well points sunk around the perimeter of the excavated area. Excavation was opened for a single trench enclosing all piers on which work was under way. After the bases of the first group of piers were completed, the excavation was extended to include a second group. Well points were advanced and the area occupied by the first group of piers was backfilled. The bases of piers 1 to 16 and piers 32 to 41 and of the two abutments are above ground-water level and were constructed in open excavation, shored only as necessary to prevent caving.

Pier 18, with base size 43 ft by 75.5 ft, is supported on three hundred and thirty-six timber piles averaging 95 ft long below cutoff level—23.5 ft below pool elevation. The bottom of base is at El. 531. Pier 19, with base size 49 ft by 75.5 ft, is supported on three hundred and eighty-four timber piles averaging 95 ft long below cutoff level. The bottom of the base is at El. 551. Greater depth of water and stronger current encountered at pier 18 required a deeper pier than at pier 19.

To provide the penetration necessary at pier 18 to withstand scour and yet to reduce the weight of the pier to a minimum, the design incorporated in the pier base a sand core 27 ft by 57.5 ft by 25 ft high. This arrangement effected a saving both in the number of supporting piles and in the volume of concrete. The 8-ft and 9-ft concrete walls surrounding the sand core, together with a cover 7 ft thick, were poured under water by a tremie. The sand core was placed simultaneously through tremie pipes so as to maintain the top of sand to conform with the rate of placing concrete walls. After pumping out the temporary cofferdam, the piles were cut off and the pier base was finished with a 12-ft thickness of concrete poured in the dry. A concrete shaft 70 ft high was then built to complete the pier.

A saving of 1,438 cu yd of concrete resulted from using the sand core instead of a solid concrete base and effected a saving of 5.1 tons per pile in dead load weight. To have constructed a solid concrete base providing the same penetration and the same maximum load per pile would have required a base 52 ft by 81 ft founded on four hundred and forty piles. On the basis of contract unit prices, such a design would have increased the cost of the pier $133,500.

Quantities and Cost

For the truss spans, 3,850,100 lb of structural silicon steel and 4,583,975 lb of carbon steel were required. In addition, 6,410,492 lb of structural steel were required in the girder spans. Also required were 2,463,500 lb of reinforcing steel, 34,087 cu yd of concrete, and 2,909 timber piles. The total cost of the project was $3,175,341.63.

Conclusions

A significant advantage of the continuous tied arch structure as compared to the conventional truss types is the economy of metal, due to a more advantageous distribution of weight for cantilever erection.

Although the subject of esthetics does not lend itself to argument based on reason only, since it is rather a matter of individual taste, an opinion is ventured that this type of structure does possess elements of beauty not found in the conventional type trusses. Similarly, the almost unobstructed river view from the main span is also a favorable feature.

The tied arch type permits erection of a self-supporting part of the structure (the trusses) spanning between piers to be utilized subsequently in completing erection of the balance of the structure. The full weight of the stress carrying framed structure must be supported on the conventional cantilever and continuous designs during erection before any closure can be made.

Acknowledgment

The Julien Dubuque Bridge is operated by the City of Dubuque Bridge Commission. The project was financed by a revenue bond issue and contributions of the state highway departments of Illinois and Iowa. The Bridge Commission was created by an Act of Congress and consists of Charles T. Landon, chairman, and Charles T. Kretschmer, William M. Clemens, Thomas M. Stampfer, and Russel E. Werner.

The cooperation and competent counsel of state and government officials were in large measure responsible for the successful completion of the project under restrictive wartime conditions. Acknowledgment is made of the able assistance of F. R. White, M. ASCE, chief engineer of the Iowa State Highway Commission; Walter A. Rosenfield, director, and W. W. Polk, M. ASCE, chief highway engineer of the Illinois Highway Commission; and Clifford Shoemaker and J. T. Voshell, Members, ASCE, district engineers of the Public Roads Administration at Kansas City, Mo., and Chicago, Ill., respectively.

The ingenious methods used by the contractors in placing foundations and in the erection of superstructure successfully accomplished the conceptions of the design. Careful and detailed analysis of all erection steps and problems and accurate fabrication of steel by the Bethlehem Steel Company were reflected in the manner in which erection proceeded. This company was represented by Jonathan Jones, M. ASCE, chief engineer; S. C. Campbell, general superintendent; and W. W. Oskin, Assoc. M. ASCE.

Contractors for the substructure, the F. J. Robers Construction Company and the La Crosse Dredging Corporation, were represented on the project by P. C. Peterson, engineer in charge. Gund-Graham Company, Melberg and Taylor, and Ulrich Willy Company were contractors for other miscellaneous work. Construction was begun on June 18, 1941, and the bridge was opened to traffic on August 31, 1943.

The bridge was planned, designed, and the construction supervised by Howard, Needles, Tammen and Bergendoff. Ned L. Ashton, Josef Sorkin, and Jacob Karol, Members, ASCE, participated in the design. Detailed plans were prepared under the supervision of C. S. Harper, M. ASCE. Ivan P. Hanson, Assoc. M. ASCE, served as resident engineer for the consultants.

DISCUSSION

Jonathan Jones,[5] M. ASCE.—This paper well exemplifies the type of descriptive account of current work which the Society should seek always to secure for its *Transactions*. Granted that current news accounts of structures in general may better be presented elsewhere, it is important that *Transactions* serve as a complete reference work for the continuing development in structure types, including their history, the advantages and disadvantages claimed, and the methods of design and execution. It is to be regretted that, in the particular field of long span bridges, *Transactions* does not now constitute such a reference work. Many of the most important bridges, which showed developments in type, extensions of span, etc., are to be found in its pages; but many others are not. The writer feels that one of the great services the Society can perform for its members would be to adopt a policy in each Division which would ensure the procurement and publication in *Transactions* of the papers necessary to constitute such a history of progress in executed works.

The authors are to be commended for having selected for this written record the features that will be of greatest benefit to a structural engineer desiring to perform analogous work.

Referring to the authors' "Conclusions," the writer is quite in agreement that the bridge possesses a pleasing form. The relative importance of aesthetic success in any given case must depend, to some extent, upon the question of whether large numbers of people will find themselves in such a position as to behold the structure in the aspect which has concerned the designer. For possibly a majority of highway bridges this condition does not hold, as they are beheld by the public from the decks only and not in elevation. In such cases as that of a highway bridge visible to railway passengers on a parallel structure, the writer has noted that a surprising number of nonengineers do note and comment upon this matter of appearance. The present structure being located adjacent to a city of some size, probably the question of appearance should be given more than ordinary weight.

The other general conclusion of the authors, however—that the type is fundamentally economical as compared with a cantilever—is not joined in by the writer. Unfortunately, no data can be assembled quickly to prove or disprove a contention of this type. There is no point in attempting to obtain approximate weights and costs of alternative types from a consideration of the recorded weights and costs of other structures. It is impossible for a designing engineer to weight the unit costs of structures of alternative types as a contractor will weight them in estimating. In recent years several important highway bridges have been built that are of a special continuous type (not that of the Dubuque bridge), the proponents of which were able to prove by careful estimates that it would show definite economy over more conventional types. When at last one purchaser took the logical step of preparing two sets of plans with equal care and equal conservatism in all regards, and issuing them for alternative bids, the prices for the conventional type were well under those for the special type.

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[5] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Inc., Bethlehem, Pa.

The writer mentions this as a "hedge" to his opinion that economy is not an advantage of the bridge type used at Dubuque; nor does he feel that the authors, although they state a basis for their conclusion, have pursued their argument to its end.

In the second paragraph under "Design Procedure," and in later description, the authors explain the three phases essential to the erection procedure: (1) The erection of the arch by cantilevering; (2) the manipulation of the end reactions and the erection and stressing of the center span tie; and (3) completion of erection of the floor steel and appurtenant details. Now with erection labor costs what they are, there is clearly no economy inherent in the taking of "three bites at the same cherry." That these three phases were necessary is true, of course, and that the separation into these phases decreased the amount of weight to be supported during the first phase is equally true; but it is very doubtful in the writer's mind that such a three-phase procedure, with the greater time required for the performance, could cost as little as the one-phase procedure required for conventional cantilever erection.

In the second paragraph under "Design Procedure," also, the authors state:

> "The form of the trusses of the central span lends itself to an economical arrangement for cantilever erection with maximum depth over intermediate supports tapering to a minimum depth at the center of the span."

This statement, however, overlooks the fact that in a conventional cantilever the depth of the main trusses over the intermediate supports would be fairly close to twice the 70-ft depth of the present structure. The depths of the arch structure were favorable for the erection of the arches, but unfavorable for the erection of the bridge as a whole. In Table 4, for instance, comparing Col. 8 with Col. 5, it will be noted that, for chords L4-L6, L6-L8, L8-L9; and U5-U7, U7-U9, and U9-U11, the total stresses under erection plus erection wind are from two to four times as great as the stresses for dead, live, and impact loading. The authors partly met this situation by increasing erection stresses 50% over the stresses of normal design. This increase was the occasion for considerable worry, and the writer would not care to erect such a structure under that assumption again. It must be remembered that erection wind is not a figment of the imagination. Wind on a loaded structure in service may be considered a hypothetical load, on the ground that during a hurricane vehicles do not travel; but nothing can stop the wind from blowing at any stage—even the most unfavorable stage—of erection, and the provision for ample wind stress at a reasonable unit is essential. Despite this increase in temporary unit stress, however, the fact remains that there is permanently built into the chords of this bridge, over the intermediate piers, a great deal of steel which will never serve any useful purpose again. It is difficult for the writer to understand how economy can necessarily be inherent in a design of which that is true.

During the erection of a typical cantilever bridge, or of a three-span continuous bridge of the same general outline as the cantilever, there is no great excess of stress over that which will be developed by dead, live, and impact loading; the exceptions being particularly one top-chord and one bottom-chord member per truss in each half span, lying between the cantilever arm and suspended span. These "wasted" members carry the moment from cantilevering only the half length of the suspended span; instead of half the length of the main span, as at Dubuque. As the erection of the typical cantilever or continuous span proceeds, all the necessary material for support of live load is put in place. After the two halves of the bridge meet at midspan, one day spent in adjusting the juncture between cantilever and suspended spans of the cantilever type, or adjusting the end reactions of the continuous type, completes the erection of the steel. Economy, and safety of erection, both seem to inhere in these types.

It would be a real contribution if an owner would some day do what was described herein—issue, for bidding, alternative plans of a continuous-tied-arch structure and a more conventional cantilever type.

A. Floris,[6] Esq.—It is not quite clear why the builders of this bridge made the bottom chord of the end spans straight and level with the suspended floor of the arched center span. Additional navigation clearance could be gained, without increase in cost, by making the top chord of the end spans straight, while arching their bottom chord. This arrangement permits the lowering of the midpiers, if the suspended floor of the center span is also raised to the level of the top chord of the end spans. A perusal of the literature on the subject shows that designers of this type of steel bridges prefer this arrangement.

In the stress analysis of the truss of the present bridge, the authors use the customary procedure of removing first the quantities not obtainable by statics. By subsequently applying the principle of virtual displacements to the base system thus formed, the necessary elastic equations are established. However, the choice of this base system and the method of obtaining the elastic equations are open to criticism. The base system is cumbersome and the derivation and solution of the elastic equations laborious. The coefficients of Eqs. 17*a*, 17*b*, and 17*c* are such that for their solution great accuracy is needed to obtain useful results.

Substantial savings in labor could have been achieved, if the authors had used more advanced methods of stress analysis. These methods, although of long standing,[7, 8, 9] have found, with one exception,[10] little favor among authors and stress analysts in the United States.

It is always possible to select the hyperstatic quantities, their direction, and point of application in such a way as to obtain as many independent elastic equations as are needed. The procedure is somewhat analogous to the method used for fixed arches and two-legged frames. In this case, by choosing the elastic center of the structure as the point of application of the hyperstatic quantities, the elastic equations containing one hyperstatic quantity only are obtained. In the solution of equations of this type, containing one unknown each, slide-rule accuracy is ample.

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[6] Asst. to Structural Engr., Pacific Elec. Ry. Co., Los Angeles, Calif.

[7] "Zur Berechnung mehrfach statisch unbestimmter Tragwerke," by Siegmund Müller, *Zentralblatt der Bauverwaltung*, 1907, p. 23.

[8] "Die graphische Statik der Baukonstruktionen," by Heinrich Müller-Breslau, Alfred Kröner, Stuttgart, 1907, Vol. II, Pt. I, p. 162.

[9] "Elastizitätsgleichungen gegenseitiger Unabhängigkeit," by Martin Grüning, *Der Eisenbau*, 1921, p. 305.

[10] "An Application of Orthogonal Moments to Problems in Statically Indeterminate Structures," by W. M. Kincaid and V. Morkovin, *Quarterly of Applied Mathematics*, 1944, p. 334.

A complete analysis of a truss similar to that of the bridge under discussion and employing the methods previously explained is given elsewhere.[11] The base system is obtained by inserting three hinges in the structure; one at the crown of the arch, and one each at the second panel of the end spans counted from the midpiers. The influence lines of this base system are determined by statics and utilized subsequently to calculate those of the hyperstatic structure.

T. H. Rust,[12] Assoc. M. ASCE.—A statically indeterminate bridge as important as the new highway bridge at Dubuque, Iowa, with piers resting on pile foundations, arouses interest about the amount and kind of settlement and movement that may occur in the piers. All foundations that depend on piles for support are subject to settlement. Appreciable settlement or rotation of the piers of this bridge will introduce stresses in the continuous trusses which may become critical, depending on the amount and the nature of the pier movements. Piles that can be driven into the stream bed to a depth of 95 ft penetrate material which could be vulnerable to settlement. More detailed information about the nature of the material in the stream bed at the pier sites would be welcome: The soil's capacity to sustain the pile loads, the method of driving, the results of pile tests, the probability of scour at these locations, facts about the soil support on vertical planes at the sides and ends of the piers, the pile plans with spacing, and the actual sizes and pile loads—all would be welcome information in the paper about the Dubuque bridge.

Structural engineers have believed that a statically indeterminate bridge, especially one of the size and the importance of this one, should be supported on piers resting on rock or some other substance closely approaching rock in its capacity to sustain the loads without appreciable settlement. Although a statically determinate simple span of steel cannot be specified with economy, under accepted working stresses, for a length much more than 650 ft, a cantilever span or perhaps some other type can be specified to obviate much of the redundant effects from pier settlement.

In a simple span of two trusses, four supports are provided, one of which is redundant, because of the portal and lateral bracing. The bracing between the trusses can be designed to lessen the effect that could result from unequal settlement of the ends of a supporting pier. Settlements of one pier with respect to the others do not affect simple and cantilever spans, as such settlements affect continuous spans. If it is planned to observe in the future the settlements of the four piers which support the continuous trusses, it may be that the necessity for such observations will be disregarded after some years of small, inconsequential settlements.

The writer has seen deterioration in 50-ft statically indeterminate concrete arch spans due to settlement of their pile foundations, and has known of the deterioration of continuous steel structures from the nonrigidity of their supports. Allowable pile loads, due to different specifications and the methods of figuring the load sustaining capacity of piles, have varied so greatly that publication of the research and field knowledge available from the Dubuque bridge piles would be welcome. Those who have had direct charge of the driving of the piles must possess much practical knowledge of the driving, which if published would be of lasting value to the engineering profession. The transfer of the pier loads of this bridge into the stream bed composed of the implied material constitutes an unknown discontinuity in load transfer, which results from the imperfect and incomplete knowledge of soil mechanics.

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[11] "Die Statik des ebenen Tragwerkes," by Martin Grüning, Julius Springer, Berlin, 1925, p. 370.

[12] Structural Engr., Chicago, Ill.

The precedent has been set for a number of years of sustaining continuous bridges on pile foundations, and the precedent is followed in this bridge. It seems that the capacity of the piles to sustain their loads without critical settlement should be supported by factual evidence as well as by a geologist's advice, in an endeavor to justify these foundations.

CHARLES W. DOHN,[13] JUN. ASCE.—High secondary stresses are developed over intermediate piers in a continuous truss system such as that used on the Dubuque bridge. This is an unusual condition following the strict interpretation of accepted specifications.[14,15,16] An analysis for secondary stresses, particularly over the intermediate piers, is thus mandatory for this rather unusual long span continuous design.

A complete discussion of the design of this bridge should include a detail of a typical joint connection, particularly near the intermediate piers. Relatively rigid joint connections, stiff members, and large deflections contribute to high secondary stresses; when these are added to all other effects, including wind, pier settlement, and temperature, they could easily become critical.

It would seem that using truss members of different materials would also contribute to large secondary stresses at some points. A silicon steel tension member designed for the same total stress as a carbon steel tension member will deflect 1.33 times as much as the carbon steel member. Such deflection could magnify secondary stress conditions. Why was silicon steel specified for some members and carbon steel for others?

The specifications previously mentioned[14,15,16] state:

> "Secondary stresses due to truss distortion or floor beam deflection usually need not be considered in any member the width of which measured parallel to the plane of distortion is less than one-tenth of its length."

Originally the speicfications were those of the American Railway Engineering Association, dated 1920. It appears that most of the members meet this condition. However, the use of members of different materials, the long and unusual continuous span, and the large deflections would seem to indicate the need for further study.

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[13] Engr., New York, N. Y.

[14] "Standard Specifications for Highway Bridges," A.A.S.H.O., Washington, D. C., 1944, p. 161, article 3.6.7.

[15] "Specifications for Steel Highway Bridges," A.R.E.A., Chicago, Ill., 1927, article 1409.

[16] "Report of Committee XV on Iron and Steel Structures," A.R.E.A., Chicago, Ill., January, 1920, clause 47.

It might be possible to increase the strength of this bridge greatly by altering certain members subject to high secondary stresses. The importance of secondary stresses in modern truss structures has been somewhat neglected in designing ordinary, standard, short span trusses after initial investigations proved their relative values. The tendency has been to assign blanket values, or to rely on a wide margin of safety factor in design. Unusually long span trusses, trusses of unusual design, and continuous trusses still warrant analysis for secondary stresses. Extreme reversals of secondary bending under moving loads might cause yielding leading to fracture at the joints of a member or, after a limited number of reversals under high stress, the member might exhibit fatigue failure.

A method extensively used to provide for secondary stresses in continuous bridges, such as the Dubuque bridge, is to rely on methods of geometric cambering to neutralize the secondary stresses. However, the loading for maximum direct stress is not necessarily the same for maximum secondary stress or maximum total stress. Influence lines for secondary bending stress in each member due to unit loads can be used to determine absolute maximum total stress. Secondary bending stress should be determined for at least the members in the vicinity of the intermediate supports of continuous truss bridges. The use of a trussed arch span with inherent increased deflection would seem conducive to increased secondary stresses.

Rough calculations indicate possible maximum secondary stresses approximating maximum primary stresses. With geometric camber such stresses cannot be entirely neutralized. Increases are allowable for combinations of secondary and axial stresses and 33⅓% is a common value. Since 25% overstress has already been allowed for dead load plus live load plus a 30-lb wind, only 8⅓% remains for secondary stress which is not enough to absorb indicated values for some members even after deducting the neutralizing effect of possible geometric camber. Whether failure could appear as fatigue failure is problematical. Reversal of secondary bending stress plus reversal of primary stress in a member would be particularly contributory to fatigue failure and a reduction in working stress rather than an increase would be in order. It appears that liberties have been taken with the factor of safety with questionable economy considering the further possibility of pier settlement and aerodynamic effects. Light earthquake shocks have been reported in the general region.

The Dubuque bridge presents a neat outline and the members are nicely proportioned and arranged. The tabulation of stresses is concise but lacks a column for secondary stress or a statement as to geometric camber. In closing the writer wishes to endorse heartily the opening paragraphs of the discussion by Jonathan Jones, M. ASCE.

Louis Balog,[17] Esq.—The layout, floor design, and bracing arrangement offered in this paper, viewed in the light of progressive developments in bridge design, invite comment on features that the writer proposes to outline herein.

Economy.—In the "Synopsis," the authors state:

> "In economy it [the Julien Dubuque Bridge] compared favorably with cantilever, tied arch, and suspension span layouts studied for the crossing."

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[17] Cons. Engr., Binghamton, N. Y.

The cantilever, being a statically determinate system, cannot utilize the tenacity and ductility of the steel; and the simple tied arch, if its features are identical with those of the bridge as built, must necessarily cost more than the continuous truss. In deciding the relative economy of a suspension span layout it is helpful to compare the Davenport (Iowa) suspension bridge (1935)[18] with the Dubuque project, since they are similar in size and clearance requirements.

The Davenport crossing is located at Bettendorf, Iowa, 80 miles south of the Dubuque bridge. Table 6 demonstrates the degree of similarity. The ratio of the substructure cost to superstructure cost is the same in both cases (line 9, Table 6). The Dubuque spans, however, cost 62% more per square foot of width between handrails (line 10, Table 6) than did the Davenport spans despite the fact that the Davenport approaches contain six 223-ft, and three 162-ft, truss spans. This economic superiority of the Davenport bridge is due mainly to its suspension span which contains about one third as much steel as the Dubuque truss span. Its floor system and stiffening trusses, respectively, contain 380 tons and 480 tons of steel in contrast to 600 tons of steel, for the bracing alone, in the Dubuque truss span. Similarly, extreme comparative economy in metal of suspension spans is demonstrated by the metal content of three other existing suspension bridges of comparable size.

TABLE 6.—COMPARATIVE ECONOMIC DATA, DAVENPORT AND DUBUQUE BRIDGE PROJECTS

Line (1)	Description (2)	Davenport (3)	Dubuque (4)
1	Length of open structure, in feet	4,955	5,759
	River Crossing:		
2	Type	Suspension	Curved truss[a]
3	Span length, in feet	370 +740 +370	345.75 +845.17 +345.75
4	Total span length, in feet	1,480	1,536.67
5	Width between handrails, in feet	25.92	31.92
	Costs in Dollars:		
6	Substructures	357,601	832,000
7	Superstructures	686,798	1,588,000
8	Total, items 6 and 7	1,044,399[b]	2,420,000
9	Ratio, line 6/line 7	0.52	0.52
10	Unit cost of spans, in dollars per square foot of deck[c]	8.13[b]	13.17

[a] Continuous curved truss, with tie. [b] Includes cost of toll plaza at the south end of the suspension span. [c] Width between handrails.

Bridges of the continuous and stiffened continuous types of advanced design, as indicated by alternate designs prepared by the writer for the Dubuque truss span, contain 20% less metal than the bridge as built.

Layout.—Lowering the mass centroid of a body increases its stability. The application of this principle in the distribution of the mass of a bridge minimizes wind and dynamic effects; and, generally, it produces an economical structure, pleasing in appearance. Considerations of statics, stability, economy, and esthetics require that parts of the bridge above the roadway be as light, and that this area have surfaces as small, as possible. The bridge should direct instantaneous attention to its cardinal purpose—a continuous road crossing a waterway. Consistent with this ideal, the designer's central objective is to lead all loads to the ground in the most natural manner.

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[18] "Building the Davenport Bridge Across the Mississippi," *Engineering News-Record*, Vol. 115, 1935, pp. 837–841.

If one of the spans of the continuous girder is twice (or more than twice) as long as the others, the required stiffness of the girder can be achieved by

1. Adding a system of stiffening members; and
2. Shaping the girder so that its form assures stiffness.

The latter effect can be accomplished either by

a. Basing the depth of the layout on the stiffness requirement of the long span; or

b. Arching the girder in the long span and introducing a tie for the horizontal reactions.

Types 1 and 2*a* are natural solutions of the problem, satisfying the foregoing basic requirements. In the writer's opinion, type 2*b*, in general, and the Dubuque truss span,[19] in particular, do not satisfy these requirements.

Study of ninety-one bridges of type 1, built and proposed, lead the writer to believe that efficient and pleasing structures result from this type, in which the stiffening system of the shallow girder can be either a compression or a tension chord, Figs. 10(*a*) and 10(*b*). Such stiffening chords may be curved, trapezoidal, or triangular in elevation and need not extend from support to support. The small wind load and other horizontal forces, received by these stiffening chords, make possible the design of pleasing overhead bracing, which assures, also, the buckling stability of these chords in the horizontal plane. Their stability in the vertical plane is governed by the spacing of the suspenders. Tension chords need not be braced. As a result of the loads acting on the girder, and of the weight of the chords themselves, such chords transmit horizontal forces at their points of attachment to the girder and therefore decrease the deflections of the long span to satisfactory values. A great variety of combinations of girder and chord properties are possible in which the magnitude of the horizontal forces can be regulated primarily by the geometry of these chords.

Type 2*a* (Fig. 10(*c*)) may have constant depth; it may have the greatest depth at midbridge, or over the intermediate supports. In the latter case the truss can be shaped so that, except for the two portals over the intermediate piers, no other visible bracing is necessary. Examples of all these forms exist.

The forms of type 2*b* (Fig. 10(*d*)) may vary between two extremes: The truss may be shallow and of nearly constant depth (except in the panels adjacent to the intermediate piers, where it is extended down below the floor to join the deck side spans), or it may be deep and an entirely through span, with greatly varying depth. The Dubuque truss span represents the latter extreme, in both truss configuration and bracing arrangement. Arching of the center span truss in this case directs attention 120 ft above the roadway to a confusion of main members and bracing members, many times as deep as the floor,[19] and raises the center of gravity from 25 ft above the bottom of the steel at piers 18 and 19, to more than 60 ft at midspan, thus decreasing the stability of the structure. The normal and vibratory effects of wind, impact, and possible earthquake forces will be greater in this bridge (since the center of gravity is high above the floor) than in bridges of types 1 and 2*a*. The centers of gravity of the latter bridges (see Fig. 10) are low and their areas exposed to wind are smaller and closer to the roadway level. In the Dubuque truss span large structural areas, high above the floor, receive considerable wind loading, that is directed down to the supports by a multitude of crisscrossing braces.

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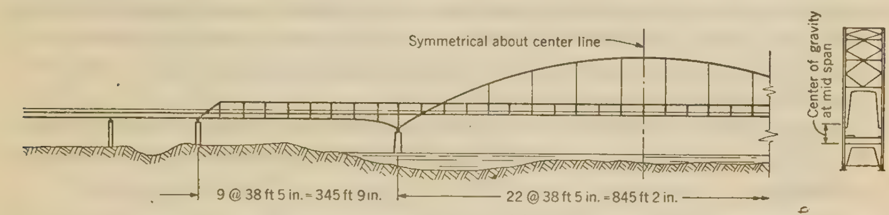
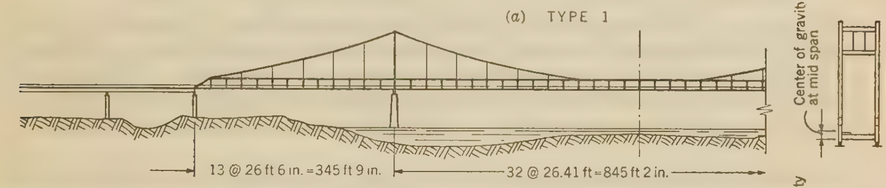
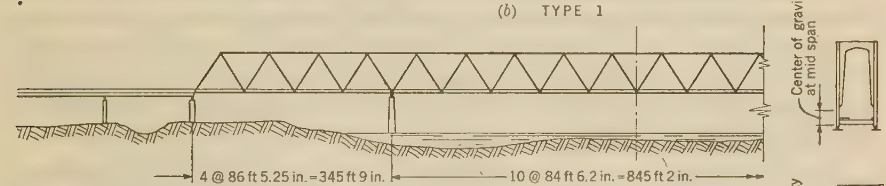
[19] *Construction Methods*, March, 1943, photograph on p. 47.


Symmetrical about center line
Center of gravity at mid span
9 @ 38 ft 5 in. = 345 ft 9 in.
22 @ 38 ft 5 in. = 845 ft 2 in.

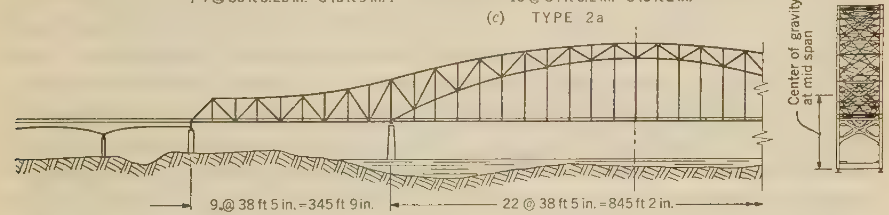
(*a*) TYPE 1


Center of gravity at mid span
13 @ 26 ft 6 in. = 345 ft 9 in.
32 @ 26.41 ft = 845 ft 2 in.

(*b*) TYPE 1


Center of gravity at mid span
4 @ 86 ft 5.25 in. = 345 ft 9 in.
10 @ 84 ft 6.2 in. = 845 ft 2 in.

(*c*) TYPE 2a


Center of gravity at mid span
9 @ 38 ft 5 in. = 345 ft 9 in.
22 @ 38 ft 5 in. = 845 ft 2 in.

(*d*) TYPE 2b; DUBUQUE

Fig. 10.—Elevations of Various Types of Trusses

In driving across this bridge, an engineer, comparing the effect of the view of this bracing with that of the lack of the truss diagonals in the center panels, will not agree to the observation in the "Synopsis" that:

> "The three-span continuous truss structure * * * presents a pleasing appearance, affords to traffic a view unobstructed by diagonal truss members across the 845-ft main channel span, * * * "

Unlike subdiagonals and an almost solid aspect of braces, main diagonals are not objectionable. They can be avoided economically, however, by the use of stiffened plate sections and welded details. Figs. 10(*a*) and 10(*b*) show such layouts. Fig. 10(*c*) shows a layout in which an unobstructed view is provided for five cars in a row (the authors providing only for two) by a continuous triangular system composed of a few, large, box shaped, stiffened plate members. The bottom chord is a box girder 8.5 ft deep. The relative depth of the top and bottom chords emphasizes the roadway and thus expresses the purpose of the structure. The properties of the bottom chord assure great savings in floor metal, handrails, and gussets; simple bracing, few joints, and reduced dead load contribute to the economy of this bridge—it contains 860 tons less metal than the bridge as built. This design demonstrates that the inclination of the members in a truss is less important than their number in avoiding both obstructions to view from the bridge and the maze of intersecting members in looking at the bridge.[20]

Adequate traffic capacity constitutes the basic requirement for the width of a bridge. The authors, however, discuss additional considerations. Under the heading, "Three-Span Through Truss Continuous Unit," they state:

> "A saving of floor beam metal would have resulted from a closer spacing of trusses, with sidewalk outside; but truss loadings would have so differed that the trusses would not have been identical and duplication of fabrication would have been sacrificed. Also, closer truss spacing would have reduced what was regarded as a desirable width-length ratio for lateral stability."

A narrower bridge may contain considerably less metal than that saved in floor beams alone, and the sacrifice of duplicate fabrication of the trusses by a sidewalk outside can be minimized by proper design to a negligible amount. The statement that lateral stability required the width used is not borne out by (*a*) the behavior of existing bridges, and (*b*) the progressive developments in bridge design since 1918.

(*a*) Lateral stability of a truss bridge is governed mainly by its torsional stiffness and the height of its center of gravity at midspan. The torsional loadings on the Dubuque truss span are much smaller than, for instance, those on the Mississippi River bridges at Memphis, Tenn. (1913), New Orleans, La. (1935), and Baton Rouge, La. (1940), which carry railway loads between, and highway loads outside, their trusses. The span lengths of these latter bridges are comparable or larger, and their unbalanced highway loads alone equal the total live load of the Dubuque truss span, in addition to which two of them carry an unbalanced railway loading. The twisting moment per linear foot acting on the Baton Rouge bridge is more than three times as large as that acting on the Dubuque truss span, yet a 3-ft-smaller width was found to be satisfactory at Baton Rouge despite its greater span length. Furthermore, a large number of truss bridges having width-length ratios up to 40% smaller than the width-length ratio of the Dubuque truss span proved themselves laterally stable for many years in service. Examples are: Colorado River Bridge, Topock, Ariz. (1890), span 990 ft; Little Colorado River "Cameron" Bridge, United States Highway No. 69 (1912), span 660 ft; Carquinez Straits Bridge, California (1927), span 1,100 ft; Cooper River Bridge, Charleston, S. C. (1929), span 1,050 ft; and Columbia River Bridge, Longview, Ore. (1930), span 1,200 ft.

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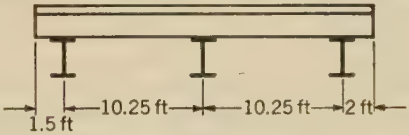
[20] *Construction Methods*, March, 1943, photograph on p. 114.

(*b*) Studies leading to the construction of bridges with triangular cross section (like the double-track railroad Ruhr River bridge at Düren, Germany (1928)), and to the construction of three-plane plate girder bridges (like those in Hamburg, Germany (1928)), had the effect of substituting investigations based on scientific facts in place of rule-of-thumb width-length ratios. These developments were followed by framing bridges of rectangular cross section for space action (since 1918 applied in reinforceing existing bridges) and by designing middle support bridges in an inverted T-arrangement. In the latter types a single truss or arch rib or cable is combined with the floor to produce an efficient bridge structure. The purposeful utilization of the torsional characteristics of various arrangements stripped the concept of the width-length ratio of structural meaning. The spacing of trusses cannot be justified, therefore, by a desirable width-length ratio, but only by investigations of the torsional rigidity and the dynamic stability of the bridge.

TABLE 7.—COEFFICIENTS OF DESIGN WHEEL LOADS FOR LOAD DISTRIBUTION ON STRINGERS



Line	Specification	A	B		C	TOTAL	
			....[a]	....[b]		....[a]	....[b]
1	AASHO; H-20 (1935).......	1.534	2.244	....	1.683	5.484	....
2	Bergendoff and Sorkin......	1.295	2.652	2.725	1.439	5.386	5.459
3	AASHO; H-20 (1941).......	1.317	2.050	....	1.439	4.806	....
4	Balog; H-20 (1935).........	1.736	1.597	1.620	1.839	5.172	5.195
5	Balog; H-20 (1941).........	1.578	1.552	1.592	1.675	4.805	4.845
6	H-20 (1935)[c]..............	1.629	1.655	1.680	1.719	5.003	5.028
7	H-20 (1941)[c]..............	1.539	1.598	1.622	1.651	4.788	4.812

[a] Corresponds to specifications of the American Association of State Highway Officials (AASHO) for the position of loads. [b] Corresponds to the position of loads giving maximum influence value. [c] Calculated by the writer by the use of "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," by Nathan M. Newmark, *Bulletin No. 304*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1938; and "Moments in I-Beam Bridges," by Nathan M. Newmark and C. P. Siess, *Bulletin No. 336*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1942.

Floor System.—The floor steel in the Dubuque truss span (curbs, crossbeams, stringers, and floor beams) weighs 850 tons, which is equal to the metal content of the trusses and the floor system of the Davenport suspension bridge. The authors state (under the heading, "Design Loads; Live Load"): "* * * The floor system was designed for an H-20 loading on 9-ft lanes in accordance with the 1935 AASHO [American Association of State Highway Officials] specifications." They state further (under the heading, "Design Procedure: Floor System") that:

> "Crossbeams, designed as continuous beams over the stringer supports, are 12-in. I at 31.8 lb in intermediate panels and 12-in. I at 40.8 lb in end panels. Stringers were designed as simple beams spanning between floor beams with maximum load reactions transmitted by the crossbeams, * * *."

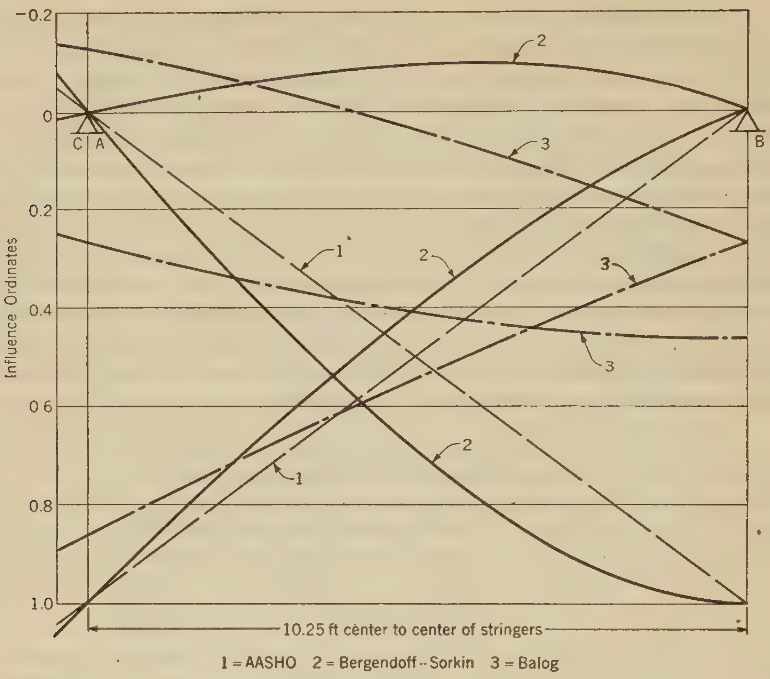
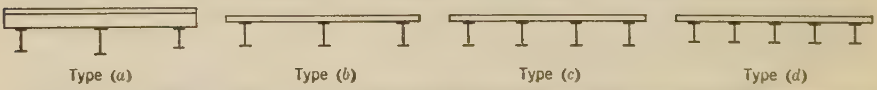


FIG. 11.—INFLUENCE LINES OF LOAD DISTRIBUTION ON STRINGERS

TABLE 8.—WEIGHT OF STRINGER METAL IN VARIOUS FLOOR DESIGNS OF THE DUBUQUE TRUSS SPAN (FLOOR BEAM SPACING, 38.42 FT)

Type	Design method	CARBON STEEL[a]		SILICON STEEL[a]		CARBON STEEL		SILICON STEEL	
		Total	Saving	Total	Saving	Section	Wt.[a]	Section	Wt.[a]
(a)	Authors......	575	....	....	....	One, 33WF125 One, 36WF150 One, 30WF116 12 I 31.8 at 4.8 ft 12 I 40.8 at 4.8 ft	575	....	....
(b)	Balog........	456	91	324[b] 79[c] 403	132	Three, 33WF125 One diaphragm (I = 2,681 in.^{4}) Two, 14WF34	456	Three, 30WF108 One diaphragm (I = 1,967 in.^{4}) Two, 14WF34	324[b] 79[c]
(c)[d]	AASHO (1935)	428	113	342	179	Four, 30WF108	432	Four, 27WF91	364
	AASHO (1941)	412	125	342	179				
	Balog........	392	141	348	174				
(d)[d]	AASHO (1935)	476	76	388	143	Five, 27WF91	455	Five, 24WF74	370
	AASHO (1941)	455	92	370	157				
	Balog........	435	107	370	157				



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[a] Weights, in pounds per foot of bridge. [b] Silicon steel. [c] Carbon steel. [d] The weights of the cross frames at midpanel are omitted from types (c) and (d) to make the comparisons equitable with the authors' design, type (a).

The 1935 AASHO specifications require that the load on the stringers "* * * shall be the reaction of the truck wheels, assuming the flooring to act as a simple beam between stringers."

Influence lines of load distribution on the stringers are presented in Fig. 11. Evaluation of these curves for H-20 (1935) and H-20 (1941) loadings in Table 7 show that the AASHO specifications (curves 1, Fig. 11) give a load distribution that is closer to the actual than are the continuous beam reactions, which the authors state were used in the design. The load distribution on the stringers and the stresses in the crossbeams are correlative. Consequently the authors' design stresses for the crossbeams also differ from the actual stresses. The stringers deflect under the loads; hence, the load distribution and the stress values used by the authors cannot occur. Table 7 indicates that 1941 AASHO specifications give the correct load on these three stringers, although they fail to give the correct distribution between them. Apart from the extensive foreign language literature on the solution of this floor problem, studies on this subject were published in the United States in 1938[21] and in June, 1942.[22]

Floor arrangements shown in Table 8 for the Dubuque truss span contain from 76 tons to 113 tons, and from 143 tons to 179 tons, less carbon steel or silicon steel, respectively, than the bridge as built. In the layouts shown in Figs. 10(*b*) and 10(*c*), the floor beam spacings are one half and one quarter, respectively, of the panel length. These floors contain 380 tons less metal than the floor as built. In 1938 the writer made an extensive comparative investigation of the type of floor system used by the authors and a five stringer arrangement. The actual transverse and longitudinal load distribution was considered in both systems, and even then type (*a*), Table 8, cost more.

Lateral and Sway Bracing.—Bracings may be designed: (1) For carrying all types of loads acting on the bridge; or (2) for carrying loads acting in their planes only. In the latter case the bracing is assumed to be pin-connected to the main trusses and the truss sections are not reduced on account of bracing participation. Although the authors assumed this latter action, they provided rigid connections. The Dubuque truss span has three lateral systems—double-intersection sway frames at every second panel point, special sway frames, and double-intersection portals, containing 600 tons of steel. In comparison, the weight of bracing metal in the bridge (Fig. 10(*c*)) is 280 tons.

The stress condition assumed by the authors in distributing loads acting on and between the main trusses is possible only if the floor beams were simple beams and the bracing were free from stress. Fig. 12(*b*) indicates the deformation of a transverse section consistent with these assumptions—near the point of application of the load the chord has the greater stress and elongation; consequently, it must move outward away from the load.[23] In 1930 and 1931, Rudolf K. Bernhard,[24] M. ASCE, reported 7,800 and 360 field measurements, respectively, on eight bridges, some framed like the Dubuque truss span proving that: (*a*) Because of eccentric floor loads, transverse sections tilt in the direction opposite to that resulting from the authors' assumptions; and (*b*) the chord stresses tend toward equality with increasing transverse stiffness of the spans (Fig. 12 (*a*)). Furthermore, he showed that, in a truss span of triangular cross section, the stresses in the chords are always equal, irrespective of the eccentricity of the load. In other words, an eccentric load is equivalent to a centric load plus a moment. The centric load produces equal stresses in the trusses and the moment rotates the section formed by the main and the lateral trusses. When such a rectangular section is rotated, without any change in the diagonal distance between the chords, the chords (the corners of the section) will remain free from stress and the sides of the section (web members and laterals) will be stressed. The sum of the stresses resulting from the centric load and the moment give the true stresses in the system. Therefore, the distribution of eccentric and lateral loads depends on the warping resistance of the span.

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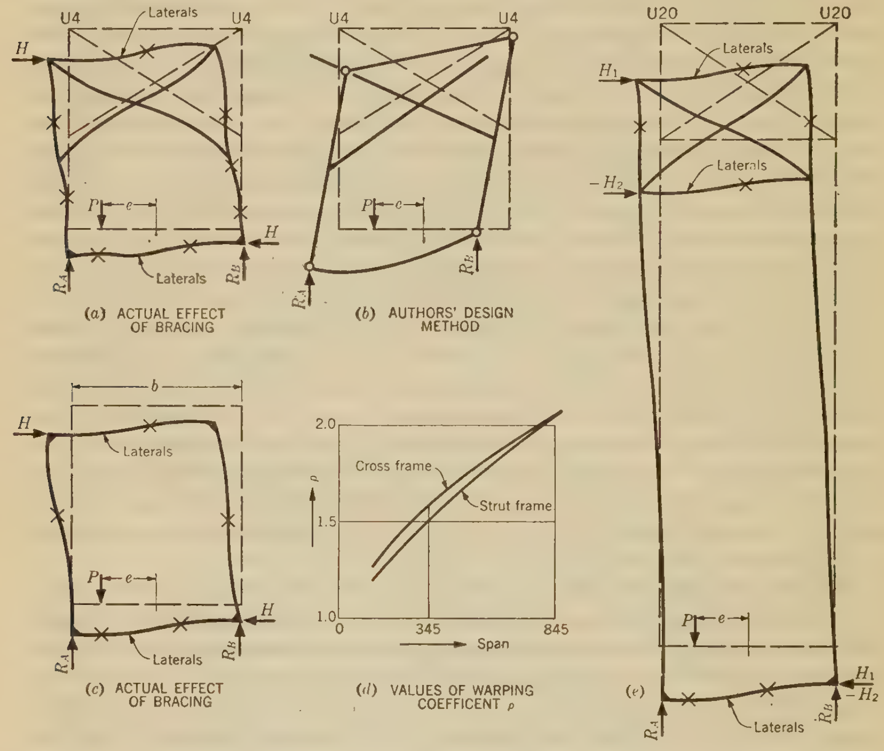
[21] "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," by Nathan M. Newmark, *Bulletin No. 304*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1938.

[22] "Moments in I-Beam Bridges," by Nathan M. Newmark and C. P. Siess, *Bulletin No. 336*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1942.

[23] "Über den Einfluss der senkrecht wirkenden Verkehrsbelastung auf die waagerechten Verbände eiserner Balkenbrücken," by W. Bergfelder, Technische Hochschule, Aachen, 1928.

[24] "Über die Verwindungssteifigkeit von zweigleisigen Eisenbahnfachwerkbrücken," by Rudolf Bernhard, *Der Stahlbau*, Vol. 3, 1930, pp. 85–92, and Vol. 4, 1931, pp. 241–244.



(*a*) ACTUAL EFFECT OF BRACING

(*b*) AUTHORS' DESIGN METHOD

(*c*) ACTUAL EFFECT OF BRACING

(*d*) VALUES OF WARPING COEFFICENT ρ

(*e*)

Fig. 12.—Effect of Bracing on the Distribution of Applied Load

In Table 9 the load distributions in the Dubuque truss span, as determined by the use of empirical warping coefficients (B) and the authors' values (S), are compared. According to the writer's computation, the lateral systems carry 46% and 71% of the moment created by a load acting at one truss only, at panel points 4 and 20, respectively. Therefore, the effect of the bracing is greater in the more flexible, longer span. Table 9 also contains distribution values for a strut frame and for a width 6 ft smaller than the 35 ft used. In the narrower bridge, bracings of the same type effect a more even distribution of an eccentric load; consequently, larger stresses are created in the bracing. The authors' method of design resulting in the same load distribution for any span length, width, and type of bracing yields results that differ considerably from those derived from measurements on actual bridges and shown by the writer in Table 9. In 1944, Louis E. Vandegrift reported measurements[25] on the three-span, continuous truss, Scioto River bridge east of Chillicothe, Ohio (1939), which further confirm the writer's opinion concerning the action of the bracing. This structure has a 24-ft roadway between the trusses and a 5-ft sidewalk outside the south truss. Both trusses were found to oscillate at practically the same frequency despite the additional sidewalk mass outside the south truss. Measurements[26] made on the Illinois River bridges at Hennepin and Lacon in Illinois (1940) indicated that the change in reactions resulting from warping the spans is more than ten times as much as that caused by raising or lowering the trusses by the same amount, proving also the effective framing action of the bracing.

TABLE 9.—EFFECT OF THE BRACING ON THE DISTRIBUTION OF LOAD (S = BERGENDOFF AND SORKIN; B = Balog)

Section	Width b, in feet	LOAD P = 1 ON TRUSS A; FIG. 12						MOMENT 0.5 $P b$ CARRIED BY:[a]			
		Reaction R_A		Reaction R_B		Reaction H		Main Trusses		Laterals	
		S	B	S	B	S	B	S	B	S	B
Fig. 12(a)......	35	1	0.770	0	0.230	0	0.193	100	54	0	46
	29	1	0.700	0	0.300	0	0.209	100	40	0	60
Fig. 12(c)......	35	1	0.795	0	0.205	0	0.171	100	59	0	41
	29	1	0.734	0	0.266	0	0.185	100	47	0	53
Fig. 12(e)......	35	1	0.644	0	0.356	0	0.096[b]	100	29	0	71

[a] Percentage of total moment. [b] $H_1 - H_2 = 0.519 - 0.423 = 0.096$.

Heinrich Müller-Breslau[27] demonstrated in 1904 that the stresses in a system composed of the main and lateral trusses of an arch differ greatly from the stresses obtained in the plain three-span truss assumed by the authors in this design. The tied arch truss bridge was the standard type of structure in Central Europe between 1880 and 1920 for both railway and highway bridges longer than 240 ft, and a wealth of experience was accumulated concerning its bracing requirements. The clear, clean, and efficient bracing systems of the double-track railroad bridges over the Rhine River in Germany at Rüdesheim

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[25] "Vibration Studies of Continuous Span Bridges," by Louis Edmond Vandegrift, *Bulletin No. 119*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio, July, 1944.

[26] "Adjustment of Continuous Bridge Affected by Warping," by L. T. Wyly, T. I. Fullenwider, and R. B. Murphy, *Engineering News-Record*, Vol. 125, 1940, p. 280.

[27] "Beiträge zur Theorie der Windverstrebungen," by Heinrich Müller-Breslau, *Zeitschrift für Bauwesen*, Vol. 54, 1904, p. 115, and Vol. 55, 1905, p. 133.

(1913), Engers (1915), and Remagen (1916), with arched spans of 566 ft, 617 ft, and 512 ft, respectively, bear testimony to this fact. None of these bridges had lateral bracing at the lower arch chord, nor any conspicous sway bracing, despite the fact that they were designed for larger wind loads and for a centric and eccentric live load seven times that of the Dubuque truss span. A single strut type of portal was found to be sufficient for rigidity and stress requirements.

Analyses and model studies made for the 1,675-ft Kill van Kull (Bayonne) arch,[28] connecting Bayonne, N. J., with Port Richmond, Staten Island, N. Y. (1931), disclosed that the sway bracing between the lateral systems at the upper and lower arch chords at every panel point throughout the span reduced the downward deflection of the truss loaded at the quarter point to 60%. When the sways in the 1,240-ft through section of the span were omitted, the deflection of the loaded truss was reduced to 55%. Therefore, the transverse bracing only in the 206-ft end deck sections of this 1,675-ft span was found to be sufficient to distribute the stresses, and to equalize the deflections, between the two arch ribs in all cases of unsymmetrical loading throughout the span. The model (1:192) also ascertained the true action of the portals; thus all unnecessary bracing was eliminated, and, consequently, the dead load was reduced, saving 600 tons of steel and improving the appearance of the bridge. This bridge was designed for a live load of 7,000 lb per ft, a horizontal transverse wind load of 2,000 lb per ft, and a temperature variation of ±55° F.

The stability conditions of the lower arch chord of the Bayonne arch and those of the Dubuque truss span are considerably different. In the former the lower arch chord carries practically all the dead load and is always highly stressed in compression; in the latter the crown section, 20% of the length of the span, is always in tension and, in the adjacent 20% lengths of the span, compression is low. These facts, in conjunction with the foregoing examples, both for the true action of the bracing and for the efficient bracing arrangements of actual large bridges, support the writer's view that the bracing of the Dubuque truss span is inconsistent with both, and with newer trends in bridge design. The broad, rigid, hangers seem not to be in keeping with the authors' emphasis on an unobstructed view to traffic, and they are inconsistent with the use of two lateral systems and sways for connecting the curved chords, because these rigid hangers carry bending and torsional stresses without benefiting the structure; wire ropes or high-strength round bars might well have been substituted.

Erection Procedure.—Under the heading, "Conclusions," the authors state:

> "A significant advantage of the continuous tied arch structure as compared to the conventional truss types is the economy of metal, due to a more advantageous distribution of the weight for cantilever erection."

Balanced against such economy, in the writer's opinion, is the fact that 345 tons of excess metal was required to meet the temporary requirements of erection. This weight constitutes 13.5% of the steel in the trusses and is equivalent to an increase in dead load to one third of the design live load. Erection considerations, being subject also to many unforseen circumstances, should not govern the design. The designer should rather concentrate his best efforts on the problem of meeting the requirements of the finished structure most economically, leaving the erector the responsibility for getting the bridge into place. If any change in the structure is required, that is subject to his approval.

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[28] "The Design, Materials and Erection of the Kill van Kull (Bayonne) Arch," by Leon S. Moisseiff, *Journal of the Franklin Institute*, Vol. 213, 1932, pp. 465–502.

Summary.—The writer believes that the river crossing at Dubuque could have been designed and erected by the use of considerably less metal than was actually used. The savings would be in floor steel, bracing members, excess steel used for erection, and increase in truss sections to support extra dead loads and wind loads that would have been avoided in other types of structures.

R. N. Bergendoff[29] and Josef Sorkin,[30] Members, ASCE.—The primary purpose of presenting this paper, giving a detailed description of the design and construction of one of the major structures across the Mississippi River, was to establish a permanent record regarding this bridge in the belief that it may prove helpful to colleagues in the profession, in the planning of structures of comparable magnitude. In a sense the urge to present this paper was a reflection of the writers' own experiences and difficulties encountered in the usual exploratory work, in planning major bridge crossings and in pursuit of determining best suitable types and features for a given site. There is an abundance of theory on design analysis of every known type of structure and, although it is valuable in the final design of a structure, theory alone offers little help in preliminary studies of comparative types, particularly when the time element is limited as has been the usual case in the past several years.

The writers were particularly pleased, therefore, in the concurrence with this logic as indicated in the opening paragraphs of Mr. Jones' discussion. It is also noted that Mr. Jones concurs in the pleasing appearance of the bridge. In this instance the structure as located is in full view of the inhabitants of the cities served on both banks of the river as well as to railway traffic across the river and along the east and west banks. The designers felt particularly gratified when the structure received the 1943 Bridge Award of the American Institute of Steel Construction in Class I for the most beautiful bridge opened to traffic in that year.

Mr. Jones questions the inherent economic advantage in the erection of this type of bridge as compared to the conventional cantilever or continuous truss construction. He concedes, however, that "Unfortunately, no data can be assembled quickly to prove or disprove a contention of this type." The writers fully agree that mere comparative weights would not be necessarily conclusive criteria. Even bids received on alternative types, although they would serve to indicate the economic preference in one particular case, would not permit drawing general conclusions, as unfortunately too many elements significant at the moment may be totally irrelevant at some other time, or at a different geographical location. It is well recognized that, aside from such fundamental costs as those of material and labor, the contractors' bids also reflect in no small measure the lack or abundance of prospective work, the chance location of the major part of their equipment necessary to the job, and various other factors of similar nature.

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[29] Cons. Engr., Howard, Needles, Tammen & Bergendoff, Kansas City, Mo., and New York, N. Y.

[30] Engr. of Design, Howard, Needles, Tammen & Bergendoff, Kansas City, Mo.

In the design of the bridge a 50% increase in stresses was allowed for erection including wind. To the best knowledge of the writers, such an increase is conventional. Mr. Jones' statement that "This increase was the occasion for considerable worry * * *" is fully appreciated; but it is believed that the structure through various construction stages was not devoid of ample safety factors. The erection stresses, although higher than normal, were still well below the elastic limit of the metal, and the structure was sectioned for a 30-lb wind against a maximum recorded wind velocity equivalent to an 11-lb wind.

The writers cannot agree with Mr. Jones' deduction that in conventional cantilever and continuous truss construction by cantilever method "* * * members carry the moment from cantilevering only the half length of the suspended span; instead of half the length of the main span, as at Dubuque." In cantilever erection of a cantilever truss the temporary chord members, introduced at the hinge of the suspended span, necessarily produce a moment at this point, which carries along the anchor arm to the support; in essence then, the final result is the same as for that of the Dubuque bridge. For purposes of record it may be of interest to note that the total amount of metal added for erection of the Dubuque bridge is 150 tons (not 345 tons, as erroneously stated in Mr. Balog's discussion) which in the knowledge of the writers compares quite favorably with conventional cantilever or continuous bridges of similar magnitude. The Mississippi River Bridge at Greenville, Miss., opened to traffic in 1940 and designed also by the writers' firm has approximately the same length of channel span, but is of conventional cantilever type. The final weight of metal added for erection purposes is comparable with that in the Dubuque span.

Mr. Floris would apparently prefer to use deck truss construction for the end spans,

> "* * * by making the top chord of the end spans straight, while arching their bottom chord. This arrangement permits the lowering of the mid-piers, if the suspended floor of the center span is also raised to the level of the top chord of the end spans."

Mr. Floris states that such an arrangement would result in "Additional navigation clearance * * * without increase in cost * * *." It is not quite clear whether Mr. Floris implies additional horizontal clearance or additional vertical clearance. Additional horizontal clearance would necessarily mean an increase in the length of the center span. Additional vertical clearance would require either an appreciable steepening of the approach grades or an increase in the height of approach, span piers, or both; in any event there is reason to believe such construction would result in additional cost. Furthermore, vertical clearance requirements under the west end span eliminated the possibility of such an alternative in this case. The writers appreciate Mr. Floris' reference to three publications in German as regards other applicable methods of stress analysis. Since, however, Mr. Floris explains "These methods * * * have found * * * little favor among authors and stress analysts in the United

States," the writers are pleased to learn that they alone did not miss the opportunity of using these methods. Mr. Floris could make a real contribution to the profession if he would translate these publications into English. The customary procedure used in the design was found neither "cumbersome" nor "laborious." All basic data preliminary to the development of Eqs. 17*a*, 17*b*, and 17*c* were analyzed by slide rule without any significant loss of accuracy. The solution of the three simultaneous equations by any accurate method certainly cannot be classed as laborious. It is the writers' considered opinion that in most instances the difference between one method of analysis and numerous others which are generally available is not material. The best method for any individual analyst is the one that best appeals to his logic. The time element, if any, is usually of secondary importance and is generally compensated by elimination of errors, because of the analyst's clear understanding of the theory.

Mr. Rust emphasizes the particular importance of stable foundations for statically indeterminate structures. The foundations for the Dubuque bridge did not involve any extraordinary problems as regards possible settlement. Piles were used primarily to afford assurance against underscour. These considerations, rather than bearing capacity, dictated the specified penetration. The piles were driven into a thick sand stratum. The design loads were limited to usual current specifications. Redrive tests indicated a bearing capacity considerably greater than that required, averaging well over twice the required capacity. The writers fully agree that statically indeterminate structures may well be avoided in instances where considerable settlement may be anticipated. It should be recognized, however, that nominal settlements are of no material significance. As was pointed out in the paper, a settlement of 1 in. in one of the end piers would result in a change of the reaction at that pier on the order of 10 kips, which is small, "percentagewise" and its effect upon the stresses in the truss members would be equally insignificant. Similarly, the same settlement of the intermediate piers would be of no great importance. Actual observations, since completion of the structure, have disclosed no evidence of settlement of the foundations.

Mr. Dohn raises the question of secondary stresses in the truss members, particularly near the intermediate piers. Contrary to Mr. Dohn's presumption there are little if any unusual features in the framing which would cause secondary stresses of extraordinary magnitude. It is also difficult to follow Mr. Dohn's deduction that the use of silicon steel will necessarily result in greater secondary stresses. Geometric cambering to neutralize the secondary stresses was used for the Dubuque bridge. It is to be appreciated that in long continuous highway bridge spans stresses due to live load are relatively small as compared to the total static stress, particularly in the region of intermediate supports; and, therefore, the effect of live loading is relatively small. As indicated in Table 4, none of the truss members is subjected to reversal because of primary loads and very few members are subject to reversal due to wind.

Mr. Balog's protracted discussion represents a great deal of effort and time. Such effort could be a distinct contribution to the original paper. Unfortunately, Mr. Balog's comments are so thoroughly misapplied as to merely confuse the factual data presented in the original paper. The discussion can be criticized on the following elemental points of recognized sound engineering practice.

1. Construction costs are not, and never have been, constant.
2. Minimum amount of material does not necessarily represent the most economical design. If so, wide flange rolled beams would have no place in bridge design.
3. Foundations on rock available at nominal depth can be designed cheaper than foundations necessarily supported on long friction piles.
4. The cost per linear foot of a span 845 ft long is more than the cost per linear foot of a span 740 ft long.
5. The ability to compute stresses and strains is not the sole qualification of a good designer. Knowledge backed by practical experience with problems dealing with properties of materials, fabrication of parts, function of details, limitations and adaptability of erection equipment, costs of material and labor, restrictions of War Department requirements, etc., is necessary to permit the designer to solve his problems properly.
6. In any design, due consideration must be taken of relative costs of labor and materials. European practice favors more labor and less material because labor is cheaper in Europe; the reverse is true in American practice.

Mr. Balog makes an extremely questionable comparison of the cost of the Dubuque bridge with that of the Bettendorf (Iowa) bridge. After arriving at a misleading difference in the cost per square foot of roadway by ignoring elemental facts, Mr. Balog accepts it as prima-facie evidence for his thesis. The Bettendorf bridge was constructed in 1935 when the *Engineering News-Record* Construction Cost Index was 196.44. The Dubuque bridge was constructed at 1941 costs when the *Engineering News-Record* Construction Cost Index was 257.84, a difference of 31.5%. The main span of the Dubuque bridge is 845.17 ft, whereas that of the Bettendorf bridge is 740 ft. An increase of the Bettendorf span to the length of the Dubuque span would result in approximately a 15% increase in the pull on the cables, height and load on the towers, and in the pull on the anchorages. Furthermore, the entire Bettendorf bridge, including approaches, is founded on shallow rock foundations. At Dubuque subsurface material for a depth of 400 ft is sand, silt, and clay. It is obvious that, transposed to the Dubuque site, the Bettendorf bridge would entail materially greater foundation costs. On the other hand, foundation costs for the Dubuque bridge would be correspondingly lower if that structure were transposed to the Bettendorf site. Taking proper account of these factors further confirms the writers' conclusions, as stated in the original paper, that the bridge, as finally constructed, compared favorably in cost to a suspension span. Mr. Balog overlooks these factors altogether and seemingly bases his conclusions as regards similarity of the two bridges on the mere fact that the ratio of cost of substructure to superstructure is 0.52 in the case of either bridge. The identity of these ratios is purely accidental, and has nothing to do with comparative economy of the structures. Misuse of perfectly good data is no reflection on the Bettendorf bridge. This bridge was designed by eminently able engineers and built by competent contractors. The type and other features of this bridge were undoubtedly dictated by sound principles and logic.

Mr. Balog takes further issue with comparative economy studies of the Dubuque bridge, stating thus:

> "The cantilever, being a statically determinate system, cannot utilize the the tenacity and ductility of the steel; * * * and * * * must necessarily cost more than the continuous truss."

Although this argument is applicable to short span structures, in which the live load forms a significant part of the total load, it is totally irrelevant in spans of the magnitude under consideration. (For the Dubuque bridge, the live load is 15% of the total load.) In long spans, introduction of articulated joints at inflection points of the continuous truss span, thus converting it into a cantilever system, will not in the least detract from the economic aspects of "tenacity and ductility" which in Mr. Balog's opinion are inherent in continuous systems only.

Mr. Balog is concerned with the position of the mass centroid in the Dubuque bridge. However, since later in his discussion he takes issue with the spacing of the trusses and maintains that the trusses could just as well be spaced closer, he apparently is quite satisfied with the stability of the bridge. Thus one part of his discussion answers the other. His argument relative to the mass centroid of the structure, if carried to its logical conclusion, would condemn every through braced arch, including the Hellgate Bridge (New York, N. Y.), Sydney (Australia) Harbor Bridge, and many other equally prominent structures. It also leads to the implication that, since the Empire State Building in New York could not possibly have the stability of a single-story frame, skyscrapers are follies. To emphasize his point Mr. Balog even cites the dangers of earthquakes. Such logic has no substance. The question is not which type of structure is infinitely more stable, but rather, "Is the structure stable?"

By the way of further comparison, Mr. Balog proposes two other types of structures in addition to the suspension bridge as shown in Fig. 10. As regards the bridge indicated in Fig. 10(*a*), a structure quite similar was actually studied for the Dubuque bridge. Owing to the span length involved, no advantage was found in this type of construction and it was dismissed primarily due to anticipated erection difficulties. Because of the similarity of this system to the one finally adopted, the writers did not deem it pertinent to mention this study in their original paper. Subsequent to the Dubuque project, consideration was again given to the use of a girder type tied arch at another site where the span was materially shorter. Designs of a trussed arch similar to that at Dubuque and of a girder arch similar to that shown in Fig. 10(*a*), were submitted to competent fabricators and erectors for their views as regards relative economy of the two structures. Their conclusions, too, were in favor of the trussed arch, principally because of necessary maintenance of channel traffic, as was also the case at Dubuque. Erection matters apparently are of no concern to Mr. Balog. To state "Erection considerations, being subject also to many unforeseen circumstances, should not govern the design," is for the engineer to adopt the proverbial habit of the ostrich, hiding his head in the sand hoping that the contractor will complete the job at a cost which will not divulge the ignorance of the engineer. Evidently Mr. Balog envisages bridge engineering as simply a matter of wrangling with "tenacity and ductility," torsion, stresses, and strains and throwing the mathematical brain child into the omniscient hands of the contractor, and hoping for the best. The outcome of such procedure is obvious. The contractor may either build the structure at a cost fully reflecting all the follies of the design or he may redesign the structure where any similarity between the original design and the final structure is purely coincidental.

The continuous truss unit propounded by Mr. Balog in Fig. 10(*c*) is thoroughly contrary to all recognized concepts of economic proportioning of truss spans. The writers cannot agree with Mr. Balog that a truss with 85-ft panels, 85-ft chords, and diagonals in excess of 100 ft in length, with the truss members unbraced in the plane of the trusses, could possibly result in an economical type of structure. In this case again the mere weight of metal is only a part of the problem. The cost of handling and transporting the long, heavy members and the field costs of splicing and erecting them are clearly of significant importance.

As regards the design of the stringers, Mr Balog misinterpreted the admittedly ambiguous statement in the paper. Actually, although the crossbeams were designed as continuous beams over the stringers, the load distribution to the stringers was substantially in accordance with the 1935 specifications of the American Association of State Highway Officials (A.A.S.H.O.). The differences in weight arrived at by Mr. Balog as indicated in Table 8 are due to serious oversights of certain factors and effects by Mr. Balog. In the first place, the weight of the stringers and crossbeams is 530 lb per ft instead of 575 lb. Furthermore, half of the sidewalk including its live load is supported by the stringer adjacent thereto. The floor systems indicated by Mr. Balog as type (*a*), Table 8, ignore this load completely. Similarly, the type (*b*) floor, Table 8 (Mr. Balog's design), would obviously require a thicker slab. The additional cost of concrete, reinforcing steel, and the cost of additional metal in the floor beams and trusses to carry the extra dead load would more than offset the gain in stringer weight. The same applies in varying degree to type (*c*) and type (*d*) floors, from Table 8, where in addition to these factors, Mr. Balog fails to account for diaphragms required by specifications and for the additional weight of connection angles as well as for the cost of additional field riveting. Therefore, Mr. Balog's comparisons are irrelevant since they lack common basis with the writers' design, and ignore details considered essential by engineers, for design as well as for construction. Mr. Balog's conclusions stated thus,

> "* * * extensive comparative investigation of the type of floor system used by the authors and a five stringer arrangement. * * * and even then type (*a*), Table 8, cost more,"

do not correspond to the numerous studies made by the writers. In fact, the writers have found that designed on a common basis, with all pertinent elements properly appraised, all conventional floor systems result in the same cost for all practical purposes. As regards the use of silicon steel for stringers, Mr. Balog fails to account for the difference in cost as compared to carbon steel. Aside from the difference in direct costs, use of silicon steel for floor systems where the live load is the principal load carried, and where the system is therefore subjected to numerous stress cycles, is seriously questioned by many engineers. In this connection it is pertinent to note the specifications of the American Railway Engineering Association thus,

> "On account of the possibility of fatigue failure, the allowable unit stresses in Article 1801 shall not apply to members that will be subjected to a large number of stress cycles during the life of the bridge and having either reversal of stress or low dead load stress. For such members the allowable unit stresses shall be reduced, and in floor systems shall not exceed those specified in Article 301 (for carbon steel)."

In discussing the lateral and sway bracing Mr. Balog cites several European bridges where sway bracing and lateral bracing at the lower arch chord were omitted. Undoubtedly, however, this is only a part of the story. Cross frames can be eliminated if strut type portals are provided in lieu thereof. It is entirely possible that such portals require less metal. However, the thing of utmost importance is not the type but the ultimate cost thereof. It is well recognized that European practice conserves metal by taking advantage of the cheaper labor. This, in fact, demonstrates the universality of the engineer's ingenuity. However, it is not by mere accident that these practices neither were originated, nor have found application, in the United States where the relative costs of labor and materials differ radically from European costs. Indiscriminate comparison of these practices is absolutely improper.

As a simple observation as to the advantage of unobstructed view, the writers stated: "The three-span continuous truss structure * * * affords to traffic a view unobstructed by diagonal truss members across the 845-ft main channel span * * *." Of course, the writers presumed the pertinent line of sight of from 4.5 ft to 5 ft above roadway level. This observation is simple, factual, and clear. Mr. Balog complains of the obstruction caused by the bracing of the arch, which is from 25 ft to 100 ft above the floor.

Mr. Balog claims his alternate designs show 20% less metal. This is contrary to results obtained by the writers in preliminary studies made for the crossing and the writers believe such extreme claims are as erroneous as the basis of the other criticisms of Mr. Balog's discussion.

The writers wish to express their appreciation to the constructive contributions of the discussers, which helped in crystallizing some of the phases of the structure described in the paper.

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